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Dynamic modelling of sma-driven flap

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1 STATIC DYNAMIC MODEL

A 2D FEA flap model was developed, but due to the low stresses at the flap components, it was further simplified such that these components can be modeled as rigid bodies; hence this work herein utilized dynamic modeling.

Each actuator, e.g. linear spring or SMA wire, is dynamically described in the same fashion. Figure 1.1 shows that each end of an actuator is fixed on a different rigid body, one at the forwards component (point F) and one at the afterwards component (point A). The two rigid bodies are connected at the joint point J.

Before developing the dynamic equations, it is necessary to define the reference systems as in table **??**.

1.1 DIRECT SOLUTION

This section is quite similar to the work done by Faria [1]. Following the notation of Ilmar [2], the length vectors are described as:

$$ {}_{R_0}\mathbf{f} = \begin{pmatrix} f_1 \\ f_2 \\ 0 \end{pmatrix} \quad {}_{R_1}\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ 0 \end{pmatrix} \quad {}_{R_0}\mathbf{r} = \begin{pmatrix} r_1 \\ r_2 \\ 0 \end{pmatrix} = {}_{R_0}\mathbf{a} - {}_{R_0}\mathbf{f} \tag{1.1} $$

Figure 1.1: Schematic of a single actuator

Notation	Reference system	Origin
R_0	Inertial reference system	leading edge
R_1	Rotational reference system of first flap	joint 1: J_1
R_i	Rotational reference system of first flap	joint i: J_i

If θ_i is the rotation angle around point J_i and $\{x_{J_i}, y_{J_i}, 0\}$ are coordinates of the point position, we have that the transformation between both reference systems is depicted by the following operator:

$$_{R_0}T_{R_1}(\mathbf{u} = \begin{vmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \tag{1.2}$$

$$_{R_0}\mathbf{a} = {}_{R_0}T_{R_1\,R_1}\mathbf{a} = \begin{vmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{pmatrix} a_1 \\ a_2 \\ 0 \end{pmatrix} = \begin{pmatrix} a_1 cos\theta - a_2 sin\theta \\ a_2 sin\theta + a_2 cos\theta \\ 0 \end{pmatrix} \tag{1.3}$$

Then from equation 1.1 we have that the distance vector $\mathbf{r}$ can be rewriten as:

$$_{R_0}\mathbf{r} = \begin{pmatrix} a_1 cos\theta - a_2 sin\theta - f_1 \\ a_1 sin\theta + a_2 cos\theta - f_2 \\ 0 \end{pmatrix} \tag{1.4}$$

1.2 Inverse solution

If it is known that the length of the actuator, $\mathbf{r}$, undergoes a deformation η (or $1+\epsilon$ if ϵ is strain), it is necessary to calculate the resultant angle θ by solving the following system:

$$\eta r_1 = a_1 cos\theta - a_2 sin\theta - f_1 \tag{1.5}$$

$$\eta r_2 = a_1 sin\theta + a_2 cos\theta - f_2 \tag{1.6}$$

which the solution is:

$$\theta = atan2(sin\theta, cos\theta) \tag{1.7}$$

where atan2 is a function that calculates the arctangent taking in to consideration the signs of the inputs in order to return the appropriate quadrant of the computed angle, and

$$cos\theta = \frac{\eta r_2 + f_2 - (\eta r_1 + f_1)a_1/a_2}{a_2 + a_1^2/a_2} \tag{1.8}$$

$$sin\theta = \frac{-\eta r_1 - f_1 + a_1 cos\theta}{a_2} \tag{1.9}$$

1.3 Torque generated by one actuator

The torque generated by each actuator is:

$$\tau = {}_{R_0}\mathbf{a} \times \mathbf{F} = (a_1 cos\theta - a_2 sin\theta)F_2 - (a_1 sin\theta + a_2 cos\theta)F_1 \tag{1.10}$$

where

$$\mathbf{F} = F\frac{{}_{R_0}\mathbf{r}}{||\mathbf{r}||} \tag{1.11}$$

Therefore

$$\tau = \frac{F}{||\mathbf{r}||}\left[(a_1 cos\theta - a_2 sin\theta)r_2 - (a_1 sin\theta + a_2 cos\theta)r_1\right] \tag{1.12}$$

F will be defined in the following section.

1.4 Angular momentum equilibrium at origin

Denoting the shape memory allot actuators with superscript ^{s}, linear actuators with superscript ^{l} and the weight with superscript ^{w}, we have that

$$\tau^s + \tau^l + \tau^w = 0 \tag{1.13}$$

where (W is a scalar that depicts the aircraft weight)

$$\tau^w = a_1^w W cos\theta \tag{1.14}$$

Substituting

$$\begin{aligned} 0 = &\frac{F^s}{||\mathbf{r}^s||}\left[(a_1^s cos\theta - a_2^s sin\theta)r_2^s - (a_1^s sin\theta + a_2^s cos\theta)r_1^s\right] \\ &+ \frac{F^l}{||\mathbf{r}^l||}\left[(a_1^l cos\theta - a_2^l sin\theta)r_2^l - (a_1^l sin\theta + a_2^l cos\theta)r_1^l\right] \\ &+ a_1^w W cos\theta \end{aligned}$$

If the dynamic simulation is force/stress-driven we have that

$$\begin{aligned} F^s = &\frac{||\mathbf{r}^s||}{(a_1^s sin\theta + a_2^s cos\theta)r_1^s - (a_1^s cos\theta - a_2^s sin\theta)r_2^s}\{\frac{F^l}{||\mathbf{r}^l||}\left[(a_1^l cos\theta - a_2^l sin\theta)r_2^l - (a_1^l sin\theta + a_2^l cos\theta)r_1^l\right] \\ &+ a_1^w W cos\theta\} \end{aligned}$$

Otherwise, if the simulation if strain-driven

$$\begin{aligned} F^l = &\frac{||\mathbf{r}^l||}{(a_1^l sin\theta + a_2^l cos\theta)r_1^l - (a_1^l cos\theta - a_2^l sin\theta)r_2^l}\{\frac{F^s}{||\mathbf{r}^s||}\left[(a_1^s cos\theta - a_2^s sin\theta)r_2^s - (a_1^s sin\theta + a_2^s cos\theta)r_1^s\right] \\ &+ a_1^w W cos\theta\} \end{aligned}$$

Since $F^l = ku = \epsilon^l ||\mathbf{r}^l||$,

$$\epsilon^l = \frac{1/k}{(a_1^l sin\theta + a_2^l cos\theta)r_1^l - (a_1^l cos\theta - a_2^l sin\theta)r_2^l} \times$$
$$\left\{ \frac{F^s}{||\mathbf{r}^s||}\left[(a_1^s cos\theta - a_2^s sin\theta)r_2^s - (a_1^s sin\theta + a_2^s cos\theta)r_1^s\right] + a_1^w W cos\theta \right\} \quad (1.15)$$

At initial conditions, where temperature is below transformation temperature, it is expected that the shape memory alloy actuator will not transform; hence the behavior is linear as:

$$F^s = \frac{E_M}{A}\epsilon^s \quad (1.16)$$

1.5 Numerical Aplication

Algorithm 1 Find inital equilibrium

1: Initialize: calculate $\tau^s_{i=0}$, $\epsilon^l_{i=0}$, ${}_{R_o}\mathbf{a}^s_{i+1}$ (eq. 1.3), $\mathbf{r}^s_{i=0}$(eq. 1.4) and $\mathbf{r}^l_{i=0}$(eq. 1.4) for $\theta = 0$.
2: **while** $|\epsilon_{i-1} - \epsilon_i| \geq$ tolerance **do**:
3: Calculate ϵ^l_{i+1} (eq. 1.15)
4: Update θ_{i+1} (eq. 1.7), $\mathbf{r}^s_{i+1}$ (eq. 1.4), ${}_{R_o}\mathbf{a}^l_{i+1}$ (eq. 1.3), ${}_{R_o}\mathbf{a}^s_{i+1}$ (eq. 1.3), $\mathbf{r}^l_{i+1}$ (eq. 1.4) and $\mathbf{F}^s_{i+1}$ (eq.)
5: i = i + 1

2 Bibliography

[1] C. T. d. Faria, "Controle da variação do arqueamento de um aerofólio utilizando atuadores de memória de forma," 2010.

[2] I. F. Santos, *Dinâmica de sistemas mecânicos: modelagem, simulação, visualização, verificação.* São Paulo: Makron, 2001.