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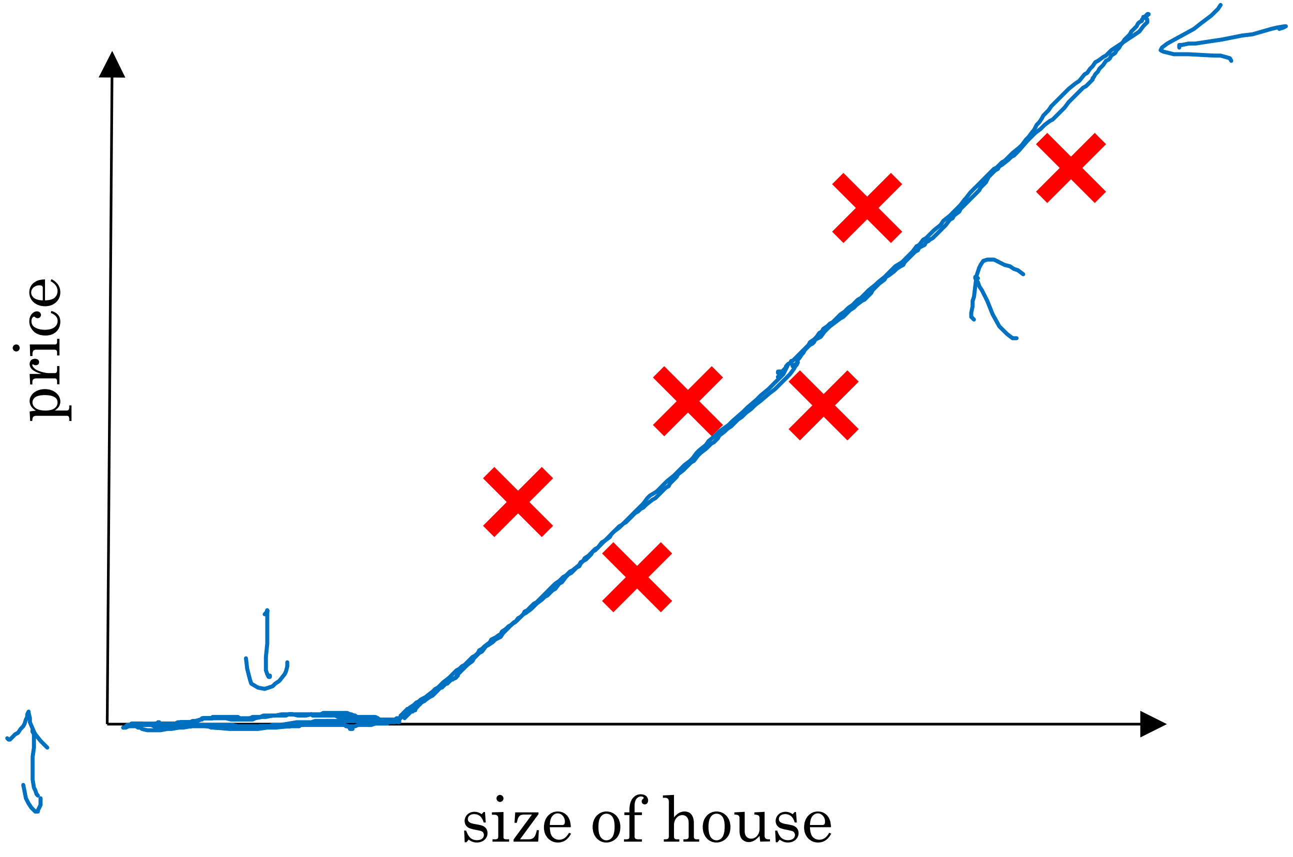
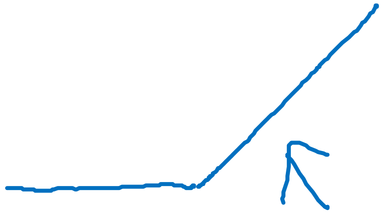
Introduction to Deep Learning

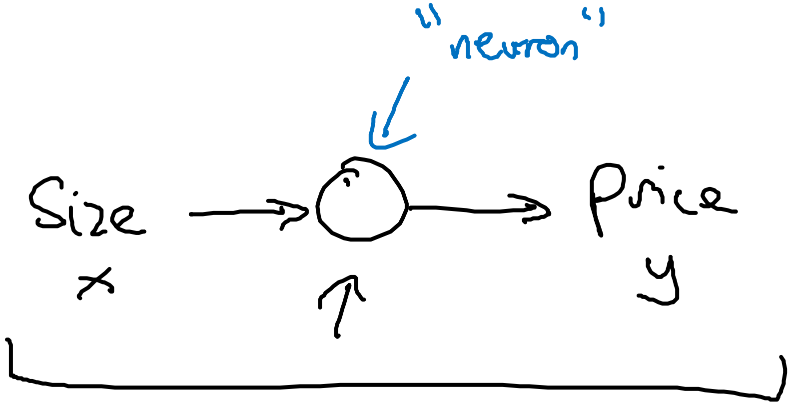
What is a Neural Network?

Housing Price Prediction

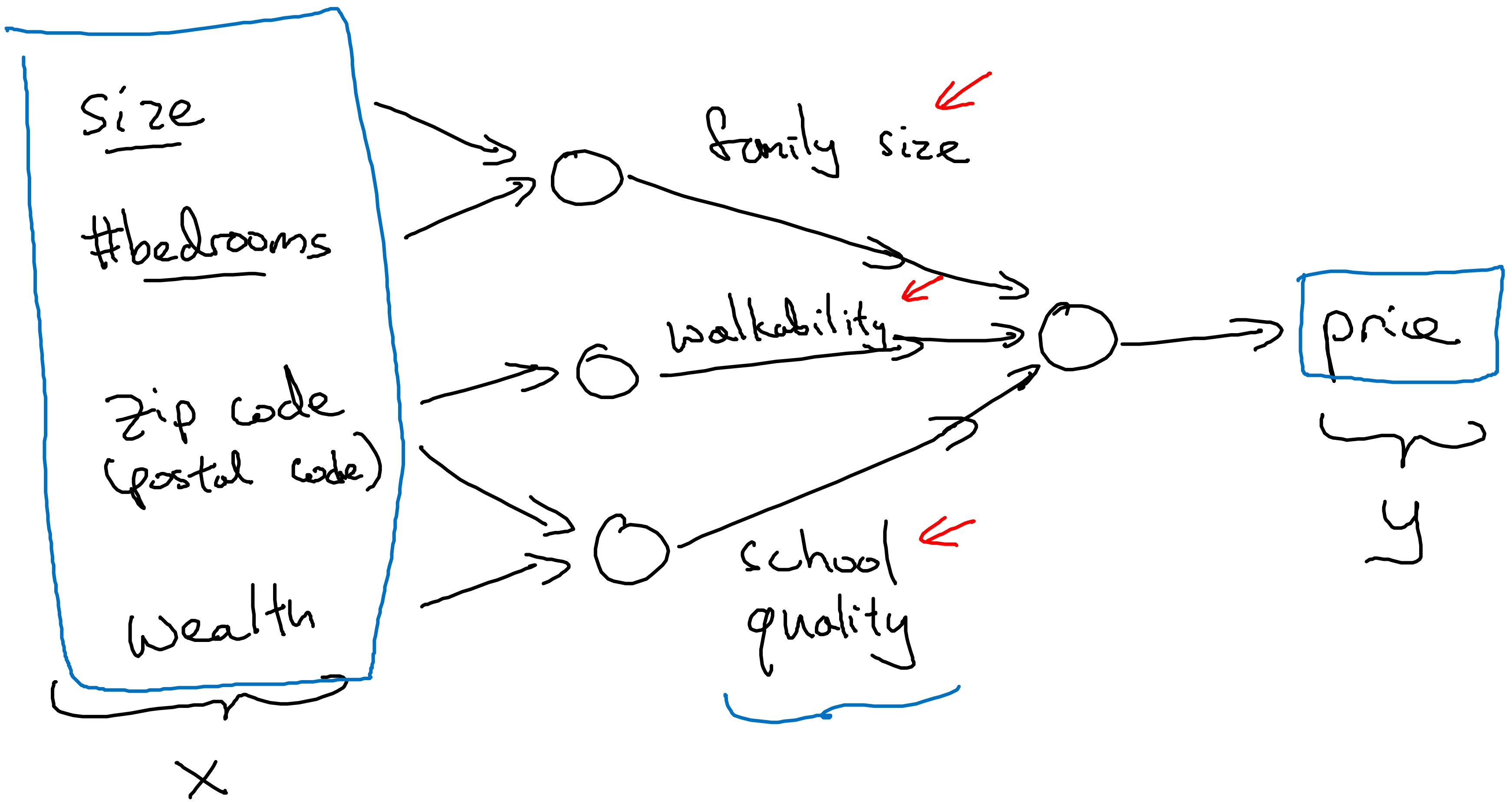




ReLU
Rectified
Linear
Unit

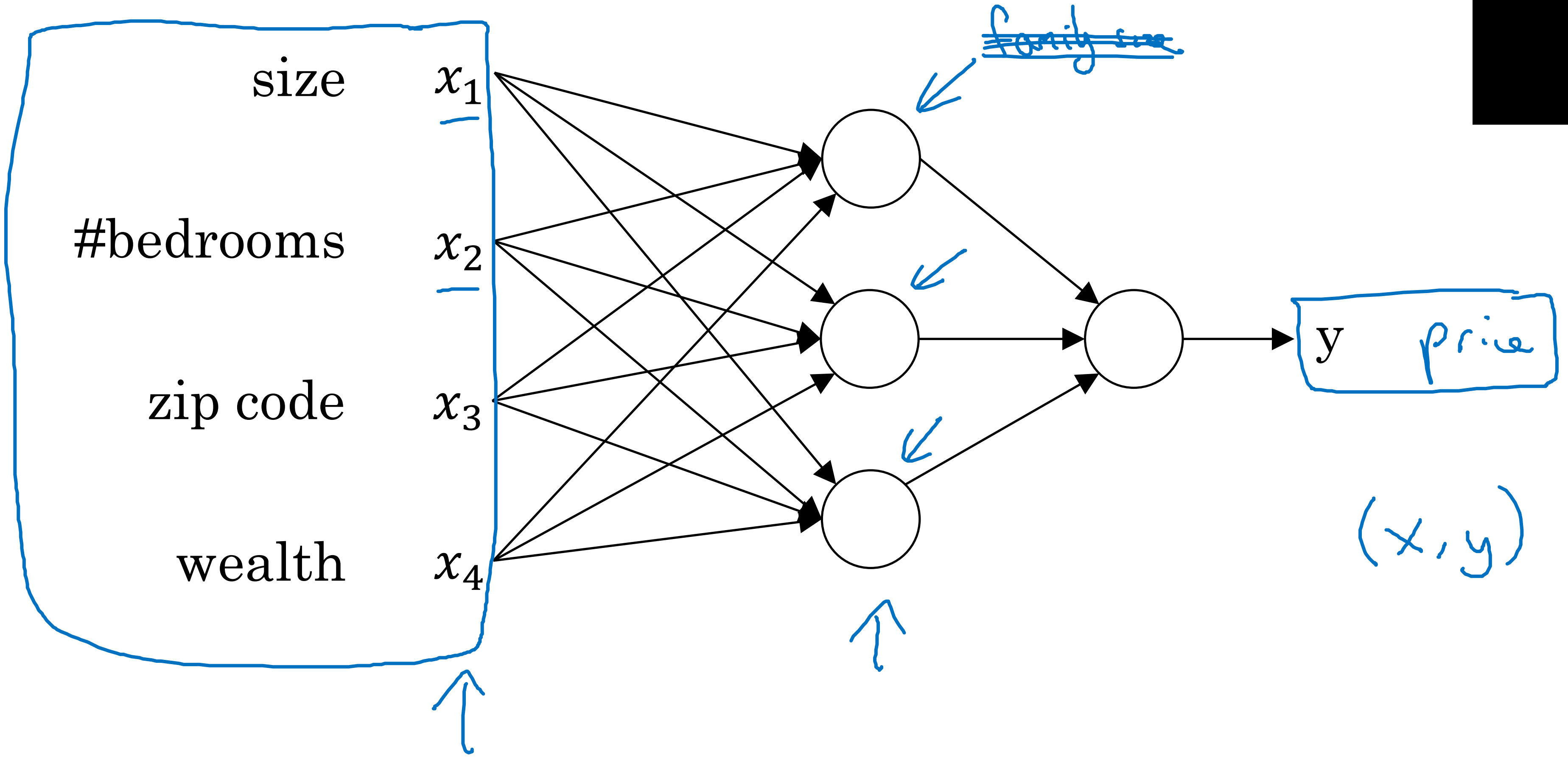


Housing Price Prediction



Housing Price Prediction

Drawing of previous Image



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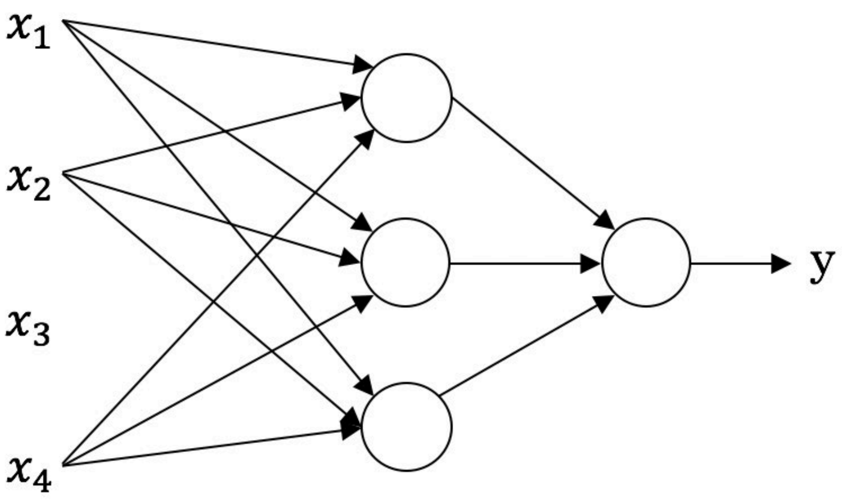
Introduction to Deep Learning

Supervised Learning with Neural Networks

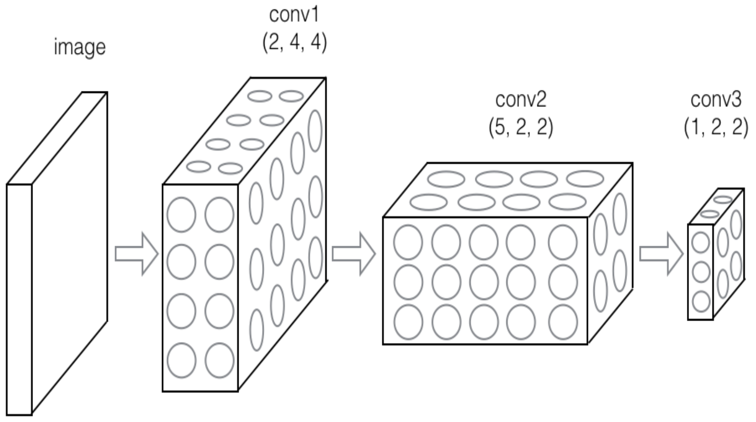
Supervised Learning

Input(x)	Output (y)	Application	
Home features	Price	Real Estate	Standard NN
Ad, user info	Click on ad? (0/1)	Online Advertising	
Image	Object (1,…,1000)	Photo tagging	CNN
Audio	Text transcript	Speech recognition	RNN
English	Chinese	Machine translation	
Image, Radar info	Position of other cars	Autonomous driving	Custom/ Hybrid

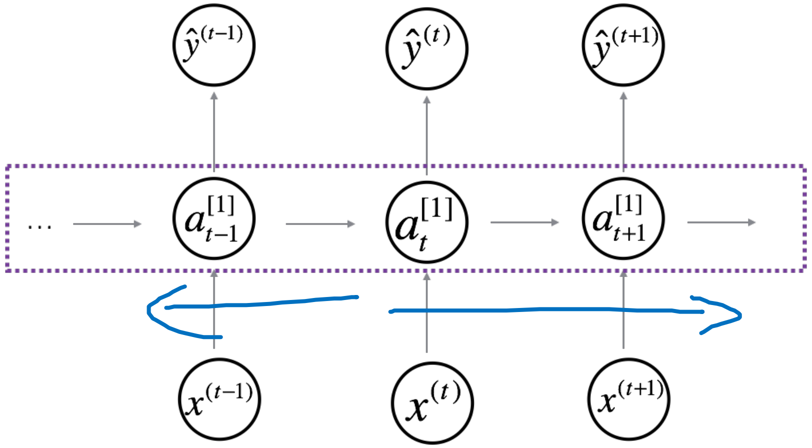
Neural Network examples



Standard NN



Convolutional NN



Recurrent NN

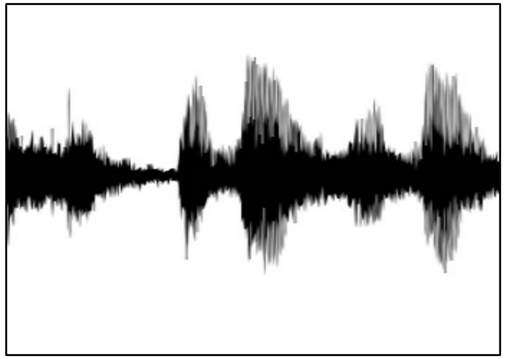
Supervised Learning

Structured Data

Size	#bedrooms	…	Price (1000$s)
2104	3		400
1600	3		330
2400	3		369
⋮	⋮		⋮
3000	4		540

User Age	Ad Id	…	Click
41	93242		1
80	93287		0
18	87312		1
⋮	⋮		⋮
27	71244		1

Unstructured Data



Audio

Image

Four scores and seven years ago…

Text

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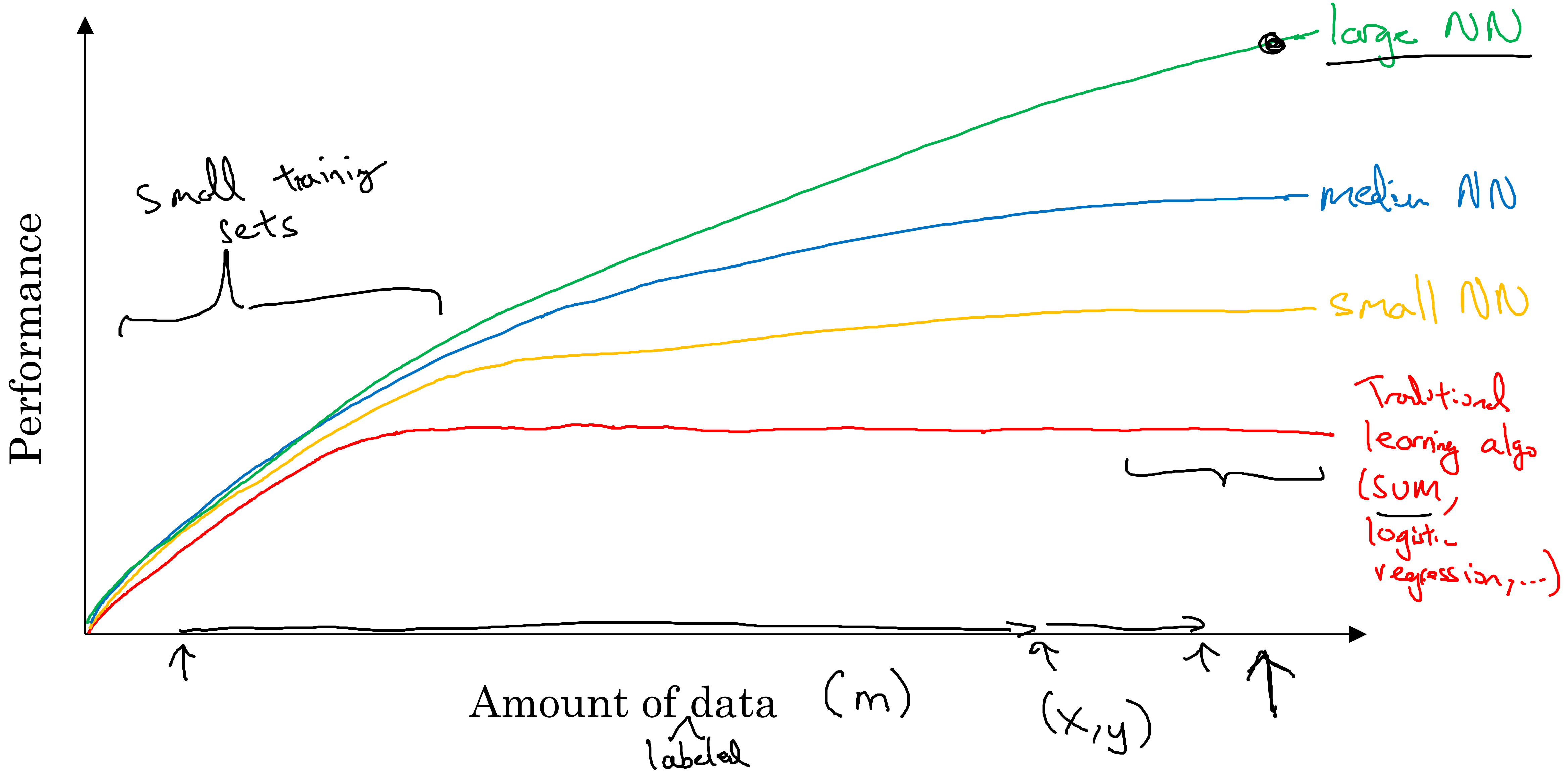
Introduction to Neural Networks

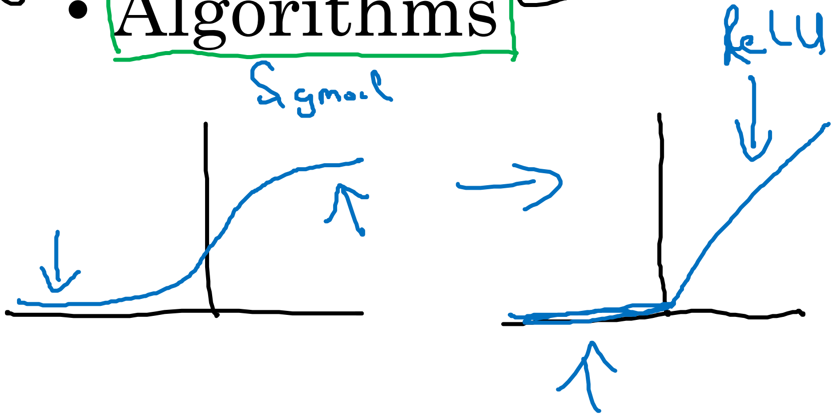
Why is Deep Learning taking off?

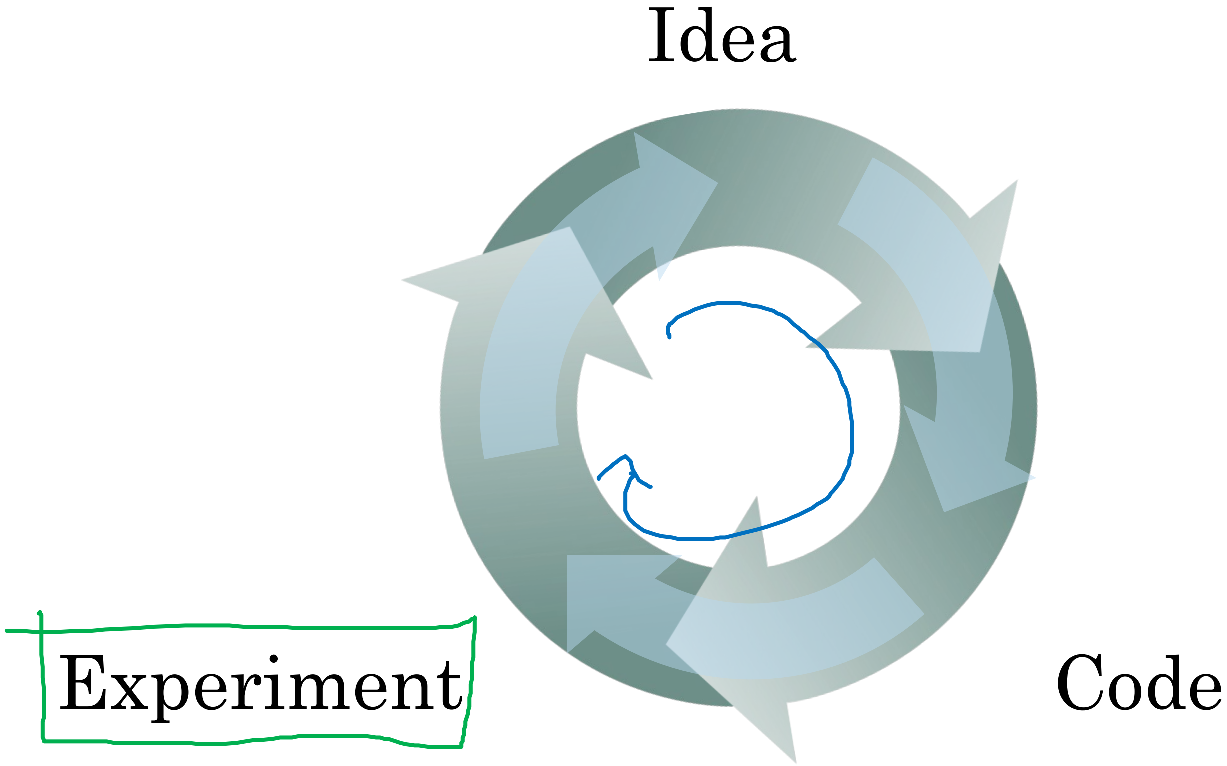
Scale drives deep learning progress



Scale drives deep learning progress

- Data
- Computation
- Algorithms





10 min
1 day
1 month

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Introduction to Neural Networks

About this Course

Courses in this Specialization

1. Neural Networks and Deep Learning
2. Improving Deep Neural Networks: Hyperparameter tuning, Regularization and Optimization
3. Structuring your Machine Learning project
4. Convolutional Neural Networks
5. Natural Language Processing: Building sequence models

Outline of this Course

Week 1: Introduction

Week 2: Basics of Neural Network programming

Week 3: One hidden layer Neural Networks

Week 4: Deep Neural Networks

Basics of Neural Network Programming

Binary Classification

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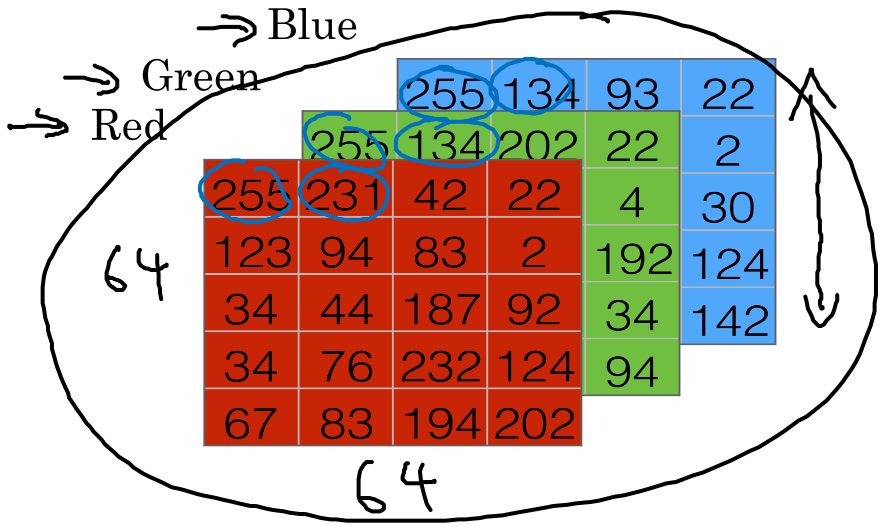
Binary Classification

64

64

1 (cat) vs 0 (non cat)

y



$$x = \begin{bmatrix} 255 \\ 231 \\ \vdots \\ \vdots \\ 255 \\ 134 \\ \vdots \end{bmatrix}$$

$$64 \times 64 \times 3 = 12288$$

$$n = n_x = 12288$$

$$x \longrightarrow y$$

Notation

(x, y) $\quad x \in \mathbb{R}^{n_x}, \; y \in \{0, 1\}$

m training examples : $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\}$

$M = M_{train}$ $\qquad M_{test} = \#\text{test examples.}$

$$X = \underbrace{\left.\begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}\right\updownarrow n_x}_{m} \qquad \xcancel{\begin{bmatrix} — & x^{(1)T} & — \\ & \vdots & \\ — & x^{(m)T} & — \end{bmatrix}}$$

$X \in \mathbb{R}^{n_x \times m}$ $\qquad X.\text{shape} = (n_x, m)$

$Y = [y^{(1)} \; y^{(2)} \; \ldots, y^{(m)}]$

$Y \in \mathbb{R}^{1 \times m}$

$Y.\text{shape} = (1, m)$

Basics of Neural Network Programming

Logistic Regression

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Logistic Regression

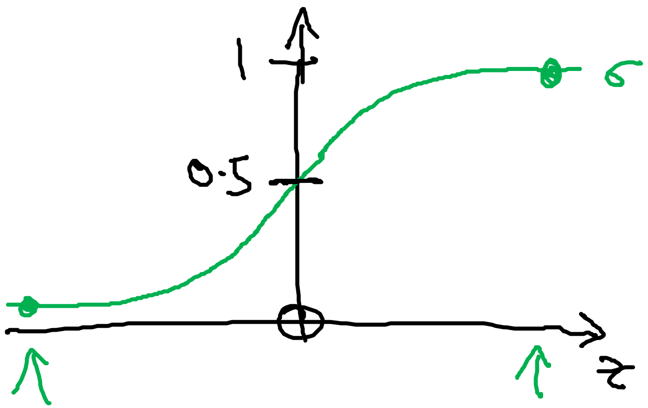
Given x, want $\hat{y} = P(y=1 \mid x)$

$0 \le \hat{y} \le 1$

$x \in \mathbb{R}^{n_x}$

Parameters: $\boxed{w} \in \mathbb{R}^{n_x}$, $\boxed{b} \in \mathbb{R}$.

Output $\hat{y} = \sigma(\underbrace{w^T x + b}_{z})$



$x_0 = 1$, $x \in \mathbb{R}^{n_x+1}$

$\hat{y} = \sigma(\theta^T x)$

$$\underbrace{\Theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{n_x} \end{bmatrix} \begin{matrix} \} \, b \leftarrow \\ \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} w \leftarrow \end{matrix}}$$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

If z large $\sigma(z) \approx \frac{1}{1+0} = 1$

If z large negative number

$$\sigma(z) = \frac{1}{1+e^{-z}} \approx \frac{1}{1+\text{Bignum}} \approx 0$$

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Basics of Neural Network Programming

Logistic Regression cost function

Logistic Regression cost function

$\hat{y}^{(i)} = \sigma(w^T x^{(i)} + b)$, where $\sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}$

$z^{(i)} = w^T x^{(i)} + b$

Given $\{(x^{(1)}, y^{(1)}),\ldots,(x^{(m)}, y^{(m)})\}$, want $\hat{y}^{(i)} \approx y^{(i)}$.

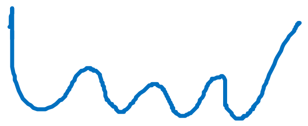
$x^{(i)}$

$y^{(i)}$

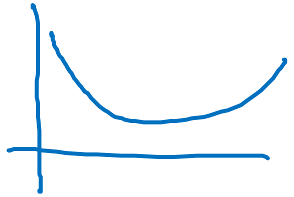
$z^{(i)}$

i-th example.

$\boxed{\text{Loss}}$ (error) function: $\mathcal{L}(\hat{y}, y) = \frac{1}{2}(\hat{y} - y)^2$



$$\boxed{\mathcal{L}(\hat{y}, y)} = -\left(\boxed{y \log \hat{y}} + (1-y)\log(1-\hat{y})\right)$$



If $y=1$: $\mathcal{L}(\hat{y}, y) = -\log \hat{y}$ ← Want $\log \hat{y}$ large, want $\hat{y}$ large.

If $y=0$: $\mathcal{L}(\hat{y}, y) = -\log(1-\hat{y})$ ← Want $\log 1-\hat{y}$ large .... want $\hat{y}$ small

$\boxed{\text{Cost}}$ function: $J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\hat{y}^{(i)} + (1-y^{(i)})\log(1-\hat{y}^{(i)})\right]$

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Basics of Neural Network Programming

Logistic Regression cost function

Logistic Regression cost function

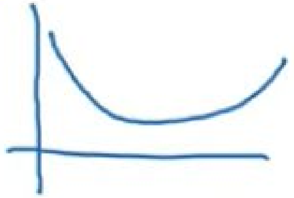
→ $\hat{y}^{(i)} = \underline{\sigma(w^T \underline{x}^{(i)} + b)}$, where $\underline{\sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}}$ $z^{(i)} = w^T x^{(i)} + b$

Given $\{\underline{(x^{(1)}, y^{(1)})}, \ldots, \underline{(x^{(m)}, y^{(m)})}\}$, want $\underline{\hat{y}^{(i)}} \approx \underline{y^{(i)}}$.

$x^{(i)}$ $y^{(i)}$ $z^{(i)}$ i-th example.

$\boxed{\text{Loss}}$ (error) function: $\mathcal{L}(\hat{y}, y) = \underline{\frac{1}{2}(\hat{y}-y)^2}$

$$\boxed{\mathcal{L}(\hat{y}, y)} = -(\boxed{y \log \hat{y}} + \underline{(1-y)\log(1-\hat{y})}) \leftarrow$$



If $\underline{y=1}$: $\mathcal{L}(\hat{y}, y) = -\log \hat{y}$ ← Want $\log \hat{y}$ large, $\underline{\text{want } \hat{y} \text{ large}}$.

If $\underline{y=0}$: $\mathcal{L}(\hat{y}, y) = -\log(1-\hat{y})$ ← Want $\log 1-\hat{y}$ large .... $\underline{\text{want } \hat{y} \text{ small}}$

$\boxed{\text{Cost}}$ function: $J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log \hat{y}^{(i)} + (1-y^{(i)})\log(1-\hat{y}^{(i)})\right]$

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Basics of Neural Network Programming

Gradient Descent

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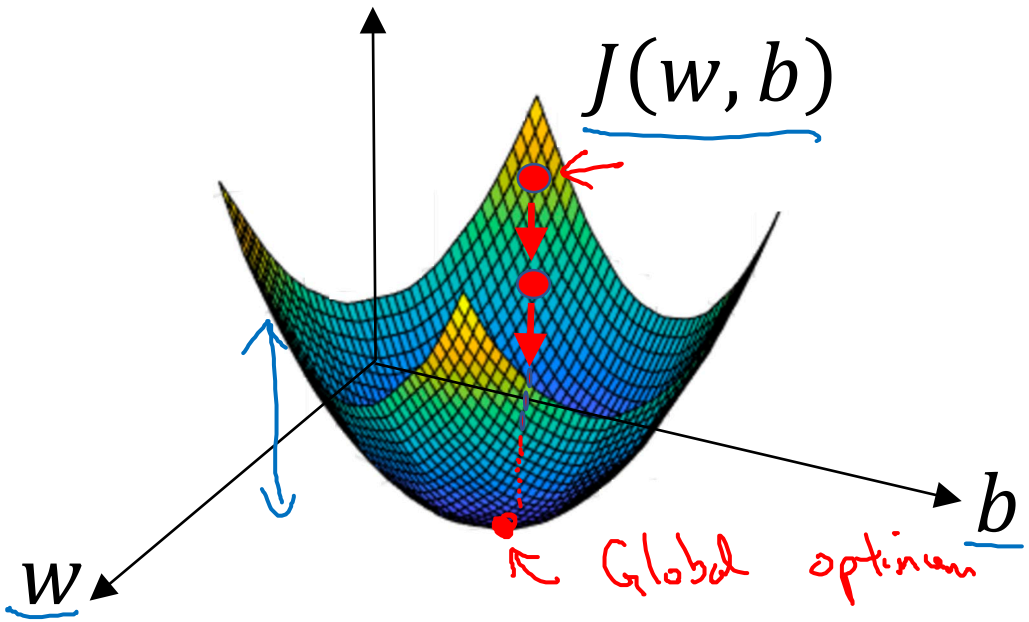
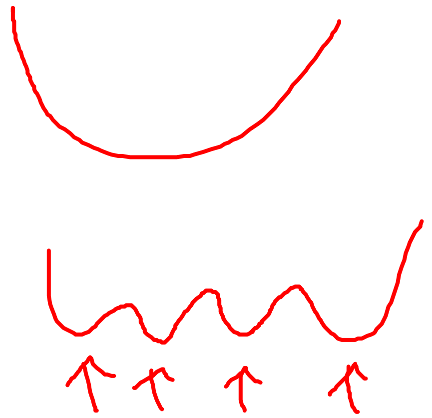
Gradient Descent

Recap: $\hat{y} = \sigma(w^T x + b)$, $\sigma(z) = \frac{1}{1+e^{-z}}$

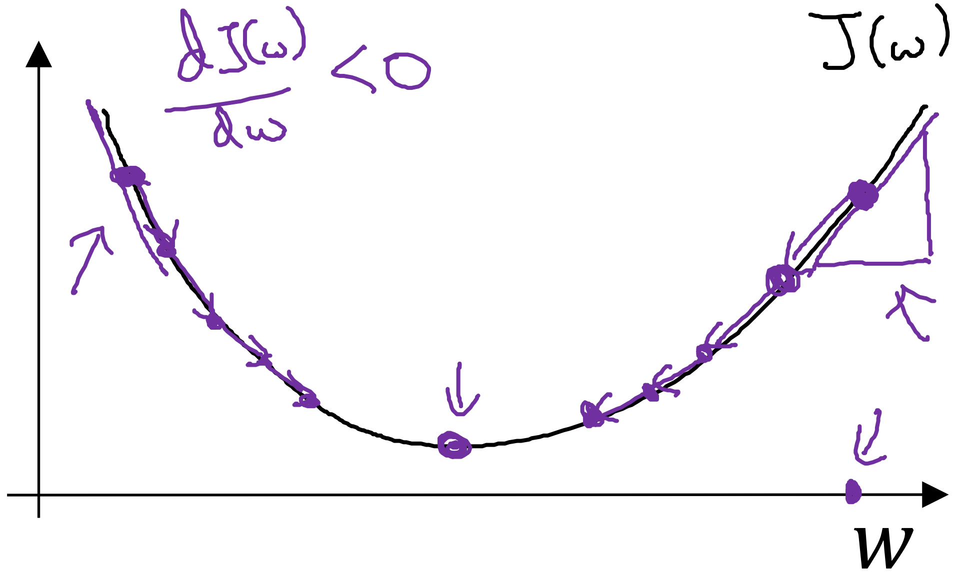
$$J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\boxed{\hat{y}^{(i)}}, \boxed{y^{(i)}}) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})$$

Want to find w, b that minimize $J(w,b)$





Gradient Descent



learning rate

Repeat {

$$w := w - \alpha \boxed{\frac{dJ(w)}{dw}}$$

}

"dw"

$$w := w - \alpha \, dw$$

$$\frac{dJ(w)}{dw} = ?$$

$J(w,b)$

$$w := w - \alpha \boxed{\frac{dJ(w,b)}{dw}} \qquad \boxed{\frac{\partial J(w,b)}{\partial w}} \rightarrow dw$$

$$b := b - \alpha \boxed{\frac{dJ(w,b)}{db}} \qquad \boxed{\frac{\partial J(w,b)}{\partial b}} \rightarrow db$$

∂ "partial derivative"

d

J

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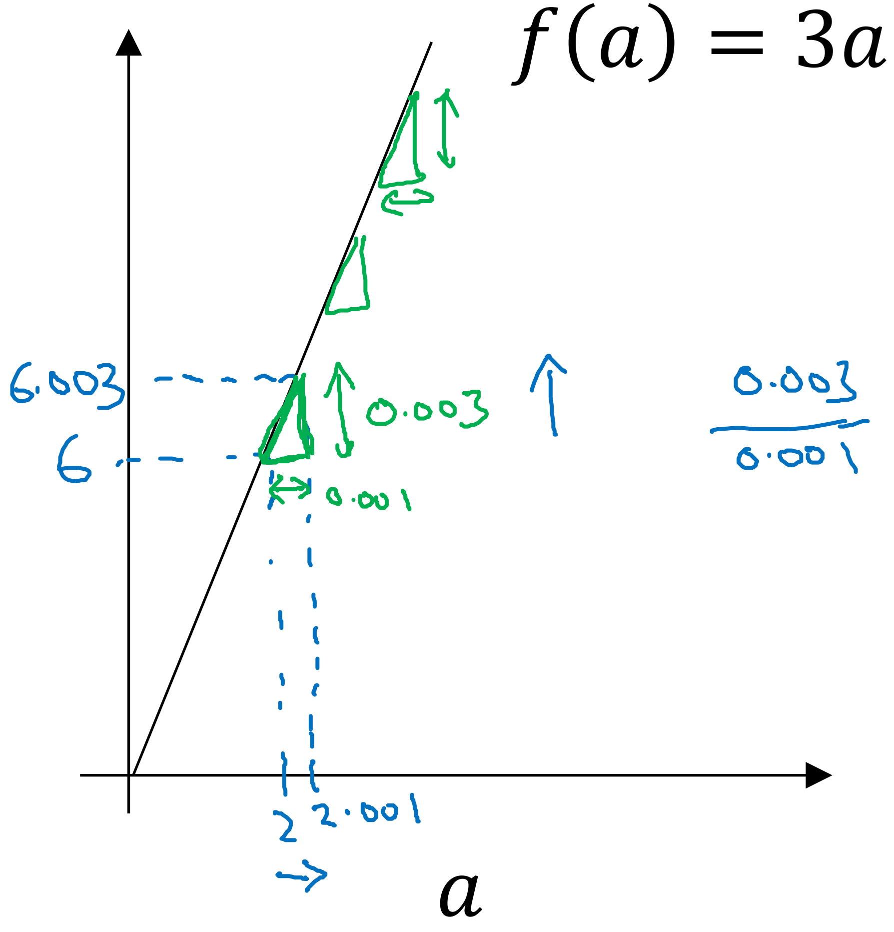
Basics of Neural Network Programming

Derivatives

Intuition about derivatives



$\frac{0.003}{0.001}$ $\frac{\text{height}}{\text{width}}$

$a = 2$ $f(a) = 6$

$a = 2.001$ $f(a) = 6.003$

slope (derivative) of $f(a)$ at $a=2$ is 3

$a = 5$ $f(a) = 15$

$a = 5.001$ $f(a) = 15.003$

slope at $a=5$ is also 3

$$\frac{d f(a)}{da} = 3 = \frac{d}{da} f(a)$$

0.001

0.00000001

0.000000000001

Basics of Neural Network Programming

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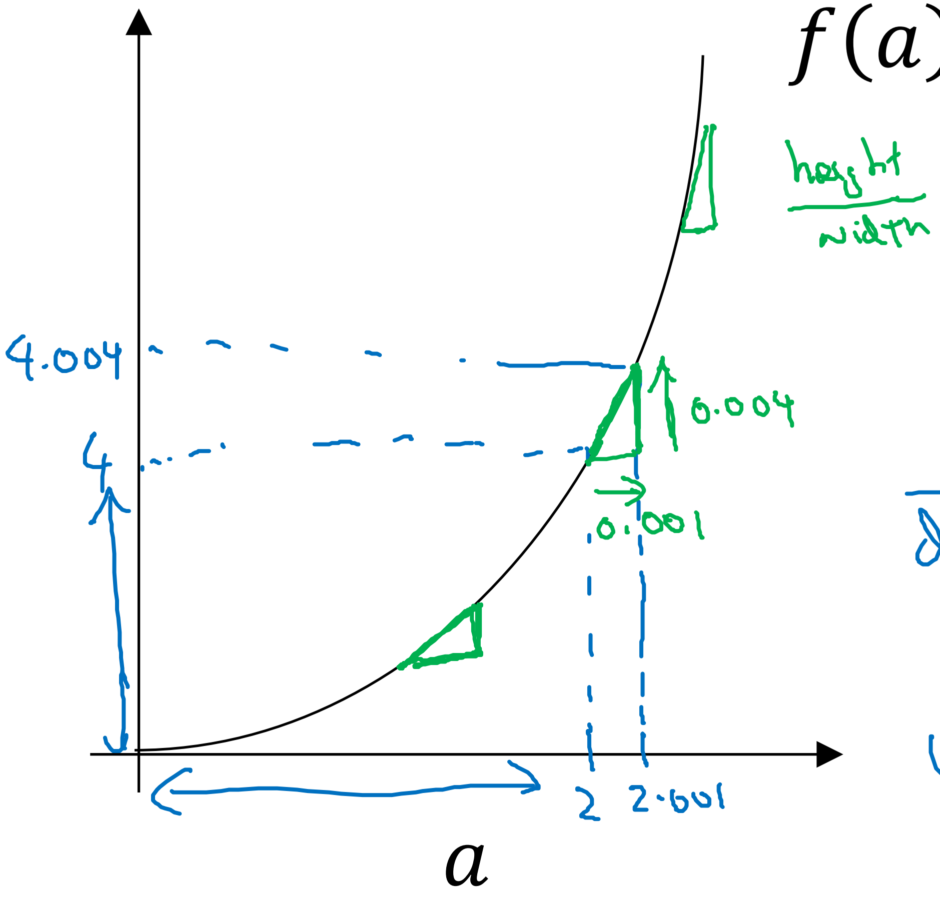
More derivatives examples

Intuition about derivatives

0.001

$0.00000\ldots01$

$f(a) = a^2$



$a = 2 \qquad f(a) = 4$

$a = 2.001 \qquad f(a) \approx 4.004$

$(4.004\boxed{001})$

slope (derivative) of f(a) at a=2 is 4.

$$\boxed{\frac{d}{da} f(a) = 4} \text{ when } \boxed{a = 2}.$$

$a = 5 \qquad f(a) = 25$

$a = 5.001 \qquad f(a) \approx 25.010$

$$\boxed{\frac{d}{da} f(a) = 10} \text{ when } \boxed{a = 5}$$

$$\frac{d}{da} a^2 = 2a$$

0.001

$(2a) \times 0.001$

$$\frac{d}{da} f(a) = \frac{d}{da} a^2 = \boxed{2a}$$

More derivative examples

$f(a) = a^2$ $\qquad \frac{d}{da} f(a) = \underbrace{2a}_{4}$ $\qquad a = 2 \quad f(a) = 4$

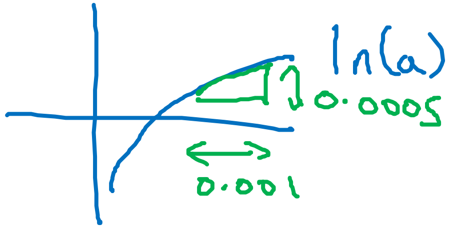
$\qquad a = 2.001 \quad f(a) \approx 4.004$

$f(a) = a^3$ $\qquad \frac{d}{da} f(a) = \underbrace{3a^2}_{3 \times 2^2 = 12}$ $\qquad a = 2 \quad f(a) = 8$

$\qquad a = 2.001 \quad f(a) \approx 8.012$

$f(a) = \log_e(a)$ $\qquad \frac{d}{da} f(a) = \frac{1}{a}$ $\qquad a = 2 \quad f(a) \approx 0.69315$

$\ln(a)$ $\qquad a = 2.001 \quad f(a) \approx 0.69365$



$\frac{d}{da} f(a) = \boxed{\frac{1}{2}}$

0.0005 $\qquad 0.0005$

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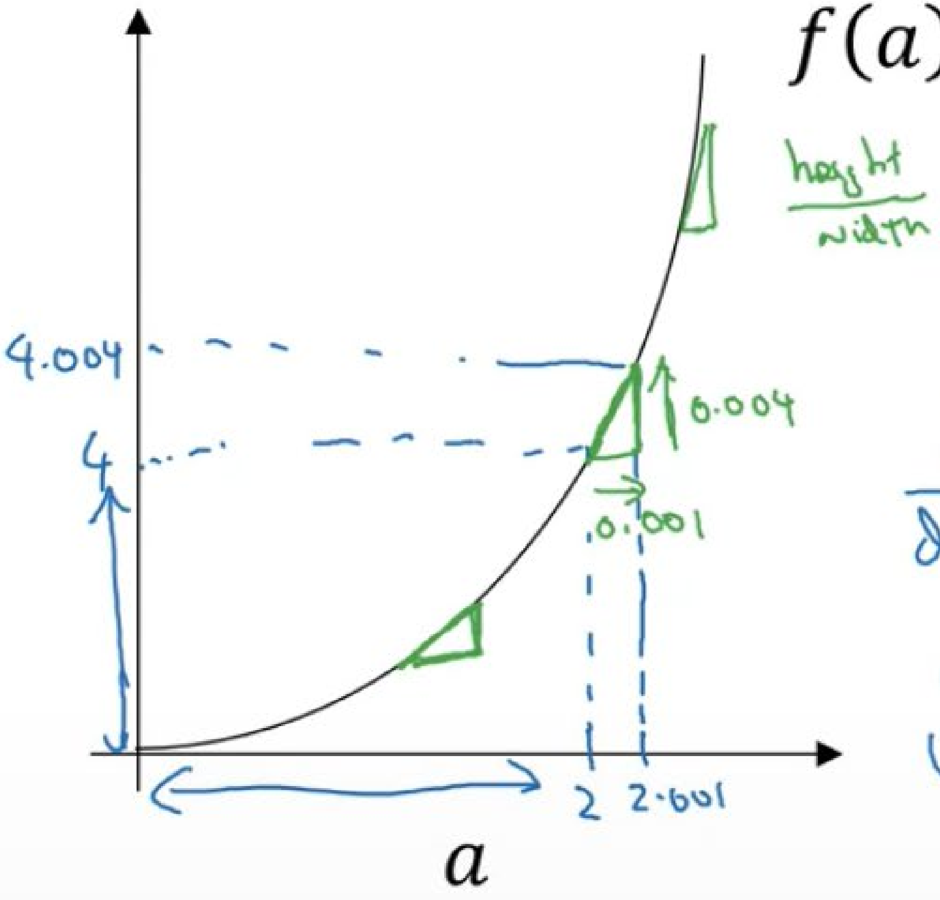
Basics of Neural Network Programming

More derivatives examples

Intuition about derivatives

$$f(a) = a^2$$



$$\frac{d}{da} a^2 = 2a$$

0.001

$(2a) \times 0.001$

0.001

0.00000....01

$a = 2$ $\qquad f(a) = 4$

$a = 2.001$ $\qquad f(a) \approx 4.004$

$(4.004\boxed{001})$

slope (derivative) of f(a) at $a=2$ is 4.

$\boxed{\frac{d}{da} f(a) = 4}$ when $\boxed{a=2}$.

$a = 5$ $\qquad f(a) = 25$

$a = 5.001$ $\qquad f(a) \approx 25.010$

$\boxed{\frac{d}{da} f(a) = 10}$ when $\boxed{a=5}$

$$\frac{d}{da} f(a) = \underbrace{\frac{d}{da} a^2}_{} = \boxed{2a}$$

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More derivative examples

$f(a) = a^2$ $\qquad \frac{d}{da} f(a) = \underbrace{2a}_{4}$ $\qquad a = 2 \quad f(a) = 4$

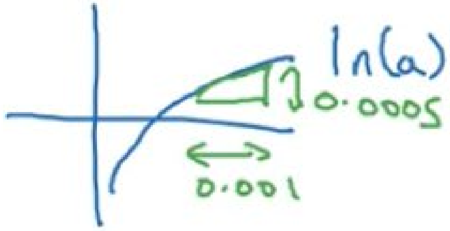
$\qquad\qquad\qquad\qquad\qquad\qquad a = 2.001 \quad f(a) \approx 4.004$

$f(a) = a^3$ $\qquad \frac{d}{da} f(a) = \underbrace{3a^2}_{3\times 2^2 = 12}$ $\qquad a = 2 \quad f(a) = 8$

$\qquad\qquad\qquad\qquad\qquad\qquad a = 2.\underline{001} \quad f(a) \approx 8.\underline{012}$

$f(a) = \log_e(a)$ $\qquad \frac{d}{da} f(a) = \frac{1}{a}$ $\qquad a = 2 \quad f(a) \approx 0.69315$

$\ln(a)$ $\qquad\qquad\qquad\qquad\qquad\qquad a = 2.\underline{001} \quad \underline{f(a)} \approx 0.69365$



$\frac{d}{da} f(a) = \boxed{\frac{1}{2}}$ $\qquad 0.0005 \longleftrightarrow \underline{0.0005}$

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Basics of Neural Network Programming

Computation Graph

Computation Graph

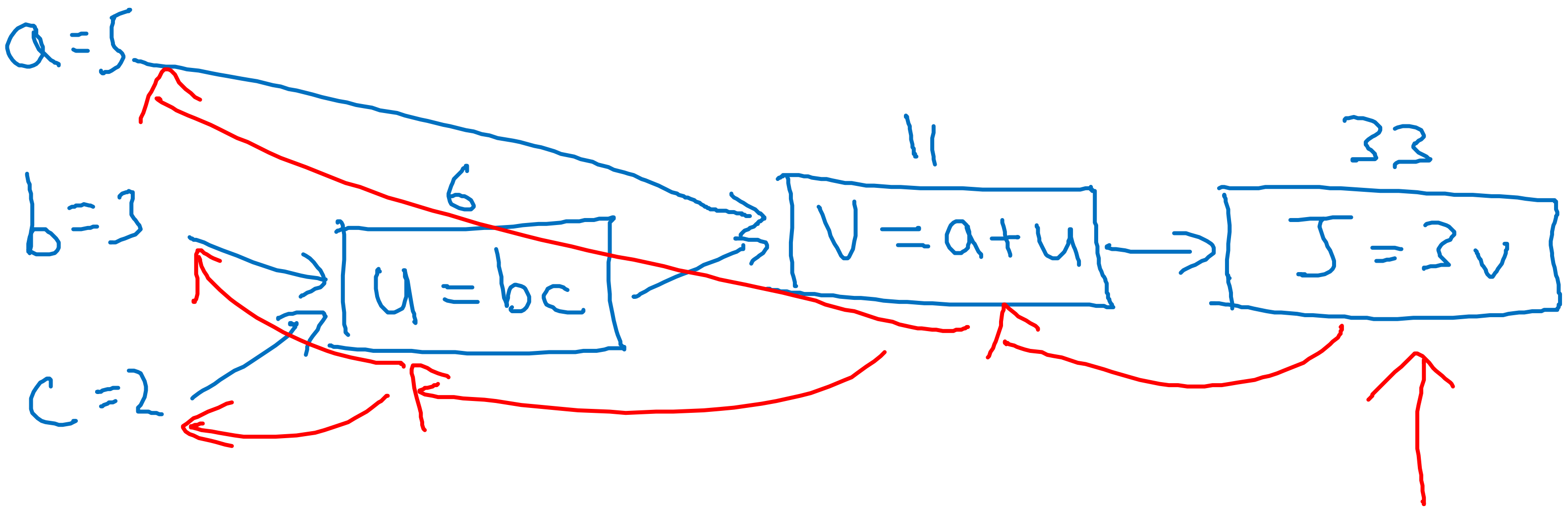
$$J(a,b,c) = \underbrace{3(\overbrace{a + \underbrace{bc}_{u}}^{})}_{J} = 3(5 + 3\times 2) = 33$$

$$\underbrace{a+bc}_{v}$$

$u = bc$

$V = a + u$

$J = 3v$



Basics of Neural Network Programming

Derivatives with a Computation Graph

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Computing derivatives

$\frac{dJ}{da}$ "da" = 3

$a = 5$

$b = 3$

$c = 2$

6

$u = bc$

11

$v = a + u$

33

$J = 3v$

$\frac{dJ}{dv}$ "dv" = 3

$J = 3v$

$v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

$f(a) = 3a$

$\frac{df(a)}{da} = \frac{df}{da} = 3$

$J = 3v$

$\frac{dJ}{dv} = 3$

$\frac{dJ}{dv} = ? = 3$

$a \rightarrow v \rightarrow J$

$\frac{dJ}{da} = 3 = \frac{dJ}{dv} \frac{dv}{da}$

3×1

$\frac{dv}{da} = 1$

$a = 5 \rightarrow 5.001$

$\rightarrow v = 11 \rightarrow 11.001$

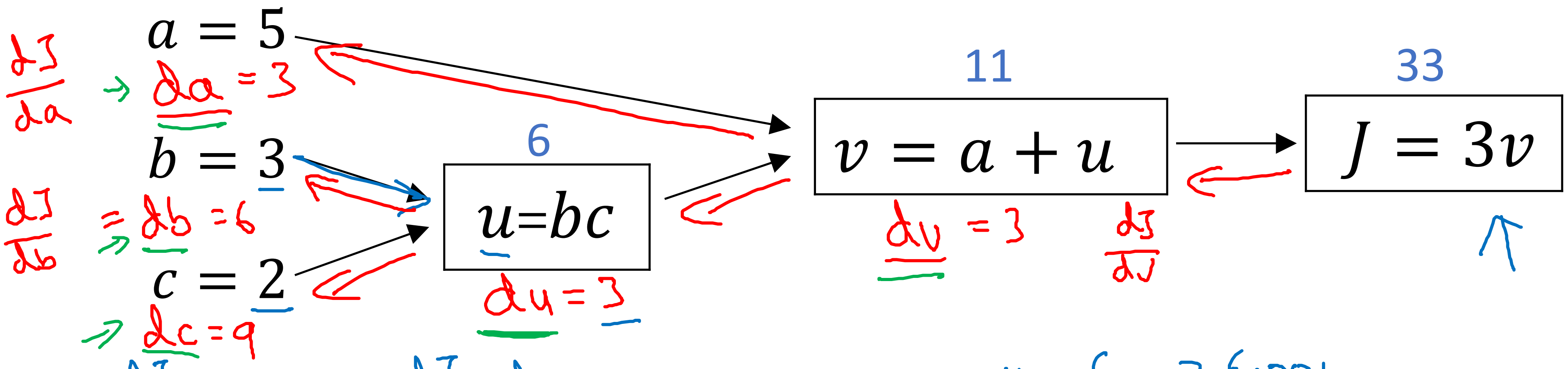
$J = 33 \rightarrow 33.003$

$\frac{d\,\mathrm{FinalOutputVar}}{d\,\mathrm{var}}$

dJdvar

"dvar"

Computing derivatives



$$\frac{dJ}{du} = 3 = \underbrace{\frac{dJ}{dv}}_{3} \cdot \underbrace{\frac{dv}{du}}_{1}$$

$$\frac{dJ}{db} = \underbrace{\boxed{\frac{dJ}{du}}}_{\to 3} \cdot \underbrace{\frac{du}{db}}_{=2} = 6$$

$$\frac{dJ}{da} = \underbrace{\boxed{\frac{dJ}{du}}}_{3} \cdot \underbrace{\frac{du}{da}}_{3} = 9 \qquad 3 \times 3$$

$u = 6 \rightarrow 6.001$

$v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

$b = 3 \rightarrow 3.001$

$u = b \cdot c = 6 \rightarrow 6.002$ $\qquad c = 2$

$J = 33.006$ $\qquad .006$

$v = 11.002$

$J = 3v$

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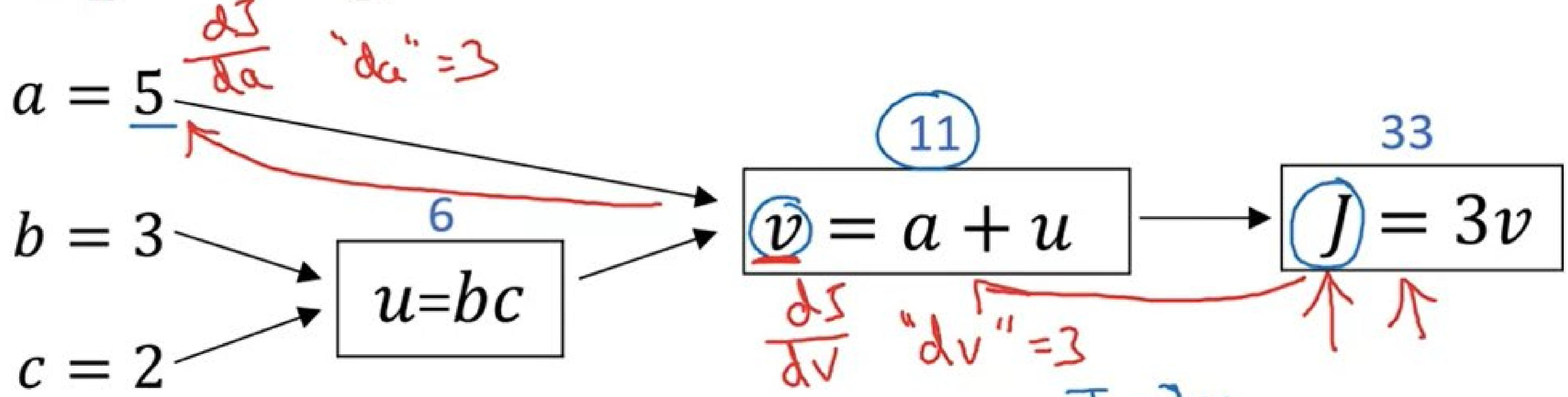
Basics of Neural Network Programming

Derivatives with a Computation Graph

Computing derivatives



$$\frac{dJ}{dv} = ? = 3$$

$$\frac{dJ}{da} = 3 = \frac{dJ}{dv}\,\frac{dv}{da}$$

3×1

$$\frac{dv}{da} = 1$$

$a \to v \to J$

$J = 3v$

$v = 11 \to 11.001$

$J = 33 \to 33.003$

$a = 5 \to 5.001$

$v = 11 \to 11.001$

$J = 33 \to 33.003$

$$\frac{d\,\text{FinalOutputVar}}{d\,\text{var}}$$

dJdvar

"dvar"

$f(a) = 3a$

$$\frac{df(a)}{da} = \frac{df}{da} = 3$$

$J = 3v$

$$\frac{dJ}{dv} = 3$$

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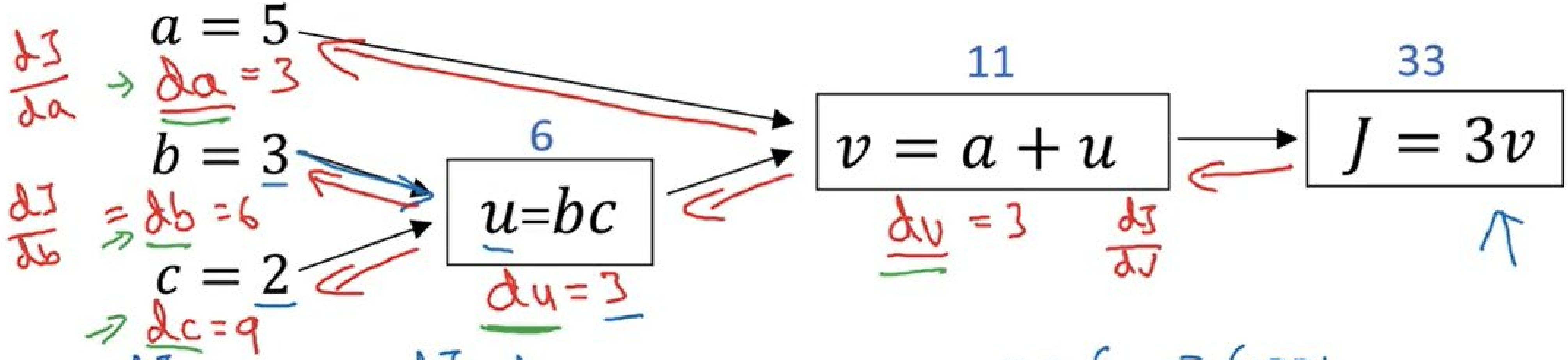
Computing derivatives



$$\frac{dJ}{du} = 3 = \underbrace{\frac{dJ}{dv}}_{3} \cdot \underbrace{\frac{dv}{du}}_{1}$$

$$\frac{dJ}{db} = \underbrace{\boxed{\frac{dJ}{du}}}_{\to 3} \cdot \underbrace{\frac{du}{db}}_{=2} = 6$$

$$\frac{dJ}{dc} = \boxed{\frac{dJ}{du}} \cdot \frac{du}{dc} = 9$$

$$3 \times 3$$

$u = 6 \to 6.001$

$v = 11 \to 11.001$

$J = 33 \to 33.003$

$b = 3 \to 3.001$

$u = b \cdot c = 6 \to 6.002$ $\quad c = 2$

$J = 33.006$ $\quad .006$

$v = 11.002$

$J = 3v$

Basics of Neural Network Programming

Logistic Regression Gradient descent

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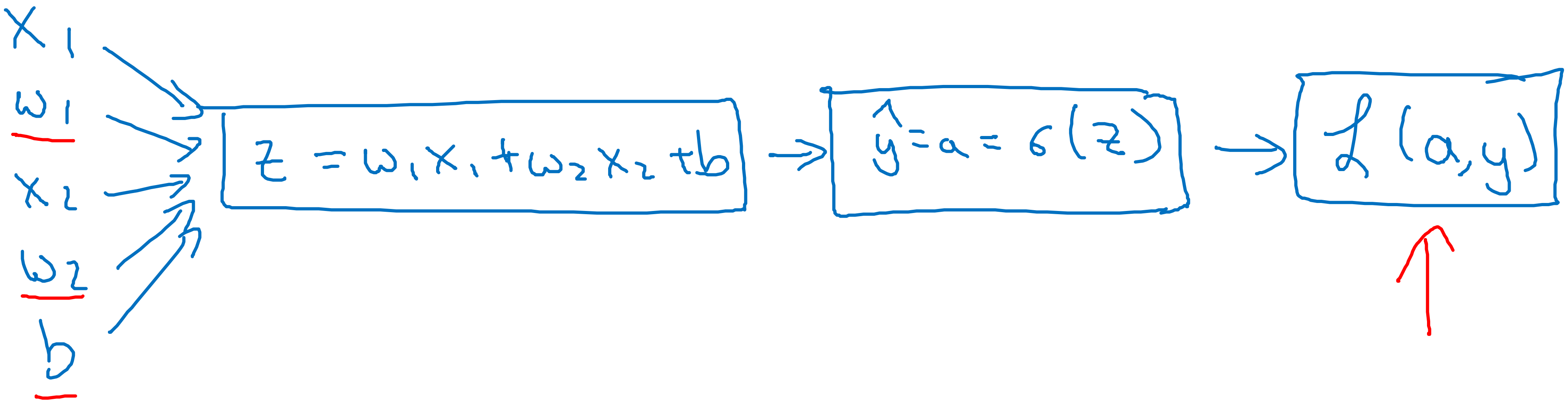
Logistic regression recap

$$z = w^T x + b$$

$$\hat{y} = a = \sigma(z)$$

$$\mathcal{L}(a, y) = -(y \log(a) + (1 - y) \log(1 - a))$$



Logistic regression derivatives

x_1, w_1, x_2, w_2, b $\rightarrow$ $\boxed{z = w_1 x_1 + w_2 x_2 + b}$ $\rightarrow$ $\boxed{a = \sigma(z)}$ $\rightarrow$ $\boxed{\mathcal{L}(a, y)}$

$$dz = \frac{d\mathcal{L}}{dz} = \frac{d\mathcal{L}(a,y)}{dz}$$

$$= a - y$$

$$= \frac{d\mathcal{L}}{da} \cdot \frac{da}{dz}$$

$$a(1-a)$$

$$\text{“}da\text{”} = \frac{d\mathcal{L}(a,y)}{da} = -\frac{y}{a} + \frac{1-y}{1-a}$$

$$\frac{\partial \mathcal{L}}{\partial w_1} = \text{“}dw_1\text{”} = x_1 \cdot dz. \qquad dw_2 = x_2 \cdot dz. \qquad db = dz.$$

$$w_1 := w_1 - \alpha\, dw_1$$

$$w_2 := w_2 - \alpha\, dw_2$$

$$b := b - \alpha\, db.$$

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Basics of Neural Network Programming

Gradient descent on m examples

Logistic regression on m examples

$$\underline{J(w,b)} = \underbrace{\frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(a^{(i)}, y^{(i)})}$$

$$(x^{(i)}, y^{(i)})$$

$$\rightarrow a^{(i)} = \hat{y}^{(i)} = \sigma(z^{(i)}) = \sigma(w^T x^{(i)} + b)$$

$$\underline{dw_1^{(i)}}, \underline{dw_2^{(i)}}, \underline{db^{(i)}}$$

$$\underline{\frac{\partial}{\partial w_1} J(w,b)} = \frac{1}{m}\sum_{i=1}^{m} \underbrace{\frac{\partial}{\partial w_1} \mathcal{L}(a^{(i)}, y^{(i)})}_{\underline{dw_1^{(i)}} \; - \; (x^{(i)}, y^{(i)})}$$

Logistic regression on m examples

$J=0;\ dw_1=0;\ dw_2=0;\ db=0$

For $i=1$ to m

$$z^{(i)} = w^T x^{(i)} + b$$

$$a^{(i)} = \sigma(z^{(i)})$$

$$J += -[y^{(i)} \log a^{(i)} + (1-y^{(i)}) \log(1-a^{(i)})]$$

$$dz^{(i)} = a^{(i)} - y^{(i)}$$

$$dw_1 += x_1^{(i)} dz^{(i)}$$

$$dw_2 += x_2^{(i)} dz^{(i)}$$

$$db += dz^{(i)}$$

$n=2$

dw_3
$\vdots$
dw_n

$J /= m$

$dw_1 /= m;\ dw_2 /= m;\ db /= m.$

$$dw_1 = \frac{\partial J}{\partial w_1}$$

$$w_1 := w_1 - \alpha\, dw_1$$

$$w_2 := w_2 - \alpha\, dw_2$$

$$b := b - \alpha\, db$$

Vectorization

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Basics of Neural Network Programming

Gradient descent on m examples

Logistic regression on m examples

$$\underline{J(w,b)} = \underbrace{\frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(a^{(i)}, y^{(i)})}$$

$(x^{(i)}, y^{(i)})$

$$\rightarrow a^{(i)} = \hat{y}^{(i)} = \sigma(z^{(i)}) = \sigma(w^T x^{(i)} + b)$$

$\underline{dw_1^{(i)}}, \underline{dw_2^{(i)}}, \underline{db^{(i)}}$

$$\underline{\frac{\partial}{\partial w_1} J(w,b)} = \frac{1}{m}\sum_{i=1}^{m} \underbrace{\frac{\partial}{\partial w_1}\mathcal{L}(a^{(i)}, y^{(i)})}_{\underline{dw_1^{(i)}} \;-\; (x^{(i)}, y^{(i)})}$$

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Logistic regression on m examples

$J = 0;\ dw_1 = 0;\ dw_2 = 0;\ db = 0$

For $i = 1$ to m

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J += -[y^{(i)} \log a^{(i)} + (1 - y^{(i)}) \log(1 - a^{(i)})]$

$dz^{(i)} = a^{(i)} - y^{(i)}$

$dw_1 += x_1^{(i)} dz^{(i)}$

$dw_2 += x_2^{(i)} dz^{(i)}$

$db += dz^{(i)}$

$n = 2$

dw_3
$\vdots$
dw_n

$J /= m$

$dw_1 /= m;\ dw_2 /= m;\ db /= m.$

$dw_1 = \frac{\partial J}{\partial w_1}$

$w_1 := w_1 - \alpha\, dw_1$

$w_2 := w_2 - \alpha\, dw_2$

$b := b - \alpha\, db$

Vectorization

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Basics of Neural Network Programming

Vectorization

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What is vectorization?

$z = \underbrace{w^T x} + b$

$w = \begin{bmatrix} \vdots \\ \vdots \end{bmatrix}$ $x = \begin{bmatrix} \vdots \\ \vdots \end{bmatrix}$

$w \in \mathbb{R}^{n_x}$

$x \in \mathbb{R}^{n_x}$

Non-vectorized:

```
z = 0
for i in range(n-x):
    z += w[i] * x[i]
z += b
```

Vectorized

$z = \underbrace{\text{np.}\underline{\text{dot}}(w, x)}_{w^T x} + b$

→ GPU
→ CPU

SIMD - single instruction multiple data.

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Basics of Neural Network Programming

More vectorization examples

Neural network programming guideline

Whenever possible, avoid explicit for-loops.

Neural network programming guideline

Whenever possible, avoid explicit for-loops.

$u = Av$

$u_i = \sum_i \sum_j A_{ij} v_j$

```
u = np.zeros((n,1))
for i ...
    for j ...
        u[i] += A[i][j] * v[j]
```

```
u = np.dot(A,v)
```

Vectors and matrix valued functions

Say you need to apply the exponential operation on every element of a matrix/vector.

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \rightarrow u = \begin{bmatrix} e^{v_1} \\ e^{v_2} \\ \vdots \\ e^{v_n} \end{bmatrix}$$

```
u = np.zeros((n,1))
for i in range(n):
    u[i]=math.exp(v[i])
```

```
import numpy as np
u = np.exp(v)

np.log(v)
np.abs(v)
np.maximum(v,0)
v**2        1/v
```

Logistic regression derivatives

$dw = np.zeros((n\_x, 1))$

$J = 0$, $\boxed{\cancel{dw1 = 0,\ dw2 = 0}}$, $db = 0$

for i = 1 to n:

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J \mathrel{+}= -[y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})]$

$dz^{(i)} = a^{(i)}(1 - a^{(i)})$

for $j = 1 \ldots n_x$
$dw_j \mathrel{+}= \ldots$

$\boxed{\cancel{\begin{aligned} dw_1 &\mathrel{+}= x_1^{(i)} dz^{(i)} \\ dw_2 &\mathrel{+}= x_2^{(i)} dz^{(i)} \end{aligned}}}$ $n_x = 2$

$dw \mathrel{+}= x^{(i)} dz^{(i)}$

$db \mathrel{+}= dz^{(i)}$

$J = J/m$, $\boxed{\cancel{dw_1 = dw_1/m,\ dw_2 = dw_2/m}}$, $db = db/m$

$dw /= m.$

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Basics of Neural Network Programming

More vectorization examples

Neural network programming guideline

Whenever possible, avoid explicit for-loops.

$u = Av$

$u_i = \sum_j A_{ij} v_j$

```
u = np.zeros((n,1))
for i ...
    for j ...
        u[i] += A[i][j]*v[j]
```

```
u = np.dot(A,v)
```

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Vectors and matrix valued functions

Say you need to apply the exponential operation on every element of a matrix/vector.

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \rightarrow u = \begin{bmatrix} e^{v_1} \\ e^{v_2} \\ \vdots \\ e^{v_n} \end{bmatrix}$$

```
u = np.zeros((n,1))
for i in range(n):
    u[i]=math.exp(v[i])
```

```
import numpy as np
u = np.exp(v)

np.log(v)
np.abs(v)
np.maximum(v,0)
v**2        1/v
```

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Logistic regression derivatives

J = 0, $\boxed{\cancel{\text{dw1 = 0, dw2 = 0}}}$, db = 0

$dw = np.zeros((n\_x, 1))$

for i = 1 to m:

$$z^{(i)} = w^T x^{(i)} + b$$

$$a^{(i)} = \sigma(z^{(i)})$$

$$J \mathrel{+}= -\left[y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})\right]$$

$$dz^{(i)} = a^{(i)}(1 - a^{(i)})$$

for $j = 1 \ldots n_x$

$dw_j \mathrel{+}= \ldots$

$$\boxed{\cancel{dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}}}$$

$$\boxed{\cancel{dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}}}$$

$n_x = 2$

$$dw \mathrel{+}= x^{(i)} dz^{(i)}$$

$$db \mathrel{+}= dz^{(i)}$$

J = J/m, $\boxed{\cancel{dw_1 = dw_1/m,\ dw_2 = dw_2/m}}$, db = db/m

$dw \mathrel{/}= m.$

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Basics of Neural Network Programming

Vectorizing Logistic Regression

Vectorizing Logistic Regression

$$z^{(1)} = \boxed{w^T x^{(1)} + b} \qquad z^{(2)} = \boxed{w^T x^{(2)} + b} \qquad z^{(3)} = w^T x^{(3)} + b$$

$$\boxed{a^{(1)}} = \sigma(z^{(1)}) \qquad \boxed{a^{(2)}} = \sigma(z^{(2)}) \qquad \boxed{a^{(3)}} = \sigma(z^{(3)})$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix} \qquad (n_x, m) \qquad \mathbb{R}^{n_x \times m}$$

$$[\;\text{———}\;]\; w^T \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$Z = \boxed{[z^{(1)} \; z^{(2)} \cdots z^{(m)}]} = \underbrace{w^T X + \underbrace{[b \; b \cdots b]}_{1 \times m}} = [\boxed{w^T x^{(1)} + b} \; \boxed{w^T x^{(2)} + b} \cdots w^T x^{(m)} + b]$$

$1 \times m$

"Broadcasting"

$$\rightarrow Z = np.dot(w.T, X) + b$$

$(1,1)$ $\mathbb{R}$

$$A = [a^{(1)} \; a^{(2)} \cdots a^{(m)}] = \sigma(Z)$$

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Basics of Neural Network Programming

Vectorizing Logistic Regression's Gradient Computation

Vectorizing Logistic Regression

$dz^{(1)} = a^{(1)} - y^{(1)}$ $\quad dz^{(2)} = a^{(2)} - y^{(2)}$ $\quad \ldots$

$$\boxed{dZ = [dz^{(1)} \; dz^{(2)} \; \ldots \; dz^{(m)}]}$$

$1 \times m$

$A = [a^{(1)} \ldots a^{(m)}] \qquad Y = [y^{(1)} \ldots y^{(m)}]$

$\rightarrow dZ = A - Y = [a^{(1)} - y^{(1)} \quad a^{(2)} - y^{(2)} \quad \ldots]$

$\rightarrow dw = 0$

$dw \mathrel{+}= x^{(1)} dz^{(1)}$

$dw \mathrel{+}= x^{(2)} dz^{(2)}$

$\vdots$

$dw \mathrel{/}= m$

$\cancel{dw_1}$

$\cancel{dw_2}$

$\vdots$

$db = 0$

$db \mathrel{+}= dz^{(1)}$

$db \mathrel{+}= dz^{(2)}$

$\vdots$

$db \mathrel{+}= dz^{(m)}$

$db \mathrel{/}= m.$

$$db = \frac{1}{m} \sum_{i=1}^{m} dz^{(i)}$$

$$= \frac{1}{m} \, \text{np.sum}(dZ)$$

$$\boxed{dw = \frac{1}{m} X \, dZ^T}$$

$$= \frac{1}{m} \begin{bmatrix} | & & | \\ x^{(1)} & \ldots & x^{(m)} \\ | & & | \end{bmatrix} \begin{bmatrix} dz^{(1)} \\ \vdots \\ dz^{(m)} \end{bmatrix}$$

$$= \frac{1}{m} \left[x^{(1)} dz^{(1)} + \ldots + x^{(m)} dz^{(m)} \right]$$

$n \times 1$

Implementing Logistic Regression

$J = 0,\ dw_1 = 0,\ dw_2 = 0,\ db = 0$

$\boxed{\text{for } i = 1 \text{ to } m:}$

$$z^{(i)} = w^T x^{(i)} + b$$

$$a^{(i)} = \sigma(z^{(i)})$$

$$J \mathrel{+}= -[y^{(i)} \log a^{(i)} + (1 - y^{(i)}) \log(1 - a^{(i)})]$$

$$dz^{(i)} = a^{(i)} - y^{(i)}$$

$$\left.\begin{aligned} dw_1 &\mathrel{+}= x_1^{(i)} dz^{(i)} \\ dw_2 &\mathrel{+}= x_2^{(i)} dz^{(i)} \end{aligned}\right\} \quad dw \mathrel{+}= x^{(i)} * dz^{(i)}$$

$$db \mathrel{+}= dz^{(i)}$$

$J = J/m,\ dw_1 = dw_1/m,\ dw_2 = dw_2/m$

$db = db/m$

for iter in range(1000):

$$Z = w^T X + b$$

$$= np.dot(w.T, X) + b$$

$$A = \sigma(Z)$$

$$dZ = A - Y$$

$$dw = \frac{1}{m} X dZ^T$$

$$db = \frac{1}{m} np.sum(dZ)$$

$$w := w - \alpha dw$$

$$b := b - \alpha db$$

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Basics of Neural Network Programming

Vectorizing Logistic Regression's Gradient Computation

Vectorizing Logistic Regression

$dz^{(1)} = a^{(1)} - y^{(1)} \qquad dz^{(2)} = a^{(2)} - y^{(2)} \qquad \cdots$

$$\boxed{dZ = [\, dz^{(1)} \;\; dz^{(2)} \; \cdots \; dz^{(m)} \,]} \leftarrow$$

$1 \times m$

$A = [\, a^{(1)} \cdots a^{(m)} \,] \qquad Y = [\, y^{(1)} \cdots y^{(m)} \,]$

$\rightarrow dZ = A - Y = [\, a^{(1)} - y^{(1)} \quad a^{(2)} - y^{(2)} \quad \cdots \,]$

$\rightarrow dw = 0$

$dw \mathrel{+}= x^{(1)} dz^{(1)}$

$dw \mathrel{+}= x^{(2)} dz^{(2)}$

$\vdots$

$dw \mathrel{/}= m$

$\cancel{dw_1}$

$\cancel{dw_2}$

$\vdots$

$db = 0$

$db \mathrel{+}= dz^{(1)}$

$db \mathrel{+}= dz^{(2)}$

$\vdots$

$db \mathrel{+}= dz^{(m)}$

$db \mathrel{/}= m$

$$\boxed{db = \frac{1}{m} \sum_{i=1}^{m} dz^{(i)} = \frac{1}{m} \, np.sum(dZ)}$$

$$\boxed{dw = \frac{1}{m} X \, dZ^{T}}$$

$$= \frac{1}{m} \begin{bmatrix} | & & | \\ x^{(1)} & \cdots & x^{(m)} \\ | & & | \end{bmatrix} \begin{bmatrix} dz^{(1)} \\ \vdots \\ dz^{(m)} \end{bmatrix}$$

$$= \frac{1}{m} \left[\, x^{(1)} dz^{(1)} + \cdots + x^{(m)} dz^{(m)} \,\right]$$

$n \times 1$

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Implementing Logistic Regression

$J = 0,\ dw_1 = 0,\ dw_2 = 0,\ db = 0$

$\boxed{\text{for } i = 1 \text{ to } m:}$

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J \mathrel{+}= -[y^{(i)} \log a^{(i)} + (1 - y^{(i)}) \log(1 - a^{(i)})]$

$dz^{(i)} = a^{(i)} - y^{(i)}$

$$\left.\begin{array}{l} dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)} \\ dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)} \end{array}\right\} \quad dw \mathrel{+}= x^{(i)} * dz^{(i)}$$

$db \mathrel{+}= dz^{(i)}$

$J = J/m,\ dw_1 = dw_1/m,\ dw_2 = dw_2/m$

$db = db/m$

for iter in range(1000):

$Z = w^T X + b$

$= np.dot(w.T, X) + b$

$A = \sigma(Z)$

$dZ = A - Y$

$dw = \frac{1}{m} X dZ^T$

$db = \frac{1}{m} np.sum(dZ)$

$w := w - \alpha dw$

$b := b - \alpha db$

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Basics of Neural Network Programming

Broadcasting in Python

Broadcasting example

Calories from Carbs, Proteins, Fats in 100g of different foods:

$$
\begin{array}{r|cccc}
 & \text{Apples} & \text{Beef} & \text{Eggs} & \text{Potatoes} \\
\hline
\text{Carb} & 56.0 & 0.0 & 4.4 & 68.0 \\
\text{Protein} & 1.2 & 104.0 & 52.0 & 8.0 \\
\text{Fat} & 1.8 & 135.0 & 99.0 & 0.9
\end{array}
= A \quad (3,4)
$$

59 cal

$\frac{56}{59} \approx 94.9\%$

0

1

Calculate % of calories from Carb, Protein, Fat. Can you do this without explicit for-loop?

```
cal = A.sum(axis = 0)
percentage = 100*A/(cal.reshape(1,4))
```

(3,4) / (1,4)

Broadcasting example

$$\begin{bmatrix}1\\2\\3\\4\end{bmatrix} + \cancel{100} \quad \begin{bmatrix}100\\100\\100\\100\end{bmatrix}$$

$$\underset{(m,n)\quad(2,3)}{\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}} + \begin{bmatrix}100 & 200 & 300\\ 100 & 200 & 300\end{bmatrix}$$

$(1,n) \rightsquigarrow (m,n)$ $\quad (2,3)$

$$\underset{(m,n)}{\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}} + \underset{(m,1)}{\begin{bmatrix}100\\200\end{bmatrix}} \begin{bmatrix}100 & 100 & 100\\ 200 & 200 & 200\end{bmatrix} =$$

$(m,1) \rightarrow (m,n)$

General Principle

$$(m, n) \quad \begin{matrix} + \\ - \\ * \\ / \end{matrix} \quad \begin{matrix} (1, n) \rightsquigarrow (m, n) \\ (m, 1) \rightsquigarrow (m, n) \end{matrix}$$

matrix

$$(m, 1) \quad + \quad \mathbb{R}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 100 = \begin{bmatrix} 101 \\ 102 \\ 103 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} + 100 = \begin{bmatrix} 101 & 102 & 103 \end{bmatrix}$$

Matlab/Octave: bsxfun

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Basics of Neural Network Programming

A note on python/ numpy vectors

Python Demo

Python / numpy vectors

```
import numpy as np

a = np.random.randn(5)

a = np.random.randn((5,1))

a = np.random.randn((1,5))

assert(a.shape = (5,1))
```

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Basics of Neural Network Programming

Explanation of logistic regression cost function (Optional)

Logistic regression cost function

$\hat{y} = \sigma(w^T x + b)$ where $\sigma(z) = \frac{1}{1+e^{-z}}$

Interpret $\hat{y} = P(y=1|x)$

If $y=1$: $P(y|x) = \hat{y}$

If $y=0$: $P(\underline{y}|x) = \underline{1 - \hat{y}}$

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Logistic regression cost function

$$\left.\begin{array}{l} \boxed{\text{If } \quad y = 1: \qquad p(y|x) = \hat{y}} \\ \boxed{\text{If } \quad y = 0: \qquad p(y|x) = 1 - \hat{y}} \end{array}\right\} \quad p(y|x)$$

$$\boxed{p(y|x) = \underbrace{\hat{y}^{y}}\ \underbrace{(1-\hat{y})^{(1-y)}}}$$

$$\text{If } y=1: \quad \underline{p(y|x) = \hat{y}} \ \underbrace{(1-\hat{y})^{0}}_{=1}$$

$$\underline{\text{If } y=0}: \quad \underline{p(y|x)} = \underbrace{\hat{y}^{0}}_{=1} \ (1-\hat{y})^{(1-0)} \quad = 1 \times (1-\hat{y}) = \underline{1-\hat{y}}$$

$$\uparrow \log \underbrace{p(y|x)} = \log \hat{y}^{y} (1-\hat{y})^{(1-y)} = y \log \hat{y} + (1-y) \log (1-\hat{y})$$

$$= -\underline{\mathcal{L}(\hat{y}, y)} \downarrow$$

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Cost on m examples

$$\underline{\log p(\text{labels in training set})} = \log \prod_{i=1}^{m} p(y^{(i)} \mid x^{(i)})$$

$$\underline{\log p(\ldots\ldots)} = \sum_{i=1}^{m} \underbrace{\log p(y^{(i)} \mid x^{(i)})}_{-\mathcal{L}(\hat{y}^{(i)},\, y^{(i)})}$$

Maximum likelihood estimation

$$= -\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$$

Cost: $\underline{J(w,b)} = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$

(minimize)

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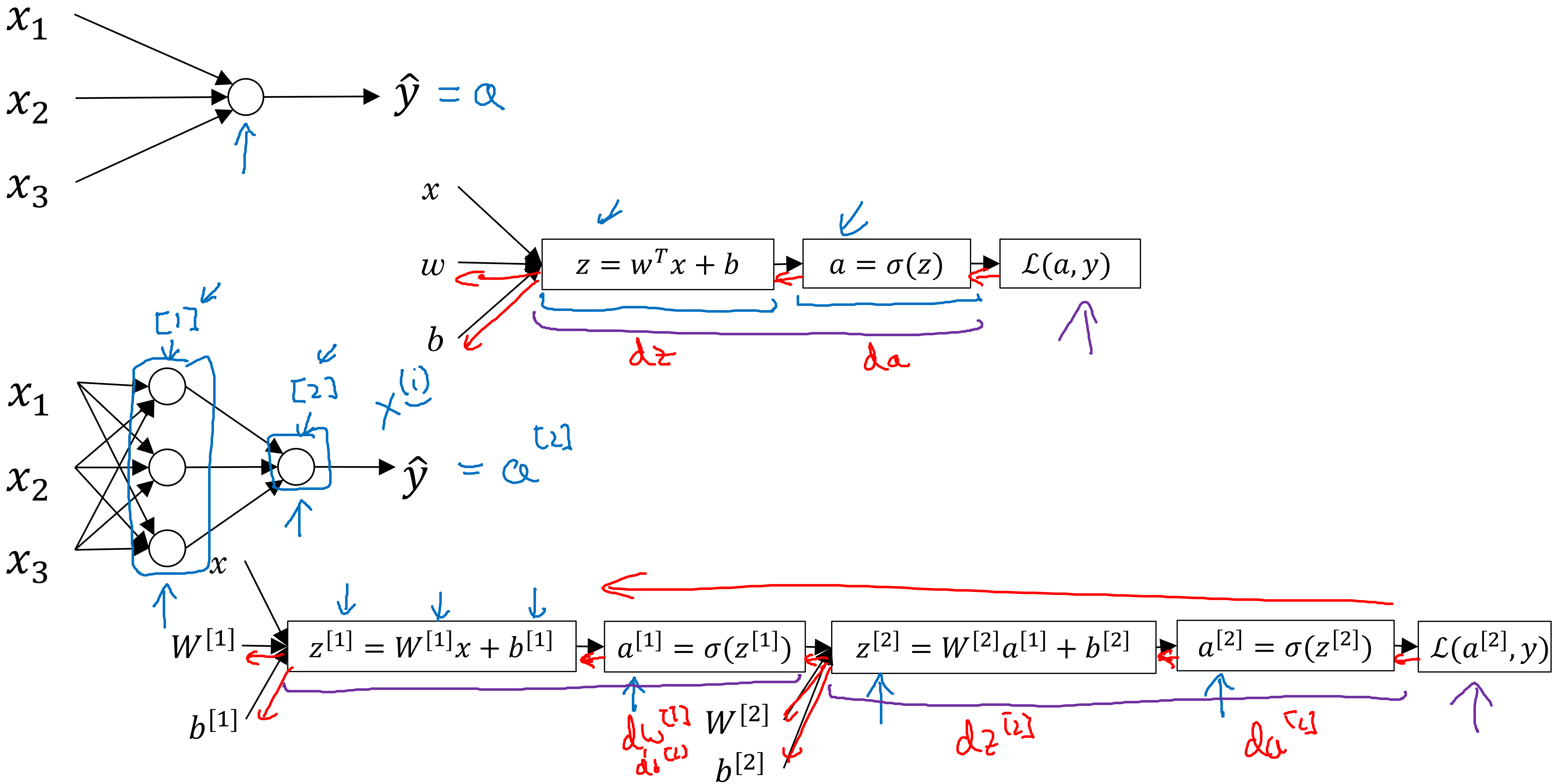
One hidden layer Neural Network

Neural Networks Overview

What is a Neural Network?



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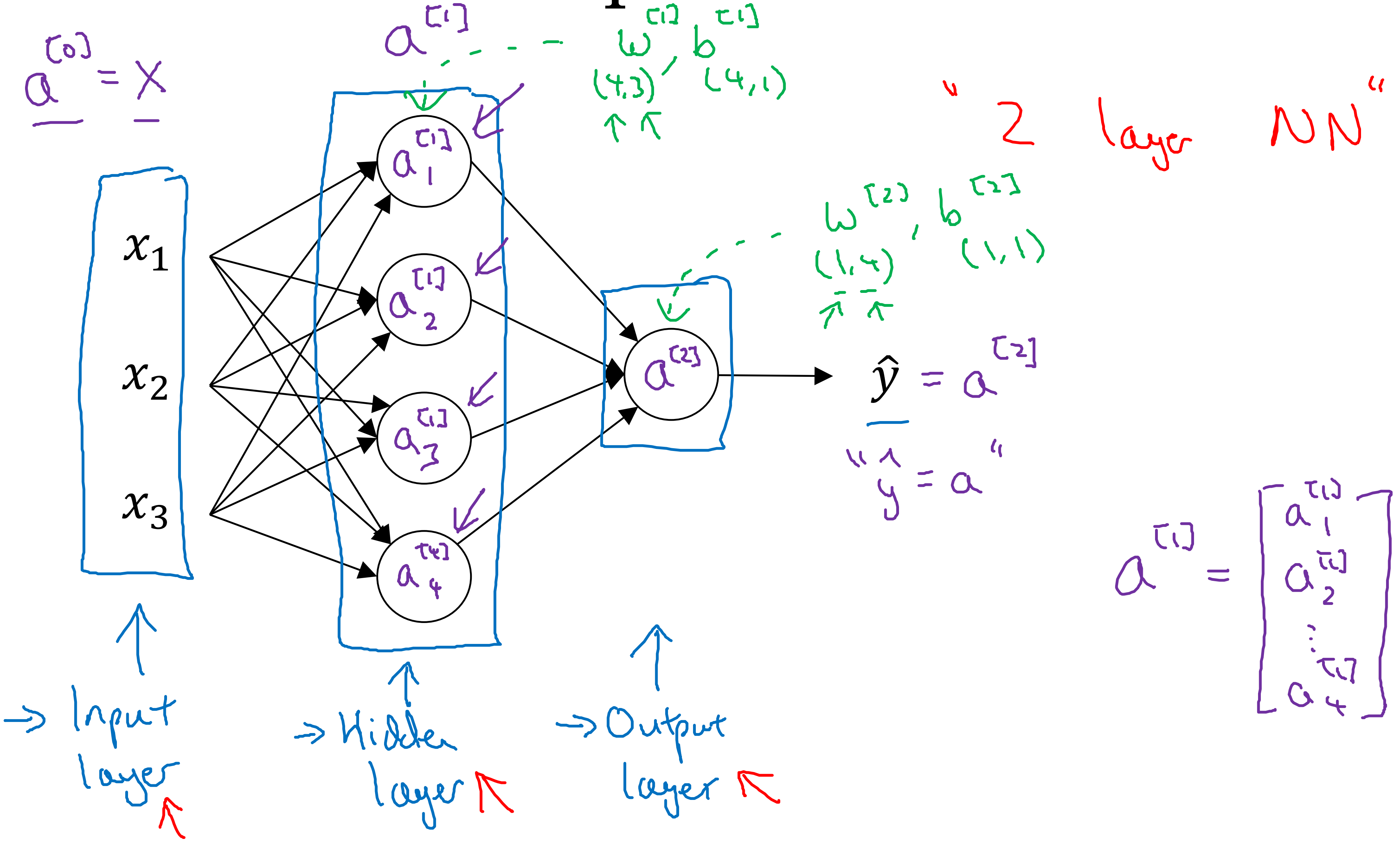
One hidden layer Neural Network

Neural Network Representation

Neural Network Representation



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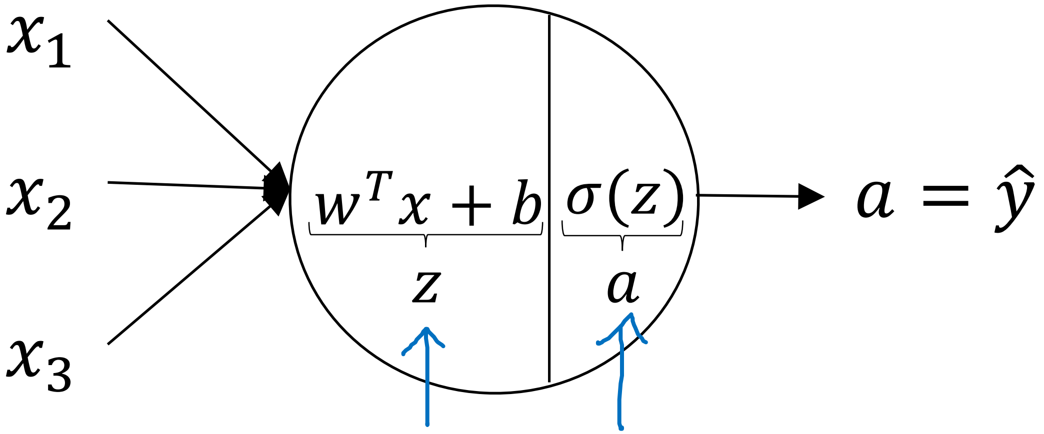
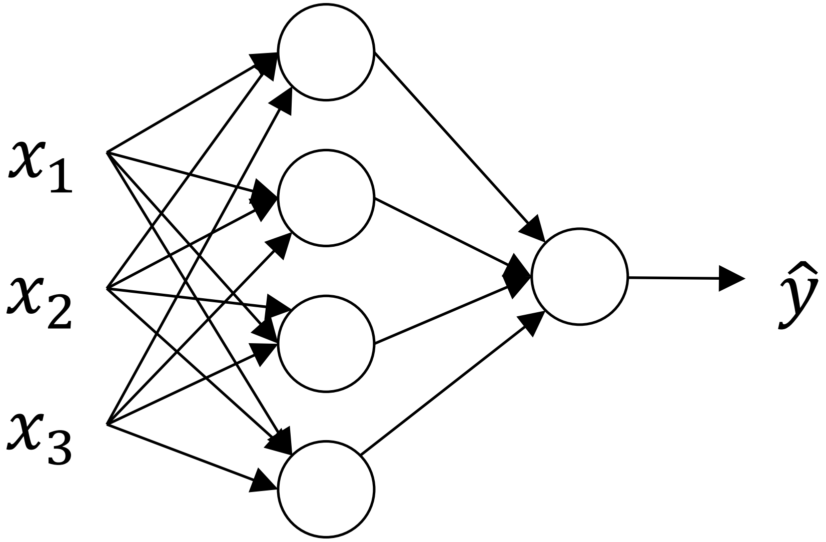
One hidden layer Neural Network

Computing a Neural Network's Output

Neural Network Representation

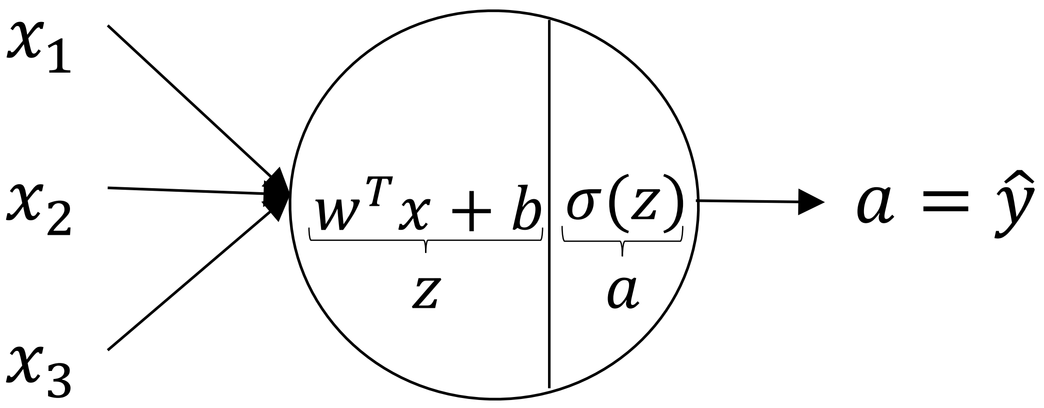




$$z = w^T x + b$$

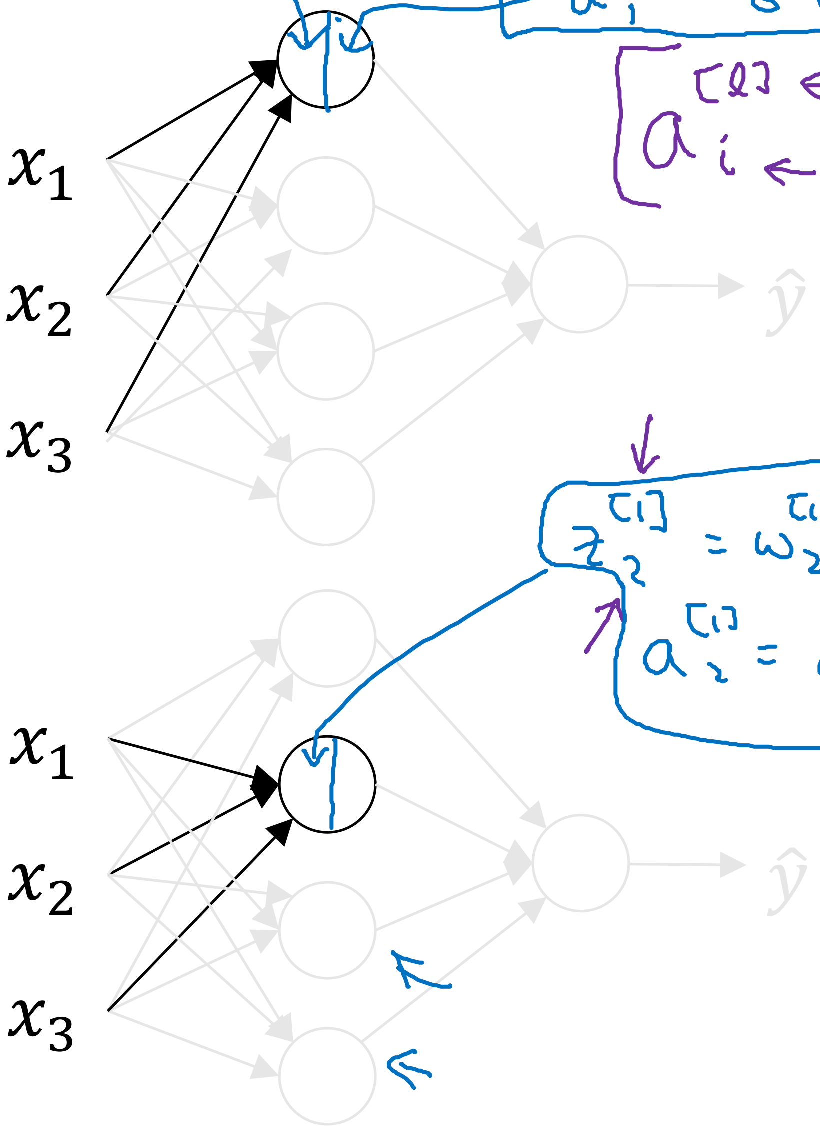
$$a = \sigma(z)$$

Neural Network Representation



$$z = w^T x + b$$

$$a = \sigma(z)$$



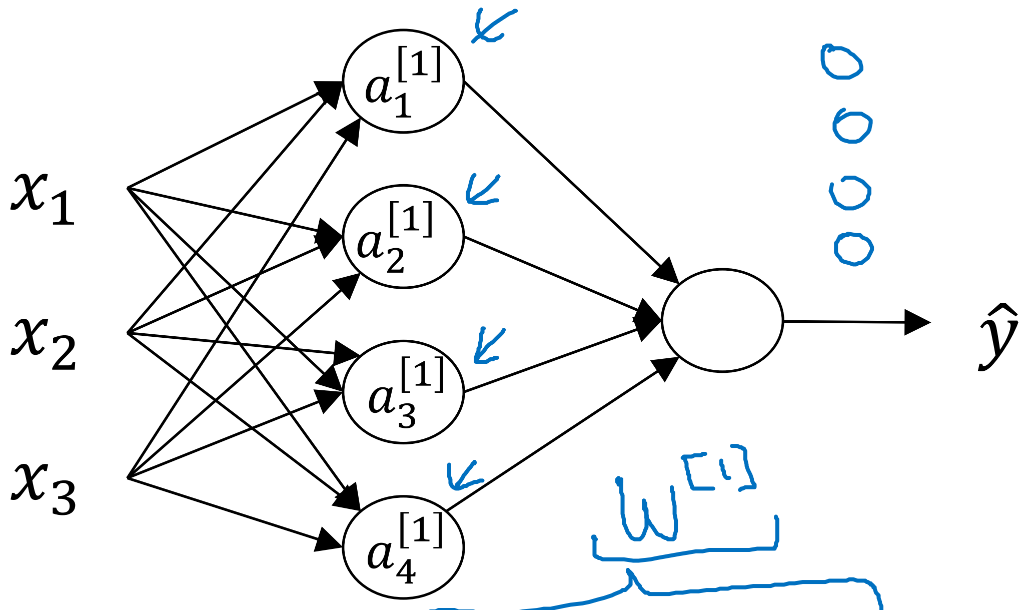
$$\boxed{\begin{aligned} z_1^{[1]} &= w_1^{[1]T} x + b_1^{[1]} \\ a_1^{[1]} &= \sigma(z_1^{[1]}) \end{aligned}}$$

$a_i^{[l]}$ ← layer

← node in layer.

$$\boxed{\begin{aligned} z_2^{[1]} &= w_2^{[1]T} x + b_2^{[1]} \\ a_2^{[1]} &= \sigma(z_2^{[1]}) \end{aligned}}$$

Neural Network Representation



$(w_1^{[1]})^T x$ $\quad a^{[1]}$

$$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]}, \quad a_1^{[1]} = \sigma(z_1^{[1]})$$

$$z_2^{[1]} = w_2^{[1]T} x + b_2^{[1]}, \quad a_2^{[1]} = \sigma(z_2^{[1]})$$

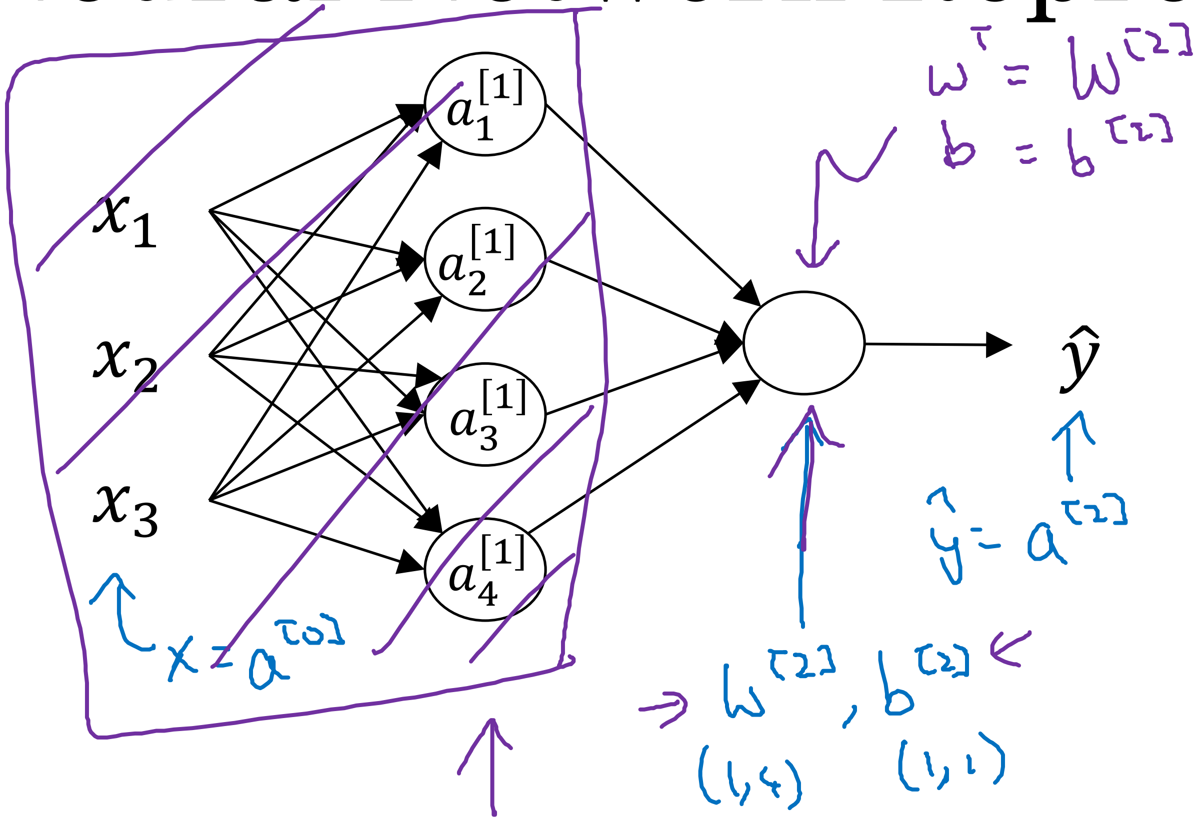
$$z_3^{[1]} = w_3^{[1]T} x + b_3^{[1]}, \quad a_3^{[1]} = \sigma(z_3^{[1]})$$

$$z_4^{[1]} = w_4^{[1]T} x + b_4^{[1]}, \quad a_4^{[1]} = \sigma(z_4^{[1]})$$

$$z^{[1]} = \overbrace{\begin{bmatrix} — w_1^{[1]T} — \\ — w_2^{[1]T} — \\ — w_3^{[1]T} — \\ — w_4^{[1]T} — \end{bmatrix}}^{W^{[1]}}_{(4,3)} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix}}_{b^{[1]}\ (4,1)} = \begin{bmatrix} w_1^{[1]T} x + b_1^{[1]} \\ w_2^{[1]T} x + b_2^{[1]} \\ w_3^{[1]T} x + b_3^{[1]} \\ w_4^{[1]T} x + b_4^{[1]} \end{bmatrix} = \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix}$$

$$a^{[1]} = \begin{bmatrix} a_1^{[1]} \\ \vdots \\ a_4^{[1]} \end{bmatrix} = \sigma(z^{[1]})$$

Neural Network Representation learning



$z = w^T x + b$

$\hat{y} = a = \sigma(z)$

Given input x:

$$\underset{(4,1)}{z^{[1]}} = \underset{(4,3)}{W^{[1]}}\, \underset{(3,1)}{\overset{a^{[0]}}{\cancel{x}}} + \underset{(4,1)}{b^{[1]}}$$

$$\underset{(4,1)}{a^{[1]}} = \sigma(\underset{(4,1)}{z^{[1]}})$$

$$\underset{(1,1)}{z^{[2]}} = \underset{(1,4)}{W^{[2]}}\, \underset{(4,1)}{a^{[1]}} + \underset{(1,1)}{b^{[2]}}$$

$$\underset{(1,1)}{a^{[2]}} = \sigma(\underset{(1,1)}{z^{[2]}})$$

One hidden layer Neural Network

Computing a Neural Network's Output

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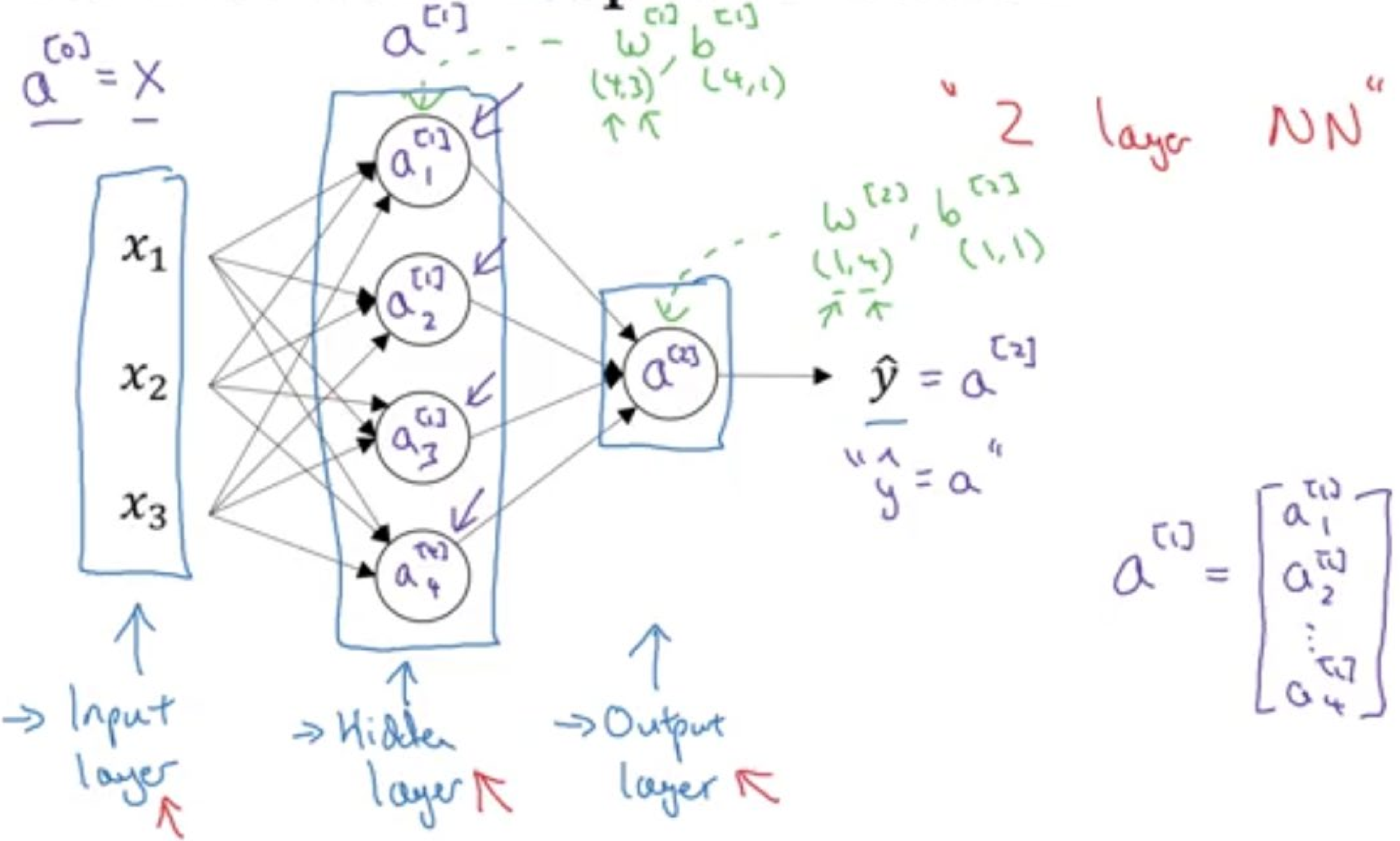
Neural Network Representation



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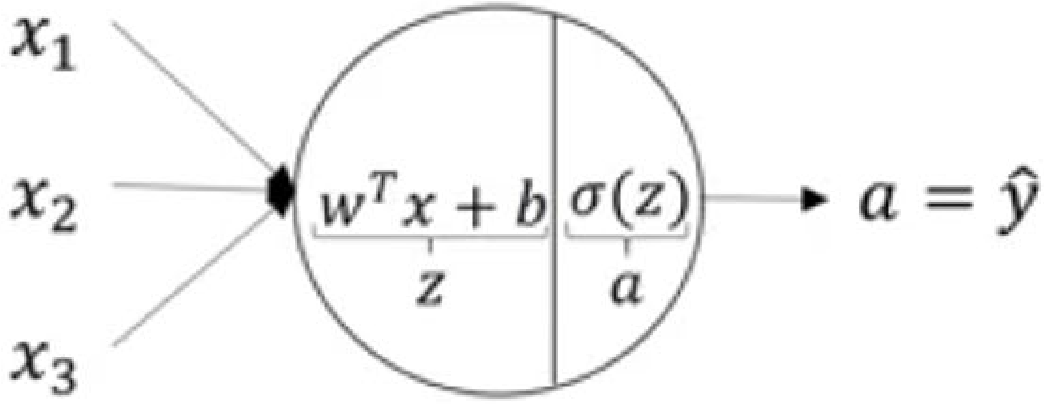
Neural Network Representation



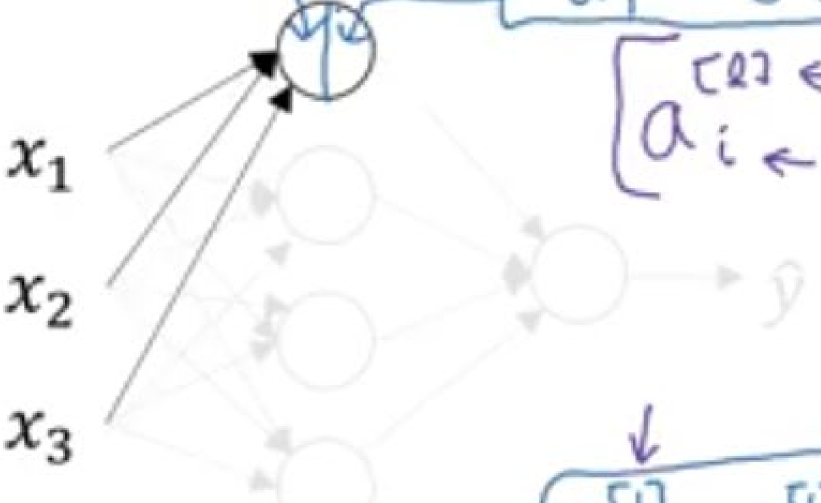
$$z = w^T x + b$$

$$a = \sigma(z)$$

$$z^{[1]}_1 = w^{[1]T}_1 x + b^{[1]}_1$$

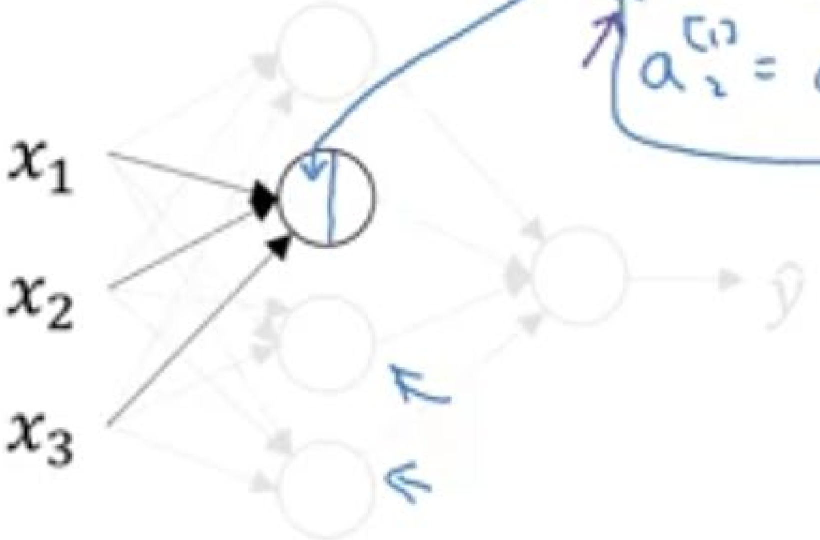
$$a^{[1]}_1 = \sigma(z^{[1]}_1)$$

$a^{[l]}_i$ ← layer, ← node in layer.



$$z^{[1]}_2 = w^{[1]T}_2 x + b^{[1]}_2$$

$$a^{[1]}_2 = \sigma(z^{[1]}_2)$$



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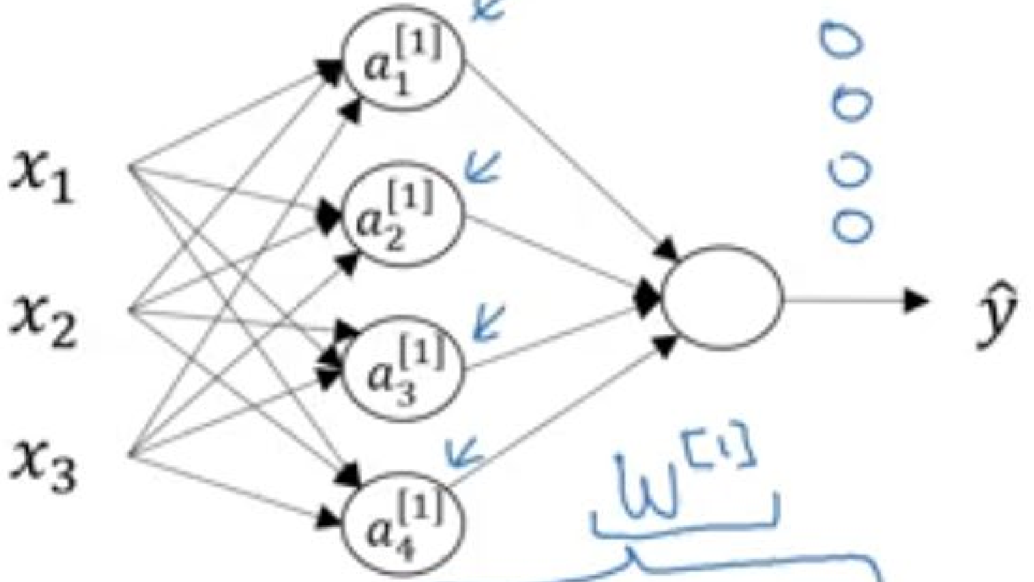
Neural Network Representation



$(w_1^{[1]})^T x \qquad a^{[1]}$

$$
\begin{aligned}
z_1^{[1]} &= w_1^{[1]T} x + b_1^{[1]}, & a_1^{[1]} &= \sigma(z_1^{[1]}) \\
z_2^{[1]} &= w_2^{[1]T} x + b_2^{[1]}, & a_2^{[1]} &= \sigma(z_2^{[1]}) \\
z_3^{[1]} &= w_3^{[1]T} x + b_3^{[1]}, & a_3^{[1]} &= \sigma(z_3^{[1]}) \\
z_4^{[1]} &= w_4^{[1]T} x + b_4^{[1]}, & a_4^{[1]} &= \sigma(z_4^{[1]})
\end{aligned}
$$

$$
z^{[1]} = \overbrace{\underbrace{\begin{bmatrix} — w_1^{[1]T} — \\ — w_2^{[1]T} — \\ — w_3^{[1]T} — \\ — w_4^{[1]T} — \end{bmatrix}}_{(4,3)}}^{W^{[1]}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix}}_{b^{[1]} \ (4,1)} = \begin{bmatrix} w_1^{[1]T} x + b_1^{[1]} \\ w_2^{[1]T} x + b_2^{[1]} \\ w_3^{[1]T} x + b_3^{[1]} \\ w_4^{[1]T} x + b_4^{[1]} \end{bmatrix} = \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix}
$$

$$
a^{[1]} = \begin{bmatrix} a_1^{[1]} \\ \vdots \\ a_4^{[1]} \end{bmatrix} = \sigma(z^{[1]})
$$

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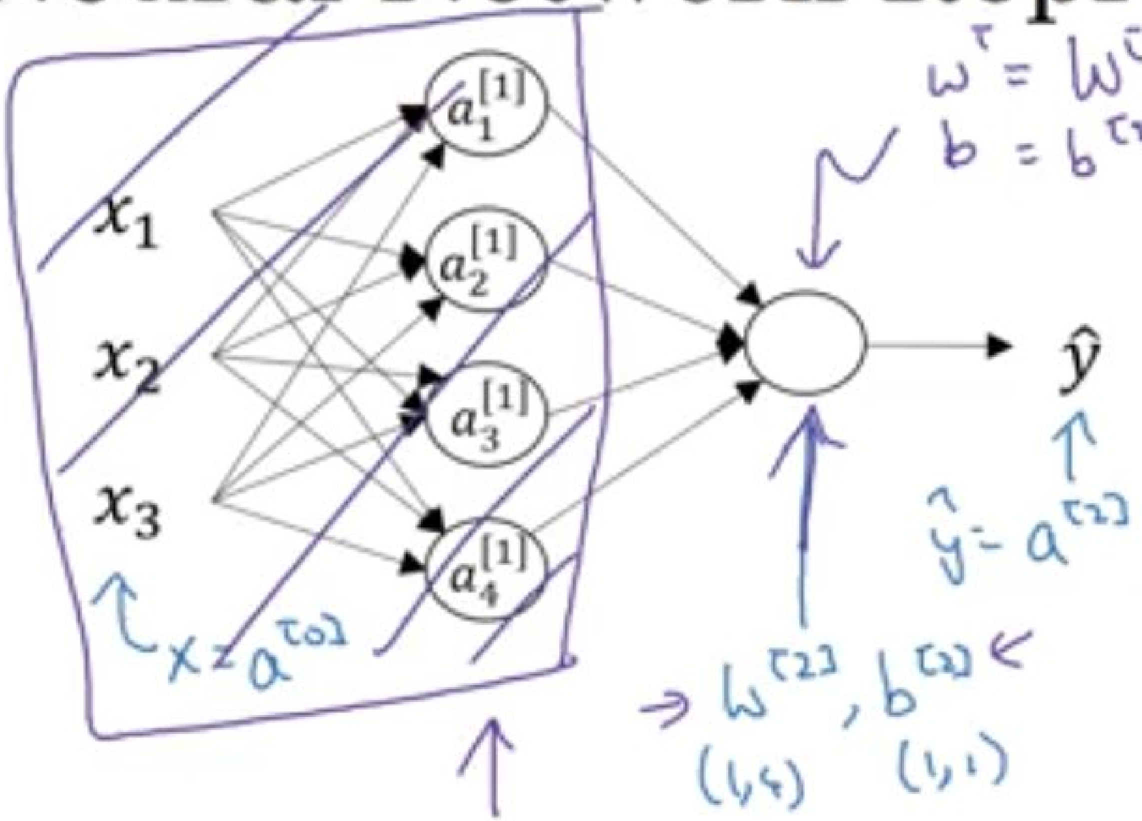
Neural Network Representation learning



$z = w^T x + b$

$\hat{y} = a = \sigma(z)$

Given input x:

$$\underset{(4,1)}{z^{[1]}} = \underset{(4,3)}{W^{[1]}} \underset{(3,1)}{\cancel{x}\, a^{[0]}} + \underset{(4,1)}{b^{[1]}}$$

$$\underset{(4,1)}{a^{[1]}} = \sigma(\underset{(4,1)}{z^{[1]}})$$

$$\underset{(1,1)}{z^{[2]}} = \underset{(1,4)}{W^{[2]}} \underset{(4,1)}{a^{[1]}} + \underset{(1,1)}{b^{[2]}}$$

$$\underset{(1,1)}{a^{[2]}} = \sigma(\underset{(1,1)}{z^{[2]}})$$

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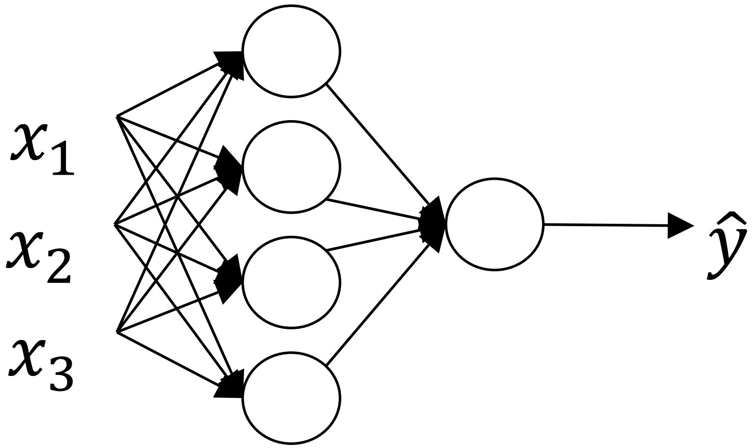
One hidden layer Neural Network

Vectorizing across multiple examples

Vectorizing across multiple examples



$$z^{[1]} = W^{[1]}x + b^{[1]}$$
$$a^{[1]} = \sigma(z^{[1]})$$
$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$
$$a^{[2]} = \sigma(z^{[2]})$$

$X \longrightarrow a^{[2]} = \hat{y}$

$x^{(1)} \longrightarrow a^{[2](1)} = \hat{y}^{(1)}$

$x^{(2)} \longrightarrow a^{[2](2)} \quad \hat{y}^{(2)}$

$\vdots$

$x^{(m)} \longrightarrow a^{[2](m)} \quad \hat{y}^{(m)}$

$a^{[2](i)}$ example i, layer 2

for $i = 1$ to m,

$z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$

$a^{[1](i)} = \sigma(z^{[1](i)})$

$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$

$a^{[2](i)} = \sigma(z^{[2](i)})$

Vectorizing across multiple examples

```
for i = 1 to m:
```

$$\boxed{z^{[1](i)}} = W^{[1]}x^{(i)} + b^{[1]}$$

$$\boxed{a^{[1](i)}} = \sigma(z^{[1](i)})$$

$$\boxed{z^{[2](i)}} = W^{[2]}a^{[1](i)} + b^{[2]}$$

$$\boxed{a^{[2](i)}} = \sigma(z^{[2](i)})$$

$$\boxed{\begin{aligned} Z^{[1]} &= W^{[1]}X + b^{[1]} \\ A^{[1]} &= \sigma(Z^{[1]}) \\ Z^{[2]} &= W^{[2]}A^{[1]} + b^{[2]} \\ A^{[2]} &= \sigma(Z^{[2]}) \end{aligned}}$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix} \quad (n_x, m)$$

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

training examples

hidden units

hidden units

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One hidden layer Neural Network

Explanation for vectorized implementation

Justification for vectorized implementation

$$z^{[1](1)} = W^{[1]}x^{(1)} + \cancel{b^{[1]}} \to 0, \quad z^{[1](2)} = W^{[1]}x^{(2)} + \cancel{b^{[1]}} \to 0, \quad z^{[1](3)} = W^{[1]}x^{(3)} + \cancel{b^{[1]}} \to 0$$

$$W^{[1]} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} \qquad W^{[1]}x^{(1)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad W^{[1]}x^{(2)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad W^{[1]}x^{(3)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

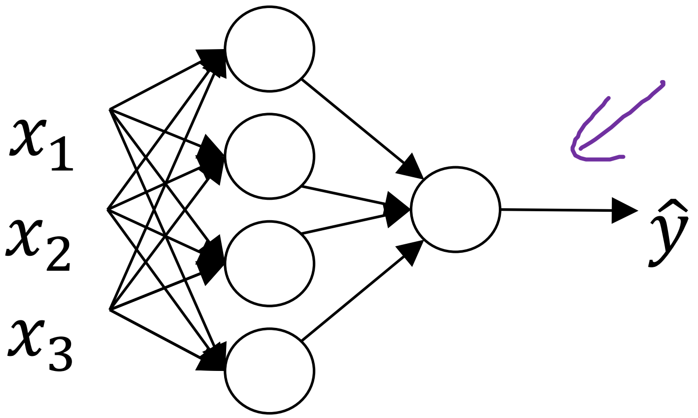
$$W^{[1]} \underbrace{\begin{bmatrix} | & | & | & \\ x^{(1)} & x^{(2)} & x^{(3)} & \cdots \\ | & | & | & \end{bmatrix}}_{X} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} | & | & | & \\ z^{[1](1)} & z^{[1](2)} & z^{[1](3)} & \cdots \\ | & | & | & \end{bmatrix} = Z^{[1]}$$

$$+b^{[1]} \quad +b^{[1]} \quad +b^{[1]}$$

$$W^{[1]}x^{(i)} = z^{[1](i)}$$

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

Recap of vectorizing across multiple examples



$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

for i = 1 to m

$$\boxed{z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}}$$
$$a^{[1](i)} = \sigma(z^{[1](i)})$$
$$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$
$$a^{[2](i)} = \sigma(z^{[2](i)})$$

$A^{[0]}$ $\quad X = a^{[0]}$ $\quad x^{(i)} = a^{[0](i)}$

$$Z^{[1]} = W^{[1]}X + b^{[1]} \leftarrow W^{[1]}A^{[0]} + b^{[1]}$$
$$A^{[1]} = \sigma(Z^{[1]})$$
$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$
$$A^{[2]} = \sigma(Z^{[2]})$$

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One hidden layer Neural Network

Explanation for vectorized implementation

Justification for vectorized implementation

$$z^{[1](1)} = W^{[1]}x^{(1)} + \cancel{b^{[1]}} \to 0, \quad z^{[1](2)} = W^{[1]}x^{(2)} + \cancel{b^{[1]}} \to 0, \quad z^{[1](3)} = W^{[1]}x^{(3)} + \cancel{b^{[1]}} \to 0$$

$$W^{[1]} = \begin{bmatrix} \text{———} \\ \text{———} \\ \text{———} \\ \text{———} \end{bmatrix} \qquad W^{[1]}x^{(1)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad W^{[1]}x^{(2)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad W^{[1]}x^{(3)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

$$W^{[1]} \underbrace{\begin{bmatrix} | & | & | & \\ x^{(1)} & x^{(2)} & x^{(3)} & \cdots \\ | & | & | & \end{bmatrix}}_{X} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} | & | & | & \\ z^{[1](1)} & z^{[1](2)} & z^{[1](3)} & \cdots \\ | & | & | & \end{bmatrix} = Z^{[1]}$$

$$+b^{[1]} \quad +b^{[1]} \quad +b^{[1]}$$

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$W^{[1]}x^{(i)} = z^{[1](i)}$$

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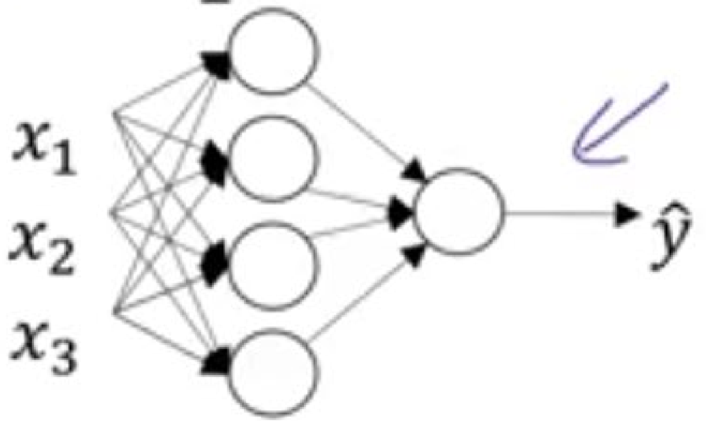
Recap of vectorizing across multiple examples



$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

```
for i = 1 to m
```

$$\boxed{z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}}$$
$$a^{[1](i)} = \sigma(z^{[1](i)})$$
$$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$
$$a^{[2](i)} = \sigma(z^{[2](i)})$$

$A^{[0]}$ $\quad X = a^{[0]}$ $\quad x^{(i)} = a^{[0](i)}$

$$Z^{[1]} = W^{[1]}X + b^{[1]} \leftarrow W^{[1]}A^{[0]} + b^{[1]}$$
$$A^{[1]} = \sigma(Z^{[1]})$$
$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$
$$A^{[2]} = \sigma(Z^{[2]})$$

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One hidden layer Neural Network

Activation functions

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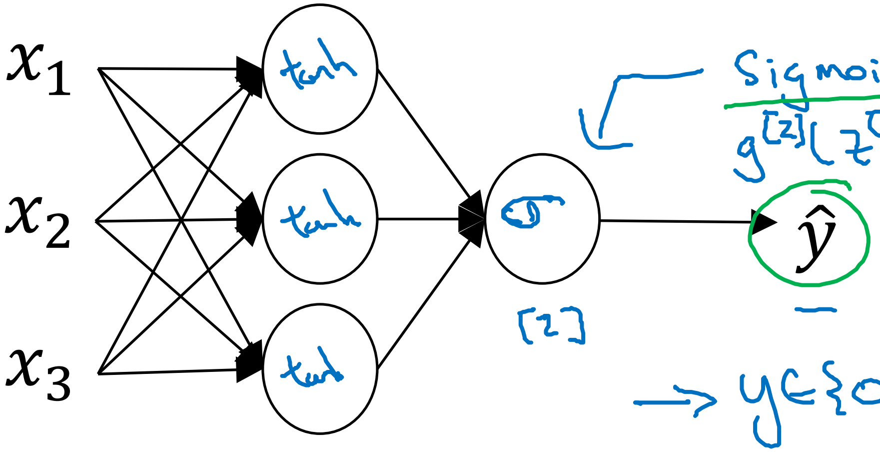
Activation functions

$g^{[1]}(z^{[1]}) = \tanh(z^{[1]})$

Sigmoid

$g^{[2]}(z^{[2]}) = \sigma(z^{[2]})$



$\rightarrow y \in \{0, 1\}$

$0 \le \hat{y} \le 1$

-1 1

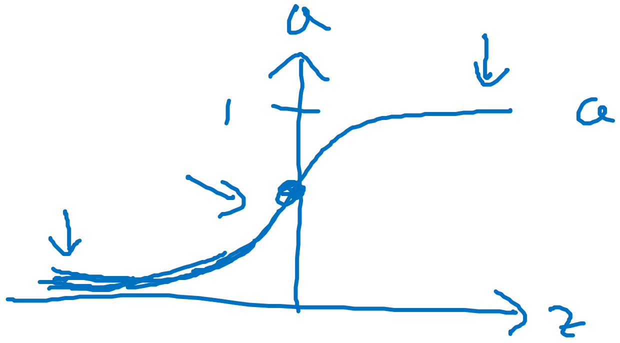
Given x:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

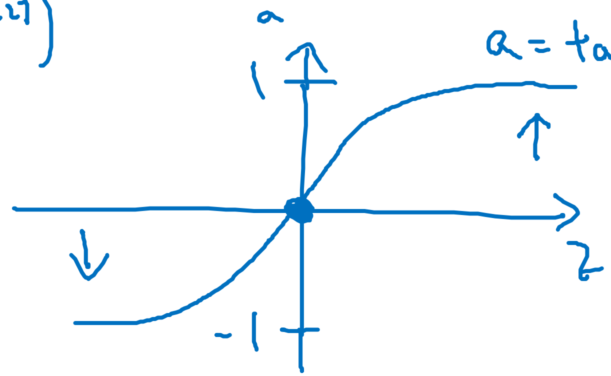
$$\rightarrow a^{[1]} = \cancel{\sigma(z^{[1]})}\ g^{[1]}(z^{[1]})$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

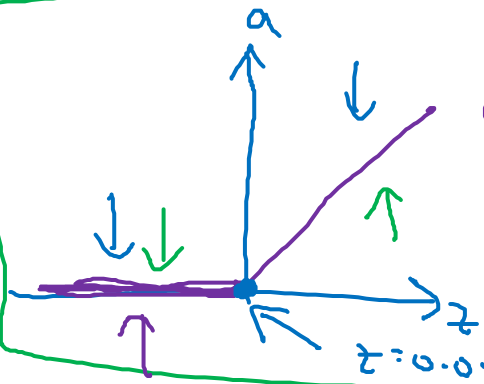
$$\rightarrow a^{[2]} = \cancel{\sigma(z^{[2]})}\ g^{[2]}(z^{[2]})$$



$a = \frac{1}{1+e^{-z}}$



$a = \tanh(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$



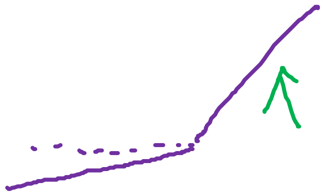
$z > 0$

$a = \max(0, z)$

ReLU

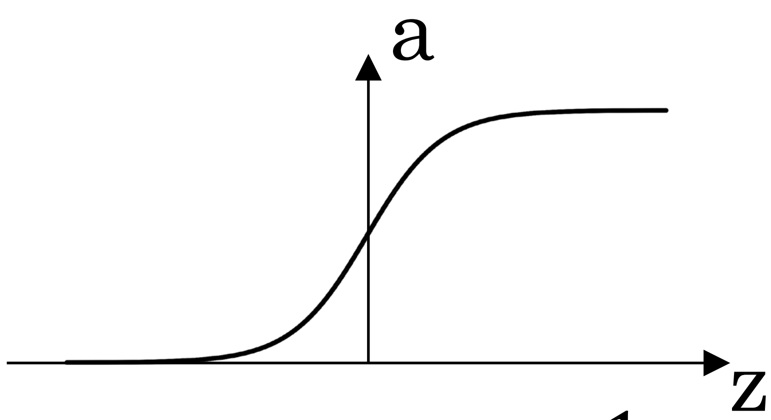
Rectified Linear Unit

$z = 0.0000000000000$

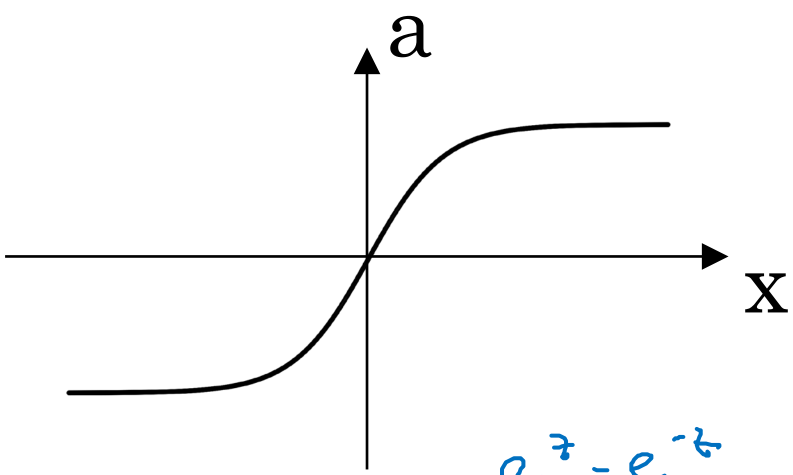


Leaky ReLU

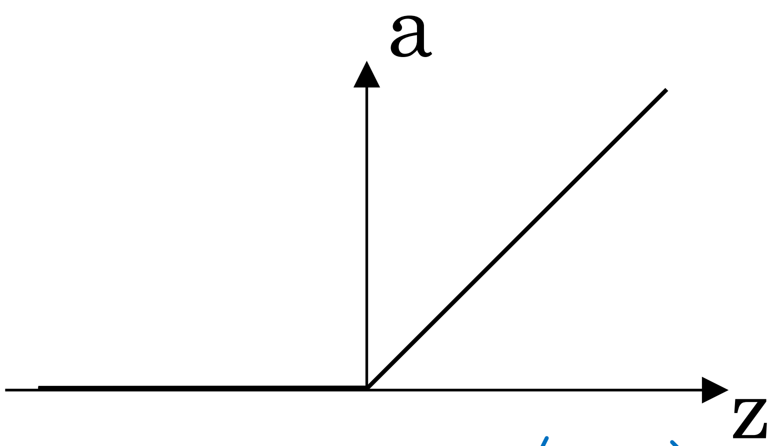
Pros and cons of activation functions



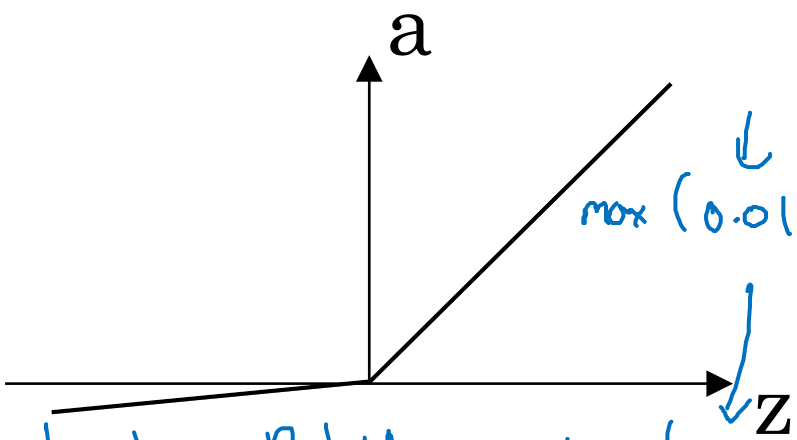
sigmoid: $a = \dfrac{1}{1 + e^{-z}}$



tanh: $a = \dfrac{e^{z} - e^{-z}}{e^{z} + e^{z}}$



ReLU $a = \max(0, z)$



$\max(0.01z, z)$

Leaky ReLU $a = \max(0.01z, z)$

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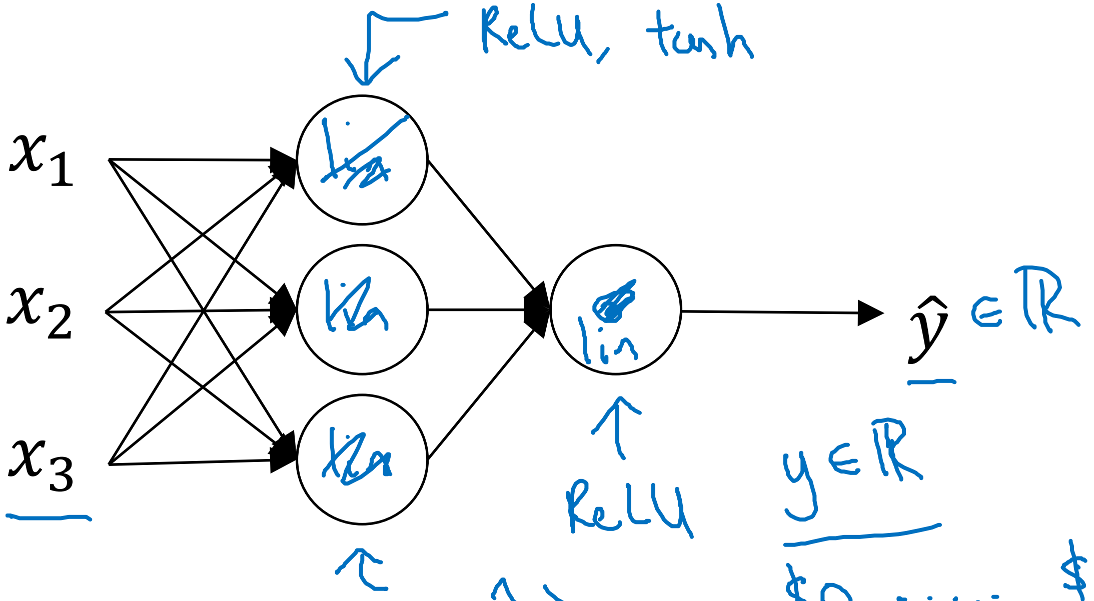
One hidden layer Neural Network

Why do you need non-linear activation functions?

Activation function



$y \in \mathbb{R}$

\$0 ..... \$1,000,000s

$\hat{y} \geq 0$.

Given x:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[1]} = \cancel{g^{[1]}(z^{[1]})}\ z^{[1]}$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = \cancel{g^{[2]}(z^{[2]})}\ z^{[2]}$$

$g(z) = z$

"linear activation function"

$$a^{[1]} = z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[2]} = z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = W^{[2]}\underbrace{(W^{[1]}x + b^{[1]})}_{a^{[1]}} + b^{[2]}$$

$$= \underbrace{(W^{[2]}W^{[1]})}_{W'}x + \underbrace{(W^{[2]}b^{[1]} + b^{[2]})}_{b'}$$

$$= W'x + b'$$

$g(z) = z$

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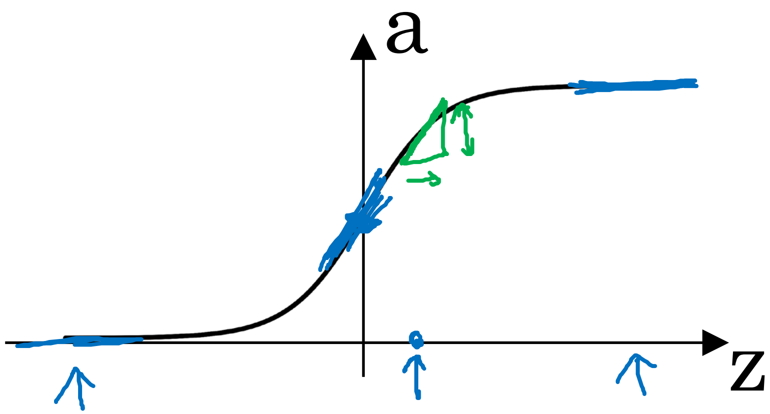
One hidden layer Neural Network

Derivatives of activation functions

Sigmoid activation function



$$g(z) = \frac{1}{1+e^{-z}}$$

$$a = g(z) = \frac{1}{1+e^{-z}}$$

$$\boxed{g'(z)} = \boxed{\frac{d}{dz} g(z)} = \text{slope of } g(x) \text{ at } z$$

$$= \frac{1}{1+e^{-z}}\left(1 - \frac{1}{1+e^{-z}}\right)$$

$$= \underbrace{g(z)\,(1-g(z))}$$

$$= \boxed{a\,(1-a)}$$

$$g'(z) = a(1-a)$$

$z = 10$. $g(z) \approx 1$

$$\frac{d}{dz} g(z) \approx 1\,(1-1) \approx 0$$

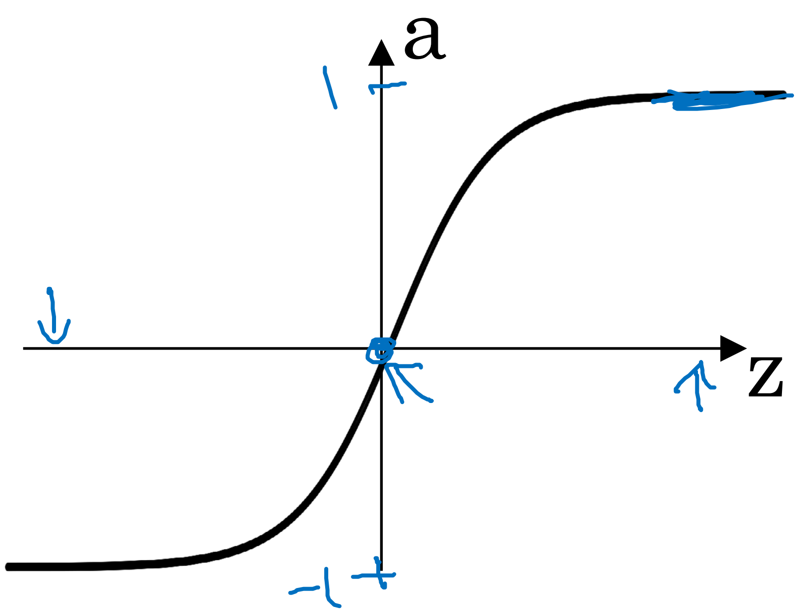
$z = -10$ $g(z) \approx 0$

$$\frac{d}{dz} g(z) \approx 0 \cdot (1-0) \approx 0$$

$z = 0$ $g(z) = \frac{1}{2}$

$$\frac{d}{dz} g(z) = \frac{1}{2}\left(1-\frac{1}{2}\right) = \frac{1}{4}$$

Tanh activation function



$$g(z) = \tanh(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

$$g'(z) = \frac{d}{dz} g(z) = \text{slope of } g(z) \text{ at } z$$
$$= 1 - (\tanh(z))^2$$

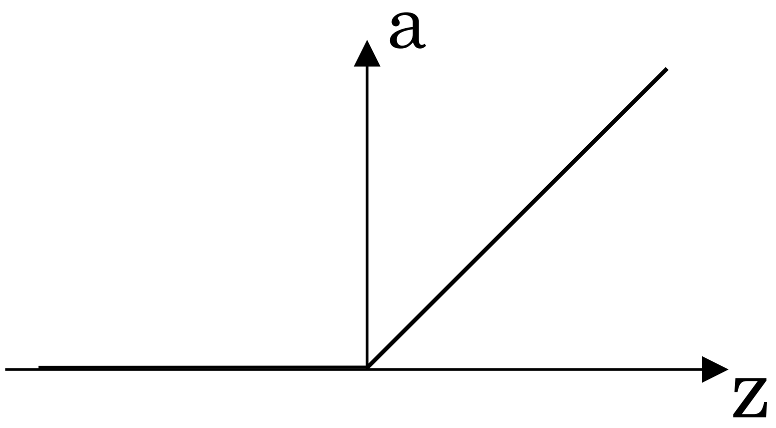
$z = 10$ $\tanh(z) \approx 1$
$g'(z) \approx 0$

$z = -10$ $\tanh(z) \approx -1$
$g'(z) \approx 0$

$z = 0$ $\tanh(z) = 0$
$g'(z) = 1$

$a = g(z)$, $g'(z) = 1 - a^2$

ReLU and Leaky ReLU

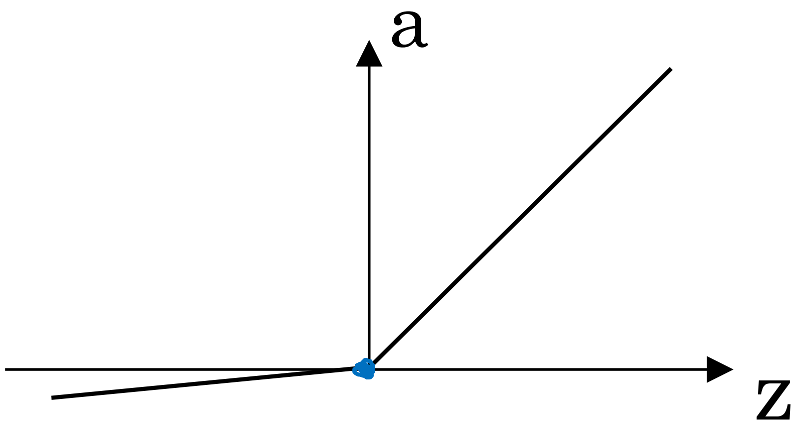


ReLU

$$g(z) = \max(0, z)$$

$$g'(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \\ \cancel{\text{undef} \quad \text{if } z = 0} \end{cases}$$

$$z = 0.00000\ldots\ldots 0$$



Leaky ReLU

$$g(z) = \max(0.01z, z)$$

$$g'(z) = \begin{cases} 0.01 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$$

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One hidden layer Neural Network

Gradient descent for neural networks

Gradient descent for neural networks

Parameters: $\underset{(n^{[1]}, n^{[0]})}{W^{[1]}}, \underset{(n^{[1]}, 1)}{b^{[1]}}, \underset{(n^{[2]}, n^{[1]})}{W^{[2]}}, \underset{(n^{[2]}, 1)}{b^{[2]}}$ $\quad n_x = n^{[0]}, n^{[1]}, \underline{n^{[2]} = 1}$

Cost function: $J(W^{[1]}, b^{[1]}, \underline{W^{[2]}}, \underline{b^{[2]}}) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$

$\hat{y} = a^{[2]}$

Gradient descent:

→ Repeat {

→ Compute predicts $(\hat{y}^{(i)}, i = 1 \ldots, m)$

$\underline{dW^{[1]}} = \frac{\partial J}{\partial W^{[1]}}, \quad \underline{db^{[1]}} = \frac{\partial J}{\partial b^{[1]}}, \ldots$

$W^{[1]} := W^{[1]} - \alpha \, dW^{[1]}$

$b^{[1]} := b^{[1]} - \alpha \, db^{[1]}$

$W^{[2]} := \ldots \quad b^{[2]} := \ldots$

}

Formulas for computing derivatives

Forward propagation:

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]})$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = g^{[2]}(Z^{[2]}) = \sigma(Z^{[2]})$$

Back propagation:

$$Y = [y^{(1)} \ y^{(2)} \dots, y^{(m)}]$$

$$dZ^{[2]} = A^{[2]} - Y$$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T}$$

$$db^{[2]} = \frac{1}{m} \text{np.sum}(dZ^{[2]}, \text{axis}=1, \text{keepdims}=\text{True})$$

$(n^{[2]},) \leftarrow$

$(n^{[2]}, 1) \leftarrow$

$$dZ^{[1]} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]}, m)} * \underbrace{g^{[1]\prime}(Z^{[1]})}_{(n^{[1]}, m)}$$

element-wise product

$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$$

$$db^{[1]} = \frac{1}{m} \text{np.sum}(dZ^{[1]}, \text{axis}=1, \text{keepdims}=\text{True})$$

$(n^{[1]}, 1)$ $(n^{[1]},)$ reshape

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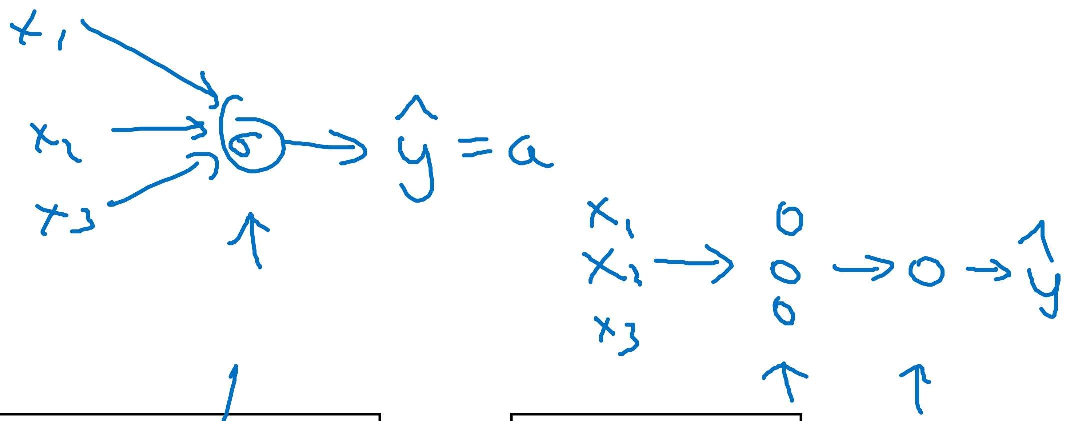
One hidden layer Neural Network

Backpropagation intuition (Optional)

Computing gradients

Logistic regression



$$
\begin{array}{l}
x \\
w \\
b
\end{array}
\rightarrow \boxed{z = w^T x + b} \rightarrow \boxed{a = \sigma(z)} \rightarrow \boxed{\mathcal{L}(a, y)}
$$

$dz = a - y$

$da = \frac{d}{da}\mathcal{L}(a,y) = -y\log a - (1-y)\log(1-a)$

$= -\frac{y}{a} + \frac{1-y}{1-a}$

$dw = dz \cdot x$

$db = dz$

$\boxed{dz = da \cdot g'(z)}$

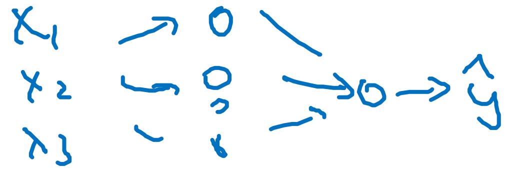
$g(z) = \sigma(z)$

$\boxed{\frac{\partial \mathcal{L}}{\partial z} = \frac{\partial \mathcal{L}}{\partial a} \cdot \boxed{\frac{da}{dz}}}$

"dz" = "da"

$\frac{d}{dz} g(z) = g'(z)$

Neural network gradients



$n_x = n^{[0]}$ $\quad n^{[1]}$ $\quad n^{[2]} = 1$

x

$W^{[2]}$ $\quad dW^{[2]}$

$b^{[2]}$ $\quad db^{[2]}$

$W^{[1]}$, $b^{[1]}$ → $\boxed{z^{[1]} = W^{[1]}x + b^{[1]}}$ → $\boxed{a^{[1]} = \sigma(z^{[1]})}$ → $\boxed{z^{[2]} = W^{[2]}x + b^{[2]}}$ → $\boxed{a^{[2]} = \sigma(z^{[2]})}$ → $\boxed{\mathcal{L}(a^{[2]}, y)}$

$dW^{[1]}$

$db^{[1]}$

$dz^{[1]} = W^{[2]T} dz^{[2]}$ $\quad da^{[1]} = \ldots$ $\quad dz^{[2]} = a^{[2]} - y$ $\quad da^{[2]}$

$* \, g^{[1]\prime}(z^{[1]})$

element wise product

$dW^{[2]} = dz^{[2]} a^{[1]T}$ $\quad$ → "$dw = dz \cdot x$"

$db^{[2]} = dz^{[2]}$

$W^{[1]}: [\;\_\_\_\_\;]$

$W^{[2]}$ $\quad (n^{[2]}, n^{[1]})$

→ $z^{[2]}, dz^{[2]}$ $\quad (n^{[2]}, 1)$ $\quad - (1,1)$

→ $z^{[1]}, dz^{[1]}$ $\quad (n^{[1]}, 1)$

foo $\quad W^{[1]}$ $\quad dW^{[1]} = dz^{[1]} \cdot x^T$

dfoo $\quad dW^{[2]}$ $\quad db^{[1]} = dz^{[1]}$

$a^{[0]T}$

$$dz^{[1]} = \boxed{W^{[2]T} dz^{[2]}} * \boxed{g^{[1]\prime}(z^{[1]})}$$

$$(n^{[1]}, 1) = (n^{[1]}, n^{[2]}) \; (n^{[2]}, 1) * (n^{[1]}, 1)$$

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Summary of gradient descent

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]} a^{[1]^T}$$

$$db^{[2]} = dz^{[2]}$$

$$dz^{[1]} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(\mathrm{z}^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T$$

$$db^{[1]} = dz^{[1]}$$

Vectorized Implementation:

$$z^{[1]} = W^{[1]} x + b^{[1]}$$

$$a^{[1]} = g^{[1]}(z^{[1]})$$

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$Z^{[1]} = W^{[1]} X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]})$$

Summary of gradient descent

$$\underline{dz^{[2]}} = \underline{a^{[2]}} - \underline{y}$$

$$dW^{[2]} = dz^{[2]} a^{[1]^T}$$

$$db^{[2]} = dz^{[2]}$$

$$\underbrace{dz^{[1]}}_{(n^{[1]},1)} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(z^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T$$

$$db^{[1]} = dz^{[1]}$$

$$\underline{dZ^{[2]} = A^{[2]} - Y}$$

$$J(\ldots) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]^T}$$

$$db^{[2]} = \frac{1}{m} np.sum(dZ^{[2]}, axis = 1, keepdims = True)$$

elementwise product

$$\underbrace{dZ^{[1]}}_{(n^{[1]},m)} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]},m)} * \underbrace{g^{[1]\prime}(Z^{[1]})}_{(n^{[1]},m)}$$

$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$$

$$db^{[1]} = \frac{1}{m} np.sum(dZ^{[1]}, axis = 1, keepdims = True)$$

One hidden layer Neural Network

Random Initialization

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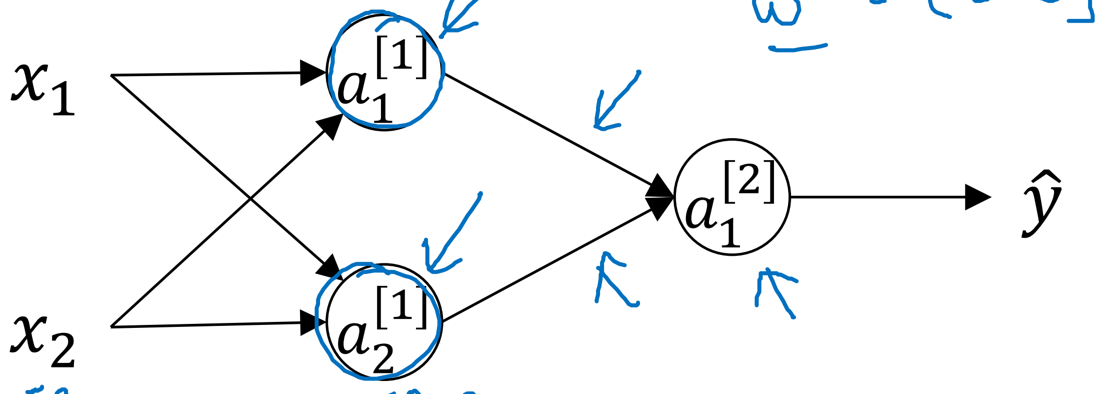
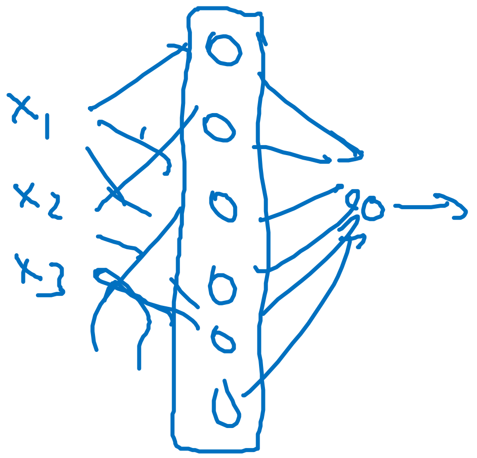
What happens if you initialize weights to zero?

$W^{[2]} = [0 \quad 0]$



$n^{[0]} = 2$ $\qquad$ $n^{[1]} = 2$

Symmetric



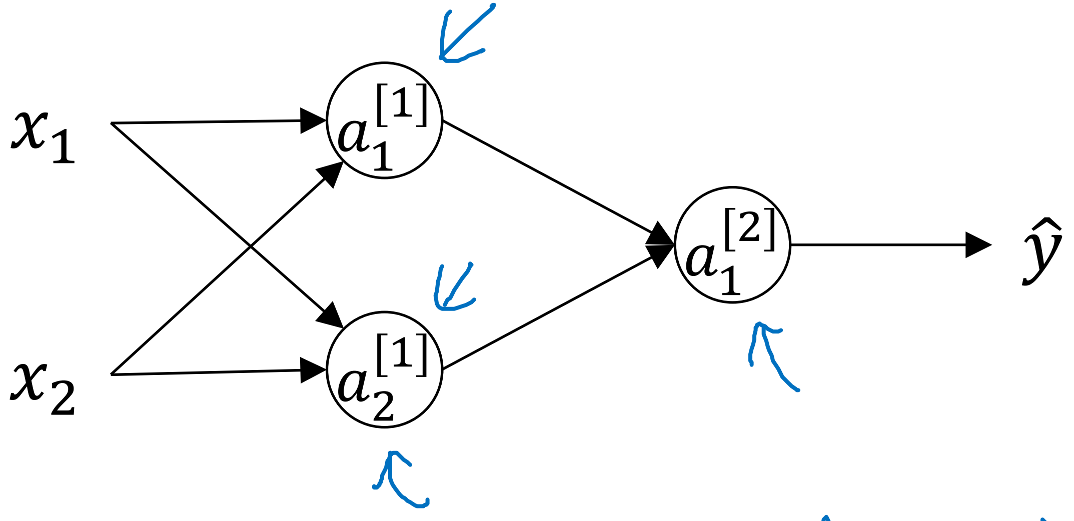
$W^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\qquad$ $b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$W^{[1]} = \begin{bmatrix} \cdots\cdots \\ \cdots\cdots \end{bmatrix}$

$a_1^{[1]} = a_2^{[1]}$ $\qquad$ $dz_1^{[1]} = dz_2^{[1]}$

$dW = \begin{bmatrix} u & v \\ u & v \end{bmatrix}$ $\qquad$ $W^{[1]} = W^{[1]} - \alpha \, dW$

Random initialization



$W^{[1]} =$ np.random.randn((2,2)) $* 0.01$

100?

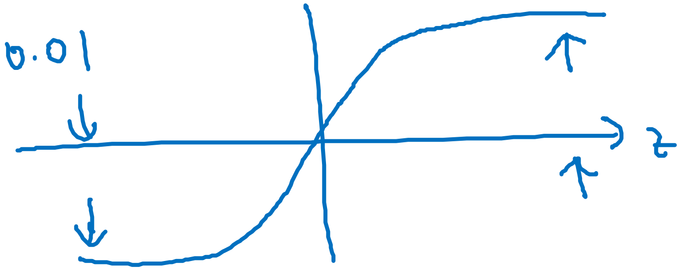
$b^{[1]} =$ np.zeros((2,1))

$W^{[2]} =$ np.random.randn((1,2)) $* 0.01$

$b^{[2]} = 0$

$z^{[1]} = W^{[1]} x + b^{[1]}$

$a^{[1]} = g^{[1]}(z^{[1]})$



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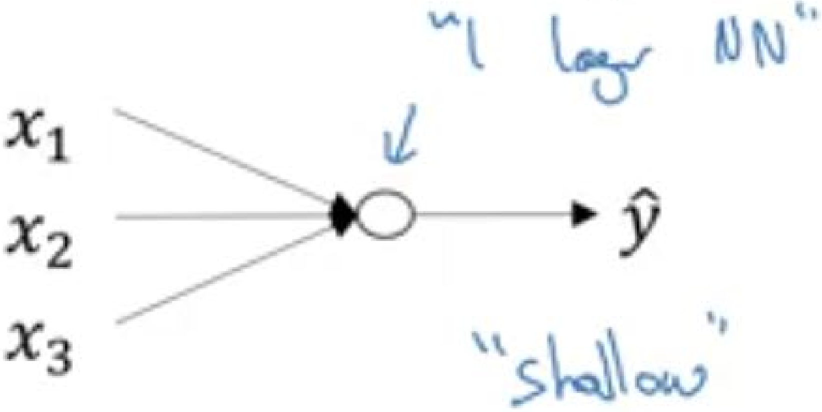
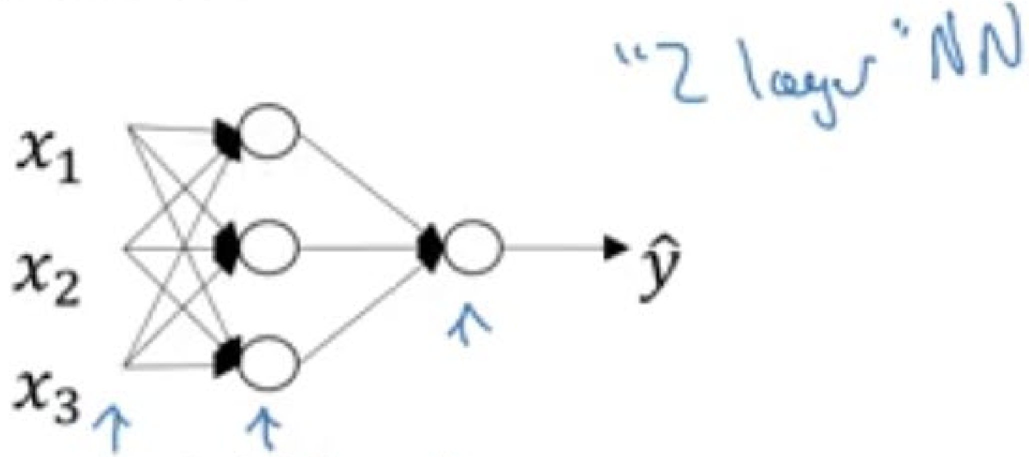
Deep Neural Networks

Deep L-layer Neural network

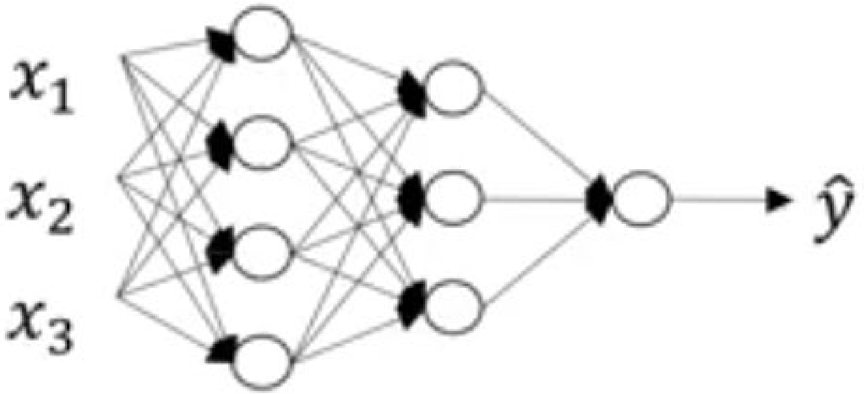
What is a deep neural network?



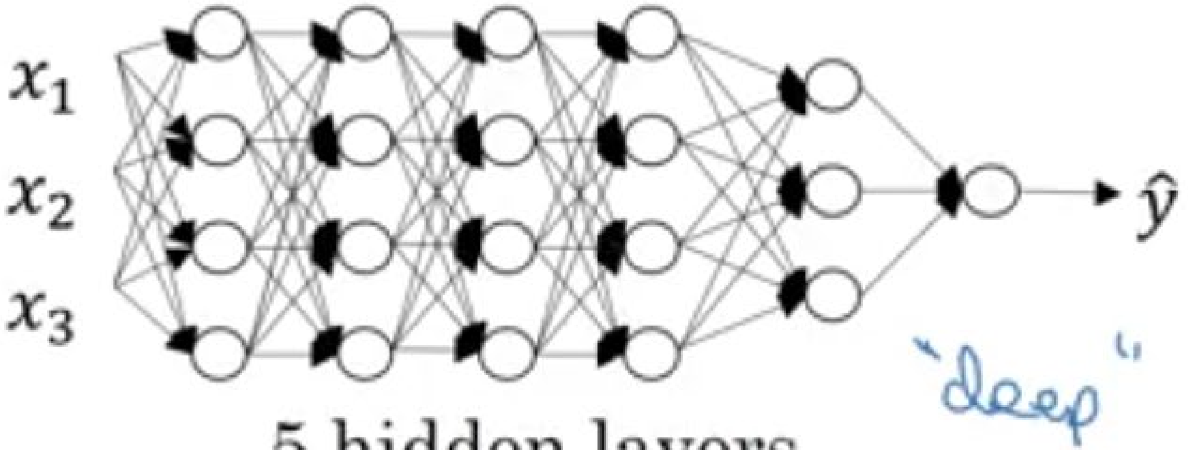
logistic regression



1 hidden layer



2 hidden layers



5 hidden layers

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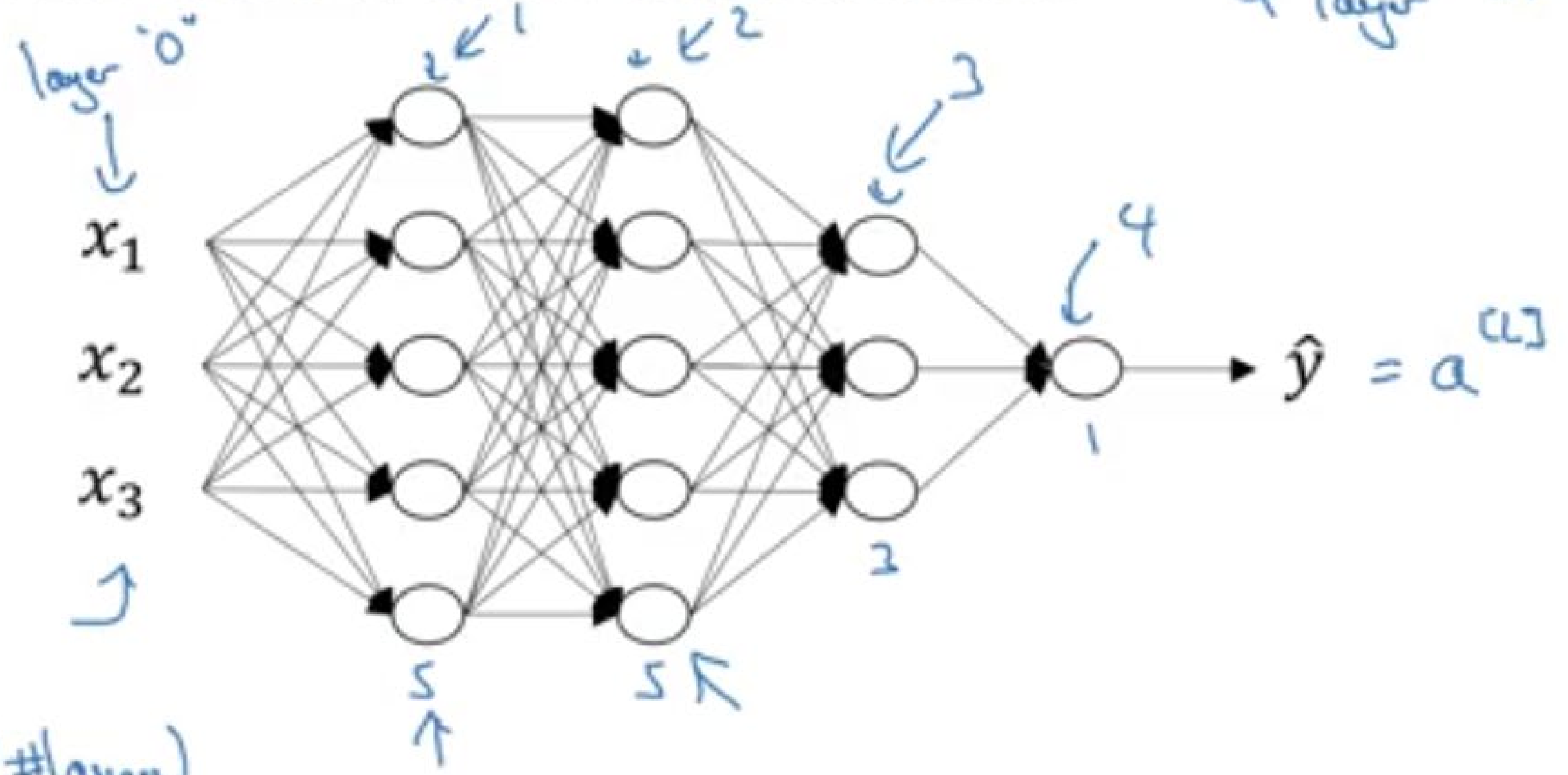
Deep neural network notation

4 layer NN

$X = a^{[0]}$



$L = 4$ (#layers)

$n^{[l]}$ = #units in layer l

$n^{[1]} = 5,\ n^{[2]} = 5,\ n^{[3]} = 3,\ n^{[4]} = n^{[L]} = 1$

$n^{[0]} = n_x = 3$

$a^{[l]}$ = activations in layer l

$a^{[l]} = g^{[l]}(z^{[l]})$, $W^{[l]}$ = weights for $\underline{z^{[l]}}$

$b^{[l]}$

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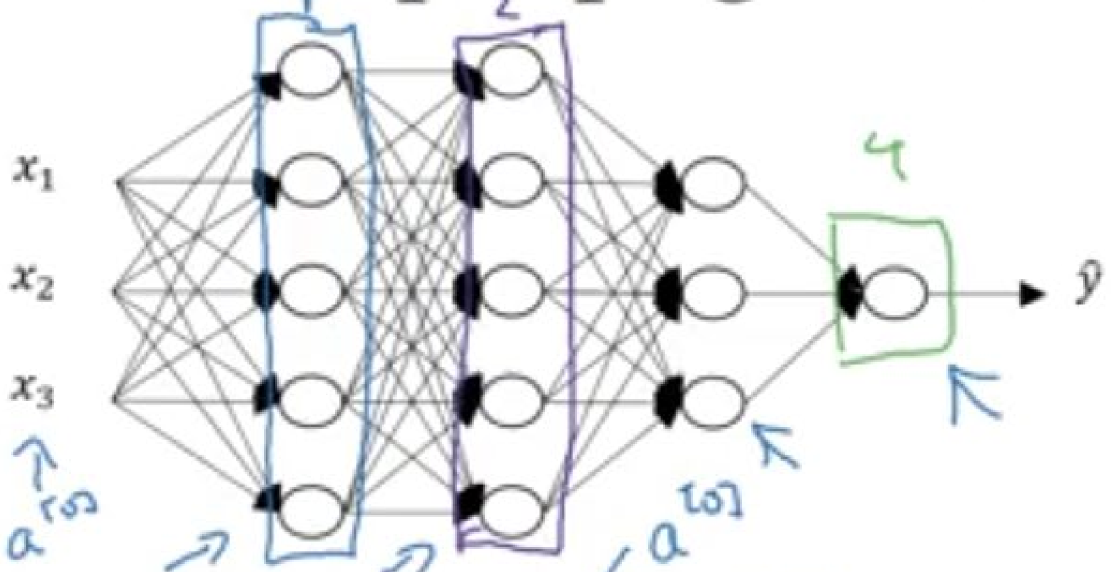
Deep Neural Networks

Forward Propagation in a Deep Network

Forward propagation in a deep network



$a^{[0]}$

x: $z^{[1]} = W^{[1]} a^{[0]} + b^{[1]}$

$a^{[1]} = g^{[1]}(z^{[1]})$

$z^{[2]} = W^{[2]} a^{[1]} + b^{[2]}$

$a^{[2]} = g^{[2]}(z^{[2]})$

$z^{[4]} = W^{[4]} a^{[3]} + b^{[4]}$, $a^{[4]} = g^{[4]}(z^{[4]}) = \hat{y}$

$\left[z^{[1](1)}, z^{[1](2)}, \ldots, z^{[1](m)} \right]$

$A^{[0]} = X$

$$\boxed{\begin{aligned} Z^{[l]} &= W^{[l]} A^{[l-1]} + b^{[l]} \\ A^{[l]} &= g^{[l]}(Z^{[l]}) \end{aligned}}$$

Vectorized:

$X = A^{[0]}$

$Z^{[1]} = W^{[1]} \cancel{X} A^{[0]} + b^{[1]}$

$A^{[1]} = g^{[1]}(Z^{[1]})$

$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$

$A^{[2]} = g^{[2]}(Z^{[2]})$

$\hat{Y} = g(Z^{[4]}) = A^{[4]}$

for $l = 1..4$

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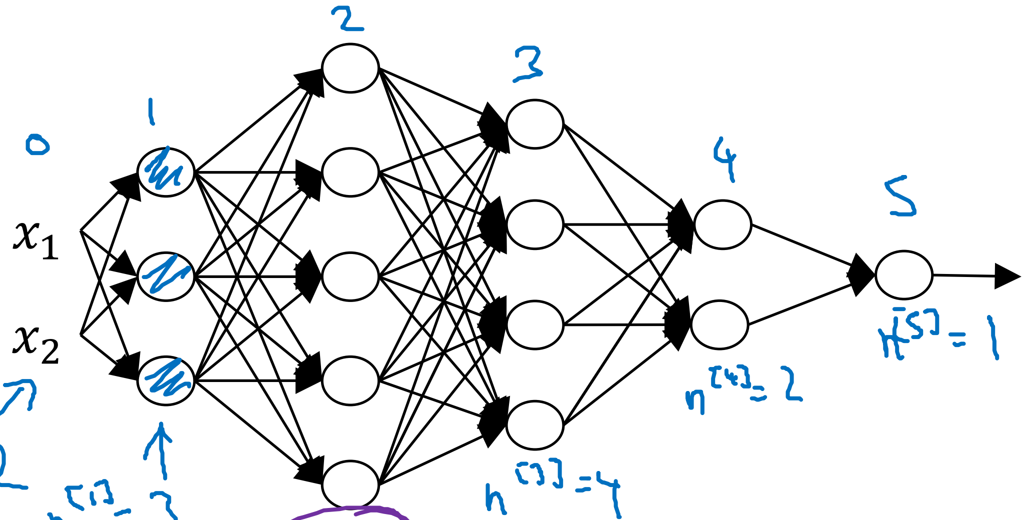
Deep Neural Networks

Getting your matrix dimensions right

Parameters $W^{[l]}$ and $b^{[l]}$

$L = 5$



$n^{[0]} = n_x = 2$, $n^{[1]} = 3$, $n^{[2]} = 5$, $n^{[3]} = 4$, $n^{[4]} = 2$, $n^{[5]} = 1$

$z^{[l]} = g^{[l]}(a^{[l]})$

$a^{[l]}$

$$z^{[1]} = \boxed{W^{[1]} \cdot x} + \boxed{b^{[1]}}$$

$(3,1) \leftarrow (3,2)\quad(2,1)\quad(3,1)$

$(n^{[1]},1)\quad(n^{[1]},n^{[0]})\quad(n^{[0]},1)\quad(n^{[1]},1)$

$W^{[1]}: (n^{[1]}, n^{[0]})$

$W^{[2]}: (5,3)\quad(n^{[2]}, n^{[1]})$

$$z^{[2]} = \boxed{W^{[2]} \cdot a^{[1]}} + \boxed{b^{[2]}}$$

$\rightarrow (5,1)\quad(5,3)\quad(3,1)\quad(5,1)$

$(n^{[2]},1)$

$W^{[3]}: (4,5)$

$W^{[4]}: (2,4)$, $W^{[5]}: (1,2)$

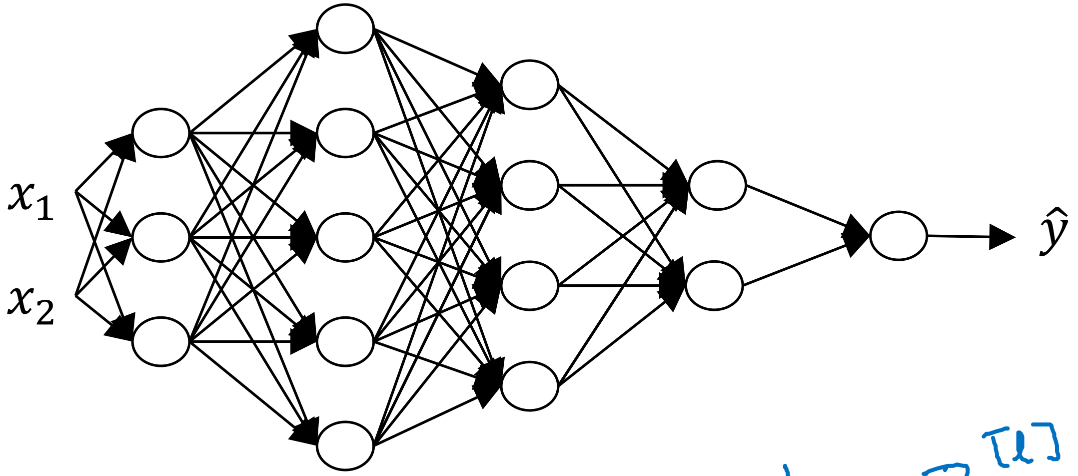
$\rightarrow W^{[l]}: (n^{[l]}, n^{[l-1]})$

$\rightarrow b^{[l]}: (n^{[l]}, 1)$

$dW^{[l]}: (n^{[l]}, n^{[l-1]})$

$db^{[l]}: (n^{[l]}, 1)$

Vectorized implementation



$$z^{[1]} = W^{[1]} \cdot x + b^{[1]}$$

$(n^{[1]},1)$ $(n^{[1]},n^{[0]})$ $(n^{[0]},1)$ $(n^{[1]},1)$

$$\begin{bmatrix} z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \end{bmatrix}$$

$$Z^{[1]} = W^{[1]} \cdot X + b^{[1]}$$

$(n^{[1]},m)$ $(n^{[1]},n^{[0]})$ $(n^{[0]},m)$ $(n^{[1]},1)$

$(n^{[1]},m)$

$z^{[l]}, a^{[l]} : (n^{[l]},1)$

$Z^{[l]}, A^{[l]} : (n^{[l]},m)$

$l=0$ $A^{[0]} = X = (n^{[0]},m)$

$dZ^{[l]}, dA^{[l]} : (n^{[l]},m)$

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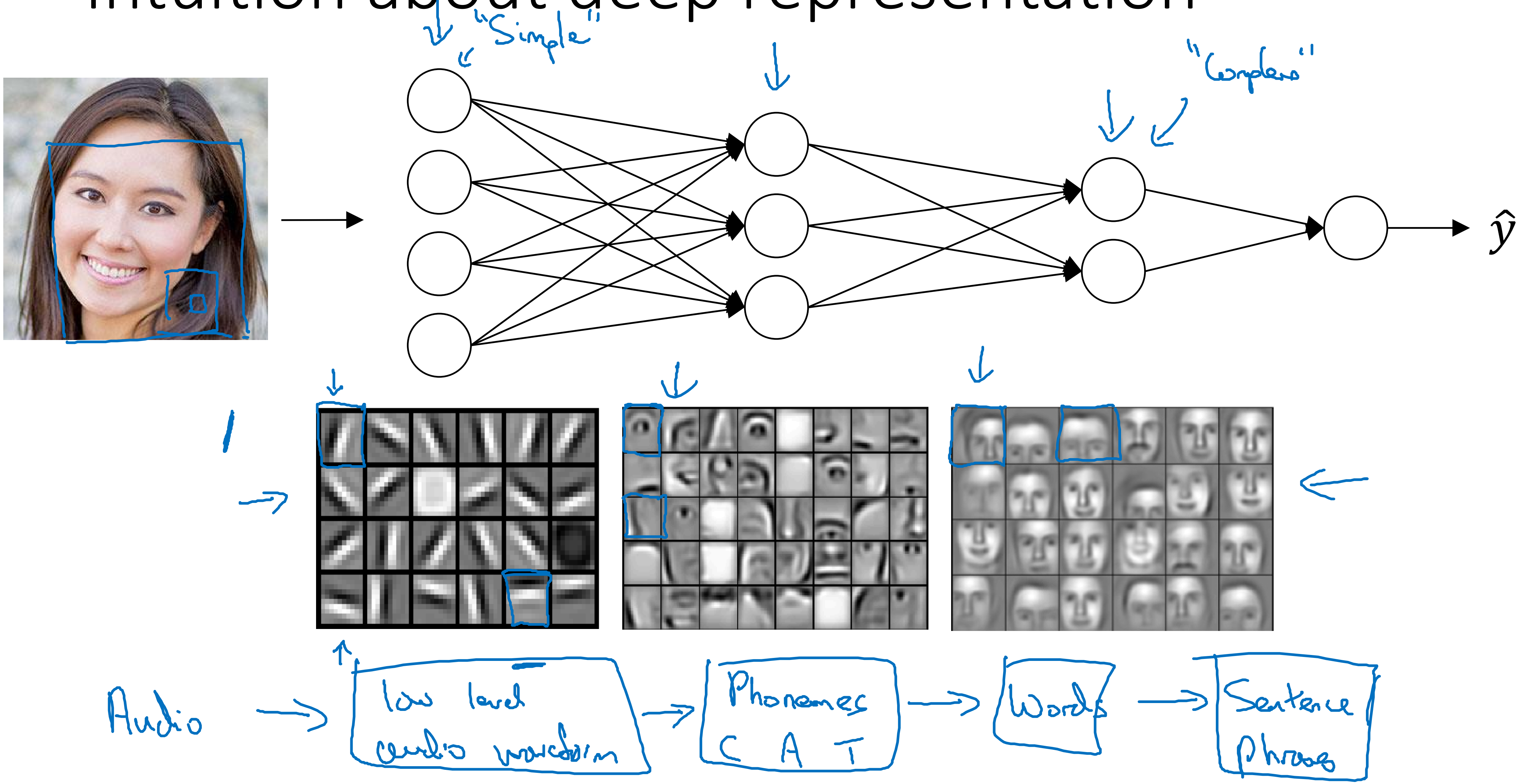
Deep Neural Networks

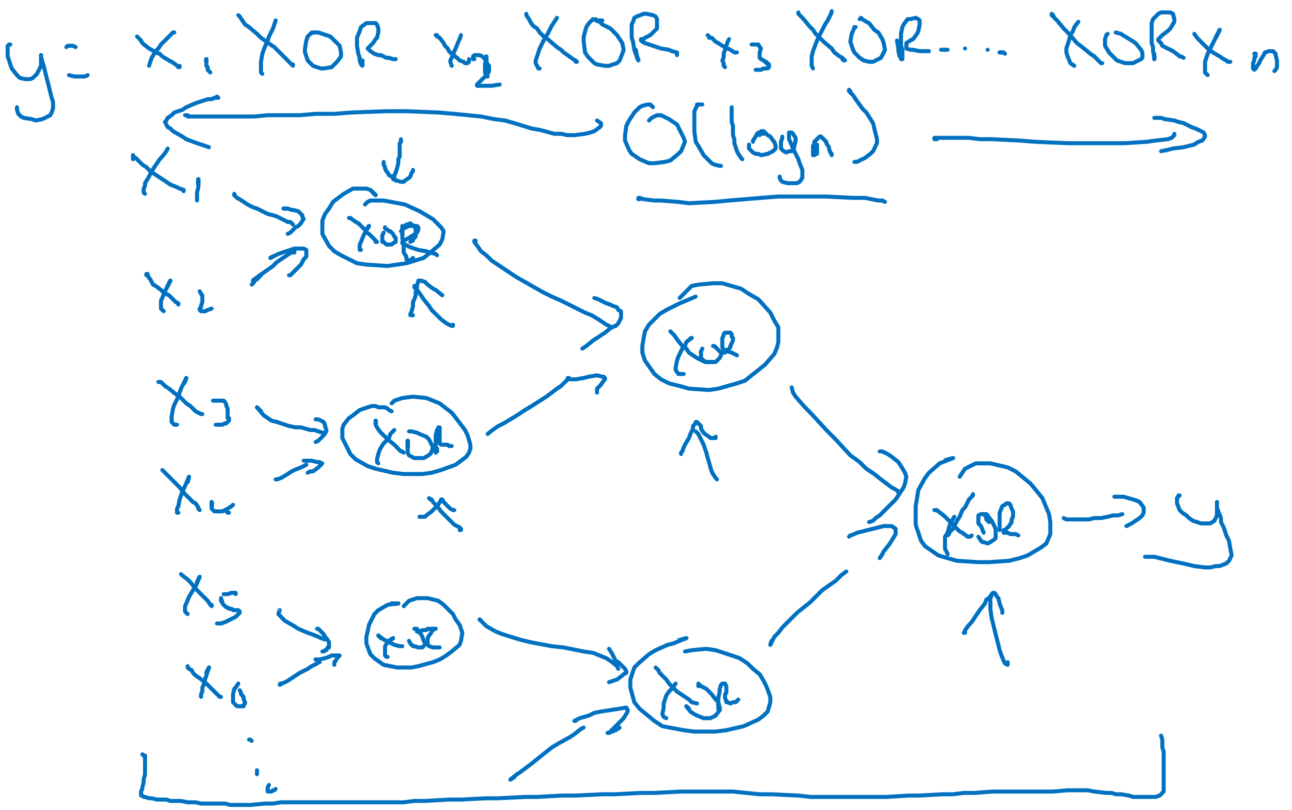
Why deep representations?

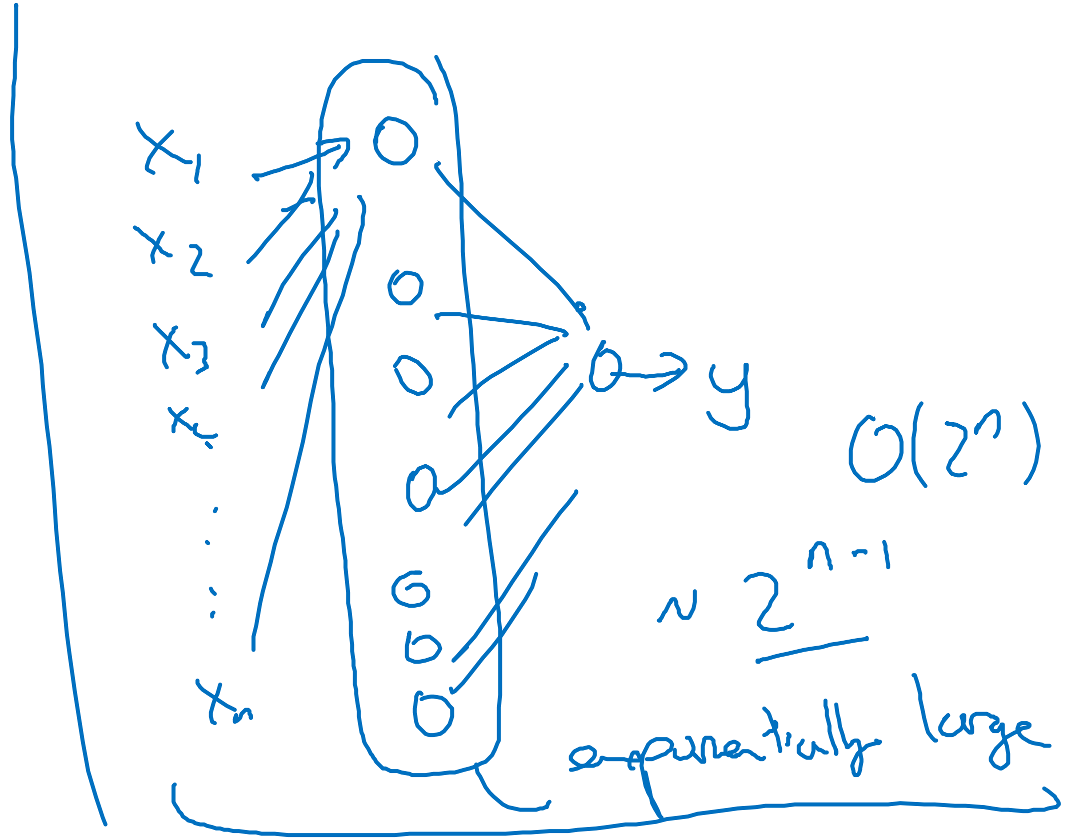
Intuition about deep representation



Circuit theory and deep learning

Informally: There are functions you can compute with a "small" L-layer deep neural network that shallower networks require exponentially more hidden units to compute.





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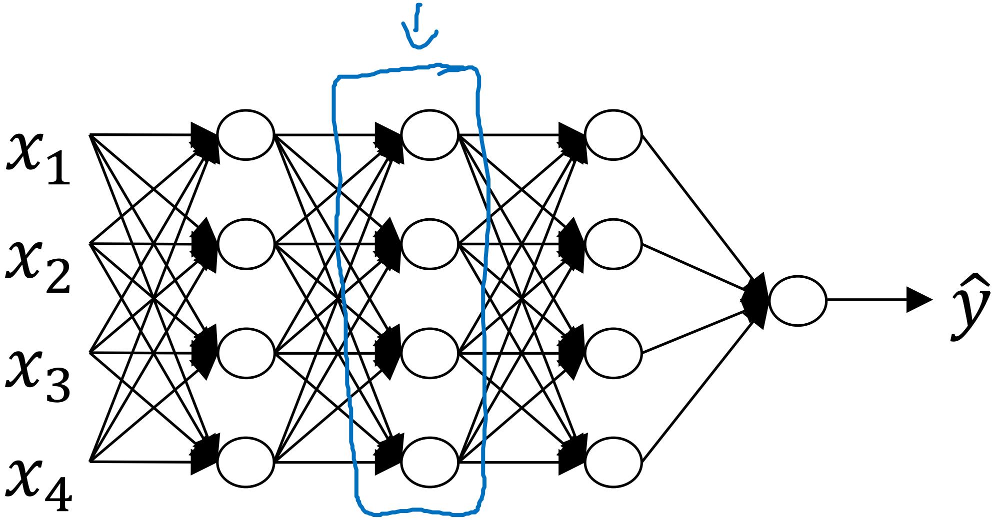
Deep Neural Networks

Building blocks of deep neural networks

Forward and backward functions



layer l: $W^{[l]}, b^{[l]}$

→ Forward: Input $a^{[l-1]}$, output $a^{[l]}$

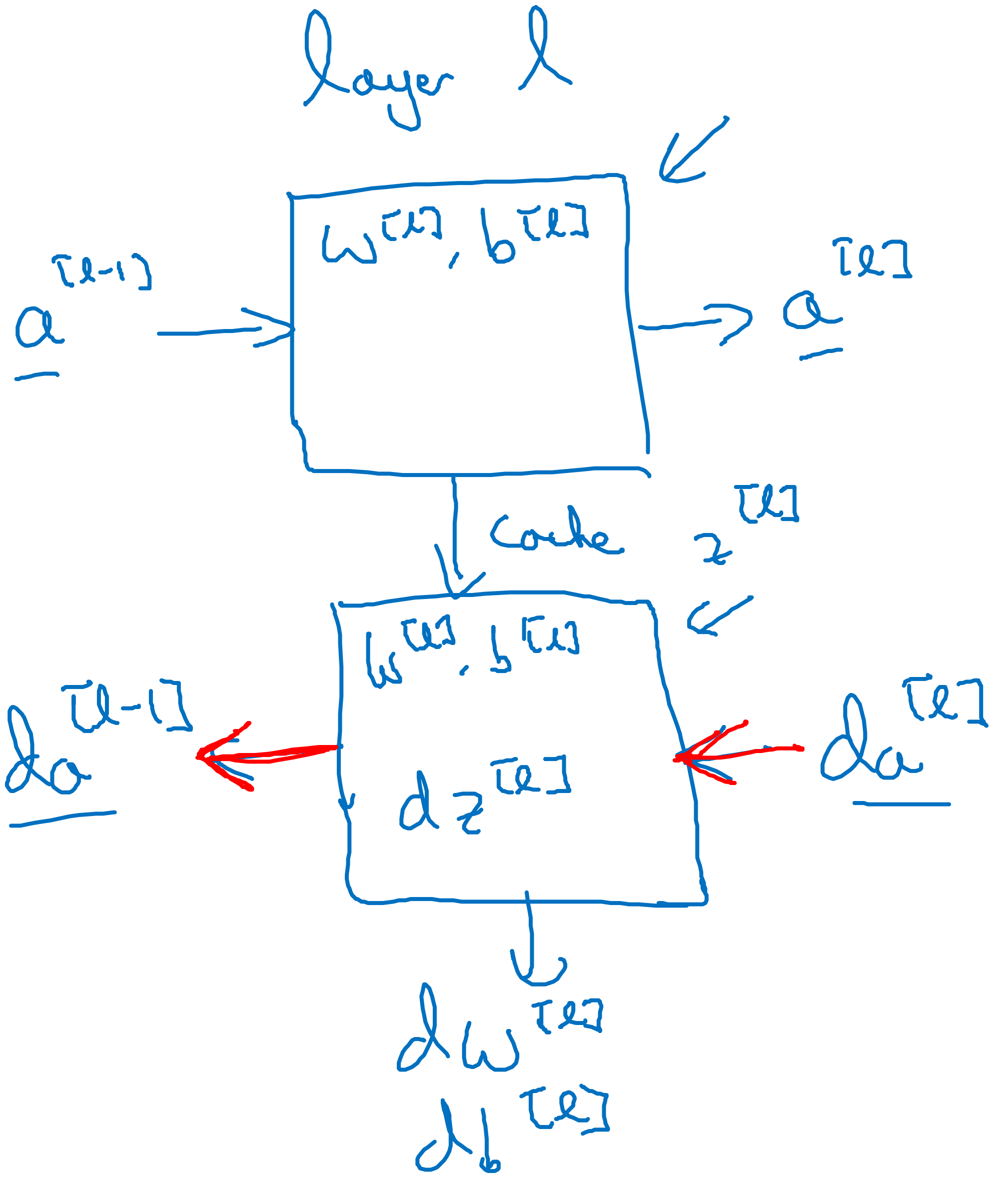
$z^{[l]} = W^{[l]} a^{[l-1]} + b^{[l]}$ cache $z^{[l]}$

$a^{[l]} = g^{[l]}(z^{[l]})$

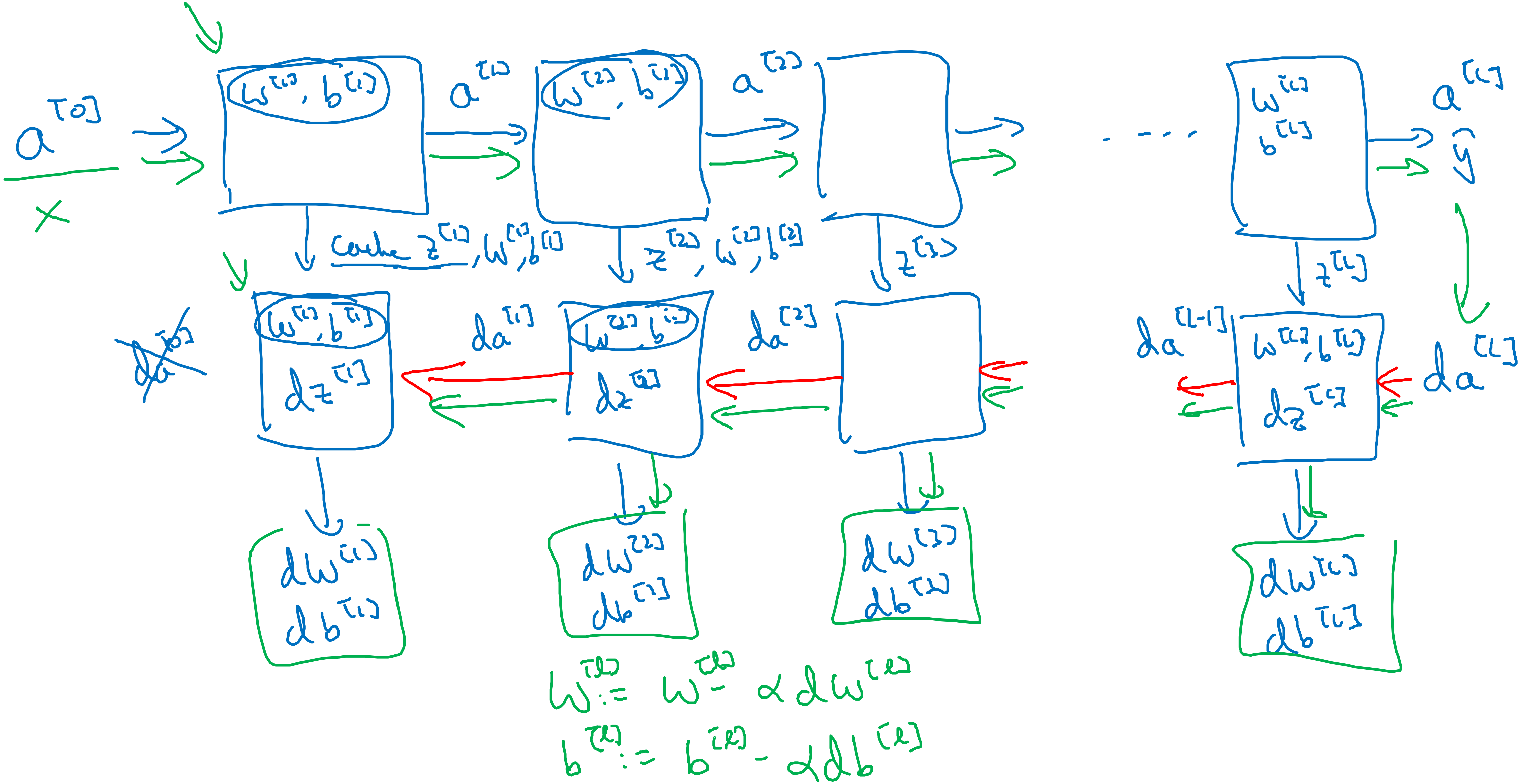
→ Backward: Input $da^{[l]}$, output $da^{[l-1]}$

cache($z^{[l]}$) $dW^{[l]}$

$db^{[l]}$



Forward and backward functions



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Deep Neural Networks

Forward and backward propagation

Forward propagation for layer l

- Input $a^{[l-1]}$ — $W^{[l]}, b^{[l]}$
- Output $a^{[l]}$, cache ($\underline{z^{[l]}}$)

$$z^{[l]} = W^{[l]} \cdot a^{[l-1]} + b^{[l]}$$

$$a^{[l]} = g^{[l]}(z^{[l]})$$

Vectorized:

$$Z^{[l]} = W^{[l]} \cdot A^{[l-1]} + b^{[l]}$$

$$A^{[l]} = g^{[l]}(Z^{[l]})$$

$a^{[0]}$

$A^{[0]}$

$X = A^{[0]} \to \square \to \square \to \square \to$

Backward propagation for layer l

→ Input $da^{[l]}$

→ Output $\boxed{da^{[l-1]}}, dW^{[l]}, db^{[l]}$

$$dz^{[l]} = da^{[l]} * g^{[l]\prime}(z^{[l]})$$

$$dW^{[l]} = dz^{[l]} \cdot a^{[l-1]}$$

$$db^{[l]} = dz^{[l]}$$

$$\boxed{da^{[l-1]}} = W^{[l]T} \cdot dz^{[l]}$$

$$dz^{[l]} = W^{[l+1]T} dz^{[l+1]} * g^{[l]\prime}(z^{[l]})$$

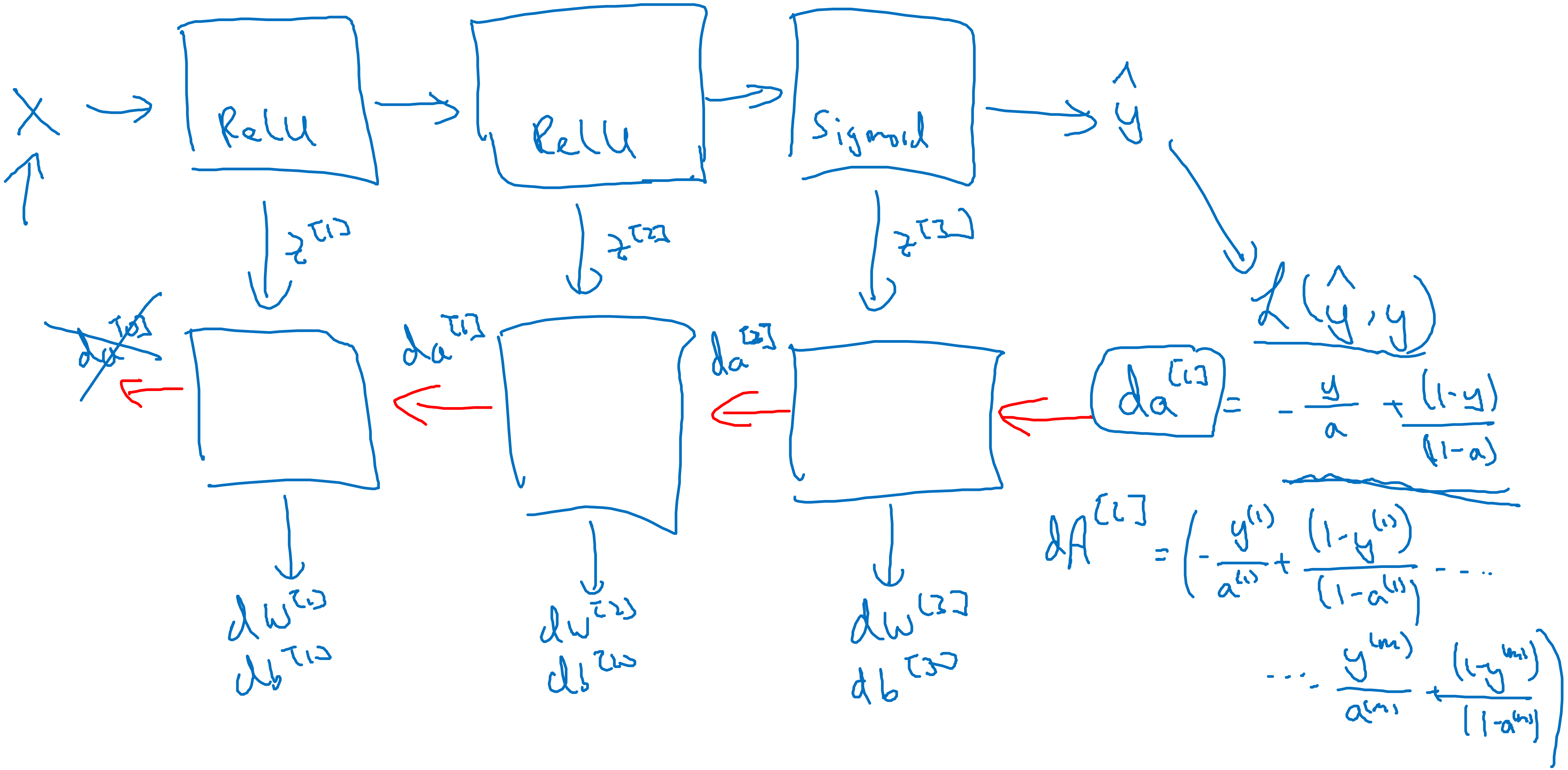
$$dZ^{[l]} = dA^{[l]} * g^{[l]\prime}(Z^{[l]})$$

$$dW^{[l]} = \frac{1}{m} dZ^{[l]} \cdot A^{[l-1]T}$$

$$db^{[l]} = \frac{1}{m} np.sum(dZ^{[l]}, axis=1, keepdims=True)$$

$$\boxed{dA^{[l-1]}} = W^{[l]T} \cdot dZ^{[l]}$$

Summary


X
ReLU
ReLU
Sigmoid
$\hat{y}$
$z^{[1]}$
$z^{[2]}$
$z^{[3]}$
$\mathcal{L}(\hat{y}, y)$
$\cancel{dx}$ $da^{[0]}$
$da^{[1]}$
$da^{[2]}$
$da^{[l]} = -\frac{y}{a} + \frac{(1-y)}{(1-a)}$
$dA^{[l]} = \left(-\frac{y^{(1)}}{a^{(1)}} + \frac{(1-y^{(1)})}{(1-a^{(1)})} \cdots \cdots -\frac{y^{(m)}}{a^{(m)}} + \frac{(1-y^{(m)})}{(1-a^{(m)})}\right)$
$dW^{[1]}$
$db^{[1]}$
$dW^{[2]}$
$db^{[2]}$
$dW^{[3]}$
$db^{[3]}$

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Deep Neural Networks

Forward and backward propagation

Forward propagation for layer l

- Input $a^{[l-1]}$
- Output $a^{[l]}$, cache ($z^{[l]}$)

$W^{[l]}, b^{[l]}$

$$z^{[l]} = W^{[l]} \cdot a^{[l-1]} + b^{[l]}$$

$$a^{[l]} = g^{[l]}(z^{[l]})$$

Vectorized:

$$Z^{[l]} = W^{[l]} \cdot A^{[l-1]} + b^{[l]}$$

$$A^{[l]} = g^{[l]}(Z^{[l]})$$

$a^{[0]}$

$A^{[0]}$

$X = A^{[0]} \rightarrow \square \rightarrow \square \rightarrow \square \rightarrow$

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Backward propagation for layer l

→ Input $da^{[l]}$

→ Output $\boxed{da^{[l-1]}}$, $dW^{[l]}$, $db^{[l]}$

$$dz^{[l]} = da^{[l]} * g^{[l]\prime}(z^{[l]})$$

$$dW^{[l]} = dz^{[l]} \cdot a^{[l-1]}$$

$$db^{[l]} = dz^{[l]}$$

$$\boxed{da^{[l-1]}} = W^{[l]T} \cdot dz^{[l]}$$

$$dz^{[l]} = W^{[l+1]T} dz^{[l+1]} * g^{[l]\prime}(z^{[l]})$$

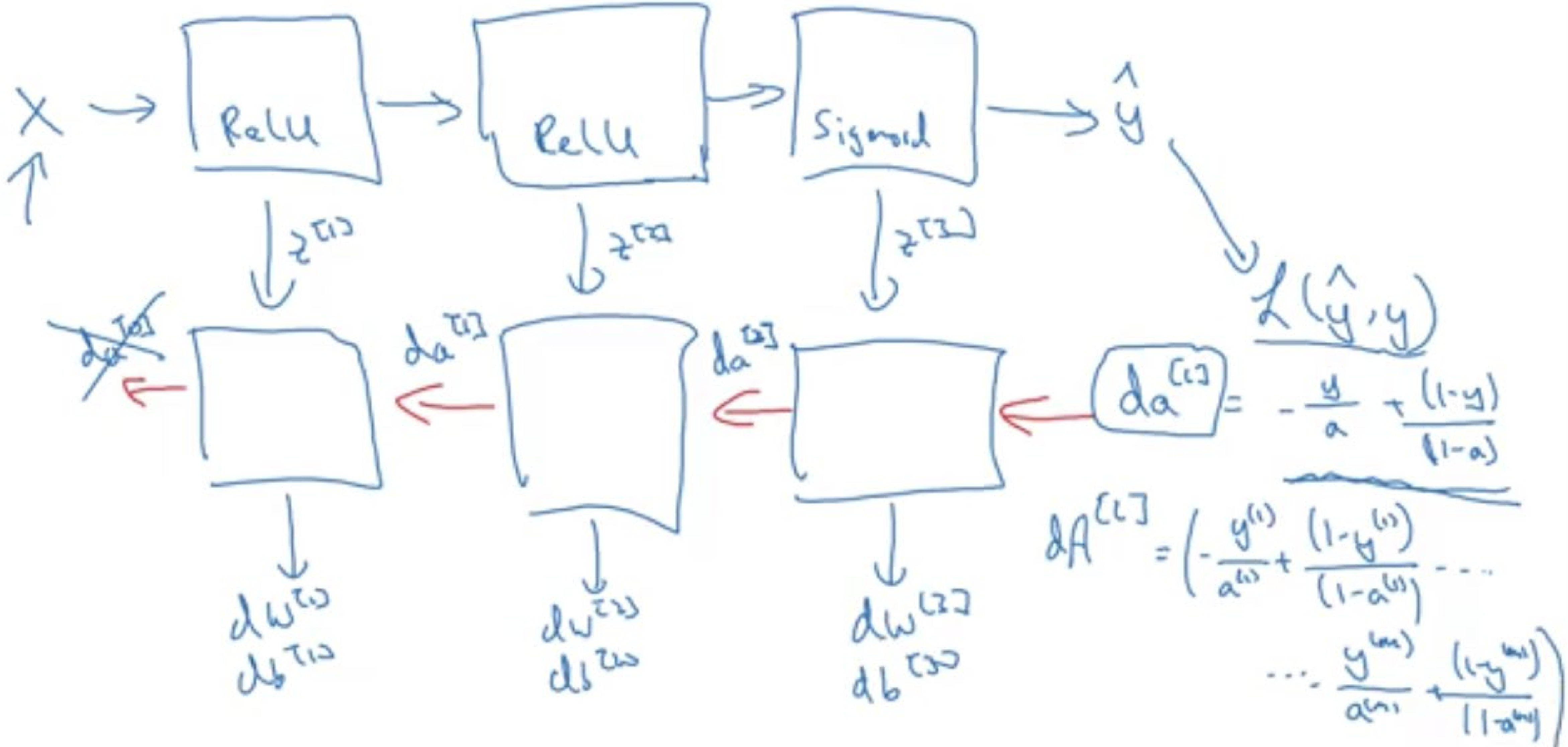
$$dZ^{[l]} = dA^{[l]} * g^{[l]\prime}(Z^{[l]})$$

$$dW^{[l]} = \frac{1}{m} dZ^{[l]} \cdot A^{[l-1]T}$$

$$db^{[l]} = \frac{1}{m} np.sum(dZ^{[l]}, axis=1, keepdims=True)$$

$$\boxed{dA^{[l-1]}} = W^{[l]T} \cdot dZ^{[l]}$$

Summary



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Deep Neural Networks

Parameters vs Hyperparameters

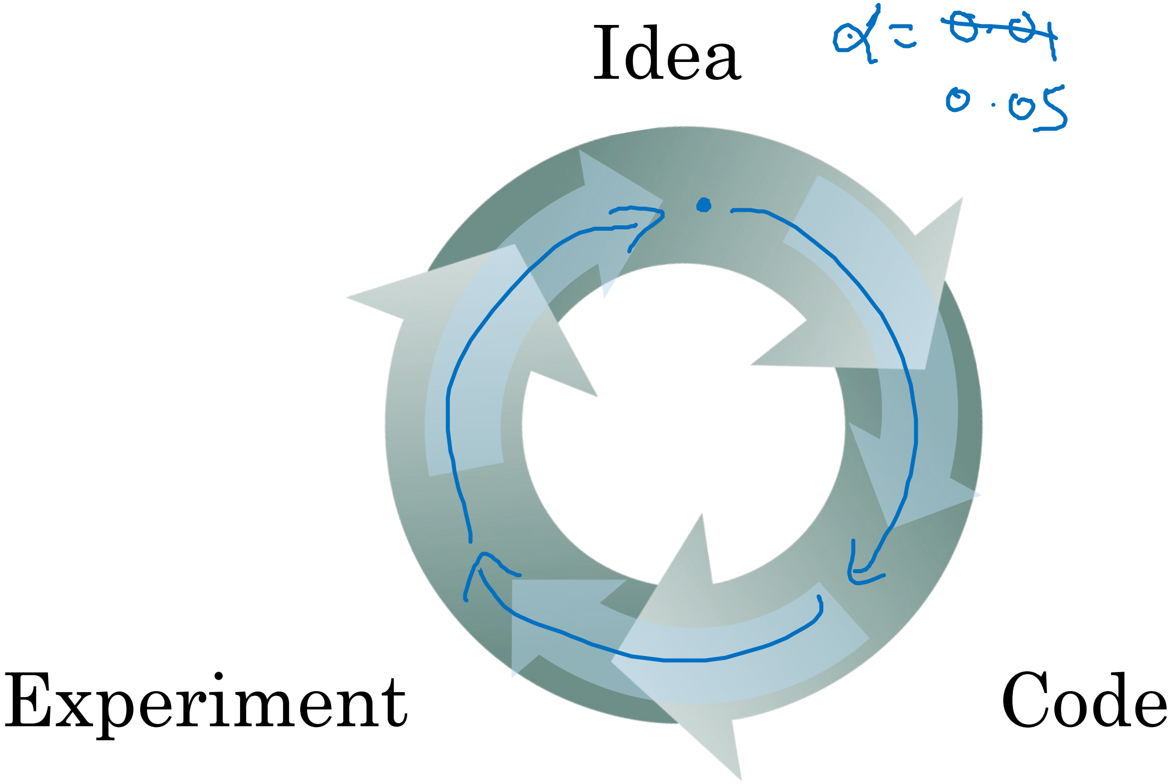
What are hyperparameters?

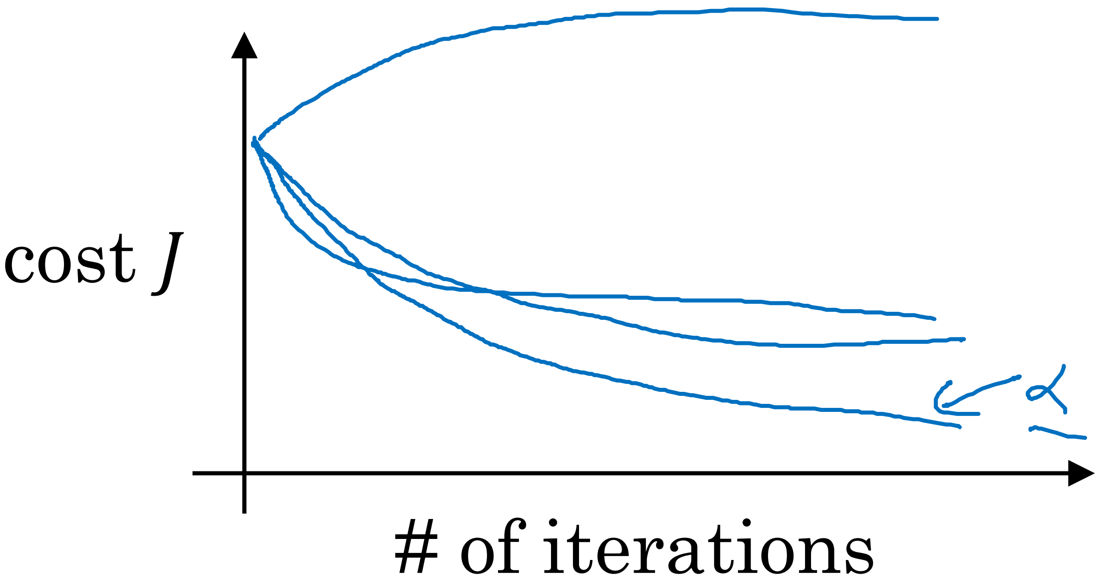
Parameters: $\underline{W^{[1]}, b^{[1]}, W^{[2]}, b^{[2]}, W^{[3]}, b^{[3]}}$ ...

$$
\text{Hyperparameters:} \left.\begin{array}{l}
\text{learning rate } \alpha \\
\text{\#iterations} \\
\text{\#hidden layers } L \\
\text{\#hidden units } n^{[1]}, n^{[2]}, \ldots \\
\text{choice of activation function}
\end{array}\right\}
$$

Later: Momentum, mini-batch size, regularizations, ...

Applied deep learning is a very empirical process





Vision, Speech, NLP, Ad, Search, Recommendation.

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Deep Neural Networks

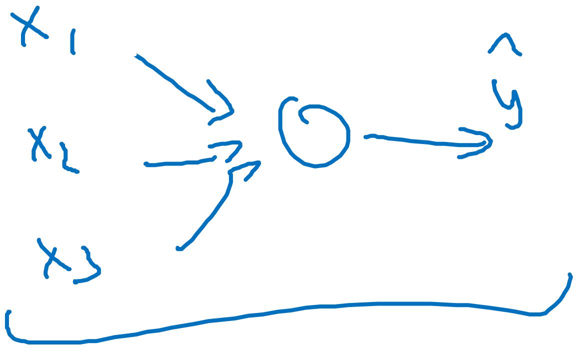
What does this have to do with the brain?

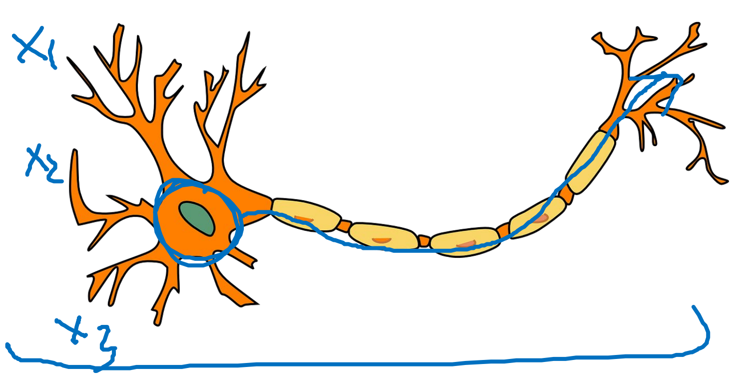
Forward and backward propagation

$$
\begin{aligned}
Z^{[1]} &= W^{[1]}X + b^{[1]} \\
A^{[1]} &= g^{[1]}(Z^{[1]}) \\
Z^{[2]} &= W^{[2]}A^{[1]} + b^{[2]} \\
A^{[2]} &= g^{[2]}(Z^{[2]}) \\
&\vdots \\
A^{[L]} &= g^{[L]}\left(Z^{[L]}\right) = \hat{Y}
\end{aligned}
$$

$$
\begin{aligned}
dZ^{[L]} &= A^{[L]} - Y \\
dW^{[L]} &= \frac{1}{m} dZ^{[L]} A^{[L]^T} \\
db^{[L]} &= \frac{1}{m} np.\mathrm{sum}(\mathrm{d}Z^{[L]}, axis = 1, keepdims = True) \\
dZ^{[L-1]} &= dW^{[L]^T} dZ^{[L]} g'^{[L]}(Z^{[L-1]}) \\
&\vdots \\
dZ^{[1]} &= dW^{[L]^T} dZ^{[2]} g'^{[1]}(Z^{[1]}) \\
dW^{[1]} &= \frac{1}{m} dZ^{[1]} A^{[1]^T} \\
db^{[1]} &= \frac{1}{m} np.\mathrm{sum}(\mathrm{d}Z^{[1]}, axis = 1, keepdims = True)
\end{aligned}
$$

"It's like the brain"





$X \longrightarrow y$