Unit 2: Differentiation

Linear approximation

In[1]:=
```
10 * 0.4 + 150
```

Out[1]= 154.

In[2]:=

$$\Delta f \approx \left.\frac{df}{dx}\right|_{x=a} \cdot \Delta x \qquad \text{for } \Delta x \text{ near } 0$$

$$f(x) \approx f'(a)(x-a) + f(a) \quad \text{for } x \text{ near } a$$

Out[2]=

$$\Delta f \approx \left.\frac{df}{dx}\right|_{x=a} \cdot \Delta x \qquad \text{for } \Delta x \text{ near } 0$$

$$f(x) \approx f'(a)(x-a) + f(a) \quad \text{for } x \text{ near } a$$

In[3]:= D[$\sqrt{x}$, x]

Out[3]= $\frac{1}{2\sqrt{x}}$

In[4]:= $\frac{1}{2\sqrt{100}}$

Out[4]= $\frac{1}{20}$

In[5]:=
```
4 * %
```

Out[5]= $\frac{1}{5}$

In[6]:= f[x_] := $x^{2.5}$

```
f '[4] * -0.03 + f[4]
```

Out[7]= 31.4

In[8]:= $D\left[\sqrt{x}, \{x, 2\}\right]$

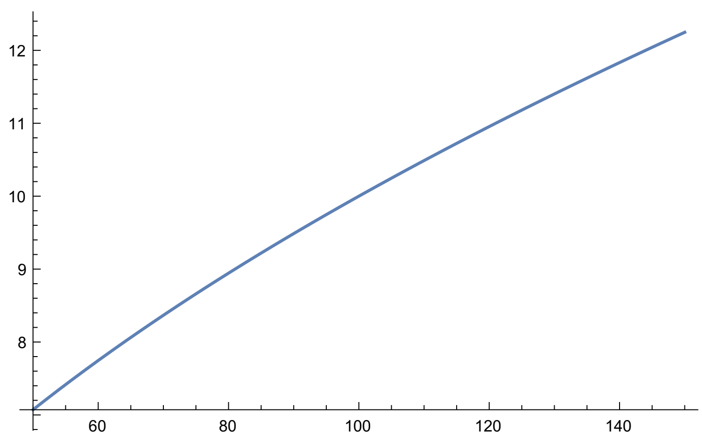
Out[8]= $-\frac{1}{4\,x^{3/2}}$

In[9]:= $-\frac{1}{4 \times 100^{3/2}}$

Out[9]= $-\frac{1}{4000}$

In[10]:= $\text{Plot}\left[\sqrt{x}, \{x, 50, 150\}\right]$

Out[10]=



In[11]:= v[x_] := 4 π * x^3 / 3

v'[r]

v'[10] * -0.03

Out[12]= $4\,\pi\,r^2$

Out[13]= -37.6991

In[14]:= v'[10]

Out[14]= 400 π

In[15]:= 20 / 400

Out[15]= $\frac{1}{20}$

Product rule

If $h(x) = f(x)\,g(x)$, then

In[1]:=
$$h'(x) = f(x)\,g'(x) + g(x)\,f'(x)$$

at all points where the derivatives $f'(x)$ and $g'(x)$ are defined.

In[2]:=
```
D[√x , x] * Cos[x] + √x * -Sin[x]
```

Out[2]=
$$\frac{\text{Cos}[x]}{2\sqrt{x}} - \sqrt{x}\ \text{Sin}[x]$$

In[3]:=
```
100 * 0.4
```

Out[3]=
```
40.
```

In[4]:=
```
100 * -0.01 + 3 * 0.4
```

Out[4]=
```
0.2
```

f (x) = x^2 Sin[x] Cos[x]

f '[x] = 2 x * Sin[x] * Cos[x]

+ x^2 * -Cos[x] * Sin[x]

+ x^2 * Sin[x] * -Sin[x]

Quotient Rule

In[1]:=
$$\text{Limit}\left[\frac{f2 * g - f * g2}{t}, t \to 0\right]$$

Out[1]=
```
Indeterminate
```

If $h(x) = \dfrac{f(x)}{g(x)}$ for all x, then

In[2]:=
$$h'(x) = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{g(x)^2}$$

at all points where f and g are differentiable and $g(x) \neq 0$.

In[3]:=
```
D[(2 + Cos[X])/(x^2 + 1), x]
```

Out[3]= $-\frac{2\,x\,(2+\text{Cos}[X])}{(1+x^2)^2}$

In[4]:=

$$\begin{aligned}
\frac{d}{dx}\sin x &= \cos x \\
\frac{d}{dx}\cos x &= -\sin x \\
\frac{d}{dx}\tan x &= \frac{1}{\cos^2 x} = \sec^2 x \\
\frac{d}{dx}\cot x &= -\frac{1}{\sin^2 x} = -\csc^2 x \\
\frac{d}{dx}\sec x &= \frac{\sin x}{\cos^2 x} = \sec x \tan x \\
\frac{d}{dx}\csc x &= -\frac{\cos x}{\sin^2 x} = -\csc x \cot x
\end{aligned}$$

Chain rule

If $h(x) = f(g(x))$, then

$$h'(x) = f'(g(x))\,g'(x)$$

at all points where the derivatives $f'(g(x))$ and $g'(x)$ are defined.

In[6]:=
```
D[120 * t + 100, t]
```

Out[6]= 120

In[7]:=
```
3 * 0.01 + 9
```

Out[7]= 9.03

In[13]:=
```
4 * (9.03 - 9) + 5
```

Out[13]= 5.12

```
Solve[5.12 == x * (2.01 - 2) + 5, x]
```

Out[15]= {{x → 12.}}

In[18]:=
```
3 * 4 * (5 * 2)
```

Out[18]= 120

Examples

In[42]:= $\mathtt{f[x\_] := \sqrt{x^3 + 2\,x + 1}}$

$\mathtt{f1[x\_] := \dfrac{1}{2\,\sqrt{x^3 + 2\,x + 1}} * (3\,x^2 + 2)}$

$\mathtt{f1[1]}$

Out[44]= $\dfrac{5}{4}$

In[61]:= $\mathtt{g[x\_] := Sin[x^2 + 2\,x]}$

$\mathtt{g1[x\_] := -Cos[x^2 + 2\,x] * (2\,x + 2)}$

$\mathtt{g1[x]}$

Out[63]= $(2 + 2\,x)\,\mathrm{Cos}[2\,x + x^2]$

Exercises

In[172]:= $\mathtt{f[x\_] := Sin\left[\dfrac{x}{x^2 + 1}\right]}$

$\mathtt{f1[x\_] := Cos[u] * u'}$

$\mathtt{f2[x\_] := Cos\left[\dfrac{x}{x^2 + 1}\right] * \dfrac{(x^2 + 1) - (x * 2 * x)}{(x^2 + 1)^2}}$

$\mathtt{f2[x] == f'[x]}$

$\mathtt{f2[x]}$

Out[175]= True

Out[176]= $\left(-\dfrac{2\,x^2}{(1 + x^2)^2} + \dfrac{1}{1 + x^2}\right)\mathrm{Cos}\left[\dfrac{x}{1 + x^2}\right]$

In[701]:= $\mathtt{k[x\_] := Cos[x] * Sin[x]^4}$

$\mathtt{u := Sin[x]^4}$

$\mathtt{k1[x\_] := -Sin[x] * u + Cos[x] * u'}$

$\mathtt{inn := Sin[x]}$

$\mathtt{auss := x^4}$

$\mathtt{k2[x\_] := -Sin[x] * Sin[x]^4 + Cos[x] * (4 * (Sin[x])^3 * Cos[x])}$

$\mathtt{k'[x] == k2[x]}$

$\mathtt{k2[x]}$

Out[707]= True

Out[708]= $4\,\mathrm{Cos}[x]^2\,\mathrm{Sin}[x]^3 - \mathrm{Sin}[x]^5$

Review

In[7]:=
```
g'[f[3]] * f'[3];
g'[5] * 3;
4 * 3

f'[g[3]] * g' (3);
f'[4] * 5;
-3 * 5
```

Out[9]= 12

Out[12]= -15

In[19]:=
```
p[i_] := 10 * i^2
p'[6]
```

Out[20]= 120