Unit 1: Derivatives

What is a derivative?

Rate of Change

```
220 - 50
```

170

```
170 / 2
```

85

Average vs. Instantaneous

$$\frac{\text{Delta } f}{\text{Delta } t}$$

$$\frac{1}{1/60}$$

60

Instantaneous approximation continued

$$\frac{220\,000 - 210\,000}{32 - 30}$$

5000

Derivative at a point

The Derivative of f(x) at x = a

$$f'(a) = \text{Lim}_{b \to a} \frac{f(b) - f(a)}{b - a}$$

A negative derivative?

```
f[t_] := 100 + 20 t - 5 t^2
f'[2]
```

0

Geometric interpretation of the derivative

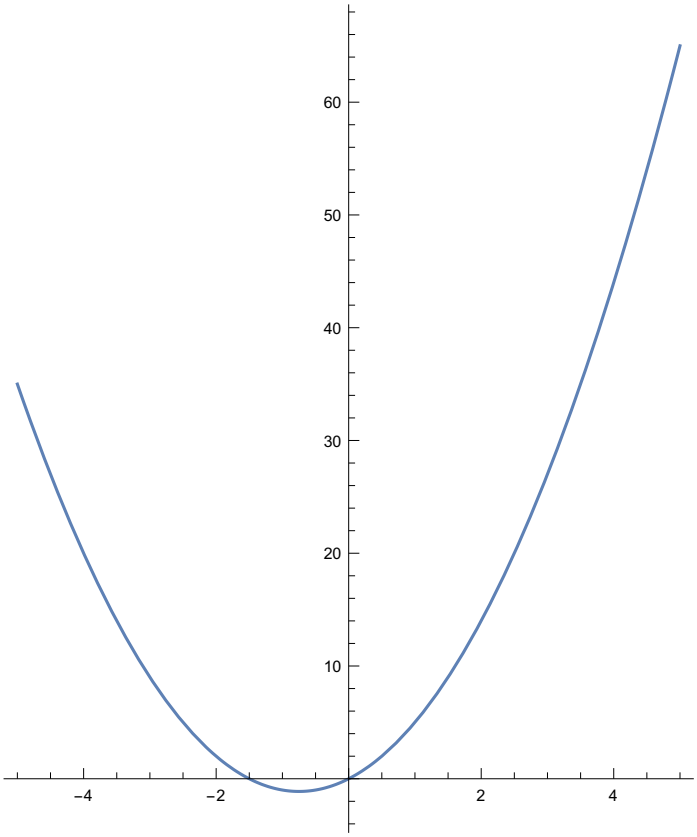
Tangent lines

Calculated using:

y-f(a)=m(x-a)

Equation of a tangent line

```
j[x_] := 2 x^2 + 3 x
Plot[j[x], {x, -5, 5}, AspectRatio → Full]
j[1]
```



5

```
j '[1]
```

7

```
j[1]
```

5

```
Simplify[y - j[1] == j'[1] * (x - 1)]
```

$7\,x == 2 + y$

Review questions

```
f[x_] := -2 x + 1
f[3]
```

-5

Calculating derivatives

Linearity

$h[x\_] := 1 / x^2$

$h'[x\_] := \lim_{\Delta x \to 0} \frac{1/(x^2 + \Delta x) - 1/x^2}{\Delta x}$

$h'[x\_] := \lim_{\Delta x \to 0} -2/x^3$

```
h[x_] := 1 / x^2
h'[s]
```

$-\frac{2}{s^3}$

Relationship between derivatives

```
f[x_] := -3/x
f'[x]
```

$\frac{3}{x^2}$

Calculation

```
f[x_] := 4 √x - 3/x^2
f'[x]
```

$\frac{6}{x^3} + \frac{2}{\sqrt{x}}$

Leibniz notation

$$\frac{dy}{dx} \text{ or } \frac{df}{dx}$$

Area of a circle

```
A[r_] := π r^2
A'[r]
A'[3]
```

$2\,\pi\,r$

$6\,\pi$

```
A2[c_] := π * (c - 2 * π)^2
A2'[c]
A2'[6 π]
```

$2\,(c - 2\,\pi)\,\pi$

$8\,\pi^2$

exercise

```
D[g^3 + 2 g^2, g] /. g → 2
f[x_] := x^3 + 2 x^2
f'[2]

3 * 2^2 + 4 * 2
```

20

20

20

Second derivatives and higher

everything lost do to power outage

Homework

Part A

Velocity

```
h[x_] := 400 - 16 x^2
(h[0] - h[2]) / (0 - 2)
```

```
Solve[h[g] == 0, g]
```

$$\frac{h[3] - h[5]}{3 - 5}$$

```
h'[5]
```

```
-32
```

```
{{g → -5}, {g → 5}}
```

```
-128
```

```
-160
```

Definition review

```
f[x_] := 1 / (2 x + 1)
f'[x]
```

$$-\frac{2}{(1 + 2\,x)^2}$$

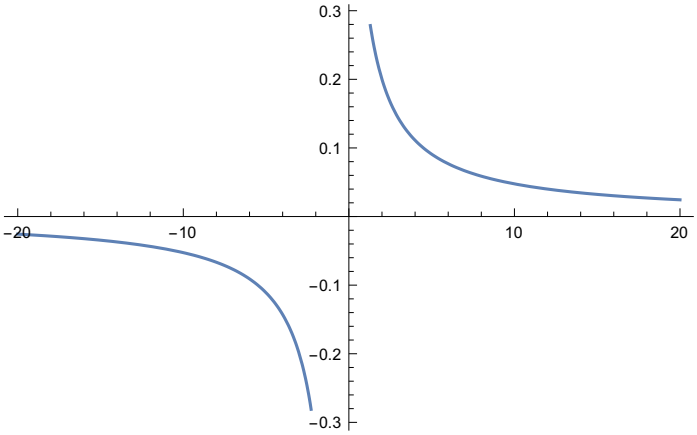
```
N[Solve[f'[x] == 1, x, Reals]]
N[Solve[f'[x] == 0, x, Reals]]
N[Solve[f'[x] == -1, x, Reals]]
```

```
{}
```

```
{}
```

```
{{x → -1.20711}, {x → 0.207107}}
```

```
Plot[f[x], {x, -20, 20}]
```




```
g[x_] := 4 x + 5
N[Solve[g[x] == 1, x, Reals]]
N[Solve[g[x] == 0, x, Reals]]
N[Solve[g[x] == -1, x, Reals]]
```

{{x → -1.}}

{{x → -1.25}}

{{x → -1.5}}

```
j[x_] := x^-7/4
j '[1]
```

$$-\frac{7}{4}$$

Tangent line

```
f[x_] := 1 / (2 x + 1)
m  := f '[1]
b  := (-1) * (m * 1 - f[1])
y = m * x + b
```

$$\frac{5}{9} - \frac{2\,x}{9}$$

Differentiability

$$\begin{cases} c * x^2 + 4\,x + 1 & x \geq 1 \\ a * x + b & x < 1 \end{cases}$$

$$\begin{cases} 1 + 4\,x + c\,x^2 & x \geq 1 \\ b + a\,x & x < 1 \\ 0 & \text{True} \end{cases}$$

```
x = 1;
c * x^2 + 4 * x + 1 == a * x + b
```

$$5 + c == a + b$$

```
2 * c * x + 4 == a
```

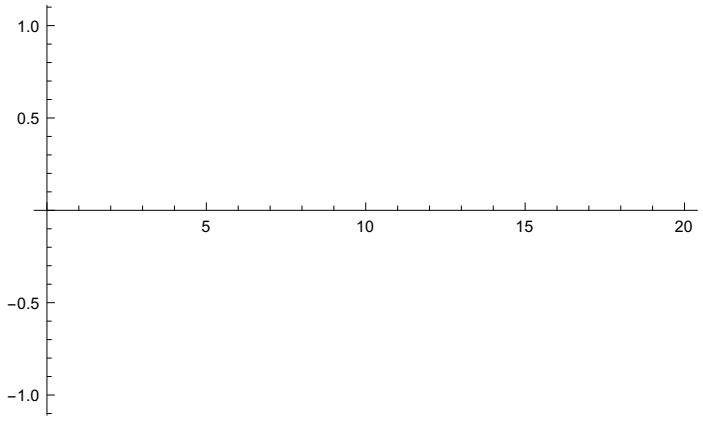
$$4 + 2\,c == a$$

```
5 + c - (4 + 2 c) == b
```

$$1 - c == b$$

Differentiability 2

```
f[x_] := Piecewise[{{a * x + b, x > 0}, {Sin[x], x ≤ 0}}]
Plot[f[x], {x, 0, 20}]
```




```
Sin[0]
```

$$0$$

Polynomials

```
D[x^10 + 3 x^5 + 2 x^3 + 4, x]
```

$$6\,x^2 + 15\,x^4 + 10\,x^9$$

```
p[x_] = Integrate[x^6 + 5 x^5 + 4 x^3, x] + 1
```

$$1 + x^4 + \frac{5\,x^6}{6} + \frac{x^7}{7}$$

```
f[x_] := { a * + x^2 + b * x + 4                  x ≤ 1
         { 5 * x^5 + 3 * x^4 + 7 * x^2 + 8 * x + 4   x > 1
```

```
x = 1;
a * + x^2 + b * x + 4 == 5 * x^5 + 3 * x^4 + 7 * x^2 + 8 * x + 4
D[a * + y^2 + b * y + 4, y]
D[5 * y^5 + 3 * y^4 + 7 * y^2 + 8 * y + 4, y]
b + 2 a x  ==  8 + 14 x + 12 x^3 + 25 x^4
```

$$4 + a + b == 27$$

$$b + 2\,a\,y$$

$$8 + 14\,y + 12\,y^3 + 25\,y^4$$

$$2\,a + b == 59$$

```
Solve[4 + a + b == 27 && 2 a + b == 59, {a, b}]
```

$$\{\{a \to 36,\ b \to -13\}\}$$

Second derivatives

```
D[3 x^2 + 2 x + 4 * √x, {x, 2}]
```

$$6 - \frac{1}{x^{3/2}}$$

```
D[-5/x + 5, {x, 2}]
```

$$-\frac{10}{x^3}$$

```
D[(x^2 + 5 x)/(x + 5), {x, 2}]
```

$$\frac{2}{5 + x} - \frac{2\,(5 + 2\,x)}{(5 + x)^2} + \frac{2\,(5\,x + x^2)}{(5 + x)^3}$$

Trig

```
D[Sin[x], {x, 103}]
```

```
-Cos[x]
```

Part B

Speedometer

```
a = 1 * π / 0.08 * 3.6
p = .22 * π / 0.08 * 3.6
```

```
141.372
```

```
31.1018
```

```
Abs[p - a] / a
```

```
0.78
```

```
f1[x_] := d * π / 0.08 * 3.6
Solve[Abs[p - f1[d]] / f1[d] == 0.05, d]
```

```
{{d → 0.209524}, {d → 0.231579}}
```

```
Abs[0.20952380952380942 - .22]
```

```
0.0104762
```

Skate Park

```
f[x_, a_, b_, c_] := a * x^2 + b * x + c
f2[x_, a_, b_] := b + 2 * a * x
Solve[f[2, a, b, c] == 1 && f[4, a, b, c] == 3 && f2[2, a, b] == -1 / 4 , {a, b, c}]
```

$$\left\{\left\{a \to \frac{5}{8}, b \to -\frac{11}{4}, c \to 4\right\}\right\}$$

```
res[x_] := 5/8 x^2 - 11/4 * x + 4
Plot[res[x], {x, 2, 4}, PlotRange → {0, 3}]
```