Unit 1: Derivatives

What is a derivative?

Rate of Change

In[ ]:= 220 - 50

Out[ ]= 170

In[ ]:= 170 / 2

Out[ ]= 85

Average vs. Instantaneous

$\frac{\text{Delta } f}{\text{Delta } t}$

In[ ]:= $\frac{1}{1/60}$

Out[ ]= 60

Instantaneous approximation continued

In[ ]:= $\frac{220000 - 210000}{32 - 30}$

Out[ ]= 5000

Derivative at a point

The Derivative of f(x) at x = a

$$f'(a) = \text{Lim}_{b\to a} \frac{f(b)-f(a)}{b-a}$$

A negative derivative?

In[ ]:= f[t_] := 100 + 20 t - 5 t^2

f '[2]

Out[ ]= 0

Geometric interpretation of the derivative

Tangent lines

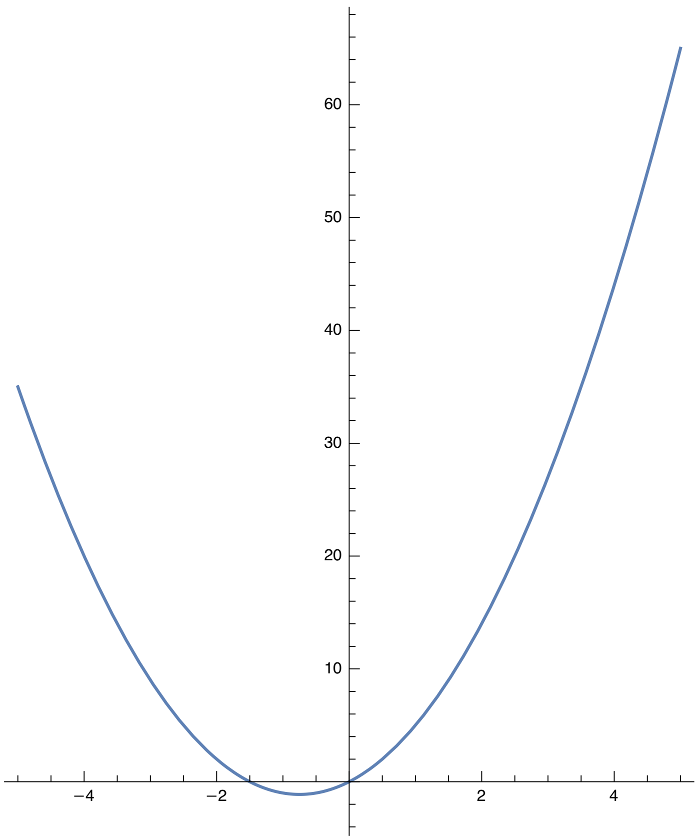
Calculated using:

y-f(a)=m(x-a)

Equation of a tangent line

In[ ]:=
```
j[x_] := 2 x^2 + 3 x
Plot[j[x], {x, -5, 5}, AspectRatio → Full]
j[1]
```

Out[ ]=



Out[ ]= 5

In[ ]:=
```
j '[1]
```

Out[ ]= 7

In[ ]:=
```
j[1]
```

Out[ ]= 5

In[ ]:=
```
Simplify[y - j[1] == j'[1] * (x - 1)]
```
Out[ ]= $7\,x == 2 + y$

Review questions

In[ ]:=
```
f[x_] := -2 x + 1
f[3]
```
Out[ ]= -5

Calculating derivatives

Linearity

In[ ]:= $h[x\_] := 1/x^2$

$$h'[x\_] := \lim_{\Delta x \to 0} \frac{1/(x^2 + \Delta x) - 1/x^2}{\Delta x}$$

$$h'[x\_] := \lim_{\Delta x \to 0} -2/x^3$$

In[ ]:=
```
h[x_] := 1 / x^2
h'[s]
```
Out[ ]= $-\frac{2}{s^3}$

Relationship between derivatives

In[ ]:= $f[x\_] := \frac{-3}{x}$

```
f'[x]
```
Out[ ]= $\frac{3}{x^2}$

Calculation

In[ ]:= $f[x\_] := 4\sqrt{x} - \frac{3}{x^2}$

```
f'[x]
```
Out[ ]= $\frac{6}{x^3} + \frac{2}{\sqrt{x}}$

Leibniz notation

$$\frac{dy}{dx} \text{ or } \frac{df}{dx}$$

Area of a circle

```
In[ ]:= A[r_] := π r^2
        A'[r]
        A'[3]
```

Out[ ]= $2\pi r$

Out[ ]= 6π

```
In[ ]:= A2[c_] := π * (c - 2 * π) ^ 2
        A2'[c]
        A2'[6 π]
```

Out[ ]= $2(c - 2\pi)\pi$

Out[ ]= $8\pi^2$

exercise

```
In[ ]:= D[g^3 + 2 g^2, g] /. g → 2
        f[x_] := x^3 + 2 x^2
        f'[2]

        3 * 2^2 + 4 * 2
```

Out[ ]= 20

Out[ ]= 20

Out[ ]= 20

Second derivatives and higher

everything lost do to power outage

Homework

Part A

Velocity

In[ ]:=
```
h[x_] := 400 - 16 x^2
(h[0] - h[2]) / (0 - 2)

Solve[h[g] == 0, g]
(h[3] - h[5]) / (3 - 5)

h'[5]
```

Out[ ]= -32

Out[ ]= {{g → -5}, {g → 5}}

Out[ ]= -128

Out[ ]= -160

Definition review

In[ ]:=
```
f[x_] := 1 / (2 x + 1)
f'[x]
```

Out[ ]= $-\frac{2}{(1+2x)^2}$

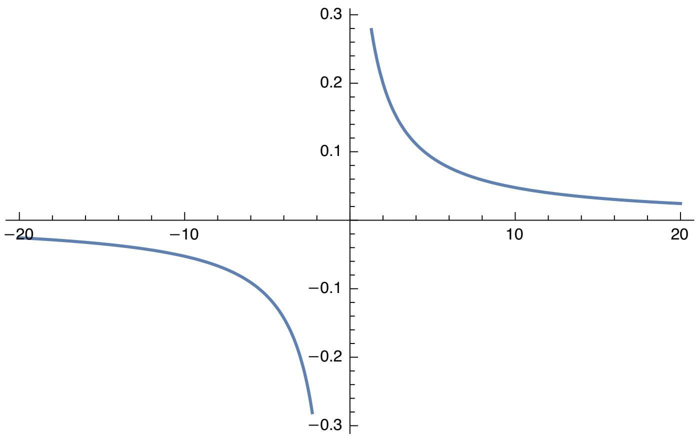
In[ ]:=
```
N[Solve[f'[x] == 1, x, Reals]]
N[Solve[f'[x] == 0, x, Reals]]
N[Solve[f'[x] == -1, x, Reals]]
```

Out[ ]= {}

Out[ ]= {}

Out[ ]= {{x → -1.20711}, {x → 0.207107}}

In[ ]:=
```
Plot[f[x], {x, -20, 20}]
```

Out[ ]=


In[ ]:=
```
g[x_] := 4 x + 5
N[Solve[g[x] == 1, x, Reals]]
N[Solve[g[x] == 0, x, Reals]]
N[Solve[g[x] == -1, x, Reals]]
```

Out[ ]= {{x → -1.}}

Out[ ]= {{x → -1.25}}

Out[ ]= {{x → -1.5}}

In[ ]:=
```
j[x_] := x^(-7/4)
j'[1]
```

Out[ ]= $-\frac{7}{4}$

Tangent line

In[ ]:=
```
f[x_] := 1 / (2 x + 1)
m := f'[1]
b := (-1) * (m * 1 - f[1])
y = m * x + b
```

Out[ ]= $\frac{5}{9} - \frac{2\,x}{9}$

Differentiability

In[ ]:= $\begin{cases} c * x^2 + 4\,x + 1 & x \geq 1 \\ a * x + b & x < 1 \end{cases}$

Out[ ]= $\begin{cases} 1 + 4\,x + c\,x^2 & x \geq 1 \\ b + a\,x & x < 1 \\ 0 & \text{True} \end{cases}$

In[ ]:=
```
x = 1;
c * x^2 + 4 * x + 1 == a * x + b
```

Out[ ]= `5 + c == a + b`

In[ ]:= `2 * c * x + 4 == a`

Out[ ]= `4 + 2 c == a`

In[ ]:= `5 + c - (4 + 2 c) == b`

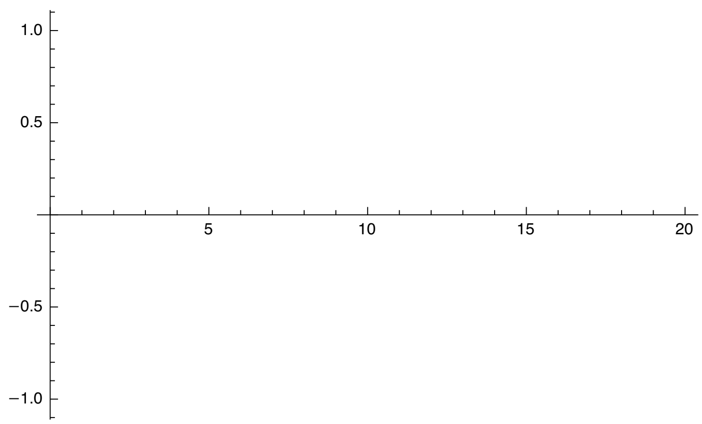
Out[ ]= `1 - c == b`

Differentiability 2

In[ ]:= f[x_] := $\begin{cases} a * x + b & x > 0 \\ \text{Sin}[x] & x \leq 0 \end{cases}$

```
Plot[f[x], {x, 0, 20}]
```

Out[ ]=



In[ ]:= `Sin[0]`

Out[ ]= `0`

Polynomials

In[ ]:= $D[x^{10} + 3 x^5 + 2 x^3 + 4, x]$

Out[ ]= $6 x^2 + 15 x^4 + 10 x^9$

In[ ]:= $p[x\_] = \text{Integrate}[x^6 + 5 x^5 + 4 x^3, x] + 1$

Out[ ]= $1 + x^4 + \frac{5 x^6}{6} + \frac{x^7}{7}$

$$f[x\_] := \begin{cases} a * + x^2 + b * x + 4 & x \leq 1 \\ 5 * x^5 + 3 * x^4 + 7 * x^2 + 8 * x + 4 & x > 1 \end{cases}$$

In[ ]:= $x = 1;$

$a * + x^2 + b * x + 4 == 5 * x^5 + 3 * x^4 + 7 * x^2 + 8 * x + 4$

$D[a * + y^2 + b * y + 4, y]$

$D[5 * y^5 + 3 * y^4 + 7 * y^2 + 8 * y + 4, y]$

$b + 2 a x == 8 + 14 x + 12 x^3 + 25 x^4$

Out[ ]= $4 + a + b == 27$

Out[ ]= $b + 2 a y$

Out[ ]= $8 + 14 y + 12 y^3 + 25 y^4$

Out[ ]= $2 a + b == 59$

In[ ]:= $\text{Solve}[4 + a + b == 27 \&\& 2 a + b == 59, \{a, b\}]$

Out[ ]= $\{\{a \to 36, b \to -13\}\}$

Second derivatives

In[ ]:= $D[3 x^2 + 2 x + 4 * \sqrt{x}, \{x, 2\}]$

Out[ ]= $6 - \frac{1}{x^{3/2}}$

In[ ]:= $D[\frac{-5}{x} + 5, \{x, 2\}]$

Out[ ]= $-\frac{10}{x^3}$

In[ ]:= $D[\frac{x^2 + 5 x}{x + 5}, \{x, 2\}]$

Out[ ]= $\frac{2}{5 + x} - \frac{2 (5 + 2 x)}{(5 + x)^2} + \frac{2 (5 x + x^2)}{(5 + x)^3}$

Trig

In[ ]:=
```
D[Sin[x], {x, 103}]
```
Out[ ]=
```
-Cos[x]
```

Part B

Speedometer

In[ ]:=
```
a = 1 * π / 0.08 * 3.6
p = .22 * π / 0.08 * 3.6
```
Out[ ]=
```
141.372
```
Out[ ]=
```
31.1018
```

In[ ]:=
```
Abs[p - a] / a
```
Out[ ]=
```
0.78
```

In[ ]:=
```
f1[x_] := d * π / 0.08 * 3.6
Solve[Abs[p - f1[d]] / f1[d] == 0.05 , d]
```
Out[ ]=
```
{{d → 0.209524}, {d → 0.231579}}
```

In[ ]:=
```
Abs[0.20952380952380942 - .22]
```
Out[ ]=
```
0.0104762
```

Skate Park

In[ ]:=
```
f[x_, a_, b_, c_] := a * x^2 + b * x + c
f2[x_, a_, b_] := b + 2 * a * x
Solve[f[2, a, b, c] == 1 && f[4, a, b, c] == 3 && f2[2, a, b] == -1 / 4 , {a, b, c}]
```
Out[ ]=

$$\left\{\left\{a \to \frac{5}{8},\ b \to -\frac{11}{4},\ c \to 4\right\}\right\}$$

In[ ]:=
```
res[x_] := 5/8 x^2 - 11/4 * x + 4
Plot[res[x], {x, 2, 4}, PlotRange → {0, 3}]
```

$$res[x\_] := \frac{5}{8} x^2 - \frac{11}{4} * x + 4$$

Out[ ]=