Unit 2: Differentiation

Linear approximation

In[ ]:= `10 * 0.4 + 150`

Out[ ]= 154.

$$\Delta f \quad \approx \frac{df}{dx}\bigg|_{x=a} \cdot \Delta x \qquad \text{for } \Delta x \text{ near } 0$$

$$f(x) \quad \approx f'(a)(x-a) + f(a) \quad \text{for } x \text{ near } a$$

In[ ]:= $D\left[\sqrt{x}, x\right]$

Out[ ]= $\frac{1}{2\sqrt{x}}$

In[ ]:= $\frac{1}{2\sqrt{100}}$

Out[ ]= $\frac{1}{20}$

In[ ]:= `4 * %`

Out[ ]= $\frac{1}{5}$

In[ ]:= `f[x_] := x^2.5`

`f'[4] * -0.03 + f[4]`

Out[ ]= 31.4

In[ ]:= $D\left[\sqrt{x}, \{x, 2\}\right]$

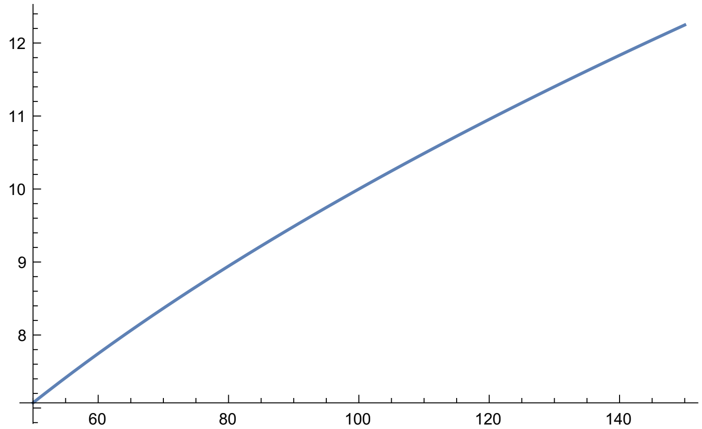
Out[ ]= $-\frac{1}{4\,x^{3/2}}$

In[ ]:= $-\frac{1}{4 \times 100^{3/2}}$

Out[ ]= $-\frac{1}{4000}$

In[•]:= Plot[$\sqrt{x}$, {x, 50, 150}]

Out[•]=



In[•]:= v[x_] := 4 π * x^3 / 3
v'[r]
v'[10] * -0.03

Out[•]= $4\pi r^2$

Out[•]= -37.6991

In[•]:= v'[10]

Out[•]= 400 π

In[•]:= 20 / 400

Out[•]= $\frac{1}{20}$

Product rule

If $h(x) = f(x)\,g(x)$, then

$$h'(x) = f(x)\,g'(x) + g(x)\,f'(x)$$

at all points where the derivatives $f'(x)$ and $g'(x)$ are defined.

In[•]:= D[$\sqrt{x}$, x] * Cos[x] + $\sqrt{x}$ * -Sin[x]

Out[•]= $\frac{\text{Cos}[x]}{2\sqrt{x}} - \sqrt{x}\ \text{Sin}[x]$

In[•]:= 100 * 0.4

Out[•]= 40.

In[ ]:= 100 * -0.01 + 3 * 0.4

Out[ ]= 0.2

f (x) = x^2 Sin[x] Cos[x]

f '[x] = 2 x * Sin[x] * Cos[x]
+ x^2 * -Cos[x] * Sin[x]
+ x^2 * Sin[x] * -Sin[x]

Quotient Rule

In[ ]:= Limit[(f2 * g - f * g2)/t, t → 0]

Out[ ]= Indeterminate

If $h(x) = \frac{f(x)}{g(x)}$ for all x, then

$$h'(x) = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{g(x)^2}$$

at all points where f and g are differentiable and $g(x) \neq 0$.

In[ ]:= D[(2 + Cos[X])/(x^2 + 1), x]

Out[ ]= $-\frac{2\,x\,(2 + \mathrm{Cos}[X])}{(1 + x^2)^2}$