Unit 1: Derivatives

What is a derivative?

Rate of Change

```
220 - 50
```

170

```
170 / 2
```

85

Average vs. Instantaneous

$$\frac{\text{Delta } f}{\text{Delta } t}$$

$$\frac{1}{1/60}$$

60

Instantaneous approximation continued

$$\frac{220\,000 - 210\,000}{32 - 30}$$

5000

Derivative at a point

The Derivative of f(x) at x = a

$$f'(a) = \text{Lim}_{b \to a} \frac{f(b) - f(a)}{b - a}$$

A negative derivative?

```
f[t_] := 100 + 20 t - 5 t^2
f'[2]
```

0

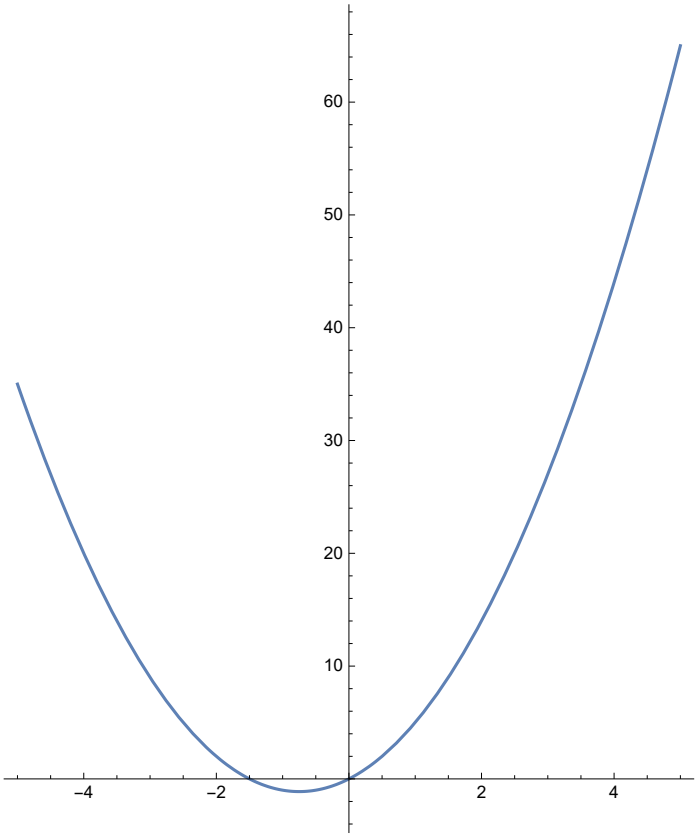
Geometric interpretation of the derivative

Tangent lines

Calculated using:
y-f(a)=m(x-a)

Equation of a tangent line

```
j[x_] := 2 x^2 + 3 x
Plot[j[x], {x, -5, 5}, AspectRatio → Full]
j[1]
```



5

```
j'[1]
```

7

```
j[1]
```

5

```
Simplify[y - j[1] == j'[1] * (x - 1)]
```

7 x == 2 + y

Review questions

```
f[x_] := -2 x + 1
f[3]
```

− 5

Calculating derivatives

Linearity

```
h[x_] := 1 / x^2
```

$$h'[x\_] := \lim_{\triangle x \to 0} \frac{1/\left(x^2 + \triangle x\right) - 1/x^2}{\triangle x}$$

$$h'[x\_] := \lim_{\triangle x \to 0} -2/x^3$$

```
h[x_] := 1 / x^2
h'[s]
```

$$-\frac{2}{s^3}$$

Relationship between derivatives

```
f[x_] := -3 / x
f'[x]
```

$$\frac{3}{x^2}$$

Calculation

```
f[x_] := 4 √x - 3 / x^2
f'[x]
```

$$\frac{6}{x^3} + \frac{2}{\sqrt{x}}$$

Leibniz notation

$$\frac{dy}{dx} \quad \text{or} \quad \frac{df}{dx}$$

Area of a circle

```
A[r_] := π r^2
A'[r]
A'[3]
```

$$2\,\pi\,r$$

$$6\,\pi$$

```
A2[c_] := π * (c - 2 * π) ^ 2
A2'[c]
A2'[6 π]
```

$$2 \left(c - 2 \pi\right) \pi$$

$$8 \pi^2$$

exercise

```
D[g^3 + 2 g^2, g] /. g → 2
f[x_] := x^3 + 2 x^2
f'[2]
```

```
3 * 2^2 + 4 * 2
```

```
20
```

```
20
```

```
20
```

Second derivatives and higher