Unit 1: Derivatives

What is a derivative?

Rate of Change

In[1]:=
```
220 - 50
```
Out[1]= 170

In[2]:=
```
170 / 2
```
Out[2]= 85

Average vs. Instantaneous

$\frac{\text{Delta } f}{\text{Delta } t}$

In[1]:= $\frac{1}{1/60}$

Out[1]= 60

Instantaneous approximation continued

In[1]:= $\frac{220\,000 - 210\,000}{32 - 30}$

Out[1]= 5000

Derivative at a point

The Derivative of f(x) at x = a

$$f'(a) = \text{Lim}_{b \to a} \frac{f(b) - f(a)}{b - a}$$

A negative derivative?

In[1]:=
```
f[t_] := 100 + 20 t - 5 t^2
f '[2]
```
Out[2]= 0

Geometric interpretation of the derivative

Tangent lines

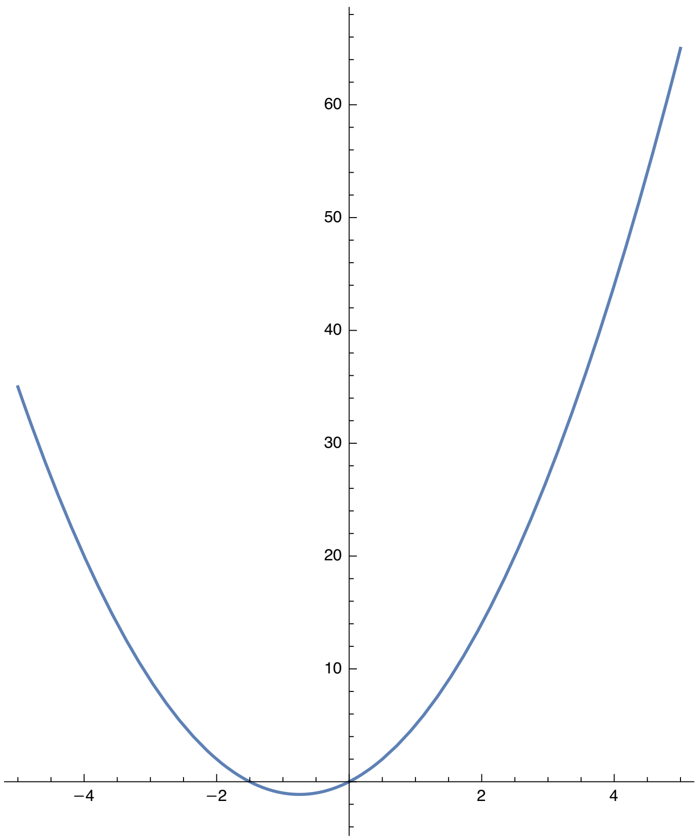
Calculated using:

y-f(a)=m(x-a)

Equation of a tangent line

In[1]:=
```
j[x_] := 2 x^2 + 3 x
Plot[j[x], {x, -5, 5}, AspectRatio → Full]
j[1]
```

Out[2]=



Out[3]= 5

In[4]:=
```
j '[1]
```

Out[4]= 7

In[5]:=
```
j[1]
```

Out[5]= 5

In[6]:=
```
Simplify[y - j[1] == j '[1] * (x - 1)]
```
Out[6]= $7x == 2 + y$

Review questions

In[1]:=
```
f[x_] := -2 x + 1
f[3]
```
Out[2]= -5

Calculating derivatives

Linearity

In[ ]:= h[x_] := $1/x^2$

$$h'[x\_] := \lim_{\Delta x \to 0} \frac{1/(x^2 + \Delta x) - 1/x^2}{\Delta x}$$

$$h'[x\_] := \lim_{\Delta x \to 0} -2/x^3$$

In[1]:=
```
h[x_] := 1/x^2
h'[s]
```
Out[2]= $-\frac{2}{s^3}$

Relationship between derivatives

In[1]:= f[x_] := $\frac{-3}{x}$

```
f'[x]
```
Out[2]= $\frac{3}{x^2}$

Calculation

In[1]:= f[x_] := $4\sqrt{x} - \frac{3}{x^2}$

```
f'[x]
```
Out[2]= $\frac{6}{x^3} + \frac{2}{\sqrt{x}}$

Leibniz notation

$$\frac{dy}{dx} \text{ or } \frac{df}{dx}$$

Area of a circle

In[1]:=
```
A[r_] := π r^2
A'[r]
A'[3]
```

Out[2]= $2\pi r$

Out[3]= 6π

In[4]:=
```
A2[c_] := π * (c - 2 * π) ^ 2
A2'[c]
A2'[6 π]
```

Out[5]= $2(c - 2\pi)\pi$

Out[6]= $8\pi^2$

exercise

In[1]:=
```
D[g^3 + 2 g^2, g] /. g → 2
f[x_] := x^3 + 2 x^2
f'[2]

3 * 2^2 + 4 * 2
```

Out[1]= 20

Out[3]= 20

Out[4]= 20

Second derivatives and higher

everything lost do to power outage

Homework

Part A

Velocity

In[1]:=
```
h[x_] := 400 - 16 x^2
(h[0] - h[2]) / (0 - 2)

Solve[h[g] == 0, g]
(h[3] - h[5]) / (3 - 5)

h'[5]
```

Out[2]= -32

Out[3]= {{g → -5}, {g → 5}}

Out[4]= -128

Out[5]= -160

Definition review

In[1]:=
```
f[x_] := 1 / (2 x + 1)
f'[x]
```

Out[2]= $-\frac{2}{(1+2x)^2}$

In[3]:=
```
N[Solve[f'[x] == 1, x, Reals]]
N[Solve[f'[x] == 0, x, Reals]]
N[Solve[f'[x] == -1, x, Reals]]
```

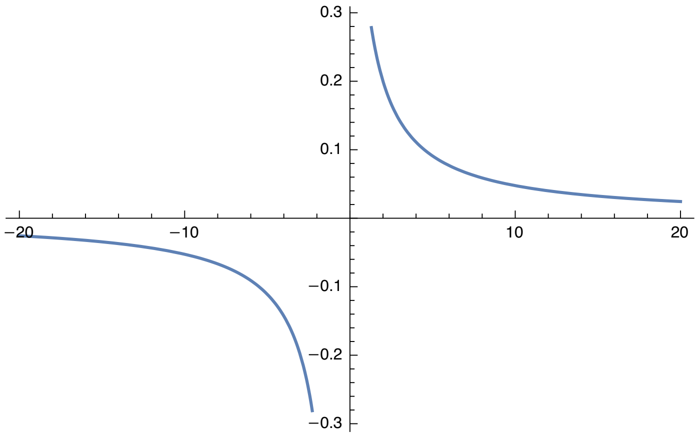
Out[3]= {}

Out[4]= {}

Out[5]= {{x → -1.20711}, {x → 0.207107}}

In[6]:=
```
Plot[f[x], {x, -20, 20}]
```

Out[6]=



In[7]:=
```
g[x_] := 4 x + 5
N[Solve[g[x] == 1, x, Reals]]
N[Solve[g[x] == 0, x, Reals]]
N[Solve[g[x] == -1, x, Reals]]
```

Out[8]= {{x → -1.}}

Out[9]= {{x → -1.25}}

Out[10]= {{x → -1.5}}

In[11]:=
```
j[x_] := x^(-7/4)
j'[1]
```

Out[12]= $-\frac{7}{4}$

Tangent line

In[1]:=
```
f[x_] := 1 / (2 x + 1)
m := f'[1]
b := (-1) * (m * 1 - f[1])
y = m * x + b
```

Out[4]= $\frac{5}{9} - \frac{2x}{9}$

Differentiability

In[1]:=
$$\begin{cases} c * x^2 + 4 x + 1 & x \geq 1 \\ a * x + b & x < 1 \end{cases}$$

Out[1]=
$$\begin{cases} 1 + 4 x + c x^2 & x \geq 1 \\ b + a x & x < 1 \\ 0 & \text{True} \end{cases}$$

In[2]:= $x = 1;$

$c * x^2 + 4 * x + 1 == a * x + b$

Out[3]= $5 + c == a + b$

In[4]:= $2 * c * x + 4 == a$

Out[4]= $4 + 2 c == a$

In[5]:= $5 + c - (4 + 2 c) == b$

Out[5]= $1 - c == b$