

Problem 42. 2012.01.05

Show that

$$\int_0^{\frac{\pi}{4}} \sin(2x) \ln(\cos x) dx = \frac{1}{4}(\ln 2 - 1),$$

and that

$$\int_0^{\frac{\pi}{4}} \cos(2x) \ln(\cos x) dx = \frac{1}{8}(\pi - \ln 4 - 2).$$

Hence evaluate

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\cos(2x) + \sin(2x)) \ln(\cos x + \sin x) dx.$$

Prerequisites.

You will need to apply Integration by Parts initially but other techniques including Integration by Substitution, the use of Harmonic Form and elementary properties of integrals will also be required. A particular result that sometimes causes some grief amongst students is the result for $\int \tan x dx$. You will need it here and if you are unsure of this result you can find it in the Key Learning Points at the end of the problem.

First Thoughts.

This looks like something to do with Integration by parts but it also looks as if something more might be needed. I will have to dip my toes in the water, explore and learn to swim!