Problem 20.   2013.01.08

(i) The functions a, b, c and d are defined by

$$a(x) = x^2 \qquad (-\infty < x < \infty)$$
$$b(x) = \ln x \qquad (x > 0)$$
$$c(x) = 2x \qquad (-\infty < x < \infty)$$
$$d(x) = \sqrt{x} \qquad (x \geq 0)$$

Write down the following composite functions, giving the domain and range of each:

$$cb, \qquad ab, \qquad da, \qquad ad.$$

(ii) The functions f and g are defined by

$$f(x) = \sqrt{x^2 - 1} \qquad (x \geq 1)$$
$$g(x) = \sqrt{x^2 + 1} \qquad (-\infty < x < \infty)$$

Determine the composite functions fg and gf, giving the domain and range of each.

(iii) Sketch the graphs of the functions h and k defined by

$$h(x) = x + \sqrt{x^2 - 1} \qquad (x \geq 1),$$
$$k(x) = x - \sqrt{x^2 - 1} \qquad (|x| \geq 1),$$

justifying the main features of the graphs, and giving the equations of any asymptotes. Determine the domain and range of the composite function kh.

Prerequisites.

You need a clear understanding of the constituent parts of a function, of the characteristics of a function, the requirements for the existence of a composite function as well as the means by which the composite function is determined. Although a detailed discussion of graph sketching appears as late as Chapter 14 you will need to draw upon many of the skills that are discussed in that section.

First Thoughts.

This calls for care in the construction of the various expressions for the composite functions and for even greater care when it comes to consideration of the various domains and ranges. Proceed with caution.