Problem 6. **2010.01.01**

Given that

$$5x^2 + 2y^2 - 6xy + 4x - 4y \equiv a(x - y + 2)^2 + b(cx + y)^2 + d$$

find the values of the constants a, b, c and d.

Solve the simultaneous equations

$$5x^2 + 2y^2 - 6xy + 4x - 4y = 9$$
$$6x^2 + 3y^2 - 8xy + 8x - 8y = 14$$

Prerequisites.

Before starting this question you should be able to solve simple sets of simultaneous equations.

First Thoughts.

This problem may not be the last word on testing your ability to solve simultaneous equations, but it surely comes close. To begin with, you will need to draw upon your knowledge of the properties of identities.

Just in case you need a reminder, let's look at something specific:

For those who are not very familiar with identities consider the problem of writing $(3 + \sqrt{5})^2$ in the form $a + b\sqrt{5}$ where a and b are integers. We need to identify the values of a and b. If we set up an identity and expand the left hand side we have

$$(3 + \sqrt{5})^2 \equiv a + b\sqrt{5}$$
$$3^2 + 6\sqrt{5} + 5 \equiv a + b\sqrt{5}$$
$$14 + 6\sqrt{5} \equiv a + b\sqrt{5}$$

The expressions on each side of the identity sign both represent a number that is the sum of a rational part and an irrational part. For the expressions to be identical, the rational parts must be equal and the irrational parts must be equal. This is because it is not possible for a rational number to be equal to an irrational number. We can therefore compare the rational parts and the irrational parts separately in order to extract a pair of simultaneous equations from which we can find the two unknowns. In this simple case the simultaneous equations give us the solutions immediately.

Therefore, $a = 14$ and $b = 6$.

Now for the problem: