

repeating myself, the only type of function that can have an inverse is a function that is one to one and onto.

If a function  $f: A \rightarrow B$  is not an "onto" function, then there are elements of the codomain  $B$  that are not images of any element of the domain  $A$ . When the mapping is reversed,  $f^{-1}$  operates to map elements of  $B$  back to set  $A$ . But, as there are elements of  $B$  upon which  $f^{-1}$  does not act  $f^{-1}$  is simply a reverse mapping but does not qualify as a function. See case 1 above.

Now look at the next problem. I chose to include this one to try to illustrate how important it is that you should know the characteristics of the functions that we use in solving problems.

**Problem 19.** 2000.01.01

To nine decimal places,  $\log_{10} 2 = 0.301029996$  and  $\log_{10} 3 = 0.477121255$ .

(i) Calculate  $\log_{10} 5$  and  $\log_{10} 6$  to three decimal places. By taking logs, or otherwise, show that  $5 \times 10^{47} < 3^{100} < 6 \times 10^{47}$ .

Hence, write down the first digit of  $3^{100}$ .

(ii) Find the first digit of each of the following numbers:  $2^{1000}$ ,  $2^{10000}$  and  $2^{100000}$ .

Prerequisites.

You will require a knowledge of the Laws of Logarithms and of the characteristics of log functions. There is a summary of these laws and of the characteristics of log functions at the start of Chapter 11.

First Thoughts.

The first bit of part (i) looks easy. I'm not sure why we need nine decimal places to start with. It isn't necessary for the first bit of part (i) so it must have something to do with the other parts of the question. We shall see!