Problem 8. **2012.02.06**

A cyclic quadrilateral ABCD has sides AB, BC, CD, and DA of lengths a, b, c and d respectively. The area of the quadrilateral is Q, and angle DAB is θ.

Find an expression for $\cos\theta$ in terms of a, b, c, d and Q. Hence show that

$$16Q^2 = 4(ad + bc)^2 - (a^2 + d^2 - b^2 - c^2)^2$$

and deduce that

$$Q^2 = (s - a)(s - b)(s - c)(s - d)$$

where $s = \frac{1}{2}(a + b + c + d)$.

Deduce a formula for the area of a triangle with sides of length a, b and c.

Prerequisites.

You will need to use some of the elementary geometry of a cyclic quadrilateral, the cosine rule and the identity $\cos^2\theta + \sin^2\theta = 1$.

First Thoughts.

Draw a diagram.