

**Problem 3.** 2004.01.01

- (i) Express  $(3 + 2\sqrt{5})^3$  in the form  $a + b\sqrt{5}$  where  $a$  and  $b$  are integers.
- (ii) Find the positive integers  $c$  and  $d$  such that  

$$\sqrt[3]{99 - 70\sqrt{2}} = c - d\sqrt{2}.$$
- (iii) Find the two real solutions of  $x^6 - 198x^3 + 1 = 0$ .

Prerequisites.

Some knowledge of rational and irrational numbers would be advantageous. The properties of identities and familiarity with factorising the difference of two squares are definitely required. Students with a good pass at GCSE should have the basic requirements.

First Thoughts.

Part (i) looks easy. I'll have a go!

Part (ii) looks easy but I'll bet it isn't. (It is when you hit on the right method, but try the usual thing first just to see how horrible it can be. You will probably stop at the second or third line of working and rightly so.)

Part (iii). Sixth order? By all accounts, there isn't a formula for 5<sup>th</sup> or higher order polynomial equations so this one must be special somehow. Ah yes, I see; it's a quadratic equation in  $x^3$ . I can't see what it has to do with parts (i) and (ii), but since it's a STEP question the probability that there is a connection is very high. Time will tell ...