Chapter 4 : Adding Up.

When I introduce this problem to a new intake of students I usually tell them "Today we are going to do some adding-up". The looks of disbelief, amazement and even pity for the poor deranged teacher have to be seen to be appreciated. In this problem we refer to "the greatest integer <u>function</u>" throughout. At the end of the problem, "adding up" has taken on a whole new meaning for the unsuspecting class.

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The square bracket notation [x] means the greatest integer less than or equal to x. For example, $[\pi] = 3, \left[\sqrt{24}\right] = 4$ and $[5] = 5$.

(i) Sketch the graph of $\sqrt{[x]}$ and show that

$$\int_0^a \sqrt{[x]}\,dx = \sum_{r=0}^{a-1} \sqrt{r}$$

when a is a positive integer.

(ii) Show that

$$\int_0^a 2^{[x]}\,dx = 2^a - 1$$

when a is a positive integer.

(iii) Determine an expression for

$$\int_0^a 2^{[x]}\,dx$$

when a is positive but not an integer.

Prerequisites.

You will need to know the formula for the sum of n terms of a geometric progression.

First Thoughts.

This seems fairly unfamiliar. It looks as if the best thing I can do is some careful point plotting and see how things turn out. Then I will just have to make things up as I go along.