Problem 22. 2011.02.07

The two sequences $a_0,\ a_1,\ a_2, \ldots$ and $b_0,\ b_1,\ b_2, \ldots$ have general terms

$$a_n = \lambda^n + \mu^n \text{ and } b_n = \lambda^n - \mu^n$$

respectively, where

$\lambda = 1 + \sqrt{2}$ and $\mu = 1 - \sqrt{2}$.

(i) Show that

$$\sum_{r=0}^{n} b_r = -\sqrt{2} + \frac{1}{\sqrt{2}} a_{n+1}$$

and give a corresponding result for

$$\sum_{r=0}^{n} a_r$$

(ii) Show that, if n is odd,

$$\sum_{m=0}^{2n} \left(\sum_{r=0}^{m} a_r \right) = \frac{1}{2} b_{n+1}^2$$

and give a corresponding result when n is even.

(iii) Show that, if n is even,

$$\left(\sum_{r=0}^{n} a_r \right)^2 - \sum_{r=0}^{n} a_{2r+1} = 2,$$

and give a corresponding result when n is odd.

Pre-requisites.

1. The theory of Geometric Progressions including the result for the sum to infinity,
2. the definition of a sequence,
3. a knowledge of the $\sum$ notation. (See page 45).

First Thoughts.

I've got all the pre-requisites so I'll just have to try it and see where it leads. I think I'll work algebraically with λ and μ rather than put in their values otherwise the working will be too long and look too complicated.