I would have spared you all this technical detail but for the fact that the issue of inverse functions appears in STEP problem 2015.2.4. It is true that the outline solution given is sufficient to answer the problem and gain full marks but ignoring the technical matters is not good practice. The reasons for the existence or non-existence of inverses are important and the reference to the principal branch of a function will appear again in Complex Analysis. As one of my aims is to prepare you as fully as possible to read Mathematics at university, I would be failing in that aim if I omitted these details.

Problem 82. 2015.02.04

(i) The continuous function f is defined by

$$\tan f(x) = x \quad (-\infty < x < \infty)$$

and $f(0) = \pi$. Sketch the curve $y = f(x)$.

(ii) The continuous function g is defined by

$$\tan g(x) = \frac{x}{1+x^2} \qquad (-\infty < x < \infty)$$

and $g(0) = \pi$. Sketch the curves $y = \frac{x}{1+x^2}$ and $y = g(x)$.

(iii) The continuous function f is defined by $h(0) = \pi$ and

$$\tan h(x) = \frac{x}{1-x^2} \qquad (x \neq \pm 1).$$

(The values of $h(x)$ at $x = \pm 1$ are such that $h(x)$ is continuous at these points.)
Sketch the curves

$$y = \frac{x}{1-x^2} \text{ and } y = h(x).$$

Prerequisites.

1. Experience of sketching discontinuous graphs and the use of left and right hand limits near discontinuities.
2. Knowledge of the graph of the inverse tangent function.
3. An in depth knowledge regarding the conditions that determine the existence of inverse functions.

First Thoughts.

This is clearly something to do with the inverse tan function but it is specified in a somewhat unusual manner. There must be a reason for this that I'm not clear about just yet so I will have to expect something unusual.