EXERCISE BOOK

Name: Abdul Musthakin **Class:**
Topic: **Teacher:**

UKMT SMC 2012

1) E ✓

2) B ✓

3) D ✓

4) B ✓

5) C ✓

6) C ✓

7) B ×D

8) A × C

9) C ✓

10) E ✓

11) D

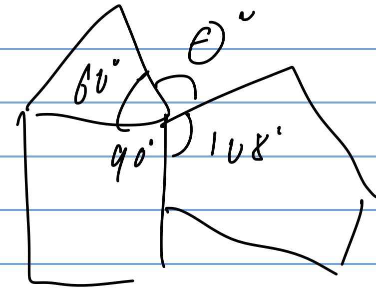
12) D ×E

13) A ×B

14) D ✓

15) B ×A

16) A ✓



$70+4\cdot 5$
$+6\cdot 7 =$
$70+20+12$
$=90+17$
$=107$

$360-60$
$-90-108$
$=210-108$
$=102$

$\frac{180(5-2)}{5}$
$=\frac{180\times 3}{5}$
$=\frac{540}{5}$
$=108$

$(10^2\cdot 10^3\cdot 10^6)^2$
$(10^{11})^2$
$=10^{22}$

$44+86+75$
$=180+75$
$=255$

$98+76+54$
$=98+130$
$=228$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 2 \\ 1 & -1 & -1 & 3 \end{array}\right]$$

$R_3 - R_2 - SR_3$
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 2 \\ 0 & -2 & 0 & 1 \end{array}\right]$$

4444 $_{10}$

5775

$y=-1/2$

$\frac{2012}{9}$

$9\,\overline{)\,2012}$ 0223 r 5

$N = 9\overset{223}{\cdots}95$

$N+1 = 9\cdots 96$

$\frac{5}{2}\cdot 1\cdot\frac{1}{2}$
$=\frac{5}{4}$

$x-\frac{1}{2}-1=1$
$x=2+\frac{1}{2}$

$= \frac{5}{2}$

$R_3 \to R_2$
$R_2 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -\frac{1}{2} \\ 1 & 1 & -1 & 2 \end{array}\right]$$

$R_3 - R_1 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & -2 & 1 \end{array}\right]$$

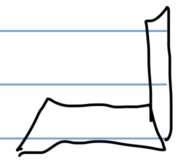
1 : 1
2 : 1+1, 2
3 : 1+1, 1+2, 1+1+1, 3

1+1+1+1+1
1+1+1+2
1+2+2
1+1+3
1+4
2+3
5

17) B
18) A
19) E
20) E
21) D
22) B
23) C
24) B
25) B

$z = -\frac{1}{2}$
$x - \frac{1}{2} - \frac{1}{2} = 1$
$x - 1 = 1$
$x = 2$
$2 \cdot -\frac{1}{4} = -\frac{1}{2}$

E G
G S
F S
G F
F E
S E



$\left(\frac{3}{6} \cdot \frac{2}{5}\right) 4$

$= 4\left[\frac{6}{30}\right]$

$= 4\left[\frac{1}{5}\right]$

$= \frac{4}{5}$

$\frac{n!}{k!(n-1)!}$

$\frac{1}{1(0)!}$

$= 1$

$\frac{2!}{2!(2!)} = 6$

$\frac{2!}{2!(1)!} = \frac{4}{4} = 1$

10 21 36 55 79

3 7 11 14 19 27

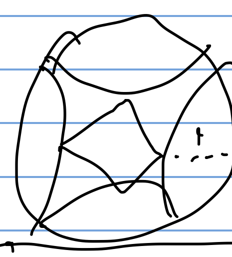
$S = \frac{d}{t}$

$1.1 S = \frac{1.1 d}{t}$

10h

10 km/h 100 km
11 km/h 100 km

$\frac{100}{11}$



Marks: (11×4) − 5 + 25

= 44 + 20

$= \boxed{64/125}$

$x = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - 2\frac{1}{2}\frac{1}{2}\cos(360-180-20)}$

$= \sqrt{\frac{1}{2} - \frac{1}{2}\cos(180-20)}$

Area:

$-6 \quad + \quad -$

$\frac{1}{2}\left[6[3-4] + -2[3-4]\right]$

$\frac{d}{1.1S} = t - x$

$9 + 12 + 3 + 4$

$= 28$

$x = 0: \quad y^2 - 4y = 12$

$(y-6)(y+2) = 0$

(0, 6)

(0, −2)

$y = 0:$

$x^2 + x = 12$

$(x-3)(x+4) = 0$

(3, 0) (−4, 0)

$S_1 = \frac{d_1}{t_1} \quad S_2 = \frac{d_2}{t_2}$

$S_2 = 1.1 S_1 \quad t_2 = t_1 - x$

$d_1 = d_2 = d \qquad d = s_1 t_1 = s_2 t_2$

$\cancel{s_1} t_1 = 1.1 \cancel{s_1} (t_1 - x)$

$t_1 = 1.1 t_1 - x$

$t_1 - 1.1 t_1 = -x$

$t_1(-0.1) = -x$

$t = \frac{x}{0.1}$

$= 10x$

$a + b + 2\sqrt{ab} = 17 + 12\sqrt{2}$

$17 = a + b, \quad ab = 72$

$a = 17 - b$

$b(17 - b) = 72$

$b^2 - 17b + 72 = 0$

$$\begin{array}{r} 17 \\ \times\ 17 \\ \hline 119 \\ 170 \\ \hline 289 \end{array}$$

$D = 17^2 - 4(72)$

$= 289 - 288$

$= 1$ ✓

$D = 54^2 - 288$

$= 1928$

$= 2^2 \cdot 482$

$= 2^3 \cdot 241$

$$\begin{array}{r} 54 \\ \times\ 54 \\ \hline 216 \\ 2700 \\ \hline 2916 \end{array} \qquad \begin{array}{r} 2916 \\ -\ 288 \\ \hline 1928 \end{array}$$

$$\begin{array}{r} 0482 \\ 4\,\overline{)1928} \end{array}$$

5: 1d : 1

5d : 1

4d : 4

3d : 3

2d : 4

1112
1121
1211
2111

122
212
221

113
131
311

23
32
14
41

$\cancel{x^2} - 16$

$= \cancel{x^2} + 2xy + y^2$

$y^2 + 2xy + 16 = 0$

$D = 4x^2 - 64$

Solutions

4) I should have gotten this question right. I obtained:

$x = 2$

$y = -\frac{1}{2}$

$z = -\frac{1}{2}$

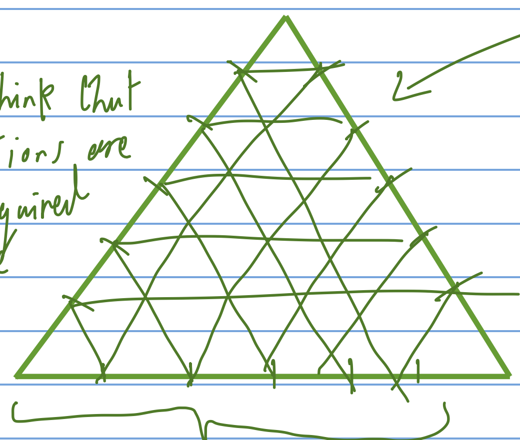
$\Rightarrow xyz = 2 \cdot -\frac{1}{2} \cdot -\frac{1}{2} = \frac{1}{2}$

However, I got $-\frac{1}{2}$, so my mistake with this question was the final calculation.

8) I attempted this question using crude visualisations.* There is a proper way to approach it, however.

An equilateral triangle may be divided into n^2 equilateral triangles, where $n \in \mathbb{Z}$ e.g. 1, 4, 9, ...

pf:



The smaller triangles are all equilateral and identical to each other

side divided into k parts; k is a positive integer

1st Row: 1
2nd Row: 3
3rd Row: 5
⋮
kth Row: $2k-1$

Total no. of triangles with k divisions:

$$\sum_{i=1}^{k} 2i-1 = \frac{\cancel{2}k(k-1)}{\cancel{2}} - k$$

$$= \boxed{k^2}$$

Q.E.D.

* I think that visualisations are still required for the final step.

So, if $3n$, where n is the number of triangles in one of those shapes, is a perfect square, then the shape could possibly be valid. Only the shapes with 3 and 12 triangles

fit this; visualisations can show that both of them would work.
So 2 of the shapes work, and the answer is C.

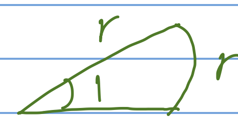
10) Whilst I answered this question correctly, I did not do so properly. I must have read the question wrong, reading the part where it says smallest possible integer as it saying largest possible integer integer instead. If I read the question correctly, my method would have have similar. The main difference is that the 5 would have been in front of the 9's:

$$N = 5\underbrace{9\ldots\ldots9}_{223} \Rightarrow N+1 = 6\underbrace{0\ldots0}_{223}$$

So the first digit of $N+1$ is 6, and my answer is still E.

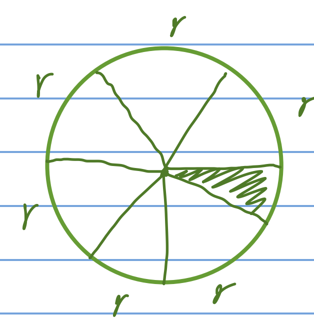
11) I left this question unanswered, because I did not believe I could find a way to approach it within a reasonable amount of time.

radius of circle: r
slant height: r



cone can be opened up into a sector with radius r and arc length r.
θ:

$$\frac{\theta}{\cancel{2\pi}}\cdot \cancel{2\pi} r = r \Rightarrow \theta = 1$$



$3r < \pi r < 3.5r$
$6r < 2\pi r < 7r$

arc length of circle

$\therefore$ max no. is 6, so answer is D.

12) I must have simply listed out all the possible sums wrong, as that is the method provided by the UKMT.

5

1+4, 4+1, 2+3, 3+2

1+1+3, 1+3+1, 3+1+1, 1+2+2, 2+1+2, 2+2+1

1+1+1+2, 1+1+2+1, 1+2+1+1, 2+1+1+1

1+1+1+1+1

However, it is the case that the possible sums for some number n is 2^{n-1}. I guess I could have guessed this from the values for $n=1$, $n=2$, and $n=3$. I do not think I would have in a reasonable amount of time. It is likely the case that there are other formulae which work (at least) up to $n=4$. Regardless, for $n=5$, the number of different ways is 16; the answer is E.

13) I could not figure this one out until I watched an animation of it on YouTube: https://m.youtube.com/watch?v=QLt0p89kdv8

After watching the animation, I can see why the correct answer was what it was. I am not sure how I made the mistake I did (thinking the second to last pointed number was 14 instead of 20). Using the correct numbers, the sum is 80, and the answer is B.

15) I am not sure how I got this one wrong. Afterwards, I was easily able to obtain the correct answer, which is $11x$. The answer is A.

$d = st$

$s_1 t_1 = s_2 t_2$

$\cancel{s_1} t_1 = 1.1 \cancel{s_1} t_2$

$t_1 = 1.1 t_2$

$t_1 - x = t_2$

$1.1 t_2 - x = t_2$

$0.1 t_2 = x$

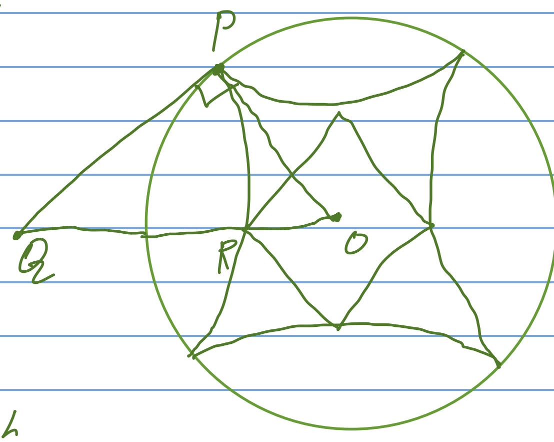
$t_2 = 10x$

$t_1 = 1.1 \cdot 10x$

$= \underline{11x}$

17) The solution to this question does not appear to be too difficult for more me to find out by myself.

O is the centre of the square and circle
Q is the centre of a quarter circle
P is where two quarter circles meet
R is vertex of square on the line OQ



$\overline{OP} = \overline{PQ} = 1$ (via what is given in the question)

$O\hat{P}Q = \frac{\pi}{2}$ ← The fact that OQ goes through vertex of square, $P\hat{O}Q$ is half of the angle of a square ($\frac{\pi}{2}$). This triangle is isoceles (two sides same length), so $O\hat{Q}P$ is $\frac{\pi}{4}$ and $O\hat{P}Q$ is $\frac{\pi}{2}$ — right angle

$\overline{OQ} = \sqrt{2}$ (pythag.)

$\overline{OR} = \sqrt{2} - 1 \because \overline{QR} = 1$ ← radius of quarter circle

diagonal of square:
$2\sqrt{2} - 2$

$l = \frac{2\sqrt{2} - 2}{\sqrt{2}}$

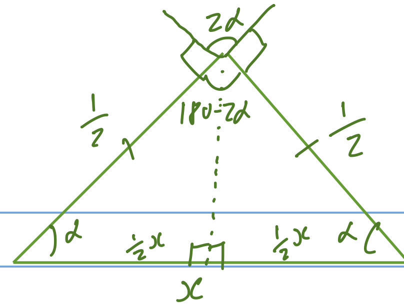
square of length l: $d = \sqrt{l^2 + l^2} = \sqrt{2l^2}$
$= l\sqrt{2}$

d is diagonal

$= \frac{4 - 2\sqrt{2}}{2} = \boxed{2 - \sqrt{2}}$

Answer is B.

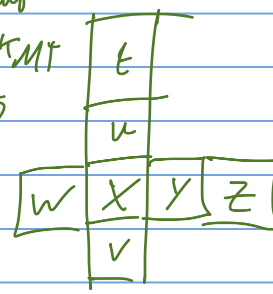
18) This question does not appear that difficult at all. I believe I could have done it by myself quite easily given a reasonable amount of time. Perhaps, the stress of being timed got to me.



$\cos(\alpha) = (\frac{1}{2}x)/(\frac{1}{2}) = x$

$\Rightarrow x = \cos(\alpha)$; the answer is A.

19) The solution does not appear to be that difficult. Of course, that is the point of UKMT questions. They are supposed to get you to think. I am not sure whether I would have been able to figure out this solution on my own, however.



$$[t+u+x+v] + [w+x+y+z] = 21+21 = 42$$

$$\underbrace{[t+u+v+w+y+z+x]}_{} + x = 42$$

$2+3+4+5+6+7+8 = 35$ $\qquad 35 + x = 42$

$\Rightarrow x = 7$

possible choices

$t+u+x+v = 21$

$t+u+v = 21 - x = 21 - 7 = 14$

$3+5+6,\ 8+4+2$ ← only valid combinations

if $8, 4, 2, 7$: 6 different permutations

6 for horizontal and vertical each $\Rightarrow 6 \times 6 = 36$ combinations

Total number of different combinations is 72, so answer is E.

Same if $3, 5, 6, 7$

$$P(3,3) = \frac{3!}{(3-3)!} = \frac{6}{1} = 6$$

20)