STEP Practice: Problem 2 (2013.01.01)

i) $x + 3\sqrt{x} - \frac{1}{2} = 0$ Let $y = \sqrt{x}$, $y \geq 0$

$y^2 + 3y - \frac{1}{2} = 0$

$2y^2 + 6y - 1 = 0$

$y = \dfrac{-6 \pm \sqrt{36+8}}{4} = \dfrac{-6 \pm 2\sqrt{11}}{4} = \dfrac{-3 \pm \sqrt{11}}{2} = \dfrac{-3+\sqrt{11}}{2}$ $\because y \geq 0$

$x = y^2 = \dfrac{[-3+\sqrt{11}]^2}{4} = \dfrac{9 - 6\sqrt{11} + 11}{4} = \dfrac{10 - 3\sqrt{11}}{2}$

ii)

a) $x + 10\sqrt{x+2} - 22 = 0$ Let $y = \sqrt{x+2}$, $y \geq 0$ $\rightsquigarrow x = y^2 - 2$

$y^2 - 2 + 10y - 22 = 0$

$y^2 + 10y - 24 = 0$

$[y+12][y-2] = 0$

$y = 2 \quad \because y \geq 0$

$x = y^2 - 2 = 4 - 2 = 2$

b) $x^2 - 4x + \sqrt{2x^2 - 8x - 3} - 9 = 0$ Let $y = x^2 - 4x$

$y + \sqrt{2y-3} - 9 = 0$ Let $z = \sqrt{2y-3}$, $z \geq 0$ $\rightsquigarrow y = \dfrac{z^2+3}{2}$

$\dfrac{z^2+3}{2} + z - 9 = 0$

$z^2 + 3 + 2z - 18 = 0$

$z^2 + 2z - 15 = 0$

$[z+5][z-3] = 0$

$z = 3 \quad \because z \geq 0$

$y = \dfrac{z^2+3}{2} = 6$

$x^2 - 4x = 6$

$x^2 - 4x - 6 = 0$

$x = \dfrac{4 \pm \sqrt{16+24}}{2} = \dfrac{4 \pm 2\sqrt{10}}{2} = 2 \pm \sqrt{10}$

Notes

This question, like many other STEP questions, use the principle of teaching you a method in the first part, which is then to be adapted to complete the later parts. Here, it is a substitution that needs to be altered in order to solve the respective equation. In this question, you must be aware of the nature of the square function — it can only output nonnegative numbers. Checking solutions via substituting them into the original equation never hurts, but considering the way we manipulated the given equations, we can be sure that our solutions are not extraneous.