Problem 59. **1998.03.04**

Show that the equation (in plane polar coordinates) $r = \cos\theta$, for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ represents a circle.

Sketch the curve $r = \cos 2\theta$ for $0 \leq \theta \leq 2\pi$, and describe the curve $r = \cos 2n\theta$, where n is an integer. Show that the area enclosed by such a curve is independent of n.

Sketch also the curve $r = \cos 3\theta$ for $0 \leq \theta \leq 2\pi$.

First Thoughts.

Well this sounds easy. I know all about the connection between Cartesian and Polar coordinates, it's easy to work out even if I couldn't remember it. In addition, I've spent quite a lot of time playing with graph drawing software so I know what most of these simple polar curves look like. $2n$ would be the number of lobes that the curve has. I didn't know the total area would be independent of n. I suppose the mechanism must be that the area of each lobe diminishes as the number of lobes increases but the enclosed area remains the same. All I need now is an integral for finding the area of one lobe and the relevant limits.