Calculus I: Review of Exponential and Logarithmic Functions

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I Exponential Functions

Exponential functions appear quite often in Calculus. Let b be a constant and satisfy $b > 0$ and $b \neq 1$. Then, we may define an exponential function $f(x)$ of some variable x as:

$$f(x) = b^x \tag{1}$$

The reason we are avoiding $b = 1$ is because that would be a constant function, equivelant to $f(x) = 1$. Having $b = 0$ would also lead to a constant function as well. Having a negative number as the base of an exponential function would require the codomain to be $\mathbb{C}$, the set of complex numbers. Let's take $b = -2$ as an example. If $f(x) = (-b)^x$, $f(x)$ would be real for x values such as $x = 2$ ($f(x) = 4$), it would be complex for x values such as $x = \frac{1}{2}$ ($f(x) = i\sqrt{2}$). We will avoid this by only allowing b to be greater than 0.

Let's take a look at some exponential functions.

Problem 1. Sketch the graph of $f(x) = 2^x$ and $g(x) = \left(\dfrac{1}{2}\right)^x$.

x	$f(x)$	$g(x)$
-2	$\frac{1}{4}$	4
-1	$\frac{1}{2}$	2
0	1	1
1	2	$\frac{1}{2}$
2	4	$\frac{1}{4}$

Let's first create a table of values for the two functions.

Now we may sketch them.

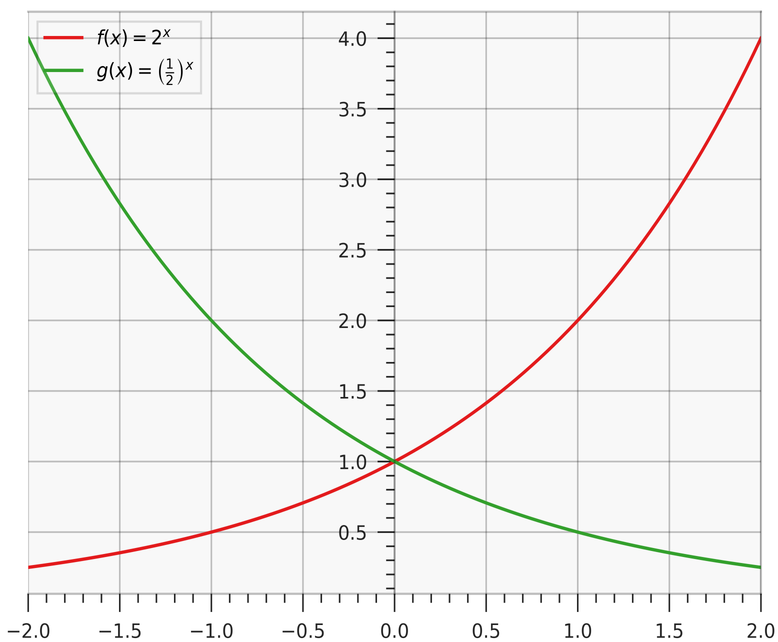

Figure 1: Exponential Functions

The graph above highlights some useful properties of exponential functions. Assuming the equations given above are satisfied, the properties of $f(x) = b^x$ are:

1. $f(0) = 1$. An exponential function will always be 1 at $x = 0$.
2. $f(x) \neq 0$. The function will never be equal to 0.
3. $f(x) > 0$. The function will always be positive.
4. The range of an exponential function is $(0, \infty)$.
5. The domain of an exponential function is $(-\infty, \infty)$.
6. If $0 < b < 1$, then:
   (a) $f(x) \to 0$ as $x \to \infty$
   (b) $f(x) \to \infty$ as $x \to -\infty$
7. If $b > 1$, then:
   (a) $f(x) \to \infty$ as $x \to \infty$
   (b) $f(x) \to 0$ as $x \to -\infty$

These are useful properties to remember throughout a Calculus course. There is a special type of exponential function, where the base is euler's number, e. This is called the natural exponential function, but is usually just referred to as the exponential function.

Definition 1. The natural exponential function is:

$$f(x) = e^x, e = 2.71828182845905\ldots$$

As $e > 0$, we know that $e^x \to \infty$ as $x \to \infty$ and $e^x \to 0$ as $x \to -\infty$. Let's look at an example with this exponential function.

Problem 2. Sketch the graph of $h(t) = 1 - 5e^{1-\frac{t}{2}}$.

t	$h(t)$
-2	-35.9453
-1	-21.4084
0	-12.5914
1	-7.2436
2	-4

Let's make a value table.

We will sketch the above function in our chosen interval.

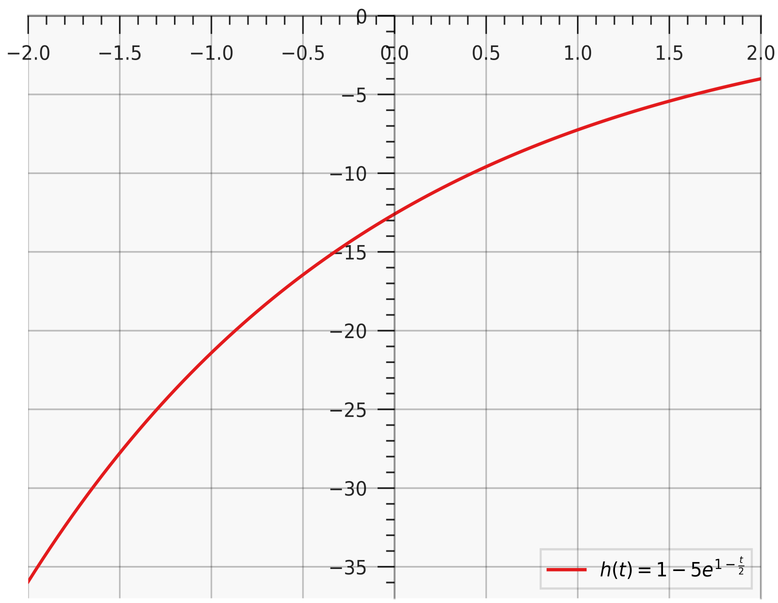

Figure 2: Natural Exponential Functions

Being able to evaluate exponential functions is quite important. Exponential functions will appear in most parts of Calculus I, particularly the natural exponential function.

II Logarithmic Functions