02-11-2024 # STEP Practice: Polynomial Graphs and Triangles

A polynomial is a finite expression of the form:

$$a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n \equiv \sum_{i=0}^{n} a_i x^i =: P\{A\}(x) \; ; \; A := \{a_0, a_1, \ldots, a_n\}$$

Let $k \in \mathbb{Z}^+$.

$$P'(x) = \frac{d}{dx}\sum_{i=0}^{n} a_i x^i = \sum_{i=0}^{n} \frac{d}{dx}\left[a_i x^i\right] = \sum_{i=0}^{n} i a_i x^{i-1}$$

$$\lim_{x\to\infty} x^{2k} = +\infty$$

$$\lim_{x\to-\infty} x^{2k} = +\infty$$

$$\lim_{x\to\infty} x^{2k+1} = +\infty$$

$$\lim_{x\to-\infty} x^{2k+1} = -\infty$$

$$\lim_{x\to\infty} x^2 = \infty \iff \left[\forall M>0 \; \exists N>0 \text{ s.t. } x>N \Rightarrow x^2>M\right]$$

$x^2 \geq 0 \quad \forall x \in \mathbb{R}$

$x^2 > x \quad \forall x > 1$

$M = \frac{1}{2}: N = \max(1, \frac{1}{2}) = 1 \quad 3>1 \Rightarrow 3^2 > 1$

$M = 2: N = \max(1,2) = 2 \quad 3>2 \Rightarrow 3^2 > 2$

pf: Given $M>0$,
Let $N = \max(1, M)$.
If $x > N$,
Then:
$x^2 > \max(1,M)^2 \geq M^2 > \underline{\underline{M}}$ Q.E.D.

$$\lim_{x\to-\infty} x^2 = \infty \iff \left[\forall M>0 \; \exists N<0 \text{ s.t. } x<N \Rightarrow x^2>M\right]$$

$x^2 > M \Rightarrow |x| > \sqrt{M} \Rightarrow x < -\sqrt{M} \quad \because \quad x < N < 0$

pf: Given $M>0$,
Let $N = -\sqrt{M}$.
If $x < N$,
Then:
$x < -\sqrt{M} \Rightarrow -x > \sqrt{M} \Rightarrow |x| > |\sqrt{M}| = \sqrt{M} \Rightarrow x^2 > \underline{\underline{M}}$ Q.E.D.

$\sin\theta \equiv \tan\theta \qquad \cos^2\theta + \sin^2\theta \equiv 1$

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \qquad \frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \tan^2\theta = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\Rightarrow \sin(A+B) + \sin(A-B) = 2\sin A\cos B$

$\sin(A+B) - \sin(A-B) = 2\cos A\sin B$

$\rightsquigarrow \sin P + \sin Q = 2\sin\left[\frac{P+Q}{2}\right]\cos\left[\frac{P-Q}{2}\right]$

$\rightsquigarrow \sin P - \sin Q = 2\cos\left[\frac{P+Q}{2}\right]\sin\left[\frac{P-Q}{2}\right]$

Let $P := A+B$

Let $Q := A-B$

$P+Q = 2A$

$A = \frac{P+Q}{2}$

$P-Q = 2B$

$B = \frac{P-Q}{2}$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

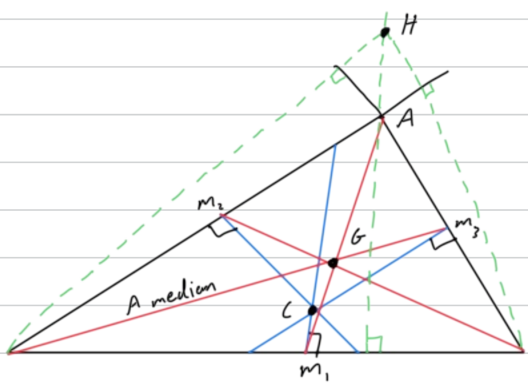
$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\Rightarrow \cos(A+B) + \cos(A-B) = 2\cos A\cos B$

$\cos(A+B) - \cos(A-B) = -2\sin A\sin B$

$\rightsquigarrow \cos P + \cos Q = 2\cos\left[\frac{P+Q}{2}\right]\cos\left[\frac{P-Q}{2}\right]$

$\rightsquigarrow \cos P - \cos Q = -2\sin\left[\frac{P+Q}{2}\right]\sin\left[\frac{P-Q}{2}\right]$




AG : GM

2 : 1

The 3 perpendicular bisectors are concurrent to the circumcentre.

A median is the line joining a vertex joining to the midpoint of the opposite side.

The medians are concurrent to the centroid of the circle.

Centre of gravity of a uniform triangular lamina

The altitudes of a triangle are concurrent at the orthocentre.

C, G, and H lie on a straight line called the Euler line.