Problem 11. 2005.01.04

(a) Given that $\cos\theta = \frac{3}{5}$ and that $\frac{3\pi}{2} \leq \theta \leq 2\pi$, show that $\sin 2\theta = -\frac{24}{25}$, and evaluate $\cos 3\theta$.

(b) Prove the identity

$$\tan 3\theta \equiv \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}.$$

Hence, evaluate $\tan\theta$, given that $\tan 3\theta = \frac{11}{2}$ and that $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$.

Prerequisites.

You need to be familiar with the use of radians for angular measure. For parts (a) and (b) you need to know basic trigonometry for angles of any magnitude and the compound angle formulae. For the final part you will need to solve a cubic equation using the factor theorem to do so. If you aren't familiar with this you may need to jump forward to the relevant section or leave the last part until later.

First Thoughts.

$\frac{3\pi}{2} \leq \theta \leq 2\pi$. I'm confident about doing part (a) and the first part of (b) because I know the compound angle formulae and the range of values given for θ tells me, indirectly, how the negative sign in $-\frac{24}{25}$ arises.

For part (b), I can develop the identity for $\tan 3\theta$ starting from $\tan 3\theta = \tan(\theta + 2\theta)$. For the next part I can use the identity to substitute for $\tan 3\theta$ in the equation and then solve for $\tan\theta$. Unusually for a STEP question, I can see the end of the question from the beginning.