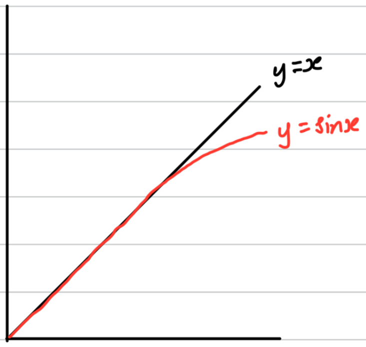
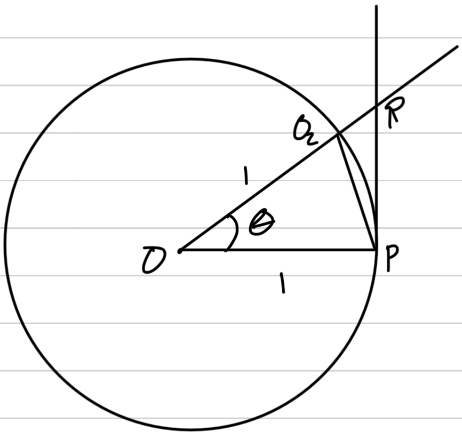
2025-02-15 STEP Practice: Differentiation of sin x





Area of triangle $OPQ_2 = \frac{1}{2}ab\sin C = \frac{1}{2}\sin\theta$

Area of sector $OPQ_2 = \frac{1}{2}r^2\theta = \frac{1}{2}\theta$

Area of triangle $OPR = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}\tan\theta$ $\quad \overline{PR} = \tan\theta$

From observation of the diagram:

$\frac{1}{2}\sin\theta < \frac{1}{2}\theta < \frac{1}{2}\tan\theta$

$\sin\theta < \theta < \tan\theta$

$\frac{1}{\sin\theta} > \frac{1}{\theta} > \frac{\cos\theta}{\sin\theta}$

$1 > \frac{\sin\theta}{\theta} > \cos\theta$ $\quad \because \sin\theta$ is increasing in the interval we are considering.

$\cos\theta < \frac{\sin\theta}{\theta} < 1$

$\lim_{\theta \to 0} \left(\cos\theta < \frac{\sin\theta}{\theta} < 1\right)$

$1^- < \frac{\sin\theta}{\theta} < 1$

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ (via squeeze rule/theorem)

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\sin'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$

$$\sin'(x) = \lim_{h\to 0} \frac{2\cos\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\cos\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{h/2}$$

$$= \lim_{h\to 0} \cos\left(x+\frac{h}{2}\right) \underbrace{\lim_{h\to 0} \frac{\sin\left(\frac{h}{2}\right)}{h/2}}_{1}$$

$$= \cos x \quad (x \text{ in radians})$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$y = \cos x$$
$$y = (1-\sin^2 x)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(1-\sin^2 x)^{-\frac{1}{2}} \cdot -2\sin x\cos x$$

$$= \frac{1}{2}\frac{1}{\cos x} \cdot -2\sin x\cos x$$
$$= -\sin x$$

$$y = \tan x = \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \frac{\cos x\cdot\cos x - \sin x\cdot -\sin x}{\cos^2 x} = \frac{\sin^2 x+\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$y = \sec x = \frac{1}{\cos x}$$

$$\frac{dy}{dx} = \frac{0\cdot\cos x - 1\cdot -\sin x}{\cos^2 x} = \frac{\sin x\cos x}{\cos^3 x} = \sec x\tan x$$

$$y = \operatorname{cosec} x = \frac{1}{\sin x}$$

$$\frac{dy}{dx} = \frac{0\cdot\sin x - 1\cdot\cos x}{\sin^2 x} = -\frac{\cos x}{\sin^2 x} = -\sec x\cot x$$

$$y = \cot x = \frac{1}{\tan x}$$

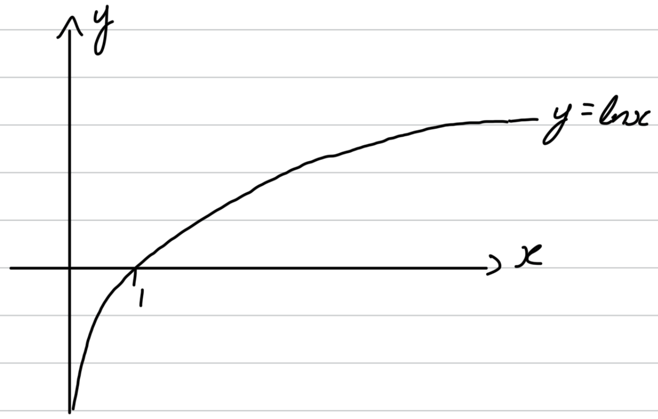
$$\frac{dy}{dx} = \frac{0\cdot\tan x - 1\cdot\sec^2 x}{\tan^2 x} = -\frac{\sec^2 x}{\tan^2 x} = -\frac{1}{\cos^2 x}\cdot\frac{\cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\operatorname{cosec}^2 x$$

$A = P\left(1 + \frac{r}{100n}\right)^{nt}$ for interest paid n times each year

A = return
P = the principal (amount invested)
r = interest rate as a % per year
t = time in years

$A = 1 \cdot \left(1 + \frac{100}{100n}\right)^n = \left(1 + \frac{1}{n}\right)^n$

$\lim_{n \to 0} \left(1 + \frac{1}{n}\right)^n = e$



$\lim_{x \to 0^+} \ln x = -\infty$

$\lim_{x \to \infty} \ln x = +\infty$

$\ln 1 = 0$

$\frac{d}{dx} \ln x = \frac{1}{x}$ $\quad \frac{d}{dx} \log_a x = \frac{d}{dx} \frac{\ln x}{\ln a} = \frac{1}{x \ln a}$

$\lim_{x \to \infty} \frac{d}{dx} \ln x = 0^+$

The domain of $\ln x$: $(0, \infty)$

The range of $\ln x$: $(-\infty, \infty)$

$\ln x$ is one to one and onto the whole range of the role number

$\ln x$ has an inverse function, which is e^x.