16-11-2024 STEP Practice: Inequalities

Negative Multiplier rule

$3 < 4$ is true

$-3 < -4$ is false

$-3 < 4$ is true

$3 < -4$ is false

If you multiply/divide an inequality by a negative number, you must reverse the inequality sign. $a > b \Rightarrow -a < -b$

Reciprocal rule

$3 < 4$ is true

$\frac{1}{3} < \frac{1}{4}$ is false

$\frac{1}{3} > \frac{1}{4}$ is true

$-3 < -4$ is true

$-\frac{1}{3} < -\frac{1}{4}$ is false

$-\frac{1}{3} > -\frac{1}{4}$ is true

$-2 < 5$ is true

$-\frac{1}{2} < \frac{1}{5}$ is false

If you take the reciprocal of both sides of an inequality, you must reverse the inequality sign, unless the signs on both sides are different.

Logarithm rule

$8 > 4$ is true

$\log_2 8 > \log_2 4$ is true

$\log_{\frac{1}{2}} 8 > \log_{\frac{1}{2}} 4$ is false

$\log_{\frac{1}{2}} 8 < \log_{\frac{1}{2}} 4$ is true

If you take a fractional-base logarithm on both sides of an inequality, you must reverse the inequality sign.

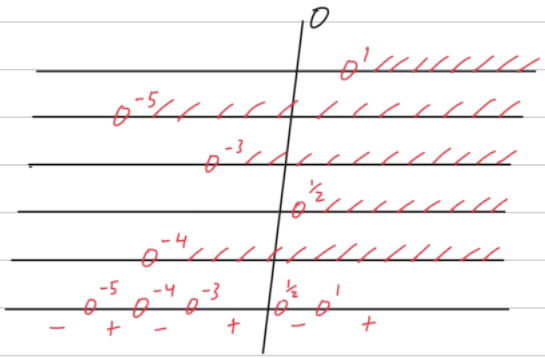
The inequality sign is reversed when a decreasing function is applied to both sides of the inequality.

A function is decreasing on the interval $[a,b]$ iff $x_2 > x_1 \Rightarrow f(x_2) < f(x_1)$ for all $x_1, x_2 \in [a,b]$.

Multiplying an inequality by a negative number $-k$ is equivalent to applying $f(x) = -kx$ to both sides of the inequality.

$$\frac{[x-1][x+5][x+3]}{[2x-1][x-4]} < 0$$

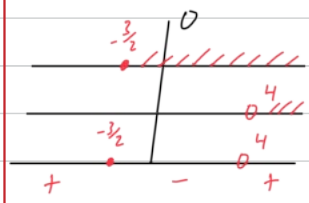
$[x-1] > 0 \Rightarrow x > 1$
$[x+5] > 0 \Rightarrow x > -5$
$[x+3] > 0 \Rightarrow x > -3$
$[2x-1] > 0 \Rightarrow x > \frac{1}{2}$
$[x+4] > 0 \Rightarrow x > -4$



$\therefore x \in (-\infty, -5) \cup (-4, 3) \cup (\frac{1}{2}, 1)$

$$\frac{2x+3}{x-4} \geq 0$$

$2x+3 \geq 0 \Rightarrow x \geq -\frac{3}{2}$
$x-4 > 0 \Rightarrow x > 4$ ← strict inequality because $x \neq 4$ (division by zero)



$\therefore x \in (-\infty, -\frac{3}{2}] \cup [4, \infty)$