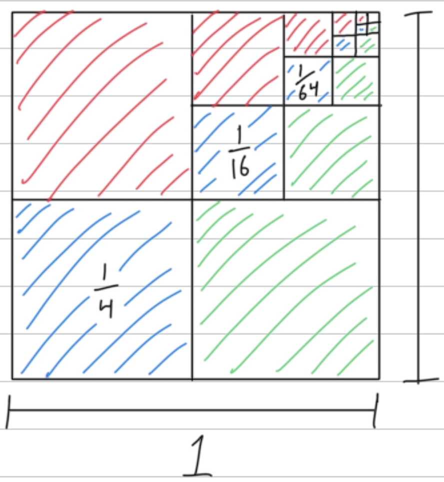
30-11-2024 STEP Practice: Convergent Series



Total Area = 1

$$\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots=\sum_{i=1}^{\infty}\left(\frac{1}{4}\right)^i=\frac{\frac{1}{4}}{1-\frac{1}{4}}=\frac{1}{4}\cdot\frac{4}{3}=\frac{1}{3}$$

$$S_\infty=\sum_{i=1}^{\infty}ar^{i-1}=\lim_{n\to\infty}\frac{a(1-r^n)}{1-r}=\frac{a}{1-r} \text{ if } |r|<1$$

$\because \lim_{n\to\infty} r^n=0$ if $|r|<1$

E.g. $2^1=2, 2^2=4, 2^3=8, \ldots$
$\left(\frac{1}{2}\right)^1=\frac{1}{2}, \left(\frac{1}{2}\right)^2=\frac{1}{4}, \left(\frac{1}{2}\right)^3=\frac{1}{8}, \ldots$

Consider: $$\sum_{i=1}^{\infty}\frac{1}{i}=1+\underbrace{\left[\frac{1}{2}+\frac{1}{3}\right]}_{>\frac{1}{2}+}+\underbrace{\left[\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right]}_{>\frac{1}{2}+}+\underbrace{\left[\frac{1}{8}+\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+\frac{1}{12}+\frac{1}{13}+\frac{1}{14}+\frac{1}{15}\right]}_{>\frac{1}{2}+}+\underbrace{\frac{1}{16}+\frac{1}{17}+\ldots}_{>\frac{1}{2}+\ldots}$$

$\sum_{i=1}^{\infty}\frac{1}{2}$ diverges $\rightsquigarrow$ $\sum_{i=1}^{\infty}\frac{1}{i}$ diverges

harmonic series

$\sum_{i=1}^{\infty}\frac{1}{i^2}=\frac{\pi^2}{6}$ ← Basel Problem

$\sum_{i=1}^{\infty}\frac{1}{i^3}$ converges because each term is smaller than the corresponding term in $\sum_{i=1}^{\infty}\frac{1}{i^2}$ (comparison test)

No one knows what it converges to, but we know that it is irrational.

Divergent series are beyond the scope of STEP, but convergent series are fairly common in problem.