Problem 4.   2003.01.02

The first question on an examination paper is:

$$\text{Solve for } x \text{ the equation } \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$

where (in the question) a and b are given non-zero real numbers. One candidate writes $x = a + b$ as the solution. Show that there are no values of a and b for which this will give the correct answer.

The next question on the examination paper is:

$$\text{Solve for } x \text{ the equation } \frac{1}{x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$$

where (in the question) a, b and c are given non-zero real numbers. The candidate uses the same technique, giving the answer as $x = a + b + c$. Show that the candidate's answer will be correct if and only if a, b and c satisfy at least one of the equations

$$a + b = 0, \qquad b + c = 0 \quad \text{or} \quad c + a = 0.$$

Prerequisites.

You need nothing more than GCSE Mathematics.

First Thoughts.

In both parts it looks as if I need to start with the addition of fractions and then consider the candidate's answers. I think I will begin with that and then see where it leads. The three conditions at the end of part two look as if they might arise from the sort of factorisation that often appears in the solution of a quadratic equation.