Simultaneous Equations.

Problem 5. 2003.01.01

It is given that

$$\sum_{r=-1}^{n} r^2 \text{ can be written in the form } pn^3 + qn^2 + rn + s,$$

where p, q, r and s are numbers. By setting $n = -1, 0, 1,$ and 2, obtain four equations that must be satisfied by p, q, r and s and hence show that

$$\sum_{r=0}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1).$$

Given that

$$\sum_{r=-2}^{n} r^3 \text{ can be written in the form } an^4 + bn^3 + cn^2 + dn + e,$$

show similarly that

$$\sum_{r=0}^{n} r^3 = \frac{1}{4}n^2(n+1)^2.$$

Prerequisites.

It is fairly obvious that you must be familiar with the Σ notation. If you are not, then now would be a good time to learn about it and a review of it is contained in the following "First Thoughts" section. Some experience of the use of identities would be useful, but isn't absolutely essential because of the directions in the question.

First Thoughts.

Overall this is a clear case of "Learn and Apply" as the second part looks like a repeat of the first. In each part the result given is an identity and therefore is true for all n. This justifies the substitution process we are told to carry out. The changes to the ranges of the indexation variable r will need some thought.