Problem 13.   2003.3.6

Show that

$$2 \sin \tfrac{1}{2}\theta \cos r\theta = \sin\left(r + \tfrac{1}{2}\right)\theta - \sin\left(r - \tfrac{1}{2}\right)\theta$$

Hence, or otherwise, find all the solutions of the equation

$$\cos a\theta + \cos(a+1)\theta + \dots + \cos(b-2)\theta + \cos(b-1)\theta = 0,$$

where a and b are positive constants with $a < b - 1$.

Prerequisites.

You will need to know the Factor Formulae. Experience of Telescoping Series would be useful, but if you haven't met them before this will be a good introduction to them.

First Thoughts.

I can see how to make the RHS into the LHS using the third of the factor formulae. There must be some substitution process that replaces r with a or b.