Differentiation.

By now, my students will have met the use of limits in the context of differentiation of polynomials from first principles, so I simply include a reminder of the definition of a derivative.

Let f be any function of x. The derivative of f with respect to x at any given point is defined to be

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

and I have no doubt that this is the most important limit in all of mathematics.

The use of derivatives in identifying the stationary points of a function is probably the first application of a derivative that students learn about. The following problem is a classic of this type.

Problem 33. 2002.01.02

Let $f(x) = x^m(x-1)^n$, where m and n are both integers greater than 1. Show that the curve $y = f(x)$ has a stationary point with $0 < x < 1$. By considering $f''(x)$, show that this stationary point is a maximum if n is even and a minimum if n is odd.

Sketch the graphs of $f(x)$ in the four cases that arise according to the values of m and n.

Prerequisites.

You will need to be able to use basic differentiation, the product rule and chain rule to find first and second derivatives and know the elementary theory about the use of first and second derivatives to locate and classify stationary points. You will also need to be able to use the results obtained to sketch graphs.

First Thoughts.

I will obviously have to find f' and f''. I may need to consider what form is best for the results so that I can use them easily to consider how n affects the shape of the graph. I imagine that some care might be needed with the sketches. It may be that drawing them all to the same scale will help to show the differences.