COMP 598 Summer (May) 2020

Assignment 2

Due Date: 18^{th} May 2020

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 11^{th} May 2020

Question 1[10 points] Show that the language

$$F = \{a^i b^j c^k | i, j, k \ge 0 \text{ and if } i = 1 \text{ then } j = k\}$$

is not regular. Show, however, that it satisfies the statement of the pumping lemma as I proved it in class, i.e. there is a p such that all three conditions for the pumping lemma are met. Explain why this does not contradict the pumping lemma.

Question 2[10 points] Are the following statements true or false? Prove your answer in each case. We have some fixed alphabet Σ with at least two letters. In the following A and B stand for languages, *i.e.* subsets of Σ^* .

- If A is regular and $A \subseteq B$ then B must be regular.
- If A and AB are both regular then B must be regular.
- If $\{A_i|i\in\mathbb{N}\}$ is an infinite family of regular sets then $\bigcup_{i=1}^{\infty}A_i$ is regular.
- If A is not regular it cannot have a regular subset.

Question 3[10 points] If L is a language over an alphabet with strictly more than one letter we define $CYC(L) = \{uv|u, v \in \Sigma^*, vu \in L\}$. Show that if L is regular then CYC(L) is also regular;[7]. Give an example of a non-regular language such that CYC(L) is regular. [3]

Question 4[10 points] Last week I incorrectly stated that the relation

$$x \equiv_L y \iff \forall z \in \Sigma^*, \ xz \in L \iff yz \in L,$$

defined for any language L is a congruence relation. Recall that a congruence relation \sim is one for which

$$x \sim y, u \sim v \Rightarrow xu \sim yv.$$

However, Rahul pointed out that this is false. Give a counter-example showing that \equiv_L is not a congruence. The correct definition that I should have given is the following

$$x \approx_L y \iff \forall u, v \in \Sigma^*, \ uxv \in L \iff uyv \in L.$$

Show that \approx_L is indeed a congruence relation. The quotient of Σ^* by \approx_L is called the *syntactic monoid* of L. The relation \equiv_L is correctly defined for the Myhill-Nerode theorem; it is only the claim that it is a congruence relation that is false.

Question 5[10 points] Which of the following pairs of formulas are equivalent? If they are not equivalent, give a transition system which shows that the formulas have different interpretations. If they are equivalent give a proof of the equivalence based on the semantics of the temporal operators.

- 1. $\Box \Diamond \phi \Rightarrow \Box \Diamond \psi$ and $\Box (\phi \Rightarrow \Diamond \psi)$.
- 2. $\Diamond(\phi \land \psi)$ and $\Diamond\phi \land \Diamond\psi$.
- 3. $\bigcirc \Diamond \phi$ and $\Diamond \bigcirc \phi$.

In the next two questions we will compare the expressive power of LTL and a version of fixed-point logic adapted to paths.

Consider the following syntax for LTL:

$$\phi ::== \mathcal{P}|\phi_1 \wedge \phi_2|\neg \phi| \bigcirc \phi|\Diamond \phi|\Box \phi|\phi_1 U\phi_2$$

where \mathcal{P} is a set of atomic propositions. The semantics is – as usual – defined in terms of infinite execution paths and the temporal operators have their usual meanings. We can enrich the language by introducing fixed point operators to get the logic μ -LTL below

$$\phi ::== \mathcal{P}|X|\phi_1 \wedge \phi_2|\neg \phi| \bigcirc \phi|\mu X.\phi(X)|\nu X.\phi(X)$$

where X stands for a variable that ranges over formulas and μ and ν are least and greatest fixed-point operators respectively. The semantics of μ and ν are given in terms of fixed points as we defined in class.

With the fixed point operators present we do not need any of the LTL operators except \bigcirc . For example, we can write $\Box \phi \equiv \nu X.\phi \wedge \bigcirc X$. We can then use negation to get \diamondsuit .

Question 6[10 points]

- 2. What would happen if you used the other fixed-point operator in your formula?
- 3. Explain why the formula $p \wedge \Box(p \Rightarrow \bigcirc \neg p) \wedge \Box(\neg p \Rightarrow \bigcirc p)$ does not express odd(p).

Question 7[10 points]

Let $\sigma_i = p^i(\neg p)p^\infty$ stand for a path where p is true for the first i steps then it is false for one state then p is true forever. Prove: given a proposition p and any LTL (not μ -LTL) formula ϕ containing n next operators, the formula ϕ has the same truth value on every σ_i with i > n.

Now show that odd(p) cannot be expressed in LTL.

Question 8[10 points] Which of the following identities hold for ω -regular expressions?

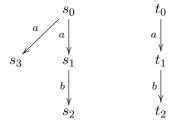
1.
$$(E_1 + E_2) \cdot F^{\omega} \equiv E_1 \cdot F^{\omega} + E_2 \cdot F^{\omega}$$

2.
$$E \cdot (F_1 + F_2)^{\omega} \equiv E \cdot F_1^{\omega} + E \cdot F_2^{\omega}$$

3.
$$(E^* \cdot F)^{\omega} \equiv E^* \cdot F^{\omega}$$
.

Either prove they are equal or give a counterexample.

Question 9[10 points] Consider the processes



Write a formula of Hennessy-Milner logic that distinguishes the states t_0 and s_0 .

The negation-free fragment of Hennessy-Milner logic is

$$\phi ::== \operatorname{true} |\phi_1 \wedge \phi_2| \phi_1 \vee \phi_2 |\langle a \rangle \phi$$

where a is an action. Show that the states t_0 and s_0 agree on all the formulas of the negation-free fragment. This should be done by induction on the structure of formulas.

Question 10[10 points]

Let $\Sigma = \{A, B\}$. Construct an NBA that accepts the set of infinite words σ over Σ that start with A and such that A occurs infinitely often in σ and between any two consecutive A's there are an odd number of B's.