Final Exam
Mike Gao
260915701
Prof. Prakash Panangaden

Question 1

$\epsilon + (b^*ba)^*b^*b$

Question 2

Minimize the dfa and check if it only has one state. If yes, check whether the state is accepting, if yes, then it accepts Σ^*

Question 3

3.1

Decidable. Since the intersection of a context free language and a regular language is context free, and we can build context-free grammar from the language. For a given CFG G, checking whether L(G) is empty or not is decidable.

3.2

It is a well known problem that $L = \Sigma^*$ is undecidable. We combine the PDA of L and DFA of R to create a new PDA. Because of the CFL-PDA equivalence, it is undecidable whether $L \cap R = \Sigma^*$.

Question 4

Context free but not regular.
The following grammar generates it:
$S \rightarrow aSc|T|\epsilon \; T \rightarrow bTc|\epsilon$
Thus it is context free. Now we need to prove that it is not regular:
Regular languages are closed under intersection. We intersect L with another regular language, we get $L \cap \{b^*c^*\} = \{b^m c^m\}$ which is not regular. According to the closure property of regular language, we get L is not regular language as well.

Question 5

5.1

Suppose you have an input on which M does not halt. This detection algorithm will run forever. Therefore it is undecidable.

5.2

Consider the following set $\overline{H_{TM}} = \{< M, w > |$M does not halt on w$\}$ Such set is non-CE as we mentioned in class. We do a reduction $\overline{H_{TM}} \leq_m FIN$

Construct a new turing machine M' with input x, and let l be length of x.

We run w on M for l steps.

There are two cases: ($\overline{H_{TM}}$) If M does not halt during the l steps, nothing is accepted. So $M' = nil \in FIN$

(H_{TM}) If M halts, all strings x with length $\geq l$ will be accepted by M', which means M' accepts infinitely many strings, so $M' \notin FIN$

We know that $\overline{H_{TM}}$ is non-CE, then FIN is non-CE as well.

Question 6

6.1

Decidable. Whether a regular language is co-finite is decidable. The regular languages are closed under complement so this is essentially asking whether regular language is finite, which is decidable.

6.2

Not decidable. We know that a set of invalid computations is a CFL. Such set is co-finite iff the Turing machine can only accept a finite set. Since it is undecidable to determine whether a given Turing machine accepts a finite or infinite number of inputs (Corollary from Rice's theorem), the statement is undecidable as well.

6.3

Not decidable by Rice's Theorem.

Question 7

7.1

True

7.2

True

7.3

True

7.4

True

7.5

False

Honor Code

I solemnly swear that I am up to no mischief. I did not consult anyone nor did I use the internet to search for answers to these questions

```
##################################################################
##################################################################
###########################%%%##########((#(((####################
#####################%%%%%#(((////////(((###(((###################
###################%%&%%%#((///////////////(((#((((((#############
####(((((#(########%&&%##(//////**********///(((((((((####((((####
#((((((((((((((#%%%&&%##(//***/************///((/((((((((((((((#(#
(((((((((((((((#%%%%&%##(//******,,,,,,,***/////(/((((((((((((((((
(((((((((((((((##%%%%%##(//******,,,,,,,***/////((((((((((((((((((
(((((((((((((((%%%%%%%%#((/*****,,,,,,,,,**////////(((((((((((((((
(((((((((((((%%%&&&%%%%##((((/***,,*,*****////////(/((((((((((((((
(((((((((((((####%&&%%&%#(//###(/***//(/***//((/%%/*//((((((((((((
(((((((((((((/####&%%(#%%##%(((#%#***(//(&&*####(##/****((((((((((
((((((((((((//(%##&&%#((/(/////##(***#*****/////((****/(((((((((((
(((((((((/////(##%&%%#(//***/%%(/***/*,/((//***(/*/*//((((((((((((
(((((((/////////##&&%#(//*(#%##(/******/(*,,**//**//////((((((((((
(((///////////////#&%##((((#%###((//**/**////////*/////////(((((((
((/////////////////&%%#(/(#%&%#((////((/*/*////***////////////((((
////////////////////@&%%(#(((#(((((///////////*********////////(((
/////////////////@@@%%&%%%#(((//*****////((#************///////(((
/////////////**#@@@@@%%%&&&%#(//((////(%((&@&#***********/////////
/////////*(&@@@@@@@@@@%%%%%%%%%%%###((((/(@@@&%**********/////////
/////%&%@@@@@@@@@@@@@@@%###(#%#((////((///@%&&@&&&%%%/****////////
(&&#%&&&&%&%&@@&@@@@@@@@@&#(//((((////(////@&@@@&@%%&&&%%%%///////
&&&&&&%#%&@&%@@&&&&&@@&@&&&#(/////////#**#&%&@@@&@&&&&&&&@&&&&#%//
@@&&&@@%&@&&&@@@&@&%&@@%@@&@&/**********(&&&&&&@%&&&%&&&&@&&&&&#%&
&@&&&&@@%&@&@@@@@&&#&&&@@@@@@@@%(******(&&&&%&@@&#%&%&%%&@@&@&&#&&
@@@&&&@@&%@@&%@@&&%%%&&&&#%%&(@@@@(,*(%&%%@%%%%&@&%&&&&@&%&&&#&&
```