Thm There is no algorithm that correctly predicts whether the output of a Turing machine with the alphabet $\{a,b\}$ is finite or infinite.

Proof Suppose there is such an algorithm $S(M)$ giving yes or no output.

If S ... M ... $S(M)$ ... S halts when run on $S(M)$? S runs forever, stuck.

Because we have a program $T(M) = \begin{cases} \text{yes} & \text{if } S(M) = \text{yes} \\ \text{loop} & \text{if } S(M) = \text{no} \end{cases}$

$T(M)$ loops if and only if $S(M)$ says yes.

A chief strategy halts

define $P(M) = \text{if } H(M) \text{ then loop}$

$P(P) = \text{if } H(P) \text{ then loop}$

If $H(P)$ then $H(P)$ loop

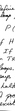
If $P(P)$ halts then H says that P loops. Both the true and false halts.

Suppose $P(P)$ halts then H says P loop. Both the false and true halts.

Suppose H is the P program built into H machine.

Turing Machine

Q states, start, accept, reject
Γ input symbols, $\Gamma \supseteq \Sigma$
Δ blank, finite tape.



The reading head can write Δ or write a new cell at a time.

$\delta: Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$

$\delta: S(L,a) = (R,b,L)$

The machine can't halt if the head keep moving one Δ.

If it moves to the Δ "a" is overwrit b, L is another Δ the left. R head keep moving to the right.

All cell stays only a finite amount of information, is proved

Easy to see $\{a^n b^n : n \geq 0\}$ can be recognized by a TM.

Configuration of a TM is a description of its state, its tape, its position of the reading head.

$\underbrace{\Delta\Delta\Delta}_{\text{tape}}\underbrace{\Delta}_{\text{state}}\underbrace{\Delta\Delta\Delta}_{\text{tape}}$ $\quad$ $\Delta\Delta\Delta