FACULTY OF SCIENCE

FINAL TAKE-HOME EXAMINATION

COMPUTER SCIENCE COMP 598

Automata Theory and Computability

Examiner: Prof. Prakash Panangaden 29th May 2020 - 31st May 2020

Instructions:

This exam has 7 questions. Please answer all questions. The maximum score for this exam is 100. The marks for each question are indicated just after each question. This is an **open book exam**: you may use any books or notes that you have, including dictionaries. You are not allowed to consult anyone or search for solutions on the internet. At the end of your solutions please type the following statement:

> I solemly swear that I am up to no mischief. I did not consult anyone nor did I use the internet to search for answers to these questions.

Please answer all questions using **LaTeX** and upload a pdf to myCourses. The questions appear on pages 1 and 2; this title page is not numbered. There are a total of three pages including this title page. You have a 3-day period in which to finish the exam. It is designed to be finished in three hours but if you are not familiar with LaTeX it may take longer; that is fine. I am not requiring a strict time limit. I will try to answer questions by email promptly during the day from 10am to 5pm on Friday and 10am to 3:30pm on Saturday and Sunday.

Question 1[10 points]
Write a regular expression for the set of all strings over the alphabet $\{a, b\}$ in which each a is immediately preceded *and* immediately followed by a b.

Question 2[10 points]
Give an algorithm to decide whether the language accepted by a DFA is Σ^*. Your algorithm description should be high-level and does not need to go into any details. For example you can say "minimize" or "determinize" without explaining how these algorithms work.

Question 3[20 points]
One of the following questions is decidable and the other is undecidable.

1. Given a CFL L and a regular language R, is $L \cap R = \emptyset$?
2. Given a CFL L and a regular language R, is $L \cap R = \Sigma^*$?

For the one that is decidable give an algorithm, for the other give an undecidability proof.

Question 4[15 points]
Consider the language $L = a^n b^m c^{n+m}$ with $n, m \geq 0$. Classify this as *one* of the three following:

1. regular,
2. context-free but not regular,
3. recursive but not context-free.

You have to prove each asssertion. For example, if you say that it is regular, you must give an NFA to recognize it; of course, in this case it is obvious that it does not belong to the other two classes. Similarly, if you claim that it is context-free, but not regular, you have to give a context-free grammar **and a proof that it is not regular**, but, of course you will not have to prove that it is recursive. If you claim that it belongs to the last class, you must show that it is not context-free (it is then immediate that it is not regular) *and* give an algorithm to recognize it.

Question 5[15 points]
The set $FIN = \{\langle M \rangle | M$ halts on finitely many inputs$\}$. Explain why this set is not decidable [2 points]. Explain why this set is not CE, use a reduction to some known non-CE set like EMPTY. $EMPTY = \{\langle M \rangle | M$ never halts on any input$\}$. [13 points].

Question 6[20 points]
Recall that a set is called *cofinite* if its complement is finite. Thus if we are talking about the natural numbers the set of numbers bigger than 17 is cofinite and the set of odd numbers is infinite but not cofinite. For this question the alphabet is fixed as $\{a, b\}$.

1. Is is decidable whether a regular language is cofinite? Assume that the regular language is defined by giving you its recognizing DFA.
2. Is it decidable whether a context-free language is cofinite?
3. Is it decidable whether a language described by a Turing machine is cofinite?

Please give *short* answers. Long answers will be penalized even if they are correct. You may invoke known theorems from the class, but not random theorems that you found on the internet.

Question 7[10 points]
Are the following statements true or false? No explanations are needed.

1. The intersection of two context-free languages can be regular.
2. The intersection of two context-free languages must be recursive.
3. Given G_1 and G_2 context-free grammars, it is undecidable whether $L(G_1) \subset L(G_2)$.
4. It is decidable whether a regular language is equal to Σ^*.
5. All programs written in Python are guaranteed to halt.