UNIVERSAL FUNCTIONS

Idea: a binary function $U(x,y)$ is said to be universal for the class of computable unary functions if $\forall f\ \forall x\ \exists y\ U(x,y) = U(x,y)$ is computable. U is called a universal function.

CO: V unary computable f $\exists y \in \mathbb{N}$ $\forall x\ U(x,y) = f(x)$

Thm: There is a binary computable function U $\forall f$ $\exists y$ $\forall x$ $U(x,y) = f(x)$

Proof: Consider all unary computable functions

Proof: Consider $\forall f\ \forall x$, compute all finite inputs

$f(0), f(1), \ldots, f(x)$

Then there exists a total computable universal function U such that $U(x,y) = f(x)$ for all unary computable functions.

Proof: Let $U(x,y)$ = the output of y on input x if y is a unary function, otherwise $U(x,y)$ = 0.

Note: U is total computable. Universal function for total computable

Let U be any computable f: $\mathbb{N} \to \mathbb{N}$

$\forall x\ \exists y\ U(x,y) = f(x)$

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