Section 13

The Post Correspondence Problem

A finite set of pairs of strings

$$S = \{ \left[\frac{x_1}{y_1}\right], \left[\frac{x_2}{y_2}\right], \ldots, \left[\frac{x_n}{y_n}\right] \}$$

As usual we use words of such tiles.

Then i pair is a sequence of tiles together, such that the concatenated top is the same as the concatenated bottom.

EXAMPLE $\left[\frac{a}{ab}\right]\left[\frac{b}{b}\right]\left[\frac{ab}{a}\right]$

$$\text{top} = abbab \quad \text{bot} = abbab$$

A solution to this instance of PCP is a sequence of position indices that "denotes" a match.

$$\begin{array}{ccc} 2 & 1 & 3 \\ \hline b & a & ab \\ b & ab & a \end{array}$$

is a solution

e.g.

$$\left\{ \left[\frac{a}{ab}\right], \left[\frac{b}{b}\right], \left[\frac{ab}{a}\right] \right\}$$

The minimal PCP instance is:

$$\left\{ \left[\frac{1}{1}\right], \left[\frac{0}{0}\right] \right\}$$

Theorem (Post) PCP is undecidable with the alphabet $\{0,1\}$.

Proof Sketch. Post reduced our problem to PCP.

Proof Sketch. Post reduced our problem to PCP.

Assume Σ and Σ has a solution

Iff UNIVERSAL(TM) and

Construct an instance that will hold one of the tiles on the "short" side and another that will hold the "long" side.

Start tile $\left[\frac{\#q_0 x_1 x_2 \ldots x_n}{\#}\right]$

For any transition of the form $\delta(q,a) = (p,b,R)$

Add $\left[\frac{qa}{bp}\right]$

For any transition of the form $\delta(q,a) = (p,b,L)$

Add $\left[\frac{qa}{pb}\right]$

$\delta(q,a) = (p,b,S)$

Add $\left[\frac{qa}{bp}\right]$ $\forall q \in F$

2 more special tiles $\left[\frac{a}{a}\right]$ $\left[\frac{\#}{\#}\right]$

3 more ending tiles $\left[\frac{\#}{\#}\right]$, $\left[\frac{\#}{\#}\right]$, $\left[\frac{\#}{\#}\right]$

enough of using these tiles $\left[\frac{\#}{\#}\right]$, $\left[\frac{\#}{\#}\right]$, $\left[\frac{\#}{\#}\right]$ $\forall a \in \{0,1\}$

$\delta(q,a) = (h,a,R)$

$\left[\frac{\#}{\#}\right]$ $\left[\frac{\#}{\#}\right]$ $\left[\frac{\#}{\#}\right]$ $\left[\frac{\#}{\#}\right]