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Lasers and laser-related equipment — Test methods for laser beam widths, divergence angles and beam propagation ratios —

Part 2:
General astigmatic beams

Lasers et équipements associés aux lasers — Méthodes d'essai des largeurs du faisceau, angles de divergence et des facteurs de limite de diffraction —

Partie 2: Faisceaux astigmatiques généraux

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Foreword

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International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

ISO 11146-2 was prepared by Technical Committee ISO/TC 172, *Optics and photonics*, Subcommittee SC 9, *Electro-optical systems.*

ISO 11146 consists of the following parts, under the general title *Lasers and laser-related equipment — Test methods for laser beam widths, divergence angles and beam propagation ratios*:

— *Part 1: Stigmatic and simple astigmatic beams*

— *Part 2: General astigmatic beams*

— *Part 3: Intrinsic and geometrical laser beam classification, propagation, and details of test methods* (Technical Report)

Introduction

The propagation properties of laser beams can be characterized by ten independent parameters when applying the method of second order moments (see ISO/TR 11146-3). Most laser beams need few parameters for a complete description due to their higher symmetry. Lasers emit beams which are stigmatic or simple astigmatic due to their resonator design.

Part 1 of ISO 11146 describes the measurement methods for stigmatic and simple astigmatic beams while this part of ISO 11146 deals with the measurement procedures for general astigmatic beams. This part of ISO 11146 is applicable to beams of unknown type. Beam characterization, based on the method of second order moments as described in Part 1 and this part of ISO 11146, is only valid within the paraxial approximation.

The theoretical description of beam characterization and propagation as well as the classification of laser beams is given in ISO/TR 11146-3, which is an informative Technical Report. The procedures for background subtraction and offset correction are also given in ISO/TR 11146-3.

In ISO 11146, the second order moments of the power (energy) density distribution function are used for the determination of beam widths. If problems are experienced in the direct measurements of these quantities, other indirect methods of measurement of second order moments may be used as long as comparable results are achievable.

In ISO/TR 11146-3, three alternative methods for beam width measurement and their correlation with the method used in this part of ISO 11146 are described. These methods are:

— variable aperture method;

— moving knife-edge method;

— moving slit method.

The problem of the dependence of the measuring result on the truncation limits of the integration area was investigated and evaluated by an international interlaboratory experiment carried out in 1997. The results of this interlaboratory testing were taken into consideration in this document.

The International Organization for Standardization (ISO) draws attention to the fact that it is claimed that compliance with this document may involve the use of a patent concerning the determination of beam characteristics by measuring along the beam caustic of the transformed beam produced by a lens as described in 5.3 and 5.4.

ISO takes no position concerning the evidence, validity and scope of this patent right.

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Lasers and laser-related equipment — Test methods for laser beam widths, divergence angles and beam propagation ratios —

Part 2:
General astigmatic beams

1   Scope

This part of ISO 11146 specifies methods for measuring beam widths (diameter), divergence angles and beam propagation ratios of laser beams. This part of ISO 11146 is applicable to general astigmatic beams or unknown types of beams. For stigmatic and simple astigmatic beams, ISO 11146-1 is applicable.

Within this part of ISO 11146, the description of laser beams is accomplished by means of the second order moments of the Wigner distribution rather than physical quantities such as beam widths and divergence angles. However these physical quantities are closely related to the second order moments of the Wigner distribution. In ISO/TR 11146-3, formulae are given to calculate all relevant physical quantities from the measured second order moments.

2   Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 11145, *Optics and optical instruments — Lasers and laser-related equipment — Vocabulary and symbols*

ISO 11146-1:2005, *Lasers and laser-related equipment — Test methods for laser beam widths, divergence angles and beam propagation ratios — Part 1: Stigmatic and simple astigmatic beams*

IEC 61040:1990, *Power and energy measuring detectors, instruments and equipment for laser radiation*

3   Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 11145, ISO 11146-1, IEC 61040 and the following apply.

NOTE   The $x-$, $y-$ and z-axes in the following definitions refer to the laboratory system (as described in Clause 4). Here and throughout this document the term "power density distribution" refers to continuous wave sources. It might be replaced by "energy density distribution" in case of pulsed sources.

3.1
generalized beam diameter
d_g
measure of the extent of the power density distribution of a beam in a cross-section at an axial location z, derived from the centred second order moments by

$$d_g = 2\sqrt{2}\sqrt{\langle x^2 \rangle + \langle y^2 \rangle} \qquad\qquad (1)$$

NOTE   This definition is similar to the beam diameter defined in ISO 11145 or ISO 11146-1. But in this context the definition is not restricted to circular power density distributions.

3.2
generalized beam waist location
$z_{0,g}$
position where the generalized beam diameter reaches its minimum value along the axis of propagation

3.3
generalized Rayleigh length
$z_{R,g}$
distance along the beam axis from the generalized beam waist where the generalized beam diameter is a factor of $\sqrt{2}$ larger than the generalized beam waist diameter

3.4
Wigner distribution
phase space distribution representing a laser beam in a transverse plane at location z

NOTE   The Wigner distribution is a function of two spatial and two angular coordinates, giving the amount of beam power propagating through the point (x,y) in the direction (Θ_x, Θ_y).

3.5
spatial first order moments of the Wigner distribution
$\langle x \rangle, \langle y \rangle$

subset of the first order moments, which can be directly obtained from measured power density distribution by

$$\langle x \rangle(z) = \frac{\displaystyle\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x,y,z)\,x \; \mathrm{d}x \; \mathrm{d}y}{\displaystyle\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x,y,z)\,\mathrm{d}x \; \mathrm{d}y} \qquad\qquad (2)$$

and

$$\langle y \rangle(z) = \frac{\displaystyle\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x,y,z)\,y \; \mathrm{d}x \; \mathrm{d}y}{\displaystyle\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x,y,z)\,\mathrm{d}x \; \mathrm{d}y} \qquad\qquad (3)$$

where $E(x,y,z)$ is the power density distribution at the specific plane z = constant.

3.6
second order moments of the Wigner distribution
$\langle x^2 \rangle, \; \langle y^2 \rangle, \; \langle xy \rangle, \; \langle \Theta_x^2 \rangle, \; \langle \Theta_y^2 \rangle, \; \langle \Theta_x \Theta_y \rangle, \; \langle x\Theta_x \rangle, \; \langle x\Theta_y \rangle, \; \langle y\Theta_x \rangle, \; \langle y\Theta_y \rangle$

ten second order moments of the Wigner distribution of the beam at location z

NOTE 1   The ten second order moments contain information on the following physical beam properties: beam size and orientation, divergence angles and their orientation, radii of curvature of the phase paraboloid and their orientation and the twist parameter. Details on these relations are given in ISO/TR 11146-3.

NOTE 2   In ISO 11146-1, the three spatial second order moments are defined as σ_x^2, σ_y^2 and σ_{xy}^2. In this part of ISO 11146 and ISO/TR 11146-3, the angular brackets are used to emphasize the coordinates of the moments. This means that $\sigma_x^2 = \langle x^2 \rangle$, $\sigma_y^2 = \langle y^2 \rangle$ and $\sigma_{xy}^2 = \langle xy \rangle$.

NOTE 3   Three angular moments $\langle \Theta_x^2 \rangle$, $\langle \Theta_y^2 \rangle$ and $\langle \Theta_x \Theta_y \rangle$ are independent of z. The other seven second order moments are, in general, functions of z.

3.7
spatial second order moments of the Wigner distribution
$\langle x^2 \rangle, \langle y^2 \rangle, \langle xy \rangle$

subset of the second order moments, which can be directly obtained from measured power density distribution by

$$\langle x^2 \rangle (z) = \frac{\int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} E(x,y,z)(x - \langle x \rangle)^2 \, \mathrm{d}x \, \mathrm{d}y}{\int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} E(x,y,z) \, \mathrm{d}x \, \mathrm{d}y} \tag{4}$$

$$\langle y^2 \rangle (z) = \frac{\int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} E(x,y,z)(y - \langle y \rangle)^2 \, \mathrm{d}x \, \mathrm{d}y}{\int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} E(x,y,z) \, \mathrm{d}x \, \mathrm{d}y} \tag{5}$$

and

$$\langle xy \rangle (z) = \frac{\int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} E(x,y,z)(x - \langle x \rangle)(y - \langle y \rangle) \, \mathrm{d}x \, \mathrm{d}y}{\int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} E(x,y,z) \, \mathrm{d}x \, \mathrm{d}y} \tag{6}$$

3.8
beam matrix
P
symmetric and positive definite 4×4 matrix containing all ten second order moments of the Wigner distribution and its elements and given by

$$\mathbf{P} = \begin{bmatrix} \langle x^2 \rangle & \langle xy \rangle & \langle x\Theta_x \rangle & \langle x\Theta_y \rangle \\ \langle xy \rangle & \langle y^2 \rangle & \langle y\Theta_x \rangle & \langle y\Theta_y \rangle \\ \langle x\Theta_x \rangle & \langle y\Theta_x \rangle & \langle \Theta_x^2 \rangle & \langle \Theta_x\Theta_y \rangle \\ \langle x\Theta_y \rangle & \langle y\Theta_y \rangle & \langle \Theta_x\Theta_y \rangle & \langle \Theta_y^2 \rangle \end{bmatrix} \tag{7}$$

3.9
effective beam propagation ratio
M_{eff}^2
invariant quantity related to the focusability of a general astigmatic beam, defined as:

$$M_{\text{eff}}^2 = \frac{4\pi}{\lambda}\left[\det(\boldsymbol{P})\right]^{\frac{1}{4}} \qquad (8)$$

where det ($\boldsymbol{P}$) is the determinant of matrix $\boldsymbol{P}$

NOTE 1   The effective beam propagation ratio M_{eff}^2 is an invariant related to the overall beam spread or the near and far field localization of the beam.

NOTE 2   For simple astigmatic beams, the effective beam propagation ratio is the geometric mean of the beam propagation ratios of the principal axes of the beam: $M_{\text{eff}}^2 = \sqrt{M_x^2 \times M_y^2}$. For stigmatic beams $M_{\text{eff}}^2 = M^2$.

3.10
intrinsic astigmatism
a

degree of how close to a stigmatic beam the general astigmatic beam can be transformed by using lenses and free spaces

$$a = \frac{8\pi^2}{\lambda^2}\left(\left(\langle x^2\rangle\langle\Theta_x^2\rangle - \langle x\Theta_x\rangle^2\right) + \left(\langle y^2\rangle\langle\Theta_y^2\rangle - \langle y\Theta_y\rangle^2\right) + 2\left(\langle xy\rangle\langle\Theta_x\Theta_y\rangle - \langle x\Theta_y\rangle\langle y\Theta_x\rangle\right)\right) - \left(M_{\text{eff}}^2\right)^2 \geqslant 0 \quad (9)$$

NOTE   Beams are classified according to their intrinsic astigmatism a, which is an invariant quantity. A beam with $a = 0$ is called intrinsic stigmatic, a beam with $a > 0$ is called intrinsic astigmatic. For simple astigmatic beams $a = (1/2)\left(M_x^2 - M_y^2\right)^2$. More details are given in ISO/TR 11146-3.

3.11
twist parameter
t

parameter related to the rotational properties of the phase front of a beam, and also to the orbital angular momentum carried by the beam

$$t = \langle x\Theta_y\rangle - \langle y\Theta_x\rangle \qquad (10)$$

NOTE   The twist parameter is invariant under propagation through free space and spherical lenses. It might be altered under propagation through cylindrical lenses.

3.12
principal axes of a power density distribution
axes of the maximum and minimum beam extent based on the centred second order moments of the power density distribution in a cross-section of the beam

[ISO 11146-1:2005]

NOTE   The axes of maximum and minimum extent are always perpendicular to each other.

3.13
orientation of a power density distribution
φ

angle between the x–axis of the laboratory system and that of the principal axis of the power density distribution which is closer to the x–axis

[ISO 11146-1:2005]

NOTE   From this definition it follows that $-\frac{\pi}{4} < \varphi < \frac{\pi}{4}$ for $|\varphi| \neq \pi/4$; if $\varphi = \pm\pi/4$, φ is defined as the angle between the x– axis and the major principal axis of the power density distribution.

3.14
beam widths
$d_{\sigma x}, d_{\sigma y}$
extent of a power density distribution in a cross-section of the beam at an axial location z along that principal axis which is closer to the x- or y-axis of the laboratory coordinate system, respectively, based on the centred second order moments of the power density distribution

NOTE   If the principal axes make the angle $\pi/4$ with the $x-$ and y-axes of the laboratory coordinate system, then $d_{\sigma x}$ is by convention the larger beam width.

[ISO 11146-1:2005]

4   Coordinate system

The $x-$, $y-$ and $z-$axes define the orthogonal space directions in the laboratory axes system and shall be specified by the user. The $z-$axis shall approximately coincide with the direction of the beam. The $x-$ and $y-$axes are transverse axes, usually horizontal and vertical, respectively. The origin of the z-axis is in a reference $x-y$ plane defined by the manufacturer, e.g. the front of the laser enclosure.

5   Test principles

5.1   General

The following test principles are valid for general astigmatic beams. For stigmatic and simple astigmatic beams ISO 11146-1 may be applied.

5.2   Spatial second order moments of the Wigner distribution

Spatial second order moments are obtained by acquisition of power density distributions by means of spatially resolving detectors, correcting the measured profiles and calculating the first and second order moments.

5.3   Second order moments of the Wigner distribution

For the determination of all ten second order moments two different measurement set-ups are required.

Eight of the ten second order moments and the sum $\left(\langle x\Theta_y\rangle + \langle y\Theta_x\rangle\right)$ are obtained by acquisition of power density distributions along the propagation axis z in different planes near the generalized waist position, calculating the three spatial second order moments of each measured power density profile and fitting three independent parabolas to them.

The difference $\left(\langle x\Theta_y\rangle - \langle y\Theta_x\rangle\right)$ is obtained from the spatial moments of a power density distribution acquired behind a cylindrical lens.

5.4   Derived quantities

The effective beam propagation ratio M_{eff}^2, the intrinsic astigmatism a, and the twist parameter t are obtained from the second order moments of the Wigner distribution according to the Equations (8) to (10).

6 Measurement arrangement and test equipment

6.1 General

The test is based on the measurement of the cross-sectional power density distribution of the entire laser beam.

6.2 Preparation

The optical axis of the measuring system should be coaxial with the laser beam to be measured. Suitable optical alignment devices are available for this purpose (e.g. aligning lasers or steering mirrors).

The aperture of the optical system should accommodate the entire cross-section of the laser beam. Clipping shall be smaller than 1 % of the total beam power or energy.

The attenuators or beam-forming optics should be mounted so that the optical axis runs through the geometrical centres. Care shall be taken to avoid systematic errors. Reflections, interference effects, external ambient light, thermal radiation or air draughts are all potential sources of error.

An evaluation shall be made to determine if the entire laser beam reaches the detector surface. For testing this, apertures of different widths can be introduced into the beam path in front of each optical component. The aperture that reduces the output signal by 5 % should have a diameter less than 0,8 times the aperture of the optical component.

6.3 Control of environment

Suitable measures such as mechanical and acoustical isolation of the test set-up, shielding from extraneous radiation, temperature stabilization of the laboratory, choice of low-noise amplifiers shall be taken to ensure that the contribution to the total probable error of the parameter to be measured is low.

Care should be taken to ensure that the atmospheric environment in high-power laser beam paths does not contain gases or vapours that can absorb the laser radiation and cause thermal distortion in the beam to be measured.

6.4 Detector system

Measurement of the cross-sectional power density distribution requires the use of a detection system with high spatial resolution and high signal-to-noise-ratio.

The accuracy of the measurement is directly related to the spatial resolution of the detector system and its signal-to-noise ratio. The latter is important for laser beams with low power densities at larger diameters (e.g. for diffracted parts of the laser beams).

For pixel based detector systems the spatial resolution, should be at least 1/20 of the smaller beam width.

In practice, noise in the wings of the power density distribution $E(x,y,z)$ may readily dominate the second order moment integral. Thus, it is usually necessary to apply background correction procedures. Refer to ISO/TR 11146-3 for further details.

The radiation detector system shall be in accordance with IEC 61040:1990, in particular with Clauses 3 and 4. Furthermore, the following points shall be noted.

— Care shall be taken to ascertain the damage thresholds of the detector surface so as to make sure that these thresholds are not exceeded by the laser beam.

— It shall be confirmed, from manufacturers' data or by measurement, that the output quantity of the detector system (e.g. the voltage) is linearly dependent on the input quantity (laser power). Any wavelength dependency, non-linearity or non-uniformity of the detector or the electronic device shall be minimized or corrected by use of a calibration procedure.

— When using a scanning device for determining the power density distribution function, care shall be taken to ensure that the laser output is temporally stable during the whole scanning period.

— When measuring pulsed laser beams, the trigger time delay of sampling as well as the measuring time interval play an important role because the beam parameters may change during the pulse. Therefore it is necessary to specify these parameters in the test report.

6.5 Beam-forming optics and optical attenuators

If the beam cross-sectional area is greater than the detector area, a suitable optical system shall be used to reduce the beam cross-sectional area on the detector surface. The change in magnification shall be taken into account during the evaluation procedure.

Optics shall be selected appropriate to wavelength.

An attenuator may be required to reduce the laser power density at the surface of the detector.

Optical attenuators shall be used when the laser output-power or power density exceeds the detector's working (linear) range or the damage threshold. Any wavelength, polarization and angular dependency, non-linearity or non-uniformity, including thermal effects of the optical attenuator, shall be minimized or corrected by use of a calibration procedure.

None of the optical elements used shall significantly influence the relative power (energy) density distribution.

7 Measurement of the second order moments

7.1 General

Before the measurements are started, the laser shall warm up for at least 1 h (unless otherwise stated by the manufacturer) to achieve thermal equilibrium. The measurements shall be carried out at the operating conditions specified by the laser manufacturer for the type of laser being evaluated.

7.2 Measurement of the second order moments of power density distributions

Spatial second order moments are calculated from measured and corrected power density distributions. The corresponding integrations are carried out on a subset of the measured data, called the integration area, because otherwise crucial noise in the data may dominate the integrals. In many cases, a proper choice of the integration area is important for obtaining reliable results. The following procedure relates the size and position of the integration area to the size and position of the measured power density distribution, which are initially unknown. Hence, an iterative procedure is required.

All integrations according to Equations (2) to (6) are performed on a rectangular integration area which is centred to the beam centroid, defined by the spatial first order moments, orientated parallel to the principal axes of the power density distribution, and sized three times the beam widths $d_{\sigma x}$ and $d_{\sigma y}$ (see Figure 1). Since the beam's centroid, orientation and widths are initially unknown, the procedure starts with an approximation for the integration area. The approximation should include the beam's extent, orientation and position. Using this integration area, initial values for the beam position, size and orientation are obtained which are used to redefine the integration area. From the new integration area, new values for the beam size, orientation and position are calculated. This procedure shall be repeated until convergence of the results is obtained.

The orientation of the principal axes of a power density distribution, or the azimuthal angle φ, can be obtained from the second order moments of the power density distribution by

$$\varphi(z) = \frac{1}{2}\arctan\left[2\langle xy\rangle \middle/ \left(\langle x^2\rangle - \langle y^2\rangle\right)\right] \tag{11}$$

for $\langle x^2\rangle \neq \langle y^2\rangle$;

for $\langle x^2\rangle = \langle y^2\rangle$, $\varphi(z)$ is obtained as:

$$\varphi(z) = \mathrm{sgn}\left(\langle xy\rangle\right) \times \frac{\pi}{4} \tag{12}$$

where

$$\mathrm{sgn}\left(\langle xy\rangle\right) = \frac{\langle xy\rangle}{|\langle xy\rangle|} \tag{13}$$

The beam widths in the direction of its principal axes are given by

$$d_{\sigma x}(z) = 2\sqrt{2}\left\{\langle x^2\rangle + \langle y^2\rangle + \gamma\left[\left(\langle x^2\rangle - \langle y^2\rangle\right)^2 + 4\left(\langle xy\rangle\right)^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}} \tag{14}$$

$$d_{\sigma y}(z) = 2\sqrt{2}\left\{\langle x^2\rangle + \langle y^2\rangle - \gamma\left[\left(\langle x^2\rangle - \langle y^2\rangle\right)^2 + 4\left(\langle xy\rangle\right)^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}} \tag{15}$$

where

$$\gamma = \mathrm{sgn}\left(\langle x^2\rangle - \langle y^2\rangle\right) = \frac{\langle x^2\rangle - \langle y^2\rangle}{|\langle x^2\rangle - \langle y^2\rangle|} \tag{16}$$
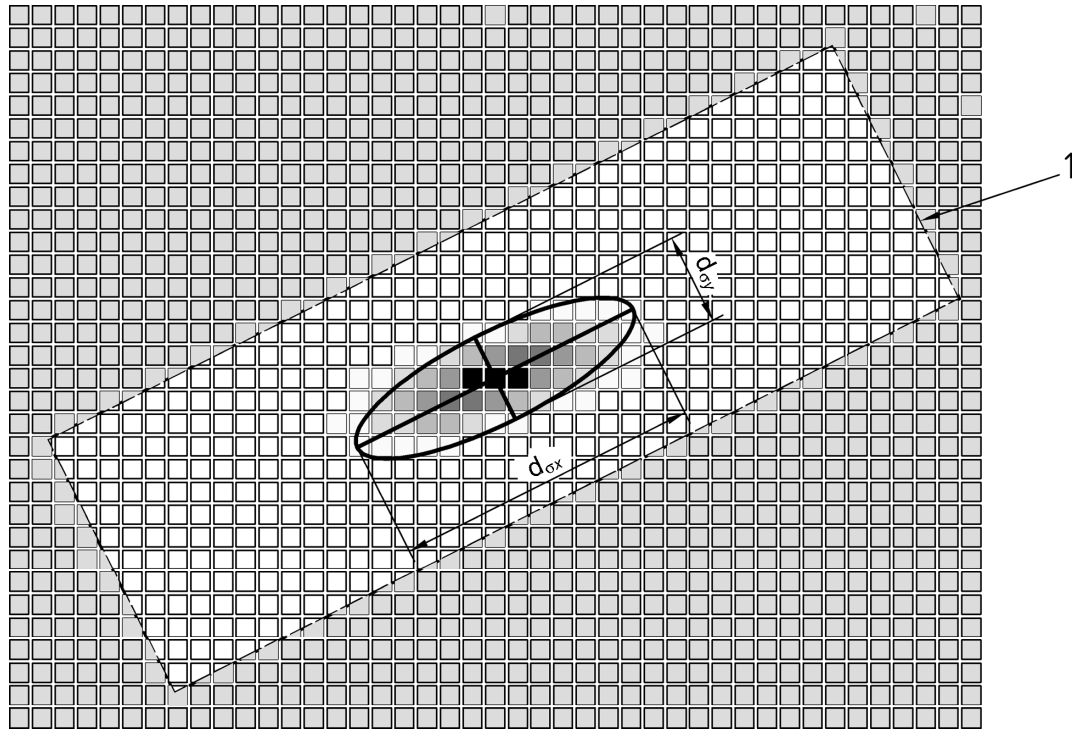
Equations (14) and (15) are valid only for $\langle x^2\rangle \neq \langle y^2\rangle$.

For $\langle x^2\rangle = \langle y^2\rangle$ they shall be replaced by

$$d_{\sigma x}(z) = 2\sqrt{2}\left(\langle x^2\rangle + \langle y^2\rangle + 2\langle xy\rangle\right)^{\frac{1}{2}} \tag{17}$$

$$d_{\sigma y}(z) = 2\sqrt{2}\left(\langle x^2\rangle + \langle y^2\rangle - 2\langle xy\rangle\right)^{\frac{1}{2}} \tag{18}$$

These calculations are performed for each measurement and the mean values and the standard deviations of the beam widths and the azimuth angle are calculated.

Key

1 outline of integration area

Figure 1 — Integration area

7.3 Measurement of all second order moments of the Wigner distribution

For sufficient and balanced accuracy of all ten second order moments, power density profiles at no less than twenty equally distributed z locations within a range of at least three generalized Rayleigh lengths before and after the generalized beam waist shall be acquired. If the generalized beam waist is not accessible for direct measurement an artificial waist shall be created by means of an aberration-free focusing lens or focusing system.

From the acquired power density distributions at distance Δz from the reference plane or the rear principal plane of the focusing system used, respectively, the spatial moments $\langle x^2 \rangle (\Delta z), \langle y^2 \rangle (\Delta z)$ and $\langle xy \rangle (\Delta z)$ shall be calculated according to 7.2. The second order moments $\langle x^2 \rangle_0$, $\langle y^2 \rangle_0$, $\langle xy \rangle_0$, $\langle \Theta_x^2 \rangle_0$, $\langle \Theta_y^2 \rangle_0$, $\langle \Theta_x \Theta_y \rangle_0$, $\langle x\Theta_x \rangle_0$, $\langle y\Theta_y \rangle_0$, and the quantity

$$s_0 = \langle x\Theta_y \rangle_0 + \langle y\Theta_x \rangle_0 \tag{19}$$

in the reference plane or the rear principal plane of the focusing system used, respectively, are obtained by fitting three independent parabolas to the spatial moments in the measurement planes according to

$$\langle x^2 \rangle (\Delta z) = \langle x^2 \rangle_0 + 2\Delta z \langle x\Theta_x \rangle_0 + \Delta z^2 \langle \Theta_x^2 \rangle_0 \tag{20}$$

$$\langle xy \rangle (\Delta z) = \langle xy \rangle_0 + \Delta z s_0 + \Delta z^2 \langle \Theta_x \Theta_y \rangle_0 \tag{21}$$

$$\left\langle y^{2}\right\rangle(\Delta z)=\left\langle y^{2}\right\rangle_{0}+2\Delta z\left\langle y\Theta_{y}\right\rangle_{0}+\Delta z^{2}\left\langle \Theta_{y}^{2}\right\rangle_{0} \tag{22}$$

The proper choice of measurement plane positions requires that the distance of the generalized beam waist position from the reference position or the rear plane of the focusing system $\Delta z_{0,g}$ and the generalized Rayleigh range $z_{R,g}$ are determined using:

$$\Delta z_{0,g}=-\frac{\left\langle x\Theta_{x}\right\rangle_{0}+\left\langle y\Theta_{y}\right\rangle_{0}}{\left\langle \Theta_{x}^{2}\right\rangle_{0}+\left\langle \Theta_{y}^{2}\right\rangle_{0}} \tag{23}$$

$$z_{R,g}=\sqrt{\frac{\left\langle x^{2}\right\rangle+\left\langle y^{2}\right\rangle}{\left\langle \Theta_{x}^{2}\right\rangle+\left\langle \Theta_{y}^{2}\right\rangle}-\frac{\left(\left\langle x\Theta_{x}\right\rangle+\left\langle y\Theta_{y}\right\rangle\right)^{2}}{\left(\left\langle \Theta_{x}^{2}\right\rangle+\left\langle \Theta_{y}^{2}\right\rangle\right)^{2}}} \tag{24}$$

To determine the second order moments $\left\langle x\Theta_{y}\right\rangle_{0}$ and $\left\langle y\Theta_{x}\right\rangle_{0}$, a cylindrical lens with a focal length f_{c} shall be placed at the distance $\Delta z=\Delta z_{0,g}+f_{c}$ from the reference plane or the rear plane of the focusing system in horizontally focusing orientation (see Figure 2).

The power density distribution in the rear focal plane of the cylindrical lens shall be acquired. The spatial second order moment $\left\langle xy\right\rangle$ calculated according to 7.2 and denoted as $\left\langle xy\right\rangle_{h}$. The cylindrical lens shall be rotated to vertically focusing orientation (see Figure 3). The power density distribution in the rear focal plane shall be acquired again, the spatial second order moment $\left\langle xy\right\rangle$ calculated according to 7.2 and denoted as $\left\langle xy\right\rangle_{v}$.

NOTE   A focal length of $f_{c}\approx z_{R,g}$ of the cylindrical lens is recommended. This ensures a proper beam size in the rear focal plane of the cylindrical lens.

The second order moments $\left\langle x\Theta_{y}\right\rangle_{0}$ and $\left\langle y\Theta_{x}\right\rangle_{0}$ in the plane $z=0$ shall then be calculated using

$$\left\langle x\Theta_{y}\right\rangle_{0}=\frac{s_{0}}{2}+\frac{1}{2f_{c}}\left(\left\langle xy\right\rangle_{v}-\left\langle xy\right\rangle_{h}\right) \tag{25}$$

$$\left\langle y\Theta_{x}\right\rangle_{0}=\frac{s_{0}}{2}-\frac{1}{2f_{c}}\left(\left\langle xy\right\rangle_{v}-\left\langle xy\right\rangle_{h}\right) \tag{26}$$

If a focusing system has been used to create an artificial waist, the following procedure shall be performed to obtain the ten second order moments in the reference plane from the already obtained ten second order moments.

Arrange all the ten second moments in a beam matrix P_{0} (see Clause 3). Calculate the beam matrix P_{ref} of the reference plane according to

$$P_{ref}=S\times P_{0}\times S^{T} \tag{27}$$

where S is the inverse geometric optical system matrix representing the ray propagation from the reference plane to the rear principal plane of the focusing system and S^{T} is the transposition of matrix S. Assuming that a thin spherical lens has been chosen and the origin of the z–axis has been placed in the centre of the lens, this system matrix is given by

$$
\boldsymbol{S} = \left\{ \begin{array}{cccc} 1-\dfrac{L}{f} & 0 & -L & 0 \\[2mm] 0 & 1-\dfrac{L}{f} & 0 & -L \\[2mm] \dfrac{1}{f} & 0 & 1 & 0 \\[2mm] 0 & \dfrac{1}{f} & 0 & 1 \end{array} \right\} \tag{28}
$$

where

L is the distance between the reference plane and the lens;

f is the focal length.

If another focusing element has been used, e.g. a thick spherical lens or a system of lenses, see ISO/TR 11146-3 on how to obtain the system matrix $\boldsymbol{S}$.

The desired second order moments in the reference plane are the elements of the matrix $\boldsymbol{P}_{\text{ref}}$.
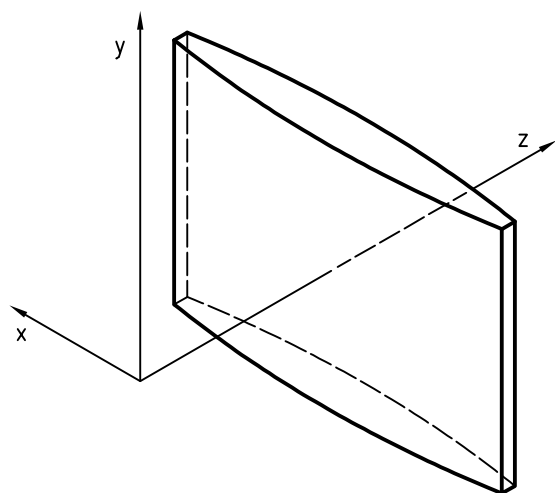

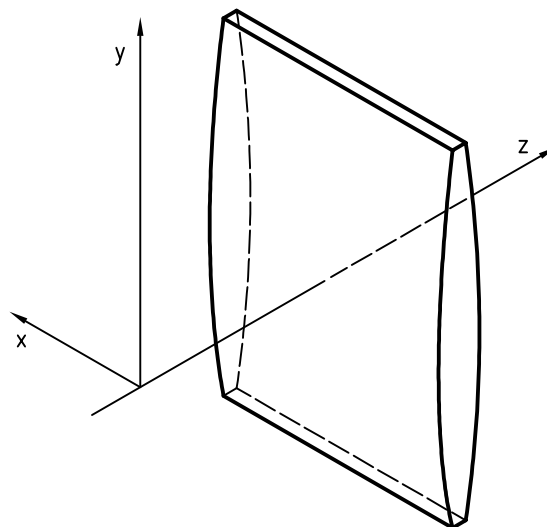**Figure 2 — Cylindrical lens in horizontally focusing orientation**

Figure 3 — Cylindrical lens in vertically focusing orientation

8   Determination of effective beam propagation ratio

The effective beam propagation ratio is calculated according to Equation (8), where $\boldsymbol{P}$ is the so-called beam matrix which contains all second order moments of the Wigner distribution which are determined as given in Clause 7. Further details are given in ISO/TR 11146-3.

9   Determination of intrinsic astigmatism

Another invariant during propagation is the intrinsic astigmatism a which is defined in Equation (9). The beams are classified due to their intrinsic astigmatism a. A beam with $a = 0$ is called intrinsic stigmatic, a beam with $a > 0$ intrinsic astigmatic. For the determination of the intrinsic astigmatism it is necessary to measure all ten second order moments of the Wigner distribution as described in Clause 7. Further details are given in ISO/TR 11146-3.

10 Determination of the twist parameter

From the second order moments of the Wigner distribution determined according to Clause 7 the twist parameter t is calculated using Equation (10).

11 Test report

The following information shall be included in the test report:

a) **General information**

    1)   reference to this part of ISO 11146 (ISO 11146-2:2005);

    2)   date of test;

    3)   name and address of test organization;

    4)   name of individual performing the test.

b) **Information concerning the tested laser**

    1)   laser type;

    2)   manufacturer;

    3)   manufacturer's model designation;

    4)   serial number.

c) **Test conditions**

    1)   laser wavelength(s) at which tested;

    2)   temperature in K (diode laser cooling fluid) (only applicable for diode lasers);

    3)   operating mode [continuous wave (cw) or pulsed];

    4)   laser parameter settings:

        ⸺ output power or energy,

        ⸺ current or energy input,

        ⸺ pulse energy,

        ⸺ pulse duration,

        ⸺ pulse repetition rate;

    5)   mode structure;

    6)   polarization;

    7)   environmental conditions.

d) Information concerning testing and evaluation

1) test equipment:

— camera   ☐,

— moving pinhole   ☐;

2) detector and sampling system:

— response time of the detector system,

— trigger delay of sampling (for pulsed lasers only),

— measuring time interval (for pulsed lasers only);

3) beam forming optics and attenuating method:

— type of attenuator,

— type of beam splitter,

— type of focusing element;

4) other optical components and devices used for the test (polarizer, monochromator, etc.);

5) other relevant parameters or characteristics of the test which have to be chosen (aperture setting, reference plane, reference axis, laboratory system).

e) Test results

1) The ten second order moments of the Wigner distribution:

Focusing element used for creating an artificial generalized waist _____

Focal length _____

At a distance from reference plane of _____

Focal length of cylindrical lens _____

Parameter	Mean value	Standard deviation
$\langle x^2 \rangle$		
$\langle y^2 \rangle$		
$\langle xy \rangle$		
$\langle \Theta_x^2 \rangle$		
$\langle \Theta_y^2 \rangle$		
$\langle \Theta_x \Theta_y \rangle$		
$\langle x\Theta_x \rangle$		
$\langle y\Theta_y \rangle$		
$\langle x\Theta_y \rangle$		
$\langle y\Theta_x \rangle$		

2) Beam propagation invariants and twist parameter (according to Clauses 8 to 10)

Parameter	Mean value	Standard deviation
Effective beam propagation ratio M_{eff}^2		
Intrinsic astigmatism a		
Twist parameter t		

If required, physical beam parameters such as beam widths and beam divergence angles may be obtained from the measured second order moments of the Wigner distribution and added to the test report. See ISO/TR 11146-3 for the relationship between the second order moments of the Wigner distribution and the physical beam parameters.

Bibliography

[1]   ISO/TR 11146-3, *Lasers and laser-related equipment — Test methods for laser beam widths, divergence angles and beam propagation ratios — Part 3: Intrinsic and geometrical laser beam classification, propagation and details of test methods*

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