
MIXTAPE
SESSIONS

Roadmap

Differential timing

- We covered mostly the simple two group case
- In the two group case, we can estimate the ATT under parallel trends using OLS with unit and time fixed effects
- If we have covariates, then we can use TWFE under restrictive assumptions, or we have other options (OR, IPW, DR)
- Now let's move to a more common scenario where we have more than two groups who get treated at various times

2x2 versus differential timing

- For this next part, similar to how we did with Sant'Anna and Zhao (2020), we will decompose TWFE to understand what it needs for unbiasedness under differential timing
- All of this is from Goodman-Bacon (2021, forthcoming) though the expression of the weights is from 2018 for personal preference
- Goodman-Bacon (2021, forthcoming) shows that parallel trends is **not enough** for TWFE to be unbiased when treatment adoption is described by differential timing
- TWFE with differential timing uses treated groups as controls – not all estimators do – and this can introduce bias

Decomposition Preview

- TWFE estimates a parameter that is a weighted average over all 2x2 in your sample
- TWFE assigns weights that are a function of sample sizes of each "group" and the variance of the treatment dummies for those groups

Decomposition (cont.)

- TWFE needs two assumptions: that the variance weighted parallel trends are zero (far more parallel trends iow) and no dynamic treatment effects (not the case with 2x2)
- Under those assumptions, TWFE estimator estimates the variance weighted ATT as a weighted average of all possible ATTs

K^2 distinct DDs

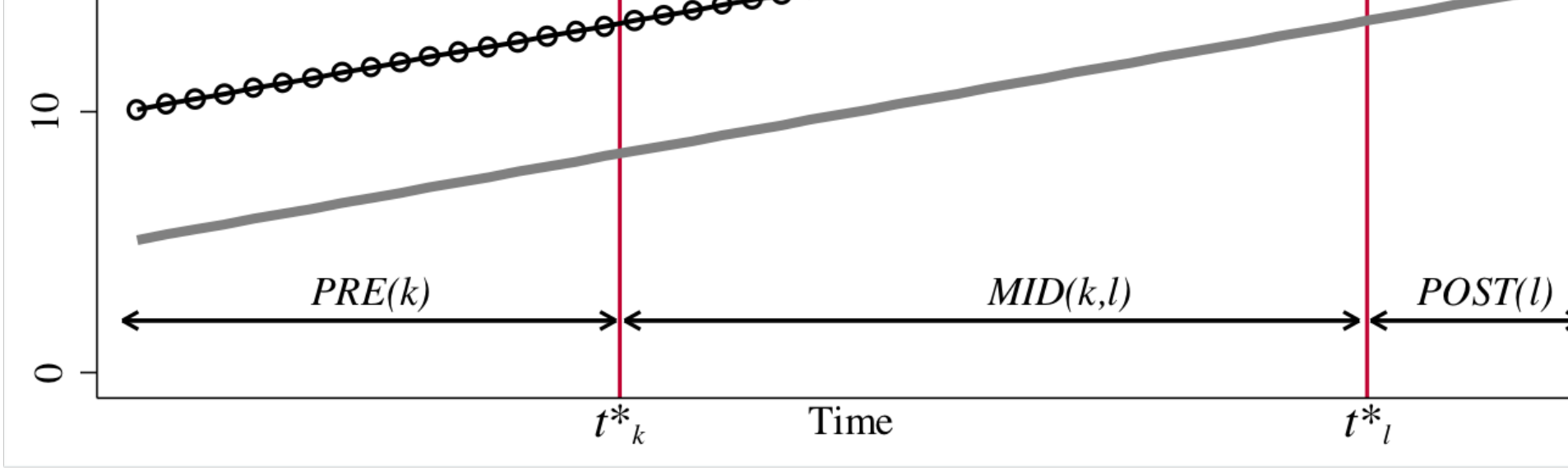
Let's look at 3 timing groups (a, b and c) and one untreated group (U). With 3 timing groups, there are 9 2x2 DDs. Here they are:

a to b	b to a	c to a
a to c	b to c	c to b
a to U	b to U	c to U

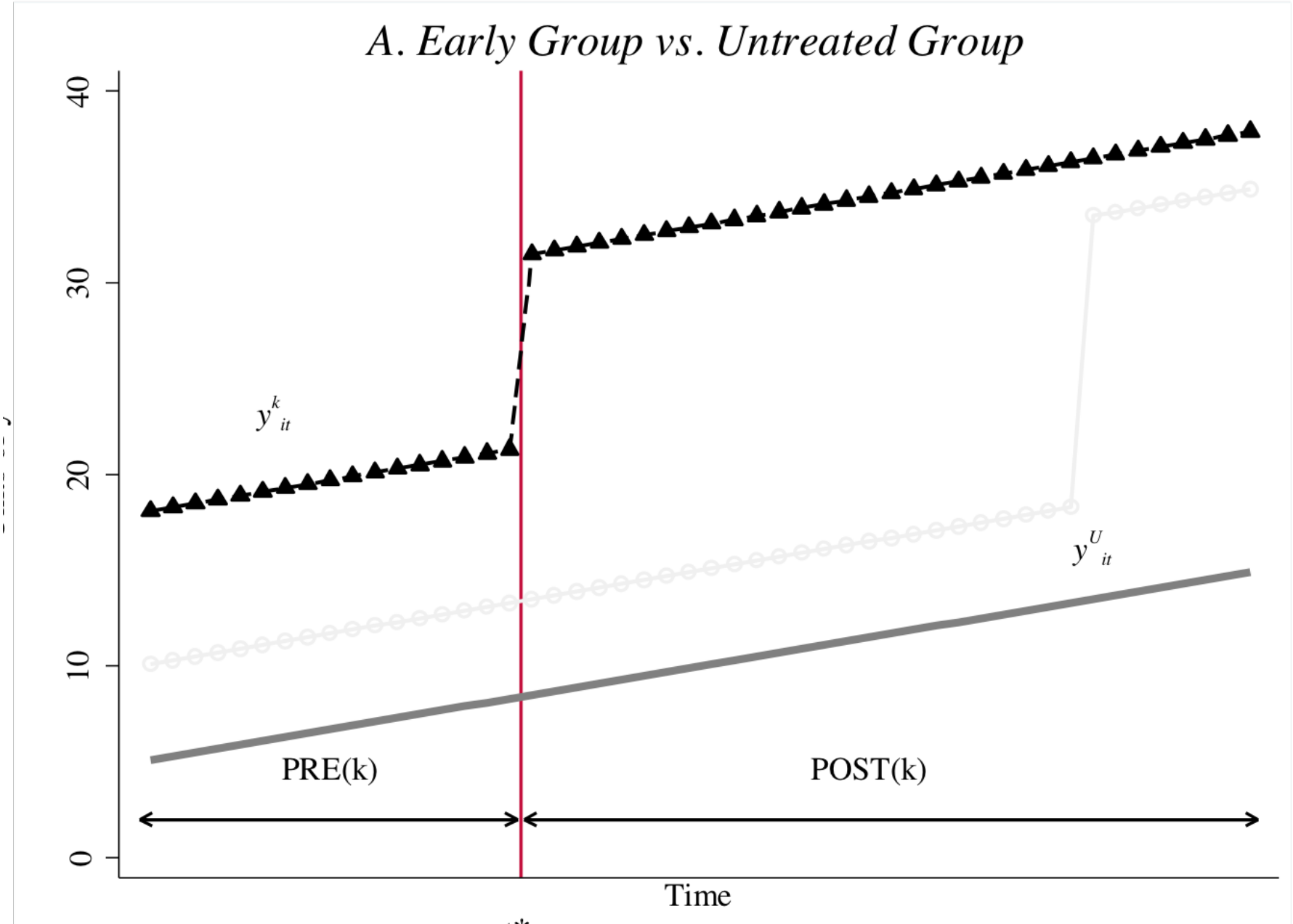
Let's return to a simpler example with only two groups – a k group treated at t_k^* and an l treated at t_l^* plus an never-treated group called the U untreated group

Terms and notation

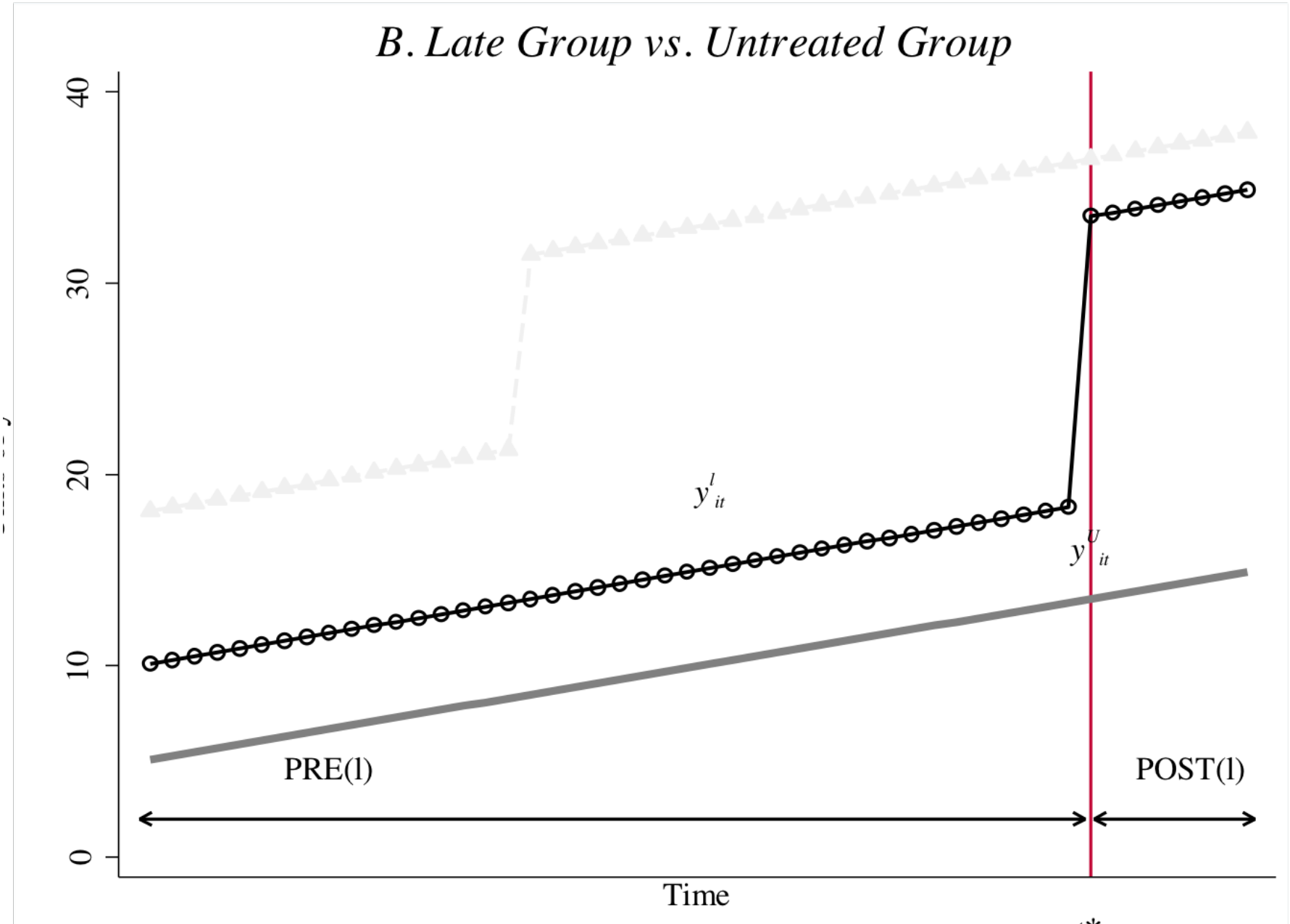
- Let there be two treatment groups (k,l) and one untreated group (U)
- k,l define the groups based on when they receive treatment (differently in time) with k receiving it earlier than l
- Denote $\overline{D}_k$ as the share of time each group spends in treatment status
- Denote $\widehat{\delta}_{jb}^{2x2}$ as the canonical 2×2 DD estimator for groups j and b where j is the treatment group and b is the comparison group


10
0
PRE(k)
MID(k,l)
POST(l)
t*k
Time
t*l

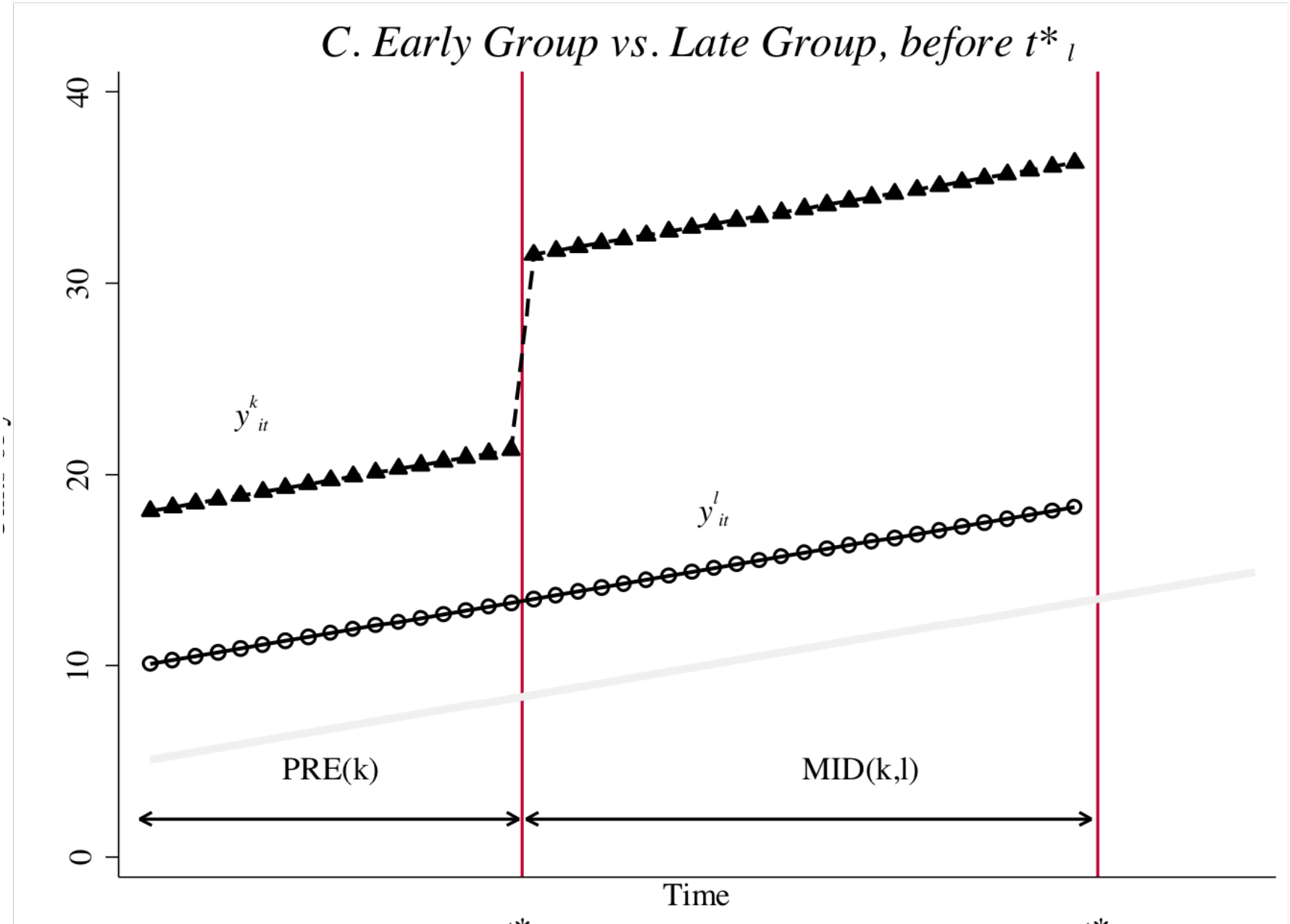
$$\widehat{\delta}_{kU}^{2x2} = \left(\overline{y}_k^{post(k)} - \overline{y}_k^{pre(k)}\right) - \left(\overline{y}_U^{post(k)} - \overline{y}_U^{pre(k)}\right)$$



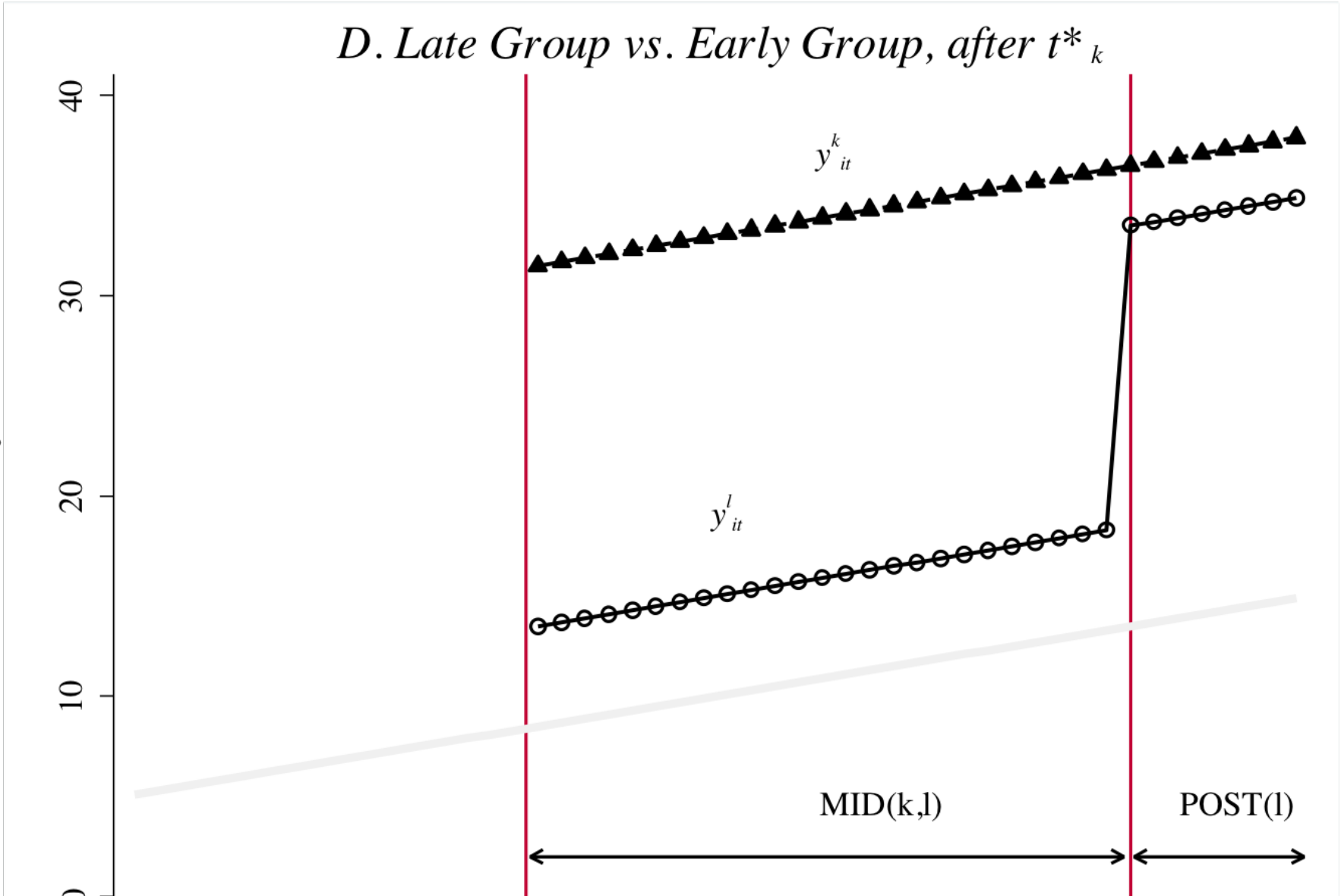
$$\widehat{\delta}_{lU}^{2x2} = \left(\overline{y}_{l}^{post(l)} - \overline{y}_{l}^{pre(l)}\right) - \left(\overline{y}_{U}^{post(l)} - \overline{y}_{U}^{pre(l)}\right)$$



$$\delta_{kl}^{2x2,k} = \left(\overline{y}_k^{MID(k,l)} - \overline{y}_k^{Pre(k,l)} \right) - \left(\overline{y}_l^{MID(k,l)} - \overline{y}_l^{PRE(k,l)} \right)$$



$$\delta_{lk}^{2x2,l} = \left(\overline{y}_{l}^{POST(k,l)} - \overline{y}_{l}^{MID(k,l)} \right) - \left(\overline{y}_{k}^{POST(k,l)} - \overline{y}_{k}^{MID(k,l)} \right)$$



Bacon decomposition

TWFE estimate yields a weighted combination of each groups' respective 2x2 (of which there are 4 in this example)

$$\widehat{\delta}^{DD} = \sum_{k \neq U} s_{kU} \widehat{\delta}^{2x2}_{kU} + \sum_{k \neq U} \sum_{l > k} s_{kl} \left[\mu_{kl} \widehat{\delta}^{2x2,k}_{kl} + (1 - \mu_{kl}) \widehat{\delta}^{2x2,l}_{lk} \right]$$

where that first 2x2 combines the k compared to U and the l to U (combined to make the equation shorter)

Third, the Weights

$$
\begin{aligned}
s_{ku} &= \frac{n_k n_u \overline{D}_k (1 - \overline{D}_k)}{\widehat{Var}(\tilde{D}_{it})} \\
s_{kl} &= \frac{n_k n_l (\overline{D}_k - \overline{D}_l)(1 - (\overline{D}_k - \overline{D}_l))}{\widehat{Var}(\tilde{D}_{it})} \\
\mu_{kl} &= \frac{1 - \overline{D}_k}{1 - (\overline{D}_k - \overline{D}_l)}
\end{aligned}
$$

where n refer to sample sizes, $\overline{D}_k(1 - \overline{D}_k)$ $(\overline{D}_k - \overline{D}_l)(1 - (\overline{D}_k - \overline{D}_l))$ expressions refer to variance of treatment, and the final equation is the same for two timing groups.

Weights discussion

- Two things to note:
  - → More units in a group, the bigger its 2x2 weight is
  - → Group treatment variance weights up or down a group's 2x2
- Think about what causes the treatment variance to be as big as possible. Let's think about the s_{ku} weights.
  - → $\overline{D} = 0.1$. Then $0.1 \times 0.9 = 0.09$
  - → $\overline{D} = 0.4$. Then $0.4 \times 0.6 = 0.24$
  - → $\overline{D} = 0.5$. Then $0.5 \times 0.5 = 0.25$
  - → $\overline{D} = 0.6$. Then $0.6 \times 0.4 = 0.24$
- This means the weight on treatment variance is maximized for *groups treated in middle of the panel*

More weights discussion

- But what about the “treated on treated” weights (i.e., $\overline{D}_k - \overline{D}_l$)
- Same principle as before - when the difference between treatment variance is close to 0.5, those 2x2s are given the greatest weight
- For instance, say $t_k^* = 0.15$ and $t_l^* = 0.67$. Then $\overline{D}_k - \overline{D}_l = 0.52$. And thus $0.52 \times 0.48 = 0.2496$.

Summarizing TWFE centralities

- Groups in the middle of the panel weight up their respective 2x2s via the variance weighting
- Decomposition highlights the strange role of panel length when using TWFE
- Different choices about panel length change both the 2x2 and the weights based on variance of treatment

Moving from 2x2s to causal effects and bias terms

Let's start breaking down these estimators into their corresponding estimation objects expressed in causal effects and biases

$$
\begin{aligned}
\widehat{\delta}_{kU}^{2x2} &= ATT_k Post + \Delta Y_k^0(Post(k), Pre(k)) - \Delta Y_U^0(Post(k), Pre) \\
\widehat{\delta}_{kl}^{2x2} &= ATT_k(MID) + \Delta Y_k^0(MID, Pre) - \Delta Y_l^0(MID, Pre)
\end{aligned}
$$

These look the same because you're always comparing the treated unit with an untreated unit (though in the second case it's just that they haven't been treated *yet*).

The dangerous 2x2

But what about the 2x2 that compared the late groups to the already-treated earlier groups? With a lot of substitutions we get:

$$\widehat{\delta}_{lk}^{2x2} = ATT_{l,Post(l)} + \underbrace{\Delta Y_l^0(Post(l), MID) - \Delta Y_k^0(Post(l), MID)}_{\text{Parallel trends bias}} - \underbrace{(ATT_k(Post) - ATT_k(Mid))}_{\text{Heterogeneity bias!}}$$

Substitute all this stuff into the decomposition formula

$$\widehat{\delta}^{DD} = \sum_{k \neq U} s_{kU} \widehat{\delta}^{2x2}_{kU} + \sum_{k \neq U} \sum_{l > k} s_{kl} \left[\mu_{kl} \widehat{\delta}^{2x2,k}_{kl} + (1 - \mu_{kl}) \widehat{\delta}^{2x2,l}_{kl} \right]$$

where we will make these substitutions

$$\begin{aligned}
\widehat{\delta}^{2x2}_{kU} &= ATT_k(Post) + \Delta Y^0_l(Post, Pre) - \Delta Y^0_U(Post, Pre) \\
\widehat{\delta}^{2x2,k}_{kl} &= ATT_k(Mid) + \Delta Y^0_l(Mid, Pre) - \Delta Y^0_l(Mid, Pre) \\
\widehat{\delta}^{2x2,l}_{lk} &= ATT_l Post(l) + \Delta Y^0_l(Post(l), MID) - \Delta Y^0_k(Post(l), MID) \\
&\quad -(ATT_k(Post) - ATT_k(Mid))
\end{aligned}$$

Notice all those potential sources of biases!

Potential Outcome Notation

$$p\,lim\,\widehat{\delta}^{TWFE}_{n\to\infty} \quad = \quad VWATT + VWPT - \Delta ATT$$

- Notice the number of assumptions needed *even* to estimate this very strange weighted ATT (which is a function of how you drew the panel in the first place).
- With dynamics, it attenuates the estimate (bias) and can even reverse sign depending on the magnitudes of what is otherwise effects in the sign in a reinforcing direction!
- Let's look at each of these three parts more closely

Variance weighted ATT

$$
\begin{aligned}
VWATT &= \sum_{k \neq U} \sigma_{kU} ATT_k(Post(k)) \\
&+ \sum_{k \neq U} \sum_{l > k} \sigma_{kl} \left[\mu_{kl} ATT_k(MID) + (1 - \mu_{kl}) ATT_l(POST(l)) \right]
\end{aligned}
$$

where σ is like s only population terms not samples.

- Weights sum to one.
- Note, if all the ATT are identical, then the weighting is irrelevant.
- But otherwise, it's basically weighting each of the individual sets of ATT we have been discussing, where weights depend on group size and variance

Variance weighted parallel trends

$$
\begin{aligned}
VWPT &= \sum_{k \neq U} \sigma_{kU} \Big[\Delta Y_k^0(Post(k), Pre) - \Delta Y_U^0(Post(k), Pre) \Big] \\
&+ \sum_{k \neq U} \sum_{l > k} \sigma_{kl} \Big[\mu_{kl} \{ \Delta Y_k^0(Mid, Pre(k)) - \Delta Y_l^0(Mid, Pre(k)) \} \\
&+ (1 - \mu_{kl}) \{ \Delta Y_l^0(Post(l), Mid) - \Delta Y_k^0(Post(l), Mid) \} \Big]
\end{aligned}
$$

There are K^2 parallel trends inside the weights. Their weighted average must equal zero.

Heterogeneity bias

$$\Delta ATT = \sum_{k \neq U} \sum_{l > k} (1 - \mu_{kl}) \Big[ATT_k(Post(l) - ATT_k(Mid)) \Big]$$

Now, if the ATT is constant over time, then this difference is zero, but what if the ATT is not constant? Then TWFE is biased, and depending on the dynamics and the VWATT, may even flip signs

Callaway and Sant'Anna 2020

- New papers are coming out focused on the issues that we are seeing with TWFE
- I'll discuss one though by Callaway and Sant'anna (2020) due to time constraints (call it CS)
- If we have time, I'll run through a simulation illustrating both the bias of TWFE and the unbiased estimation of this CS estimator
- Interesting ancestry – CS is a descendent of Abadie (2005) from earlier

Preliminary

CS considers identification, aggregation, estimation and inference procedures for ATT in DD designs with

1. multiple time periods
2. variation in treatment timing (i.e., differential timing)
3. parallel trends only holds after conditioning on observables

When might you use this estimator

Probably in the very situations describing your own study

1. When treatment effects heterogenous by time of adoption
2. When treatment effects change over time
3. When shortrun effects more pronounced than longrun effects
4. When treatment effect dynamics differ if people are first treated in a recession relative to expansion years

Group-time ATT is the parameter of interest in CS

$$ATT(g,t) = E[Y_t^1 - Y_t^0 | G_g = 1]$$

Group-time ATT

Group-time ATT is the ATT for a specific group and time

- Groups are basically cohorts of units treated at the same time
- CS will calculate an ATT per group/time which will be the sum of all $T - t_k$ for all groups (i.e., a lot)
- Group-time ATT estimates are not determined by the estimation method one adopts (first difference or FE) bc they are simple differences in means
- Does not directly restrict heterogeneity with respect to observed covariates, timing or the evolution of treatment effects over time
- Provides a way to aggregate over these to get a single ATT
- Inference is the bootstrap

Notation

- T periods going from $t = 1, \ldots, T$
- Units are either treated ($D_t = 1$) or untreated ($D_t = 0$) but once treated cannot revert to untreated state
- G_g signifies a group and is binary. Equals one if individual units are treated at time period t.
- C is also binary and indicates a control group unit equalling one if "never treated" (can be relaxed though to "not yet treated")
  - $\rightarrow$ Recall the problem with TWFE on using treatment units as controls
- Generalized propensity score enters into the estimator as a weight:

$$\widehat{p(X)} = Pr(G_g = 1 | X, G_c + C = 1)$$

Assumptions

Assumption 1: Sampling is iid (panel data)

Assumption 2: Conditional parallel trends (for either never treated or not yet treated)

$$E[Y_t^0 - Y_{t-1}^0 | X, G_g = 1] = [Y_t^0 - Y_{t-1}^0 | X, C = 1]$$

Assumption 3: Irreversible treatment

Assumption 4: Common support (propensity score)

Assumption 5: Limited treatment anticipation (i.e., treatment effects are zero pre-treatment)

CS Estimator (the IPW version)

$$ATT(g,t) = E\left[\left(\frac{G_g}{E[G_g]} - \frac{\frac{\hat{p}(X)C}{1-\hat{p}(X)}}{E\left[\frac{\hat{p}(X)C}{1-\hat{p}(X)}\right]}\right)(Y_t - Y_{g-1})\right]$$

This is the inverse probability weighting estimator. Alternatively, there is an outcome regression approach and a doubly robust. Sant'Anna recommends DR. Notice hw CS doesn't use already-treated as controls.

Staggered adoption (i.e., universal coverage)

Proof.

Remark 1: In some applications, eventually all units are treated, implying that C is never equal to one. In such cases one can consider the "not yet treated" ($D_t = 0$) as a control group instead of the "never treated?" ($C = 1$). □

Aggregated vs single year/group ATT

- The method they propose is really just identifying very narrow ATT per group time.
- But we are often interested in more aggregate parameters, like the ATT across all groups and all times
- They present two alternative methods for building “interesting parameters”
- Inference from a bootstrap

Stata simulation

Let's now review a simulation in Stata which can be downloaded from my github repo called **`baker.do`**.

Pedro Sant'Anna for the win

- Now a word from a good friend – Pedro Sant'Anna. Legend!
- He'll be discussing deChaisemartin and D'Haultfoeiller (2020) because:
  - → He's a good guy
  - → He's a great presenter
  - → I fell behind

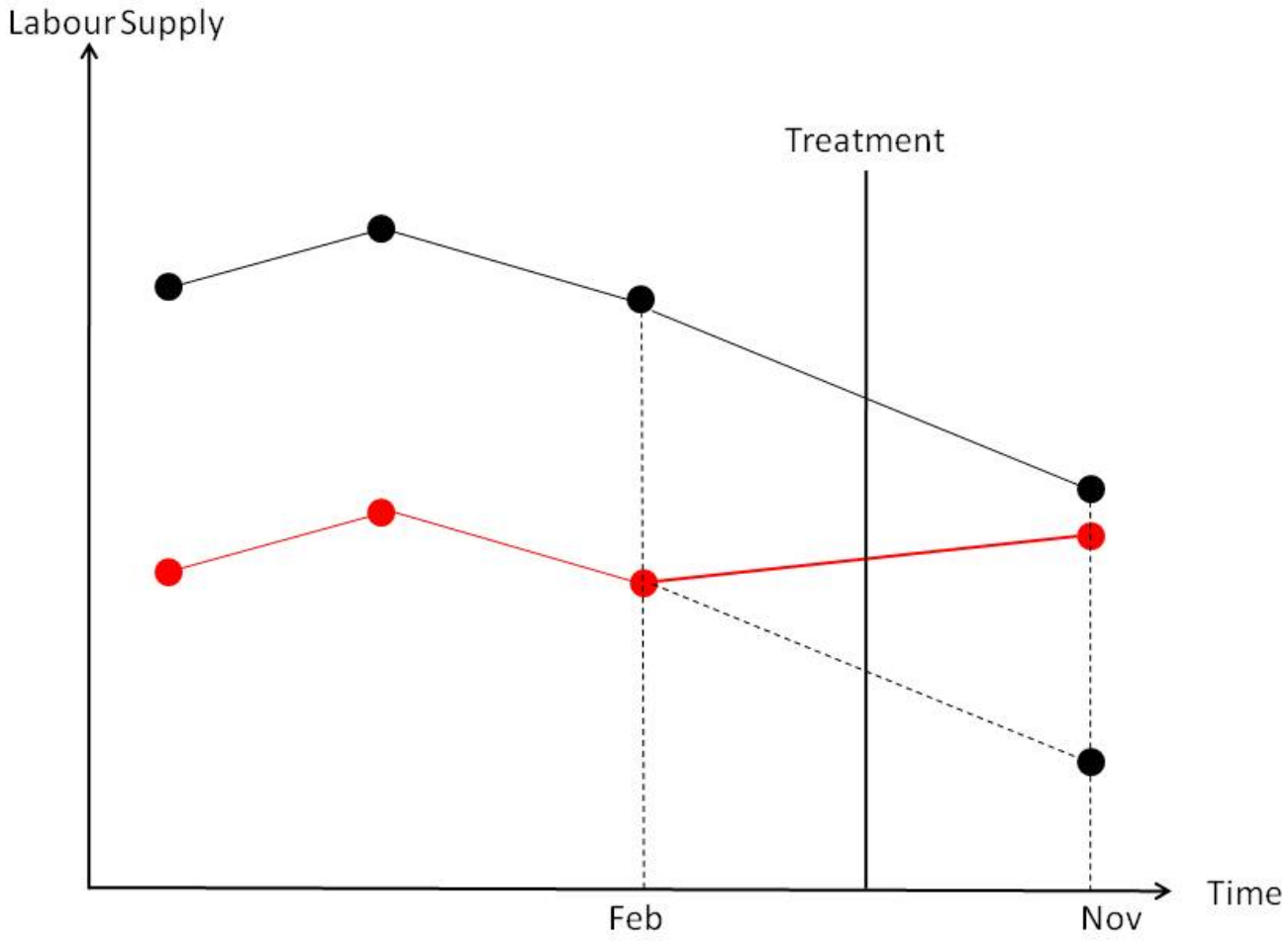
de Chaisemartin and D’Haultfoeulle

Now a word from our sponsor (Pedro San’tAnna)

Pre-trends

- The identifying assumption for all DD designs is parallel trends
- Parallel trends cannot be directly verified because technically one of the parallel trends is an unobserved counterfactual
- But one often will check a hunch for parallel trends using pre-trends
- But, even if pre-trends are the same one still has to worry about other policies changing at the same time (omitted variable bias)

Plot the raw data when there's only two groups



Event study regression

- Including leads into the DD model is an easy way to analyze pre-treatment trends
- Lags can be included to analyze whether the treatment effect changes over time after assignment
- The estimated regression would be:

$$Y_{its} = \gamma_s + \lambda_t + \sum_{\tau=-2}^{-q} \gamma_\tau D_{s\tau} + \sum_{\tau=0}^{m} \delta_\tau D_{s\tau} + x_{ist} + \varepsilon_{ist}$$

→ Treatment occurs in year 0
→ Includes q leads or anticipatory effects
→ Includes m leads or post treatment effects

Medicaid and Affordable Care Act example

- Miller, et al. (2019) examine a rollout of Medicaid under the Affordable Care Act
- They link large-scale survey data with administrative death records
- 9.3 reduction in annual mortality caused by Medicaid expansion
- Driven by a reduction in disease-related deaths which grows over time

(a) Medicaid Eligibility

(b) Medicaid Coverage

(c) Uninsured

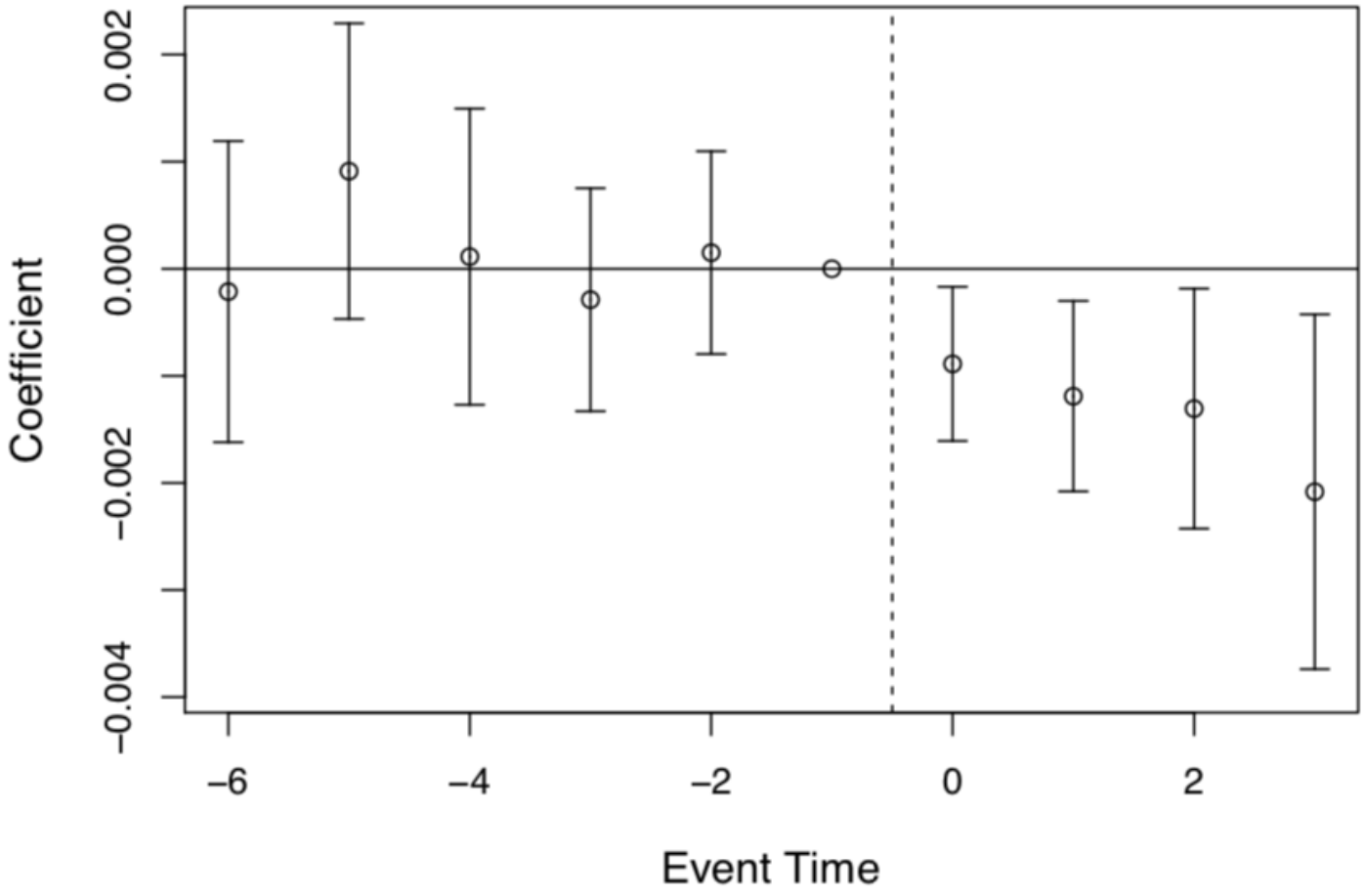


Figure: Miller, et al. (2019) estimates of Medicaid expansion's effects on on annual mortality

Differential timing complicates plotting sample averages

- New Jersey treated in late 1992, New York in late 1993, Pennsylvania never treated
- Pre-treatment:
  - → New Jersey: <1992
  - → New York: <1993
  - → Pennsylvania: undefined
- So how do we check parallel leads?

Early efforts at event studies

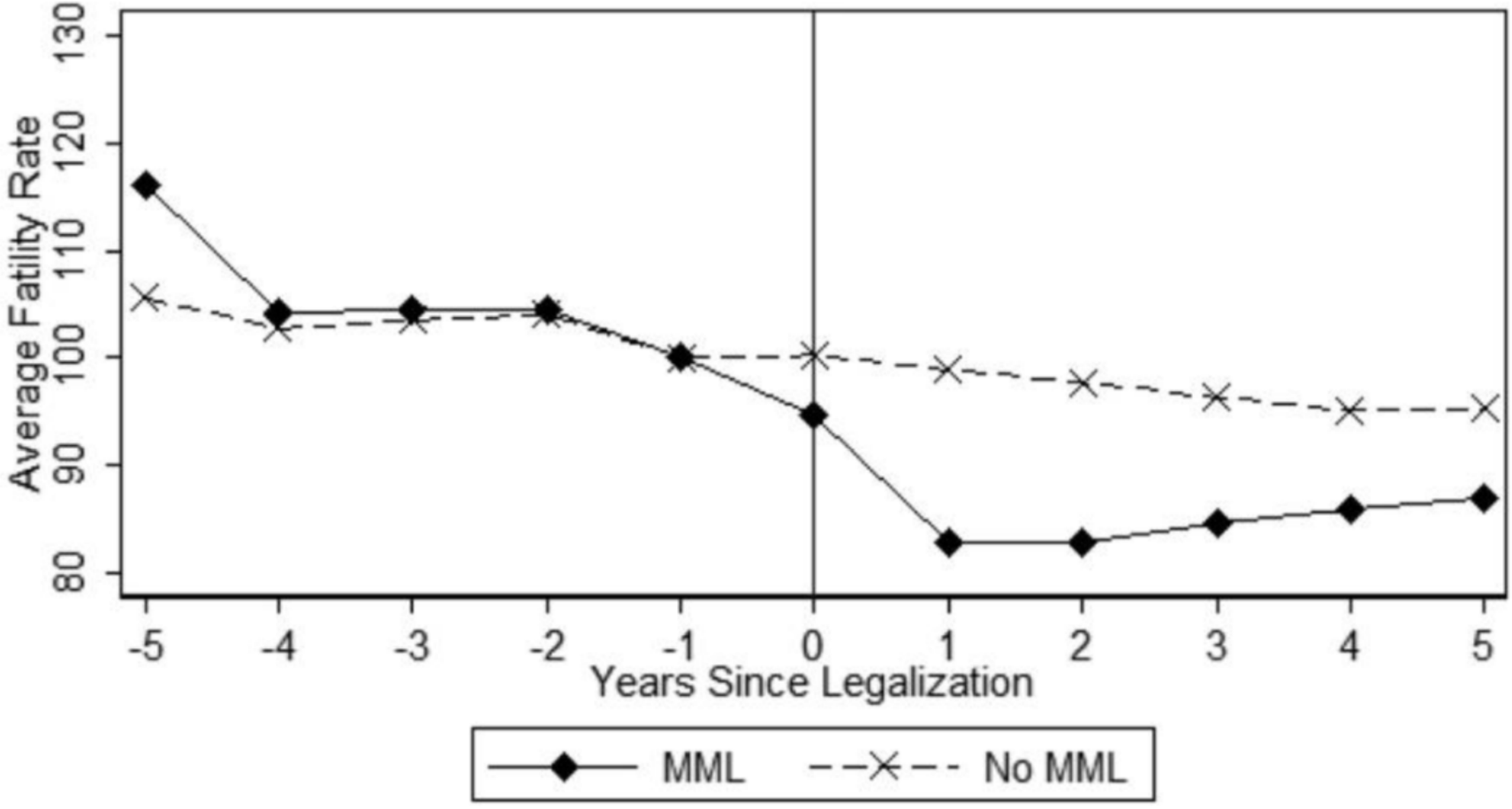


Figure: Anderson, et al. (2013) display of raw traffic fatality rates for re-centered treatment states and control states with randomized treatment dates

Randomized control counties to receive arbitrary dates as treatment can be misleading

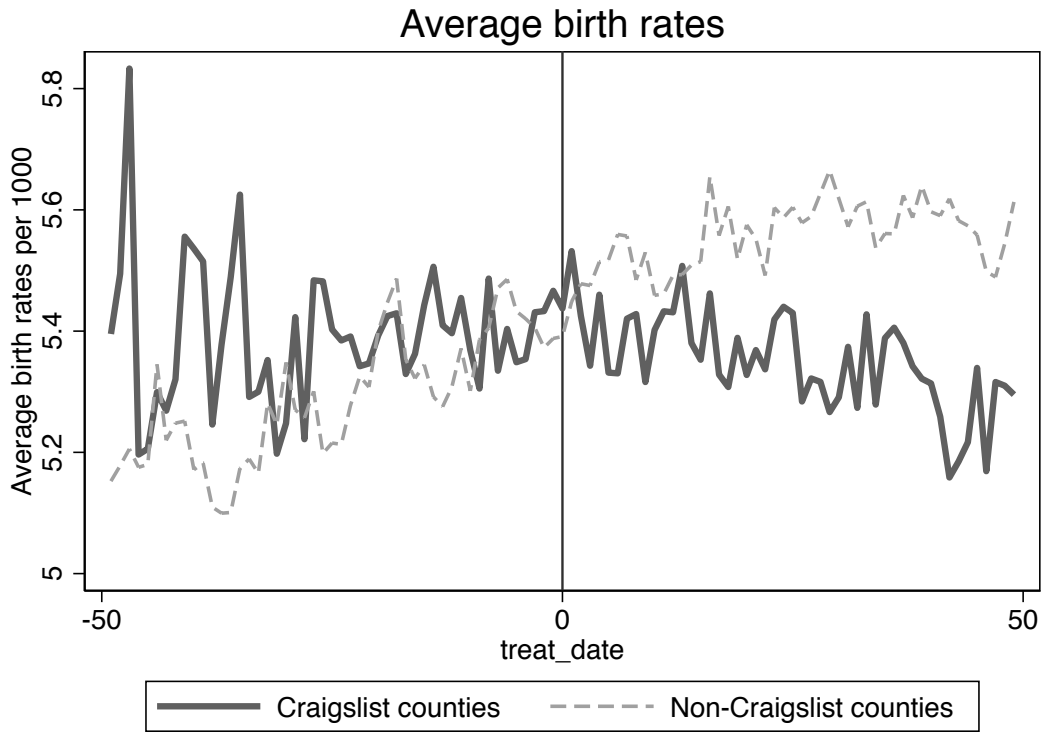
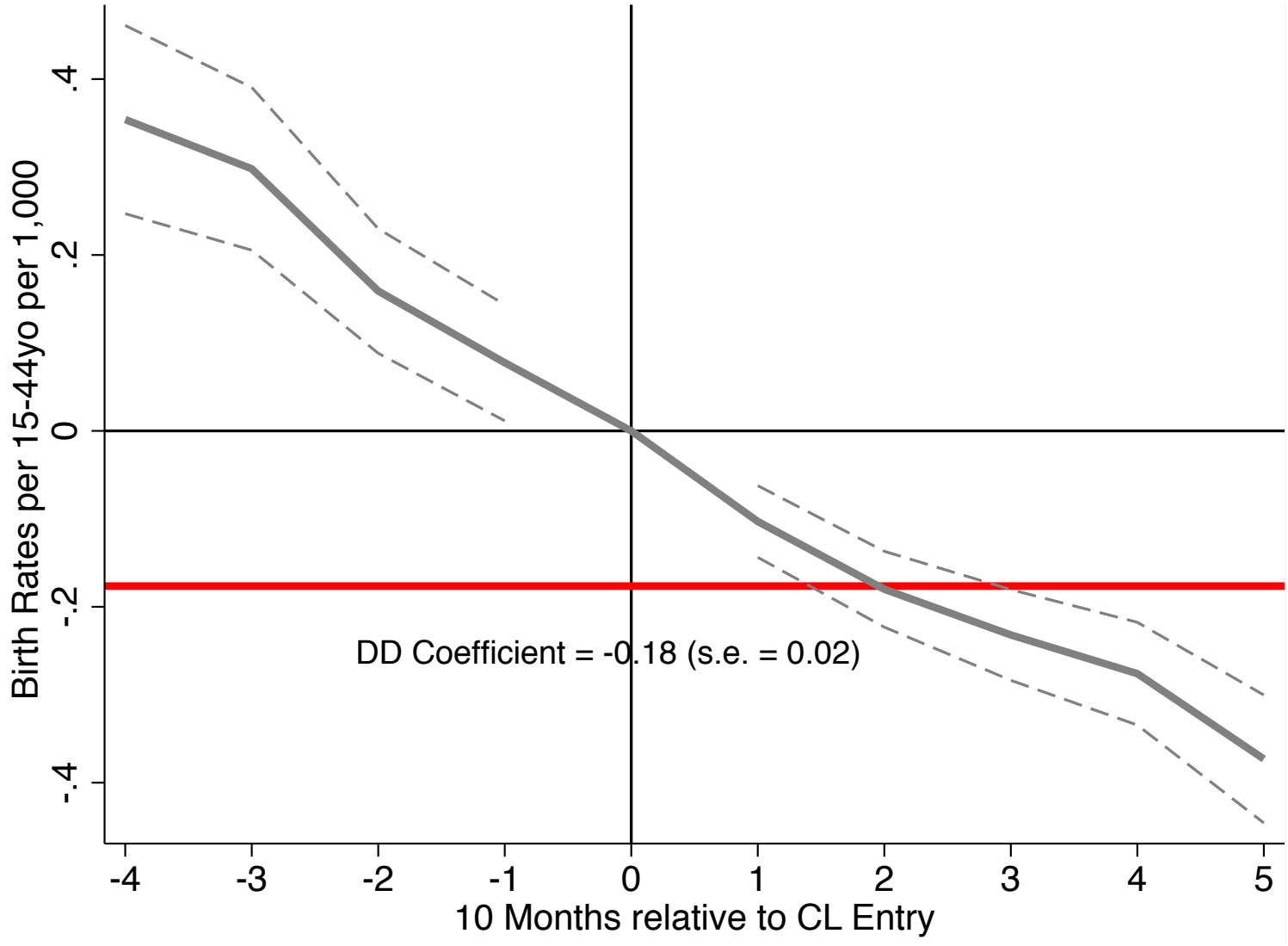


Figure: From one of my studies. Looks decent right?



Same data as a couple slides ago, leads don't look good

Sun and Abraham 2020

- Recall our discussion of event studies estimated with TWFE under differential timing
- Now that we know about the biases of TWFE when estimating aggregate DD parameters, let's revisit event studies under differential timing
- Callaway and Sant'Anna (2020) propose alternative estimators for event studies that estimate group-time ATT in relative event time
- But now we will discuss Sun and Abraham (2020) [SA] which is like a blend of Goodman-Bacon's decomposition and Callaway and Sant'anna alternative estimator to TWFE

Summarizing

- Goodman-Bacon (2021, forthcoming) focused on decomposition of TWFE to show bias under differential timing
- Callaway and Sant'anna (2020) presents alternative estimator that yields unbiased estimates of group-time ATTs which can be aggregated or put into event study plots
- Sun and Abraham (SA) is like a combination of the two papers

Summarizing (cont.)

1. SA is a decomposition of the population regression coefficient on event study leads and lags with differential timing estimated with TWFE
2. They show that the population regression coefficient is "contaminated" by information from other leads and lags
3. SA presents an alternative estimator that is not so dissimilar to CS

Summarizing (cont.)

- Problems seem to occur with DD when we introduce treatment effect heterogeneity
- Under treatment effect heterogeneity, spurious non-zero positive lead coefficients even when there is no pretrend
- This problem is exacerbated by the TWFE related weights as under some scenarios, the weights sum to zero and“cancel out” the treatment effects from other periods
- They present a 3-step TWFE based alternative estimator which addresses the problems that they find

Summarizing (cont.)

- Only decomposition of TWFE estimating dynamic leads and lags (Goodman-Bacon focused on a “static” specification)
- Contamination of coefficients on leads and lags by treatment effects depends on the magnitude of the weights on the true group-time ATT, or “cohort-specific ATT”
- Weights are a function of cohort composition
- Examining weights lets you gauge how treatment effect heterogeneity would interact with potential non-zero and non-convex weighting in population regression coefficients on the leads and lags

Difficult notation sadly

- When treatment occurs at the same time, we say they are part of the same cohort, e
- If we bin the data, then a lead or lag l will appear in the bin g so sometimes they use g instead of l or $l \in g$
- Building block is the "cohort-specific ATT" or $CATT_{e,l}$ – same thing as CS group-time ATT
- Estimate $CATT_{e,l}$ with population regression coefficient μ_l

Difficult notation (cont.)

- At each time t there are two possible treatment status $D_{i,t} \in \{0, 1\}$ over $T + 1$ time periods
- Path of treatment status scales exponentially with T and an take on 2^{T+1} possible values
- They focus on irreversible treatment where treatment status is non-decreasing sequence of zeroes and ones

Difficult notation (cont.)

- If a group is never treated, the ∞ symbol is used to either describe the group ($E_i = \infty$) or the potential outcome (Y^{∞})
- $Y_{i,t}^{\infty}$ is is the potential outcome for unit i if it had never received treatment (versus received it later), also called the baseline outcome
- Other counterfactuals are possible – maybe unit i isn't "never treated" but treated later in counterfactual

More difficult notation (cont.)

- Treatment effects are the difference between the observed outcome relative to the never-treated counterfactual outcome: $Y_{i,t} - Y_{i,t}^{\infty}$
- We can take the average of treatment effects at a given relative time period across units first treated at time $E_i = e$ (same cohort) which is what we mean by $CATT_{e,l}$
- Doesn't use t index time ("calendar time"), rather uses l which is time until or time after treatment date e ("relative time")
- Think of it as $l =$year - treatment date

Definition 1

Definition 1: The cohort-specific ATT l periods from initial treatment date e is:

$$CATT_{e,l} = E[Y_{i,e+l} - Y^{\infty}_{i,e+l} | E_i = e]$$

Identifying assumption 1

Assumption 1: Parallel trends in baseline outcomes:
$E[Y_{i,t}^{\infty} - Y_{i,s}^{\infty} | E_i = e]$ is the same for all $e \in supp(E_i)$ and for all s, t and is equal to $E[Y_{i,t}^{\infty} - Y_{i,s}^{\infty}]$

Interesting SA comment: Never-treated units are likely to differ from ever-treated units in many ways; think of a Roy model. What does it imply that they chose not to get treated? It may imply net negative treatment effects and that could mean they may not share the same evolution of baseline outcomes as the treatment groups. If you think they are unlikely to satisfy this assumption, then drop them. Almost like a synthetic control approach.

Assumption 2

Assumption 2: No anticipator behavior in pre-treatment periods:
There is a set of pre-treatment periods such that
$E[Y^{e}_{i,e+l} - Y^{\infty}_{i,e+l} | E_i = e] = 0$ for all possible leads.

Basically means that potential outcomes prior to treatment at baseline by on average the same. This means there is no pre-trends, essentially. This is most plausible if the full treatment paths are not known to the units (e.g., Craigslist opening erotic services without announcement)

Assumption 3

Assumption 3: Treatment effect homogeneity: For each relative time period l, the $CATT_{e,l}$ doesn't depend on the cohort and is equal to $CATT_l$.

Assumption 3 requires each cohort experience the same path of treatment effects. Treatment effects need to be the same across cohorts in every relative period for homogeneity to hold, whereas for heterogeneity to occur, treatment effects just need to differ across cohorts in one relative time period. Doesn't preclude dynamic treatment effects, though. It just imposes that cohorts share the same treatment path.

Treatment effect heterogeneity

- Assumption 3 is violated when different cohorts experience different paths of treatment effects
- Cohorts may differ in their covariates which affect how they respond to treatment (e.g., if treatment effects vary with age, and there is variation in age across units first treated at different times, then there will be heterogeneous treatment effects)
- Doesn’t rule out parallel trends

TWFE Regression

$$Y_{i,t} = \alpha_i + \delta_t + \sum_{g \in G} \mu_g 1\{t - E_i \in g\} + \varepsilon_{i,t}$$

They say E_i is the initial time of a binary variable absorbing treatment for unit i. Fixed effects should be obvious. μ_g is the population regression coefficient on the leads and lags that we want to estimate. We estimate this using OLS and get $\widehat{\mu_g}$.

We are interested in the properties of μ_g under differential timing as well as whether there are any never-treated units

Specifying the leads and lags

How will we specify the $1\{t - E_i \in g\}$ term? SA considers a couple:

1. Static specification:

$$Y_{i,t} = \alpha_i + \delta_t + \mu_g \sum_{l \geq 0} D_{i,t}^l + \varepsilon_{i,t}$$

2. Dynamic specification:

$$Y_{i,t} = \alpha_i + \delta_t + \sum_{l=-K}^{-2} \mu_l D_{i,t}^l + \sum_{l=0}^{L} \mu_l D_{i,t}^l + \varepsilon_{i,t}$$

Multicollinearity

Dynamic specification requires deciding which leads to drop. They recommend dropping two: $l = -1$ and some other one (they seem to favor $l = -4$). The reason is twofold. You drop one of them to avoid multicollinearity in the relative time indicators. You drop a second one because of the multicollinearity coming from the linear relationship between TWFE and the relative period indicators.

Trimming and binning

- First some terms: trimming and binning, I do both in the Mixtape when analyzing Cheng and Hoekstra (2013)
- Binning means placing all “distant” relative time indicators into a single one. Done because of the sparseness of units in such distant bins. So if there’s 3 distant leads and lags that aren’t balanced, combine them all into the last lead and lag
- Trimming means excluding any relative period for which you don’t have balance in relative time. This creates a balanced panel “in relative time”, but imbalanced panel length overall.
- They’ll analyze both and how they affect $\widehat{\mu_g}$ estimation using TWFE

Interpreting $\widehat{\mu_g}$ under no to all assumptions

Proposition 1 (no assumptions): The population regression coefficient on relative period bin g is a linear combination of differences in trends from its own relative period $l \in g$, from relative periods $l \in g'$ of other bins $g' \neq g$, and from relative periods excluded from the specification (e.g., trimming).

$$
\begin{aligned}
\mu_g \quad = \quad & \underbrace{\sum_{l \in g} \sum_{e} w^g_{e,l} (E[Y_{i,e+l} - Y^{\infty}_{i,0} | E_i = e] - E[Y^{\infty}_{i,e+l} - Y^{\infty}_{i,0}])}_{\text{Good stuff}} \\
+ \quad & \underbrace{\sum_{g' \neq g} \sum_{l \in g'} \sum_{e} w^g_{e,l} (E[Y_{i,e+l} - Y^{\infty}_{i,0} | E_i = e] - E[Y^{\infty}_{i,e+l} - Y^{\infty}_{i,0}])}_{\text{Bleh – Other included relative time}} \\
+ \quad & \underbrace{\sum_{l \in g^{excl}} \sum_{e} w^g_{e,l} (E[Y_{i,e+l} - Y^{\infty}_{i,0} | E_i = e] - E[Y^{\infty}_{i,e+l} - Y^{\infty}_{i,0}])}_{\text{More bleh – Excluded}}
\end{aligned}
$$

Superscript g associates the weight with coefficient μ_g. The weight associated with cohort e in relative period l is equal to the population regression coefficient on the $1\{t - E_i \in g\}$ from regression $D^l_{i,t} \times 1\{E_i = e\}$ on all bin indicators included in the regression and TWFE. Just the mechanics of double demeaning from TWFE

Weight ($w^g_{e,l}$) summation cheat sheet

1. For relative periods of μ_g own $l \in g$, $\sum_{l \in g} \sum_e w^g_{e,l} = 1$
2. For relative periods belonging to some other bin $l \in g'$ and $g' \neq g$, t $\sum_{l \in g'} \sum_e w^g_{e,l} = 0$
3. For relative periods not included in G, $\sum_{l \in g^{excl}} \sum_e w^g_{e,l} = -1$

Estimating the weights

Regress $D_{i,t}^{l} \times 1\{E_i = e\}$ on:

1. all bin indicators included in the main TWFE regression,
2. $\{1\{t - E_i \in g\}\}_{g \in G}$(i.e., leads and lags) and
3. the unit and time fixed effects

Interpretation of coefficients under parallel trends only

Proposition 2: Under the parallel trends only, the population regression coefficient on the indicator for relative period bing g is a linear combination of $CATT_{e,l\in g}$ as well as $CATT_{d,l'}$ from other relative periods $l' \notin g$ with the same weights stated in Proposition 1:

$$\begin{aligned}\mu_g \quad = \quad & \underbrace{\sum_{l\in g}\sum_{e} w^g_{e,l} CATT_{e,l}}_{\text{Desirable}} \\ & + \underbrace{\sum_{g'\neq g, g'\in G}\sum_{l'\in g'}\sum_{e} w^g_{e,l'} CATT_{e,l'}}_{\text{Undesirable – other specified bins}} \\ & + \underbrace{\sum_{l'\in g^{excl}}\sum_{e} w^g_{e,l'} CATT_{e,l'}}_{\text{Undesirable – excluded relative time indicators}}\end{aligned}$$

Comment on Proposition 2

The coefficient μ_g can be written as an average of $CATT_{e,l}$ from own periods but also $CATT_{e,l'}$ from other periods.

The weights are still functions of cohort comparisons, like in Proposition 1, which means μ_g can be written as non-convex averages of not only $CATT_{e,l}$ from own periods $l \in g$, but also $CATT_{e,l'}$ from other periods.

Means μ_g could in fact be the wrong sign to all $CATT_{e,l \in g}$.

Weights can help us gauge the severity of this problem.

When the weights have larger magnitude, treatment effect heterogeneity matters more as a particular $CATT_{e,l}$ can drive the overall estimates. But when weights are uniform, treatment effect heterogeneity matters less.

Interpretation under parallel trends and no anticipation

Proposition 3: If parallel trends holds and no anticipation holds for all $l < 0$ (i.e., no anticipatory behavior pre-treatment), then the population regression coefficient μ_g for g is a linear combination of post-treatment $CATT_{e,l'}$ for all $l' \geq 0$.

$$\begin{aligned}
\mu_g \quad = \quad & \sum_{l' \in g, l' \geq 0} \sum_{e} w^g_{e,l'} CATT_{e,l'} \\
& + \sum_{g' \neq g, g' \in G} \sum_{l' \in g', l' \geq 0} \sum_{e} w^g_{e,l'} CATT_{e,l'} \\
& + \sum_{l' \in g^{excl}, l' \geq 0} \sum_{e} w^g_{w,l'} CATT_{e,l'}
\end{aligned}$$

Proposition 3 comment

Notice how once we impose zero pre-treatment treatment effects, those terms are gone (i.e., no $l \in g, l < 0$). But the second term remains unless we impose treatment effect homogeneity (homogeneity causes terms due to weights summing to zero to cancel out). Thus μ_g may be non-zero for pre-treatment periods *even though parallel trends hold in the pre period.*

Proposition 4

Proposition 4: If parallel trends and treatment effect homogeneity, then $CATT_{e,l} = ATT_l$ is constant across e for a given l, and the population regression coefficient μ_g is equal to a linear combination of $ATT_{l \in g}$, as well as $ATT_{l' \notin g}$ from other relative periods

$$\begin{aligned} \mu_g &= \sum_{l \in g} w_l^g ATT_l \\ &\quad + \sum_{g' \neq g} \sum_{l' \in g'} w_{l'}^g ATT_{l'} \\ &\quad + \sum_{l' \in g^{excl}} w_{l'}^g ATT_{l'} \end{aligned}$$

Proposition 4 comment

The weight $w_l^g = \sum_e w_{e,l}^g$ sums over the weights $w_{e,l}^g$ from Proposition 1 and is equal to the population regression coefficient from the following auxiliary regression:

$$D_{i,t}^l = \alpha_i + \lambda_t + \sum_{g \in G} w_l^g \cdot 1\{t - E_i \in g\} + u_{i,t}$$

which regresses $D_{i,t}^l$ on all bin indicators and TWFE

On binning

- Many propose either binning or trimming to create “balanced” panels (in relative event time)
- But SA notes that binning in simulations creates uninterpretable weights (due to the binned $CATT_{e,l'}$ inclusion in μ_g), whereas trimming creates weights that are more reasonable
- This may be because trimming subtracts the corresponding $CATT_{e,l'}$ from μ regression coefficient

Intuition for contamination

- Stupid notation make Hulk smash!
- Let's do a simple toy example instead

Balanced panel $T = 2$ with cohorts $E_i \in \{1, 2\}$. We drop two relative time periods to avoid multicollinearity, so we will include bins $\{-2, 0\}$ and drop $\{-1, 1\}$.

Toy example

$$\mu_{-2} = \underbrace{CATT_{2,-2}}_{\text{own period}} + \underbrace{\frac{1}{2}CATT_{1,0} - \frac{1}{2}CATT_{2,0}}_{\text{other included bins}}$$
$$+ \underbrace{\frac{1}{2}CATT_{1,1} - CATT_{1,-1} - \frac{1}{2}CATT_{2,-1}}_{\text{Excluded bins}}$$

- Parallel trends gets us to all of the $CATT$
- No anticipation makes $CATT = 0$ for all $l < 0$ (all $l < 0$ cancel out)
- Homogeneity cancels second and third terms
- Still leaves $\frac{1}{2}CATT_{1,1}$ – you chose to exclude a group with a treatment effect

Lesson: drop the relative time indicators on the left, not things on the right, bc lagged effects will contaminate through the excluded bins

Interaction-weighted estimator

- They propose an interacted weighted estimator (IW) as a consistent estimator for μ_g
- Estimator uses either never-treated as controls or “last cohort treated” if no never-treated (contra CS which uses “not yet treated”)
- No covariates bc this is a regression with fixed effects and time-varying covariates create own biases, although they note you can plug in CS for the DD calculation and recover $CATT$ that way
- The interaction is a TWFE regression specification that interacts relative period indicators with cohort/group indicators, excluding indicators for never-treated cohorts

Interaction-weighted estimator

- **Step one**: Do this DD regression and hold on to $\widehat{\delta}_{e,l}$

$$Y_{i,t} = \alpha_i + \lambda_t + \sum_{e \notin C} \sum_{l \neq -1} \delta_{e,l} (1\{E_i = e\} \cdot D_{i,t}^l) + \varepsilon_{i,t}$$

Can use never-treated or last-treated cohort. Drop always treated. The $\delta_{e,l}$ is a DD estimator for $CATT_{e,l}$ with particular choices for pre-period and cohort controls

Interaction-weighted estimator

- **Step two**: Estimate weights using sample shares of each cohort in the relevant periods:

$$Pr(E_i = e | E_i \in [-l, T - l])$$

IW estimator

- **Step three**: Take a weighted average of estimates for $CATT_{e,l}$ from Step 1 with weight estimates from step 2

$$\widehat{v}_g = \frac{1}{|g|} \sum_{l \in g} \sum_{e} \widehat{\delta}_{e,l} \widehat{Pr}\{E_i = e | E_i \in [-l, T-l]\}$$

Consistency and Inference

- Under parallel trends and no anticipation, $\widehat{\delta}_{e,l}$ is consistent, and sample shares are also consistent estimators for population shares.
- Thus IV estimator is consistent for a weighted average of $CATT_{e,l}$ with weights equal to the share of each cohort in the relevant period(s).
- They show that each IW estimator is asymptotically normal and derive its asymptotic variance. Doesn't rely on bootstrap like CS.

DD Estimator of CATT

Definition 2: DD estimator with pre-period s and control cohorts C estimates $CATT_{e,l}$ as:

$$\widehat{\delta_{e,l}} = \frac{E_N[(Y_{i,e+l} - Y_{i,s}) \times 1\{E_i = e\}]}{E_N[1\{E_i = e\}]} - \frac{E_N[(Y_{i,e+l} \times 1\{E_i \in C\}]}{E_N[1\{E_i \in C\}]}$$

Proposition 5: If parallel trends and no anticipation both hold for all pre-periods, then the DD estimator using any pre-period and non-empty control cohorts (never-treated or not-yet-treated) is an unbiased estimate for $CATT_{e,l}$

Software

- **Stata**: eventstudyinteract (can be installed from ssc)
- **R**: did2s (see `https://asjadnaqvi.github.io/DiD/docs/02_R/`)

Conclusion of SA

- Bacon shows the TWFE coefficient on the static parameter is “contaminated” by other periods leads and lags
- Three strong assumptions needed for TWFE to be unbiased: parallel trends, no anticipation, and treatment homogeneity
- Three step interaction-weighted estimator is an alternative
- Doesn’t restrict to treatment profile homogeneity
- Callaway and Sant’Anna (2020) and Sun and Abraham (2020) use different controls, but under certain situations (no covariates, never treated) they are the same (“nested”)

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