Day 2: On Weights and Clusters

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Design-Based Regression Inference
Spring 2024

Outline

1. Heterogeneous Treatment Effects

2. Clustered Standard Errors

Whose Treatment Effect is it Anyway?

- On Monday we contrasted design vs. outcome-model strategies in a constant-effect world (i.e. with a causal model of $y_i = \beta x_i + \varepsilon_i$)
  - Of course the real world is messier: more realistic is $y_i = \beta_i x_i + \varepsilon_i$ (or more complicated forms of effect heterogeneity)

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- Today we'll see another difference: how design-based vs. model-based regression/IV weigh together heterogeneous effects
  - Bottom line: design avoids recent concerns over "negative weights"...
  - ... at least as long as you don't have multiple treatments!

Why So Negative?

- A recent TWFE literature (e.g. de Chaisemartin and D'Haultfoeuille '20; Goodman-Bacon '21; Borusyak et al. '23) shows that some regressions identify $\beta = E[\psi_i \beta_i]/E[\psi_i]$ for possibly negative ψ_i

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- It turns out that such ψ_i also arise in design-based specifications, and they can also be negative
  - But sign reversals are impossible in design-based specs: then we also have $\beta = E[\phi_i \beta_i]/E[\phi_i]$ for "ex-ante" ϕ_i which are always non-negative

Simple Setup

- Suppose a researcher estimates by OLS:

$$y_i = \beta x_i + w_i'\gamma + e_i$$

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- Assume appropriate asymptotics for OLS to consistently estimate:

  $$\beta = \frac{E[\tilde{x}_i y_i]}{E[\tilde{x}_i^2]} = \frac{E[\tilde{x}_i x_i \beta] + E[\tilde{x}_i \varepsilon_i]}{E[\tilde{x}_i^2]}$$

  where $\tilde{x}_i$ are residuals from the population regression of x_i on w_i

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  ASSUMPTION 2: $E[x_i \mid \varepsilon_i, \beta_i, w_i] = w_i' \lambda$

  - Treatment is conditionally mean-independent of potential outcomes, with a linear *expected treatment* $E[x_i \mid w_i]$ (e.g. the propensity score)
  - E.g. a stratified experiment, where x_i is randomly assigned within strata dummied out in w_i
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- The second assumption yields a design-based OLS specification

  - Stronger (sufficient) condition: $x_i \mid (\varepsilon_i, \beta_i, w_i) \overset{iid}{\sim} G(w_i)$

Ex-Post Weights

- Since $E[\tilde{x}_i \varepsilon_i] = 0$, the OLS estimand has an average-effect representation under either assumption:

$$\beta = \frac{E[\psi_i \beta_i]}{E[\psi_i]}, \qquad \psi_i = \tilde{x}_i x_i$$

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- The ex-post weights are the end of the story for β under Assumption 1. But in design-based specifications we can take one more step
  - In experiments, who is in the effective control group is *random*. Before treatment is drawn, everyone expects the same weight!

Ex-Ante Weights

- Using the law of iterated expectations, we can also write:

$$\beta = \frac{E[E[\psi_i \mid w_i, \beta_i]\beta_i]}{E[E[\psi_i \mid w_i, \beta_i]]} \equiv \frac{E[\phi_i \beta_i]}{E[\phi_i]}$$

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  - $E[\tilde{x}_i x_i \mid w_i, \beta_i] = E[\tilde{x}_i^2 \mid w_i, \beta_i] + E[\tilde{x}_i \mid w_i, \beta_i] w_i' \lambda = Var(x_i \mid w_i, \beta_i) + 0$

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- Hence: sign reversals cannot occur in design-based OLS specifications

Interpretation

- Even if we formulate a design-based regression in terms of constant effects, the estimand is still reasonable under heterogeneous effects
  - Not necessarily true for outcome models (makes sense: we were just modeling ε_i! But additional models on β_i need not help)

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- Of course, the ϕ_i-weighted estimand may not be most of interest!
    - If $Cov(\phi_i, \beta_i) \approx 0$, we'll still get something close to $E[\beta_i]$
    - Otherwise, ϕ_i-weighting has desirable efficiency properties (Goldsmith-Pinkham et al. 2024)
    - Large class of alternative propensity-score-based estimators for other estimands under the stronger design assumption

General Setting

- Borusyak and Hull (2024) extend ex ante / ex post weights to:
  1. A more general causal model: potential outcomes $y_i(x)$ and $y_i = y_i(x_i)$
  2. IV: design-based assumption is then $E[z_i \mid y_i(\cdot), w_i] = w_i'\lambda$

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- For convex ex-ante weights in IV we require first-stage *monotonicity*: that x_i is non-decreasing in z_i for all units regardless of $y_i(\cdot)$
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- Framework is general, allowing for "formula" IVs (e.g. shift-share)
    - We'll see more about this in Friday's class

Multiple Treatments: Contamination Bias

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- They derive alternative estimators which avoid contamination bias while maintaining the nice efficiency properties of OLS weighting
  - Ultimately, becomes an empirical question of how important bias is

Example: Project STAR

- Krueger (1999) studies the STAR RCT, which randomized 12k students in 80 public elementary schools in Tennessee (!) to one of 3 classroom types:
    1. Regular-sized (20-25 students) – Control
    2. Small (13-17 students) – Treatment 1
    3. Regular-sized with a teaching aide – Treatment 2

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- Kids were randomized within schools, so the propensity of assignment to each treatment varied by school
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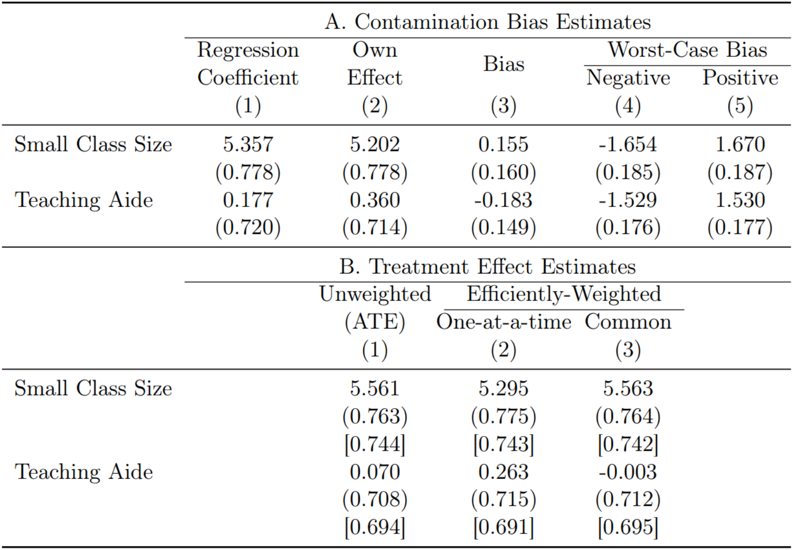
- We find significant *potential* for contamination bias: lots of treatment effect heterogeneity and variation in contamination weights
  - But actual contamination bias is minimal: $Corr(effects, weights) \approx 0$

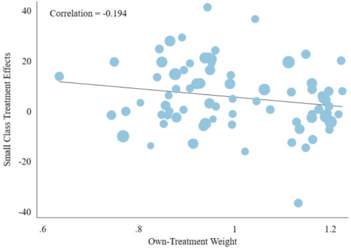
Project STAR, Revisited



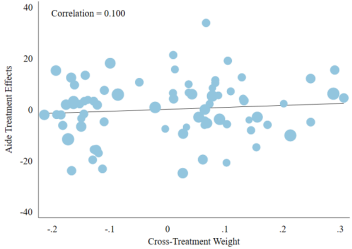
	A. Contamination Bias Estimates				
	Regression Coefficient	Own Effect	Bias	Worst-Case Bias	
				Negative	Positive
	(1)	(2)	(3)	(4)	(5)
Small Class Size	5.357	5.202	0.155	-1.654	1.670
	(0.778)	(0.778)	(0.160)	(0.185)	(0.187)
Teaching Aide	0.177	0.360	-0.183	-1.529	1.530
	(0.720)	(0.714)	(0.149)	(0.176)	(0.177)

	B. Treatment Effect Estimates		
	Unweighted	Efficiently-Weighted	
	(ATE)	One-at-a-time	Common
	(1)	(2)	(3)
Small Class Size	5.561	5.295	5.563
	(0.763)	(0.775)	(0.764)
	[0.744]	[0.743]	[0.742]
Teaching Aide	0.070	0.263	-0.003
	(0.708)	(0.715)	(0.712)
	[0.694]	[0.691]	[0.695]

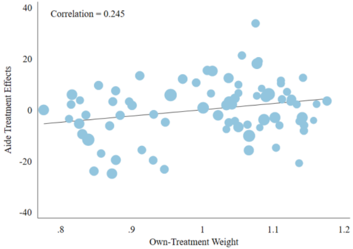
STAR Regression Weights vs. Treatment Effects



Panel A: Small Class
Own-Treatment Weight



Panel B: Aide
Cross-Treatment Weight



Panel C: Aide
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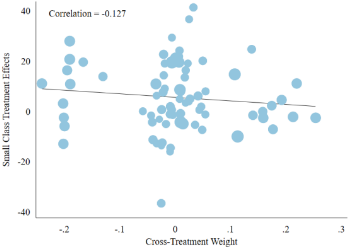

Panel D: Small Class
Cross-Treatment Weight

Outline

1. Heterogeneous Treatment Effects✓

2. Clustered Standard Errors

OLS Asymptotics: Review

- Where do SEs come from? OLS $\hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$ can be rewritten:

$$\sqrt{N}(\hat{\beta} - \beta) = \left(\frac{\mathbf{X}'\mathbf{X}}{N}\right)^{-1} \left(\frac{\mathbf{X}'\boldsymbol{\varepsilon}}{\sqrt{N}}\right)$$

  where $\mathbf{Y} = \mathbf{X}\beta + \boldsymbol{\varepsilon}$ stacks observations of the population regression

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  - Under rather mild conditions (a LLN), $\frac{\mathbf{X}'\mathbf{X}}{N} \xrightarrow{p} E\left[\frac{1}{N}\sum_i X_i X_i'\right]$

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  - We need to zero out some covariances to make progress

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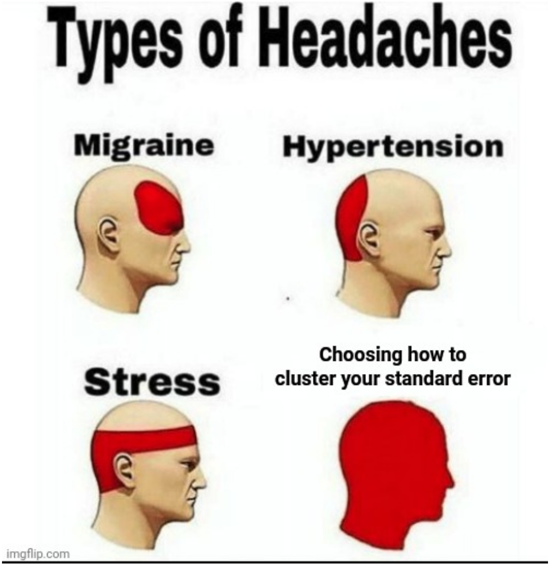
- This is what's going on under the hood when you ", *cluster(c)*"!

Easy, Right?



Source: Khoa Vu (of course)

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- This leads to the popular (and sometimes misused) heuristic: cluster at the level of treatment / identifying variation
  - See Abadie et al. (2023) for a more complete version of this argument

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