Day 3: On Formulas and Models

Peter Hull

Design-Based Regression Inference
Spring 2024

Outline

1. Formula Treatments/Instruments

2. Structural Models

Beyond Simple Treatments/Instruments

- We've seen how a regression of y_i on x_i and w_i identifies a convex average of treatment effects when $E[x_i \mid y_i(\cdot), w_i] = E[x_i \mid w_i] = w_i'\gamma$
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- Let's build up to these slowly...

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  - Can use $s_i g_i$ as an IV controlling for s_i, given exclusion/monotonicity

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- Now suppose $g_i \mid y(\cdot), s, q \overset{iid}{\sim} G(q_i)$: e.g., a stratified RCT
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- Key point: the design of exogenous shocks g_i + knowledge of the "formula" $s_i g_i$ tells us what controls are needed for identification

Shift-Share Instruments

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- Cool new twist: we can use design to "translate" shocks from one level (e.g. industries) to estimate effects at another (e.g. regions)!

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  - In Autor et al. (2014), this means controlling for the sum-of-shares interacted with period FE

Example: Autor et al. (2014)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Coefficient	-0.596	-0.489	$\boxed{-0.267}$	-0.314	-0.310	-0.290	-0.432
	(0.114)	(0.100)	$\boxed{(0.099)}$	(0.107)	(0.134)	(0.129)	(0.205)
Regional controls							
Autor et al. (2013) controls	✓	✓	✓		✓	✓	✓
Start-of-period mfg. share	✓						
Lagged mfg. share		✓	✓	✓	✓	✓	✓
Period-specific lagged mfg. share			✓	✓	✓	✓	✓
Lagged 10-sector shares					✓		✓
Local Acemoglu et al. (2016) controls						✓	
Lagged industry shares							✓
SSIV first stage F-stat.	185.6	166.7	123.6	272.4	64.6	63.3	27.6
# of region-periods	1,444	1,444	1,444	1,444	1,444	1,444	1,444
# of industry-periods	796	794	794	794	794	794	794

From Borusyak et al. (2022); check out my SSIV mixtape for more!

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  3. Compute expected instrument as $\mu_i = \frac{1}{L} \sum_\ell z_i^{(\ell)}$

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- They use the differential timing of high-speed rail (HSR) construction in China, conditional on construction plans, as exogenous shocks

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- They use the differential timing of high-speed rail (HSR) construction in China, conditional on construction plans, as exogenous shocks
  - Basic idea: permute which planned lines opened by some date conditional on line observables to generate counterfactual shocks

Example: Borusyak and Hull (2023)

- BH are interested in estimating the effect of market access on employment by leveraging changes in the transportation network
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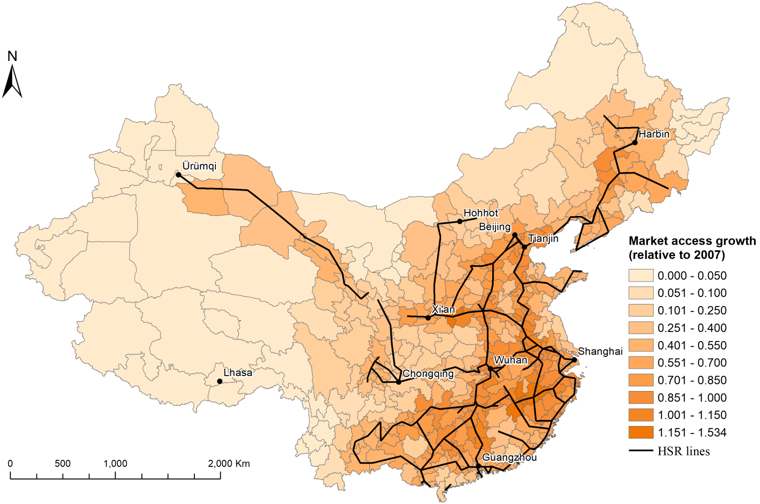
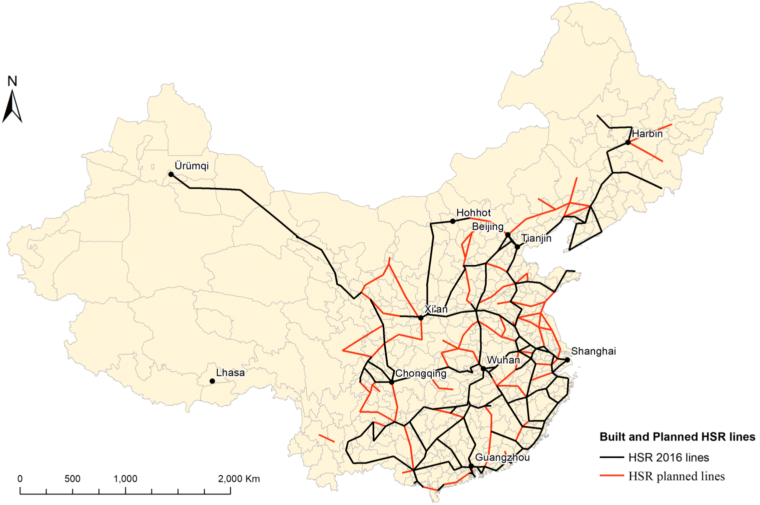
  - Basic idea: permute which planned lines opened by some date conditional on line observables to generate counterfactual shocks
  - Then either control for or recenter by expected market access growth

HSR Lines and Market Access



Naive OLS compares dark ("treatment") vs light ("control") regions

HSR Lines and Counterfactuals



Counterfactuals permute which lines opened by 2016, conditional on length

BH Estimates

	Unadjusted OLS (1)	Recentered IV (2)	Controlled OLS (3)
Panel A. No Controls			
Market Access Growth	0.232	0.081	0.069
	(0.075)	(0.098)	(0.094)
		[-0.315, 0.328]	[-0.209, 0.331]
Expected Market Access Growth			0.318
			(0.095)
Panel B. With Geography Controls			
Market Access Growth	0.132	0.055	0.045
	(0.064)	(0.089)	(0.092)
		[-0.144, 0.278]	[-0.154, 0.281]
Expected Market Access Growth			0.213
			(0.073)
Recentered	No	Yes	Yes
Prefectures	274	274	274

Large effect for naive OLS goes away with recentering/controlling

- Once adjusting for μ_i, auxilliary controls don't matter ($\implies$ balance)

Inference Can Be Tricky with Formulas

- Common exposure to the exogenous shocks g make $z_i = f_i(g, s)$ correlated across i, potentially in complicated ways
  - E.g. for shift-share $z_i = \sum_k s_{ik} g_k$, if unit i and j are far apart in space but close in terms of $(s_{ik})_{k=1}^K$ and $(s_{jk})_{k=1}^K$ then $Cov(z_i, z_j) > 0$

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- For other $f_i(\cdot)$, Borusyak and Hull '23 propose randomization inference
  - Use the counterfactual g to simulate the distribution of test statistics under the null and check if the actual test is in the tails

Outline

1. Formula Treatments/Instruments✓

2. Structural Models

Models as Extrapolators

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  - In fact, we've already seen this: our Day-1 constant-effect model of $y_i = \beta x_i + \varepsilon_i$ can be understood as extrapolating simply across all i
  - Other (nonlinear) procedures can sometimes be seen as imposing different extrapolations to the same underlying (design-based) variation

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  - Different extrapolation of missing CATE: a feature or a bug?

ExtrapoLATEing

- Consider the simplest design-based IV story: binary x_i, binary z_i, no controls. IV identifies LATE:

$$\beta^{IV} = E[y_i(1) - y_i(0) \mid x_i(1) > x_i(0)]$$

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  - Actually not quite true: can identify avg. $y_i(1)$ of always-takers (w/ $x_i(1) = x_i(0) = 1$), avg. $y_i(0)$ of never-takers (w/ $x_i(1) = x_i(0) = 0$), as well as avg. $y_i(1)$ & $y_i(0)$ separately for compliers
  - By adding a (semi-)parametric model of selection, we can extrapolate these objects to identify other parameters, e.g., ATE $E[y_i(1) - y_i(0)]$

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Suppose we have a z_i which is as-good-as-randomly assigned + excludable

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Key point: the model allows us to extrapolate "local" IV variation to estimate more "policy relevant" parameters

- When z_i has limited support, the model is doing more "work"
- With full support, we have "identification at infinity" (w/o a model)

Linking Back to LATE

Kline and Walters (2019) formalize this extrapolation logic in the familiar Imbens and Angrist (1994) setup

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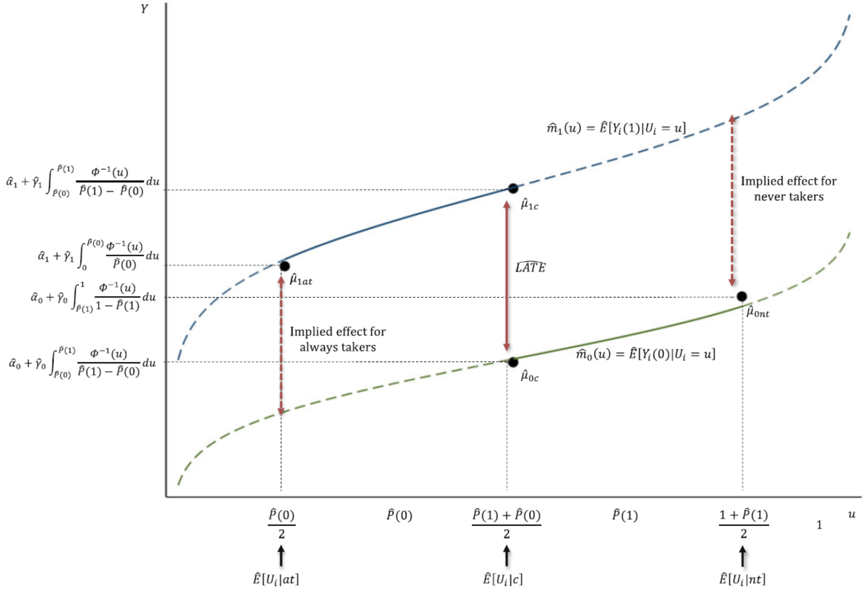
All of this can be extended to conditional as-good-as-random assignment

- Design knowledge gives you reduced-form estimands; then plug these into a model to get more!

Heckit Extrapolation of IV Moments



"Heckit" model: $E[Y_i(d)|U_i] = \alpha_d + \gamma_d \Phi^{-1}(U_i)$

Summing Up

- You can do a lot with a solid design-based identification strategy
  - Give a clear *ex ante* rationalization for controls in a linear regression/IV
  - Have confidence in the level of standard error clustering
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Thanks for a Great Class!