Instrumental Variables

Heterogeneous Effects

MIXTAPE SESSIONS

Roadmap

- The LATE Theorem
  - Potential Outcome Setup
  - Theorem and Extensions
- Characterizing Compliers
  - Outcomes
  - Covariates
- Marginal Treatment Effects
  - Continuous Instruments
  - Discrete Instruments

From Constant to Heterogeneous Effects

So far we have implicitly been considering models w/ constant effects

- $Y_i = \alpha + \beta D_i + \varepsilon_i$ implies $\partial Y / \partial D = \beta$ for all observations i
- What if this model is *misspecified*? I.e. what if $Y_i = \alpha + \beta_i D_i + \varepsilon_i$?

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Formalized in the (Nobel-winning!) Imbens and Angrist '94 LATE thm.

- Using a general potential outcomes framework...

Potential Outcome Setup

Let $Y_i(0)$ and $Y_i(1)$ denote individual i's potential outcomes given a binary treatment $D_i \in \{0, 1\}$

- Observed outcomes: $Y_i = (1 - D_i)Y_i(0) + D_iY_i(1)$

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Imbens-Angrist' insight: we can also do this for an IV first stage:

- Let $D_i(0)$ and $D_i(1)$ denote individual i's potential *treatment* given a binary *instrument* $Z_i \in \{0, 1\}$:

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Under what assumptions can we causally interpret `ivreg2 Y (D=Z)`?

Imbens and Angrist (1994) Assumptions

1. *As-good-as-random assignment*: $Z_i \perp (Y_i(0), Y_i(1), D_i(0), D_i(1))$

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4. *Monotonicity*: $D_i(1) \geq D_i(0)$ for all i (i.e., almost-surely)
   - $\rightarrow$ The instrument can only shift the treatment in one direction

Local Average Treatment Effect (LATE) Identification

Imbens and Angrist showed that under these assumptions:

$$\beta^{IV} = E[Y_i(1) - Y_i(0) \mid D_i(1) > D_i(0)]$$

The IV estimand β^{IV} identifies a LATE: the average treatment effect $Y_i(1) - Y_i(0)$ among *compliers*: those with $1 = D_i(1) > D_i(0) = 0$

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- Monotonicity rules out the presence of *defiers*, with $D_i(1) < D_i(0)$

$\Longrightarrow$ Different (valid) IVs can identify different LATEs!

What Does This Mean *Practically*?

Two conceptually distinct considerations: *internal* vs. *external* validity

- Context of an IV, and who the compliers likely are, may matter
- Usual “overidentification” test logic fails: two valid IVs may have different estimands! (see Kitagawa (2015) for alternative tests)

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In addition to as-good-as-random assignment / exclusion, we may need to worry about monotonicity when we do IV

- Sensible in earlier lottery / natural experiment / panel examples
- Maybe questionable in judge IVs (coming soon!)

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- Angrist/Imbens '95: multivalued (ordered) D_i, saturated covariates
- Angrist/Graddy/Imbens '00: continuous D_i (supply/demand setup)
- Heckman/Vytlicil '05: continuous Z_i (more on this soon)
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Recent discussions highlight importance of including flexible controls

- E.g. Słoczyński '20, Borusyak and Hull '21, Mogstad et al. '22
- If monotonicity only holds conditional on X_i, may need Z_i-by-X_i interactions (which may lead to many-weak problems...)

Roadmap

Who Are the Compliers?

Characterizing the i that make up the IV estimand (w/$D_i(1) > D_i(0)$) is key for understanding internal vs. external validity

- Unfortunately we can't identify compliers directly: we only observe $D_i(1)$ (when $Z_i = 1$) or $D_i(0)$ (when $Z_i = 0$), not both together!

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It turns out we can still characterize compliers by their outcomes ($Y_i(0)$ and $Y_i(1)$) and other observables X_i

- Comparing $E[X_i \mid D_i(1) > D_i(0)]$ to $E[X_i]$ can maybe shed light on how $E[Y_i(1) - Y_i(0) \mid D_i(1) > D_i(0)]$ compares to $E[Y_i(1) - Y_i(0)]$

Outcomes

Computing $E[Y_i(1) \mid D_i(1) > D_i(0)]$ is surprisingly easy in the IA setup

- Define $W_i = Y_i D_i$, and note that this new outcome has potentials with respect to D_i of $W_i(1) = Y_i(1)$ and $W_i(0) = 0$

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- Thus IV with W_i as the outcome identifies $E[W_i(1) - W_i(0) \mid D_i(1) > D_i(0)] = E[Y_i(1) \mid D_i(1) > D_i(0)]$

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Similar logic shows that IV with $Y_i(1 - D_i)$ as the outcome and $1 - D_i$ as the treatment identifies $E[Y_i(1) \mid D_i(1) > D_i(0)]$

- So easy to do! And extends to covariates / multiple IVs...

Characterizing Charter Lottery Complier $Y_i(0)$'s

TABLE 6—POTENTIAL-OUTCOME GAPS IN URBAN AND NONURBAN AREAS

	Urban				Nonurban			
Subject	Treatment effect (1)	$E_u[Y_0 \mid D=0]$ (2)	λ_0^u (3)	λ_1^u (4)	Treatment effect (5)	$E_n[Y_0 \mid D=0]$ (6)	λ_0^n (7)	λ_1^n (8)
Panel A. Middle school								
Math	$\boxed{0.483^{***}}$	$\boxed{-0.399^{***}}$	0.077	0.560***	$\boxed{-0.177^{**}}$	$\boxed{0.236^{***}}$	0.010	−0.143***
	$\boxed{(0.074)}$	$\boxed{(0.011)}$	(0.049)	(0.054)	$\boxed{(0.074)}$	$\boxed{(0.007)}$	(0.061)	(0.042)
N	4,858				2,239			
ELA	0.188***	−0.422***	0.118**	0.306***	−0.148***	0.260***	0.102**	−0.086***
	(0.064)	(0.012)	(0.054)	(0.049)	(0.048)	(0.007)	(0.050)	(0.030)
N	4,551				2,323			
Panel B. High school								
Math	0.557***	−0.371***	0.074	0.602***	0.065	0.241***	0.207	0.271***
	(0.164)	(0.021)	(0.099)	(0.151)	(0.146)	(0.008)	(0.145)	(0.041)
N	3,743				432			
ELA	0.417***	−0.369***	−0.004	0.410***	0.064	0.250***	0.237	0.301***
	(0.140)	(0.018)	(0.096)	(0.119)	(0.151)	(0.008)	(0.152)	(0.051)
N	4,858				435			

Source: Angrist, Pathak, and Walters (2013)

Covariates

For covariates X_i (not affected by D_i) we can follow a similar trick:

- Either IV'ing X_iD_i on D_i or IV'ing $X_i(1-D_i)$ on $1-D_i$ identifies complier characteristics $E[X_i \mid D_i(1) > D_i(0)]$
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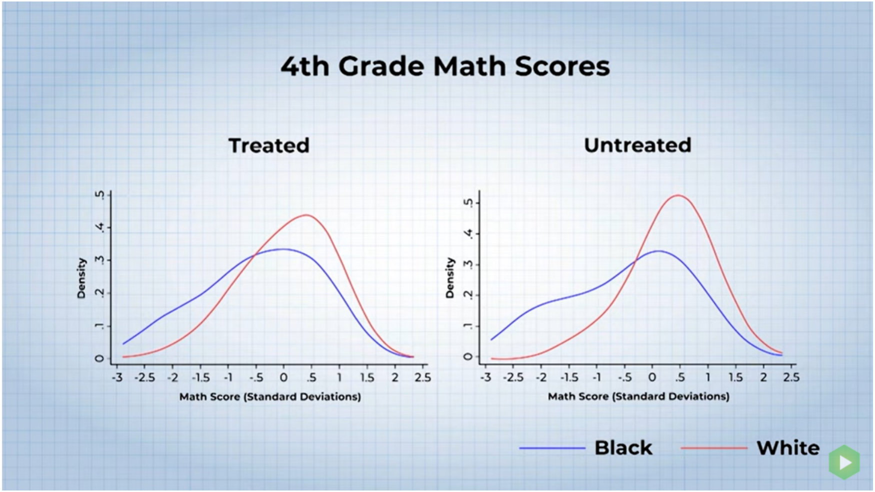
Abadie (2003) gives a slicker (but a bit more involved) approach to estimating any function of $(Y_i(0), Y_i(1), X_i)$ for compliers

- Involves weighting by $\kappa = 1 - \frac{D_i(1-Z_i)}{1-E[Z_i|W_i]} - \frac{(1-D_i)Z_i}{E[Z_i|W_i]}$ where W_i are any necessary "design controls" (e.g. lottery risk sets)
- You can do some really cool stuff with this!

Black/White Potential Outcomes, Pre-Charter



Source: Josh Angrist Nobel Lecture (2021)

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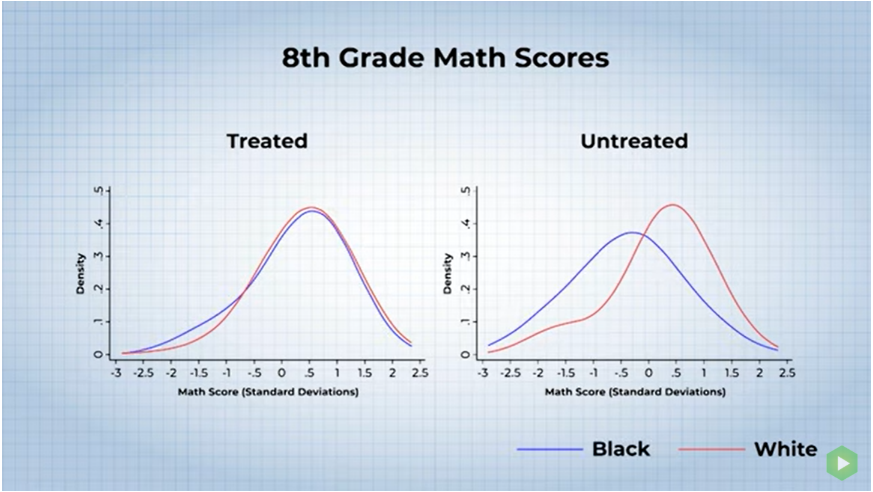

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Roadmap

Heckman and Vytlicil (2005, 2007, 2010, 2013...)

If we have a Z_i that varies continuously, we might learn more about how treatment effects vary with compliance

- Different types of i may “respond” at different margins of Z_i

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Heckman-Vytlicil write $D_i = \mathbf{1}[p(Z_i) \geq U_i]$, with $U_i \mid Z_i \sim U(0,1)$

- $p(z) = Pr(D_i = 1 \mid Z_i = z)$ is the treatment propensity score
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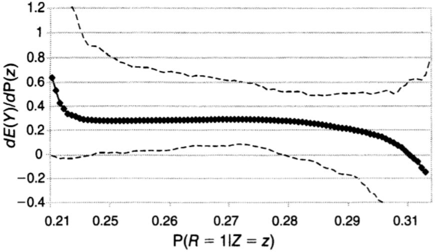
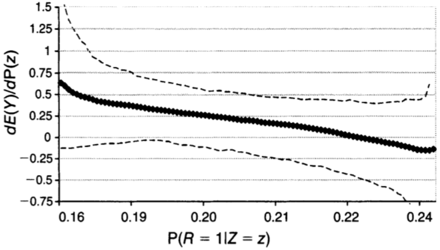
Now we can consider how $Y_i(1) - Y_i(0)$ varies continuously with U_i ...

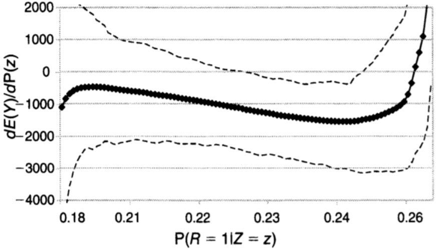
Doyle (2007): MTEs of Foster Care Removal



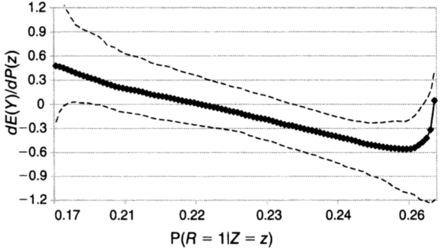
A. Delinquency MTE



B. Teen Motherhood MTE



C. Earnings MTE



D. Employment MTE

Local Instrumental Variables

Heckman (2000) shows that MTEs are identified by “local IV”:

$$E[Y_i(1) - Y_i(0) \mid U_i = p] = \frac{\partial E[Y_i \mid p(Z_i) = p]}{\partial p}$$

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- Suggests we flexibly estimate $p(z) = Pr(D_i = 1 \mid Z_i = z)$, $E[Y_i \mid p(Z_i)]$, and then take the derivative of the latter
- In practice this is often done parametrically, and with controls

What if We Don't Have Continuous Instruments?

A fascinating recent literature considers intermediate cases of Imbens-Angrist and Heckman-Vytlacil:

- Discrete (binary/multivalued) Z_i, with parametric/shape restrictions to trace out (or maybe bound) the MTE curve
- Effectively using a model to "extrapolate" from local variation, maybe to identify more policy-relevant parameters

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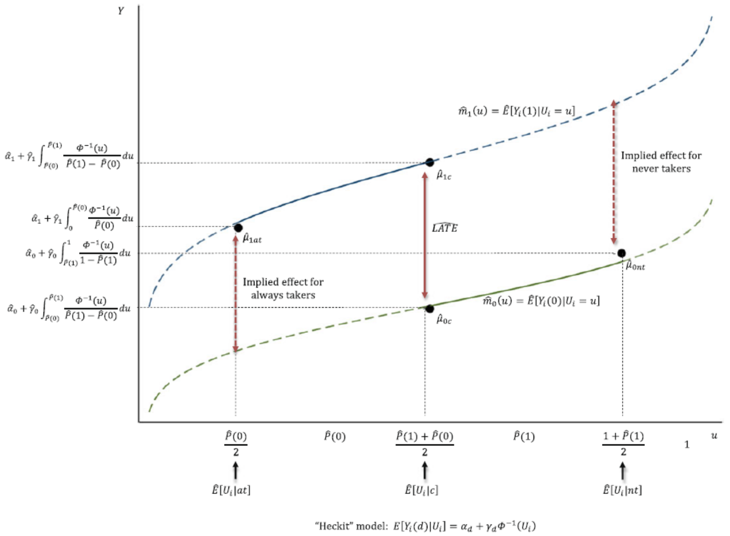
Some examples: Brinch et al. (2017), Mogstad et al. (2018), Kline and Walters (2019), Hull (2020), Arnold et al. (2021), Kowalski (2022)...

- Lots more to do here (especially on the practical side)

How Parametric “Heckit” Models Extrapolate LATEs



Source: Kline and Walters (2019)