On Linear Logic, Functional Programming, and Attack Trees

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Abstract. This paper has two main contributions, the first is a new linear logical semantics of causal attack trees in four-valued truth tables. Our semantics is very simple and expressive, supporting specializations, and combines in an interesting way the *ideal* and *filter* semantics of causal attack trees. Our second contribution is Lina, a new embedded, in Haskell, domain specific functional programming language for conducting threat analysis using attack trees. Lina has many benefits over existing tools, for example, Lina allows one to specify attack trees very abstractly increasing reuse, and is compositional allowing one to break down complex attack trees into smaller ones that can be reasoned about and analyzed incrementally. Furthermore, Lina supports automatically proving properties, such as equivalences and specializations, about attack trees using Maude and the semantics introduced in this paper.

1 Introduction

Attack trees are perhaps the most popular graphical model used to conduct threat analysis of both physical and virtual secure systems. They were made popular by Bruce Schneier in the late nineties [16]. In those early years attack trees were studied and used as a syntactic tool to help guide analysis. However, as systems grew more complex the need for a semantics of attack trees become apparent, after all, without a proper semantics how can we safely manipulate attack trees, extend the expressivity of attack trees, or compare them?

A number of different models of attack trees have been proposed: a model in boolean algebras [11,10,15], series-parallel pomsets [12], Petri nets [13], and tree automata [1]. There have also been various extensions, such as, adding sequential composition [6], and defense nodes [9,10]. All of these models and extensions have their benefits, but at the heart of them all is logic.

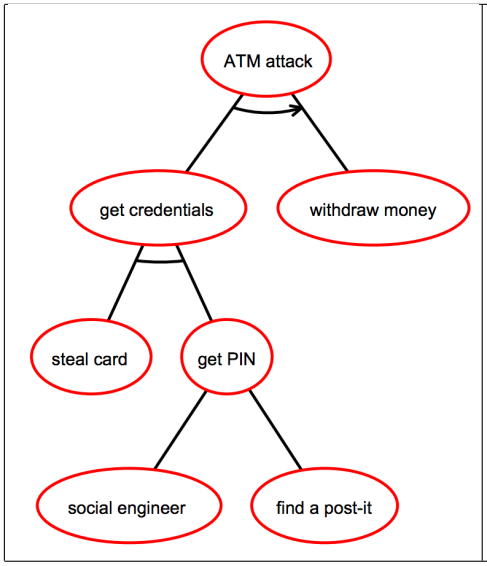
The model in boolean algebras was the first and most elegant model of attack trees, but it failed to capture the process notion of attack trees, that is, the fact that base attacks are actual processes that need to be carried out, and the branching nodes composed these processes in different ways. Thus, the community moved towards models of resources like parallel-series pomsets, Petri nets, and automata. However, the complexity of these models increased, and hence, comparing the models is difficult which makes it hard to decide which to use

and under which circumstances. The need for a means of recovering the elegant logical model of attack trees becomes apparent.

Linear Logic. It is fitting that attack trees are the most popular model used in threat analysis, because one of the most widely studied logics used to reason about resources is *linear logic* which is an excellent candidate for modeling attack trees. In fact, Horne et al.[5] has already produced a number of interesting results. Most importantly, they show that attack trees can be modeled as formulas in linear logic, and then one can prove properties between attack trees by proving implications between them. Furthermore, by studying attack trees from a linear logical perspective they introduce a new property between attack trees called *specializations*. Prior to their paper the literature was primarily concerned with equality between attack trees, but the logical semantics of attack trees reveal how one can break these equalities up into directional rewrite rules. An attack tree is a *specialization* of another if the former is related to the later via these rewrite rules. The logical semantics model the rewrite rules as implications.

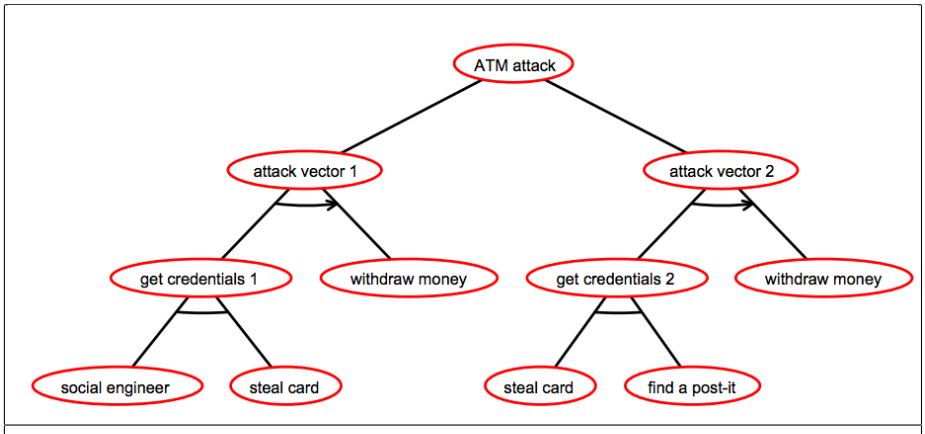
This paper has two main contributions, the first is a new simple linear logical semantics of causal attack trees – attack trees with sequential composition – in four-valued truth tables. We show that our semantics is surprisingly expressive. It supports specializations, and even lays outside of the semantics proposed by Horne et al.[5], because it combines in an interesting way what they call the *ideal* and *filter* semantics of causal attack trees. However, the most appealing aspect of this semantics is that it is extremely simple.

Functional Programming. Our second contribution is Lina, a new domain specific functional programming language for conducting threat analysis using attack trees. Consider the example attack trees in Fig. 1 and Fig. 1. Both of




```
seq_node "ATM attack"
  (and_node "get credientials"
    (base_na "steal card")
    (or_node "get PIN"
      (base_na "social engineer")
      (base_na "find a post-it")))
    (base_na "withdraw money")
```

Fig. 1. Attack Tree for an ATM attack from Figure 1 Kordy et al. [8] with its corresponding Lina script




```
or_node "ATM attack"
   (seq_node "attack vector 1"
     (and_node "get credentials 1"
       (base_na "social engineer")
       (base_na "steal card"))
     (base_na "withdraw money"))
   (seq_node "attack vector 2"
     (and_node "get credentials 2"
       (base_na "steal card")
       (base_na "find a post-it"))
     (base_na "withdraw money"))
```

Fig. 2. Attack Tree for an ATM attack from Figure 2 of Kordy et al. [8] and its corresponding Lina script

these contain actual Lina programs for each of the corresponding attack trees, in fact every example in this paper are Lina programs. Lina supports causal attack trees both with attributes or without, thus, there are two types of base attacks: base attacks with attributes, denoted `base_wa`, and base attacks with no attributes, denoted `base_na`. Lina is designed to be extremely simple, and actually reflect the typical pseudocode found throughout the literature. However, Lina is more than just a simple definitional language.

Lina is an embedded domain specific language whose host language is the Haskell programming language [7]. So, why Haskell? As security researchers and professionals we are in the business of verifying the correctness of various systems. Thus, our tools should be taking advantage of verification tools to insure that our constructions, tools, and analysis are correct. By embedding Lina into Haskell we are able to take advantage of cutting edge verification tools while conducting threat analysis. For example, right out the box Lina supports property-based randomized testing using QuickCheck [2], and refinement types in Liquid Haskell [17] to verify properties of our attack trees or the attribute domains used while analyzing attack trees. Furthermore, Haskell's advanced type system helps catch bugs while we develop our attack trees and their attribute domains

as a side-effect of type checking. Finally, functional programs are short, but not obfuscated, and hence, allow for very compact and trustworthy programs.

That being said, we are designing Lina so that it can be used with very little Haskell experience. It is our hope that one will be able to make use of Lina without having to know how to write Haskell programs, and we plan to develop new tooling to support this.

Lina is approaching threat analysis from a programming language perspective. This approach leads to a number of new advances. First, as Gadyatskaya and Trujillo-Rasua [4] argue, as a community we need to start building more automated means of conducting threat analysis, and there is no better way to build or connect automated tools than a programming language. Lina is perfect as a target for new tools, and it can be connected to existing tools fairly easily. In fact, Lina already supports automation using the automatic rewrite system Maude [3], for example, the two attack trees in Fig. 1 can be automatically proven equivalent to each other in Lina. This is similar to Krody's [8] SPTool, but Lina goes further and supports more than one backend rewrite system, for example, Lina is the first tool to support automatically proving specializations of attack trees. The user can choose which backend they wish to use.

2 Causal Attack Trees

We begin by introducing causal attack trees. This formulation of attack trees was first proposed by Jhawar et al. [6] where they called them SAND attack trees, but sequential composition does not always meet the same properties as conjunction, for example, classically it is a self dual operator, thus, we follow Horne et al.'s lead [5] and call them causal attack trees.

Definition 1. *Suppose $\mathbb{B}$ is a set of base attacks whose elements are denoted by b. Then an **attack tree** is defined by the following grammar:*

$$A, B, C, T := b \mid \mathsf{OR}(A, B) \mid \mathsf{AND}(A, B) \mid \mathsf{SEQ}(A, B)$$

Equivalence of attack trees, denoted by $A \approx B$, is defined as follows:

$\mathsf{OR}(A, A) \approx A$	$\mathsf{OR}(\mathsf{OR}(A, B), C) \approx \mathsf{OR}(A, \mathsf{OR}(B, C))$
$\mathsf{OR}(A, B) \approx \mathsf{OR}(B, A)$	$\mathsf{AND}(\mathsf{AND}(A, B), C) \approx \mathsf{AND}(A, \mathsf{AND}(B, C))$
$\mathsf{AND}(A, B) \approx \mathsf{AND}(B, A)$	$\mathsf{SEQ}(\mathsf{SEQ}(A, B), C) \approx \mathsf{SEQ}(A, \mathsf{SEQ}(B, C))$
	$\mathsf{AND}(A, \mathsf{OR}(B, C)) \approx \mathsf{OR}(\mathsf{AND}(A, B), \mathsf{AND}(A, C))$
	$\mathsf{SEQ}(A, \mathsf{OR}(B, C)) \approx \mathsf{OR}(\mathsf{SEQ}(A, B), \mathsf{SEQ}(A, C))$

Throughout the sequel we will show that the previous rules are sound with respect to our new model, but just as Horne et al. [5] we will then show that there are properties of attack trees that these rules do not support, but our semantics allows.

3 A Quaternary Semantics for Causal Attack Trees

Kordy et al. [10] gave a very elegant and simple semantics of attack-defense trees in boolean algebras. Unfortunately, while their semantics is elegant it does not

capture the resource aspect of attack trees, it allows contraction, and it does not provide a means to model sequential composition. In this section we give a semantics of attack trees in the spirit of Kordy et al.'s using a four valued logic. This section was formally verified in the Agda Proof Assistant [14][4].

The propositional variables, elements of the set PVar, of our quaternary logic, denoted by P, Q, R, and D, range over the set $\mathsf{4} = \{0, \frac{1}{4}, \frac{1}{2}, 1\}$. We think of 0 and 1 as we usually do in boolean algebras, but we think of $\frac{1}{4}$ and $\frac{1}{2}$ as intermediate values that can be used to break various structural rules. In particular we will use these values to prevent exchange for sequential composition from holding, and contraction from holding for parallel and sequential composition.

Definition 2. *The logical connectives of our four valued logic are defined as follows:*

Parallel and Sequential Conjunction:

$P \odot_4 Q = 1$, *where neither* P *nor* Q *are* 0
$P \odot_4 Q = 0$, *otherwise*

$P \rhd_4 Q = 1$, *where* $P \in \{\frac{1}{2}, 1\}$ *and* $Q \neq 0$
$P \rhd_4 Q = \frac{1}{4}$, *where* $P = \frac{1}{4}$ *and* $Q \neq 0$
$P \rhd_4 Q = 0$, *otherwise*

Choice:

$P \sqcup_4 Q = \mathsf{max}(P, Q)$

These definitions are carefully crafted to satisfy the necessary properties to model attack trees. Comparing these definitions with Kordy et al.'s [10] work we can see that choice is defined similarly, but parallel composition is not a product – ordinary conjunction – but rather a linear tensor product, and sequential composition is not actually definable in a boolean algebra, and hence, makes heavy use of the intermediate values to insure that neither exchange nor contraction hold.

We use the usual notion of equivalence between propositions, that is, propositions ϕ and ψ are considered equivalent, denoted by $\phi \equiv \psi$, if and only if they have the same truth tables. In order to model attack trees the previously defined logical connectives must satisfy the appropriate equivalences corresponding to the equations between attack trees. These equivalences are all proven by the following result.

Lemma 1 (Properties of the Attack Tree Operators in the Quaternary Semantics).

(Symmetry) For any P *and* Q, $P \bullet Q \equiv Q \bullet P$, *for* $\bullet \in \{\odot_4, \sqcup_4\}$.

(Symmetry for Sequential Conjunction) It is not the case that, for any P *and* Q, $P \rhd_4 Q \equiv Q \rhd_4 P$.

[4] The formalization can be found at `https://github.com/MonoidalAttackTrees/ATLL-Formalization`

(Associativity) For any P, Q, and R, $(P \bullet Q) \bullet R \equiv P \bullet (Q \bullet R)$, for $\bullet \in \{\odot_4, \rhd_4, \sqcup_4\}$.

(Contraction for Parallel and Sequential Conjunction) It is not the case that for any P, $P \bullet P \equiv P$, for $\bullet \in \{\odot_4, \rhd_4\}$.

(Distributive Law) For any P, Q, and R, $P \bullet (Q \sqcup_4 R) \equiv (P \bullet Q) \sqcup_4 (P \bullet R)$, for $\bullet \in \{\odot_4, \rhd_4\}$.

Proof. Symmetry, associativity, contraction for choice, and the distributive law for each operator hold by simply comparing truth tables. As for contraction for parallel composition, suppose $P = \frac{1}{4}$. Then by definition $P \odot_4 P = 1$, but $\frac{1}{4}$ is not 1. Contraction for sequential composition also fails, suppose $P = \frac{1}{2}$. Then by definition $P \rhd_4 P = 1$, but $\frac{1}{2}$ is not 1. Similarly, symmetry fails for sequential composition. Suppose $P = \frac{1}{4}$ and $Q = \frac{1}{2}$. Then $P \rhd_4 Q = \frac{1}{4}$, but $Q \rhd_4 P = 1$.

At this point it is quite easy to model attack trees as formulas. The following defines their interpretation.

Definition 3. *Suppose $\mathbb{B}$ is some set of base attacks, and $\nu : \mathbb{B} \to \mathsf{PVar}$ is an assignment of base attacks to propositional variables. Then we define the interpretation of attack trees to propositions as follows:*

$$\begin{array}{lcllcl} [\![b \in \mathbb{B}]\!] & = & \nu(b) & [\![\mathsf{SEQ}(A,B)]\!] & = & [\![A]\!] \rhd_4 [\![B]\!] \\ [\![\mathsf{AND}(A,B)]\!] & = & [\![A]\!] \odot_4 [\![B]\!] & [\![\mathsf{OR}(A,B)]\!] & = & [\![A]\!] \sqcup_4 [\![B]\!] \end{array}$$

We can use this semantics to prove equivalences between attack trees.

Lemma 2 (Equivalence of Attack Trees in the Quaternary Semantics). *Suppose $\mathbb{B}$ is some set of base attacks, and $\nu : \mathbb{B} \to \mathsf{PVar}$ is an assignment of base attacks to propositional variables. Then for any attack trees A and B, if $A \approx B$, then $[\![A]\!] \equiv [\![B]\!]$.*

Proof. This proof holds by induction on the form of $A \approx B$.

This is a very simple and elegant semantics, but it also leads to a more substantial theory.

4 Specialization in the Quaternary Semantics

The quaternary semantics introduced in the previous section does indeed capture all of the equivalences of attack trees, but it also supports proving specializations. Consider the example attack trees in Fig. 3. Attack tree C is a sound specialization of attack tree A, and attack tree B is a sound specialization of attack tree A. Attack tree C requires the attacker to break into the system before they can steal the backup, but attack tree A does not require this. Then attack tree B has dropped bribing the sysadmin and simply requires the attacker to just steal the backups. Notice that none of the attack trees in Fig. 3 are equivalent. So how do we prove these specializations are sound?

A.

```
and_node "obtain secret"
 (or_node "obtain encryped file"
    (base_na "bribe sysadmin")
    (base_na "steal backup"))
 (seq_node "obtain password"
    (base_na "break into system")
    (base_na "install keylogger"))
```

B.

```
seq_node "break in, obtain secret"
 (base_na "break into system")
 (and_node "obtain secret inside"
   (base_na "install keylogger")
   (base_na "steal backup"))
```

C.

```
or_node "obtain secret"
 (and_node "obtain secret via sysadmin"
  (base_na "bribe sysadmin")
  (seq_node "obtain password"
    (base_na "break into system")
    (base_na "install keylogger")))
  (seq_node "break in, obtain secrect"
    (base_na "break into system")
    (and_node "obtain secret inside"
      (base_na "install keylogger")
      (base_na "steal backup")))
```

Fig. 3. Encrypted Data Attack from Figure 1 (A), Figure 3 (B), and Figure 2 (C) of Horne et al. [5]

We simply define a notion of entailment in the quaternary semantics. Denote by $P \leqslant_4 Q$ the obvious ordering on $\mathbf{4}$. Then we have the following result immediately.

Lemma 3 (Entailment in the Quaternary Semantics). *$P \equiv Q$ if and only if $P \leqslant_4 Q$ and $Q \leqslant_4 P$*

This result shows that we can now break up the equivalence of attack trees into directional properties captured here by entailments, and hence, every equivalence proved in the previous section can also be used directionally.

We may now formally define when a attack tree is a sound specialization of another attack tree.

Definition 4. *An attack tree A is a sound specialization of an attack B if and only if $[\![A]\!] \leqslant_4 [\![B]\!]$.*

The next result proves some additional properties in the quaternary semantics that can be used to reason about attack trees.

Lemma 4 (Properties of Entailment in the Quaternary Semantics).
Ideal Entailments:
$((a \odot_4 b) \rhd_4 (c \odot_4 d)) \leqslant_4 ((a \rhd_4 c) \odot_4 (b \rhd_4 d))$
$((a \odot_4 b) \rhd_4 c) \leqslant_4 (a \odot_4 (b \rhd_4 c))$
$(a \rhd_4 (b \odot_4 c)) \leqslant_4 (b \odot_4 (a \rhd_4 c))$
$(a \rhd_4 b) \leqslant_4 (a \odot_4 b)$

Filter Entailments:
$((a \rhd_4 c) \odot_4 (b \rhd_4 d)) \leqslant_4 ((a \odot_4 b) \rhd_4 (c \odot_4 d))$
$(a \odot_4 (b \rhd_4 c)) \leqslant_4 ((a \odot_4 b) \rhd_4 c)$

Choice Entailments:
$a \leqslant_4 (a \sqcup_4 b)$
$b \leqslant_4 (a \sqcup_4 b)$

Each of the above entailments are due to Horne et al. [5]. They introduce two types of semantics called the *ideal semantics* and the *filter semantics*. The former satisfies all of the entailments in Lemma 1 and the left side of Lemma 4, but the latter is similar, however, satisfying the right side of Lemma 4. They were able to show that the ideal entailments allow one to prove properties of attack trees with different attribute domains than the filter entailments.

In comparison with their work the semantics presented here is a blend of the ideal and filter semantics. It primarily consists of the ideal semantics, but as we can see from Lemma 4 the first two axioms are actually logical equivalences. This implies that this semantics can prove properties that neither the ideal nor the filter semantics can capture. However, we do not yet know which attribute domains correspond to this semantics. We can isolate ourselves to just the ideal or the filter semantics, but what attribute domains can we reason about in this semantics when we blend them? This is left to future work.

We can now formally prove that the attack tree C is a specialization of attack tree A, and that attack tree B is a specialization of attack tree A from Fig. 3.

Example 1. First, consider the following assignment:

$a :=$ `"bribe sysadmin"` $\quad b :=$ `"break into system"`
$c :=$ `"install keylogger"` $\quad d :=$ `"steal backup"`

Then we have the following interpretations:

$$\begin{aligned}[\![A]\!] &= [\![\mathsf{AND}(\mathsf{OR}(a,d),\mathsf{SEQ}(b,c))]\!] \\ &= (a \sqcup_4 d) \odot_4 (b \rhd_4 c)\end{aligned} \qquad \begin{aligned}[\![B]\!] &= [\![\mathsf{SEQ}(b,\mathsf{AND}(c,d))]\!] \\ &= b \rhd_4 (c \odot_4 d)\end{aligned}$$

$$\begin{aligned}[\![C]\!] &= [\![\mathsf{OR}(\mathsf{AND}(a,\mathsf{SEQ}(b,c)),\mathsf{SEQ}(b,\mathsf{AND}(c,d)))]\!] \\ &= (a \odot_4 (b \rhd_4 c)) \sqcup_4 (b \rhd_4 (c \odot_4 d))\end{aligned}$$

We reuse the same names for base attacks across the interpretations above. Finally, we have the following two entailments:

$\underline{[\![C]\!] \leqslant_4 [\![A]\!]}$:

$$\begin{aligned} & (a \odot_4 (b \rhd_4 c)) \sqcup_4 (b \rhd_4 (c \odot_4 d)) \\ \leqslant_4\ & (a \odot_4 (b \rhd_4 c)) \sqcup_4 (b \rhd_4 (d \odot_4 c)) \\ \leqslant_4\ & (a \odot_4 (b \rhd_4 c)) \sqcup_4 (d \odot_4 (b \rhd_4 c)) \\ \leqslant_4\ & (a \sqcup_4 d) \odot_4 (b \rhd_4 c)\end{aligned}$$

$\underline{[\![B]\!] \leqslant_4 [\![A]\!]}$:

$$\begin{aligned} & b \rhd_4 (c \odot_4 d) \\ \leqslant_4\ & b \rhd_4 (c \odot_4 (a \sqcup_4 d)) \\ \leqslant_4\ & b \rhd_4 ((a \sqcup_4 d) \odot_4 c) \\ \leqslant_4\ & (a \sqcup_4 d) \odot_4 (b \rhd_4 c)\end{aligned}$$

Notice that neither $[\![A]\!] \leqslant_4 [\![C]\!]$ nor $[\![A]\!] \leqslant_4 [\![B]\!]$ hold, and thus, equivalences cannot prove the previous properties.

5 Lina: An EDSL for Conducting Threat Analysis using Causal Attack Trees

All of the models mentioned in this paper have been incorporated into a new embedded domain specific language (EDSL) for conducting threat analysis called Lina[5]; which means small, young palm tree, but we constructed the name by combining the words linear and attack.

Lina is embedded inside of Haskell, a statically typed functional programming language. The most important property of any EDSL is that they subsume the entirety of their host language, and can be prototyped quite rapidly. Haskell has several advantages, like Lina's ability to utilize Haskell's cutting edge verification tools, and its strong type system for catching bugs quickly. In addition, Haskell has several tools that make building EDSLs more easily, for example, type classes.

Lina currently supports three types of causal attack trees:

- Process Attack Trees: these are attack trees with no attributes at all,
- Attributed Process Attack Trees: these are attack trees with attributes on the base attacks only. This is an intermediate representation used to build full attack trees.
- Full Attack Trees: these are attributed process attack trees with an associated attribute domain.

Internally, we represent causal attack trees by a simple data type, called `IAT`, whose nodes are labeled with an integer identifier we call `ID`. We then define each type of attack tree as a record (labeled tuple):

```
-- Attributed Process Attack Tree
data APAttackTree attribute label =
 APAttackTree {
  process_tree :: IAT,
  labels :: B.Bimap label ID,
  attributes :: M.Map ID attribute
 }
```

```
-- Process Attack Tree
type PAttackTree label = APAttackTree () label

-- Full Attack Tree
data AttackTree attribute label = AttackTree {
      ap_tree :: APAttackTree attribute label,
      configuration :: Conf attribute
}
```

A `B.Bimap` is a dictionary where we can efficiently lookup `ID`s given a `label` or efficiently lookup `label`s given an `ID`, a `M.Map` is a typical dictionary, and `()` is the unit type.

This design has several benefits. Internal attack trees are very easy to translate to various backends, especially formulas because we can use the `ID`s on base attacks as atomic formulas – which has its own benefits discussed below – and modifying labels and attributes is more efficient than having them labeled on the

[5] Lina is under active development and its implementation can be found online at `https://github.com/MonoidalAttackTrees/Lina`

trees themselves. The previous data types reveal that actually all attack trees are attributed process attack trees where a process attack tree simply does not use the attributes. This allows Lina to offer a uniform syntax for specifying each type of attack tree.

One important aspect of the definition of the various forms of attack trees is that the types `label` and `attribute` are actually type variables, and thus, our definition of attack trees is very general, in fact, `label` and `attribute` can be instantiated with any type whose elements are comparable. This property is captured by ad-hoc polymorphism using type classes in Haskell, and are checked during type checking.

Conducting threat analysis using attack trees requires them to be associated with an attribute domain. Typically, an attribute domain is a set together with operations for computing the attribute of the branching nodes of an attack tree given attributes on the base attacks. In Lina attribute domains are defined by a type, here called `attribute`, and a configuration:

```
data Conf attribute = (Ord attribute) => Conf {
  orOp  :: attribute -> attribute -> attribute,
  andOp :: attribute -> attribute -> attribute,
  seqOp :: attribute -> attribute -> attribute
}
```

Utilizing higher-order functions we can define configurations quite easily, and quite generically. For example, here is the configuration that computes the minimum attribute for choice nodes, the maximum attribute for parallel nodes, and takes the sum of the children nodes as the attribute for sequential nodes:

```
minMaxAddConf :: (Ord attribute,Semiring attribute) => Conf attribute
minMaxAddConf = Conf min max (.+.)
```

Notice here that this configuration will work with any type at all whose elements are comparable and form a semiring, thus, making configurations generic and reusable. This includes types like `Integer` and `Double`.

The definitional language for attributed process attack trees of type `APAttackTree attribute label` is described by the following grammar:

$$at ::= \texttt{base\_na label} \mid \texttt{base\_wa attribute label} \mid \texttt{or\_node label at1 at2} \\ \mid \texttt{and\_node label at1 at2} \mid \texttt{seq\_node label at1 at2}$$

A full example of the definition of an attributed process attack tree for attacking an autonomous vehicle can be found in Fig. 4. The definition of `vehicle_attack` begins with a call to `start_PAT`. Behind the scenes all of the `ID`'s within the internal attack tree are managed implicitly, and this requires the internals of Lina to work within a special state-based type. The function `start_PAT` initializes this state.

Finally, we can define the vehicle attack tree as follows:

```
vehicle_AT :: AttackTree Double String
vehicle_AT = AttackTree vehicle_attack minMaxMaxConf
```

This attack tree associates the vehicle attack attributed process attack tree with a configuration called `minMaxMaxConf` that simply takes the minimum as the

```
import Lina.AttackTree

vehicle_attack :: APAttackTree Double String
vehicle_attack = start_PAT $
  or_node "Autonomous Vehicle Attack"
    (seq_node "external sensor attack"
       (base_wa 0.2 "modify street signs to cause wreck")
       (and_node "social engineering attack"
          (base_wa 0.6 "pose as mechanic")
          (base_wa 0.1 "install malware")))
    (seq_node "over night attack"
       (base_wa 0.05 "Find address where car is stored")
       (seq_node "compromise vehicle"
          (or_node "break in"
             (base_wa 0.8 "break window")
             (base_wa 0.5 "disable door alarm/locks"))
          (base_wa 0.1 "install malware")))
```

Fig. 4. Lina Script for an Autonomous Vehicle Attack

attribute of choice nodes, and the maximum as the attribute of every parallel and sequential node.

Two features that Lina has that other tools lack is its ability to abstract the definitions of attack trees, and it is highly compositional, because it is embedded inside of a functional programming language. Consider the following abstraction of `vehicle_attack`:

```
vehicle_AT' :: Conf Double -> AttackTree Double String
vehicle_AT' conf = AttackTree vehicle_attack conf
```

Here the configuration has been abstracted. This facilitates experimentation because the security practitioner can run several different forms of analysis on the same attack tree using different attribute domains.

Attack trees in Lina can also be composed, and hence, complex trees can be broken down into smaller ones, then studied in isolation. This helps facilitate correctness, and offers more flexibility. As an example, in Fig. 5 we break up `vehicle_attack` into several smaller attack trees. We can see in the example that if we wish to use an already defined attack tree in one we are defining, then we can make use of the `insert` function. As we mentioned above, behind the scenes Lina maintains a special state that tracks the identifiers of each node, thus, when one wishes to insert an existing attack tree, which will have its own identifier labeling, into a new tree, then that internal state must be updated, thus, `insert` carries out this updating. Lina is designed so that the user never has to encounter that internal state.

So far we have introduced Lina's basic design and definitional language for specifying causal attack trees, and we have already begun seeing improvements over existing tools, however, Lina has so much more to offer. We now introduce Lina's support for reasoning and performing analysis on causal attack trees.

Kordy et al. [8] introduce the SPTool, an equivalence checker for causal attack trees, that makes use of the rewriting logic system Maude [3] which allows one to specify rewrite systems, but also systems of equivalences. Kordy et al. specify the

```
se_attack :: APAttackTree Double String
se_attack = start_PAT $
  and_node "social engineering attack"
     (base_wa 0.6 "pose as mechanic")
     (base_wa 0.1 "install malware")
```

```
bi_attack :: APAttackTree Double String
bi_attack = start_PAT $
  or_node "break in"
     (base_wa 0.8 "break window")
     (base_wa 0.5 "disable door alarm/locks")
```

```
cv_attack :: APAttackTree Double String
cv_attack = start_PAT $
  seq_node "compromise vehicle"
    (insert bi_attack)
    (base_wa 0.1 "install malware")
```

```
es_attack :: APAttackTree Double String
es_attack = start_PAT $
  seq_node "external sensor attack"
       (base_wa 0.2 "modify street signs to cause
                     wreck")
       (insert se_attack)
```

```
on_attack :: APAttackTree Double String
on_attack = start_PAT $
  seq_node "over night attack"
     (base_wa 0.05 "Find address where car
                    is stored")
     (insert cv_attack)
```

```
vehicle_attack'' :: APAttackTree Double String
vehicle_attack'' = start_PAT $
  or_node "Autonomous Vehicle Attack"
    (insert es_attack)
    (insert on_attack)
```

Fig. 5. The Autonomous Vehicle Attack Decomposed

equivalences for causal attack trees from Jhawar et al.'s [6] work in Maude, and then use Maude's querying system to automatically prove equivalences between causal attack trees. This is a great idea, and we incorporate it into Lina, but we make several advancements over SPTool.

Lina includes a general Maude interface, and allows one to easily define new Maude backends, where a *Maude backend* corresponds to a Maude specification of a particular rewrite system. Currently, Lina has two Maude backends: equivalences for causal attack trees, and the multiplicative attack tree linear logic (MATLL). The former is essentially the exact same specification as the SPTool, but the latter corresponds to the quaternary semantics defined in Section 3 and Section 4, specifically, this backend is defined as a rewrite system that includes all of the rules from Lemma 1 and Lemma 4.

Attributed process attack trees are converted into the following syntax:

$$\text{(Maude Formula) } F := \texttt{ID} \mid F1; F2 \mid F1.F2 \mid F1 + F2$$

This is done by simply converting the internal attack tree into the above syntactic form. For example, the Maude formula for the autonomous vehicle attack from Fig. 4 is `(0 ; (1 . 2)) || (5 ; ((6 || 7) ; 2))`, where each integer corresponds to the identifier of the base attacks. Note that the base attack `2` appears twice, this is because this base attack appears twice in the original attack tree. This syntax is then used to write the Maude specification for the various backends.

The full Maude specification for the causal attack tree equivalence checker can be found in Appendix A. However, their specification only supports proving equivalences, but what about specializations? Lina supports proving specializations between attack trees using the MATLL Maude backend. Its full Maude specification can be found in Fig. 6. The axioms `a1` through `a5` are actually

```
mod MATLL is

protecting LOOP-MODE .

sorts Formula .
subsort Nat < Formula .

op _||_  : Formula Formula -> Formula [ctor assoc comm] .
op _._   : Formula Formula -> Formula [ctor assoc comm prec 41] .
op _;_   : Formula Formula -> Formula [ctor assoc prec 40] .

var a b c d : Formula .

rl [a1]          : a . (b || c)       => (a . b) || (a . c) .
rl [a1Inv]       : (a . b) || (a . c) => a . (b || c) .
rl [a2]          : a ; (b || c)       => (a ; b) || (a ; c) .
rl [a2Inv]       : (a ; b) || (a ; c) => a ; (b || c) .
rl [a3]          : (b || c) ; a       => (b ; a) || (c ; a) .
rl [a3Inv]       : (b ; a) || (c ; a) => (b || c) ; a .
rl [a4]          : (a . b) ; c        => a . (b ; c) .
rl [a4Inv]       : a . (b ; c)        => (a . b) ; c .
rl [a5]          : (a ; b) . (c ; d)  => (a . c) ; (b . d) .
rl [a5Inv]       : (a . c) ; (b . d)  => (a ; b) . (c ; d) .
rl [switch]      : a ; (b . c)        => b . (a ; c) .
rl [seq-to-para] : a ; b              => a . b .
endm
```

Fig. 6. Maude Specification for MATLL

equivalences, but the last two rules are not. The existence of inverses may cause Maude try additional paths during search, and hence, may increase the search time. We are looking into alternatives. At this point we can use these backends to reason about attack trees.

The programmer can make queries to Lina by first importing one or more Lina modules, and then making a query using Haskell's REPL – read, evaluate, print, loop – called GHCi. Consider the example Lina program in Fig. 7, these are the attack trees from Fig. 3. Then an example Lina session is as follows:

```
> :load source/Lina/Examples/Specializations.hs
...
Ok, modules loaded
> is_specialization enc_data2 enc_data1
True
>
```

In this session we first load the Lina script from Fig. 7 which is stored in the file `Specializations.hs`. Then we ask Lina if `enc_data2` is a specialization of `enc_data1`, and Lina responds `True`, thus, automating the proof given in Example 1. In addition to reasoning about attack trees Lina also support analysis of attack trees.

Currently, Lina supports the several types of analysis: evaluating attack trees, querying the attack tree for the attribute value of a node, projecting out the set of attacks from an attack tree, computing the maximal and minimal attack.

When one defines an attack tree that tree is left unevaluated, that is, the attribute dictionary associated with the attack tree only has attributes recorded

```
import Lina.AttackTree
import Lina.Maude.MATLL

-- A
enc_data1 :: PAttackTree String
enc_data1 = start_PAT $
  and_node "obtain secret"
    (or_node "obtain encryped file"
       (base_na "bribe sysadmin")
       (base_na "steal backup"))
    (seq_node "obtain password"
       (base_na "break into system")
       (base_na "install keylogger"))

-- C
enc_data2 :: PAttackTree String
enc_data2 = start_PAT $
  or_node "obtain secret"
    (and_node "obtain secret via sysadmin"
       (base_na "bribe sysadmin")
          (seq_node "obtain password"
             (base_na "break into system")
             (base_na "install keylogger")))
    (seq_node "break in, then obtain secrect"
       (base_na "break into system")
       (and_node "obtain secret from inside"
         (base_na "install keylogger")
         (base_na "steal backup")))
```

Fig. 7. Full Lina Script for Attack Trees A and C from Fig. 3

for the base attacks. If one wishes to know the attribute values at the branching nodes, then one must evaluate the attack tree, which then populates the attribute dictionary with the missing attributes. For example, we may evaluate the attack tree for the autonomous vehicle attack from Fig. 4, and query the tree for the attributes at various nodes:

```
> let (Right e_vat) = eval vehicle_AT
> e_vat <@> "social engineering attack"
0.6
>
```

Here we first evaluate the attack tree `vehicle_AT` giving it the name `e_vat`, and then we use the attributed query combinator `<@>` to ask for the attribute at the parallel node labeled with `"social engineering attack"`. Note that the evaluator, `eval`, uses the configuration associated with the attack tree to compute the values at each branching node. It is also possible to project out various attacks from an attack tree.

In Lina an *attack* corresponds to essentially an attack tree with no choice nodes. We call its data type `Attack attribute label`. An attack does not have any choice nodes, because they are all split into multiple attacks; one for each child node. For example, the set of possible attacks for the autonomous vehicle attack from Fig. 4 can be found in Fig. 8. Lina can compute these automatically using the `get_attacks` command. Finally, given the set of attacks for the au-

```
SEQ("external sensor attack",0.6)
        ("modify street signs to cause wreck",0.2)
        (AND("social engineering attack",0.6)
                ("pose as mechanic",0.6)
                ("install malware",0.1))
```

```
SEQ("over night attack",0.8)
        ("Find address where car is stored",0.05)
        (SEQ("compromise vehicle",0.8)
                ("break window",0.8)
                ("install malware",0.1))
```

```
SEQ("over night attack",0.5)
        ("Find address where car is stored",0.05)
        (SEQ("compromise vehicle",0.5)
                ("disable door alarm/locks",0.5)
                ("install malware",0.1))
```

Fig. 8. Set of Possible Attacks for an Autonomous Vehicle Attack

tonomous vehicle attack we can also compute the set of minimal and maximal attacks. For example, consider the following session:

```
> min_attacks.get_attacks $ vehicle_AT
[SEQ("over night attack",0.5)
("Find address where car is stored",0.05)
(SEQ("compromise vehicle",0.5)
("disable door alarm/locks",0.5)
("install malware",0.1))]
```

In this session we first apply `get_attacks` to `vehicle_AT` to compute the set of possible attacks, and then we compute the minimal attack from this set.

6 Conclusion and Future Work

We made two main contributions: a new four-valued truth table semantics of causal attack trees that supports specializations of attack trees, and a new embedded, in Haskell, domain specific programming language called Lina for specifying, reasoning, and analyzing attack trees.

We plan to lift the quaternary semantics into a natural deduction system based on the logic of bunched implications, and then study proof search within this new system. Lina is under active development, and we have a number of extensions planned, for example, adding support for attack-defense trees, attack(-defense) graphs, attack nets, a GUI for viewing the various models, and a SMT backend. Finally, it is necessary for number of case studies to be carried out within Lina to be able to support the types of analysis required for real world applications.

7 Acknowledgments

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A Maude Specification for Causal Attack Trees

```
mod Causal is

protecting LOOP-MODE .

sorts Formula .
subsort Nat < Formula .

op _||_    : Formula Formula -> Formula [ctor assoc comm] .
op _._     : Formula Formula -> Formula [ctor assoc comm] .
op _;_     : Formula Formula -> Formula [ctor assoc] .
op EQ(_,_) : Formula Formula -> Bool .

var P Q R S : Formula .

eq P . (Q || R)  = (P . Q)  || (P . R) .
eq P ; (Q || R)  = (P ; Q)  || (P ; R) .
eq (Q || R) ; P  = (Q ; P)  || (R ; P) .

ceq EQ(P,Q) = true
  if P = Q .
eq EQ(P,Q) = false .

endm
```