$costvar$, c
$termvar$, x, y, z, f
$baseAttackVars$, N
$baseAttackTVars$, n
$index$, i, j, k

$$
\begin{array}{lll}
A,\ B,\ C,\ E,\ F,\ D,\ T & ::= & \\
& | & N \\
& | & A \odot B \\
& | & A \oplus B \\
& | & A \rhd B \\
& | & A \sqcup B \\
& | & A \rightharpoonup B \\
& | & B \leftharpoonup A \\
& | & A \multimap B \\
& | & (A) \\
& | & A \\
\\
p & ::= & \\
& | & - \\
& | & x \\
& | & p_1 \oplus p_2 \\
& | & p_1 \odot p_2 \\
& | & p_1 \rhd p_2 \\
& | & p_1 \sqcup p_2 \\
& | & (p) \\
\\
t,\ s & ::= & \\
& | & \mathsf{n} \\
& | & x \\
& | & t_1 \odot t_2 \\
& | & \mathsf{let}\ p = t_1\ \mathsf{in}\ t_2 \\
& | & t_1 \rhd t_2 \\
& | & t_1 \sqcup t_2 \\
& | & t_1 \oplus t_2 \\
& | & \mathsf{dist}\ x_{11}, x_{12}\ \mathsf{with}\ x_1\ \mathsf{in}\ t \\
& | & \mathsf{contract}\ x_{11}, x_{12}\ \mathsf{with}\ x_1\ \mathsf{in}\ t \\
& | & \lambda x.t \\
& | & \lambda_l x.t \\
& | & \lambda_r x.t \\
& | & t_1\ t_2 \\
& | & \mathsf{app}_r\ t_1\ t_2 \\
& | & \mathsf{app}_l\ t_1\ t_2 \\
& | & (t) \\
\\
\Gamma,\ \Delta & ::= & \\
& | & \cdot \\
& | & A \\
& | & x : A \\
& | & \Gamma \mid \Gamma' \\
& | & \Gamma, \Gamma' \\
& | & \Gamma; \Gamma' \\
& | & \Gamma_1 . \Gamma_2
\end{array}
$$

$$
\begin{array}{lcl}
 & | & (\Gamma) \\
 & | & \Gamma
\end{array}
$$

$$
\begin{array}{lcl}
\Phi,\ \Psi & ::= & \\
 & | & \cdot \\
 & | & t : A \\
 & | & \Phi, \Phi' \\
 & | & (\Phi) \\
 & | & \Phi
\end{array}
$$

$\boxed{\Gamma_1 \vdash \Gamma_2}$ Context Morphisms

$$\frac{}{\Gamma \vdash \Gamma} \quad \text{C\_ID}$$

$$\frac{\Gamma_1 \vdash \Gamma_2 \quad \Gamma_2 \vdash \Gamma_3}{\Gamma_1 \vdash \Gamma_3} \quad \text{C\_C}$$

$$\frac{}{(\Gamma_1 \mid \Gamma_2) \mid \Gamma_3 \vdash \Gamma_1 \mid (\Gamma_2 \mid \Gamma_3)} \quad \text{C\_A1}$$

$$\frac{}{\cdot \mid \Gamma_2 \vdash \Gamma_2} \quad \text{C\_M1}$$

$$\frac{}{\Gamma_1 \mid \cdot \vdash \Gamma_1} \quad \text{C\_M2}$$

$$\frac{}{\Gamma_1 \mid A, B \mid \Gamma_2 \vdash \Gamma_1 \mid B, A \mid \Gamma_2} \quad \text{C\_E1L}$$

$$\frac{}{\Gamma_1 \mid A.B \mid \Gamma_2 \vdash \Gamma_1 \mid B.A \mid \Gamma_2} \quad \text{C\_E2L}$$

$$\frac{}{\Gamma_1 \mid A \mid \Gamma_2 \vdash \Gamma_1 \mid A.A \mid \Gamma_2} \quad \text{C\_C1L}$$

$$\frac{}{\Gamma_1 \mid A.A \mid \Gamma_2 \vdash \Gamma_1 \mid A \mid \Gamma_2} \quad \text{C\_C2L}$$

$$\frac{}{\Gamma_1 \mid A; (\Delta_1.\Delta_2) \mid \Gamma_2 \vdash \Gamma_1 \mid (A; \Delta_1).(A; \Delta_2) \mid \Gamma_2} \quad \text{C\_D1L}$$

$$\frac{}{\Gamma_1 \mid (A; \Delta_1).(A; \Delta_2) \mid \Gamma_2 \vdash \Gamma_1 \mid A; (\Delta_1.\Delta_2) \mid \Gamma_2} \quad \text{C\_D2L}$$

$$\frac{}{\Gamma \mid A \vdash \Gamma} \quad \text{C\_WEAK}$$

$\boxed{\Phi_1 \vdash \Phi_2}$ Context Morphisms

$$\frac{}{\Phi \vdash \Phi} \quad \text{CC\_ID}$$

$$\frac{\Phi_1 \vdash \Phi_2 \quad \Phi_2 \vdash \Phi_3}{\Phi_1 \vdash \Phi_3} \quad \text{CC\_C}$$

$$\frac{}{(\Phi_1, \Phi_2), \Phi_3 \vdash \Phi_1, (\Phi_2, \Phi_3)} \quad \text{CC\_A1}$$

$$\frac{}{\cdot, \Phi_2 \vdash \Phi_2} \quad \text{CC\_M1}$$

$$\frac{}{\Phi_1, \cdot \vdash \Phi_1} \quad \text{CC\_M2}$$

$$\frac{}{\Phi_1, x_1 : A, x_2 : B, \Phi_2 \vdash \Phi_1, x_2 : B, x_1 : A, \Phi_2} \quad \text{CC\_E}$$

$\boxed{\Gamma \vdash T}$ Valid Attack Trees

$$\frac{}{T \vdash T} \quad \text{AT\_VAR}$$

$$\frac{}{\cdot \vdash N} \quad \text{AT\_NODE}$$

$$\frac{\Gamma \vdash T_1 \quad \Delta \vdash T_2}{\Gamma, \Delta \vdash T_1 \odot T_2} \quad \text{AT\_PARAI}$$

$$\frac{\Gamma \vdash T_1 \quad \Delta \vdash T_2}{\Gamma . \Delta \vdash T_1 \sqcup T_2} \quad \text{AT\_CHOICEI}$$

$$\frac{\Gamma \vdash T_1 \quad \Delta \vdash T_2}{\Gamma ; \Delta \vdash T_1 \rhd T_2} \quad \text{AT\_SEQI}$$

$\boxed{\Gamma \vdash A}$ Attack Tree Logic (ATL)

$$\frac{}{B \vdash B} \quad \text{L\_VAR}$$

$$\frac{}{\cdot \vdash N} \quad \text{L\_NODE}$$

$$\frac{\Gamma_1 \vdash \Gamma_2 \quad \Gamma_2 \vdash A}{\Gamma_1 \vdash A} \quad \text{L\_CTX}$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma, \Delta \vdash A \odot B} \quad \text{L\_PARAI}$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma . \Delta \vdash A \sqcup B} \quad \text{L\_CHOICEI}$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma ; \Delta \vdash A \rhd B} \quad \text{L\_SEQI}$$

$$\frac{\Gamma \vdash A \odot B \quad \Delta_1 \mid A, B \mid \Delta_2 \vdash C}{\Delta_1 \mid \Gamma \mid \Delta_2 \vdash C} \quad \text{L\_PARAE}$$

$$\frac{\Gamma \vdash A \sqcup B \quad \Delta_1 \mid A.B \mid \Delta_2 \vdash C}{\Delta_1 \mid \Gamma \mid \Delta_2 \vdash C} \quad \text{L\_CHOICEE}$$

$$\frac{\Gamma \vdash A \rhd B \quad \Delta_1 \mid A; B \mid \Delta_2 \vdash C}{\Delta_1 \mid \Gamma \mid \Delta_2 \vdash C} \quad \text{L\_SEQE}$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B} \quad \text{L\_LIMPI}$$

$$\frac{\Gamma \vdash A \multimap B \quad \Delta \vdash A}{\Gamma, \Delta \vdash B} \quad \text{L\_LIMPE}$$

$$\frac{\Gamma ; A \vdash B}{\Gamma \vdash A \rightharpoonup B} \quad \text{L\_RLIMPI}$$

$$\frac{\Gamma \vdash A \rightharpoonup B \quad \Delta \vdash A}{\Gamma ; \Delta \vdash B} \quad \text{L\_RLIMPE}$$

$$\frac{A ; \Gamma \vdash B}{\Gamma \vdash B \leftharpoondown A} \quad \text{L\_LLIMPI}$$

$$\frac{\Gamma \vdash B \leftharpoondown A \quad \Delta \vdash A}{\Delta ; \Gamma \vdash B} \quad \text{L\_LLIMPE}$$

$\boxed{\Gamma \vdash t : T}$ Valid Attack Tree Type Theory (ATTT)

$$\frac{}{x : T \vdash x : T}\ \text{ATT\_VAR}$$

$$\frac{}{\cdot \vdash \mathsf{n} : N}\ \text{ATT\_NODE}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Delta \vdash t_2 : T_2}{\Gamma, \Delta \vdash t_1 \odot t_2 : T_1 \odot T_2}\ \text{ATT\_PARAI}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Delta \vdash t_2 : T_2}{\Gamma . \Delta \vdash t_1 \sqcup t_2 : T_1 \sqcup T_2}\ \text{ATT\_CHOICEI}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Delta \vdash t_2 : T_2}{\Gamma ; \Delta \vdash t_1 \rhd t_2 : T_1 \rhd T_2}\ \text{ATT\_SEQI}$$

$\boxed{\Gamma \vdash t : A}$ Attack Tree Type Theory (ATTT)

$$\frac{}{x : B \vdash x : B}\ \text{T\_VAR}$$

$$\frac{}{\cdot \vdash \mathsf{n} : N}\ \text{T\_NODE}$$

$$\frac{\Gamma_1 \mid x_1 : A, x_2 : B \mid \Gamma_2 \vdash t : C}{\Gamma_1 \mid x_2 : A, x_1 : B \mid \Gamma_2 \vdash t : C}\ \text{T\_EX1}$$

$$\frac{\Gamma_1 \mid x_1 : A . x_2 : B \mid \Gamma_2 \vdash t : C}{\Gamma_1 \mid x_2 : A . x_1 : B \mid \Gamma_2 \vdash t : C}\ \text{T\_EX2}$$

$$\frac{\Gamma_1 \mid \Gamma_2 \vdash t : B \quad x \notin \mathsf{FV}(t)}{\Gamma_1 \mid x : A \mid \Gamma_2 \vdash t : B}\ \text{T\_WEAK}$$

$$\frac{\Gamma_1 \mid x_1 : A; (\Delta_1 . \Delta_2) \vdash t : D \quad x_2 \notin \mathsf{FV}(t)}{\Gamma_1 \mid (x_1 : A; \Delta_1).(x_2 : A; \Delta_2) \vdash t : D}\ \text{T\_DIST1}$$

$$\frac{\Gamma_1 \mid (x_{11} : A; \Delta_1).(x_{12} : A; \Delta_2) \mid \Gamma_2 \vdash t : D \quad \Delta_1 \neq \cdot \quad \Delta_2 \neq \cdot}{\Gamma_1 \mid x_1 : A; (\Delta_1 . \Delta_2) \vdash \mathsf{dist}\, x_{11}, x_{12}\, \mathsf{with}\, x_1\, \mathsf{in}\, t : D}\ \text{T\_DIST2}$$

$$\frac{\Gamma_1 \mid x_1 : A \mid \Gamma_2 \vdash t : B \quad x_2 \notin \mathsf{FV}(t)}{\Gamma_1 \mid (x_1 : A . x_2 : A) \mid \Gamma_2 \vdash t : B}\ \text{T\_CONTRACT1}$$

$$\frac{\Gamma_1 \mid x_1 : A . x_2 : A \mid \Gamma_2 \vdash t : B}{\Gamma_1 \mid x : A \mid \Gamma_2 \vdash \mathsf{contract}\, x_1, x_2\, \mathsf{with}\, x\, \mathsf{in}\, t : B}\ \text{T\_CONTRACT2}$$

$$\frac{\Gamma \vdash t_1 : A \quad \Delta \vdash t_2 : B}{\Gamma, \Delta \vdash t_1 \odot t_2 : A \odot B}\ \text{T\_PARAI}$$

$$\frac{\Gamma \vdash t_1 : A \quad \Delta \vdash t_2 : B}{\Gamma . \Delta \vdash t_1 \sqcup t_2 : A \sqcup B}\ \text{T\_CHOICEI}$$

$$\frac{\Gamma \vdash t_1 : A \quad \Delta \vdash t_2 : B}{\Gamma ; \Delta \vdash t_1 \rhd t_2 : A \rhd B}\ \text{T\_SEQI}$$

$$\frac{\Gamma \vdash t_1 : A \odot B \quad \Delta_1 \mid x_1 : A, x_2 : B \mid \Delta_2 \vdash t_2 : C}{\Delta_1 \mid \Gamma \mid \Delta_2 \vdash \mathsf{let}\,(x_1 \odot x_2) = t_1\, \mathsf{in}\, t_2 : C}\ \text{T\_PARAE}$$

$$\frac{\Gamma \vdash t_1 : A \sqcup B \quad \Delta_1 \mid x_1 : A . x_2 : B \mid \Delta_2 \vdash t_2 : C}{\Delta_1 \mid \Gamma \mid \Delta_2 \vdash \mathsf{let}\,(x_1 \sqcup x_2) = t_1\, \mathsf{in}\, t_2 : C}\ \text{T\_CHOICEE}$$

$$\frac{\Gamma \vdash t_1 : A \rhd B \quad \Delta_1 \mid x_1 : A; x_2 : B \mid \Delta_2 \vdash t_2 : C}{\Delta_1 \mid \Gamma \mid \Delta_2 \vdash \mathsf{let}\,(x_1 \rhd x_2) = t_1\, \mathsf{in}\, t_2 : C}\ \text{T\_SEQE}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x.t : A \multimap B}\ \textsc{T\_limpI}$$

$$\frac{\Gamma \vdash t_1 : A \multimap B \quad \Delta \vdash t_2 : A}{\Gamma, \Delta \vdash t_1\, t_2 : B}\ \textsc{T\_limpE}$$

$$\frac{\Gamma; x : A \vdash t : B}{\Gamma \vdash \lambda_r x.t : A \rightharpoonup B}\ \textsc{T\_RlimpI}$$

$$\frac{\Gamma \vdash t_1 : A \rightharpoonup B \quad \Delta \vdash t_2 : A}{\Gamma; \Delta \vdash \mathsf{app}_r\, t_1\, t_2 : B}\ \textsc{T\_RlimpE}$$

$$\frac{x : A; \Gamma \vdash t : B}{\Gamma \vdash \lambda_l x.t : B \leftharpoonup A}\ \textsc{T\_LlimpI}$$

$$\frac{\Gamma \vdash t_1 : B \leftharpoonup A \quad \Delta \vdash t_2 : A}{\Delta; \Gamma \vdash \mathsf{app}_l\, t_1\, t_2 : B}\ \textsc{T\_LlimpE}$$

$\boxed{t_1 \rightsquigarrow t_2}$

$$\frac{}{\mathsf{let}\,(x_1 \odot x_2) = t_1 \odot t_2\,\mathsf{in}\, t_3 \rightsquigarrow [t_1/x_1][t_2/x_2]t_3}\ \textsc{R\_paraBeta}$$

$$\frac{}{\mathsf{let}\,(x_1 \sqcup x_2) = t_1 \sqcup t_2\,\mathsf{in}\, t_3 \rightsquigarrow [t_1/x_1][t_2/x_2]t_3}\ \textsc{R\_choiceBeta}$$

$$\frac{}{\mathsf{let}\,(x_1 \rhd x_2) = t_1 \rhd t_2\,\mathsf{in}\, t_3 \rightsquigarrow [t_1/x_1][t_2/x_2]t_3}\ \textsc{R\_seqBeta}$$

$$\frac{}{(\lambda x.t_2)\, t_1 \rightsquigarrow [t_1/x]t_2}\ \textsc{R\_beta}$$

$$\frac{}{\mathsf{app}_r\,(\lambda_r x.t_2)\, t_1 \rightsquigarrow [t_1/x]t_2}\ \textsc{R\_betaR}$$

$$\frac{}{\mathsf{app}_l\,(\lambda_l x.t_2)\, t_1 \rightsquigarrow [t_1/x]t_2}\ \textsc{R\_betaL}$$

$$\frac{}{\mathsf{let}\,(x_1 \odot x_2) = (\mathsf{let}\,(y_1 \odot y_2) = t_1\,\mathsf{in}\, t_2)\,\mathsf{in}\, t_3 \rightsquigarrow \mathsf{let}\,(y_1 \odot y_2) = t_1\,\mathsf{in}\,(\mathsf{let}\,(x_1 \odot x_2) = t_2\,\mathsf{in}\, t_3)}\ \textsc{R\_paraCC}$$

$$\frac{}{\mathsf{let}\,(x_1 \odot x_2) = (\mathsf{let}\,(y_1 \sqcup y_2) = t_1\,\mathsf{in}\, t_2)\,\mathsf{in}\, t_3 \rightsquigarrow \mathsf{let}\,(y_1 \sqcup y_2) = t_1\,\mathsf{in}\,(\mathsf{let}\,(x_1 \odot x_2) = t_2\,\mathsf{in}\, t_3)}\ \textsc{R\_paraCCC}$$

$$\frac{}{\mathsf{let}\,(x_1 \odot x_2) = (\mathsf{let}\,(y_1 \rhd y_2) = t_1\,\mathsf{in}\, t_2)\,\mathsf{in}\, t_3 \rightsquigarrow \mathsf{let}\,(y_1 \rhd y_2) = t_1\,\mathsf{in}\,(\mathsf{let}\,(x_1 \odot x_2) = t_2\,\mathsf{in}\, t_3)}\ \textsc{R\_paraSCC}$$

$$\frac{}{\mathsf{let}\,(x_1 \sqcup x_2) = (\mathsf{let}\,(y_1 \sqcup y_2) = t_1\,\mathsf{in}\, t_2)\,\mathsf{in}\, t_3 \rightsquigarrow \mathsf{let}\,(y_1 \sqcup y_2) = t_1\,\mathsf{in}\,(\mathsf{let}\,(x_1 \sqcup x_2) = t_2\,\mathsf{in}\, t_3)}\ \textsc{R\_choiceCC}$$

$$\frac{}{\mathsf{let}\,(x_1 \sqcup x_2) = (\mathsf{let}\,(y_1 \odot y_2) = t_1\,\mathsf{in}\, t_2)\,\mathsf{in}\, t_3 \rightsquigarrow \mathsf{let}\,(y_1 \odot y_2) = t_1\,\mathsf{in}\,(\mathsf{let}\,(x_1 \sqcup x_2) = t_2\,\mathsf{in}\, t_3)}\ \textsc{R\_choicePCC}$$

$$\frac{}{\mathsf{let}\,(x_1 \sqcup x_2) = (\mathsf{let}\,(y_1 \rhd y_2) = t_1\,\mathsf{in}\, t_2)\,\mathsf{in}\, t_3 \rightsquigarrow \mathsf{let}\,(y_1 \rhd y_2) = t_1\,\mathsf{in}\,(\mathsf{let}\,(x_1 \sqcup x_2) = t_2\,\mathsf{in}\, t_3)}\ \textsc{R\_choiceSCC}$$

$$\frac{}{\mathsf{let}\,(x_1 \rhd x_2) = (\mathsf{let}\,(y_1 \rhd y_2) = t_1\,\mathsf{in}\, t_2)\,\mathsf{in}\, t_3 \rightsquigarrow \mathsf{let}\,(y_1 \rhd y_2) = t_1\,\mathsf{in}\,(\mathsf{let}\,(x_1 \rhd x_2) = t_2\,\mathsf{in}\, t_3)}\ \textsc{R\_seqCC}$$

$$\frac{}{\mathsf{let}\,(x_1 \rhd x_2) = (\mathsf{let}\,(y_1 \odot y_2) = t_1\,\mathsf{in}\, t_2)\,\mathsf{in}\, t_3 \rightsquigarrow \mathsf{let}\,(y_1 \odot y_2) = t_1\,\mathsf{in}\,(\mathsf{let}\,(x_1 \rhd x_2) = t_2\,\mathsf{in}\, t_3)}\ \textsc{R\_seqPCC}$$

$$\frac{}{\mathsf{let}\,(x_1 \rhd x_2) = (\mathsf{let}\,(y_1 \sqcup y_2) = t_1\,\mathsf{in}\, t_2)\,\mathsf{in}\, t_3 \rightsquigarrow \mathsf{let}\,(y_1 \sqcup y_2) = t_1\,\mathsf{in}\,(\mathsf{let}\,(x_1 \rhd x_2) = t_2\,\mathsf{in}\, t_3)}\ \textsc{R\_seqCCC}$$

$$\frac{}{(\mathsf{let}\,(x_1 \odot x_2) = t_1\,\mathsf{in}\, t_2)\, t_3 \rightsquigarrow \mathsf{let}\,(x_1 \odot x_2) = t_1\,\mathsf{in}\,(t_2\, t_3)}\ \textsc{R\_paraACC}$$

$$\frac{}{(\mathsf{let}\,(x_1 \sqcup x_2) = t_1\,\mathsf{in}\, t_2)\, t_3 \rightsquigarrow \mathsf{let}\,(x_1 \sqcup x_2) = t_1\,\mathsf{in}\,(t_2\, t_3)}\ \textsc{R\_choiceACC}$$

$$\frac{}{(\mathsf{let}\,(x_1 \rhd x_2) = t_1 \,\mathsf{in}\, t_2)\; t_3 \rightsquigarrow \mathsf{let}\,(x_1 \rhd x_2) = t_1 \,\mathsf{in}\,(t_2\; t_3)} \quad \text{R\_{\scriptsize SEQ}ACC}$$

$\boxed{\Gamma \vdash \Phi}$

$$\frac{}{x : B \vdash x : B} \quad \text{F\_{\scriptsize VAR}}$$

$$\frac{\Gamma_1 \vdash \Gamma_2 \quad \Gamma_2 \vdash \Phi_2 \quad \Phi_2 \vdash \Phi_1}{\Gamma_1 \vdash \Phi_1} \quad \text{F\_{\scriptsize CTX}}$$

$$\frac{\Gamma_2 \vdash \Phi_1, t : A, \Phi_3 \quad \Gamma_1 \mid x : A \mid \Gamma_3 \vdash \Phi_2}{\Gamma_1 \mid \Gamma_2 \mid \Gamma_3 \vdash \Phi_1, [t/x]\Phi_2, \Phi_3} \quad \text{F\_{\scriptsize CUT}}$$

$$\frac{\Gamma_1 \mid x_1 : A, x_2 : B \mid \Gamma_2 \vdash \Phi}{\Gamma_1 \mid y : A \odot B \mid \Gamma_2 \vdash \mathsf{let}\,(x_1 \odot x_2) = y \,\mathsf{in}\, \Phi} \quad \text{F\_{\scriptsize PARA}L}$$

$$\frac{\Gamma \vdash t_1 : A, \Phi_1 \quad \Delta \vdash t_2 : B, \Phi_2}{\Gamma, \Delta \vdash t_1 \odot t_2 : A \odot B, \Phi_1, \Phi_2} \quad \text{F\_{\scriptsize PARA}R}$$

$$\frac{\Gamma_1 \mid x_1 : A; x_2 : B \mid \Gamma_2 \vdash \Phi}{\Gamma_1 \mid y : A \rhd B \mid \Gamma_2 \vdash \mathsf{let}\,(x_1 \rhd x_2) = y \,\mathsf{in}\, \Phi} \quad \text{F\_{\scriptsize SEQ}L}$$

$$\frac{\Gamma \vdash t_1 : A, \Phi_1 \quad \Delta \vdash t_2 : B, \Phi_2}{\Gamma; \Delta \vdash t_1 \rhd t_2 : A \rhd B, \Phi_1, \Phi_2} \quad \text{F\_{\scriptsize SEQ}R}$$

$$\frac{\Gamma_2 \vdash t_1 : A, \Phi_1 \quad \Gamma_1 \mid x : B \mid \Gamma_3 \vdash \Phi_2}{\Gamma_1 \mid y : A \multimap B, \Gamma_2 \mid \Gamma_3 \vdash \Phi_1, [y\; t_1/x]\Phi_2} \quad \text{F\_{\scriptsize LIMP}L}$$

$$\frac{\Gamma, x : A \vdash t : B, \Phi \quad x \notin \mathsf{FV}(\Phi)}{\Gamma \vdash \lambda x.t : A \multimap B, \Phi} \quad \text{F\_{\scriptsize LIMP}R}$$

$$\frac{\Gamma_2 \vdash t_1 : A, \Phi_1 \quad \Gamma_1 \mid x : B \mid \Gamma_3 \vdash \Phi_2}{\Gamma_1 \mid \Gamma_2, y : A \rightharpoonup B \mid \Gamma_3 \vdash \Phi_1, [y\; t_1/x]\Phi_2} \quad \text{F\_R{\scriptsize LIMP}L}$$

$$\frac{\Gamma; x : A \vdash t : B, \Phi \quad x \notin \mathsf{FV}(\Phi)}{\Gamma \vdash \lambda_r x.t : A \rightharpoonup B, \Phi} \quad \text{F\_R{\scriptsize LIMP}R}$$

$$\frac{\Gamma_2 \vdash t_1 : A, \Phi_1 \quad \Gamma_1 \mid x : B \mid \Gamma_3 \vdash \Phi_2}{\Gamma_1 \mid y : A \leftharpoonup B, \Gamma_2 \mid \Gamma_3 \vdash \Phi_1, [y\; t_1/x]\Phi_2} \quad \text{F\_L{\scriptsize LIMP}L}$$

$$\frac{x : A; \Gamma \vdash t : B, \Phi \quad x \notin \mathsf{FV}(\Phi)}{\Gamma \vdash \lambda_l x.t : B \leftharpoonup A, \Phi} \quad \text{F\_L{\scriptsize LIMP}R}$$

$$\frac{\Gamma \vdash \Phi_1, t_1 : A, t_2 : B, \Phi_2}{\Gamma \vdash \Phi_1, t_1 \oplus t_2 : A \oplus B, \Phi_2} \quad \text{F\_{\scriptsize PAR}R}$$

$$\frac{\Gamma_1 \mid x_1 : A \vdash \Phi_1 \quad x_2 : B \mid \Gamma_2 \vdash \Phi_2}{\Gamma_1 \mid z : A \oplus B \mid \Gamma_2 \vdash (\mathsf{let}\, x_1 \oplus - = z \,\mathsf{in}\, \Phi_1), (\mathsf{let}\, - \oplus x_2 = z \,\mathsf{in}\, \Phi_2)} \quad \text{F\_{\scriptsize PAR}L}$$

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Definition rules:        97 good    0 bad
Definition rule clauses: 151 good    0 bad
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