$costvar$, c
$termvar$, x, y, z, f
$baseAttackVars$, N
$baseAttackTVars$, n
$index$, i, j, k

$$
\begin{array}{lcl}
A,\ B,\ C,\ E,\ F,\ D,\ T & ::= & \\
 & | & N \\
 & | & A \odot B \\
 & | & A \oplus B \\
 & | & A \rhd B \\
 & | & A \sqcup B \\
 & | & A \rightharpoonup B \\
 & | & B \leftharpoondown A \\
 & | & A \multimap B \\
 & | & (A) \\
 & | & A \\
 & & \\
p & ::= & \\
 & | & - \\
 & | & x \\
 & | & p_1 \oplus p_2 \\
 & | & p_1 \odot p_2 \\
 & | & p_1 \rhd p_2 \\
 & | & p_1 \sqcup p_2 \\
 & | & (p) \\
 & & \\
t,\ s & ::= & \\
 & | & \mathsf{n} \\
 & | & x \\
 & | & t_1 \odot t_2 \\
 & | & \mathsf{let}\ p = t_1\ \mathsf{in}\ t_2 \\
 & | & t_1 \rhd t_2 \\
 & | & t_1 \sqcup t_2 \\
 & | & t_1 \oplus t_2 \\
 & | & \mathsf{dist}\ x_{11}, x_{12}\ \mathsf{with}\ x_1\ \mathsf{in}\ t \\
 & | & \mathsf{contract}\ x_{11}, x_{12}\ \mathsf{with}\ x_1\ \mathsf{in}\ t \\
 & | & \lambda x.t \\
 & | & \lambda_l x.t \\
 & | & \lambda_r x.t \\
 & | & t_1\ t_2 \\
 & | & \mathsf{app}_r\ t_1\ t_2 \\
 & | & \mathsf{app}_l\ t_1\ t_2 \\
 & | & (t) \\
 & & \\
\Gamma,\ \Delta & ::= & \\
 & | & \cdot \\
 & | & A \\
 & | & x : A \\
 & | & \Gamma(\Gamma') \\
 & | & \Gamma, \Gamma' \\
 & | & \Gamma; \Gamma' \\
 & | & \Gamma_1 \,.\, \Gamma_2
\end{array}
$$

$$
\begin{array}{lcl}
 & | & \Gamma_1 \circ \Gamma_2 \\
 & | & (\Gamma) \\
 & | & \Gamma
\end{array}
$$

$$
\begin{array}{lcl}
\Phi,\ \Psi & ::= & \\
 & | & \cdot \\
 & | & t : A \\
 & | & \Phi, \Phi' \\
 & | & (\Phi) \\
 & | & \Phi \\
 & | & \Phi(\Phi')
\end{array}
$$

$\boxed{\Gamma_1 \vdash \Gamma_2}$ Context Morphisms

$$\frac{}{\Gamma \vdash \Gamma}\ \textsc{C\_id}$$

$$\frac{\Gamma_1 \vdash \Gamma_2 \quad \Gamma_2 \vdash \Gamma_3}{\Gamma_1 \vdash \Gamma_3}\ \textsc{C\_c}$$

$$\frac{}{(\Gamma_1 \circ \Gamma_2) \circ \Gamma_3 \vdash \Gamma_1 \circ (\Gamma_2 \circ \Gamma_3)}\ \textsc{C\_a1}$$

$$\frac{}{\Gamma \circ \cdot \vdash \Gamma}\ \textsc{C\_u1}$$

$$\frac{}{\cdot \circ \Gamma \vdash \Gamma}\ \textsc{C\_u2}$$

$$\frac{}{\Gamma(A, B) \vdash \Gamma(B, A)}\ \textsc{C\_e1}$$

$$\frac{}{\Gamma(A \,.\, B) \vdash \Gamma(B \,.\, A)}\ \textsc{C\_e2}$$

$$\frac{}{\Gamma(A; (\Delta_1 \,.\, \Delta_2)) \vdash \Gamma((A; \Delta_1) \,.\, (A; \Delta_2))}\ \textsc{C\_d1}$$

$$\frac{}{\Gamma((A; \Delta_1) \,.\, (A; \Delta_2)) \vdash \Gamma(A; (\Delta_1 \,.\, \Delta_2))}\ \textsc{C\_d2}$$

$$\frac{}{\Gamma(A) \vdash \Gamma}\ \textsc{C\_Weak}$$

$\boxed{\Phi_1 \vdash \Phi_2}$ Context Morphisms

$$\frac{}{\Phi \vdash \Phi}\ \textsc{CC\_id}$$

$$\frac{\Phi_1 \vdash \Phi_2 \quad \Phi_2 \vdash \Phi_3}{\Phi_1 \vdash \Phi_3}\ \textsc{CC\_c}$$

$$\frac{}{(\Phi_1, \Phi_2), \Phi_3 \vdash \Phi_1, (\Phi_2, \Phi_3)}\ \textsc{CC\_a1}$$

$$\frac{}{\cdot, \Phi_2 \vdash \Phi_2}\ \textsc{CC\_m1}$$

$$\frac{}{\Phi_1, \cdot \vdash \Phi_1}\ \textsc{CC\_m2}$$

$$\frac{}{\Phi(x_1 : A, x_2 : B) \vdash \Phi(x_2 : B, x_1 : A)}\ \textsc{CC\_e}$$

$\boxed{\Gamma \vdash T}$ Valid Attack Trees

$$\frac{}{T \vdash T}\ \textsc{AT\_var}$$

$$\frac{}{\cdot \vdash N}\ \text{AT\_NODE}$$

$$\frac{\Gamma \vdash T_1 \quad \Delta \vdash T_2}{\Gamma, \Delta \vdash T_1 \odot T_2}\ \text{AT\_PARAI}$$

$$\frac{\Gamma \vdash T_1 \quad \Delta \vdash T_2}{\Gamma \,.\, \Delta \vdash T_1 \sqcup T_2}\ \text{AT\_CHOICEI}$$

$$\frac{\Gamma \vdash T_1 \quad \Delta \vdash T_2}{\Gamma; \Delta \vdash T_1 \rhd T_2}\ \text{AT\_SEQI}$$

$\boxed{\Gamma \vdash A}$ Attack Tree Logic (ATL)

$$\frac{}{B \vdash B}\ \text{L\_VAR}$$

$$\frac{}{\cdot \vdash N}\ \text{L\_NODE}$$

$$\frac{\Gamma_1 \vdash \Gamma_2 \quad \Gamma_2 \vdash A}{\Gamma_1 \vdash A}\ \text{L\_CTX}$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma, \Delta \vdash A \odot B}\ \text{L\_PARAI}$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma \,.\, \Delta \vdash A \sqcup B}\ \text{L\_CHOICEI}$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma; \Delta \vdash A \rhd B}\ \text{L\_SEQI}$$

$$\frac{\Gamma \vdash A \odot B \quad \Delta(A, B) \vdash C}{\Delta(\Gamma) \vdash C}\ \text{L\_PARAE}$$

$$\frac{\Gamma \vdash A \sqcup B \quad \Delta(A \,.\, B) \vdash C}{\Delta(\Gamma) \vdash C}\ \text{L\_CHOICEE}$$

$$\frac{\Gamma \vdash A \rhd B \quad \Delta(A; B) \vdash C}{\Delta(\Gamma) \vdash C}\ \text{L\_SEQE}$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}\ \text{L\_LIMPI}$$

$$\frac{\Gamma \vdash A \multimap B \quad \Delta \vdash A}{\Gamma, \Delta \vdash B}\ \text{L\_LIMPE}$$

$$\frac{\Gamma; A \vdash B}{\Gamma \vdash A \rightharpoonup B}\ \text{L\_RLIMPI}$$

$$\frac{\Gamma \vdash A \rightharpoonup B \quad \Delta \vdash A}{\Gamma; \Delta \vdash B}\ \text{L\_RLIMPE}$$

$$\frac{A; \Gamma \vdash B}{\Gamma \vdash B \leftharpoonup A}\ \text{L\_LLIMPI}$$

$$\frac{\Gamma \vdash B \leftharpoonup A \quad \Delta \vdash A}{\Delta; \Gamma \vdash B}\ \text{L\_LLIMPE}$$

$\boxed{\Gamma \vdash t : T}$ Valid Attack Tree Type Theory (ATTT)

$$\frac{}{x : T \vdash x : T}\ \text{ATT\_VAR}$$

$$\frac{}{\cdot \vdash \mathsf{n} : N} \quad \text{ATT\_NODE}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Delta \vdash t_2 : T_2}{\Gamma, \Delta \vdash t_1 \odot t_2 : T_1 \odot T_2} \quad \text{ATT\_PARAI}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Delta \vdash t_2 : T_2}{\Gamma \mathbin{.} \Delta \vdash t_1 \sqcup t_2 : T_1 \sqcup T_2} \quad \text{ATT\_CHOICEI}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Delta \vdash t_2 : T_2}{\Gamma ; \Delta \vdash t_1 \rhd t_2 : T_1 \rhd T_2} \quad \text{ATT\_SEQI}$$

$\boxed{\Gamma \vdash t : A}$ Attack Tree Type Theory (ATTT)

$$\frac{}{x : B \vdash x : B} \quad \text{T\_VAR}$$

$$\frac{}{\cdot \vdash \mathsf{n} : N} \quad \text{T\_NODE}$$

$$\frac{\Gamma(x_1 : A, x_2 : B) \vdash t : C}{\Gamma(x_2 : A, x_1 : B) \vdash t : C} \quad \text{T\_EX1}$$

$$\frac{\Gamma(x_1 : A \mathbin{.} x_2 : B) \vdash t : C}{\Gamma(x_2 : A \mathbin{.} x_1 : B) \vdash t : C} \quad \text{T\_EX2}$$

$$\frac{\Gamma(x_1 : A; (\Delta_1 \mathbin{.} \Delta_2)) \vdash t : D \quad x_2 \notin \mathsf{FV}(t)}{\Gamma((x_1 : A; \Delta_1) \mathbin{.} (x_2 : A; \Delta_2)) \vdash t : D} \quad \text{T\_DIST1}$$

$$\frac{\Gamma((x_{11} : A; \Delta_1) \mathbin{.} (x_{12} : A; \Delta_2)) \vdash t : D \quad \Delta_1 \neq \cdot \quad \Delta_2 \neq \cdot}{\Gamma(x_1 : A; (\Delta_1 \mathbin{.} \Delta_2)) \vdash \mathsf{dist}\, x_{11}, x_{12}\, \mathsf{with}\, x_1\, \mathsf{in}\, t : D} \quad \text{T\_DIST2}$$

$$\frac{\Gamma \vdash t_1 : A \quad \Delta \vdash t_2 : B}{\Gamma, \Delta \vdash t_1 \odot t_2 : A \odot B} \quad \text{T\_PARAI}$$

$$\frac{\Gamma \vdash t_1 : A \quad \Delta \vdash t_2 : B}{\Gamma \mathbin{.} \Delta \vdash t_1 \sqcup t_2 : A \sqcup B} \quad \text{T\_CHOICEI}$$

$$\frac{\Gamma \vdash t_1 : A \quad \Delta \vdash t_2 : B}{\Gamma ; \Delta \vdash t_1 \rhd t_2 : A \rhd B} \quad \text{T\_SEQI}$$

$$\frac{\Gamma \vdash t_1 : A \odot B \quad \Delta(x_1 : A, x_2 : B) \vdash t_2 : C}{\Delta(\Gamma) \vdash \mathsf{let}\,(x_1 \odot x_2) = t_1\, \mathsf{in}\, t_2 : C} \quad \text{T\_PARAE}$$

$$\frac{\Gamma \vdash t_1 : A \sqcup B \quad \Delta(x_1 : A \mathbin{.} x_2 : B) \vdash t_2 : C}{\Delta(\Gamma) \vdash \mathsf{let}\,(x_1 \sqcup x_2) = t_1\, \mathsf{in}\, t_2 : C} \quad \text{T\_CHOICEE}$$

$$\frac{\Gamma \vdash t_1 : A \rhd B \quad \Delta(x_1 : A; x_2 : B) \vdash t_2 : C}{\Delta(\Gamma) \vdash \mathsf{let}\,(x_1 \rhd x_2) = t_1\, \mathsf{in}\, t_2 : C} \quad \text{T\_SEQE}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x.t : A \multimap B} \quad \text{T\_LIMPI}$$

$$\frac{\Gamma \vdash t_1 : A \multimap B \quad \Delta \vdash t_2 : A}{\Gamma, \Delta \vdash t_1\, t_2 : B} \quad \text{T\_LIMPE}$$

$$\frac{\Gamma ; x : A \vdash t : B}{\Gamma \vdash \lambda_r x.t : A \rightharpoonup B} \quad \text{T\_RLIMPI}$$

$$\frac{\Gamma \vdash t_1 : A \rightharpoonup B \quad \Delta \vdash t_2 : A}{\Gamma ; \Delta \vdash \mathsf{app}_r\, t_1\, t_2 : B} \quad \text{T\_RLIMPE}$$

$$\frac{x : A; \Gamma \vdash t : B}{\Gamma \vdash \lambda_l x.t : B \leftharpoonup A} \quad \textsc{T\_LlimpI}$$

$$\frac{\Gamma \vdash t_1 : B \leftharpoonup A \quad \Delta \vdash t_2 : A}{\Delta; \Gamma \vdash \mathsf{app}_l\ t_1\ t_2 : B} \quad \textsc{T\_LlimpE}$$

$\boxed{t_1 \rightsquigarrow t_2}$

$$\frac{}{\mathsf{let}\,(x_1 \odot x_2) = t_1 \odot t_2\ \mathsf{in}\ t_3 \rightsquigarrow [t_1/x_1][t_2/x_2]t_3} \quad \textsc{R\_paraBeta}$$

$$\frac{}{\mathsf{let}\,(x_1 \sqcup x_2) = t_1 \sqcup t_2\ \mathsf{in}\ t_3 \rightsquigarrow [t_1/x_1][t_2/x_2]t_3} \quad \textsc{R\_choiceBeta}$$

$$\frac{}{\mathsf{let}\,(x_1 \rhd x_2) = t_1 \rhd t_2\ \mathsf{in}\ t_3 \rightsquigarrow [t_1/x_1][t_2/x_2]t_3} \quad \textsc{R\_seqBeta}$$

$$\frac{}{(\lambda x.t_2)\ t_1 \rightsquigarrow [t_1/x]t_2} \quad \textsc{R\_beta}$$

$$\frac{}{\mathsf{app}_r\ (\lambda_r x.t_2)\ t_1 \rightsquigarrow [t_1/x]t_2} \quad \textsc{R\_betaR}$$

$$\frac{}{\mathsf{app}_l\ (\lambda_l x.t_2)\ t_1 \rightsquigarrow [t_1/x]t_2} \quad \textsc{R\_betaL}$$

$$\frac{}{\mathsf{let}\,(x_1 \odot x_2) = (\mathsf{let}\,(y_1 \odot y_2) = t_1\ \mathsf{in}\ t_2)\ \mathsf{in}\ t_3 \rightsquigarrow \mathsf{let}\,(y_1 \odot y_2) = t_1\ \mathsf{in}\ (\mathsf{let}\,(x_1 \odot x_2) = t_2\ \mathsf{in}\ t_3)} \quad \textsc{R\_paraCC}$$

$$\frac{}{\mathsf{let}\,(x_1 \odot x_2) = (\mathsf{let}\,(y_1 \sqcup y_2) = t_1\ \mathsf{in}\ t_2)\ \mathsf{in}\ t_3 \rightsquigarrow \mathsf{let}\,(y_1 \sqcup y_2) = t_1\ \mathsf{in}\ (\mathsf{let}\,(x_1 \odot x_2) = t_2\ \mathsf{in}\ t_3)} \quad \textsc{R\_paraCCC}$$

$$\frac{}{\mathsf{let}\,(x_1 \odot x_2) = (\mathsf{let}\,(y_1 \rhd y_2) = t_1\ \mathsf{in}\ t_2)\ \mathsf{in}\ t_3 \rightsquigarrow \mathsf{let}\,(y_1 \rhd y_2) = t_1\ \mathsf{in}\ (\mathsf{let}\,(x_1 \odot x_2) = t_2\ \mathsf{in}\ t_3)} \quad \textsc{R\_paraSCC}$$

$$\frac{}{\mathsf{let}\,(x_1 \sqcup x_2) = (\mathsf{let}\,(y_1 \sqcup y_2) = t_1\ \mathsf{in}\ t_2)\ \mathsf{in}\ t_3 \rightsquigarrow \mathsf{let}\,(y_1 \sqcup y_2) = t_1\ \mathsf{in}\ (\mathsf{let}\,(x_1 \sqcup x_2) = t_2\ \mathsf{in}\ t_3)} \quad \textsc{R\_choiceCC}$$

$$\frac{}{\mathsf{let}\,(x_1 \sqcup x_2) = (\mathsf{let}\,(y_1 \odot y_2) = t_1\ \mathsf{in}\ t_2)\ \mathsf{in}\ t_3 \rightsquigarrow \mathsf{let}\,(y_1 \odot y_2) = t_1\ \mathsf{in}\ (\mathsf{let}\,(x_1 \sqcup x_2) = t_2\ \mathsf{in}\ t_3)} \quad \textsc{R\_choicePCC}$$

$$\frac{}{\mathsf{let}\,(x_1 \sqcup x_2) = (\mathsf{let}\,(y_1 \rhd y_2) = t_1\ \mathsf{in}\ t_2)\ \mathsf{in}\ t_3 \rightsquigarrow \mathsf{let}\,(y_1 \rhd y_2) = t_1\ \mathsf{in}\ (\mathsf{let}\,(x_1 \sqcup x_2) = t_2\ \mathsf{in}\ t_3)} \quad \textsc{R\_choiceSCC}$$

$$\frac{}{\mathsf{let}\,(x_1 \rhd x_2) = (\mathsf{let}\,(y_1 \rhd y_2) = t_1\ \mathsf{in}\ t_2)\ \mathsf{in}\ t_3 \rightsquigarrow \mathsf{let}\,(y_1 \rhd y_2) = t_1\ \mathsf{in}\ (\mathsf{let}\,(x_1 \rhd x_2) = t_2\ \mathsf{in}\ t_3)} \quad \textsc{R\_seqCC}$$

$$\frac{}{\mathsf{let}\,(x_1 \rhd x_2) = (\mathsf{let}\,(y_1 \odot y_2) = t_1\ \mathsf{in}\ t_2)\ \mathsf{in}\ t_3 \rightsquigarrow \mathsf{let}\,(y_1 \odot y_2) = t_1\ \mathsf{in}\ (\mathsf{let}\,(x_1 \rhd x_2) = t_2\ \mathsf{in}\ t_3)} \quad \textsc{R\_seqPCC}$$

$$\frac{}{\mathsf{let}\,(x_1 \rhd x_2) = (\mathsf{let}\,(y_1 \sqcup y_2) = t_1\ \mathsf{in}\ t_2)\ \mathsf{in}\ t_3 \rightsquigarrow \mathsf{let}\,(y_1 \sqcup y_2) = t_1\ \mathsf{in}\ (\mathsf{let}\,(x_1 \rhd x_2) = t_2\ \mathsf{in}\ t_3)} \quad \textsc{R\_seqCCC}$$

$$\frac{}{(\mathsf{let}\,(x_1 \odot x_2) = t_1\ \mathsf{in}\ t_2)\ t_3 \rightsquigarrow \mathsf{let}\,(x_1 \odot x_2) = t_1\ \mathsf{in}\ (t_2\ t_3)} \quad \textsc{R\_paraACC}$$

$$\frac{}{(\mathsf{let}\,(x_1 \sqcup x_2) = t_1\ \mathsf{in}\ t_2)\ t_3 \rightsquigarrow \mathsf{let}\,(x_1 \sqcup x_2) = t_1\ \mathsf{in}\ (t_2\ t_3)} \quad \textsc{R\_choiceACC}$$

$$\frac{}{(\mathsf{let}\,(x_1 \rhd x_2) = t_1\ \mathsf{in}\ t_2)\ t_3 \rightsquigarrow \mathsf{let}\,(x_1 \rhd x_2) = t_1\ \mathsf{in}\ (t_2\ t_3)} \quad \textsc{R\_seqACC}$$

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Definition rules:        77 good    0 bad
Definition rule clauses: 114 good    0 bad
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