Short Paper: Proposing a New Foundation of Attack Trees in Monoidal Categories

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Abstract

This short paper introduces a new research direction studying at the intersection of threat analysis using attack trees and interactive theorem proving using linear logic. Currently, the project has developed a new semantics of attack trees in dialectica spaces, a well-known model of intuitionistic linear logic, which offers a new branching operator to attack trees. Then by exploiting the Curry-Howard-Lambek correspondence the project seeks to develop a new domain-specific linear functional programming language called Lina – for Linear Threat Analysis – for specifying and reasoning about attack trees.

1. Introduction

What do propositional logic, multisets, directed acyclic graphs, source sink graphs (or parallel-series pomsets), Petri nets, and Markov processes all have in common? They are all mathematical models of attack trees – see the references in (Kordy et al. 2014; Jhawar et al. 2015) – but also, they can all be modeled in some form of a symmetric monoidal category[1] (Tzouvaras 1998; Brown et al. 1991; Fiore and Campos 2013; Albasini et al. 2010) – for the definition of a symmetric monoidal category see Appendix A. Taking things a little bit further, monoidal categories have a tight correspondence with linear logic through the beautiful Curry-Howard-Lambek correspondence (Barr 1991). This correspondence states that objects of a monoidal category correspond to the formulas of linear logic and the morphisms correspond to proofs of valid sequents of the logic. I propose that attack trees – in many different flavors – be modeled as objects in monoidal categories, and hence, as formulas of linear logic.

Suppose we are the computer security team of a small hospital, and we need to assess the potential for an intruder to gain root access of the all important medical records Unix server. So we build an attack tree to assess all of the potential ways one could gain root access to the server. Such an attack tree might look something like the following:

```
⊔ "Obtain Root Privileges" 10
  (⊔ "Access System Console" 30
    (⊔ "Enter Computer Center" 30
      (leaf "Break In to Computer Center" 80)
      (leaf "Unattended Guest" 30))
    (leaf "Corrupt Operator" 50))
  (⊔ "Obtain Root Password" 10
    (⊔ "Naive Approach" 40
      (▷ "Guess Password" 50
        (leaf "Obtain Password File" 30)
        (leaf "Encounter Guessable Password" 20))
      (leaf "Look Over Sys. Admin. Shoulder" 40))
    (⊔ "Sophisticated Approach" 10
      (leaf "Trojan Horse Sys. Admin. Account" 10)
      (leaf "Corrupt Sys. Admin." 80))
```

In the interest of saving space we present our attack trees in script form. Each node is labeled with a goal of the attacker and a cost for executing such an attack. In the example above there are two types of branching nodes, those labeled with $\sqcup$ which stands for a choice between attacks, and those labeled with $\triangleright$ which stands for sequential composition of attacks. The above attack tree shows that the only reasonable attack for an intruder is to try and send a trojan horse to the system administrator with a cost of 10.

The example above implies that attack trees are a modeling tool, originally proposed by Bruce Schneier (Schneier 1999), which are used to assess the threat potential of a security critical system. Attack trees have since been used to analyze the threat potential of many types of security critical systems, for example, cybersecurity of power grids (Ten et al. 2007), wireless networks (Reinhardt et al. 2012), and many others. Attack trees consists of several goals, usually specified in English prose, for example, "compromise safe" or "obtain administrative privileges", where the root is the ultimate goal of the attack and each node coming off of the root is a refinement of the main goal into a subgoal. Then each subgoal can be further refined. The leaves of an attack tree make up the set of base attacks.

Attack trees for real-world security scenarios can grow to be quite complex. The attack tree presented in (Ten et al. 2007) to access the security of power grids has twenty-nine nodes with sixty counter measures attached to the nodes throughout the tree. The details of the tree spans several pages of appendix. The attack tree developed for the border gateway protocol has over a hundred nodes (Convery et al. 2003), and the details of the tree spans ten pages. Manipulating such large trees without a formal semantics can be dangerous.

One of the leading questions the field is seeking to answer is "what is an attack tree?" That is, what is a mathematical foundation of attack trees? There have been numerous attempts at answering this question. For example, attack trees have been based on propo-

[1] I provide a proof that the category of source sink graphs is monoidal in Appendix B.

PLAS '16 October 24, 2016, Vienna, Austria

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sitional logic and De Morgan Algebras (Kordy et al. 2014; Kordy et al 2012; Piètre-Cambacédès and Bouissou 2010), multisets (Mauw and Oostdijk 2006), Petri nets (McDermott 2001), tree automata (Camtepe and Yener 2007), and series parallel graphs (Jhawar et al. 2015). **There is currently no known semantics of attack trees based in category theory**.

By far the most intuitive foundation of attack trees is propositional logic or De Morgan algebras, however, neither of these properly distinguish between attack trees with repeated subgoals. If we consider each subgoal as a **resource** then the attack tree using a particular resource twice is different than an attack tree where it is used only once. The multiset semantics of attack trees was developed precisely to provide a resource conscious foundation (Mauw and Oostdijk 2006). The same can be said for the Petri nets semantics (McDermott 2001). A second benefit of a semantics based in multisets, Petri nets, and even tree automata is that operators on goals in attack trees are associated with concurrency operators from process algebra. That is, the goals of an attack tree should be thought of as being run concurrently. Furthermore, when moving to these alternate foundations **the intuitiveness and elegance of the propositional logic semantics is lost**. Lastly, existing work has focused on specifically what an attack tree is, and has not sought to understand what the theory of attack trees is.

By modeling attack trees in monoidal categories we obtain a sound mathematical model, a resource conscious logic for reasoning about attack trees, and the means of constructing a functional programming language for defining attack trees (as types), and constructing semantically valid transformations (as programs) of attack trees.

Linear logic was first proposed by Girard (Girard 1987) and was quickly realized to be a theory of resources. In linear logic, every hypothesis must be used exactly once. Thus, formulas like $A \otimes A$ and A are not logically equivalent – here $\otimes$ is linear conjunction. This resource perspective of linear logic has been very fruitful in computer science and lead to linear logic being a logical foundation of processes and concurrency where formulas may be considered as processes. Treating attack trees as concurrent processes is not new; they have been modeled by event-based models of concurrency like Petri nets and partially-ordered multisets (pomsets) (Jhawar et al. 2015; Mauw and Oostdijk 2006). In fact, pomsets is a model in which events (the resources) can be executed exactly once, and thus, has a relationship with linear logic (Retoré 1997). However, connecting linear logic as a theory of attack trees is novel, and strengthens this perspective.

In this short paper I introduce a newly funded research project[2] investigating founding attack trees in monoidal categories, and through the Curry-Howard-Lambek correspondence deriving a new domain-specific functional programming language called Lina for Linear Threat Analysis. Note that this paper introduces an ongoing research project, and thus, we do not currently have the complete story, but we feel that the community will find this project of interest, and the project would benefit from the feedback of the community. I begin by defining an extension – inspired by our semantics – of the attack trees given in (Jhawar et al. 2015) in Section 2. Then I introduce a new semantics of attack trees in dialectica spaces, which depends on a novel result on dialectica spaces, in Section 3. The final section, Section 4, discusses Lina and some of the current problems the project seeks to answer.

[2] This material is based upon work supported by the National Science Foundation CRII CISE Research Initiation grant, "CRII:SHF: A New Foundation for Attack Trees Based on Monoidal Categories", under Grant No. 1565557.

2. Attack Trees

(Jhawar et al. 2015) introduce attack trees with sequential composition, but here I consider an extension of their definition. One of the projects ultimate goals is to extend attack trees with even more operators driven by our choice of semantics. The syntax for attack trees is defined in the following definition.

Definition 1. *The following defines the syntax of **Attack Trees** given a set of base attacks $b \in \mathsf{B}$:*

$$t ::= b \mid t_1 \odot t_2 \mid t_1 \sqcup t_2 \mid t_1 \rhd t_2 \mid t_1 \otimes t_2$$

I denote unsynchronized non-communicating parallel composition of attacks by $t_1 \odot t_2$, choice between attacks by $t_1 \sqcup t_2$, sequential composition of attacks by $t_1 \rhd t_2$, and a new operator called unsynchronized interacting parallel composition denoted $t_1 \otimes t_2$.

The following rules define the attack tree equivalence relation:

$$\frac{}{(t_1 \,\mathsf{op}\, t_2)\,\mathsf{op}\, t_3 = t_1 \,\mathsf{op}(t_2 \,\mathsf{op}\, t_3)}\ \text{ASSOC}$$

$$\frac{}{t_1 \,\mathsf{op_S}\, t_2 = t_2 \,\mathsf{op_S}\, t_1}\ \text{SYM}$$

$$\frac{}{(t_1 \sqcup t_2) \odot t_3 = (t_1 \odot t_3) \sqcup (t_2 \odot t_3)}\ \text{DIST}_1$$

$$\frac{}{(t_1 \sqcup t_2) \rhd t_3 = (t_1 \rhd t_3) \sqcup (t_2 \rhd t_3)}\ \text{DIST}_2$$

where $\mathsf{op} \in \{\odot, \otimes, \rhd, \sqcup\}$ *and* $\mathsf{op_S} \in \{\odot, \otimes, \sqcup\}$.

The syntax given in the previous definition differs from the syntax used by Jhawar et al. (Jhawar et al. 2015). First, I use infix binary operations, while they use prefix n-ary operations. However, it does not sacrifice any expressivity, because each operation is associative, and parallel composition, choice, and interacting parallel composition are symmetric. Thus, Jhawar et al.'s definition of attack trees can be embedded into the ones defined here.

The second major difference is that the typical parallel composition operator found in attack trees is modeled here by unsynchronized non-communicating parallel composition which happens to be a symmetric tensor product, and not a disjunction. This is contrary to the literature, for example, the parallel operation of Jhawar et al. defined on source sink graphs (Jhawar et al. 2015) can be proven to be a coproduct – see Appendix B – and coproducts categorically model disjunctions. Furthermore, parallel composition is modeled by multiset union in the multiset semantics, but we can model this as a coproduct. However, in the semantics given in the next section if we took parallel composition to be a coproduct, then the required isomorphisms necessary to model attack trees would not exist.

The third difference is that I denote the choice between executing attack t_1 or attack t_2, but not both, by $t_1 \sqcup t_2$ instead of using a symbol that implies that it is a disjunction. This fits very nicely with the semantics of Jhawar et al., where they collect the attacks that can be executed into a set. The semantics I give in the next section models choice directly.

The fourth, and final, difference is that I extend the syntax with a new operator called unsynchronized communicating parallel composition. The attack $t_1 \otimes t_2$ states that t_1 interacts with the attack t_2 in the sense that processes interact. Modeling interacting attacks allows for the more refined modeling of security critical systems, for example, it can be used to bring social engineering into the analysis where someone communicates malicious information or commands to a unsuspecting party. As a second example, interacting parallel composition could be used to model interacting bot nets.

Finally, the equivalence relation is essentially the equivalence given in Jhaware et al. (Jhawar et al. 2015) – Theorem 1.

3. Semantics of Attack Trees in Dialectica Spaces

I now introduce a new semantics of attack trees that connects their study with a new perspective of attack trees that could highly impact future research: intuitionistic linear logic, but it also strengthens their connection to process calculi. This section has been formalized in the proof assistant Agda[3]. The semantics is based on the notion of a dialectica space:

Definition 2. *A **dialectica space** is a triple (A, Q, δ) where A and Q are sets and $\delta : A \times Q \to \mathsf{3}$ is a multi-relation where $\mathsf{3} = \{0, \bot, 1\}$ and $\bot$ represents undefined.*

Dialectica spaces can be seen as the intuitionistic cousin (de Paiva 2006) of Chu spaces (Pratt 1999). The latter have be used extensively to study process algebra and as a model of classical linear logic, while dialectica spaces and their morphisms form a categorical model of intuitionistic linear logic called $\mathsf{Dial}_3(\mathsf{Sets})$ (originally due to (de Paiva 1987)); I do not introduce dialectica space morphisms here, but the curious reader can find the definition in the formal development. I will use the intuitions often used when explaining Chu spaces as processes to explain dialectica spaces as processes, but it should be known that these intuitions are due to Pratt and Gupta (Gupta 1994).

Intuitively, a dialectica space, (A, Q, δ), can be thought of as a process where A is the set of actions the process will execute, Q is the set of states the process can enter, and for $a \in A$ and $q \in Q$, $\delta(a, q)$ indicates whether action a can be executed in state q.

The interpretation of attack trees into dialectica spaces is as follows:

Parallel Composition. Suppose $\mathcal{A} = (A, Q, \alpha)$ and $\mathcal{B} = (B, R, \beta)$ are two dialectica spaces. Then we can construct – due to de Paiva (de Paiva 2014) – the dialectica space $\mathcal{A} \odot \mathcal{B} = (A \times B, Q \times R, \alpha \odot \beta)$ where $(\alpha \odot \beta)((a, b), (q, r)) = \alpha(a, q) \otimes_3 \beta(b, r)$ and $\otimes_3$ is the symmetric tensor product definable on $\mathsf{3}$[4]. Thus, from a process perspective we can see that $\mathcal{A} \odot \mathcal{B}$ executes actions of $\mathcal{A}$ and actions of $\mathcal{B}$ in parallel. Parallel composition is associative and symmetric.

Choice. Suppose $\mathcal{A} = (A, Q, \alpha)$ and $\mathcal{B} = (B, R, \beta)$ are two dialectica spaces. Then we can construct the dialectica space $\mathcal{A} \sqcup \mathcal{B} = (A + B, Q + R, \alpha \sqcup \beta)$ where $(\alpha \sqcup \beta)(i, j) = \alpha(i, j)$ if $i \in A$ and $j \in Q$, $(\alpha + \beta)(i, j) = \beta(i, j)$ if $i \in B$ and $j \in R$, otherwise $(\alpha + \beta)(i, j) = 0$. Thus, from a process perspective we can see that $\mathcal{A} \sqcup \mathcal{B}$ executes either an action of $\mathcal{A}$ or an action of $\mathcal{B}$, but not both. Choice is symmetric and associative, but it is not a coproduct, because it is not possible to define the corresponding injections.

Sequential Composition. Suppose $\mathcal{A} = (A, Q, \alpha)$ and $\mathcal{B} = (B, R, \beta)$ be two dialectica spaces. Then we can construct – due to de Paiva (de Paiva 2014) – the dialectica space $\mathcal{A} \rhd \mathcal{B} = (A \times B, Q \times R, \alpha \rhd \beta)$ where $(\alpha \rhd \beta)((a, b), (q, r)) = \alpha(a, q) \mathbin{\mathsf{land}} \beta(i, r)$, and land is lazy conjunction defined for $\mathsf{3}$[5]. This is a non-symmetric conjunctive operator, and thus, sequential composition is non-symmetric. This implies that from a process perspective $\mathcal{A} \rhd \mathcal{B}$ will first execute the actions of $\mathcal{A}$ and then execute actions of $\mathcal{B}$ in that order. Sequential composition is associative.

Interacting Parallel Composition. Suppose $\mathcal{A} = (A, Q, \alpha)$ and $\mathcal{B} = (B, R, \beta)$ are two dialectica spaces. Then we can construct the dialectica space $\mathcal{A} \otimes \mathcal{B} = (A \times B, (B \to Q) \times (A \to R), \alpha \otimes \beta)$ where $B \to Q$ and $A \to R$ denote function spaces, and $(\alpha \otimes \beta)((a, b), (f, g)) = \alpha(a, f(b)) \wedge \beta(b, g(a))$. From a process perspective the actions of $\mathcal{A} \otimes \mathcal{B}$ are actions from $\mathcal{A}$ and actions of $\mathcal{B}$, but the states are pairs of maps $f : B \to Q$ and $g : A \to R$ from actions to states. This is the point of interaction between the processes. This operator is symmetric and associative.

At this point it is straightforward to define an interpretation $[\![t]\!]$ of attack trees into $\mathsf{Dial}_3(\mathsf{Sets})$. Soundness with respect to this model would correspond to the following theorem.

Theorem 3 (Soundness)**.** *If $t_1 \rightsquigarrow t_2$, then $[\![t_1]\!]$ is isomorphic to $[\![t_2]\!]$ in $\mathsf{Dial}_3(\mathsf{Sets})$.*

Those familiar with Chu spaces and their application to process algebra may be wondering how treating dialectica spaces as processes differs. The starkest difference is that in this model process simulation is modeled by morphisms of the model, but this is not possible in Chu spaces. In fact, to obtain the expected properties of processes a separate notion of bi-simulation had to be developed for Chu spaces (Gupta 1994). However, I took great care to insure that the morphisms of our semantics capture the desired properties of process simulation, and hence, attack trees.

The ability to treat morphisms as process simulation was not easy to achieve. The definition of choice in the semantics presented here actually is the definition given for Chu spaces (Gupta 1994), but Brown et al. use the coproduct defined for dialectica spaces to model choice in Petri nets. However, taking the coproduct for choice here does not lead to the isomorphisms $(\mathcal{A} \sqcup \mathcal{B}) \rhd \mathcal{C} \cong (\mathcal{A} \rhd \mathcal{C}) \sqcup (\mathcal{B} \rhd \mathcal{C})$ and $(\mathcal{A} \sqcup \mathcal{B}) \odot \mathcal{C} \cong (\mathcal{A} \odot \mathcal{C}) \sqcup (\mathcal{B} \odot \mathcal{C})$, thus, we will not be able to soundly model attack trees. I have found that if choice is modeled using the definition from Chu spaces (Gupta 1994) then we obtain these isomorphisms which is a novel result[6].

This semantics can be seen as a generalization of some existing models. Multisets, pomsets, and Petri nets can all be modeled by dialectica spaces (Brown et al. 1991; Gupta 1994). However, there is a direct connection between dialectica spaces and linear logic which may lead to a logical theory of attack trees.

4. Lina: A Domain Specific PL for Threat Analysis

The second major part of this project is the development of a staticly-typed domain-specific linear functional programming language for specifying and reasoning about attack trees called Lina for Linear Threat Analysis. Lina will consist of a core language and a surface language.

The project views attack trees as consisting of two layers: a logic layer and a quantitative layer. The former is described by the definition of attack trees in the previous section, but the latter is the layer added atop of the logical layer used when conducting analysis, for example, computing the set of attacks with minimal cost. Thus, there are two types of proofs about attack trees. Proofs about the logical layer will be checked using a linear type system, but proofs about the quantitative layer will be mostly numerical. In this section I largely concentrate on the logical layer which is the current focus of the project.

[3] The complete formalization can be found at `https://github.com/heades/dialectica-spaces/tree/PLAS16` which is part of a general library for working with dialectica spaces in Agda developed with Valeria de Paiva.

[4] See the formal development for the full definition: `https://github.com/heades/dialectica-spaces/blob/PLAS16/concrete-lineales.agda#L328`

[5] See the formal development for the full definition: `https://github.com/heades/dialectica-spaces/blob/PLAS16/concrete-lineales.agda#L648`

[6] For the proofs see the formal development: `https://github.com/heades/dialectica-spaces/blob/PLAS16/concurrency.agda#L70` and `https://github.com/heades/dialectica-spaces/blob/PLAS16/concurrency.agda#L150`

Lina's core will consist of a language for defining attack trees, and this language will consist of the two layers, but one benefit of the layered view of attack trees is that the logical layer can be projected out, and hence, reasoning about the logical layer using a linear type system may completely ignore the quantitative layer. The following two sections describe both of these concepts.

4.1 Lina's Core: Defining Attack Trees

The attack tree for assessing the risk of becoming root on a Unix machine from the introduction is actually written in Lina's definition language. However, there is one simplification that was made. The data on branching nodes actually are binary functions, but when those functions are constant we omit their arguments.

We can see from the example that Lina treats the nodes of the attack tree as combinators, either $\mathsf{leaf}\ l\ q$ or $c\ l\ q$, where c is a branching node symbol, l is a label, for example, a string, and q is a quantitative expression. The types of l and q depend on the type of the tree itself, for example, the type of the tree above is $\mathsf{AttackTree\ String\ Double}$, and thus, labels are strings, and leafs are constant doubles, but the quantitative data on branching nodes has type $\mathsf{Double} \to \mathsf{Double} \to \mathsf{Double}$. Thus, one novelty of Lina is that data at nodes can be higher order, but the data at branching nodes is always a binary function whose first argument is the data from the left tree, and the second argument is the data from the right tree. Hence, making it easier and more precise to compute costs across the tree. In the example above, we actually used the functions $\lambda x.\lambda y.x + y$ and $\lambda x.\lambda y.\mathsf{min}\ x\ y$ on the sequential composition and choice branching nodes respectively. Finally, branching nodes have two additional arguments, t_1 and t_2, which are the left and right trees.

Throughout the remainder of this section I give a brief overview of the preliminary design of Lina's language for defining attack trees. The following defines the syntax (d ranges over any double, and s ranges over any string):

(Quantitative Types)	$Q := \mathsf{Double} \mid Q \to Q$
(Numeric Operators)	$\mathsf{op} := + \mid - \mid * \mid /$
(Quantitative Expressions)	$q := x \mid d \mid \lambda x.q \mid q_1\ q_2 \mid q_1\ \mathsf{op}\ q_2 \mid \mathsf{min}\ q_1\ q_2 \mid \mathsf{rec}\ q_0\ \mathsf{of}\ q_1, q_2$
(Label Types)	$L := \mathsf{String} \mid \mathsf{Double}$
(Labels)	$l := s \mid d$
(Kinds)	$k := \mathsf{AttackTree}\ L\ Q \mid k_1 \to k_2$
(Attack Tree Combinators)	$c := \otimes \mid \odot \mid \triangleright \mid \sqcup$
(Attack Trees)	$t := x \mid \lambda x.t \mid t_1\ t_2 \mid \mathsf{leaf}\ l\ q \mid c\ l\ q$

Typing for this language is straightforward, and to save space we do not give every rule. The typing rules for the quantitative language corresponds to the simply typed λ-calculus with doubles, numeric operators, and a recursor in the spirit of Gödel's system T, and thus, I omit the rules here, but denote the typing judgment by $\Delta \vdash q : Q$, where Δ is a typing context consisting of pairs $x : Q$; for the complete set of rules see Appendix C. Typing labels is trivial, and the typing judgment is denoted by $\vdash l : L$. Finally, the following rules defines the kinding rules for attack trees:

$$\frac{}{\Gamma_0, x : k, \Gamma_1 \vdash x : k}\ \text{K\_VAR} \qquad \frac{\Gamma, x : k_0 \vdash t : k_1}{\Gamma \vdash \lambda x.t : k_0 \to k_1}\ \text{K\_FUN}$$

$$\frac{\Gamma \vdash t_1 : k_0 \quad \Gamma \vdash t_0 : k_0 \to k_1}{\Gamma \vdash t_0\ t_1 : k_1}\ \text{K\_APP}$$

$$\frac{\vdash l : L \quad \cdot \vdash q : Q}{\Gamma \vdash \mathsf{leaf}\ l\ q : \mathsf{AttackTree}\ L\ Q}\ \text{K\_LEAF}$$

$$\frac{\Gamma \vdash t_1 : \mathsf{AttackTree}\ L\ Q \quad \vdash l : L \quad \Gamma \vdash t_2 : \mathsf{AttackTree}\ L\ Q \quad \cdot \vdash q : Q \to Q \to Q}{\Gamma \vdash c\ l\ q\ t_1\ t_2 : \mathsf{AttackTree}\ L\ Q}\ \text{K\_COMB}$$

Functions over attack trees will allow for the definition of attack tree schemas that one could use to build up a library of attack trees. At this point one could speak of evaluating attack trees which would correspond to normalizing the tree to its combinator form, but we could also speak about evaluating the data of the tree, but how this is done is left for future work.

Lina's attack tree definition language technically lives at the type level. We can see that the logical layer of each attack tree corresponds to a linear type. When attack trees get large it makes sense to want to restructure the tree to gain new insights, but existing tools do not support this in such a way that one knows that the tree obtained after restructuring is semantically equivalent to the original tree. Due to the Curry-Howard-Lambek correspondence Lina will come equipped with a linear type system that can be used to register semantically valid transformations of attack trees as programs between linear types, but there is a big hurdle that first must be crossed.

4.2 Lina's Core: A Linear Type System

The current main focus of the project is the design and analysis of Lina's core type system. Types in Lina will correspond to attack trees while programs correspond to semantically valid transformations of attack trees, thus, a question we must answer then is **how do we sufficiently represent the model of attack trees in $\mathsf{Dial}_3(\mathsf{Sets})$ as a linear logic?** The problem is the fact that Lina will require both commutative (parallel composition and choice) and non-commutative monoidal operators (sequencing).

Supporting both commutative and non-communicative operators within the same linear logic has been a long standing question. A starting point might be with Reedy's LLMS which has already been shown to have a categorical model in $\mathsf{Dial}_3(\mathsf{Sets})$ by de Paiva (de Paiva 2014). In fact, the definition of non-interacting parallel composition given here is due to her model. However, we have taken a new path which we also approach categorically, and then syntactically.

To accommodate both a commutative and non-commutative tensor product we isolate exchange in the same way that Girard isolated weakening and contraction using a comonad. In this section I give a brief overview of how this is done. I begin with the notion of a Lambek category.

Definition 4. *A **Lambek category** is a monoidal category $(\mathcal{C}, I, \triangleright, \alpha, \lambda, \rho)$ where $\triangleright : \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}$ is the non-commutative tensor product, and $\alpha_{A,B,C} : (A \triangleright B) \triangleright C \longrightarrow A \triangleright (B \triangleright C)$, $\lambda_A : I \triangleright A \longrightarrow A$, and $\rho_A : A \triangleright I \longrightarrow A$ are all natural transformations subject to several coherence diagrams*[7].

We call the previous category a Lambek category to pay homage to Joachim Lambek and his work on the Lambeck calculus (Lambek 1958) which is a non-commutative substructural logic that predates linear logic. The traditional definition of a Lambek category also requires that the monoidal category be biclosed, but we do not concern ourselves here with closed categories.

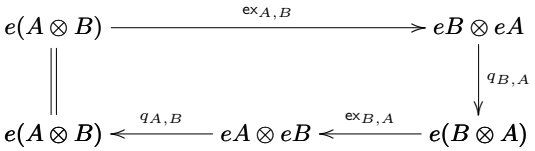
In our model exchange will be considered as an effect, and so we isolate it inside a comonad. This will allow for the definition of a commutative tensor product.

Definition 5. *A **Lambek category with exchange** is a Lambek category $(\mathcal{C}, I, \triangleright, \alpha, \lambda, \rho)$ equipped with a monoidal comonad (e, ε, δ) where $e : \mathcal{C} \longrightarrow \mathcal{C}$ is a monoidal endofunctor*[8]*, and $\varepsilon_A : eA \longrightarrow A$ and $\delta_A : eA \longrightarrow e^2A$ are natural transformations. In addition,*

[7] The coherence diagrams are equivalent to the ones in the definition of a symmetric monoidal category modulo symmetry; see the appendix for the complete definition of a symmetric monoidal category.

[8] For the full definition of a monoidal functor see Definition 9 in Appendix A.

there is a natural transformation $\mathsf{ex}_{A,B} : e(A \rhd B) \longrightarrow eB \rhd eA$. *Each of these morphisms are subject to the several coherence diagrams which I omit due to space. Most importantly, the following diagram must commute:*



The previous diagram can be seen as a form of invertibility for $\mathsf{ex}_{A,B}$.

The previous definition is largely based on how weakening and contraction are modeled by the of-course exponential, and how exchange operates in the coKleisli category of (e, ε, δ). The coKleisli category contains as objects all of the objects of $\mathcal{C}$, but has as morphisms all the morphisms of $\mathcal{C}$ whose source is of the form eA for some A. The coKleisli category is best viewed as the world inside a comonad. That is, it contains all of the structure of the ambient category, but also the additional effects the comonad provides. Thus, the coKleisli category of the exchange comonad should be a symmetric monoidal category, and indeed it is.

Lemma 6. *Suppose $\mathcal{C}$ is a Lambek category with exchange. Then the coKleisli category of the exchange comonad is symmetric monoidal.*

It is now should be more straightforward to construct a term assignment from this model. The following rules define a preliminary definition of a natural deduction term assignment:

$$\frac{}{x : T \vdash x : T}\ \text{VAR} \qquad \frac{}{\cdot \vdash \mathsf{unit} : \mathsf{I}}\ \text{UNITI}$$

$$\frac{\Gamma_2 \vdash s_1 : \mathsf{I} \quad \Gamma_1 \vdash s_2 : T}{\Gamma_1, \Gamma_2 \vdash \mathsf{let\ unit} = s_1 \ \mathsf{in}\ s_2 : T}\ \text{UNITE}$$

$$\frac{\Gamma_1 \vdash s_1 : T_1 \quad \Gamma_2 \vdash s_2 : T_2}{\Gamma_1, \Gamma_2 \vdash s_1 \rhd s_2 : T_1 \rhd T_2}\ \text{TENI}$$

$$\frac{\Gamma_2 \vdash s_1 : T_1 \rhd T_2 \quad \Gamma_1, x : T_1, y : T_2, \Gamma_3 \vdash s_2 : T_3}{\Gamma_1, \Gamma_2, \Gamma_3 \vdash \mathsf{let}\ x \rhd y = s_1 \ \mathsf{in}\ s_2 : T_3}\ \text{TENE}$$

$$\frac{\Gamma_2 \vdash s_1 : \mathsf{e}\ T_1 \rhd \mathsf{e}\ T_2 \quad \Gamma_1, w : \mathsf{e}\ T_2, x : \mathsf{e}\ T_1, \Gamma_2 \vdash s : T}{\Gamma_1, \Gamma_2, \Gamma_3 \vdash \mathsf{ex}\ s_1\ \mathsf{with}\ w, x\ \mathsf{in}\ s : T}\ \text{EX}$$

$$\frac{\Gamma \vdash s : \mathsf{e}\ T}{\Gamma \vdash \mathbf{derelict}\, s : T}\ \text{DERELICTION}$$

$$\frac{\overrightarrow{\Gamma_i \vdash s_i : \mathsf{e}\ T_i} \quad x_1 : \mathsf{e}\ T_1, \ldots, x_i : \mathsf{e}\ T_i \vdash s : T}{\Gamma \vdash \mathsf{promote}\ \overrightarrow{s}\ \mathsf{for}\ \overrightarrow{x_i}\ \mathsf{in}\ s : T}\ \text{PROMOTION}$$

The previous rules are based on the natural deduction formalization of intuitionistic linear logic due to (Benton et al. 1992).

We can interpret both sequential and interacting parallel composition as types. That is, we can interpret $t_1 \rhd t_2$ as $T_1 \rhd T_2$, and $t_1 \otimes t_2$ as $\mathsf{e}\ T_1 \rhd \mathsf{e}\ T_2$. However, accommodating the other attack tree branching connectives in addition to these in linear logic is left for future work.

4.3 Lina's Surface Language

So far I have only mentioned Lina's core language, because this has been the focus of the first year of the project. The surface language will have two main objectives: simplicity and automation. It will also be paired with an IDE specifically geared toward threat analysis. I can only give a brief account of the projects ideas for the surface language, but experiments will need to be carried out before its final design will be settled.

Simplicity. At the IDE level the project plans to capitalize on the graphical languages category theory provides (Selinger 2009) to allow for the registration of semantically valid transformations of attack trees. Thus, preventing a security specialist from having to learn the Lina programming language. The surface language will rely on local type inference to make programming easier, but this may require a new local type inference algorithm for intuitionistic linear logic.

Automation. Reasoning at the quantitative level could be highly benefited from automated theorem proving which we plan to interface with Lina. Furthermore, there has been recent work (Huistra 2016; Sheyner et al. 2002; Vigo et al. 2014; Wolters 2016) on exploring automatically constructing attack trees from a specification which the project plans to investigate interfacing with Lina, because once you have the tree one may want to analyze it themselves. Furthermore, these automated approaches use sophisticated graph rewriting algorithms, it might be possible to reframe this work in terms of linear logic and potentially find new ways to automatically construct attack trees using automated theorem proving.

5. Conclusion and Future Work

The project described here seeks to develop a new semantics to attack trees that can be leveraged to design a new domain-specific programming language for reasoning and analyzing attack trees to assess the threat of security critical systems.

I showed that attack trees (Section 2) can be given a semantics in dialectica spaces (Section 3), thus, relating the study of attack trees to the study of intuitionistic linear linear logic. Then I showed that this model can be abstracted into an alternate categorical model in Lambek categories that is easier to translate into a type system by exploiting the Curry-Howard-Lambek correspondence (Section 4.2). Finally, I introduced the preliminary design for a domain-specific functional programming language called Lina for Linear Threat Analysis (Section 4) to be used to develop a new tool to conduct threat analysis using attack trees. This tool will include the ability to design and formally reason about attack trees using interactive and automated theorem proving as well as graphical reasoning tools.

There is still a lot of work to be done. A forth coming paper will fully explore the alternate categorical model in Lambek categories both categorically and syntactically. In addition, Lina's core design needs to be further developed and case studies need to be conducted to assess its effectiveness, and to understand where automation will be most useful. Finally, Lina's surface language and accompanying IDE still needs to be designed and developed.

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Appendix

A. Symmetric Monoidal Categories

This appendix provides the definitions of both categories in general, and, in particular, symmetric monoidal closed categories. We begin with the definition of a category:

Definition 7. *A **category**, $\mathcal{C}$, consists of the following data:*

- *A set of objects $\mathcal{C}_0$, each denoted by A, B, C, etc.*
- *A set of morphisms $\mathcal{C}_1$, each denoted by f, g, h, etc.*
- *Two functions src, the source of a morphism, and tar, the target of a morphism, from morphisms to objects. If $\mathsf{src}(f) = A$ and $\mathsf{tar}(f) = B$, then we write $f : A \to B$.*
- *Given two morphisms $f : A \to B$ and $g : B \to C$, then the morphism $f; g : A \to C$, called the composition of f and g, must exist.*
- *For every object $A \in \mathcal{C}_0$, the there must exist a morphism $\mathsf{id}_A : A \to A$ called the identity morphism on A.*
- *The following axioms must hold:*
  - *(Identities) For any $f : A \to B$, $f; \mathsf{id}_B = f = \mathsf{id}_A; f$.*
  - *(Associativity) For any $f : A \to B$, $g : B \to C$, and $h : C \to D$, $(f; g); h = f; (g; h)$.*

Categories are by definition very abstract, and it is due to this that makes them so applicable. The usual example of a category is the category whose objects are all sets, and whose morphisms are set-theoretic functions. Clearly, composition and identities exist, and satisfy the axioms of a category. A second example is pre-ordered sets, $(A, \leq)$, where the objects are elements of A and a morphism $f : a \to b$ for elements $a, b \in A$ exists iff $a \leq b$. Reflexivity yields identities, and transitivity yields composition.

Symmetric monoidal categories pair categories with a commutative monoid like structure called the tensor product.

Definition 8. *A **symmetric monoidal category (SMC)** is a category, $\mathcal{M}$, with the following data:*

- *An object I of $\mathcal{M}$,*
- *A bi-functor $\otimes : \mathcal{M} \times \mathcal{M} \to \mathcal{M}$,*

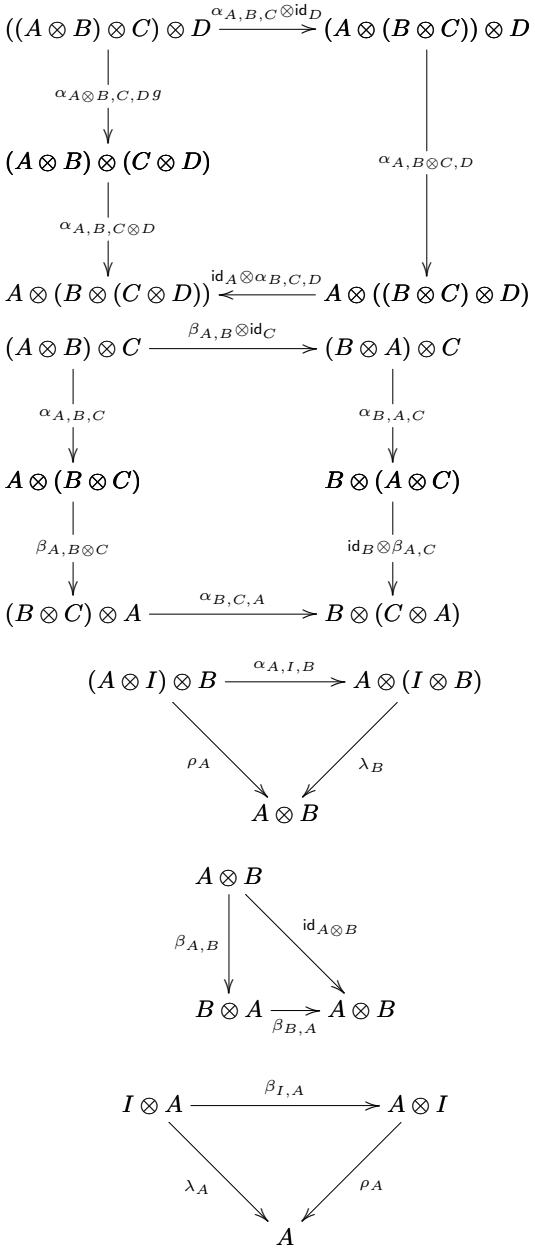
- *The following natural isomorphisms:*

$$\lambda_A : I \otimes A \to A$$
$$\rho_A : A \otimes I \to A$$
$$\alpha_{A,B,C} : (A \otimes B) \otimes C \to A \otimes (B \otimes C)$$

- *A symmetry natural transformation:*

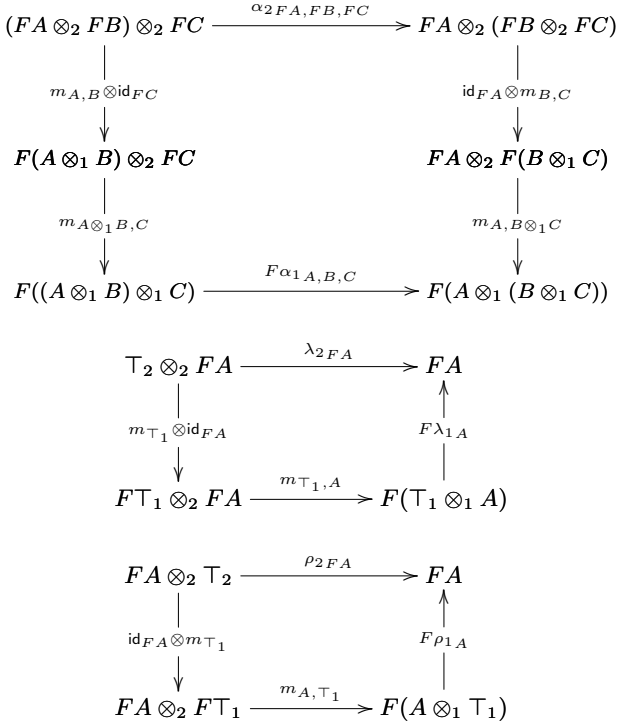
$$\beta_{A,B} : A \otimes B \to B \otimes A$$

- *Subject to the following coherence diagrams:*



Monoidal categories posses additional structure, and hence, ordinary functors are not enough, thus, the notion must also be extended.

Definition 9. *Suppose we are given two monoidal categories* $(\mathcal{M}_1, \top_1, \otimes_1, \alpha_1, \lambda_1, \rho_1)$ *and* $(\mathcal{M}_2, \top_2, \otimes_2, \alpha_2, \lambda_2, \rho_2)$*. Then a* ***monoidal functor*** *is a functor* $F : \mathcal{M}_1 \longrightarrow \mathcal{M}_2$*, a map* $m_{\top_1} : \top_2 \longrightarrow F\top_1$ *and a natural transformation* $m_{A,B} : FA \otimes_2 FB \longrightarrow F(A \otimes_1 B)$ *subject to the following coherence conditions:*



B. Source Sink Graphs are Symmetric Monoidal

In this appendix I show that the category of source-sink graphs defined by Jhawar et al. (Jhawar et al. 2015) is symmetric monoidal. First, recall the definition of source-sink graphs and their homomorphisms.

Definition 10. *A* ***source-sink graph*** *over* B *is a tuple* $G = (V, E, s, z)$*, where* V *is the set of vertices,* E *is a multiset of labeled edges with support* $E^* \subseteq V \times \mathsf{B} \times V$*,* $s \in V$ *is the unique start,* $z \in V$ *is the unique sink, and* $s \neq z$*.*

Suppose $G = (V, E, s, z)$ *and* $G' = (V', E', s', z')$*. Then a* ***morphism between source-sink graphs****,* $f : G \to G'$*, is a graph homomorphism such that* $f(s) = s'$ *and* $f(z) = z'$*.*

Suppose $G = (V, E, s, z)$ and $G' = (V', E', s', z')$ are two source-sink graphs. Then given the above definition it is possible to define sequential and non-communicating parallel composition of source-sink graphs where I denote disjoint union of sets by $+$ (p 7. (Jhawar et al. 2015)):

Sequential Composition :
$G \triangleright G' = ((V \setminus \{z\}) + V', E^{[s'/z]} + E', s, z')$

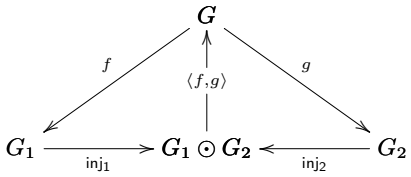
Parallel Composition :
$G \odot G' = ((V \setminus \{s, z\}) + V', E^{[s'/s, z'/z]} + E', s', z')$

It is easy to see that we can define a category of source-sink graphs and their homomorphisms. Furthermore, it is a symmetric monoidal category were parallel composition is the symmetric tensor product. It is well-known that any category with co-products is symmetric monoidal where the co-product is the tensor product.

I show here that parallel composition defines a co-product. This requires the definition of the following morphisms:

$$\mathsf{inj}_1 : G_1 \to G_1 \odot G_2$$
$$\mathsf{inj}_2 : G_2 \to G_1 \odot G_2$$
$$\langle f, g \rangle : G_1 \odot G_2 \to G$$

In the above $f : G_1 \to G$ and $g : G_2 \to G$ are two source-sink graph homomorphisms. Furthermore, the following diagram must commute:



Suppose $G_1 = (V_1, E_1, s_1, z_1)$, $G_2 = (V_2, E_2, s_2, z_2)$, and $G = (V, E, s, z)$ are source-sink graphs, and $f : G_1 \to G$ and $g : G_2 \to G$ are source-sink graph morphisms – note that $f(s_1) = g(s_2) = s$ and $f(z_1) = g(z_2) = z$ by definition. Then we define the required co-product morphisms as follows:

$$
\begin{array}{l}
\mathsf{inj}_1 : V_1 \to (V_1 \setminus \{s_1, z_1\}) + V_2 \\
\mathsf{inj}_1(s_1) = s_2 \\
\mathsf{inj}_1(z_1) = z_2 \\
\mathsf{inj}_1(v) = v, \text{otherwise}
\end{array}
$$

$$
\begin{array}{l}
\mathsf{inj}_2 : V_2 \to (V_1 \setminus \{s_1, z_1\}) + V_2 \\
\mathsf{inj}_2(v) = v
\end{array}
$$

$$
\begin{array}{l}
\langle f, g\rangle : (V_1 \setminus \{s_1, z_1\}) + V_2 \to V \\
\langle f, g\rangle(v) = f(v), \text{ where } v \in V_1 \\
\langle f, g\rangle(v) = g(v), \text{ where } v \in V_2
\end{array}
$$

It is easy to see that these define graph homomorphisms. All that is left to show is that the diagram from above commutes:

$$
\begin{array}{rcl}
(\mathsf{inj}_1; \langle f, g\rangle)(s_1) & = & \langle f, g\rangle(\mathsf{inj}_1(s_1)) \\
& = & g(s_2) \\
& = & s \\
& = & f(s_1)
\end{array}
$$

$$
\begin{array}{rcl}
(\mathsf{inj}_1; \langle f, g\rangle)(z_1) & = & \langle f, g\rangle(\mathsf{inj}_1(z_1)) \\
& = & g(z_2) \\
& = & z \\
& = & f(z_1)
\end{array}
$$

Now for any $v \in V_1$ we have the following:

$$
\begin{array}{rcl}
(\mathsf{inj}_1; \langle f, g\rangle)(v) & = & \langle f, g\rangle(\mathsf{inj}_1(v)) \\
& = & f(v)
\end{array}
$$

The equation for inj_2 is trivial, because inj_2 is the identity.

C. Typing for Lina's Quantitative Expressions

$$\frac{}{\Delta_0, x : Q, \Delta_1 \vdash x : Q} \quad \text{Q\_VAR}$$

$$\frac{}{\Delta \vdash d : \mathsf{Double}} \quad \text{Q\_DOUBLE}$$

$$\frac{\Delta, x : Q_0 \vdash q : Q_1}{\Delta \vdash \lambda x.q : Q_0 \to Q_1} \quad \text{Q\_FUN}$$

$$\frac{\Delta \vdash q_1 : Q_0 \quad \Delta \vdash q_0 : Q_0 \to Q_1}{\Delta \vdash q_0\ q_1 : Q_1} \quad \text{Q\_APP}$$

$$\frac{\Delta \vdash q_1 : \mathsf{Double} \quad \Delta \vdash q_2 : \mathsf{Double}}{\Delta \vdash q_1\ \mathsf{op}\ q_2 : \mathsf{Double}} \quad \text{Q\_NOP}$$

$$\frac{\Delta \vdash q_0 : \mathsf{Double} \quad \Delta \vdash q_1 : Q \quad \Delta \vdash q_2 : Q \to Q}{\Delta \vdash \mathsf{rec}\ q_0\ \mathsf{of}\ q_1, q_2 : Q} \quad \text{Q\_REC}$$