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Abstract. TODO

1 Introduction

2 A Ternary Semantics for SAND Attack Trees

References

1. Ravi Jhawar, Barbara Kordy, Sjouke Mauw, Saša Radomirović, and Rolando Trujillo-Rasua. Attack trees with sequential conjunction. In Hannes Federrath and Dieter Gollmann, editors, *ICT Systems Security and Privacy Protection*, volume 455 of *IFIP Advances in Information and Communication Technology*, pages 339–353. Springer International Publishing, 2015.

Appendix

.1 SSG Semantics

In this appendix I show that the category of source-sink graphs defined by Jhawar et al. [1] is symmetric monoidal. First, recall the definition of source-sink graphs and their homomorphisms.

Definition 1. A source-sink graph over B is a tuple G = (V, E, s, z), where V is the set of vertices, E is a multiset of labeled edges with support $E^* \subseteq V \times B \times V$, $s \in V$ is the unique start, $z \in V$ is the unique sink, and $s \neq z$.

Suppose G = (V, E, s, z) and G' = (V', E', s', z'). Then a **morphism between** source-sink graphs, $f : G \to G'$, is a graph homomorphism such that f(s) = s' and f(z) = z'.

Suppose G = (V, E, s, z) and G' = (V', E', s', z') are two source-sink graphs. Then given the above definition it is possible to define sequential and non-communicating parallel composition of source-sink graphs where I denote disjoint union of sets by + (p 7. [1]):

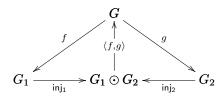
Sequential Composition :
$$G\rhd G'=((V\setminus\{z\})+V',E^{[s'/z]}+E',s,z')$$
 Parallel Composition :
$$G\odot G'=((V\setminus\{s,z\})+V',E^{[s'/s,z'/z]}+E',s',z')$$

It is easy to see that we can define a category of source-sink graphs and their homomorphisms. Furthermore, it is a symmetric monoidal category were parallel composition is the symmetric tensor product. It is well-known that any category with co-products is symmetric monoidal where the co-product is the tensor product.

I show here that parallel composition defines a co-product. This requires the definition of the following morphisms:

$$\begin{aligned} &\inf_1:G_1\to G_1\odot G_2\\ &\inf_2:G_2\to G_1\odot G_2\\ &\langle f,g\rangle:G_1\odot G_2\to G \end{aligned}$$

In the above $f:G_1\to G$ and $g:G_2\to G$ are two source-sink graph homomorphisms. Furthermore, the following diagram must commute:



Suppose $G_1=(V_1,E_1,s_1,z_1),\ G_2=(V_2,E_2,s_2,z_2),$ and G=(V,E,s,z) are source-sink graphs, and $f:G_1\to G$ and $g:G_2\to G$ are source-sink graph morphisms – note that $f(s_1)=g(s_2)=s$ and $f(z_1)=g(z_2)=z$ by definition. Then we define the required co-product morphisms as follows:

$$\begin{split} &\inf_1: V_1 \to (V_1 \setminus \{s_1, z_1\}) + V_2 \\ &\inf_1(s_1) = s_2 \\ &\inf_1(z_1) = z_2 \\ &\inf_1(v) = v, \text{ otherwise} \\ &\inf_2: V_2 \to (V_1 \setminus \{s_1, z_1\}) + V_2 \\ &\inf_2(v) = v \\ &\langle f, g \rangle : (V_1 \setminus \{s_1, z_1\}) + V_2 \to V \\ &\langle f, g \rangle (v) = f(v), \text{ where } v \in V_1 \\ &\langle f, g \rangle (v) = g(v), \text{ where } v \in V_2 \end{split}$$

It is easy to see that these define graph homomorphisms. All that is left to show is that the diagram from above commutes:

$$\begin{split} (\mathsf{inj_1};\langle f,g\rangle)(s_1) &= \langle f,g\rangle(\mathsf{inj_1}(s_1)) \\ &= g(s_2) \\ &= s \\ &= f(s_1) \\ \\ (\mathsf{inj_1};\langle f,g\rangle)(z_1) &= \langle f,g\rangle(\mathsf{inj_1}(z_1)) \\ &= g(z_2) \\ &= z \\ &= f(z_1) \end{split}$$

Now for any $v \in V_1$ we have the following:

$$\begin{aligned} (\mathsf{inj_1}; \langle f, g \rangle)(v) &= \langle f, g \rangle (\mathsf{inj_1}(v)) \\ &= f(v) \end{aligned}$$

The equation for inj_2 is trivial, because inj_2 is the identity.