Deducing options' discount factors and implicit dividend rates

Sebastiano Monti, ID: 2052399
(Dated: April 1, 2022)

This report describes two techniques that have been used in order to extract some important information on options, for a fixed dividend-paying asset. The first one, is to find the discount factor of call and put options at two different strike prices, for fixed maturities using box spread strategy. The latter one is to deduce the implicit dividend rate of call and put options for fixed (at the money) strike prices and maturities using put-call parity.

DESCRIPTION OF THE COMPANY

The company that is taken into consideration for this report is International Business Machines Corporation, better known as IBM.

IBM is an American multinational technology corporation headquartered in Armonk, New York, with operations in over 171 countries. The company began in 1911, founded in Endicott, New York, by trust businessman Charles Ranlett Flint, as the "Computing-Tabulating-Recording Company" (CTR) and later renamed "International Business Machines" in 1924. The current, CEO and chairman of the company is Arvind Krishna.

Among the inventions that made IBM so important are the automated teller machine (ATM), the floppy disk, the hard disk drive, the magnetic stripe card, the relational database, the SQL programming language, the UPC barcode, and dynamic random-access memory (DRAM).

IBM Corporation provides integrated solutions and services worldwide. The company operates through four business segments: Software, Consulting, Infrastructure, and Financing.

The Software segment offers hybrid cloud platform and software solutions, such as Red Hat, an enterprise open-source solutions; software for business automation, AIOps and management, integration, and application servers; data and artificial intelligence solutions; and security software and services for threat, data, and identity. This segment also provides transaction processing software that supports clients' mission-critical and on-premise workloads in banking, airlines, and retail industries

IBM Cloud includes infrastructure as a service (IaaS), software as a service (SaaS) and platform as a service (PaaS) offered through public, private and hybrid cloud delivery models. IBM also provides Cloud Data Encryption Services (ICDES), using cryptographic splitting to secure customer data.

Hardware designed by IBM include IBM's Power microprocessors, which are employed inside many console gaming systems; IBM Secure Blue, an encryption hardware that can be built into microprocessors, and TrueNorth, a neuromorphic CMOS integrated circuit. Other services provided by IBM are, for example, SPSS, a software package used for statistical analysis; IBM Watson, a technology platform that uses natural language processing and machine learning to reveal insights from large amounts of unstructured data and IBM Cognos Analytics, an infrastructure for the New York City Police Department that performs data visualizations of CompStat crime data.

Recently, the company is engaged in research about quantum computing and, in January 2019, introduced its first commercial quantum computer, IBM Q System One.

Research has been a part of the corporation since its founding and now IBM Research constitutes the largest industrial research organization in the world, with 12 labs on 6 continents. In terms of investment, IBM's research and development expenditure totals several billion dollars each year. Current research includes a collaboration with the University of Michigan to see computers act as an academic advisor for undergraduate computer science and engineering students at the university, and a partnership with AT&T, combining their cloud and Internet of Things (IoT) platforms to make them interoperable and to provide developers with easier tools. The company is also involved in research into advanced algorithms and machine learning and their decision-making processes. To that end, the company recently released an analysis tool for how and why algorithms make decisions while scanning for biases in automated decision-making.

IBM has been a leading proponent of the Open Source Initiative, and began supporting Linux in 1998. The company invests billions of dollars in services and software based on Linux through the IBM Linux Technology Center.

The Consulting segment offers business transformation services, including strategy, business process design and operations, data and analytics, and system integration services; technology consulting services; and application and cloud platform services. The Infrastructure segment provides on-premises and cloud-based server and storage solutions for its clients' mission-critical and regulated workloads; and support services and solutions for hybrid cloud infrastructure. The Financing segment offers lease, installment payment, loan financing, and short-term working capital financing services.

The current IBM's market capitalization is estimated to be worth 115,38 billions $, with an enterprise value

of 163,27 billions $. The majority holders of the group are the internal members, who altogether hold the 0,10% of the shares and some institutions (among which, the most relevant are Vanguard Group Inc., Blackrock Inc., and State Street Corporation) which hold the 57,09% of the shares.

METHODS

First of all, since one of the objectives of this report was to deduce the dividend policy of the asset, a company which surely provides dividend payments has been chosen, in particular, IBM. In order to ensure this, and for all the collections of data, site [1] has been taken as reference.

Box spread

The first interesting quantities that have been computed are the *discount rates* of IBM options for different fixed maturities T. In order to compute these, two calls and two puts options, with two different strike prices: K_1 and K_2, have been chosen for each fixed maturity. In particular, the strikes have been carefully chosen in order to satisfy $K_1 << K_2$.

Denoting the price of one share of underlying with S, and since $(S-K_1)^+ - (K_1-S)^+ = S-K_1$ and $-(S-K_2)^+ + (K_2-S)^+ = K_2-S$, the following identity holds:

$$(S-K_1)^+ - (S-K_2)^+ + (K_2-S)^+ - (K_1-S)^+ = K_2-K_1 \quad (1)$$

Applying thus *box spread* strategy, for a fixed T, by:

- Buying a call with strike K_1 and selling a call with strike K_2;
- Buying a put with strike K_2 and selling a put with strike K_1;

it is possible to obtain the price of the box spread, written in Equation 2.

$$p^C_{K_1} - p^C_{K_2} + p^P_{K_2} - p^P_{K_1} = (K_2-K_1)\, D_0(T) \quad (2)$$

Where $p^C_{K_1}$ ($p^C_{K_2}$) and $p^P_{K_1}$ ($p^P_{K_2}$) respectively denote the prices of call and put options for a fixed strike K_1 (K_2) and $D_0(T)$ is the discount factor corresponding to the chosen maturity T.

Thus, the discount factor can be simply deduced by inverting the above formula, as:

$$D_0(T) = \frac{p^C_{K_1} - p^C_{K_2} + p^P_{K_2} - p^P_{K_1}}{K_2-K_1} \quad (3)$$

put-call parity

At this point, it is possible to proceed with the calculation of the *dividend factor* through the *put-call parity* strategy, by choosing different put and call options with strike price K, at the money, for each different maturity.

Assuming the possibility to find quotes for call and put options with same strikes K and maturities T, the put-call parity for a fixed maturity involves the following strategy:

- Buying a call with strike price K;
- Buying a put with strike price K.

The payoff of such a strategy is given by:

$$(S-K)^+ - (K-S)^+ = S-K \quad (4)$$

While its market price is:

$$p^C_0 - p^P_0 = D_0(T)\,(F_0(T)-K) \quad (5)$$

Where p^C_0 and p^P_0 denote respectively the prices of the chosen call and put for fixed strike and maturity, and $F_0(T)$ is the forward contract price.

Thus, the forward contract's price can be computed by inverting Equation 5 as follows.

$$F_0(T) = \frac{p^C_0 - p^P_0}{D_0(T)} + K \quad (5)$$

But, for a dividend-paying asset, $F_0(T)$ is also equal to the following.

$$F_0(T) = S\, e^{RT} - d = \frac{S}{D_0(T)} - d \quad (6)$$

Where R denotes the interest rate per month and d is the implicit dividend.

Denoting with q the dividend rate and assuming dividends to follow an exponential proportionality relation, as shown below:

$$d = S\, e^{qT} \quad (7)$$

It is thus possible to compute $F_0(T)$ from Equation 5 by substituting the corresponding values from the collected data and $D_0(T)$ with the discount value found in previous section. Then, compute d and q by using respectively Equations 6 and 7.

RESULTS

All the following data associated to the dividend-paying asset, chosen for this report, have been collected by consulting site [1].

The price of one IBM share at 2022-03-29, 01:25PM EDT was

$$S = 130,71\$$$

The maturities that have been taken into consideration for the calculations are: $T = \{1\,month, 3\,months, 6\,months, 1\,year\}$, respectively with expiration dates set to: April 29 2022; June 17 2022; October 21 2022 and January 20 2023.

Discount rates calculation

For each value of maturity, the main features of chosen call and put options with strikes K_1 and K_2, are summarized in Tables I, II, III, IV.

$T = 1\,month$		
	Strike price $K_1 = 110,00\,\$$	
	Call	Put
Contract name	IBM220429C00110000	IBM220429P00110000
Last trade date	2022-03-28 9:39AM EDT	2022-03-29 10:36AM EDT
Last price $p_0^{C;P}$	20,60$	0,24$
	Strike price $K_2 = 136,00\,\$$	
	Call	Put
Contract name	IBM220429C00136000	IBM220429P00136000
Last trade date	2022-03-29 9:59AM EDT	2022-03-23 3:00PM EDT
Last price $p_0^{C;P}$	2,20$	8,50$

TABLE I. Main features of 1 month maturity options.

$T = 3\,months$		
	Strike price $K_1 = 60,00\,\$$	
	Call	Put
Contract name	IBM220617C00060000	IBM220617P00060000
Last trade date	2022-03-02 4:14PM EDT	2022-03-18 3:14PM EDT
Last price $p_0^{C;P}$	63,85$	0,03$
	Strike price $K_2 = 220,00\,\$$	
	Call	Put
Contract name	IBM220617C00220000	IBM220617P00220000
Last trade date	2021-10-29 12:51PM EDT	2021-10-25 9:53AM EDT
Last price $p_0^{C;P}$	0,23$	95,85$

TABLE II. Main features of 3 months maturity options.

By substituting the values in Equation 3, it has been possible to find the *discount rates* for all fixed maturities. The results are summarized in Table V.

$T = 6\,months$		
	Strike price $K_1 = 85,00\,\$$	
	Call	Put
Contract name	IBM221021C00085000	IBM221021P00085000
Last trade date	2022-03-09 4:39PM EDT	2022-03-21 1:51PM EDT
Last price $p_0^{C;P}$	42,20$	0,95$
	Strike price $K_2 = 160,00\,\$$	
	Call	Put
Contract name	IBM221021C00160000	IBM221021P00160000
Last trade date	2022-03-29 9:50AM EDT	2022-03-28 12:28PM EDT
Last price $p_0^{C;P}$	1,10$	32,55$

TABLE III. Main features of 6 months maturity options.

$T = 1\,year$		
	Strike price $K_1 = 55,00\,\$$	
	Call	Put
Contract name	IBM230120C00055000	IBM230120P00055000
Last trade date	2021-10-19 12:01PM EDT	2021-11-01 11:04AM EDT
Last price $p_0^{C;P}$	87,20$	0,44$
	Strike price $K_2 = 220,00\,\$$	
	Call	Put
Contract name	IBM230120C00220000	IBM230120P00220000
Last trade date	2021-10-28 1:49PM EDT	2021-08-25 5:34PM EDT
Last price $p_0^{C;P}$	0,40$	88,25$

TABLE IV. Main features of 1 year maturity options.

Dividend rates calculation

At this point, in order to evaluate the dividend policy of the company, a call and a put with strike prices at the money have been chosen for each maturity. Their main features are summarized in Table VI.

By substituting in Equation 5 the corresponding values of the new options, contained in Table VI, and using the corresponding discount rates, summarized in Table V, the forward contract's prices $F_0(T)$ can be obtained. Moreover, manipulating Equations 6 and 7, it is also possible to compute implicit dividends d and dividend rates q associated to each maturity. In particular, in the calculation of q, T has been normalized as the number of maturity months divided by the number of months in one year ($T\backslash 12$).

The obtained results are summarized in Table VII.

CONCLUSIONS

As it can be observed in Table V, greater than one discount rates have been obtained for 1 month and one year maturities, implying the existence of negative interest

Maturity T	$K_2 - K_1$	$D_0(T)$
1 month	26$	1,025
3 months	160$	0,997
6 months	75$	0,969
1 year	165$	1,058

TABLE V. Discount rates' values for each maturity.

Strike price $K = 130,00\,\$$		
	$T = 1\,month$	
	Call	Put
Contract name	IBM220429C00130000	IBM220429P00130000
Last trade date	2022-03-29 2:10PM EDT	2022-03-29 1:22PM EDT
Last price $p_0^{C;P}$	4,35$	3,51$
	$T = 3\,months$	
	Call	Put
Contract name	IBM220617C00130000	IBM220617P00130000
Last trade date	2022-03-29 2:39PM EDT	2022-03-29 3:34PM EDT
Last price $p_0^{C;P}$	5,60$	5,05$
	$T = 6\,months$	
	Call	Put
Contract name	IBM221021C00130000	IBM221021P00130000
Last trade date	2022-03-25 2:25PM EDT	2022-03-16 1:07PM EDT
Last price $p_0^{C;P}$	8,80$	14,38$
	$T = 1\,year$	
	Call	Put
Contract name	IBM230120C00130000	IBM230120P00130000
Last trade date	2022-03-29 1:36PM EDT	2022-03-25 9:34AM EDT
Last price $p_0^{C;P}$	10,20$	13,06$

TABLE VI. Main features of at the money options.

rates R. These results are not normal in this particular situation and this may be due to multiple factors. Probably, the main one is that the formulas for box spread and put-call parity, work only when applied to European options [2]. Since from site [1] it cannot be ensured that the chosen options for this report are European ones, incorrect results may be expected. Furthermore, the models we considered do not take into account some more sophisticated dynamics that happen in the market.

As it can be observed, wrong results for 1 month and 1 year maturities propagated hence to the corresponding implicit dividends and dividend rates.

Navigating in the statistics section of [1], it can be found that the estimated forward annual dividend rate of IBM company is equal to $6,56$. Moreover, by looking at historical data, it can be seen that dividends are provided with three months periodicity, with the last date being February 10, 2022 with $1,64$ dividend.

Maturity T	$F_0(T)$	d	q
1 month	130,82$	-3,35$	\
3 months	130,55$	0,62$	-21,422
6 months	124,24$	10,60$	-5,024
1 year	127,30$	-3,78$	\

TABLE VII. Dividend rates' values for each maturity.

[1] https://finance.yahoo.com/quote/IBM?p=IBM
[2] Hull, John C. *Options futures and other derivatives*. Pearson Education India, 2003.