Visualization of Greeks and implied volatility surface and call options' pricing

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This report is subdivided into a theoretical part and a more practical one. In the first part, the aim is to visualize the three dimensional shapes of the Greeks functions, from the Black-Scholes' model, by fixing some parameters. The purpose of the latter part is instead to visualize the three dimensional shape of the smile effect for the implied volatility, by collecting real data of call options with different maturities from the web. After doing this, it is finally possible to compute the volatility from historical data, comparing it with the implied volatility and pricing the (at the money) options using the Black-Scholes' formula.

BRIEF DESCRIPTION OF GREEKS AND VOLATILITY SURFACE

Starting from European call options pricing formulas of the Black-Scholes' model (1), (2), it is possible to calculate some interesting functions known as *Greek letters*.

$$c_t = S_t \mathcal{N}(d_1) - K e^{-r(T-t)} \mathcal{N}(d_2) \tag{1}$$

$$d_1 = \frac{\log(\frac{S_t}{K}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \quad d_2 = d_1 - \sigma\sqrt{T-t} \tag{2}$$

First recall that in above formulas c_t represents the price of the European call option according to Black-Scholes' model, S_t is the initial stock price of the chosen asset, K is the strike price of the call, T represents its maturity, r is the interest rate, σ is the volatility and $\mathcal{N}(x)$ represents the cumulative function of a normal randomly distributed variable.

The main utility of Greeks is that they quantify different aspects of the risk in an option position. Thus, a trader has to manage the Greeks in such a way that all possible risks become acceptable. Assuming Black-Scholes' model, the following formulas identifies the Greeks for a European call option on a non dividend paying asset.

$$\Delta \equiv \frac{\partial c_t}{\partial S_t} = \mathcal{N}(d_1) \tag{3}$$

$$\Gamma \equiv \frac{\partial^2 c_t}{\partial S_t^2} = \frac{\partial \Delta}{\partial S_t} = \frac{\varphi(d_1)}{S_t \sigma \sqrt{T-t}} \quad \varphi(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} \tag{4}$$

$$\rho \equiv \frac{\partial c_t}{\partial r} = K(T-t)\mathcal{N}(d_2) e^{-r(T-t)} \tag{5}$$

$$\Theta \equiv \frac{\partial c_t}{\partial t} = -\frac{S_t \varphi(d_1)\sigma}{2\sqrt{T-t}} - rK\mathcal{N}(d_2)e^{-r(T-t)} \tag{6}$$

$$\nu \equiv \frac{\partial c_t}{\partial \sigma} = S_t \varphi(d_1)\sqrt{T-t} \tag{7}$$

Since the volatility σ is a parameter that cannot be directly observed but only estimated from historical data of the stock price, in the Black-Scholes' formula and in the Greek letters, the volatility has to be intended as an implied volatility, that is, a volatility implied by option prices observed in the market. More precisely, the value of this quantity is the is number that, substituted to σ in (1), gives the spot market price of a European call option on a non dividend paying asset. Unfortunately, since (1) is not invertible in σ, only an iterative procedure can be used in order to deduce implied volatility. Thus, the important difference between historical and implied volatility is that the former is backward looking on time, while the second is forward looking and is used to monitor the market's opinion on the volatility of a certain stock.

Plotting the implied volatility of some options with a fixed maturity in function of their stock prices, is possible to obtain a figure that is known as *volatility smile*. Instead, a three dimensional plot of the implied volatility in function of both the strike prices of the options and their time to maturity $\tau \equiv T - t$ is known as *volatility surface*. Both these figures will be visualized in the next sections.

The main use of these graphs for traders, is to allow for non lognormality of the probability distribution of an underlying asset at any given future time.

METHODS

Visualization of Greeks

For this very first part, the strike price, the interest rate and the volatility have been set to some fixed values. In particular, it has been set:

$$K = 100\$ \qquad r = 1\% \qquad \sigma = 20\%$$

Then, stock price and time to maturity have been set to two different arrays of values:

$$S_t = [60;\ 65;\ 70;\ \ldots;\ 140]\,\$$$

$$\tau = [0,1;\ 0,2;\ 0,3;\ \ldots;\ 0,9;\ 1;\ 2;\ 3;\ 4;\ 5]\,\textit{years}$$

The Greeks have been then computed through a VBA script that implements functions (3), (4), (5), (6), (7) by taking in input the above set of parameters and finally, each of the functions have been plotted in Excel as a three dimensional surface in function of S_t and τ.

As last task, in order to see how the shapes of the functions vary changing the volatility, the same procedure has been repeated also for $\sigma = 10\%$ and $\sigma = 30\%$.

Implied volatility surface and smile and option pricing

For this second part, it has been chosen a non dividend paying asset. In particular, since `finance.yahoo.com` does not specify the type of the options (Americans, Europeans, ...), Alibaba Group Holding Limited (BABA) has been chosen as asset of reference (information on the company can be found in the first report) and call options are assumed to be European.

After, a set of maturities have been fixed, and for each of them, the whole set of call options available in site [2] has been stored in an Excel file. A small extract of dataset for some call options with maturity $T \approx 1\,month$ is showed in table III. Where the mid prices have been computed taking the average of bid and ask prices.

The maturities that have been chosen for this analysis are $T \approx [1;\, 3;\, 6;\, 9]\,months$ and $T \approx [1;\, 2]\,years$, that correspond to:

$$\tau = [0,08;\, 0,25;\, 0,50;\, 0,75;\, 1;\, 2]\,years$$

In order to visualize the smile effect, the column of all implied volatility has then been plotted in function of the respective strike price.

By plotting the implied volatility in function also of the time to maturity τ, a three dimensional plot of the volatility surface has been obtained.

At this point, historical data on the call options have been collected for each maturity. A brief extract of one of these datasets, for one month maturity is showed in table IV. Daily returns have been then calculated from daily adjusted closing prices p^{AC} using the following formula:

$$return_t = \frac{p_t^{AC} - p_{t-1}^{AC}}{p_{t-1}^{AC}} \tag{8}$$

Daily volatility σ_{daily} has been obtained by computing the standard deviation of the array of daily returns and historical volatility is then given by:

$$\sigma_{historical} = \sqrt{252}\,\sigma_{daily} \tag{9}$$

Where 252 are the days of open market in a year.

Historical volatility have been compared with respective implied volatility associated to at the money options for each maturity and finally, the same options have then been priced using Black-Scholes' formula (1), (2), both using implied and historical volatility and European market's interest rates.

RESULTS

The plots of the Greeks are grouped in figures 2 and 3 below. In particular, in all these graphs it can be observed that, at the increasing of time to maturity, the surfaces become smoother and smoother.

The implied volatility surface is displayed in figure 1. From this graph, it is already possible to observe a sort of convexity of the plotted surface, that leads, in two dimensions, to the characteristic smile shape for the implied volatility.

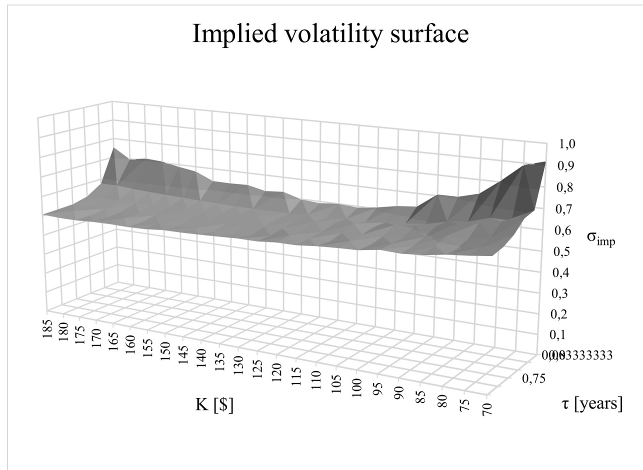

FIG. 1. Three dimensional plot of implied volatility $\sigma_{implied}$ as function of K and τ.

The smiles at each maturity are visualized in figure 4, together with the respective values of historical volatility. In these graphs, the differences between the implied volatility of at the money call options and the historical volatility for a fixed maturity can also be observed. For clarification, these quantities are also listed in table I. Note that the stock price of one BABA share at April 05, 2022 was $S_t = 110,90\$$.

T	$\tau[years]$	K [\$]	$\sigma_{implied}$	$\sigma_{historical}$	r (EU)
1 month	0,08	111,00	0,6019	1,3086	-0,41567
3 months	0,25	110,00	0,5614	0,8577	-0,45755
6 months	0,50	110,00	0,5774	0,7290	-0,37715
9 months	0,75	110,00	0,5588	0,6417	-0,24351
1 year	1,00	110,00	0,5435	0,5766	-0,08732
2 years	2,00	110,00	0,5508	0,5020	0,54343

TABLE I. Main features of at the money call options for each fixed maturity.

In table I are also indicated the interest rates of European market associated to each maturity, which have been used in order to price at the money call options using the Black-Scholes' formula. In particular, option prices have been calculated both for implied ($c_t^{implied}$) and historical ($c_t^{historical}$) volatility. Pricing results are listed and compared with options' mid prices in table II.

T	$c_t^{implied}$ [$]	$c_t^{historical}$ [$]	Mid price [$]
1 month	5,97	14,98	8,13
3 months	7,77	14,27	11,80
6 months	10,61	15,29	16,45
9 months	14,11	17,28	19,83
1 year	20,42	21,87	21,20
2 years	75,51	74,96	29,73

TABLE II. Pricing of at the money call options for each fixed maturity using Black-Scoles' formula with European interest rates and both historical and implied volatility.

CONCLUSIONS

Greek letters have been visualized in three dimensional plots. From the graphs it can be observed that by increasing both the time to maturity and the volatility, the shapes of the surfaces become smoother.

The presence of implied volatility smile have been also checked, together with the visualization of implied volatility surface. In the two dimensional plots of smiles, the difference between historical volatility and implied volatility can be be observed, with particular attention to the one of at the money call options, which have also been priced using Black-Scholes' formula.

Pricing results are indeed better, compared to the ones obtained with one step binomial model in the first report. They are still a bit different compared to the mid prices, but overall good; a part for the 2 years maturity call, which is highly overpriced with respect to the associated mid price. The use of historical and implied volatility gives similar results for the prices. Perhaps, higher precision is obtained using historical volatility, comparing the results with the mid prices.

[1] Hull, John C. *Options futures and other derivatives.* Pearson Education India, 2003.

[2] https://finance.yahoo.com/quote/BABA?p=BABA&.tsrc=fin-srch

Contract name	Last trade date	Strike [$]	Last price [$]	Bid [$]	Ask [$]	Implied volatility	Mid price [$]
BABA220506C00045000	2022-04-01 11:38AM EDT	45,00	68,10	65,70	67,90	1,5996	66,80
BABA220506C00055000	2022-03-30 2:29PM EDT	55,00	61,40	55,60	57,80	1,2070	56,70
BABA220506C00070000	2022-03-30 12:19PM EDT	70,00	48,80	41,30	42,45	0,9180	41,88
BABA220506C00075000	2022-04-01 1:24PM EDT	75,00	36,76	36,60	37,70	0,8906	37,15
BABA220506C00080000	2022-04-04 9:59AM EDT	80,00	34,84	31,95	32,65	0,8076	32,3
BABA220506C00085000	2022-04-05 10:26AM EDT	85,00	28,10	27,10	27,95	0,7341	27,53
BABA220506C00090000	2022-04-05 11:02AM EDT	90,00	23,10	22,80	23,45	0,7075	23,13
BABA220506C00095000	2022-04-05 10:39AM EDT	95,00	19,00	18,75	19,65	0,7065	19,20
BABA220506C00099000	2022-03-30 12:10PM EDT	99,00	21,70	15,50	16,00	0,6483	15,75
BABA220506C00100000	2022-04-05 10:51AM EDT	100,00	15,20	14,60	15,25	0,6355	14,93

TABLE III. Some features of 1 month maturity call options.

Date	Open price [$]	Higher price [$]	Lower price [$]	Close price [$]	Adj. close price [$]	Volume	Daily returns
04/04/2022	115,70	118,22	113,38	117,50	117,50	39088500	\
01/04/2022	117,62	118,95	109,75	110,20	110,20	55787700	-0,062
31/03/2022	115,03	115,09	108,71	108,80	108,80	31240200	-0,013
30/03/2022	115,54	120,10	115,54	116,58	116,58	28381700	0,072
29/03/2022	118,36	119,60	115,74	116,71	116,71	28060900	0,001
28/03/2022	113,86	116,23	111,92	115,09	115,09	30168000	-0,014
25/03/2022	110,53	113,86	109,40	112,99	112,99	42396500	-0,018
24/03/2022	112,86	116,50	110,82	115,15	115,15	55850200	0,019
23/03/2022	115,70	124,11	112,68	117,24	117,24	81605700	0,018
22/03/2022	114,01	118,24	112,37	114,99	114,99	88193400	-0,019

TABLE IV. Some historical data of 1 month maturity call options.

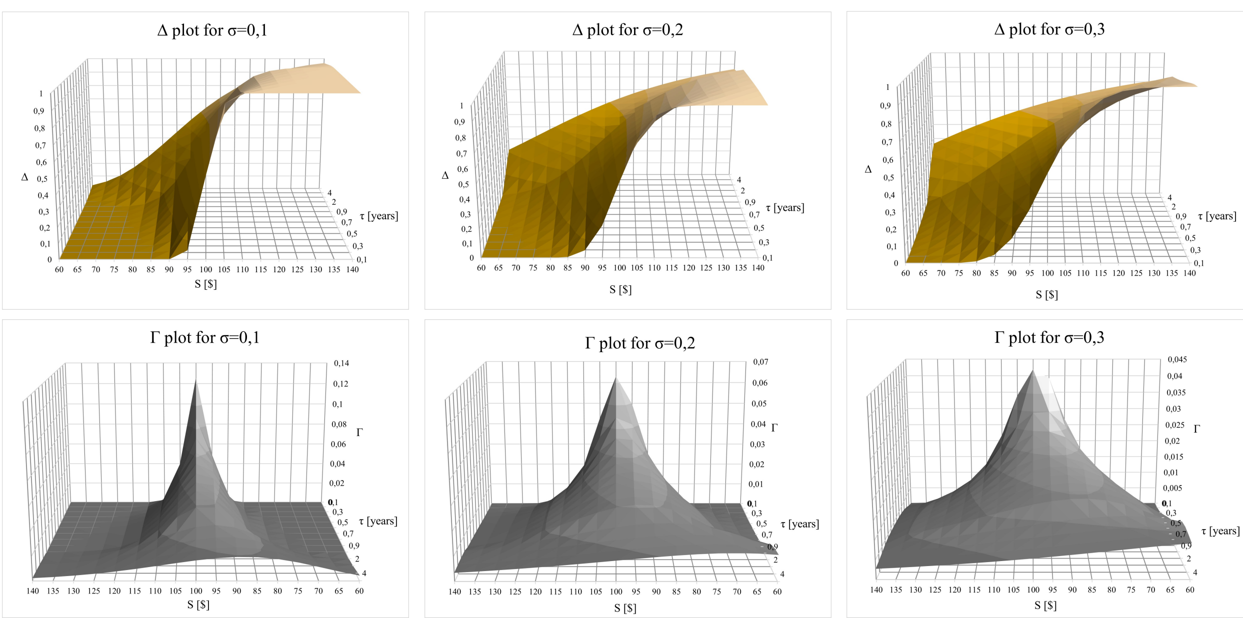

FIG. 2. Three dimensional plots of Greek letters Δ and Γ as functions of S_t and τ, for $\sigma = [0,1;\, 0,2;\, 0,3]$.

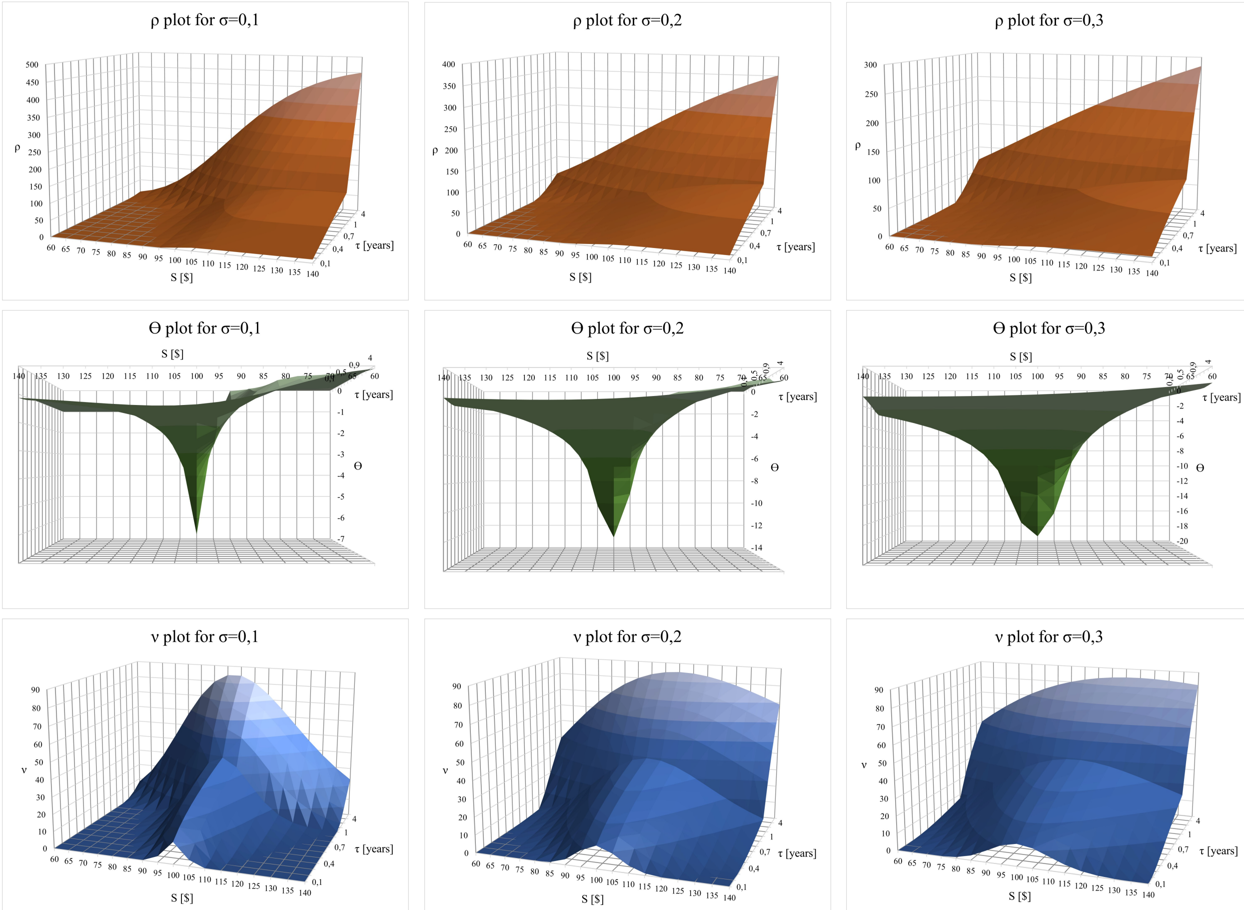

FIG. 3. Three dimensional plots of Greek letters ρ, Θ and ν as functions of S_t and τ, for $\sigma = [0,1;\ 0,2;\ 0,3]$.

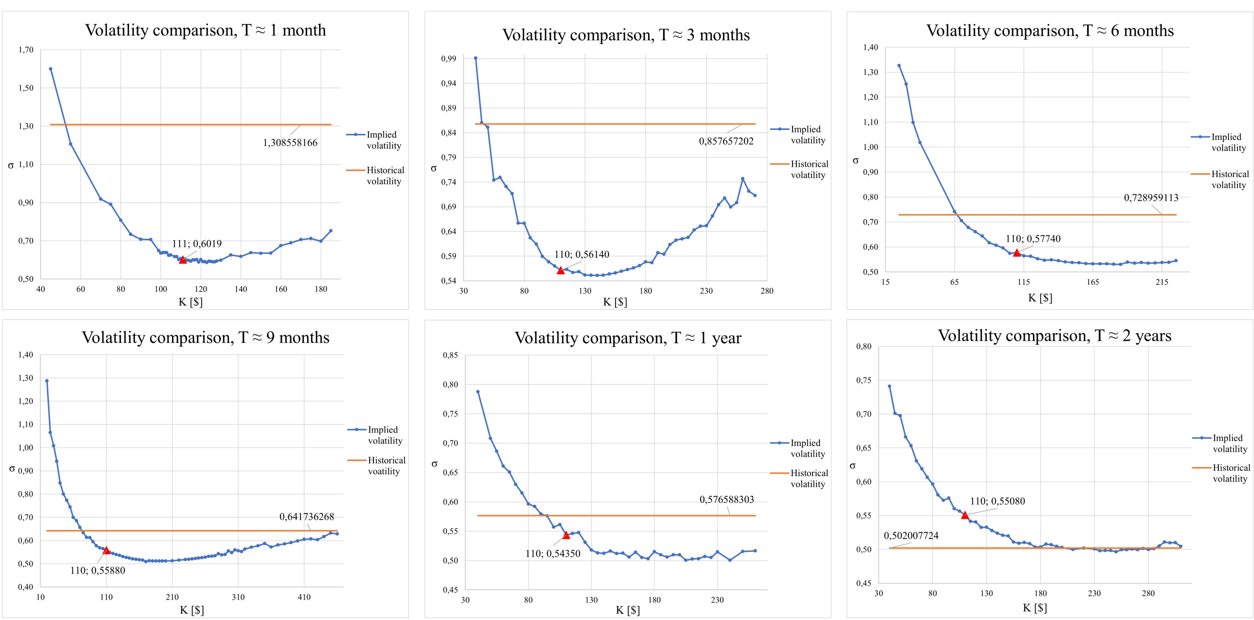

FIG. 4. Plots of implied volatility $\sigma_{implied}$ and historical volatility $\sigma_{historical}$ as functions of K for fixed maturity. The red triangles represent the implied volatility of at the money call options.