Value at Risk Estimation

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The aim of this report is to calculate the so called value at risk (to which will refer as VaR, for brevity) of an equibalanced portfolio of two assets, using different techniques. Each of the exploited methods, obviously required the knowledge of some historical data on the involved assets.

DESCRIPTION OF THE COMPANIES

The two chosen companies for the construction of the portfolio are Alibaba Group Holding Limited (BABA) and Twitter Inc. (TWTR). For the first company, a brief description was already given in the first report. Hence we provide only at describing the latter one.

Twitter was created by Jack Dorsey, Noah Glass, Biz Stone, and Evan Williams in March 2006 and launched in July of that year and operates as a platform for public self-expression and conversation in real-time. Its main basement is in San Francisco, California.

The company's primary product is Twitter, a platform that allows users to consume, create, distribute, and discover content. It also provides promoted products that enable advertisers to promote brands, products, and services, as well as enable advertisers to target an audience based on various factors, including who an account follows and actions taken on its platform, such as Tweets created and engagement with Tweets.

Its promoted products consist of promoted ads and Twitter Amplify, Follower Ads, and Twitter takeover. In addition, the company offers monetization products for creators, including Tips to directly send small one-time payments on Twitter using various payment methods, including bitcoin; Super Follows, a paid monthly subscription, which includes bonus content, exclusive previews, and perks as a way to support and connect with creators on Twitter; and Ticketed Spaces to support creators on Twitter for their time and effort in hosting, speaking, and moderating the public conversation on Twitter Spaces.

Further, it offers products for developers and data partners comprising Twitter Developer Platform, a platform that enables developers to build tools for people and businesses using its public application programming interface; and paid access to Twitter data for partners with commercial use cases.

The majority holders of the company are the insiders, who all together hold the 12,42% of the shares; and some institutions, that all together hold the 80,63% of the shares and among which, the most important are: Vanguard Group Inc. (9,22%), Morgan Stanley (9,19%) and Blackrock Inc. (6,79%).

On April 25, 2022, the Twitter board of directors agreed to a $44 billion buyout by Elon Musk, the CEO of SpaceX and Tesla.

BRIEF DESCRIPTION OF VALUE AT RISK CALCULATION TECHNIQUES

Volatility is a useful estimation of risk in a portfolio, however it does not take into account the direction of an investment's movement. A stock, for example, can be volatile also when it rapidly jumps higher, however this does not represent a risk for an investor, because it brings gains. The risk, instead, is represented by the concrete possibility of loosing money. For this purpose it becomes useful to introduce the concept of value at risk (VaR), which basically responds to the practical question "how much can i loose in case things goes bad?".

In each of the techniques that have been used in order to compute VaR, three components are always required: a time window in the future in which we want to predict our possible losses, a confidence level for the estimation (typically set to high values in order to be almost sure not to face losses) and a loss percentage.

In this report, three important techniques have been exploited for the computation of VaR: the variance-covariance method, the Monte Carlo simulation and the historical method.

Variance-Covariance Method

The main assumption in this technique is that the stock returns r_j are normally distributed. This does not represent a limitation in the case, like ours, we consider a portfolio consisting of two assets, because the (weighted) sum of two normally distributed variables is again a normally distributed variable. This method requires thus the only calculation of the expected value $E[r]$ and the standard deviation σ_r of the asset returns, which, again, are supposed to be normally distributed. From fixed time horizon T and confidence level (in our case 95%, 99% ad 99,5%) it is then possible, using the tables for a standardized normal distribution, to get the abscissa x_{cl} associated to that confidence level and calculate the VaR as:

$$VaR_{cl}^{Norm} = \sigma x_{cl}\sqrt{T} \tag{1}$$

Monte Carlo Simulation

Monte Carlo method is used in order to generate a certain number of randomly distributed trials from a standard fixed normal distribution, simulating future stock price returns. From the the generated samples is then possible to construct an histogram of frequencies and finally obtain the VaR by computing the percentiles Q_{cl}^{MC} corresponding to the fixed confidence levels of the simulated distribution. VaR for a fixed time horizon, is just obtained by multiplying the above quantity for the square root of T.

$$VaR_{cl}^{MC} = Q_{cl}^{MC}\sqrt{T} \tag{2}$$

Historical Method

To exploit this technique it is necessary to collect a reasonable amount of historical data on the considered assets, in order to reconstruct the past history of stock price returns. Assuming that, also in the future, the returns will continue to follow the very same distribution of historical data, mutatis mutandis it is possible to calculate VaR in the same way as it is done for Monte Carlo simulation.

$$VaR_{cl}^{Hist} = Q_{cl}^{Hist}\sqrt{T} \tag{3}$$

METHODS

First of all, historical data on the two chosen assets (BABA and TWTR) have been collected consulting sites [2], [3] fixing a time window of six months. Starting from respective adjusted closing prices p^{AC}, daily returns r_t at time t have been calculated using the following relation.

$$r_t = \frac{p_t - p_{t-1}}{p_{t-1}} \tag{4}$$

An equibalanced portfolio has been constructed by simply assigning two weights w_{BABA}, w_{TWTR} to both assets, with the constraint that $w_{BABA} + w_{TWTR} = 1$. The only possible choice for an equibalanced portfolio is that $w_{BABA} = w_{TWTR} = 0,5$.

The expected value and the variance of the returns have then been computed for each asset simply by using the corresponding built-in Excel functions. Exploiting then the linearity of the expected value $E[X]$, for the portfolio we have that:

$$E_{portf}[r] = w_{BABA}E_{BABA}[r] + w_{TWTR}E_{TWTR}[r] \tag{5}$$

Instead, for the Variance of the portfolio it holds that:

$$\begin{aligned} Var_{portf}[r] &= w_{BABA}^2 Var_{BABA}[r] + w_{TWTR}^2 Var_{TWTR}[r] + \\ &+ 2\rho_{BABA,TWTR} w_{BABA} w_{TWTR} \sqrt{Var_{BABA}[r] Var_{TWTR}[r]} \end{aligned} \tag{6}$$

Where $\rho_{BABA,TWTR}$ represent the correlation coefficient between the returns of the two assets.

The daily volatility can be then computed by taking the square root of the corresponding variances:

$$\sigma = \sqrt{Var[r]} \tag{7}$$

In alternative, to calculate the expected value, the variance and the volatility of the equibalanced portfolio, it is equivalently possible to use the same techniques used for a single asset, using the vector of portfolio daily returns computed by the following.

$$r_{portf,\,t} = w_{BABA} r_{BABA,\,t} + w_{TWTR} r_{TWTR,\,t}\ \forall t \tag{8}$$

The volatility of the assets and the portfolio has also been computed using the exponentially weighted moving average (EWMA) model, which is a particular case of the model using standard deviation. In EWMA model, the weights associated to the returns of a fixed asset decrease exponentially with a factor λ as we move back in time. In our particular case, we used RiskMetrics EWMA, originally created by JP Morgan, that fixes $\lambda = 0,94$.

It can be shown that this wheighting scheme leads to the following recursive formula for updating volatility estimates.

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)r_{t-1}^2 \tag{9}$$

Where, in place of σ_0 when $t = 1$, it has been substituted the value of daily volatility calculated with the previous method.

At this point we have all the elements to calculate VaR with all of the techniques explained above. In particular, a column of time horizons $T = 1, 2, 3, \ldots, 100$ have been created and three values for the confidence level have been fixed: 95%, 99% and 99,5%.

VaR using variance-covariance method has been computed for the assets and the portfolio using equation 1 and substituting to the volatility both the respective flat daily volatility and the volatility calculated with the EWMA model, corresponding to the respective time horizon.

VaR using Monte Carlo simulation has been computed by generating 1000 samples of both the assets and portfolio returns. From the the percentiles of the generated distributions, VaR has been computed using equation 2 for all considered time horizons.

Finally, the distributions followed by assets' and portfolio's historical returns of past six months, have been used in order to deduce historical VaR as showed in equation 3 for all considered confidence levels and time horizons.

RESULTS

For sake of brevity it not possible to list in this report all the data that have been collected for the calculations.

However, small subsets of historical data of both assets has been resumed in tables II, III. In any case, data can be consulted from sites [2], [3].

Computed expected values, variances and volatility of the returns are listed in table I.

Asset	$E[r]$ [\$]	$Var[r]$ [\$]	σ [\$]
BABA	0,005	0,002	0,046
TWTR	0,001	0,001	0,035
Portfolio	0,003	0,001	0,034

TABLE I. Computed expected values, variances and volatility.

In particular, a correlation coefficient between BABA and TWTR returns $\rho = 0,422$ have been found.

Histograms of the returns are plotted in figures (), (), () below.

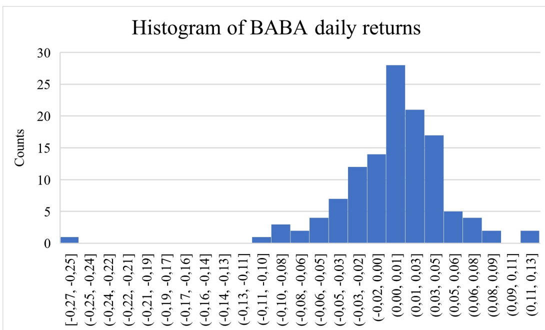

FIG. 1. Histogram of BABA daily returns.

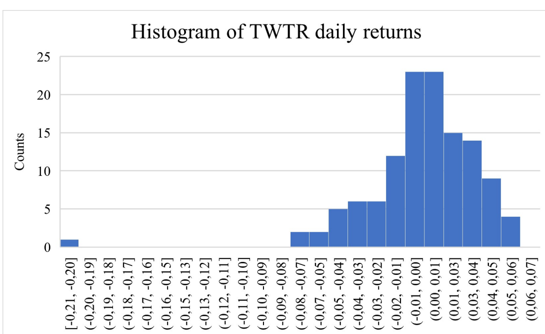

FIG. 2. Histogram of TWTR daily returns.

The results obtined from the calculations of VaR are finally resumed in plots 4 below.

CONCLUSIONS

As it can be seen from plots in figure 4, Normal VaR is often an optimistic estimation of value at risk and thus it may lead to some losses and non realistic previsions. In

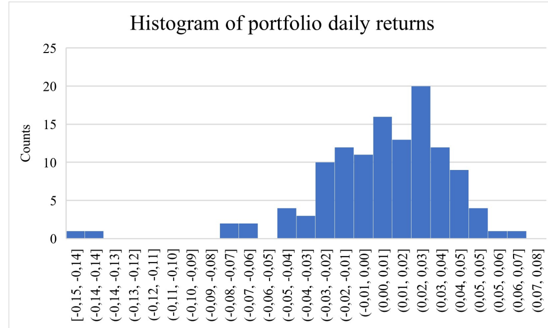

FIG. 3. Histogram of portfolio daily returns.

fact, returns may not necessarily follow a normal distribution over time, especially during periods characterized by an high volatility.

It can be also observed that, for an high number of samples, the model used in order to generate Monte Carlo random samples leads to a distribution of returns that is very similar to a normal distribution, and this is also demonstrated by the fact that normal and Monte Carlo curves in figure 4 are very narrow.

The VaR values obtained through the EWMA estimation of the volatility leads to very different shapes of the curves. This is maybe due to the fact that this is the only technique that takes into account the past history of the assets.

[1] Hull, John C. *Options futures and other derivatives*. Pearson Education India, 2003.
[2] https://finance.yahoo.com/quote/BABA?p=BABA&.tsrc=fin-srch
[3] https://finance.yahoo.com/quote/TWTR?p=TWTR&.tsrc=fin-srch

Date	Open price [$]	Higher price [$]	Lower price [$]	Close price [$]	Adj. close price [$]	Daily returns	σ_{EWMA} [$]
May 02, 2022	96,41	101,32	95,15	101,21	101,21	\	\
Apr 29, 2022	101,53	103,52	96,66	97,09	97,09	-0,041	0,046
Apr 28, 2022	88,62	90,98	86,94	90,91	90,91	-0,064	0,046
Apr 27, 2022	85,32	89,41	84,78	88,32	88,32	-0,028	0,047
Apr 26, 2022	85,87	86,24	83,28	83,99	83,99	-0,049	0,046
Apr 25, 2022	82,7	86,17	81,8	85,84	85,84	0,022	0,046
Apr 22, 2022	86,5	90,68	86,31	86,49	86,49	0,008	0,045
Apr 21, 2022	89,55	90,01	85,4	85,99	85,99	-0,006	0,044
Apr 20, 2022	93,48	93,7	89,27	89,41	89,41	0,040	0,044
Apr 19, 2022	92,29	93,95	90,95	93,5	93,5	0,046	0,043

TABLE II. Some historical data of BABA.

Date	Open price [$]	Higher price [$]	Lower price [$]	Close price [$]	Adj. close price [$]	Daily returns	σ_{EWMA} [$]
May 02, 2022	48,7	49,76	48,57	49,14	49,14	\	\
Apr 29, 2022	49,45	50,25	48,74	49,02	49,02	-0,002	0,035
Apr 28, 2022	49,01	49,92	47,96	49,11	49,11	0,002	0,033
Apr 27, 2022	49,06	49,26	47,82	48,64	48,64	-0,010	0,032
Apr 26, 2022	51,57	51,62	49,43	49,68	49,68	0,021	0,032
Apr 25, 2022	51,02	52,29	50,24	51,7	51,7	0,041	0,031
Apr 22, 2022	47,77	49,73	47,49	48,93	48,93	-0,054	0,032
Apr 21, 2022	45,51	47,94	45,5	47,08	47,08	-0,034	0,033
Apr 20, 2022	46,34	47,11	45,03	46,72	46,72	-0,008	0,034
Apr 19, 2022	47,3	48,5	45	46,16	46,16	-0,012	0,033

TABLE III. Some historical data of TWTR.

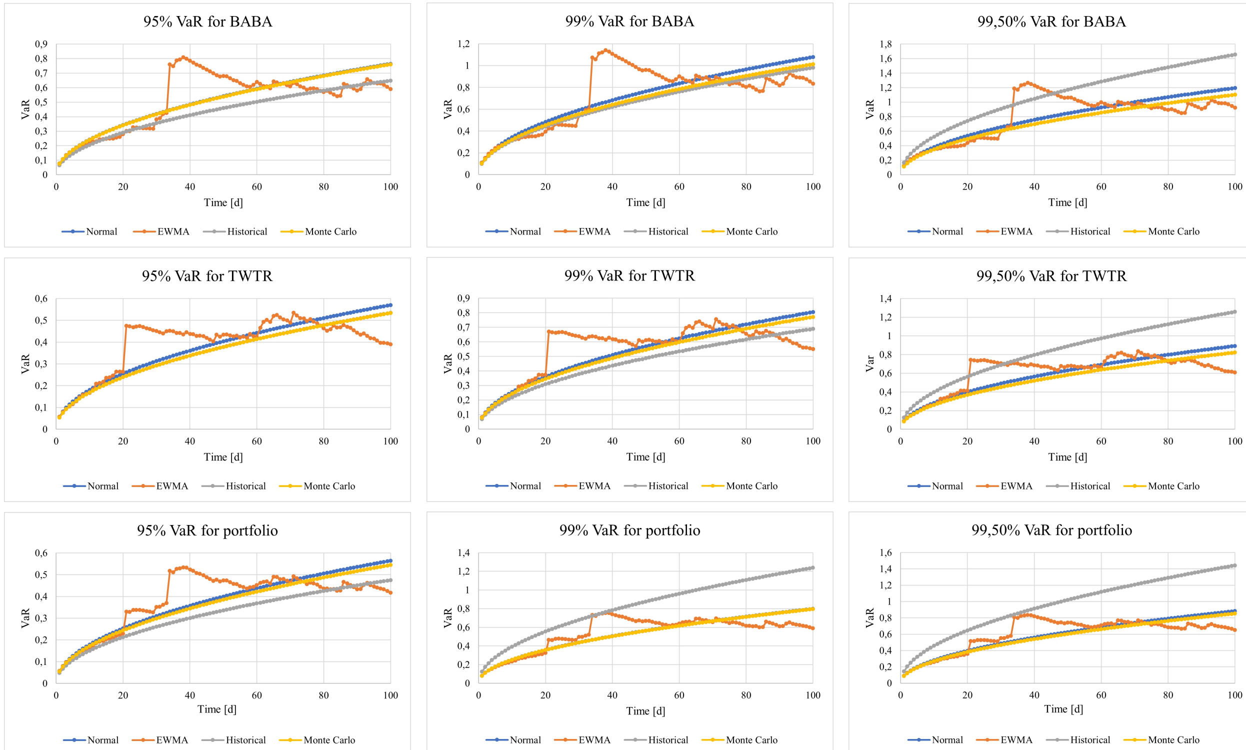

FIG. 4. Comparison between normal, Monte Carlo and historical methods for evaluation of VaR for BABA and TWTR assets and the equibalanced portfolio.