Pricing of a call option using static binomial model

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Binomial option pricing model consists in an iterative procedure that allows, through the specification of some historical data, to evaluate the price of an option during the time between the sign of the contract and its maturity. As the name suggests, at each time step, only two outcomes are possible with a certain probability: a move up, or a move down by certain percentage amounts. Considering many time steps, the representation of the model results in a binomial tree, which is particularly useful for pricing American options. One fundamental concept that must be imposed in order to price the options is the absence of arbitrage opportunities, which conduces, at the end, to a risk-neutral evaluation. Taking this model as a reference, the main purpose of this report is to price an American call option.

DESCRIPTION OF THE COMPANY

The company that is taken into consideration for this report is Alibaba group holding limited (abbreviated as BABA). Alibaba is a group that, through its subsidiaries, provides technology infrastructures and marketing reach to merchants, brands, retailers, and other businesses to engage with their users and customers in the People's Republic of China and internationally. It was founded on 28 June 1999 in Hangzhou, Zhejiang by Jack Ma, who has been the former executive chairman until 10 September 2019, when was substituted by Daniel Zhang.

The group operates through four segments: core commerce, cloud computing, digital media and entertainment, and innovation initiatives and others. Its main activities consist in consumer to consumer, business to consumer and business to business sales services via web portals, as well as electronic payment services, shopping search engines and cloud computing services. It owns and operates a diverse portfolio of companies around the world in numerous business sectors.

Alibaba.com has three main services: the English language portal Alibaba.com, which handles sales between importers and exporters from more than 240 countries and regions, the Chinese portal 1688.com, which manages domestic business to business trade in China, and transaction-based retail website AliExpress.com, which allows smaller buyers from all over the world to buy small quantities of goods (mostly small Chinese businesses offering products) at wholesale prices. Similar to eBay, sellers on Aliexpress can be either companies or individuals.

In 2003, Alibaba launched Taobao Marketplace, offering a variety of products for retail sales and becoming soon China's largest customer to customer online shopping platform. Taobao's growth was attributed to offering free registration and commission-free transactions using a free third-party payment platform. In April 2008, Taobao introduced a spin-off, Taobao Mall (later Tmall.com), an online retail platform to complement the Taobao portal, offering global brands to an increasingly affluent Chinese consumer base.

In 2013, Alibaba and six others large Chinese logistics companies, established a company called Cainiao for delivery of packages in China, announcing also the will to build a global logistics network.

In 2017, Alibaba started opening a chain of supermarkets, named Hema, as part of the company's "new retail strategy", where customers can either order in the store or online for delivery in under 30 minutes.

In September 2009, the company launched Alibaba Cloud, aiming to build a cloud computing service platform, including e-commerce data mining, e-commerce data processing, and data customization; resulting in being the largest high-end cloud computing company in China and developing also technologies geared towards urban solutions such as streamlining traffic, detecting accidents and improving transport efficiency through artificial intelligence.

Regarding the FinTech sector and entertainment services, in 2004, the Alibaba Group launched Alipay, a third-party online payment platform and created a new live entertainment business unit under its digital media and entertainment group, which focuses on ticketing, content creation and live experiences.

The current market capitalization is estimated to be worth $323,04\,billions\,\$$, with an enterprise value of $267,32\,billions\,\$$. The majority holders of the group are the internal members, who altogether hold the $0,02\%$ of the shares and some institutions (among which, the most relevant are Goldman Sachs Group Inc., Primecap Management Company and HSBC Holdings Plc) which hold the $18,94\%$ of the shares.

METHODS

Binomial pricing model is particularly suited for pricing American options for the reason that, contrarily to European ones, they can be executed at any time between the purchase and the expiration date. For this reason, one BABA share with price S_0 and two BABA call options have been chosen consulting site [1]. One option with maturity $T = 3\,months$ and expiration date fixed

for 17 June 2022, while the other with $T = 6\,months$ and expiration date 16 September 2022, paying attention in choosing at the money strike prices K for the options. Alibaba company does not provide dividends, thus the formulas that have been used for the analysis, do not take into account the dividend payments.

Then, using the main features describing the options, mid prices have been calculated by averaging the values of bid and ask and considered as target prices for the model.

Once chosen the call options, daily historical data of the company have been inspected in order to calculate other important parameters of the model. In particular, for $T = 3\,months$ (respectively, $T = 6\,months$) all working days between 22 December 2021 and 21 March 2022 (22 September 2021 and 21 March 2022) have been taken into consideration.

Firstly, the vector of daily adjusted closing prices $\overrightarrow{S}$, has been used in order to calculate the vector of daily returns $\overrightarrow{return}$ as shown in (1). The components of these vectors at any open market day t, are denoted respectively: S_t and $return_t$.

$$return_t = \frac{S_t - S_{t-1}}{S_t} \tag{1}$$

Starting from the vector of daily returns, daily and annual volatility have been then computed through the relations:

$$\sigma_{daily} = stddev(\overrightarrow{return}) \qquad \sigma_{year} = \sqrt{252}\,\sigma_{daily} \tag{2}$$

Where $stddev$ denotes the standard deviation of the set of daily returns vector's components and 252 is the total number of open market days in one year.

At this point, u and d parameters of the binomial model have been computed using the following relations.

$$u = e^{\sigma_{year}\sqrt{T}} \qquad d = e^{-\sigma_{year}\sqrt{T}} \tag{3}$$

Where, from now on, T does not any more represent the number of months before contract's maturity, but instead, that number normalized by the the total number of months in a year: $T = 3\backslash 12 = 0,25$ in case of 3 months maturity and $T = 6\backslash 12 = 0,5$ in case of 6 months maturity.

Subsequently, consulting site [2], it has been possible to select US dollar LIBOR interesting rates R over 3 and 6 months estimated in 22 March 2022, in order to compute the capitalisation factors through simple compounding (sc) and simple discounting (sd) as shown below.

$$sc = 1 + RT \qquad sd = \frac{1}{sc} = \frac{1}{1 + RT} \tag{4}$$

The risk-neutral probability for the option's price to go through a move up, or a move down in the final price has been calculated in the following way.

$$q = \frac{e^{RT} - d}{u - d} \tag{5}$$

It is important to remark that q represents a probability that is given by the trend of the market, not by speculative subjective guesses and, as a probability, it is useful to check whether it is $0 < q < 1$ in order also to be sure that any opportunity of arbitrage is avoided.

Finally, from the value of q, it has been possible to apply the risk-neutral pricing formula in order to price the calls in case of non dividend paying asset as follows.

$$p_0^{call} = e^{-RT}\,(q\,f^u + (1-q)\,f^d) \tag{6}$$

Where the possible payoffs f^u and f^d have been computed through the following formulas.

$$f^u = (S_0\,u - K)^+ \qquad f^d = (S_0\,d - K)^+ \tag{7}$$

In which $(x)^+$ denotes the positive part of x, also equal to $max(0; x)$

RESULTS

All the following data have been collected consulting the sites [1, 2].

The price of one BABA share at 2022-03-22, 01:04PM EDT was $S_0 = 116,49\$$ and the main features of the chosen call options are summarized in Tables I and II below.

Contract name	BABA220617C00115000
Last trade date	2022-03-22 12:37PM EDT
Maturity T	3 months
Strike price K [$]	115,00
Last price [$]	14,25
Bid [$]	14,40
Ask [$]	14,80
Mid price [$]	**14,60**

TABLE I. Main features of 3 months maturity option.

For sake of brevity, only few historical data have been reported in Table V, however they are fully accessible at site [1].

The other relevant quantities described in the previous section and calculated from historical data are summarized in Tables III and IV below.

The final results of the estimated prices are, for the 3 months maturity call option: $p_0^{call_3} = 28,69\$$, while for the 6 months maturity call option: $p_0^{call_6} = 32,52\$$.

Contract name	BABA220916C00115000
Last trade date	2022-03-22 12:24PM EDT
Maturity T	6 months
Strike price K [$]	115,00
Last price [$]	18,30
Bid [$]	18,50
Ask [$]	19,65
Mid price [$]	**19,08**

TABLE II. Main features of 6 months maturity option.

σ_{daily}	0,062
σ_{year}	0,981
$T\,[year^{-1}]$	0,25
u	1,633
d	0,612
R	0,95371%
sc	1,002
sd	0,998
q	0,382
f^u [$]	75,259
f^d [$]	0

TABLE III. Relevant calculated quantities of 3 months maturity option.

CONCLUSIONS

The prices of the call options calculated through the binomial model, have to be compared to the respective mid prices calculated at the beginning, from the main features of the options. What it can be observed is that, in both cases, the binomial model over-estimates the prices of the call options. In particular, the over-estimation with respect to the mid prices is of about $+96,61\%$ for the 3 months maturity option and $+70,44\%$ for the 6 months maturity option. These results support the fact that the binomial model does not predict real case data with a great precision, however it is surely useful in order to have a first estimation of a call price and put the basis for more sophisticated models based on continuous time representation.

[1] https://finance.yahoo.com/quote/BABA?p=BABA
[2] https://www.global-rates.com/en/interest-rates/libor/libor.aspx

σ_{daily}	0,050
σ_{year}	0,789
$T\,[year^{-1}]$	0,5
u	1,747
d	0,572
R	1,38457%
sc	1,007
sd	0,993
q	0,370
f^u [$]	88,506
f^d [$]	0

TABLE IV. Relevant calculated quantities of 6 months maturity option.

Date	Open price [$]	Higher price [$]	Lower price [$]	Close price [$]	Adj. close price [$]	Volume	Daily returns
22/12/2021	117,09	118,66	115,15	117,81	117,81	24502000	\
23/12/2021	116,80	119,59	115,39	118,66	118,66	17563300	0,007
27/12/2021	117,80	121,80	116,10	116,59	116,59	17864400	-0,017
28/12/2021	116,26	116,42	114,47	114,80	114,80	17052100	-0,015
29/12/2021	113,80	114,20	110,38	112,09	112,09	21100500	-0,024
30/12/2021	112,63	124,30	112,30	122,99	122,99	45437100	0,097
31/12/2021	121,23	122,91	118,59	118,79	118,79	24287000	-0,034
03/01/2022	119,38	121,95	115,82	120,38	120,38	16775300	0,013
04/01/2022	119,53	120,87	115,77	119,56	119,56	20824000	-0,007
05/01/2022	118,00	126,62	117,70	121,16	121,16	36651100	0,013

TABLE V. Some historical data of 3 months maturity option.