EECS 461 PROBABILITY & STATISTICS

TUES SEPT 6 2022

ASSIGNMENT #2

MORGAN BERGEN

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1.4.2. YOU HAVE A SIX-SIDED DIE THAT YOU ROLE ONCE

LET $R_i$ DENOTE THE EVENT THAT YOU ROLL IS $i$.

LET $G_j$ DENOTE THE EVENT THAT THE ROLL IS GREATER THAN $j$.

LET $E$ DENOTE THE EVENT THAT THE ROLL OF THE DIE IS EVEN NUMBERED.

A WHAT IS $P[R_3 \mid G_1]$, THE CONDITIONAL PROBABILITY THAT 3 IS ROLLED GIVEN THAT THE ROLL IS GREATER THAN 1.

DEF CONDITIONAL PROB. CORRESPOND TO A MODIFIED PROB. MODEL THAT REFLECTS PARTIAL INFO ABOUT THE OUTCOME OF AN EXPERIMENT. THE MODIFIED MODEL HAS A SMALLER SAMPLE SPACE THAN THE ORIGINAL MODEL.

$P[A]$ A PRIORI PROBABILITY OF A.

$P[R_3 \mid G_1]$ THE PROBABILITY THAT 3 IS ROLLED, GIVEN THAT THE ROLL IS $> 1$.

SAMPLE SPACE IS $S = \{1, 2, 3, 4, 5, 6\}$

$R_3 = \{3\}$

$G_1 = \{2, 3, 4, 5, 6\}$

$P[R_3] = 1/6$

$P[G_1] = 5/6$

$G_1 \cap R_3 = \{3\} = R_3$

$P[G_1 \cap R_3] = P[R_3] = 1/6$

$$P[R_3 \mid G_1] = \frac{P[R_3 G_1]}{P[G_1]} = \frac{P[R_3]}{P[G_1]} = \frac{1/6}{5/6} = \frac{1}{5} = 20\%$$

$$\boxed{\therefore P[R_3 \mid G_1] = 20\%}$$

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1.4.4. PHONESMART IS HAVING A SALE ON BANANAS.

IF YOU BUY 1 BANANA AT FULL PRICE, YOU GET A SECOND AT HALF PRICE.

WHEN COUPLES COME TO BUY A PAIR OF PHONES, SALES OF APRICOTS & BANANAS ARE EQUALLY LIKELY.

GIVEN THAT THE FIRST PHONE SOLD IS A BANANA,

THE SECOND IS TWICE AS LIKELY TO BE A BANANA RATHER THAN AN APRICOT.

WHAT IS THE PROBABILITY THAT A COUPLE BUYS A PAIR OF BANANAS?

LET $B_N$ DENOTE THE EVENT THAT THE NTH PHONE THAT IS SOLD IS A BANANA

& LET $A_N$ DENOTE THE EVENT THAT THE NTH PHONE THAT IS SOLD IS AN APRICOT

IN ORDER TO DETERMINE THE PROB. THAT A COUPLE BUYS A PAIR OF BANANAS,

WE MUST FIND $P[B_1 B_2]$

$P[A_1 A_2] = ?$

$P[A_1 B_2] = ?$

$P[B_1 A_2] = ?$

$P[B_1 B_2] = ?$

HOWEVER WE KNOW THAT THE FOLLOWING IS TRUE, GIVEN OUR UNKNOWNS.

– $P[A_1 A_2] + P[A_1 B_2] + P[B_1 A_2] + P[B_1 B_2] = 1$ <=> TOTAL PROBABILITIES = 1

$P[A_1] = P[B_1] = 1/2$

$P[A_2] = P[B_2] = 1/2$ <=> "SALES OF APRICOTS & BANANAS ARE EQUALLY LIKELY"

– $P[A_1] = P[A_1 A_2] + P[A_1 B_2] = 1/2$

– $P[A_2] = P[A_1 A_2] + P[B_1 A_2] = 1/2$

– $\dfrac{P[B_1 B_2]}{P[B_1]} = 2\left(\dfrac{P[B_1 A_2]}{P[B_1]}\right)$ <=> "GIVEN FIRST PHONE SOLD IS A BANANA, THE SECOND PHONE IS TWICE AS LIKELY TO BE A BANANA"

=> $\dfrac{P[B_1 B_2]}{\cancel{P[B_1]}} = 2\left(\dfrac{P[B_1 A_2]}{\cancel{P[B_1]}}\right)$

=> $\dfrac{P[B_1 B_2]}{2} = P[B_1 A_2]$

REPLACE

REPLACE

$P[A_1 A_2] + P[A_1 B_2] + P[B_1 A_2] + P[B_1 B_2] = 1$ => $P[A_1 A_2] + P[A_1 B_2] + \frac{3}{2} P[B_1 B_2] = 1$

$P[A_1] = P[A_1 A_2] + P[A_1 B_2] = 1/2$ => $P[A_1 A_2] + P[A_1 B_2] = 1/2$

$P[A_2] = P[A_1 A_2] + P[B_1 A_2] = 1/2$ => $P[A_1 A_2] + 1/2\, P[B_1 B_2] = 1/2$

$$\boxed{P[A_1 A_2] + P[A_1 B_2]} + \frac{3}{2} P[B_1 B_2] = 1$$

$$P[A_1 A_2] + P[A_1 B_2] = 1/2$$

$$1/2 + 3/2\, P[B_1 B_2] = 1$$

$$-1/2 \qquad\qquad -1/2$$

$$3/2\, P[B_1 B_2] = 1/2$$

$$\therefore \implies P[B_1 B_2] = 1/3$$

THEREFORE PROB. THAT A COUPLE BUYS A PAIR OF BANANAS IS $33.\bar{3}\%$.

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1.5.2

**1.5.2** For the telephone usage model of Example 1.18, let $B_m$ denote the event that a call is billed for $m$ minutes. To generate a phone bill, observe the duration of the call in integer minutes (rounding up). Charge for $M$ minutes $M = 1, 2, 3, \ldots$ if the exact duration $T$ is $M - 1 < t \leq M$. A more complete probability model shows that for $m = 1, 2, \ldots$ the probability of each event $B_m$ is

$$P[B_m] = \alpha(1-\alpha)^{m-1}$$

where $\alpha = 1 - (0.57)^{1/3} = 0.171$.

(a) Classify a call as long, $L$, if the call lasts more than three minutes. What is $P[L]$?

(b) What is the probability that a call will be billed for nine minutes or less?

M IS THE PROB. THAT A CALL IS BILLED FOR MORE THAN 3 MINUTES,

"3 OR FEWER BILLED MINUTES"

$$P[L] = 1 - P[B_m \leq 3]$$

BREAK UP

$$= 1 - P[B_1] - P[B_2] - P[B_3]$$

SINCE WE KNOW $P[B_m] = \alpha(1-\alpha)^{m-1}$

$P[B_1] \Rightarrow P[B_1] = \alpha(1-\alpha)^{1-1}$
$= \alpha(1-\alpha)^0$
$= \alpha(1)$
$= \alpha$

$\Rightarrow P[B_1] = \alpha$

$P[B_2] \Rightarrow P[B_2] = \alpha(1-\alpha)^{2-1}$
$P[B_2] = \alpha(1-\alpha)$

$P[B_3] \Rightarrow P[B_3] = \alpha(1-\alpha)^{3-1}$
$P[B_3] = \alpha(1-\alpha)^2$

$$P[L] = 1 - P[B_1] - P[B_2] - P[B_3]$$

$$= 1 - \alpha - \alpha(1-\alpha) - \alpha(1-\alpha)^2$$

$$= (1-\alpha)^3$$

WHERE $\alpha = 1 - (0.57)^{1/3} = 0.171$
$\alpha = 0.171$

$$= (1 - 0.171)^3$$

$$\boxed{P[L] \approx 0.5697227\ldots}$$

$\therefore$ THE PROB. THAT A CALL WILL LAST MORE THAN 3 MIN. IS APPROX. 57%.

(B) PROB. BILL FOR 9 MIN OR LESS IS

$$P[B_m \leq 9] = \sum_{i=1}^{9} \alpha(1-\alpha)^{i-1} = 1 - \alpha - \alpha(1-\alpha) - \alpha(1-\alpha)^2 - \alpha(1-\alpha)^3 - \alpha(1-\alpha)^4 - \alpha(1-\alpha)^5 - \alpha(1-\alpha)^6 - \alpha(1-\alpha)^7 - \alpha(1-\alpha)^8$$

$$= 1 - (0.171) - 0.171(1-0.171) - 0.171(1-0.171)^2 \ldots - 0.171(1-0.171)^8$$

$$= \cancel{1 - (0.5697)^3}$$

$\therefore$ PROB. BILL FOR 9 MIN OR LESS IS = 81%.

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1.6.2 EVENTS A & B ARE EQUIPROBABLE (3)

ARE MUTUALLY EXCLUSIVE (2)

ARE INDEPENDENT (1)

THUS WHAT IS $P[A]$

(3) $P[AB] = P[A]\ P[B]$

(2) $P[AB] = 0$

(1) $P[A] = P[B]$

THUS $0 = P[AB] = P[A]\,P[B] = P[A]\ P[A]$

$0 = P[A]\,P[A]$

$$\boxed{\begin{array}{l} \therefore\ 0 = P[A] \\ \&\ 0 \neq P[B] \end{array}}$$

## 1.6.6

**1.6.6** In an experiment, $C$ and $D$ are independent events with probabilities $P[C] = 5/8$ and $P[D] = 3/8$.

(a) Determine the probabilities $P[C \cap D]$, $P[C \cap D^c]$, and $P[C^c \cap D^c]$.

(b) Are $C^c$ and $D^c$ independent?

(A)

$$P[C \cap D] = P[C]\,P[D] = (5/8)(3/8) = 15/64$$

COMPLEMENT

$$P[C \cap D^c] = P[C] - P[C \cap D] = (5/8) - (15/64) = 25/64$$

$$P[C \cup D^c] = P[C] + P[D^c] - P[C \cap D^c]$$

$$= 5/8 + (1 - 3/8) - 25/64$$

$$P[C \cup D^c] = 55/64$$

(B) YES, BY DEF EVENTS C & D ARE INDEP. IFF $P[CD] = P[C]\,P[D]$

**EECS 461 Probability and Statistics**
Fall Semester 2022
Assignment **#2** Due **6 September 2022**

Reading: Sections 1.4-1.7, and 2.1-2.3 in Yates/Goodman

Do all of the Quizzes in the Reading assignment (including Quiz 1.7 on MATLAB), but do *not* hand them in. Answers to the Quizzes are on the book's website (search Yates Goodman Wiley)

For all problems from the book, you should use the method(s) from the corresponding section to solve the problem. For example, you should use a tree diagram (section 2.1) to solve problem 2.1.4.

1. Problem 1.4.2, part (a) only, p. 31.
2. Problem 1.4.4, p. 32.
3. Problem 1.5.2, p. 33.
4. Problem 1.6.2, p. 33.
5. Problem 1.6.6, p. 33.
6. Problem 2.1.4, p. 57.
7. Problem 2.1.6, p. 57.
8. Problem 2.2.6, p. 59.
9. Problem 2.2.12, p. 60.
10. Problem 2.3.2, p. 60. And by the way, the Celtics *DID* win 8 straight beginning in 1959 and *DID* win 10 of 11 starting in 1959! Bill Russell, who recently died, played on all of those teams (plus the 1957 NBA champion Celtics, as a rookie). Wilt Chamberlain played on the 1967 NBA champion Philadelphia 76ers. Boston did not make it to the finals that year.
11. Problem 2.3.4, p. 60. Express your answer in terms of $p$, which you know to be 0.5 or greater. Apologies for these last 2 problems to those of you who dislike sports.

  