EECS 461 PROBABILITY & STATISTICS

CONTINUOUS RANDOM VARIABLES & MULTIPLE RANDOM VARIABLES

PROBLEMS 1-5 THURS OCT 13 2022

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(1) PROBLEM 4.5.6 PAGE 156

4.5.6. X IS AN ERLANG (N, λ) RANDOM VARIABLE WITH PARAMETER $\lambda = 1/3$ AND EXPECTED VALUE $E[X] = 15$

$$f_X(x) = \begin{cases} \dfrac{\lambda^N x^{N-1} e^{-\lambda x}}{(N-1)!} & ; \quad x \geq 0 \\ 0 & ; \quad \text{OTHERWISE} \end{cases}$$

(A) WHAT IS THE VALUE OF THE PARAMETER N?

$$E(x) = \int_{-\infty}^{\infty} x f_X(x)\, dx = \int_0^{\infty} \frac{x \cdot \lambda^N x^{N-1} e^{-\lambda x}}{(N-1)!} dx \quad \overset{\text{SUBSTITUTE}}{\left[\begin{matrix} T = \lambda x \\ \lambda\, dx = dT \end{matrix} \right]} \quad = \frac{\lambda^N}{(N-1)!} \int_0^{\infty} x^N \cdot e^{-\lambda x} dx$$

$$= \frac{\cancel{\lambda^N}}{(N-1)!} \int_0^{\infty} \frac{x^N}{\cancel{\lambda^N}} \cdot e^{-T} \frac{dT}{\lambda} = \frac{1}{\lambda (N-1)!} \int_0^{\infty} T^N e^{-T} dT = \frac{1}{\lambda (N-1)!} \sqrt{N+1}$$

$$\frac{N}{\lambda} = \frac{N!}{\lambda (N-1)!}$$

IF $E(X) = 15$ THEN $\dfrac{N}{\lambda} \Rightarrow 15\left(\frac{1}{3}\right) = 5$, $\boxed{\text{PARAMETER VALUE IS } 5}$

(B) WHAT IS THE PDF OF X?

$$\boxed{f_X(x) = \begin{cases} \dfrac{x^4 e^{-x/3}}{3^5 \cdot 4!} & ; \quad x \geq 0 \\ 0 & ; \quad \text{OTHERWISE} \end{cases}}$$

(C) WHAT IS $\mathrm{VAR}[X]$?

$$\mathrm{VAR}[X] = \overset{?}{E(x^2)} - (\overset{15}{E(x)})^2$$

$$E(x^2) = \frac{1}{3^N \cdot 4!} \int_0^{\infty} x^6 \cdot e^{-x/3} dx = \overset{\text{SUBSTITUTE}}{\left[\begin{matrix} T = x/3 \\ dx = 3\,dT \end{matrix} \right]} = \frac{1}{3^5 \cdot 4!} \int_0^{\infty} 3T^6 \cdot e^{-T} \cdot 3\,dT$$

$$= \frac{9}{4!} \int_0^{\infty} \frac{T^6}{e^T} dT = \frac{9}{4!} (7!) = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 1890$$

$$\mathrm{VAR}[X] = E(x^2) - (E(x))^2 = 1890 - 225 = 1665$$

$$\boxed{\mathrm{VAR}[X] = 1665}$$

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2. FIND EACH OF THE FOLLOWING PROBABILITIES FOR A GAUSSIAN RANDOM VARIABLE. SHOW ALL WORK & GIVE NUMERICAL VALUES. THESE ARE GOOD RULES OF THUMB TO REMEMBER FOR GAUSSIAN RANDOM VARIABLES.

(A) THE PROBABILITY THAT THE RV IS WITHIN ONE STANDARD DEVIATION (EITHER WAY) FROM THE MEAN.

μ POPULATION MEAN

σ POPULATION STANDARD DEVIATION

$$\boxed{P((\mu - 1\sigma) < X < (\mu + 1\sigma)) = 68\%}$$

(B) THE PROBABILITY THAT THE RV IS WITHIN TWO STANDARD DEVIATIONS (EITHER WAY) FROM THE MEAN.

$$\boxed{P((\mu - 2\sigma) < X < (\mu + 2\sigma)) = 95\%}$$

(C) THE PROBABILITY THAT THE RV IS WITHIN THREE STANDARD DEVIATIONS (EITHER WAY) FROM THE MEAN.

$$\boxed{P((\mu - 3\sigma) < X < (\mu + 3\sigma)) = 99.7\%}$$

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(3) PROBLEM 4.6.4 PARTS (a) & (b) ONLY PAGE 158

4.6.4 IN EACH OF THE FOLLOWING CASES, Y IS A GAUSSIAN RANDOM VARIABLE. FIND THE EXPECTED VALUE $\mu = E[Y]$

(A) Y HAS A STANDARD DEVIATION $\sigma = 10$ & $P[Y \leq 10] = 0.933$

$$P[Y < y] = 0.933$$

$$\Rightarrow P\left(\frac{Y-\mu}{\sigma} < \frac{y_0 - \mu}{\sigma}\right) = 0.933$$

$$\frac{y_0 - \mu}{\sigma} = \text{INVNORM}(0.933)$$

$$\mu = 10 - 10(\text{INVNORM}(0.933))$$

$$\boxed{\mu = -4.98513}$$

(b) $P[Y > 5] = 1/2$

$\boxed{\mu = 5}$ BECAUSE THE DISTRIBUTION IS SYMMETRIC

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4. THE LIFETIME OF A COMPUTER CIRCUIT BOARD CAN BE MODELED AS A GAUSSIAN RANDOM VARIABLE WITH A MEAN OF 2500 HOURS. IF 95% OF THE BOARDS ARE TO LAST AT LEAST 2400 HOURS, WHAT IS THE LARGEST VARIANCE THAT THE RANDOM VARIABLE CAN HAVE?

$\mu = 2500$ USING NORMAL DISTRIBUTION $P[Z < z] = 0.05$ $1 = 0.95 + 0.05$

$x = 2400$

$$z = \frac{x - \mu}{\sigma}$$

$$-1.645 = \frac{x - \mu}{\sigma}$$

$$-1.645 = \frac{2400 - 2500}{\sigma}$$

$$\sigma = 60.79$$

$$\boxed{\sigma^2 = 3695.42}$$

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5. A MANUFACTURER OF PRECISION RESISTORS PRODUCES RESISTORS WHOSE VALUES FOLLOW A GAUSSIAN DISTRIBUTION WITH A MEAN OF 500 OHMS & A VARIANCE OF 2 OHMS. IN ADVERTISING LITERATURE, THE MANUFACTURER WANTS TO CLAIM "THE PROBABILITY OF ONE OF OUR RESISTORS HAVING A VALUE OUTSIDE THE RANGE FROM 500 - X OHMS TO 500 + X OHMS IS 1 IN A MILLION" WHAT SHOULD SHE USE FOR THE VALUE OF X?

$\boxed{X = 0}$ ANY VAL OF X THAT IS NOT ZERO WOULD BE GREATER THAN 0.000001 FOR MEAN & VAR GIVEN