EECS 461 Probability and Statistics
Fall Semester 2022
Assignment **#8** Due **18 October 2022** Back to Tuesday.

Reading: Sections 4.7 - 4.8, 5.1 in Yates/Goodman

Do all of the Quizzes in the Reading assignment but do *not* hand them in. Answers to the Quizzes are on the book's website (search Yates Goodman Wiley)

For all problems from the book, you should use the method(s) from the corresponding section to solve the problem.

1. For a 1-hour midterm exam, 80% of the students finish the exam and hand it in by the end of the hour, and 20% have to turn them in at the end of the hour without finishing the exam. For the students who finish by the end of the hour, their turn-in times are uniformly distributed between 45 and 60 minutes. Let T be the random variable of the turn-in times of the students, in minutes.
    a. Find the PDF of T.
    b. Find the CDF of T.
    c. Find the expected value of T.
2. Problem 4.7.6, p. 160.
3. Problem 5.1.4, p. 207. Hint: To show that it is a valid joint CDF, you need to show that all of the properties of Theorem 5.1 hold AND that the joint CDF is monotonic non-decreasing, which would involve Theorem 5.2.

EECS 461 PROBABILITY & STATISTICS

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TUES OCT 18 2022

MORGAN BERGEN

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1. FOR A 1-HOUR MIDTERM 80% OF THE STUDENTS FINISH THE EXAM & HAND IT IN BY THE END OF THE HOUR, 20% HAVE TO TURN THEM IN AT THE END OF THE HOUR WITHOUT FINISHING THE EXAM. FOR THE STUDENTS WHO FINISH BY THE END OF THE HOUR, THEIR TURN-IN TIMES ARE UNIFORMLY DISTRIBUTED BETWEEN 45 & 60 MINUTES. LET T BE THE RANDOM VARIABLE OF THE TURN IN TIMES OF THE STUDENTS, IN MINUTES.

A. FIND THE PDF OF T

PDF OF UNIFORM DISTRIBUTION IS AS FOLLOWS

$$\boxed{f(T) = \frac{1}{b-a} = \frac{1}{60-45} = \frac{1}{15}, \quad \text{IFF} \quad 45 \le T \le 60}$$

$b = 60$ & $a = 45$

B. FIND THE CDF OF T

$$\boxed{F(T \le t) = \int_{45}^{t} f(x)\,dx = \int_{45}^{t} \frac{1}{15}\,dx = \left[\frac{x}{25}\right]\Bigg|_{45}^{t} = \frac{t-45}{15}}$$

C. FIND THE EXPECTED VALUE OF T (WE KNOW $f(t) = 1/15$)

$$E[T] = \int_{45}^{60} t\, f(t)\,dt = \left[\frac{t^2}{30}\right]\Bigg|_{45}^{60} = \frac{(60 \cdot 60) - (45 \cdot 45)}{30} = 52.5$$

$$\boxed{E[T] = 52.5 \text{ MIN}}$$

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2. PROBLEM 4.7 6. P. 160

4.7.6. WHEN YOU MAKE A PHONE CALL THE LINE IS BUSY WITH PROBABILITY 0.2 & NO ONE ANSWERS WITH PROBAB 0.3. THE RANDOM VARIABLE X DESCRIBES THE CONVERSATION TIME (IN MINUTES) OF A PHONE CALL THAT IS ANSWERED. X IS AN EXPONENTIAL RANDOM VARIABLE WITH $E[X] = 3$ MINUTES. LET THE RANDOM VARIABLE W DENOTE THE CONVERSATION TIME (IN SECONDS) OF ALL CALLS ($W = 0$ WHEN THE LINE IS BUSY OR THERE IS NO ANSWER)

A. WHAT IS $F_W(w)$?

RANDOM VAR IS $W = \begin{cases} 60x, & \text{IF ALL CALLS ARE ANSWERED} \\ 0, & \text{OTHERWISE} \end{cases}$

EXPONENTIAL VAR IS $X_3 = \dfrac{1}{E[X_3]}$

FOR 1 MINUTE = 60 SECONDS

FOR 3 MINUTES = 180 SECONDS

$$F_x(w) = \begin{cases} \frac{1}{180} e^{-x/180}, & x \geq 0 \\ 0, & \text{OTHERWISE} \end{cases}$$

$\Downarrow$

$$F_x(w) = \begin{cases} 1 - e^{-x/180}, & x \geq 0 \\ 0, & \text{OTHERWISE} \end{cases}$$

$E[X] = 180$ SEC

$$F_W(w) = P(A^c) + P(A)\, F_{W/A}(w)$$

$P(A^c)$ - PROBABILITY OF THE PHONE CALL THAT IS ANSWERED

$P(A)$ - PROBABILITY OF PHONE CALL NOT ANSWERED OR BUSY

$$F_W(w) = 0.5 + 0.5 f_x(w)$$

$$\boxed{\text{CDF} \quad F_W(w) = \begin{cases} 0.5 + 0.5 f_w(w), & w \geq 0 \\ 0, & w < 0 \end{cases}}$$

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4.7.6. WHEN YOU MAKE A PHONE CALL THE LINE IS BUSY WITH PROBABILITY 0.2 & NO ONE ANSWERS WITH PROBAB 0.3. THE RANDOM VARIABLE X DESCRIBES THE CONVERSATION TIME (IN MINUTES) OF A PHONE CALL THAT IS ANSWERED. X IS AN EXPONENTIAL RANDOM VARIABLE WITH $E[X] = 3$ MINUTES. LET THE RANDOM VARIABLE W DENOTE THE CONVERSATION TIME (IN SECONDS) OF ALL CALLS ($W = 0$ WHEN THE LINE IS BUSY OR THERE IS NO ANSWER)

B. $f_W(w)$?

PDF
$$f_W(w) = \begin{cases} \frac{1}{2}\delta(w) + \frac{1}{2} f_X(w), & x \geq 0 \\ 0, & \text{OTHERWISE} \end{cases}$$

$$f_W(w) = \frac{1}{2}\delta(w) + \frac{1}{2} f_X(w)$$

$$E[W] = \int_{\infty}^{\infty} w f_X(w)\, dw$$

$$= \frac{1}{2} E[X] = \frac{180}{2}$$

$$\boxed{E[W] = 90}$$

C. WHAT ARE $E[W]$ & $VAR[W]$

$$E[W^2] = \int_{\infty}^{\infty} w^2 f_W(w)\, dw = \frac{1}{2}\int_{-\infty}^{\infty} w^2 f_W(w)\, dw$$

$$\boxed{E[W^2] = \frac{1}{2} \frac{E[W^2]}{2}}$$

$$VAR[W] = E[W^2] - (E[W])^2$$

$$\boxed{VAR[W] = \frac{1}{2} V(X) + \left(\frac{E[X]}{2}\right)^2}$$

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3. PROBLEM 5.1.4

5.1.4 RANDOM VARIABLES X & Y HAVE CDF $F_X(x)$ & $F_Y(y)$
IS $F(x, y) = F_X(x) F_Y(y)$ A VALID CDF?
EXPLAIN YOUR ANSWER

$$f_Y(y) = \frac{D}{DY} F_Y(y)$$

$$f_X(x) = \frac{D}{DX} F_X(x)$$

THEOREM 5.1 STATES THAT $\forall$ RANDOM VARIABLE XY

(A) $0 \leq F_{X,Y}(x, y) \leq 1$
(B) $F_{X,Y}(\infty, \infty) = 1$
(C) $F_X(x) = F_{X,Y}(x, \infty)$
(D) $F_Y(y) = F_{X,Y}(\infty, y)$
(E) $F_{X,Y}(x, -\infty) = 0$
(F) $F_{X,Y}(-\infty, y) = 0$
(G) IF $x \leq x_1$ & $y \leq y_1$ THEN $F_{X,Y}(x_1, y_1)$

$$f_{X,Y}(x, y) = f_X(x) f_Y(y)$$

$$F_{X,Y}(x, y) = \iint f_{X,Y}(x, y) \, DX DY = F_X(x) F_Y(y)$$

BOTH $F_X(x)$ & $F_Y(y)$ ARE INDEPENDENT BECAUSE IT SATISFIES THE TWO D CDF