EECS 461 Probability and Statistics

Fall Semester 2022

Assignment **#11** Due **8 November 2022** (Yes, this is the day after the midterm exam)

Reading: Sections 5.9 - 5.10 in Yates/Goodman

Do all of the Quizzes in the Reading assignment but do *not* hand them in. Answers to the Quizzes are on the book's website (search Yates Goodman Wiley)

For all problems from the book, you should use the method(s) from the corresponding section to solve the problem.

1. A random voltage is measured at 2 time instants. Let the RVs X and Y represent those 2 measurements. Both X and Y are Gaussian with mean=0 and variance=4 watts. These 2 measurements are determined to be uncorrelated. Write the joint PDF of the 2 measurements.
2. Problem 5.9.10, p. 215.
3. Problem 5.10.2, p. 215.

EECS 461 PROBABILITY & STOCHASTIC PROCESSES

CH 5 MULTIPLE RANDOM VARIABLES — 5.9 – 5.10

MORGAN BERGEN TUES NOV 1 2022

1. A RANDOM VOLTAGE IS MEASURED AT 2 TIME INSTANTS. LET THE RVs X & Y REPRESENT THOSE 2 MEASUREMENTS. BOTH X & Y ARE GAUSSIAN WITH MEAN = 0 AND VARIANCE = 4 WATTS THESE TWO MEASUREMENTS ARE DETERMINED TO BE UNCORRELATED. WRITE THE JOINT PDF OF THE 2 MEASUREMENTS.

THE JOINT PDF MUST LOOK AS FOLLOWS

$$F_{X,Y}(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx\,dy = 1$$

IF UNCORRELATED $\quad \text{COV}[X,Y] = 0 = \rho_{X,Y}$

IF MEAN IS 0 $\quad \mu_X = E[X] = 0, \quad \mu_Y = E[Y] = 0$

IF VARIANCE IS 4 $\quad \sigma_X^2 = \text{VAR}[X] = 4, \quad \sigma_Y^2 = \text{VAR}[Y] = 4$

$\sigma_X = 2 \qquad \sigma_Y = 2$

THUS FOR THE PDF IS GIVEN BY

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}\exp\left[\frac{-1}{2(1-\rho^2)}S\right]$$

$$= \frac{1}{2\pi(2)(2)\sqrt{1-0^2}}\exp\left[-\frac{1}{2(1-0^2)}\left[\left(\frac{x-\mu_X}{\sigma_X}\right)^2+\left(\frac{y-\mu_Y}{\sigma_Y}\right)^2-\frac{2\rho(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y}\right]\right]$$

$$= \frac{1}{2\pi(2)(2)\sqrt{1-0^2}}\exp\left[-\frac{1}{2(1-0^2)}\left[\left(\frac{x-\mu_X}{\sigma_X}\right)^2+\left(\frac{y-\mu_Y}{\sigma_Y}\right)^2-\frac{2\rho(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y}\right]\right]$$

$$\boxed{f_{X,Y}(x,y) = \frac{1}{8\pi\sqrt{1}}\exp\left[-\frac{1}{2}\left[\left(\frac{x}{2}\right)^2+\left(\frac{y}{2}\right)^2\right]\right] \quad \therefore}$$

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2. PROBLEM 5.9.10 PAGE 215

UNDER WHAT CONDITIONS ON THE CONSTANTS, a, b, c, d IS $f(x,y)$ A JOINT GAUSSIAN PDF?

$$f(x,y) = de^{-(a^2x^2 + bxy + c^2y^2)}$$

∴ ACCORDING TO DEF 5.10 A GAUSSIAN PDF WITH PARAMETERS a, b, c, d ALL MUST CONFORM TO THE CONDITIONS THAT a, b, c, d MUST BE EQUAL THE EXPECTED VALUES, STANDARD DEVIATIONS, AND CORRELATION COEFFICIENT OF X & Y, $a, c, d > 0$ & $-1 < b \leq 1$

3. PROBLEM 5.10.2 PAGE 215

WHEN ORDERING A PERSONAL COMPUTER, A CUSTOMER CAN ADD THE FOLLOWING FEATURES TO THE BASIC CONFIGURATION: (1) ADDITIONAL MEMORY (2) FLAT PANEL DISPLAY (3) PROFESSIONAL SOFTWARE (4) WIRELESS MODEM A RANDOM COMPUTER ORDER HAS FEATURE i WITH PROBABILITY $p_i = 2^{-i}$ INDEPENDENT OF OTHER FEATURES. IN AN HOUR IN WHICH THREE COMPUTERS ARE ORDERED, LET N_i EQUAL THE NUMBER OF COMPUTERS WITH FEATURE i.

(A) FIND THE JOINT PMF $P_{N_1,N_2,N_3,N_4}(n_1,n_2,n_3,n_4)$

$$P_{N_i}(n_i) = p_i = 2^{-i}$$

$$P_{N_1}(n_i) = 2^{-1}, \quad P_{N_2}(n_2) = 2^{-2}, \quad P_{N_3}(n_3) = 2^{-3}, \quad P_{N_4}(n_4) = 2^{-4}$$

$$P_{N_1N_2N_3N_4}(n_1,n_2,n_3,n_4) = P_{N_1}(n_1)P_{N_2}(n_2)P_{N_3}(n_3)P_{N_4}(n_4) = 2^{-1}\cdot 2^{-2}\cdot 2^{-3}\cdot 2^{-4}$$

∴ $P_{N_1N_2N_3N_4}(n_1,n_2,n_3,n_4) = 2^{-10}$

(B) WHAT IS THE PROBABILITY OF SELLING A COMPUTER WITH NO ADDITIONAL FEATURES

$$1 - P_{N_i}(n_i) = 1 - 2^{-i}$$

$$1 - P_{N_i}(n_i) = \prod_{i=1}^{4}\left(1 - P_{N_i}(n_i)\right) = \left(1 - P_{N_1}(n_1)\right)\left(1 - P_{N_2}(n_2)\right)\left(1 - P_{N_3}(n_3)\right)\left(1 - P_{N_4}(n_4)\right)$$

$$= \left(1 - 2^{-1}\right)\left(1 - 2^{-2}\right)\left(1 - 2^{-3}\right)\left(1 - 2^{-4}\right)$$

$$\approx 0.307617$$

∴ THUS PROB OF SELLING A COMP WITH NO ADDITIONAL FEATURES 0.307617

(C) WHAT IS THE PROBABILITY OF SELLING A COMPUTER WITH AT LEAST THREE ADDITIONAL FEATURES?

$$P_{N_1N_2N_3}(n_1,n_2,n_3) + P_{N_1N_2N_4}(n_1,n_2,n_4) + P_{N_1N_3N_4}(n_1,n_3,n_4) + P_{N_2N_3N_4}(n_1,n_2,n_3,n_4)$$

$$\Rightarrow \left(2^{-1}2^{-2}2^{-3}\right) + \left(2^{-1}2^{-2}2^{-4}\right) + \left(2^{-1}2^{-3}2^{-4}\right) + \left(2^{-2}2^{-3}2^{-4}\right) + \left(2^{-10}\right)$$

$$\approx 0.0303$$

∴ THUS PROB OF SELLING A COMP WITH AT LEAST THREE ADDITIONAL FEATURES 0.0303