EECS 461 PROBABILITY & STATISTICS

CONTINUOUS RANDOM VARIABLES & PAIRS OF RANDOM VARIABLES

HOMEWORK 06

OCTOBER 1ST 2022

MORGAN BERGEN

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1. CONSIDER A DISCRETE RANDOM VAR (DRV) H WITH PROBABILITY MASS FUNCTION (PMF) GIVEN BY:

h	0	1	2	3	4
$P_H(h)$	0.1	0.3	0.2	0.2	0.2

WE WILL BE CONSIDERING SEVERAL POSSIBLE VALUES FOR A BLIND ESTIMATE (ESTIMATE WITHOUT OBSERVATION) FOR THIS RANDOM VARIABLE. FOR EACH CASE BELOW FIND...

PROBABILITY MASS FUNCTION PMJ

(i) THE VALUE OF THE MEAN SQUARED ERROR (MSE) OF THE ESTIMATE

~~DISCRETE~~ RANOM VARIABLE DRV

$RV := \{X \mid x \in \text{RANGE}(x)\}$

S ASSIGNING 0 & 1 TO SETS A IN S

$\forall A \subset S$ THE MODEL GIVES US $P[A] \mid 0 \leq P[A] \leq 1$

WHEN WE OBSERVE ONE OF THESE NUMBERS

THE SET OF $\forall \exists$ OBSERVATIONS S IS THE SAMPLE SPACE

PROBABILITY FUNCTIONS COMPRISING OF

$\mathbb{R}^{\pm}$ ASSIGNMENTS TO OUTCOME $\in S$

REF. TO THESE $\mathbb{R}^{\pm}$ WE REFER THE OBSERVATION AS A RANDOM VARIABLE X

$S_x \in X \parallel S_y \in Y$ RANGE

PROB MODELS ALWAY BEGIN WITH AN EXPERIMENT $\forall X \rightarrow$ RELATED TO AN EXP.

$\therefore$ 3 TYPES OF $\rightarrow$ RELATIONSHIPS ARE AS FOLLOWS

1. RV IS THE OBSERVATION $RV = RV$

   RV = OBSERVATION OF OUTCOME

   AN ELEM OF S_X

2. RV IS A FUNCTION OF THE OBSERVATION

   $P[\cdot]$ DEFINED ON S AS THE FUNC()

LET $RV = X$ BY THE FUNC $X(s)$ THAT MAPS THE SAMPLE OUTCOMES TO THE $\mathbb{R}^{\pm}$ OF RV
$\{X = x\}$ $\exists$ A SET OF SAMPLE PTS. $s \in S$ FOR WHICH $X(s) = x$

$$\Rightarrow \{X = x\} = \{s \in S \mid X(s) = x\}$$

PMF OF RV X EXPRESSES THE PROBABILITY MODEL OF AN EXPERIMENT AS A MATH. FUNC(). THE FUNC() IS THE PROBABILITY $P[X = x]$ $\forall$ $\mathbb{R}^{+} x$

RECALL DUR = $\mathbb{R}^{\pm}$ $0 \leq P[A] \leq 1$ TO $\forall \exists$ OUTCOME $\in S$ WHEN WE HAVE A DISCRETE

$$P_X(x) = P[X = x]$$

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$X = x$:= AN EVENT CONSISTING OF ALL OUTCOMES OF S OF THE UNDERLYING EXPERIMENT FOR WHICH $X(s) = x$.

X := SET OF EVENTS / A SET OF SETS OF OUTCOMES

OUTCOME IS AN OBSERVATION

EVENT IS A SET OF OUTCOMES

FOR UNDERLYING E

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(i) THE VALUE OF THE MEAN SQUARED ERROR (MSE) OF THE ESTIMATE

(ii) THE PROBABILITY THAT THE ESTIMATE WILL BE CORRECT

MINIMUM MEAN SQUARED ERROR ESTIMATE OF RANDOM VAR X

$$P_H(h) = \begin{cases} 0.1 & h = 0 \\ 0.3 & h = 1 \\ 0.2 & h = 2, 3, 4 \\ 0 & \text{OTHERWISE} \end{cases}$$

FIND THE VALUE OF THE MEAN SQUARED ERROR OF THE ESTIMATE

$$MSE = Var[H] = E[(X - \mu_X)^2]$$

EXPECTED VALUE

VARIANCE

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WE WILL BE CONSIDERING SEVERAL POSSIBLE VALUES FOR A BLIND ESTIMATE (ESTIMATE WITHOUT OBSERVATION) FOR THIS RANDOM VARIABLE. FOR EACH CASE BELOW FIND...

(i) THE VALUE OF THE MEAN SQUARED ERROR (MSE) OF THE ESTIMATE

We will be considering several possible values for a blind estimate (estimate without observation) for this random variable. For each case below, find (i) the value of the mean squared error (MSE) of the estimate and (ii) the probability that the estimate will be correct.

a. In this first case, use the value of the blind estimate that minimizes the mean squared error (MSE).

b. Now use the median of H as the blind estimate.

c. Finally, use the mode of H as the blind estimate.

d. Summarize your results in a table. Which estimate do you think is the "best"?

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2 IS THE FOLLOWING A VALID CUMULATIVE DISTRIBUTION FUNCTION (CDF) FOR A CONTINUOUS RANDOM VARIABLE (CRV) T?

$$F_T(t) = \frac{t^2 + t}{t^2 + 1} \quad \text{FOR} \quad 0 \leq t \leq \infty$$

$F_T(t)$ IS 0 ELSEWHERE. JUSTIFY YOUR ANSWER

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3 THE CDF OF THE CONTINUOUS RANDOM VARIABLE W IS

$$F_W(w) = \begin{cases} 0 & w < -5 \\ \frac{w+5}{8} & -5 \le w < -3 \\ \frac{1}{4} & -3 \le w < 3 \\ \frac{1}{4} + \frac{3(w-3)}{8} & 3 \le w < 5 \\ 1 & w \ge 5 \end{cases}$$

SKETCH THE CDF BEFORE ATTEMPTING THE PARTS OF THE PROBLEM

(A) WHAT IS $P[W \le 4]$?

(B) WHAT IS $P[-2 < W \le 2]$

(C) WHAT IS $P[W > 0]$?

(D) WHAT IS THE VALUE OF A SUCH THAT $P[W \le A] = 1/2$?

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4. CONSIDER THE FOLLOWING CDF FOR CONTINUOUS RANDOM VARIABLE G

$$F_G(g) = K(g+2) \quad \text{FOR} \quad -2 < g \leq \quad \text{FOR SOME CONSTANT } K$$

FOR $g \leq -2$, $F_G(g) = 0$, & FOR $g > 3$, $F_G(g) = 1$

(A) FIND THE VALUE OF K & SKETCH THE RESULTING CDF

(B) FIND $P[1 < G \leq 2.5]$ USING THIS CDF

(C) FIND $P[G=1]$

(D) FIND $f_G(g)$, THE PROBABILITY DENSITY FUNCTION (PDF) FOR G, & SKETCH IT

(E) FIND $P[1 < G \leq 2.5]$ USING THIS PDF

(F) CALCULATE THE MEAN & STANDARD DEVIATION OF G DIRECTLY FROM THE PDF

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5. A CONTINUOUS RANDOM VARIABLE B HAS PDF GIVEN BY:

$f_B(b) = b^2 + Kb + 1$ FOR $0 < b \leq 2$ FOR SOME CONSTANT K

$f_B(b) = 0$ OTHERWISE

(A) FIND THE VALUE OF K & SKETCH THE RESULTING PDF. YOU MAY WANT TO PLOT IT WITH MATLAB FIRST; IF YOU DO YOU CAN JUST SUBMIT A PRINTOUT OF THAT PLOT

(B) FIND THE MEAN & STANDARD DEVIATION OF B

(C) FIND & SKETCH/PLOT THE CDF OF B

(6) THE LIFE OF A CERTAIN KIND OF BATTERY IS AN EXPONENTIALLY DISTRIBUTED RANDOM VARIABLE WITH MEAN OF 250 HOURS. WHAT IS THE PROBABILITY THAT SUCH A BATTERY WILL LAST AT MOST 200 HOURS?