PROBABILITY & STOCTASTIC

MORGAN BERGEN

SEPTEMBER 15 2022

QUIZ 4.3

RANDOM VARIABLE X HAS THE PROBABILITY DENSITY FUNCTION

$$f_X(x) = \begin{cases} cxe^{-x/2} & x \geq 0 \\ 0 & \text{OTHERWISE} \end{cases}$$

SKETCH PDF,

$$1 = \int_{-\infty}^{\infty} f_X(x)\,dx = \int_0^{\infty} cxe^{-x/2}\,dx$$

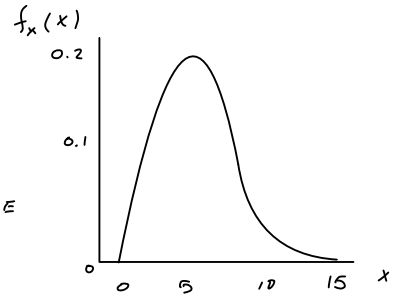
$$1 = -2cxe^{-x/2}\Big|_0^{\infty} + \int_0^{\infty} 2ce^{-x/2}\,dx$$

$$= -4ce^{-x/2}\Big|_0^{\infty}$$

$$1 = 4c$$

$$c = 1/4$$

THEREFORE, $f_X(x) = \begin{cases} (x/4)e^{-x/2} & x \geq 0 \\ 0 & \text{OTHERWISE} \end{cases}$



$N = 2$

$\lambda = 1/2$

(B) FIND THE CDF $F_X(x)$

TO FIND THE CDF $F_X(x)$ WE NOTE X IS A NONNEGATIVE RANDOM VARIABLE SO THAT $F_X(x) = 0$ FOR ALL $x < 0$, FOR $x \geq 0$

$$F_X(x) = \int_0^x f_X(y)\,dy = \int_0^y \frac{y}{4} e^{-y/2}\,dy = -\frac{y}{2}e^{-y/2}\Big|_0^x + \int_0^x \frac{1}{2} e^{-y/2}\,dy$$

$$= 1 - \frac{x}{2}e^{-x/2} - e^{-x/2}$$

THE COMPLETE EXPRESSION FOR THE CDF IS,