

EECS 461 Probability and Statistics
Fall Semester 2022
Assignment #10 COMPLETE NOW Due 1 November 2022

Reading: Sections 5.6 - 5.9 in Yates/Goodman (This is the *entire* reading assignment)

Do all of the Quizzes in the Reading assignment but do *not* hand them in. Answers to the Quizzes are on the book's website (search Yates Goodman Wiley)

For all problems from the book, you should use the method(s) from the corresponding section to solve the problem.

1. Recall the air support roof example from class, in which G represented inside barometric pressure and H represented outside barometric pressure. The joint PDF was given as:

$$f_{G,H}(g, h) = c/g \text{ for } 27 \leq h \leq g \leq 33$$

and 0 otherwise, with c approximately 1.724. The G and H marginals were derived in class. For each part below, show your work and give numerical values for all of your answers.

- a. Find $E[G]$ and $E[H]$.
 - b. Find $E[GH]$ and $\text{Cov}[G, H]$.
 - c. Find the variance of both G and H .
 - d. Find the variance of $(G + H)$.
 - e. Find the correlation coefficient $\rho_{G,H}$.
 - f. Find $E[G - H]$ and give a practical physical interpretation for this expectation.
2. Manufacture of a widget requires welding 2 joints and tightening 3 bolts. Let X be the number of defective welds and Y be the number of improperly tightened bolts. From past experience, the joint PMF is given below.

| --- | $x = 0$ | $x = 1$ | $x = 2$ | $x = 3$ |
|---------|---------|---------|---------|---------|
| $y = 0$ | 0.840 | 0.030 | 0.020 | 0.010 |
| $y = 1$ | 0.060 | 0.010 | 0.008 | 0.002 |
| $y = 2$ | 0.010 | 0.005 | 0.004 | 0.001 |

- a. Find the expected number of defective welds, the expected number of improperly tightened bolts, and the mean total number of manufacturing problems.
- b. Find the correlation (not correlation coefficient) and covariance of X and Y .
- c. Find the variance of both X and Y .
- d. Find the correlation coefficient $\rho_{X,Y}$.

- e. Are X and Y independent? Justify your answer mathematically.
3. The length L and width W of a rectangle have joint PDF given by:
 $f_{L,W}(l, w) = 2e^{-(l+2w)}$ for $l \geq 0, w \geq 0$ and 0 otherwise.
- a. Find the correlation of L and W : $E[LW]$, which is also the expected area of the rectangle.
- b. Are L and W independent? Justify your answer mathematically.
4. A random voltage is measured at 2 time instants. Let the RVs X and Y represent those 2 measurements. Both X and Y are Gaussian with mean=0 and variance=4 watts. These 2 measurements are determined to be uncorrelated. Write the joint PDF of the 2 measurements.