EECS 461 PROBABILITY & STATISTICS

TUES SEPT 12 2022

ASSIGNMENT # 3

MORGAN BERGEN

2.1.4 You have two biased coins.

Coin A comes up heads with probability 1/4.

Coin B comes up heads with probability 3/4.

However you are not sure which is which, so you choose a coin randomly and you flip it.

If the flip is heads, you guess that the flipped coin is B; otherwise you guess that the flipped coin is A.

What is the probability P[C] that your guess is correct?

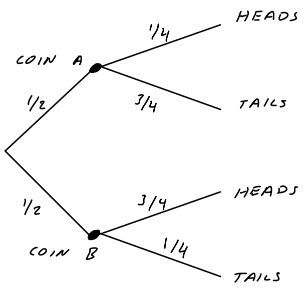
WE KNOW THAT THE CHANCE OF AN EVENT OCCURING WILL BE EQUAL TO 3/4

THIS QUESTION IS IN REGARDS TO A SEQUENTIAL EXPERIMENT

WE WILL CONSTRUCT A SAMPLE TREE TO FIND THE CONDITIONAL PROBABILITY

B H P[C] PROBABILITY IS CORRECT

A T



$C_A H$ 1/8

$C_A T$ 3/8

$C_B H$ 3/8

$C_B T$ 1/8

$$P[C] = P[C_B H \; C_A T]$$

$$= P[C_B H] + P[C_A T]$$

$$= 3/8 + 3/8$$

$$P[C] = 3/4$$

PROBABILITY THAT YOU ARE CORRECT IS 75%

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2.1.6 A machine produces photo detectors in pairs. Tests show that the first photo detector is acceptable with probability 3/5. When the first photo detector is acceptable, the second photo detector is acceptable with probability 4/5. If the first photo detector is defective, the second photo detector is acceptable with probability 2/5.

(a) Find the probability that exactly one photo detector of a pair is acceptable.

(b) Find the probability that both photo detectors in a pair are defective.

A)

P[ONE PHOTO DETECTOR PAIR IS ACCEPTED] =

P[FIRST PHOTO DETECTOR IS ACCEPTED & SECOND IS DETECTED] +

P[FIRST PHOTO DETECTOR IS ACCEPTED & SECOND IS DETECTED]

$P[PD_1 \ A \ \& \ PD_2 \ D] = \frac{3}{5}\left(1 - \frac{4}{5}\right) = 0.12$

$P[PD_1 \ A] = \frac{3}{5}$

$P[PD_2 \ A \ \text{IFF} \ PD_1 \ A] = \frac{4}{5}$

$P[PD_1 \ D \ \& \ PD_2 \ A] = \left(1 - \frac{3}{5}\right)\frac{2}{5} = 0.16$

$P[PD_2 \ A \ \text{IFF} \ PD_2 D] = \frac{2}{5}$

$P[PD_N \ A \ \text{IFF} \ PD_N \ A] = 0.12 + 0.16 = 0.28$

PROBABILITY THAT EXACTLY ONE PAIR IS ACCEPTABLE IS 28%.

0.12 + 0.16 = 0.28

(B) $P[PD_1 \ D \ \& \ PD_2 \ D] = P[PD_1 \ D] * P[PD_2 \ D] = \frac{2}{5} \cdot \frac{3}{5} = 0.24$

$P[PD_1 \ D] = \left(1 - \frac{3}{5}\right) = \frac{2}{5}$

$P[PD_2 \ D] = \left(1 - \frac{2}{5}\right) = \frac{3}{5}$

24% PROBABILITY THAT BOTH ARE DEFECTIVE

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2.2.6 In a game of poker, you are dealt a five-card hand.

(a) What is the probability [P| Rs] that your hand has only red cards?
(b) What is the probability of full house" with three-of-a-kind and two-of-a-kind

TOTAL NO. OF CARDS = 52

C	C	C	C	C	C	C	C	C	C	C	C	C
H	H	H	H	H	H	H	H	H	H	H	H	H
D	D	D	D	D	D	D	D	D	D	D	D	D
S	S	S	S	S	S	S	S	S	S	S	S	S
A_R/A_B	K_R/K_B	Q_R/Q_B	J_R/J_B	$10_R/10_B$	$9_R/9_B$	$8_R/8_B$	$7_R/7_B$	$6_R/6_B$	$5_R/5_B$	$4_R/4_B$	$3_R/3_B$	$2_R/2_B$

13 VALUES

4 SUITS

2 COLORS

S = 13 BLACK CARDS   S + C = 26

C = 13 BLACK CARDS

H = 13 RED CARDS   H + D = 26

D = 13 RED CARDS

$$P[\text{RED HAND} \mid S] = \frac{\text{NO. OF OUTCOMES RED}}{\text{NO. OF OUTCOMES}} = \frac{657280}{2598960} = 0.0253 = \boxed{2.53\%}$$

26 CHOOSES 5 $= \dfrac{26!}{5!\,(21)!} = 657280$

52 CHOOSES 5 $= \dfrac{52!}{5!\,(47!)} = 2598960$

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2.2.6 In a game of poker, you are dealt a five-card hand.

(a) What is the probability [P| Rs] that your hand has only red cards?
(b) What is the probability of full house" with three-of-a-kind and two-of-a-kind

(B)

A FULL HOUSE IS 3 OF A KIND

K K K 2 2

13 VALUES $\xrightarrow{\text{THEN}}$ 12 VALUES

3 OUT OF 4 2 OUT OF 4

$$13 \cdot \binom{4}{3} \cdot 12 \cdot \binom{4}{2}$$

$$\frac{\text{NO. OF OUTCOMES FOR FULL HOUSE}}{\text{NO. OF OUTCOMES}} = \frac{13 \cdot \binom{4}{3} \cdot 12\binom{4}{2}}{2598960} = \frac{3744}{2598960} = 0.00144$$

$$= 0.144\%$$

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2.2.12 A basketball team has three pure centers, four pure forwards, four pure guards, and one swingman who can play either guard or forward. A pure position player can play only the designated position. If the coach must start a lineup with one center, two forwards, and two guards, how many possible lineups can the coach choose?

LINE UP SWINGPERSON GUARD $= \binom{3}{1} \cdot \binom{4}{2} \cdot \binom{4}{1} = 72$

$+$

LINEUP NO SWING PERSON $= \binom{3}{1} \cdot \binom{4}{2} \cdot \binom{4}{2} = 108$

$+$

LINEUP SWING PERSON $= \binom{3}{1}\binom{4}{1}\binom{4}{2} = 72$

$= \boxed{252 \text{ POSSIBLE LINEUPS}}$

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2.3.2 The Boston Celtics have won 16 NBA championships over approximately 50 years. Thus it may seem reasonable to assume that in a given year the Celtics win the title with probability p = 16/50 = 0.32, independent of any other year. Given such a model, what would be the probability model, what would be the probability of the Celtics winning eight straight championships beginning in 1959? Also, what would be the probability of the Celtics winning the title in 10 out of 11 years, starting in 1959? Given your answers, do you trust this simple probability model?

$P[C_W] = 0.32$

$P[C_L] = 1 - 0.32 = 0.68$

$P[C_W]^8 = 0.32^8 = 0.0001$

$(P[C_W])^{10} \cdot 11 \cdot 0.68 = 0.32^{10} \cdot 11 \cdot 0.68 = \boxed{0.0000084 \text{ PROB}}$

I DO NOT TRUST THIS MODEL BECAUSE IT MODELS AN ISOLATED SYSTEM ONE THAT DOES NOT ACCOUNT FOR THE (IE QUALITY OF COACH, INDIVIDUAL PLAYERS, WHEATHER THESE WINNINGS WILL BE AT HOME OR AWAY, ETC.)

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2.3.4 In a game between two equal teams, the home team wins with probability p > 1/2. In a best of three playoff series, a team with the home advantage has a game at home, followed by a game away, followed by a home game if necessary. The series is over as soon as one team wins two games. What is P[H], the probability that the team with the home advantage wins the series? Is the home advantage increased by playing a three-game series rather than a one-game playoff? That is, is it true that P|H] ≥ p for all p > 1/2?

3 PERMUTATIONS {WW, WLW, LWW}

$P[p > 1/2] = (1-p)$

$P[H] = p \cdot p + p(1-p) \cdot p + (1-p) \cdot p \cdot p = p^2 + 2p^2(1-p)$

$P[H] > p > 1/2 \rightarrow P[H] = 3p^2 - 2p^3$

IF $p = 1/2$, $P[H] \cdot p = 0.5 \Rightarrow 3(1/2)^2 - 2(1/2)^3 = 1/2$

IF $p > 1/2$ THEN $P[H] > 1/2$, YES

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2.5.2 Following Quiz 2.3, suppose the communication link has different error probabilities for transmitting 0 and 1. When a 1 is sent, it is received as a 0 with probability 0.01. When a 0 is sent, it is received as a 1 with probability 0.03. Each bit in a packet is still equally likely to be a 0 or 1. Packets have been coded such that if five or fewer bits are received in error, then the packet can be decoded. Simulate the transmission of 100 packets, each contain- ing 100 bits. Count the number of packets decoded correctly.

(A)

LET S = SENT , R = RECIEVED

GIVEN $P[0S] = P[1S] = 0.5$

$$P[1S \mid 1R] = \frac{P[1S \mid 1R]}{P[1S]} = \frac{P[1S \mid 1R]}{P[1S \mid 1R] + P[1S \mid 0R]} = \frac{(0.95)(0.5)}{(0.95)(0.5) + (0.05 \cdot 0.5)} = 0.95 \Rightarrow \boxed{95\%}$$

(B)

$$P[0S \mid 0R] = \frac{P[0S \mid 0R]}{P[0S]} = \frac{P[0S \mid 0R]}{P[0S \mid 0R] + P[0S \mid 1R]} = \frac{(0.99 \cdot 0.5)}{(0.99 \cdot 0.5) + (0.01 \cdot 0.5)} = 0.99 \rightarrow \boxed{99\%}$$