EECS 461 Probability and Statistics

Fall Semester 2022

Assignment #1 Due 30 August 2022

Morgan Bergen

Reading: EECS 461 Syllabus plus "A Message to Students from the Authors" (p. xi) and Chapter 1 in Yates/Goodman

Do all of the Quizzes in the Reading assignment (including Quiz 1.7 on MATLAB), but do not hand them in. Answers to the Quizzes are on the book's website (search Yates Goodman Wiley)

The following questions are intended to ensure that you have read and understood the information in the course information sheet (syllabus) that was handed out on the first day of class (the syllabus is also posted on Canvas).

1. **Send an email message to me with "461 your last name" in the subject line. You may include any questions that you might have, but do not include your answers to any of the other questions/problems of this assignment or of Assignment 0; those answers must be turned in on paper on the due date.**

2. **Where is my office and what are my office hours?**

Where: 2001C Eaton

When: Thursday from 1:30 - 2:30 pm, Wednesday from 3:00 - 4:00 pm, or via appt.

3. **Suppose you have the following scores - what will your course score be?**

   Homework Average: 85%

   Quiz Average: 75%

   Midterm Exam: 65 out of 80 possible points

   Final Exam: 96 out of 120 possible points

4. **If you have a course score of 90 Course Points (out of the 102 possible), will you get an A- for the course?**
5. **What material are you allowed to bring to exams? What material will be provided to you?**

The materials you will provide are reference sheets (e.g. tables).

The materials I may provide are,

Midterm exam - one one-sided 8 x 11 with ½ inch margined sheet of notes

Final exam - one two-sided 8 x 11 with ½ inch margined sheet of notes

6. **What must you do if you will not be able to attend one of the exams?**

Contact Dr. Petr via email, phone, in-person in advance and provide documentation (the documentation can be provided after the exam if need be).

7. **How will you get solutions for the weekly homework assignments?**

The text book companion website www.wiley.com/college/yates

The url I that ^ link redirects to is then this for a breakdown of sections

https://bcs.wiley.com/he-bcs/Books?action=index&itemId=1118324560&bcsId=8677

8. **What will we do during the Monday afternoon discussion sessions?**

On Monday Nov 7 we will be taking the midterm exam (tentative). However on regular days we will be reviewing over problems, reviewing over material, and be given the opportunity to ask questions regarding the course material, example problems, and homework problems. This will be done in an informal group setting where we get some problem solving practice.

9. **What are the dates (some tentative at this point) for the Midterm Exam and the Final Exam? It would be a good idea to put them on your calendar/planner/phone *now*.**

Midterm Exam 1 - Nov 7 2022 5:00 - 7:00 pm
Final Exam 1 - Dec 10 2022 at 1:30 - 4:00 pm

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The following questions are on the reading assignment for this week. For book problems, note that Problem X.Y. indicates that the question has to do with material from chapter X, section Y.

10. **What is the main message that you got from "A Message to Students from the Author**
11. **Problem 1.1.2, p. 29 in Yates/Goodman.**
12. **Problem 1.2.2, p. 30. For parts c, d, f, and g, give an explanation in addition to the yes/no answer.**
13. **Problem 1.2.6, p. 30. Note that the sample space is uncountably infinite. To make your life easier, make each partition consist of just 2 outcomes (sets). Give a word description of how the partition might be useful. For example, if we are interested in determining that neither resistor value is too high, a partition could be A, = (R, <100, R2 < 100) and A2 = (R, ≥100) U (R2≥100}. You may not use this example as one of your 4 partitions.**
14. **Problem 1.3.6, p. 31.**
15. **Problem 1.3.10, part (a) only, p. 31**
16. **Weather forecasters throw around rain probabilities like crazy. Let A represent the event that it rains tomorrow and B represent the event that it rains the next day. Suppose P[A] <P[B] and PIAT≥0.5.**
    1. **What are bounds on P[A U B] and on PLAN B]? Show/describe how you obtained your bounds. You may want to use Venn diagrams.**
    2. **Suppose your local weather forecaster predicts rain tomorrow with 85% chance and rain the day after with 90% chance. What are bounds on the chance of rain BOTH days? What are bounds on the chance of rain EITHER day?**
17. **Problem 1.4.2, part (a) only, p. 31**
18. **Problem 1.4.4, p. 32.**
19. **Problem 1.5.2, p. 33.**
20. **Problem 1.6.2, p. 33.**
21. **Problem 1.6.6, p. 33.**

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(3)

HOMEWORK	85 %.	8.5 / 10 PTS
QUIZZES	75 %.	9 / 10 PTS
MIDTERM	65/80	32.5 / 40 PTS
FINAL	96/120	32 / 40 PTS
		82/102 PTS
∴ FINAL GRADE		80.39 %

(4)

A	93, 102	PTS
A-	90, 93	PTS
B+	87, 90	PTS
B	83, 87	PTS
B-	80, 83	PTS

IF YOU 90/102 YOU WILL GET AN A-, GIVEN PENDING OR EXTERNAL CIRCUMSTANCES ARE NON OCCURING.

QUESTION #10

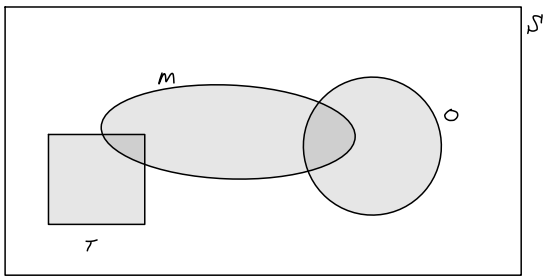
I AM GOING TO BE THE STUDENT WHO WILL PUT IN MORE HRS. INTO THIS COURSE THAN OTHER COURSES IM CURRENTLY TAKING. THE MATH IS NOT AS RIGOROUS AS ITS APPLICATIONS ARE IN SOLVING PROBLEMS. I WILL NEED REPEATED EXPOSURE TO DO WELL & REPEATED PRACTICE TO DO EXCELLENT. TAKING THE MIDTERM & FINAL ARE LIKE RACES, & LIKE RACES I HAVE TO TRAIN. DO THE PROBLEMS & KEEP ANSWERING UNTILL YOU COMPREHEND THE HOW & WHY TO A SPECIFIC SOLUTION. INTRODUCTION TO DATA SCIENCE (FOR MY PERSONAL EX) WILL UTILIZE THE CONCEPTS COVERED.

QUESTION #11 — 1.1.2 PAGE 29

FOR GERLANDA'S PIZZA IN QUIZ 1.1, ANSWER THESE QUESTIONS

A PIZZA IS EITHER TUSCAN (T) OR NEPOLITAN (N) SO $\{N, T\}$.

N IS EVERYTHING OUTSIDE THE SQUARE BOX (T).



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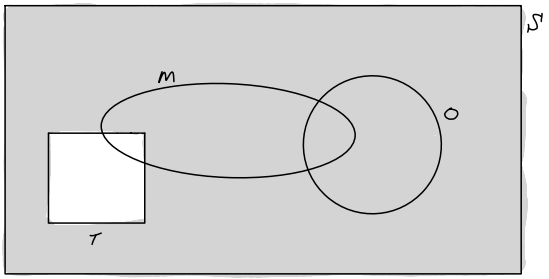


$N = T^c$

(a) ARE N & M MUTUALLY EXCLUSIVE?

TWO SETS ARE MUTUALLY EXCLUSIVE IFF $N \cap M = \phi$, MEANING THEIR DISJOINTED.

$N = T^c$ & $T^c \cap M \neq \phi \iff N \cap M \neq \phi$ THEREFORE NO, THEY ARE NOT MUTUALLY EXCLUSIVE.

$x \in T^c$ & $x \in N$

(8) ARE N, T, AND M COLLECTIVELY EXHAUSTIVE?

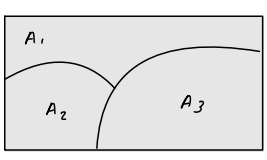
A PIZZA IS EITHER N OR T, THUS $N \cup T = S$

AND TO BE COLLECTIVELY EXAUSTIVE ∀ SETS MUST EQUAL S (THE UNIVERSAL SET. THUS $N \cup T \cup M = S$

THUS YES.

COLLECTIVELY EXHAUSTIVE SETS

A COLLECTION OF SETS $A_1 \cup A_2 \cup A_3 \cup \dots \cup A_N = S$



$\bigcup_{i=1}^{N} A_i = A_1 \cup A_2 \cup \dots \cup A_N$

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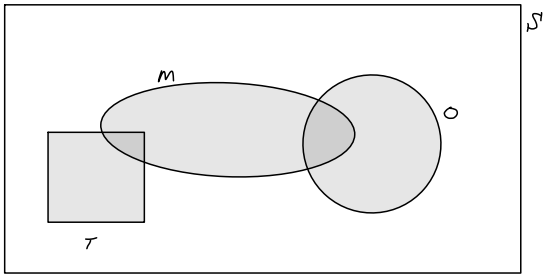
QUESTION #11 — 1.1.2 PAGE 29

FOR GERLANDA'S PIZZA IN QUIZ 1.1, ANSWER THESE QUESTIONS

A PIZZA IS EITHER TUSCAN (T) OR NEPOLITAN (N) SO $\{N, T\}$.

N IS EVERYTHING OUTSIDE THE SQUARE BOX (T).



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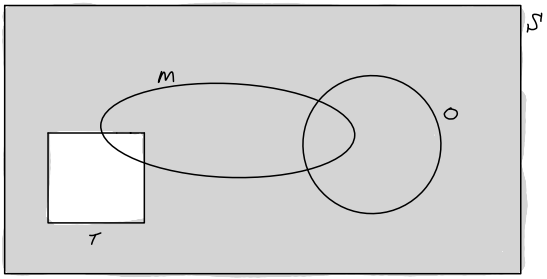


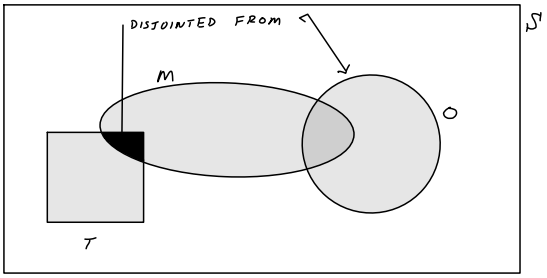
$N = T^c$

(C) ARE T & O MUTUALLY EXCLUSIVE? STATE THE CONDITION IN WORDS.

IN ORDER FOR TWO SETS TO BE MUTUALLY EXCLUSIVE TWO SETS MUST BE DISJOINTED, THIS MEANS EVERYTIME ANYONE ORDERS PIZZA THEY NEVER GET ONIONS, HOWEVER THAT IS NOT THE CASE.

(D) DOES GERLANDA'S MAKE TUSCAN PIZZAS WITH MUSHROOMS & ONIONS?

$M \cap T$ AND O ARE DISJOINTED, THUS NO.



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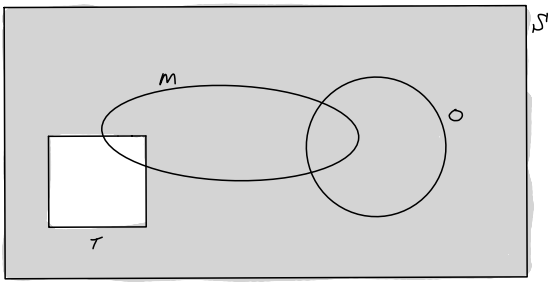
(E) DOES GERLANDA'S MAKE NEAPOLITAN PIZZA THAT HAVE NEITHER MUSHROOMS OR ONIONS?



$N = T^c$

YES, $N = T^c$ & $(T \cup M \cup O)^c$

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12. Problem 1.2.2, p. 30. For parts c, d, f, and g, give an explanation in addition to the yes/no answer.

AN INTEGRATED CIRCUIT FACTORY HAS THREE MACHINES X, Y, Z. TEST ONE INTEGRATED CIRCUIT PRODUCED BY THE MACHINE EITHER A CIRCUIT IS ACCEPTABLE (a) OR IT FAILS (f). AN OBSERVATION IS A SEQUENCE OF THREE TEST RESULTS CORRESPONDING TO THE CIRCUITS FROM THE MACHINES X, Y, Z RESPECTIVELY. FOR EXAMPLE, aaf IS THE OBSERVATION THAT THE CIRCUITS FROM X & Y PASS THE TEST, & THE CIRCUIT FROM Z FAILS THE TEST

(A) WHAT ARE THE ELEMENTS OF THE SAMPLE SPACE OF THIS EXPERIMENT?

S := A MUTUALLY EXCLUSIVE, COLLECTIVELY EXHAUSTIVE SET OF ALL POSSIBLE OUTCOMES.

$S = \{aaa, aaf, afa, aff, faa, faf, ffa, fff\}$

(B) WHAT ARE THE ELEMENTS OF THE SETS

$Z_F = \{$ CIRCUIT FROM Z FAILS $\}$

$X_A = \{$ CIRCUIT FROM X IS ACCEPTABLE $\}$

$S = \{aaa, aaf, afa, aff, faa, faf, ffa, fff\}$

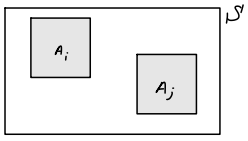
$Z_F = \{aaf, aff, faf, fff\}$

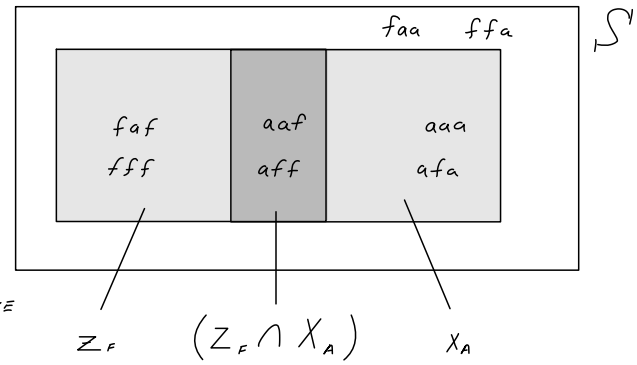
$X_A = \{aaa, aaf, afa, aff\}$

(C) ARE Z_F & X_A MUTUALLY EXCLUSIVE?

MUTUALLY EXCLUSIVE / DISJOINTED

$A_i \cap A_j = \emptyset, \quad i \neq j$





Z_F & X_A $\neg$! MUTUALLY EXCLUSIVE

$\Rightarrow (Z_F \cap X_A) \neq \emptyset$

$\Rightarrow (Z_F \cap X_A) = \{aaf, aff \mid aaf \text{ & } aff \in S, Z_F, X_A\}$

MEANING THE TWO SETS ARE NOT DISJOINTED, THEY BOTH SHARE THE ELEMENTS aaf & aff

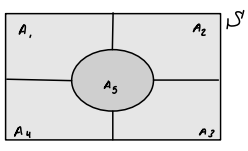
(D) ARE Z_F & X_A COLLECTIVELY EXHAUSTIVE?

IN ORDER FOR Z_F & X_A TO BE COLLECTIVELY EXHAUS. $Z_F \cup X_A = S$, HOWEVER $(Z_F \cup X_A) \neq S$

COLLECTIVELY EXAUSTIVE ALSO THE CARDINALITIES OF THE SETS ARE NOT EQUAL; $|Z_F \cup X_A| \neq |S|$

$A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_N = S$

$X_A = \{aaa, aaf, afa, aff\}$



$S = \{aaa, aaf, afa, aff, faa, faf, ffa, fff\}$

$Z_F = \{aaf, aff, faf, fff\}$

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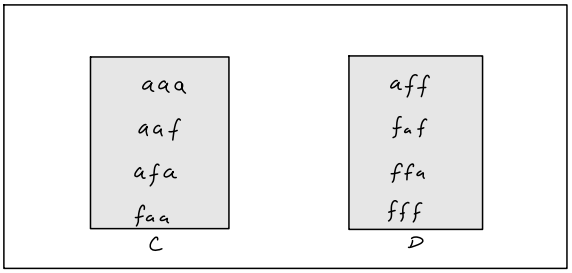

S

$C := \{$ MORE THAN ONE CIRCUIT ACCEPTABLE $\}$

$C = \{aaa, aaf, afa, faa\}$

$S = \{aaa, aaf, afa, aff, faa, faf, ffa, fff\}$

$D = \{aff, faf, ffa, fff\}$

$D := \{$ AT LEAST TWO CIRCUITS FAIL $\}$

(E) WHAT ARE THE ELEMENTS OF THE SETS

$C := \{$ MORE THAN ONE CIRCUIT ACCEPTABLE $\}$

$D := \{$ AT LEAST TWO CIRCUITS FAIL $\}$

$C = \{aaa, aaf, afa, faa\}$

$D = \{aff, faf, ffa, fff\}$

(F) ARE C & D MUTUALLY EXCLUSIVE?

YES, BECAUSE $(C \cap D) = \emptyset$, $C \neq D$

(G) ARE C & D COLLECTIVELY EXAUSTIVE?

YES, BECAUSE $(C \cup D) = S$, AND CARDINALITIES ARE EQUAL.

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13. PROBLEM 1.2.6 P. 30

NOTE THAT THE SAMPLE SPACE IS UNCOUNTABLY INFINITE.

MAKE EACH PARTITION CONSIST OF 2 OUTCOMES (SETS).

GIVE A WORD DESCRIPTION OF HOW THE PARTITION MIGHT BE USEFUL.

EX. IF WE ARE INTERESTED IN DETERMINING THAT NEITHER RESISTOR VALUE IS TOO HIGH,

A PARTITION COULD BE $A_1 = \{R_1 < 100,\ R_2 < 100\}$

$$A_2 = (R_1 \geq 100\} \cup \{R_2 \geq 100\}$$

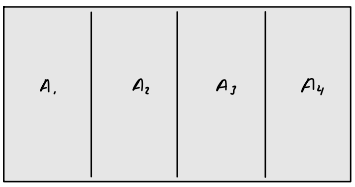
YOU MAY NOT USE THE EX. AS ONE OF YOUR 4 PARTITIONS.

1.2.6 THE SAMPLE SPACE OF AN EXPERIMENT CONSISTS OF THE MEASURED RESISTANCES OF TWO RESISTORS.

GIVE 4 EX. OF PARTITIONS.

PARTITIONED DEFN.

$$\bigcap_{i=1}^{N} A_i = A_1 \cap A_2 \cap \cdots \cap A_N$$



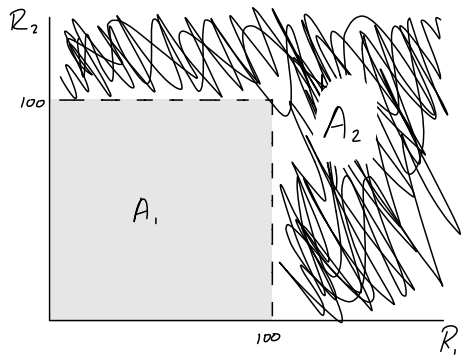
THUS $\forall\ A_i \cap A_j = \emptyset \quad i \neq j$ - MUTUALLY EXCLUSIVE

$\not\forall\ \forall A_i \cup A_j = S$ - COLLECTIVELY EXHAUSTIVE

EXAMPLE GIVEN

$-A_1 = \{R_1 < 100,\ R_2 < 100\}$

$-A_2 = (R_1 \geq 100\} \cup \{R_2 \geq 100\}$



EX 1

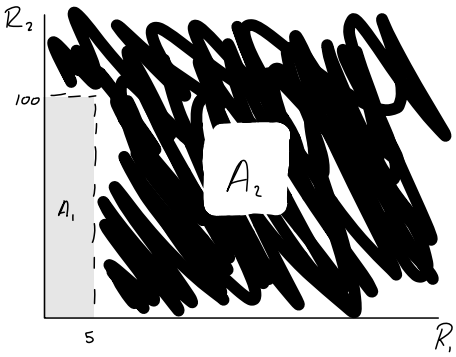
$A_1 = \{R_1 < 5,\ R_2 < 100\}$

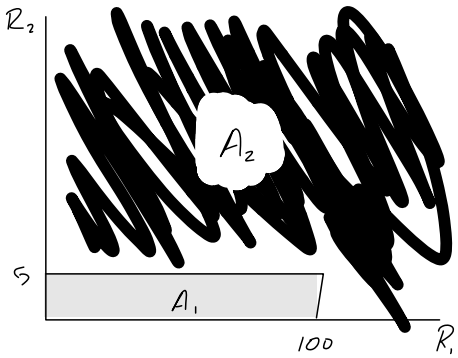
$A_2 = (R_1 \geq 5\} \cup \{R_2 \geq 100\}$

EX 2

$A_1 = \{R_1 < 100,\ R_2 < 5\}$

$A_2 = (R_1 \geq 100\} \cup \{R_2 \geq 5\}$





EX 3

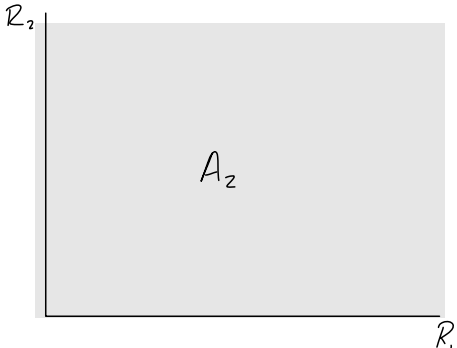
$A_1 = \{\emptyset\}$

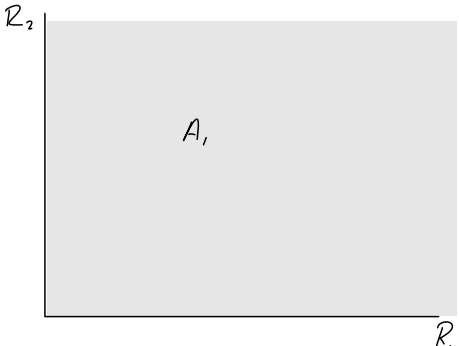
$A_2 = \{R_1 \geq 0 \cup R_2 \geq 0\}$

EX 4

$A_1 = \{R_1 \geq 0 \cup R_2 \geq 0\}$

$A_2 = \{\emptyset\}$





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14. PROBLEM 1.3.6. P. 31

THERE ARE TWO TYPES OF CELLULAR PHONES

(H) HANDHELD PHONES THAT YOU CARRY

(M) MOBILE PHONES THAT ARE MOUNTED TO VEHICLES

PHONE CALLS CAN BE CLASSIFIED BY

(F) A FAST TRAVELING SPEED

(W) A SLOW TRAVELING SPEED

MONITOR A CELLULAR PHONE CALL & OBSERVE THE TYPE OF TELEPHONE & THE SPEED OF THE USER

THE PROBABILITY MODEL FOR THIS EXPERIMENT HAS THE FOLLOWING INFO

$\{(H), (M)\}$ $\{(F), (W)\}$

$P[F] = 0.5$

$P[HF] = 0.2 = P[H \cap F]$

$P[MW] = 0.1 = P[M \cap W]$

WHAT IS THE SAMPLE SPACE OF THE EXPERIMENT?

SAMPLE SPACE $S := \{HF, MF, HW, MW\}$

$H := \{HF, HW\}$

$M := \{MF, MW\}$

FIND THE FOLLOWING PROBABILITIES

$P[W] = 0.5$, BECAUSE WE KNOW THAT WE CAN EITHER HAVE A SLOW OR FAST PHONE

THUS $P[F] + P[W] = 1$

$0.5 + P[W] = 1$

$P[W] = 0.5$

$P[M \cap F] = P[F] - P[H \cap F] \Rightarrow P[F] = P[H \cap F] + P[M \cap F]$

$= 0.5 - 0.2$

$P[M \cap F] = 0.3$

THIS WILL HELP US FIND M, WHICH WILL IN TURN HELP US FIND H.

$P[M] = P[M \cap W] + P[M \cap F]$

$P[M] = 0.3 + 0.1$

$P[M] = 0.4$

SINCE WE CAN ONLY HAVE HANDHELD OR MOBILE THEIR SUM MUST EQUAL 1

THUS, $P[H] + P[M] = 1$

$P[H] + 0.4 = 1$

$P[H] = 0.6$

$P[F] = 0.5$ $P[W] = 0.5$

$P[H \cap F] = 0.2$ $P[H \cap W] =$

$P[M \cap F] = 0.3$ $P[M \cap W] = 0.1$

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16. Weather forecasters throw around rain probabilities like crazy. Let A represent the event that it rains tomorrow and B represent the event that it rains the next day. Suppose P[A]<P[B] and P[A] ≥ 0. 5.

a. What are bounds on P[A ∪ B] and on P[A ∩ B]? Show/describe how you obtained your bounds. You may want to use Venn diagrams.

b. Suppose your local weather forecaster predicts rain tomorrow with 85% chance and rain the day after with 90% chance. What are bounds on the chance of rain BOTH days? What are bounds on the chance of rain EITHER day?

A. $P[A] < P[B]$

$P[A] \geq 0.5$

$P[B] > P[A] \geq 0.5$

$P[B] \geq 0.5$

THUS MAKING

$P[A] + P[B] < 1$

$P[A \cup B] = P[A] + P[B] - P[A \cap B]$

$0 < P[A \cup B] < 1$

$0.5 \leq P[A \cap B] < 1$

B. $P[T] = 0.85 \quad 0 \leq P[T \cap D] \leq 0.85$ - BOTH

$P[D] = 0.9 \quad 0.85 \leq P[T \cup D] \leq 0.9$

FOR THE BOUNDS GIVEN ARE FOR EITHER

$0 \leq P[T] \leq 0.85$ "OR" $0 \leq P[D] \leq 0.9$

↑ LOWER ↑ UPPER ∪ ↑ LOWER ↑ UPPER

BECAUSE WE KNOW $P[T \cup D] = P[T] + P[D]$