EECS 461 PROBABILITY & STATISTICS

CONTINUOUS RANDOM VARIABLES

MORGAN BERGEN

4.1 CONTINOUS SAMPLE SIZE

A RANDOM VARIABLE X IS CONTINUOUS IF THE RANGE S_X CONSISTS OF ONE OR MORE INTERVALS.

FOR EACH $x \in S_X$, $P[X = x] = 0$

RANGE IS THE SET OF $\mathbb{R}$

DIFFERENT FROM DRV : UNCOUNTABLY INFINITE (VS COUNTABLE

DEF 4.1 CUMULATIVE DISTRIBUTION FUNCTION (CDF)

THE CUMULATIVE DISTRIBUTIVE FUNCTION (CDF) OF A VARIABLE X IS

$$F_X(x) = P[X \le x]$$

GRAPHS ALL OF THE CDF START AT ZERO ON THE LEFT & END AT ONE ON THE RIGHT

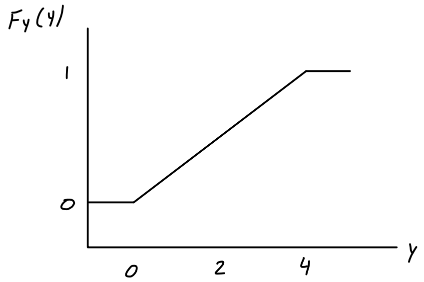
THE PROBABILITY THAT THE RANDOM VARIABLE IS IN AN INTERVAL IS THE DIFFERENCE IN THE CDF EVALUATED AT THE ENDS OF THE INTERVAL.

QUIZ 4.2

CUMULATIVE DISTRIBUTION FUNCTION OF THE RANDOM VARIABLE Y IS,

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ y/4 & 0 \le y \le 4 \\ 1 & y > 4 \end{cases}$$

SKETCH THE CDF & CALCULATE THE FOLLOWING PROBABILITIES:



(A) $P[Y \le -1] = F_Y(-1) = 0$

(B) $P[Y \le 1] = F_Y(1) = 1/4$

(C) $P[2 < Y \le 3] = F_Y(3) - F_Y(2) = 3/4 - 2/4 = 1/4$

(D) $P[Y > 1.5] = 1 - F_Y(1.5) = 8/8 - 3/8 = 5/8$

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DEF 4.3 PROBABILITY DENSITY FUNCTION

THE PDF OF A CONTINUOUS RANDOM VARIABLE X IS

$$f_X(x) = \frac{D F_X(x)}{DX}$$

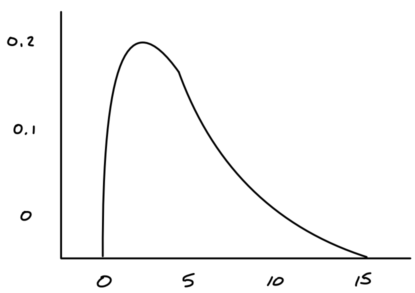
$$P[x_1 < X \le x_2] = \int_{x_1}^{x_2} f_X(x)\, dx$$

QUIZ 4.3 RANDOM VARIABLE X HAS THE PROBABILITY DENSITY FUNCTION

$$f_X(x) = \begin{cases} cxe^{-x/2} & x \ge 0 \\ 0 & \text{OTHERWISE} \end{cases} \qquad f_X(x) = \begin{cases} (x/4)xe^{-x/2} & x \ge 0 \\ 0 & \text{OTHER.} \end{cases}$$

SKETCH THE CDF & FIND THE FOLLOWING PROBABILITIES

$$1 = \int_{-\infty}^{\infty} F_X(x)\, dx = \int_0^{\infty} cxe^{-x/2}\, dx = \cancel{-2ce^{-x/2}\Big|_0^{\infty}} + \int_0^{\infty} 2ce^{-x/2}\, dx$$



$$= \int_0^{\infty} 2ce^{-x/2}\, dx$$

$$= -4ce^{-x/2}\Big|_0^{\infty}$$

$$1 = 4c$$

$$c = 1/4$$

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4.5 FAMILIES OF CONTINUOUS RANDOM VARIABLES

DEF 4.5 UNIFORM RANDOM VARIABLES

Y IS A UNIFORM (a, b) RANDOM VARIABLE IS THE PDF OF X IS

$$f_Y(y) = \begin{cases} \dfrac{1}{(b-a)} & a \leq x \leq b \quad \& \quad b > a \\ 0 & \text{OTHERWISE} \end{cases}$$

THM

IF X IS A UNIFORM (a, b) RANDOM VARIABLE

(A) THE CDF X IS $$F_X(x) = \begin{cases} 0 & x \leq a \\ (x-a)/(b-a) & a < x \leq b \\ 1 & x > b \end{cases}$$

(B) THE EXPECTED VALUE OF X IS $E[X] = (b+a)/2$

(C) THE VARIANCE OF X IS $Var[X] = (b-a)^2/12$

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QUIZ	4.5	PAGE	138
	4.6		145
	4.7		152
	5.1		166
	5.2		169

QUIZ 4.6

X IS THE GAUSSIAN $(0,1)$ RANDOM VARIABLE & Y IS THE GAUSSIAN $(0,2)$ RANDOM VARIABLE SKETCH THE PDFs $f_X(x)$ & $f_Y(y)$ ON THE SAME AXES & FIND

(A) $P[-1 < X \leq 1]$

$f_X(x) \quad (0,1)$

$f_Y(y) \quad (0,2)$