EECS 461 PROBABILITY & STATISTICS

ASSIGNMENT # 8

TUES OCT 18 2022

MORGAN BERGEN

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1. FOR A 1-HOUR MIDTERM 80% OF THE STUDENTS FINISH THE EXAM & HAND IT IN BY THE END OF THE HOUR, 20% HAVE TO TURN THEM IN AT THE END OF THE HOUR WITHOUT FINISHING THE EXAM. FOR THE STUDENTS WHO FINISH BY THE END OF THE HOUR, THEIR TURN-IN TIMES ARE UNIFORMLY DISTRIBUTED BETWEEN 45 & 60 MINUTES. LET T BE THE RANDOM VARIABLE OF THE TURN IN TIMES OF THE STUDENTS, IN MINUTES.

A. FIND THE PDF OF T

PDF OF UNIFORM DISTRIBUTION IS AS FOLLOWS

$$\boxed{f(T) = \frac{1}{b-a} = \frac{1}{60-45} = \frac{1}{15}, \quad \text{IFF} \quad 45 \le T \le 60}$$

$b = 60$ & $a = 45$

B. FIND THE CDF OF T

$$\boxed{F(T \le t) = \int_{45}^{t} f(x)\,dx = \int_{45}^{t} \frac{1}{15}\,dx = \left[\frac{x}{25}\right]\Bigg|_{45}^{t} = \frac{t-45}{15}}$$

C. FIND THE EXPECTED VALUE OF T (WE KNOW $f(t) = 1/15$)

$$E[T] = \int_{45}^{60} t\, f(t)\,dt = \left[\frac{t^2}{30}\right]\Bigg|_{45}^{60} = \frac{(60 \cdot 60) - (45 \cdot 45)}{30} = 52.5$$

$$\boxed{E[T] = 52.5 \text{ MIN}}$$

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2. PROBLEM 4.7 6. P. 160

4.7.6. WHEN YOU MAKE A PHONE CALL THE LINE IS BUSY WITH PROBABILITY 0.2 & NO ONE ANSWERS WITH PROBAB 0.3. THE RANDOM VARIABLE X DESCRIBES THE CONVERSATION TIME (IN MINUTES) OF A PHONE CALL THAT IS ANSWERED. X IS AN EXPONENTIAL RANDOM VARIABLE WITH $E[X] = 3$ MINUTES. LET THE RANDOM VARIABLE W DENOTE THE CONVERSATION TIME (IN SECONDS) OF ALL CALLS ($W = 0$ WHEN THE LINE IS BUSY OR THERE IS NO ANSWER)

A. WHAT IS $F_W(w)$?

RANDOM VAR IS $W = \begin{cases} 60x, & \text{IF ALL CALLS ARE ANSWERED} \\ 0, & \text{OTHERWISE} \end{cases}$

EXPONENTIAL VAR IS $X_3 = \dfrac{1}{E[X_3]}$

FOR 1 MINUTE = 60 SECONDS

FOR 3 MINUTES = 180 SECONDS

$$F_x(w) = \begin{cases} \dfrac{1}{180} e^{-x/180}, & x \geq 0 \\ 0, & \text{OTHERWISE} \end{cases}$$

$$\Downarrow$$

$$F_x(w) = \begin{cases} 1 - e^{-x/180}, & x \geq 0 \\ 0, & \text{OTHERWISE} \end{cases}$$

$E[X] = 180$ SEC

$$F_W(w) = P(A^c) + P(A)\, F_{W/A}(w)$$

$P(A^c)$ - PROBABILITY OF THE PHONE CALL THAT IS ANSWERED

$P(A)$ - PROBABILITY OF PHONE CALL NOT ANSWERED OR BUSY

$$F_W(w) = 0.5 + 0.5 f_x(w)$$

$$\boxed{\text{CDF} \quad F_W(w) = \begin{cases} 0.5 + 0.5 f_w(w), & w \geq 0 \\ 0, & w < 0 \end{cases}}$$

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B. $f_W(w)$?

PDF
$$f_W(w) = \begin{cases} \frac{1}{2}\delta(w) + \frac{1}{2} f_X(w) & , \quad x \geq 0 \\ 0 & , \quad \text{OTHERWISE} \end{cases}$$

$$f_W(w) = \frac{1}{2}\delta(w) + \frac{1}{2} f_X(w)$$

$$E[W] = \int_{\infty}^{\infty} w f_X(w)\, dw$$

$$= \frac{1}{2} E[X] = \frac{180}{2}$$

$$\boxed{E[W] = 90}$$

C. WHAT ARE $E[W]$ & $VAR[W]$

$$E[W^2] = \int_{\infty}^{\infty} w^2 f_W(w)\, dw = \frac{1}{2}\int_{-\infty}^{\infty} w^2 f_W(w)\, dw$$

$$\boxed{E[W^2] = \frac{1}{2} \quad \frac{E[W^2]}{2}}$$

$$VAR[W] = E[W^2] - (E[W])^2$$

$$\boxed{VAR[W] = \frac{1}{2} V(X) + \left(\frac{E[X]}{2}\right)^2}$$

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3. PROBLEM 5.1.4

5.1.4 RANDOM VARIABLES X & Y HAVE CDF $F_X(x)$ & $F_Y(y)$
IS $F(x, y) = F_X(x) F_Y(y)$ A VALID CDF?
EXPLAIN YOUR ANSWER

$$f_Y(y) = \frac{D}{DY} F_Y(y)$$

$$f_X(x) = \frac{D}{DX} F_X(x)$$

THEOREM 5.1 STATES THAT $\forall$ RANDOM VARIABLE XY

(A) $0 \le F_{X,Y}(x, y) \le 1$

(B) $F_{X,Y}(\infty, \infty) = 1$

(C) $F_X(x) = F_{X,Y}(x, \infty)$

(D) $F_Y(y) = F_{X,Y}(\infty, y)$

(E) $F_{X,Y}(x, -\infty) = 0$

(F) $F_{X,Y}(-\infty, y) = 0$

(G) IF $x \le x_1$ & $y \le y_1$ THEN $F_{X,Y}(x_1, y_1)$

$$f_{X,Y}(x, y) = f_X(x) f_Y(y)$$

$$F_{X,Y}(x, y) = \iint f_{X,Y}(x, y)\, DX\, DY = F_X(x) F_Y(y)$$

BOTH $F_X(x)$ & $F_Y(y)$ ARE INDEPENDENT BECAUSE IT SATISFIES THE TWO D CDF