EECS 461 PROBABILITY & STATISTICS

HOMEWORK 05

SEPT 27 2022

MORGAN BERGEN

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1. THE RUNTIME (ROUNDED TO THE NEAREST SECOND) OF A GIVEN PROGRAM ON A GIVEN COMPUTING PLATFORM IS OBSERVED MANY TIMES, AND THE PARTIAL TABLE OF PMF & CDF OF THE RANDOM VARIABLE R REPRESENTING RUN TIME HAS BEEN CONSTRUCTED.

TIME	PMF	CDF
5	—	—
6	0.19	0.27
7	0.35	—
8	0.15	—
9	—	0.91
10	—	—

$\rightarrow$

$P(5) = 0.08 \quad C(5) = 0.08$

$C(7) = 0.62$

$C(8) = 0.77$

$P(9) = 0.14$

$P(10) = 0.09 \quad C(10) = 1$

A. COMPLETE THE TABLE BY DETERMINING THE MISSING VALUES

B. FROM THE CDF OF R, DETERMINE THE PROBABILITY THAT THE RUNTIME WILL BE IN THE HALF-OPEN INTERVAL $(7, 9]$.

LET PMF $= P(x)$ & CDF $= C(x)$

$P(5) = 0.27 - 0.19 = 0.08$

$C(5) = 0.08$

$C(7) = 0.27 + 0.35 = 0.62$

$C(8) = 0.62 + 0.15 = 0.77$

$P(9) = 0.91 - 0.77 = 0.14$

$P(10) = 1 - 0.91 = 0.09$

$C(10) = 0.08 + 0.19 + 0.35 + 0.15 + 0.14 + 0.09 = 1$

$(7, 9]$

OVER INTERVAL $P(7 < R \leq 9) = P(R \leq 9) - P(R < 7)$

$= 0.91 - 0.27$

$= 0.64$

$\boxed{64\%}$

3. DETERMINE THE MEAN, MEDIAN, & MODE OF THE RANDOM VARIABLE R FROM PROBLEM 1. RECALL THAT MEDIAN & MODE MAY NOT BE UNIQUE.

MEAN $= \sum_{R=5}^{10} (R \cdot P(R)) = (5 \cdot 0.08) + (6 \cdot 0.19) + (7 \cdot 0.35) + (8 \cdot 0.15) + (9 \cdot 0.14) + (10 \cdot 0.09) = 7.35$

$\boxed{\text{MEAN} = 7.35}$

MEDIAN $= C(R) \geq 0.5$

& $C(7) = 0.62$, THUS $\boxed{\text{MEDIAN} = 7}$

$\boxed{\text{MODE} = 7}$

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2. THE FOLLOWING VALUES REPRESENT THE VOLTAGE ACROSS A GIVEN RESISTOR (ROUND TO THE NEAREST VOLT) IN A VERY NOISY CIRCUIT, IN THE ORDER THE VALUES WERE OBSERVED.

45, 32, 38, 47, 51, 33, 42, 28, 35, 44, 37, 29, 28, 32, 35, 43, 30, 41, 32, 40

PUT THE VALUES IN INCREASING ORDER, THEN FIND THE MEAN, MEDIAN, & MODE OF THIS SET OF VOLTAGES. RECALL THAT MEDIAN & MODE MAY NOT BE UNIQUE.

28, 28, 29, 30, $\underline{32, 32, 32}$, 33, 35, 35, 37, 38, 40, 41, 42, 43, 44, 45, 47, 51

$$\frac{\sum_{N=1}^{N} \{e\}}{N} = \frac{371}{10} = \boxed{37.1 = \text{MEAN}}$$

$\boxed{\text{MEAN} = 36}$

$\boxed{\text{MODE} = 32}$

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4. CALCULATE THE MEAN, VARIANCE, AND STANDARD DEVIATION OF THE DISCRETE RANDOM VARIABLE B FROM PROBLEM 8 OF ASSIGNMENT 4.

B	PMF P(B)	B · P(B)	B² · P(B)
1	0.4379	0.437956	0.437956
2	0.21897	0.437956	0.58392
3	0.14598	0.4379562	1.313865
4	0.10949	0.837956	1.7518248
5	0.087591	0.437955	2.189775

μ MEAN

σ^2 VARIANCE

σ STANDARD DEV

$$\boxed{\mu = \sum_{B=1}^{5} B \cdot P(B) = 2.589}$$

$$\sigma^2 = \sum_{B=1}^{5} B(P(B)) - \mu^2 = 6.2773408 - 6.70291 = -0.44889\ ?$$

b	p(b)	b(p(b))	b²(p(b))	μ mean	σ - std dev	σ² - variance
1	0.4379562044	0.4379562044	0.4379562044	12.8102189784088	6.08935891302364	37.0802919716204
2	0.656934306589781	1.31386861317956	2.62773722635912			
3	0.802919708049635	2.40875912414891	7.22627737244672			
4	0.912408759144526	3.6496350365781	14.5985401463124			
5	1.00000000002044	5.0000000001022	25.000000000511			

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5. CALCULATE THE MEAN OF THE DISCRETE RANDOM VARIABLES:

   A. W FROM PROBLEM 3 OF ASSIGNMENT 4

   B. A FROM PROBLEM 4 OF ASSIGNMENT 4

   C. R FROM PROBLEM 5 OF ASSIGNMENT 4

   D. A FROM PROBLEM 6 OF ASSIGNMENT 4

   E. W FROM PROBLEM 7 OF ASSIGNMENT 4

HAVE YET TO RECIEVE WORK BACK FROM PREVIOUS ASSIGNMENT TO VERIFY IT'S VALIDITY THUS I WILL WAIT IN ORDER TO COMPUTE RESULTS & ATTACH TO AMENDMENT

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6. PROBLEM 3.6.6. PAGE 114

Suppose that a cellular phone costs $20 per month with 30 minutes of use in- cluded and that each additional minute of use costs $0.50. If the number of minutes you use in a month is a geometric random variable M \Vith expected value of E[M]=l/p=30minutes,\vhatisthePlVIF of C, t he cost of the phone for one month?

PMF $$P_M(m) = \begin{cases} (1-p)^{m-1} p & m = 1, 2, 3, \dots \\ 0 & \text{OTHERWISE} \end{cases}$$

$$P_{COST}(20) = P[M \le 30] = \sum_{m=1}^{30} (1-p)^{m-1} \cdot p$$

$$C = 20 + \frac{(m-30)}{2}$$

$$2C = 40 + m - 30$$

$$2C - 10 = m$$

$$M = 2C - 10$$

$$P_{COST}(C) = P_M(M)$$

$$P_{COST}(C) = P_M(2C-10) \quad , \quad C = 20.5, 21, 21.5, 22, 22.5, \dots,$$

$$P_C(C) = \begin{cases} 1 - (1-p)^{30} & C = 20 \\ (1-p)^{(2C-10)-1} p & C = 20.5, 21, 21.5, 22, 22.5 \dots \end{cases}$$

7. PROBLEM 3.7.8. PAGE 115

A new cellular phone billing plan costs $15 per month plus $1 for each minute of use. If the number of minutes you use the phone in a month is a geometric random variable v1ith expected value l /p, 'vhat is the expected monthly cost E[CJ of the phone? For 'vhat values of p is this billing plan preferable to the billing plan of Prob- lem 3.6.6 and Problem 3.7.7?

$$P_m(m) = \begin{cases} (1-p)^{m-1} p & m = 1, 2, \ldots \\ 0 & \text{OTHERWISE} \end{cases}$$

$C = 15 + M$

$C \geq 16$

$P[C = c] = P[M = c - 15]$

$$P_c(c) = \begin{cases} (1-p)^{c-16} p & c = 16, 17, 18, 19, \ldots, c+1 \\ 0 & \text{OTHERWISE} \end{cases}$$

$$E[C] = E[15 + M]$$

$$= 15 + E[m]$$

$$\boxed{\therefore \text{ MONTHLY COST} = 15 + \frac{1}{p}}$$

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8. CALCULATE THE VARIANCE & STANDARD DEVIATION OF THE FOLLOWING DISCRETE RANDOM VARIABLES:

A. W FROM PROBLEM 3 OF ASSIGNMENT 4

B. A FROM PROBLEM 4 OF ASSIGNMENT 4

C. R FROM PROBLEM 5 OF ASSIGNMENT 4

D. A FROM PROBLEM 6 OF ASSIGNMENT 4

E. W FROM PROBLEM 7 OF ASSIGNMENT 4

HAVE YET TO RECIEVE WORK BACK FROM PREVIOUS ASSIGNMENT TO VERIFY IT'S VALIDITY THUS I WILL WAIT IN ORDER TO COMPUTE RESULTS & ATTACH TO AMENDMENT

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9. PROBLEM 3.8.8, PAGE 116

GIVEN A RANDOM VARIABLE X WITH AN EXPECTED VALUE μ_X & VARIANCE σ_X^2
FIND THE EXPECTED VALUE & VARIANCE OF

$$Y = \frac{X - \mu_X}{\sigma_X}$$

$V(x) = \sigma_x^2$

$E(x) = \mu_x$

$$Y = \frac{x - \mu_x}{\sigma_x}$$

$$E(Y) = \frac{1}{\sigma_x}\left(E(x) - \mu_x\right) = \frac{\mu_x - \mu_x}{\sigma_x} = 0$$

$$V(Y) = \frac{V(x)}{\sigma_x^2} = 1$$

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10. PROBLEM 3.9.2 PAGE 116

WRITE A MATLAB FUNCTION X = SHIPWEIGHT8(m) THAT PRODUCES m RANDOM SAMPLE VALUES OF THE PACKAGE WEIGHT X WITH PMF GIVEN EXAMPLE 3.27.

x.m

x.m 〉 No Selection

```
%% write a MATLAB function y = shipweight8(m) that produces m random sample values of the package weight y with PMF given in Eyample 3.27

%% E [Y] = 0.15(100 + 190 + 270 + 340) + 0.10(400) + 0.30(500) = 325 cent s

clc; clear all; close all;
y = [100 190 270 340 400 500];
output = prob(y)

disp('the eypected cost of shipping is = "+E+" cents' );

function output = prob(y)
    for i = 1:length(y)
        if (y(i) >= 100 && y(i) <= 340)
            output(i) = 0.15;
        elseif (y(i) == 400)
            output(i) = 0.10;
        elseif (y(i) == 500)
            output(i) = 0.30;
        else
            output(i) = 0;
        end
    end
end
```

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