EECS 461 Probability and Statistics
Fall Semester 2022
Assignment **#6** Due **4 October 2022**

Reading: Sections 3.9, 4.1 - 4.6 in Yates/Goodman

Do all of the Quizzes in the Reading assignment but do *not* hand them in. Answers to the Quizzes are on the book's website (search Yates Goodman Wiley)

For all problems from the book, you should use the method(s) from the corresponding section to solve the problem.

1. Consider a discrete random variable (DRV) H with probability mass function (PMF) given by:

h	0	1	2	3	4
$P_H(h)$	0.1	0.3	0.2	0.2	0.2

We will be considering several possible values for a blind estimate (estimate without observation) for this random variable. For each case below, find (i) the value of the mean squared error (MSE) of the estimate and (ii) the probability that the estimate will be correct.

   a. In this first case, use the value of the blind estimate that minimizes the mean squared error (MSE).

   b. Now use the median of H as the blind estimate.

   c. Finally, use the mode of H as the blind estimate.

   d. Summarize your results in a table. Which estimate do you think is the "best"?

2. Is the following function a valid cumulative distribution function (CDF) for a continuous random variable (CRV) T?

   $$F_T(t) = \frac{t^2 + t}{t^2 + 1} \quad \text{for } 0 \le t < \infty$$

   $F_T(t)$ is 0 elsewhere. Justify your answer.

3. Problem 4.2.4, p. 154. Sketch the CDF before attempting the parts of the problem.

4. Consider the following CDF for continuous random variable G.

   $$F_G(g) = K(g + 2) \quad \text{for } -2 < g \le 3 \text{ for some constant } K.$$

   For $g \le -2$, $F_G(g) = 0$, and for $g > 3$, $F_G(g) = 1$.

   a. Find the value of K and sketch the resulting CDF.

   b. Find $P[1 < G \le 2.5]$ using this CDF.

    c.   Find $P[G = 1]$.

    d.   Find $f_G(g)$, the probability density function (PDF) for G, and sketch it.

    e.   Find $P[1 < G \le 2.5]$ using this PDF.

    f.   Calculate the mean and standard deviation of G directly from the PDF.

5.   A continuous random variable B has PDF given by:

$$f_B(b) = b^2 + Kb + 1 \quad \text{for } 0 < b \le 2 \quad \text{for some constant } K, \text{ and } f_B(b) \text{ is 0 otherwise.}$$

    a.   Find the value of K and sketch the resulting PDF. You may want to plot it with MATLAB first; if you do, you can just submit a printout of that plot.

    b.   Find the mean and standard deviation of B.

    c.   Find and sketch/plot the CDF of B.

6.   The life of a certain kind of battery is an exponentially distributed random variable with mean of 250 hours. What is the probability that such a battery will last at most 200 hours?

7.   Problem 4.5.6, p. 156.

8.   Problem 4.5.8, p. 156. Note that we are not given the range of U, but you are given its mean. Using that information, you should be able to work this problem.

9.   Problem 4.6.4, parts (a) and (b) only, p. 158.