EECS 461 PROBABILITY & STATISTICS

TUES SEPT 6 2022

ASSIGNMENT #2

MORGAN BERGEN

1.4.2. YOU HAVE A SIX-SIDED DIE THAT YOU ROLE ONCE

LET R_i DENOTE THE EVENT THAT YOU ROLL IS i.

LET G_j DENOTE THE EVENT THAT THE ROLL IS GREATER THAN j.

LET E DENOTE THE EVENT THAT THE ROLL OF THE DIE IS EVEN NUMBERED.

A WHAT IS $P[R_3 \mid G_1]$, THE CONDITIONAL PROBABILITY THAT 3 IS ROLLED GIVEN THAT THE ROLL IS GREATER THAN 1.

DEF CONDITIONAL PROB. CORRESPOND TO A MODIFIED PROB. MODEL THAT REFLECTS PARTIAL INFO ABOUT THE OUTCOME OF AN EXPERIMENT. THE MODIFIED MODEL HAS A SMALLER SAMPLE SPACE THAN THE ORIGINAL MODEL.

$P[A]$ A PRIORI PROBABILITY OF A.

$P[R_3 \mid G_1]$ THE PROBABILITY THAT 3 IS ROLLED, GIVEN THAT THE ROLL IS >1.

SAMPLE SPACE IS $S = \{1, 2, 3, 4, 5, 6\}$

$R_3 = \{3\}$

$G_1 = \{2, 3, 4, 5, 6\}$

$P[R_3] = 1/6$

$P[G_1] = 5/6$

$G_1 \cap R_3 = \{3\} = R_3$

$P[G_1 \cap R_3] = P[R_3] = 1/6$

$$P[R_3 \mid G_1] = \frac{P[R_3 G_1]}{P[G_1]} = \frac{P[R_3]}{P[G_1]} = \frac{1/6}{5/6} = \frac{1}{5} = 20\%$$

$$\boxed{\therefore P[R_3 \mid G_1] = 20\%}$$

EECS 461 PROBABILITY & STATISTICS

TUES SEPT 6 2022

ASSIGNMENT #2

MORGAN BERGEN

1.4.4. PHONESMART IS HAVING A SALE ON BANANAS.
IF YOU BUY 1 BANANA AT FULL PRICE, YOU GET A SECOND AT HALF PRICE.
WHEN COUPLES COME TO BUY A PAIR OF PHONES, SALES OF APRICOTS & BANANAS ARE EQUALLY LIKELY.
GIVEN THAT THE FIRST PHONE SOLD IS A BANANA,
THE SECOND IS TWICE AS LIKELY TO BE A BANANA RATHER THAN AN APRICOT.
WHAT IS THE PROBABILITY THAT A COUPLE BUYS A PAIR OF BANANAS?

LET B_N DENOTE THE EVENT THAT THE NTH PHONE THAT IS SOLD IS A BANANA
& LET A_N DENOTE THE EVENT THAT THE NTH PHONE THAT IS SOLD IS AN APRICOT

IN ORDER TO DETERMINE THE PROB. THAT A COUPLE BUYS A PAIR OF BANANAS,
WE MUST FIND $P[B_1 B_2]$

$P[A_1 A_2] = ?$

$P[A_1 B_2] = ?$

$P[B_1 A_2] = ?$

$P[B_1 B_2] = ?$

HOWEVER WE KNOW THAT THE FOLLOWING IS TRUE, GIVEN OUR UNKNOWNS.

– $P[A_1 A_2] + P[A_1 B_2] + P[B_1 A_2] + P[B_1 B_2] = 1 \iff$ TOTAL PROBABILITIES $= 1$

$P[A_1] = P[B_1] = 1/2$ $\iff$ "SALES OF APRICOTS & BANANAS ARE EQUALLY LIKELY"

$P[A_2] = P[B_2] = 1/2$

– $P[A_1] = P[A_1 A_2] + P[A_1 B_2] = 1/2$

– $P[A_2] = P[A_1 A_2] + P[B_1 A_2] = 1/2$

– $\dfrac{P[B_1 B_2]}{P[B_1]} = 2\left(\dfrac{P[B_1 A_2]}{P[B_1]}\right)$ $\iff$ "GIVEN FIRST PHONE SOLD IS A BANANA, THE SECOND PHONE IS TWICE AS LIKELY TO BE A BANANA"

$\Rightarrow \dfrac{P[B_1 B_2]}{\cancel{P[B_1]}} = 2\left(\dfrac{P[B_1 A_2]}{\cancel{P[B_1]}}\right)$

$\Rightarrow \dfrac{P[B_1 B_2]}{2} = P[B_1 A_2]$

REPLACE

REPLACE

$P[A_1 A_2] + P[A_1 B_2] + P[B_1 A_2] + P[B_1 B_2] = 1 \Rightarrow P[A_1 A_2] + P[A_1 B_2] + \frac{3}{2} P[B_1 B_2] = 1$

$P[A_1] = P[A_1 A_2] + P[A_1 B_2] = 1/2 \qquad \Rightarrow P[A_1 A_2] + P[A_1 B_2] = 1/2$

$P[A_2] = P[A_1 A_2] + P[B_1 A_2] = 1/2 \qquad \Rightarrow P[A_1 A_2] + 1/2\, P[B_1 B_2] = 1/2$

EECS 461 PROBABILITY & STATISTICS

TUES SEPT 6 2022

ASSIGNMENT #2

MORGAN BERGEN

$$\boxed{\mathcal{P}[A_1 A_2] + \mathcal{P}[A_1 B_2]} + \frac{3}{2}\mathcal{P}[B_1 B_2] = 1$$

$$\mathcal{P}[A_1 A_2] + \mathcal{P}[A_1 B_2] = 1/2$$

$$1/2 + 3/2\,\mathcal{P}[B_1 B_2] = 1$$
$$-1/2 \qquad\qquad -1/2$$
$$3/2\,\mathcal{P}[B_1 B_2] = 1/2$$

$\therefore \implies \mathcal{P}[B_1 B_2] = 1/3$

THEREFORE PROB. THAT A COUPLE BUYS A PAIR OF BANANAS IS $33.\overline{3}\%$.

EECS 461 PROBABILITY & STATISTICS

TUES SEPT 6 2022

ASSIGNMENT #2

MORGAN BERGEN

1.5.2

1.5.2 For the telephone usage model of Example 1.18, let B_m denote the event that a call is billed for m minutes. To generate a phone bill, observe the duration of the call in integer minutes (rounding up). Charge for M minutes $M = 1, 2, 3, \dots$ if the exact duration T is $M - 1 < t \le M$. A more complete probability model shows that for $m = 1, 2, \dots$ the probability of each event B_m is

$$P[B_m] = \alpha(1-\alpha)^{m-1}$$

where $\alpha = 1 - (0.57)^{1/3} = 0.171$.

(a) Classify a call as long, L, if the call lasts more than three minutes. What is $P[L]$?

(b) What is the probability that a call will be billed for nine minutes or less?

M IS THE PROB. THAT A CALL IS BILLED FOR MORE THAN 3 MINUTES,

"3 OR FEWER BILLED MINUTES"

$$P[L] = 1 - P[B_m \le 3]$$

BREAK UP

$$= 1 - P[B_1] - P[B_2] - P[B_3]$$

SINCE WE KNOW $P[B_m] = \alpha(1-\alpha)^{m-1}$

$$P[B_1] \implies P[B_1] = \alpha(1-\alpha)^{1-1}$$
$$= \alpha(1-\alpha)^0$$
$$= \alpha(1)$$
$$= \alpha$$
$$\implies P[B_1] = \alpha$$

$$P[B_2] \implies P[B_2] = \alpha(1-\alpha)^{2-1}$$
$$P[B_2] = \alpha(1-\alpha)$$

$$P[B_3] \implies P[B_3] = \alpha(1-\alpha)^{3-1}$$
$$P[B_3] = \alpha(1-\alpha)^2$$

$$P[L] = 1 - P[B_1] - P[B_2] - P[B_3]$$

$$= 1 - \alpha - \alpha(1-\alpha) - \alpha(1-\alpha)^2$$

$$= (1-\alpha)^3$$

WHERE $\alpha = 1 - (0.57)^{1/3} = 0.171$

$\alpha = 0.171$

$$= (1 - 0.171)^3$$

$$\boxed{P[L] \approx 0.5697227\ldots}$$

$\therefore$ THE PROB. THAT A CALL WILL LAST MORE THAN 3 MIN. IS APPROX. 57%.

(B) PROB. BILL FOR 9 MIN OR LESS IS

$$P[B_m \le 9] = \sum_{i=1}^{9} \alpha(1-\alpha)^{i-1} = 1 - \alpha - \alpha(1-\alpha) - \alpha(1-\alpha)^2 - \alpha(1-\alpha)^3 - \alpha(1-\alpha)^4 - \alpha(1-\alpha)^5 - \alpha(1-\alpha)^6 - \alpha(1-\alpha)^7 - \alpha(1-\alpha)^8$$

$$= 1 - (0.171) - 0.171(1-0.171) - 0.171(1-0.171)^2 \ldots - 0.171(1-0.171)^8$$

$$= 1 - (0.5697)^3$$

$\therefore$

PROB. BILL FOR 9 MIN OR LESS IS = 81%.

1.6.2 EVENTS A & B ARE EQUIPROBABLE (3)

ARE MUTUALLY EXCLUSIVE (2)

ARE INDEPENDENT (1)

THUS WHAT IS $P[A]$

(3) $P[AB] = P[A]\,P[B]$

(2) $P[AB] = 0$

(1) $P[A] = P[B]$

THUS $0 = P[AB] = P[A]\,P[B] = P[A]\,P[A]$

$0 = P[A]\,P[A]$

$$\boxed{\begin{array}{l} \therefore\ 0 = P[A] \\ \&\ 0 \neq P[B] \end{array}}$$

EECS 461 PROBABILITY & STATISTICS

TUES SEPT 6 2022

ASSIGNMENT #2

MORGAN BERGEN

1.6.6

1.6.6 In an experiment, C and D are independent events with probabilities $P[C] = 5/8$ and $P[D] = 3/8$.

(a) Determine the probabilities $P[C \cap D]$, $P[C \cap D^c]$, and $P[C^c \cap D^c]$.

(b) Are C^c and D^c independent?

(A)

$$P[C \cap D] = P[C]\,P[D] = (5/8)(3/8) = 15/64$$

COMPLEMENT

$$P[C \cap D^c] = P[C] - P[C \cap D] = (5/8) - (15/64) = 25/64$$

$$P[C \cup D^c] = P[C] + P[D^c] - P[C \cap D^c]$$

$$= 5/8 + (1 - 3/8) - 25/64$$

$$P[C \cup D^c] = 55/64$$

(B) YES, BY DEF EVENTS C & D ARE INDEP. IFF $P[CD] = P[C]\,P[D]$