EECS 461 PROBABILITY & STATISTICS

ASSIGNMENT 04

SEPTEMBER 20 2022

A process has 5 components connected as follows. Component 1 is followed (series) by components 2, 3, and 4 connected in parallel, followed by (series) component 5. Component 1 has reliability 0.99; components 2, 3, and 4 have reliabilities of 0.96, 0.92, and 0.85 (respectively); and component 5 has reliability of 0.95. What is the overall reliability of this process?

C_1 RELIABILITY $= 0.99$

C_2 RELIABILITY $= 0.96$

C_3 RELIABILITY $= 0.92$

C_4 RELIABILITY $= 0.85$

C_5 RELIABILITY $= 0.95$

$$C_1 \cdot C_2 \cdot C_3 \cdot C_4 \cdot C_5 \Rightarrow 0.99 \cdot 0.96 \cdot 0.92 \cdot 0.85 \cdot 0.95 = 0.706$$

OVERALL RELIABILITY IS THE PRODUCT OF $\forall$ THUS

$\boxed{\text{RELIABILITY OF THE PROCESS IS } 0.706}$

We wish to modify the cellular telephone coding system in Example 2.21 in order to reduce the number of errors. In particular, if there are two or three zero in the received sequence of 5 bits, we will say that a deletion (event D) occurs. Otherwise, if at least 4 zeroes are received, the receiver decides a zero was sent, or if at least 4 ones are received, the receiver decides a one was sent. We say that an error occurs if i was sent and the receiver decides j ≠ i was sent. For this modified protocol, what is the probability P[E] of an error? What is the probability of P[D] of a deletion?

$$P[E] = \sum_{p=3}^{5} \binom{5}{N} q^{N}(1-q)^{5-N}$$

$$P[K,5] = \binom{5}{K} p^{K}(1-p)^{5-K}, \quad K = 0,1,2,3,4,5$$

$$P[C] = P[S_5, 5] + P[S_4, 5]$$

$$= p^5 + 5p^4(1-p)$$

$$= (.9)^5 + 5(.9)^4 - (1-.9)$$

$$P[C] = 0.91854$$

$$P[E] = 5p(1-p)^4 + (1-p)^5$$

$$= 5(.9)(1-.9)^4 + (1-.9)^5$$

$$\boxed{P[E] = 0.00046}$$

$$P[D] = P(S_{3,5}) + P(S_{2,5})$$

$$= 10p^3(1-p)^2 + 10p^2(1-p)^3$$

$$= 10(0.9)^3(1-.9)^2 + 10(.9)^2 - (1-.9)^3$$

$$\boxed{P[D] = 0.081}$$

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Some drivers from Lawrence to KC stop in KC and others continue to another destination. We know t hat, on average, 5% of the drivers arriving in KC from Lawrence continue to St. Louis. If we stop drivers arriving in KC from Lawrence and ask what their destination is, what is the probability that we will have to s top exactly 30 drivers before finding one who is traveling to St. Louis? Identify the family of PMF that you used.

DRIVER PROBABILITY $P = 0.05$

PROBABILITY MASS FUNCTION PMF

$f_x(x) = (1-p)^x p$, $x = 30$ DRIVERS WHO ARE TRAVELING TO ST. LOUIS

$f_x(30) = (1-0.05)^{30}(0.05)$

$$\boxed{f_x(30) \doteq 0.01073}$$

Widgets coming off a production line are sampled and tested randomly in batches of 3 items, with each item being declared either defective or working. From prior experience, it is known that, on average, 10 of every 100 widgets will be defective. If A is the random variable that is the number of defective widgets in a batch, develop the Probability Mass Function (PMF) of A, identify which family it belongs to, and graph it.

$P(X = x) = \binom{3}{x} 0.1^x (0.9^{3-x})$ ← BINOMIAL DISTRIBUTION WHERE $N=3$ & PROB IS 0.1

x	P(x)
0	0.729
1	0.243
2	0.027
3	0.001

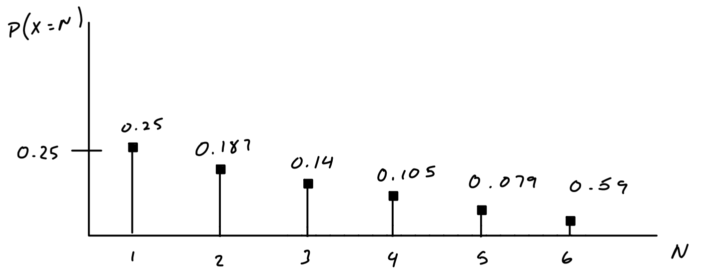
A box contains 3 blue balls and 1 red ball. The experiment is as follows. Balls are picked in succession until the red ball is picked. Any time a blue ball is picked, it is discarded and another ball is picked from the box. Let R be the RV of the number of picks to get the red ball. Develop the PMF of R, identify which family it belongs to, and graph it.

$P(\text{RED}) = 0.25$
$P(!\text{RED}) = 0.75$

DRAW 1 RED $= P(\text{RED}) = 0.25$
DRAW 2 RED $= P(!\text{RED}) \cdot P(\text{RED}) = 0.1875$
DRAW 3 RED $= P(!\text{RED}) \cdot P(!\text{RED}) \cdot P(\text{RED}) = (0.75)^2 \cdot (0.25) = 0.14062$
DRAW N RED $= P(!\text{RED})^{N-1} \cdot P(\text{RED}) = (0.75)^{N-1} \cdot 0.25$

PMF

$\therefore P[x = N] = 0.25\,(0.75)^{N-1}, \quad N = 1, \ldots, \infty$



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Telephone calls arrive at a call center at an average rate of 300 calls per hour. What is the probability that no more than 2 calls will arrive during any given minute? State any assumptions that you used.

300 CALLS IN AN HOUR
1 CALL EVERY 5 SECONDS $\longrightarrow \dfrac{e^{-5} \cdot 5^{x}}{x!}$ X IS NO MORE THAN 2 CALLS

$$P[x \le 2] = P[x=2] + P[x=1] + P[x=0]$$

$$= \frac{e^{-5}(5)^2}{2!} + \frac{e^{-5}(5)^1}{1!} + \frac{e^{-5}(5)^0}{0!}$$

$$\simeq 0.12465\,20195\ldots$$

At Newark airport, your jet joins a line as the tenth jet waiting for takeoff. At Newark, takeoffs and landings are synchronized to the minute. In each one minute interval, an arriving jet lands with probability p = ⅔, independent of an arriving jet blocks any waiting jet from taking off in that one-minute interval. However, if there is no arrival, then the waiting jet at the head of the line takes off. Each take-off requires exactly one minute.

(a) Let L1 denote the number of jets that land before the jet at the front of the line takes off. Find the PMF PL1(l)

(b) Let W denote the number of minutes you wait until your jet takes off. Find the P[W = 10]. Note that if no jets land for ten minutes, then one waiting jet will take off each minute and W = 10).

(c) What is the PMF of W?

(A) IF PROBABILITY OF EACH JET IS $2/3$
THEN TAKE OFF MUST BE $P_{L_1}(l) = 2/3^{\,l}$

(B) NO JET LANDS IN TEN MINUTES, THEN $(1/3)^{10} \simeq 0.0000169$

(C) PMF OF W : $P(W=w) = (1/3)^{10}\left((2/3)^{(w-10)}\right)$ SUCH THAT w IS GREATER THAN OR EQUAL TO W.

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$$\simeq 0.124652019 5 \ldots$$

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A discrete RV B has PMF PB(b) = C/b for b = 1, 2, 3, 4, 5 and 0 otherwise, for some constant C.

a. Find the value of C.

b. Sketch the PMF of B.

c. Find and sketch the Cumulative Distribution Function (CDF) of B

$$P_B(b) = \begin{cases} \frac{c}{b}, & \text{FOR } b = 1, \dots, 5 \\ 0, & \text{OTHERWISE} \end{cases}$$

(A) $$1 = \sum_{b=1}^{5} P_B(b) = \frac{C}{1} + \frac{C}{2} + \frac{C}{3} + \frac{C}{4} + \frac{C}{5}$$

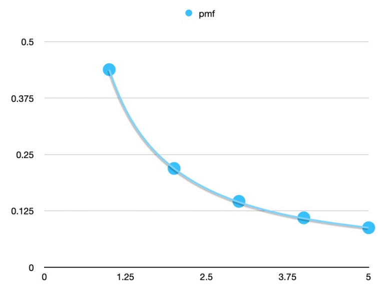
$$1 = \frac{137C}{60}$$

$$C = \frac{60}{137}$$

THUS $$P_B(b) = \begin{cases} \frac{60}{137b}, & \text{IF } b = 1, \dots, 5 \\ 0, & \text{OTHERWISE} \end{cases}$$

(B)

b	PMF, $P(B = b)$
1	$\frac{60}{137 \cdot 1}$
2	$\frac{60}{137 \cdot 2}$
3	$\frac{60}{137 \cdot 3}$
4	$\frac{60}{137 \cdot 4}$
5	$\frac{60}{137 \cdot 5}$



b	pmf
1	0.4379562044
2	0.218978102189781
3	0.145985401459854
4	0.109489051094891
5	0.0875912408759124

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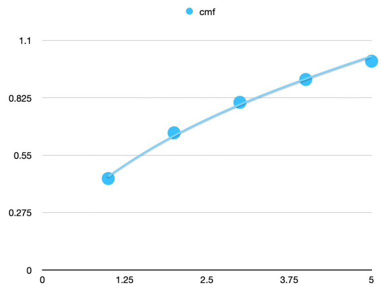
(c)

b	CMF, $P(B \le b)$
1	$\frac{60}{137 \cdot 1}$
2	$\frac{60}{137 \cdot 1} + \frac{60}{137 \cdot 2}$
3	$\frac{60}{137 \cdot 1} + \frac{60}{137 \cdot 2} + \frac{60}{137 \cdot 3}$
4	$\frac{60}{137 \cdot 1} + \frac{60}{137 \cdot 2} + \frac{60}{137 \cdot 3} + \frac{60}{137 \cdot 4}$
5	$\frac{60}{137 \cdot 1} + \frac{60}{137 \cdot 2} + \frac{60}{137 \cdot 3} + \frac{60}{137 \cdot 4} + \frac{60}{137 \cdot 5}$



b	CMF ~~pmf~~
1	0.4379562044
2	0.656934306589781
3	0.802919708049635
4	0.912408759144526
5	1.00000000002044