SEQUENTIAL EXPERIMENTS

COUNTING METHODS

EX   HOW MANY POSSIBLE SEQUENCES OF 1000 BITS (1s & 0s) HAVE 997 1s?

WE ARE CHOOSING 997 POSITION NUMBERS FROM $M = 1000$ POSSIBLE, WITHOUT REPLACEMENT
COMBINATIONS.

$N = 1000$ POSITIONS
$K = 997$ POSITIONS

$$\binom{1000}{997} = \frac{1000!}{997!\,(3!)} = 166,167,000$$

SAMPLING WITH REPLACEMENT
THIS IS N REPETITIONS OF THE SAME SUBEXPERIMENT.
THESE SUB EXP CALLED INDEPENDENT TRIAL

THEOREM 2.5

FOR N REPETITIONS OF A SUBEXPERIMENT WITH SAMPLE SPACE
$S_{sub} = \{S_0, \dots, S_{m-1}\}$ THE SAMPLE SPACE S OF A SEQ. EXP. HAS M^N OUTCOMES.

EX   EECS IS LENGTH−N BINARY SEQUENCES
N INDEPENDENT TRIALS WITH COLLECTION HAVING $M = 2$ ITEMS (0s & 1s)
IF ALL 1000-BIT SEQUENCES ARE EQUALLY LIKELY, THEN THE PROB. OF A SEQUENCE
CONTAINING 997 1s IS...

$$\frac{\#\ 997\text{-}1\ \text{SEQ.}}{\text{TOTAL}\ \#\ \text{POSSIBLE}} = \frac{\binom{1000}{997}}{2^{1000}} = 1.55 \times 10^{-23}$$

CAN GENERALIZE TO THE CASE WITH M OUTCOMES FOR EACH SUB-EXPERIMENT

THEOREM 2.7

FOR N REPETITIONS OF A SUBEXPERIMENT WITH SAMPLE SPACE
$S = \{S_0, \dots, S_{m-1}\}$ THE NUMBER OF LENGTH $N = N_0 + \cdots + N_{m-1}$
OBSERVATIONAL SEQUENCES WITH S_i APPEARING N_i TIMES IS

$$\binom{N}{N_0, \dots, N_{m-1}} = \frac{N!}{N_0!\,N_1! \cdots N_{m-1}!}$$

2.3 INDEPENDENT TRIALS

INDEPENDENT TRIALS ARE IDENTICAL SUB EXPERIMENTS IN A SEQUENTIAL EXPERIMENT.
PROBABILITY MODELS OF ALL THE SUBEXPERIMENTS ARE IDENTICAL & INDEPENDENT OF
THE OUTCOMES OF PREVIOUS SUBEXPERIMENTS.

BINARY SUBEXPERIMENTS

APPLIED ANYTIME THE SAMPLE SPACE OF SUB-EXPERIMENTS CONSISTS OF 2 ITEMS.

1, 0 : DIGITAL COMM

S, F : FAILURE ANALYSIS

W, L : GAME ANALYSIS

$S' = \{0, 1\}$

THM 2.8

THE PROBABILITY OF N_0 FAILURES & N_1 SUCCESSES IN $N = N_0 + N_1$ INDEPENDENT TRIALS IS

$$P[E_{N_0,N_1}] = \underbrace{\binom{N}{N_1}(1-p)^{N-N_1}p^{N_1}}_{\text{IN TERMS OF } N_1} = \underbrace{\binom{N}{N_0}(1-p)^{N_0}p^{N-N_0}}_{\text{IN TERMS OF } N_0}$$

EX COMPUTING JOB RAN ON $N = 1000$ PARALLEL PROCESSORS

EACH PROCESSOR HAS A PROBABILITY OF FAILURE DURING JOB OF $\varepsilon = 10^{-3}$

ALL INDEPENDENT PROCESSORS.

$$P[\text{SUCCESS}] = P[\text{0 FAILURES}] = (1-\varepsilon)^N = 0.999^{1000} = 0.368$$

$$P[\text{FAILURE}] = (1 - 0.368) = 0.632$$

NOW WE HAVE $m = 2$ BACK UP PROCESSORS THAT INSTANTLY & AUTOMATICALLY TAKE PLACE TAKE THE PLACE OF A FAILED PROCESSOR

(BACK UP PROCESSORS NEVER FAILURE)

JOB FAILURE PROBABILITY IS THE PROBABILITY $>$ m FAILURES

$= 1 - P[\leq m \text{ FAILURES}]$

$$P[\text{0 FAILURES}] = (1-\varepsilon)^N = 0.368$$

$$P[\text{1 FAILURES}] = \binom{N}{1}(1-\varepsilon)^N(\varepsilon)^1 = 0.368$$

$$P[\text{2 FAILURES}] = \binom{N}{2}(1-\varepsilon)^{N-2}(\varepsilon)^2 = 0.184$$

$$P[\text{FAIL}] = 1 - 0.368 - 0.368 - 0.184 = 0.08$$

EX 2 PARITY CHECK CODING

BINARY DATA DIVIDED INTO 8-BIT "WORDS" $P[\text{BIT ERROR}] = p = 0.01$

WHAT IS THE PROBABILITY THAT AN 8-BIT WORD HAS AT LEAST 1 [UNDETECTED] BIT ERROR?

$$P[\text{WORD ERROR}] = 1 - P[\text{0 BIT ERRORS}]$$

$$= 1 - (0.99) = 7.73 \times 10^{-2}$$

PARITY CHECK CODING

MITIGATE UNDETECTED BIT / PROCESSOR ERRORS

FOR EACH 8-BIT WORD, ADD A 9^{TH} BIT SO THAT THE $\sum$ OF THE 1s (PARITY) IN 9-BIT RESULT IS EVEN

01011100 $\longrightarrow$ 01011100 | 0

00110001 $\longrightarrow$ 00110001 | 0

IF ANY SINGLE BIT IS CHANGED (BIT ERROR)

THE PARITY RESULT IS ADD & WE HAVE <u>DETECTED</u> THAT BIT ERROR

ALSO TRUE FOR ANY ODD NUMBER OF BIT ERRORS.

BUT AN EVEN NUMBER OF BIT ERRORS IS <u>NOT</u> DETECTED.

WHAT IS THE PROBABILITY OF UNDETECTED BIT ERRORS.

PROBABILITY OF K ERRORS

$$P[\text{K ERROR}] = \binom{9}{K} p^{K}(1-p)^{9-K}$$

$$P[\text{UNDETECTED ERRORS}] = P[\text{2 BIT ERRORS}] + P[4] + P[6] + P[8]$$

$$= \frac{9!}{7!\,(9-7)!}(0.01)^2 + \frac{9!}{4!\,(9-4)!}(0.01)^4(0.09)^5 + \cdots$$

$$\simeq 1^{ST} \text{ TERM} = 3.36 \times 10^{-3}$$

THM 2.9

A SUBEXPERIMENT HAS A SAMPLE SPACE $S_{SUB} = \{S_0, \dots, S_{m-1}\}$ WITH $P[S_i] = P_i$

FOR $N = N_0 + \cdots + N_{m-1}$ INDEPENDENT TRIALS, THE PROBABILITY OF N_i OCCURENCES OF S_i

$i = 0, 1, \dots, m-1$, IS

$$P[E_{N_0, \dots, N_{m-1}}] = \binom{N}{N_0, \dots, N_{m-1}} P_0^{N_0} \cdots P_{m-1}^{N_{m-1}}$$

WIDGETS CAN HAVE T_1 OR T_2 MANUFACTURING DEFECTS, NEVER BOTH

LET $P[T_1] = 0.05$ & $P[T_2] = 0.10$

WHAT IS THE PROBABILITY THAT A RUN OF 100 WIDGETS