Probability and Stochastic Processes

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1 Probability axioms

A probability model assigns a number between 1 and 0 to every event. The probability of the union of mutually exclusive evnts is the sum of the probabilities of the event in the union. Set theory representation of a sample space S as the universal set, outcomes s that are the elements of the universal set S, and events A that are the sets of elements. We must complete the model by incorporating P[A] to $\forall a \, \exists A$.

1.1 Relative frequency notion of probability

With respect to the physical idea of an experiment, the probability of an event is the proportion of time that event is observed in a large number of runs of the experiment.

1.2 Axioms of probability

Probability measure is a function that maps events in the sample space to $\mathbb{R}$ The entire theory of probability will be built upon the following three axioms.

Probability measure $\mathcal{P}[\cdot] \mid 0 \leq \mathcal{P} \in \mathbb{R} \leq 1$

Axiom 1 $\forall A, \mathcal{P}[A] \geq 0$

Axiom 2 $\mathcal{P}[\mathcal{S}] = 1$

Axiom 3 $A_1, A_2, A_3, \ldots, A_n$ for any countable collection of mutually exclusive events

$\exists \quad \mathcal{P}[A_1 \cup A_2 \cup A_3 \cup \ldots] = \mathcal{P}[A_1] + \mathcal{P}[A_2] + \cdots + \mathcal{P}[A_n]$

1.3 Theorems of Probability