EECS 461 PROBABILITY & STOCHASTIC PROCESSES

CH 5 MULTIPLE RANDOM VARIABLES

OCTOBER 18 2022

MORGAN BERGEN

WE CONSIDER EXPERIMENTS THAT PRODUCE A COLLECTION OF RANDOM VARIABLES $X_1, X_2, \dots, X_N$, WHERE N CAN BE $\forall \mathbb{Z}^+$

DEF 5.2 JOINT PROBABILITY MASS FUNCTION (PMF)

THE JOINT PMF OF DISCRETE RANDOM VARIABLES X & Y IS

$$P_{X,Y}(x,y) = P[X = x, Y = y]$$

THE RANGE TO DENOTE THE SET OF POSSIBLE VALUES OF THE PAIR (X,Y) THAT IS,

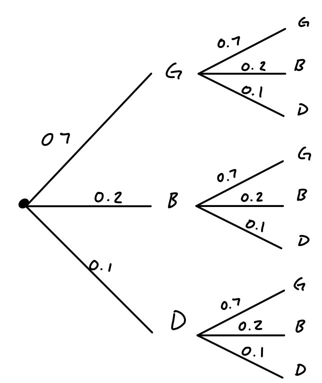
$$S_{X,Y} = \{(x,y) \mid P_{X,Y}(x,y) > 0\}$$

PROPERTIES EXTENSION OF UNIVARIABLE PMF

$$P_{X,Y}(x,y) \geq 0 \quad \text{FOR ALL } (x,y)$$

$$\sum_{(x,y) \in S_{X,Y}} P_{X,Y}(x,y) = 1$$

TREE DIAGRAM TO GET PMF

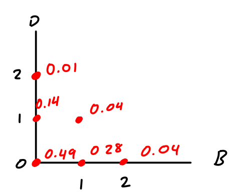


B	D	
		0.49
0	0	0.14
1	0	0.07
0	1	0.14
1	0	0.04
2	0	0.02
1	1	0.07
0	1	0.02
1	1	0.01
0	2	1.0

PLOT PROB IN TABLE

D \ B	0	1	2
0	0.49	0.28	0.04
1	0.14	0.04	/
2	0.01	/	/

PLOT ON GRAPH



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THM 5.3 FOR DRVs X & Y & $\forall$ SET A IN X, Y PLANE

PROBABILITY OF EVENT α $\{(x, y) \in A\}$ IS

$$P[\alpha] = \sum_{(x,y) \in A} P_{X,Y}(x,y)$$

EX WIDGETS

LET EVENT $\alpha = \{(B, D) \mid B + D = 2\}$

5.3 MARGINAL PMF

FOR DISCRETE RANDOM VARIABLES, THE MARGINAL PMFs $P_X(x)$ & $P_Y(y)$ ARE PROBABILITY MODELS FOR THE INDIVIDUAL RANDOM VARIABLES X & Y BUT THEY DO NOT PROVIDE A COMPLETE PROBABILITY MODEL FOR THE PAIR X, Y

JOINT PMF $P_{X,Y}(x,y)$: PROB MODEL ABOUT X, Y AND THEIR RELATIONSHIP

INFO ABOUT INDIVIDUAL X OR Y RV WOULD BE THEIR INDIVIDUAL PMFs...

CALLED MARGINAL PMFs : $P_X(x)$ & $P_Y(y)$

CAN GET MARGINALS DIRECTLY FROM THE JOINT

METHOLOGY (DIFFERENT APPROACH FROM TEXT)

RECALL **THM 1.9** FOR $\forall$ EVENT A & A PARTITION $\{B_1, B_2, \ldots, B_m\}$

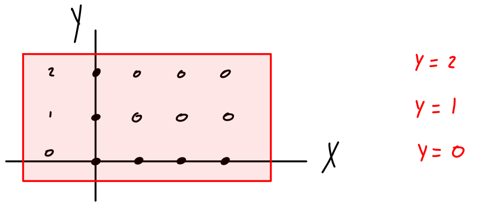
$$P[A] = \sum_{i=1}^{m} P[A, B_i]$$

IN CONTEXT OF JOINT PMF $P_{X,Y}(x,y)$

$\{Y = y_i\}$ IS AN EVENT & THE SET OF EVENTS

$\{Y = y_i\}$ FOR ALL $y_i \in S_Y$ IS A PARTITION OF $S_{X,Y}$

ILLUSTRATION



$$P[X = x_j] = \sum_{y_i = S_Y} P[X = x_i, Y = y_i]$$

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EXPRESSING AS PMFs

$$P_X(x_i) = \sum_{y_i \in S_Y} P_{X,Y}(x_i, y_i)$$

REMOVING SUBSCRIPTS

$$P_X(x) = \sum_{y \in S_Y} P_{X,Y}(x, y)$$

SIMILARLY: $$P_Y(y) = \sum_{x \in S_X} P_{X,Y}(x, y)$$

NOW IF $P_{X,Y}(x,y)$ IS EXPRESSED IN TABLE FORM, WE ARE JUST SUMMING OVER ROWS & COLS TO GET MARGINAL PMF, WE DID THIS BEFORE

EX WIDGETS

D \ B	0	1	2	$P_D(d)$
0	0.49	0.28	0.04	0.81
1	0.14	0.04		0.18
2	0.01			0.01
$P_B(b)$	0.64	0.32	0.04	

JOINT PDF (CRVs)

SIMILAR TO SINGLE CRVs & ANALOGOUS TO DRVs, WE HAVE

DEFN 5.3 $$F_{X,Y}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{X,Y}(a,b)\, db\, da$$

THM 5.5