Homework 3
EECS 510
2022 Fall

1. (10 points) Show that the language $L \coloneqq \{0^n 1 0^n \mid n \in \mathbf{N}\}$ is not regular.

> **Solution:** For any $n \in \mathbf{N}$, let $w = 0^n 1 0^n$. For any splitting of $w = xyz$ where $|xy| \leq n$ and $y \neq \epsilon$, it must be the case $y = 0^k$ for some $k \in \mathbf{N}$ with $k > 0$. By the pumping lemma for regular languages, the string $xz = xy^0 z = 0^{n-k} 1 0^n$ must be in the language $L$. But $n - k \neq n$. Therefore, $xz \notin L$, and the language $L$ cannot be regular.

2. (10 points) Show that the language $L \coloneqq \{0^n 1^m \mid m \geq n\}$ is not regular.

> **Solution:** For any $n \in \mathbf{N}$, let $w = 0^n 1^n$. For any splitting $w = xyz$ with $|xy| \leq n$ and $y \neq \epsilon$, it must be the case that $y = 0^k$ for some $k \in \mathbf{N}$ with $k > 0$. By the pumping lemma for regular languages, the string $xy^2 z = 0^{n+k} 1^n$ must be in the language $L$. But $n + k > n$. Therefore, $xy^2 z \notin L$, and the language is not regular.

3. (10 points) Show that the language $L \coloneqq \{0^n 1^{2n} \mid n \in \mathbf{N}\}$ is not regular.

> **Solution:** For any $n \in \mathbf{N}$, let $w = 0^n 1^{2n}$. For any splitting $w = xyz$ with $|xy| \leq n$ and $y \neq \epsilon$, it must be the case that $y = 0^k$ for some $k \in \mathbf{N}$ with $k > 0$. By the pumping lemma for regular languages, the string $xz = xy^0 z = 0^{n-k} 1^{2n}$ must be in the language $L$. Yet $2n$ is not twice $n - k$. Therefore, $xz \notin L$, and the language is not regular.