

ALPHABET IS DENOTED BY Σ IS A NON EMPTY FINITE SET.

BINARY ALPHABET $\Sigma =$

GENETIC ALPHABET

UNICODE ALPHABET UTF-8

A STRING / WORD IS A FINITE SEQUENCE OF SYMBOLS CHOSEN FROM THE ALPHABET

THE EMPTY STRING IS THE STRING WITH ZERO OCCURRENCES OF SYMBOLS

ϵ - EMPTY STRING

Σ - ALPHABET

CONCATENATION OF STRINGS DENOTED $x \cdot y$ OR xy

POWERS OF AN ALPHABET $\Sigma^0 = \{\epsilon\}$

$\Sigma^1 = \{0, 1\}$

$\Sigma^2 = \{00, 01, 10, 11\}$

$\Sigma^3 = \{000, 001, 010, 011, 100, 101, 110, 111\}$

$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \dots$ ALL STRINGS OVER Σ

A LANGUAGE L IS ANY SUBSET OF Σ^*

$\Sigma^*, \emptyset, \{\epsilon\}$ ARE ALWAYS LANGUAGES OVER Σ

$\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \dots$ ALL NON-EMPTY STRINGS OVER Σ

EXAMPLE LANGUAGE $\{0^n 1^n : n \in \mathbb{N}\}$

$\{10, 11, 101, 111, \dots\}$

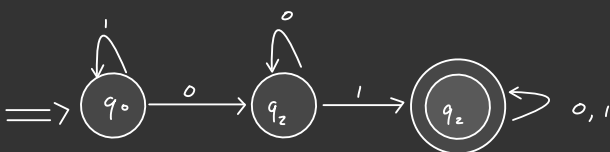
BINARY STRINGS THAT REPRESENT PRIME #S

UNIVERSALITY OF STRINGS VS. OPTIMIZATION

DFA - DETERMINISTIC FINITE AUTOMATA

- (1) FINITE SET OF STATES
- (2) AN ALPHABET Σ
- (3) TRANSITION FUNCTION $\delta: Q \times \Sigma \rightarrow Q$
- (4) START STATE $q_0 \in Q$
- (5) A SET OF FINAL / ACCEPTING STATES $\subseteq Q$

DFA GRAPHICAL NOTATION



AUTOMATA THAT ACCEPTS STRINGS WHICH CONTAIN 01

TRANSITION TABLE

	0	1
q_0	q_1	q_0
q_1	q_1	q_2
$*q_2$	q_2	q_2

$$\delta(\text{STATE}, \text{CHARACTER}) = \text{STATE}$$

$$\delta(\text{STATE}, \text{STRING}) = \text{STATE}$$

BASE RECURSIVE BASE CASE

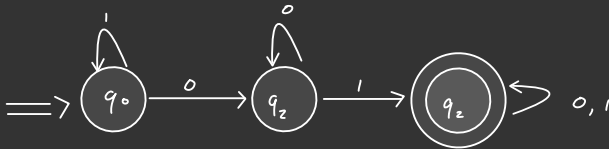
$$\delta(q, \varepsilon) := q \quad \exists q \in Q$$

ECUR

$$\delta(q, u a) := \delta(\delta(q, u), a)$$

u := STRING PREFIX

a := TRAILING CHAR



$$\hat{\delta}(q_0, 1011)$$

$$\hat{\delta}(q_0, \varepsilon) = q_0$$

$$\hat{\delta}(q_0, 1) = \delta(q_0, 1) = q_0$$

$$\hat{\delta}(q_0, 10) = \delta(\hat{\delta}(q_0, 1), 0) = \delta(q_0, 0) = q_1$$

$$\hat{\delta}(q_0, 101) = \delta(\hat{\delta}(q_0, 10), 1) = \delta(q_1, 1) = q_2$$

$$\hat{\delta}(q_0, 1011) = \delta(\hat{\delta}(q_0, 101), 1) = \delta(q_2, 1) = q_2$$

EXAMPLE DFA TO ACCEPT THE LANGUAGE $L = \{w : w \text{ CONTAINS AS MANY } 0\text{s AS } 1\text{s}\}$
AN EVEN NUMBER OF 0s AS 1s

q_0 : EVEN # OF 0s & EVEN # OF 1s

EX: 0011, 11, 00

q_1 : EVEN # OF 0s & ODD # OF 1s

q_2 : ODD # OF 0s & EVEN # OF 1s

q_3 : ODD # OF 0s & ODD # OF 1s

THE LANGUAGE OF A DFA $A = (Q, \Sigma, \delta, q_0, F)$

$$L(A) = \{w : \hat{\delta}(q_0, w) \in F\}$$

STARTING IN THE INITIAL STATE & FOLLOWING THE TRANSITIONS FOR w
IF THIS LANDS YOU ON A FINAL STATE w IS IN THE LANGUAGE.

EXAMPLE DFA TO ACCEPT THE LANGUAGE $L = \{w : w \text{ CONTAINS AS MANY } 0\text{'s AS } 1\text{'s}\}$
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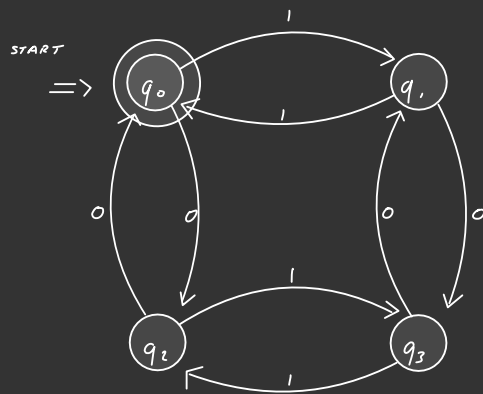
q_2 : ODD # OF 0s & EVEN # OF 1s

q_3 : ODD # OF 0s & ODD # OF 1s

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IF THIS LANDS YOU ON A FINAL STATE w IS IN THE LANGUAGE.

EX 1011 : NOT IN THE LANGUAGE

$$\hat{\delta}(q_0, 1011) = q_3 \notin F$$

0110 : IS IN THE LANGUAGE

$$\hat{\delta}(q_0, 0110) = q_0 \text{ IS A FINAL STATE}$$