Answer the following questions to the best of your ability. Your answers should be both thorough and complete. Do your own work. Good luck!

Name _____

1. (5 points) Write a context-free grammar for the language $\{a^m b^n c^m \mid m, n \geq 1\}$.

> **Solution:**
>
> $$S \rightarrow aSc \mid aBc$$
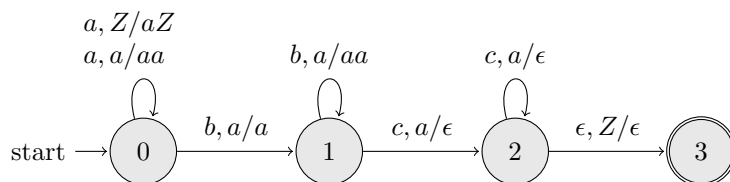> $$B \rightarrow bB \mid b$$

2. (5 points) Write a context-free grammar for the language over $\{i, e\}$ where for every string in the language all prefixes have at least as many is as es. Informally, this is the language of C-style nested `if-then` and `if-then-else` statements with i representing an `if` statement and e representing an `else` statement.

> **Solution:**
>
> $$S \rightarrow iSeS \mid iS \mid \epsilon$$

3. (5 points) Draw a graphical representation of a PDA for the language $\{a^m b^n c^m \mid m, n \geq 1\}$.

Solution: The PDA below is defined as $P = (\{0, 1, 2, 3\}, \{a, b, c\}, \{a, Z\}, \delta, 0, Z, \{4\})$ and accepts the language either by final state or by empty stack.

$a, Z/aZ$
$a, a/aa$ $b, a/aa$ $c, a/\epsilon$

start $\longrightarrow$ (0) $\xrightarrow{b, a/a}$ (1) $\xrightarrow{c, a/\epsilon}$ (2) $\xrightarrow{\epsilon, Z/\epsilon}$ ((3))

4. (5 points) Draw a graphical representation of a PDA for the language $\{0^m 1^n \mid 1 \le m \le n\}$.

Solution: The PDA below is defined as $P = (\{0, 1, 2\}, \{0, 1\}, \{0, Z\}, \delta, 0, Z, \{2\})$ and accepts the language by final state.

$0, Z/0Z$
$0, 0/00$ $1, 0/\epsilon$ $1, Z/Z$

start $\longrightarrow$ (0) $\xrightarrow{\;1, 0/\epsilon\;}$ (1) $\xrightarrow{\;1, Z/Z\;}$ ((2))