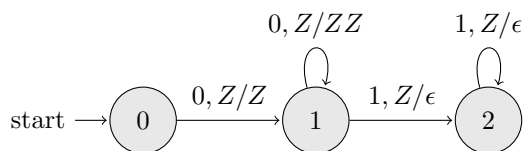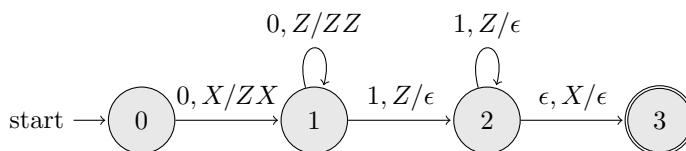Design PDAs to accept the following languages by either final state or empty stack, whichever is most convenient for the particular language. In the solutions, both acceptance styles are presented in order to cover more student answers. You only need to provide one acceptance style per question.

1. (5 points) $\{0^n 1^n \mid n \geq 1\}$

> **Solution:** The PDA $P_N = (\{0,1,2\}, \{0,1\}, \{Z\}, \delta_N, 0, Z)$ accepts the language by empty stack with δ_N given by the diagram below.
>
> $$0, Z/ZZ \qquad 1, Z/\epsilon$$
> $$\text{start} \longrightarrow \boxed{0} \xrightarrow{0, Z/Z} \boxed{1} \xrightarrow{1, Z/\epsilon} \boxed{2}$$
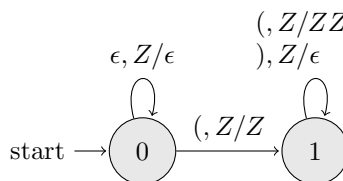>
> The PDA $P_F = (\{0,1,2,3\}, \{0,1\}, \{X,Z\}, \delta_F, 0, X, \{3\})$ accepts the language by final state with δ_F given by the diagram below.
>
> $$0, Z/ZZ \qquad 1, Z/\epsilon$$
> $$\text{start} \longrightarrow \boxed{0} \xrightarrow{0, X/ZX} \boxed{1} \xrightarrow{1, Z/\epsilon} \boxed{2} \xrightarrow{\epsilon, X/\epsilon} \boxed{3}$$
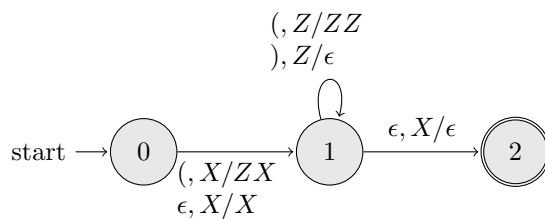
2. (5 points) the language of balanced parentheses

> **Solution:** The PDA $P_N = (\{0,1\}, \{(,)\}, \{Z\}, \delta_N, 0, Z)$ accepts the language by empty stack with δ_N given by the diagram below.
>
> $$\epsilon, Z/\epsilon \qquad \qquad (, Z/ZZ \quad), Z/\epsilon$$
> $$\text{start} \longrightarrow \boxed{0} \xrightarrow{(, Z/Z} \boxed{1}$$
>
> The PDA $P_F = (\{0,1,2\}, \{(,)\}, \{X,Z\}, \delta_F, 0, X, \{2\})$ accepts the language by final state with δ_F given by the diagram below.
>
> $$(, Z/ZZ \quad), Z/\epsilon$$
> $$\text{start} \longrightarrow \boxed{0} \xrightarrow[\epsilon, X/X]{(, X/ZX} \boxed{1} \xrightarrow{\epsilon, X/\epsilon} \boxed{2}$$

3. (5 points) Think of i as an **if** statement and e as an **else** branch. Then the language over $\{i, e\}$ where all prefixes of every string contain at least as many is then es represents the language of every valid C-style **if**/**else** nesting.

Solution: The PDA $P_F = (\{0,1\}, \{i,e\}, \{Z\}, \delta_F, 0, Z, \{1\})$ accepts the language by final state with δ_F given by the diagram below.

$$i, Z/ZZ$$
$$e, Z/\epsilon$$

start $\longrightarrow$ (0) $\xrightarrow{i, Z/Z}$ ((1))
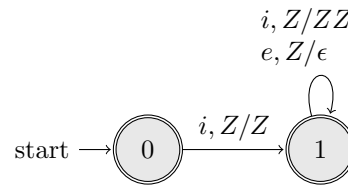
The PDA $P_N = (\{0,1,2\}, \{i,e\}, \{Z\}, \delta_N, 0, Z)$ accepts the language by empty stack with δ_N given by the diagram below.

$$i, Z/ZZ$$
$$\epsilon, Z/\epsilon \qquad e, Z/\epsilon \qquad \epsilon, Z/\epsilon$$

start $\longrightarrow$ (0) $\xrightarrow{i, Z/Z}$ (1) $\xrightarrow{\epsilon, Z/\epsilon}$ (2)