Stats 3Y03 Summary

Note: R might be on the final:\$

Chapter 1

Categorical variable: qualitative variable, such as funny; limited number of options

- e.g. Blood type, Political party
- It can still be a number if the number doesn't describe a quantity
- Ordinal: Values that can be ordered, such as academic grade
- Nominal: Values that cannot be ordered, such as brand name

Types of variables

Numerical variable: quantitative variable, such as position

Continuous: decimalsDiscrete: integer

Univariate Data: single variable

Bivariate Data: 2 variables (not required in this course)

Multivariate Data: more than 2 variables

Probability: average of population is from average of sample

Inferential statistics: average of sample is from average of population

Sampling Frame: list of things in a list that can be sampled

- telemarketers' sample frame is the people with a phone number in the phone book/phone archive of the company
- when doing a culture study of farms, the sample frame could even be a map

Enumerative study:

- identifiable goal
- well-defined, unchanging sample frame
- enumerate (explain, evaluate, describe) a condition that exists with the existing population

Analytic study:

- focused on improvement of the process which created the results and which will continue creating results in the future
- no well-defined sampling frame

Target population

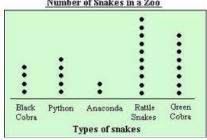
- population you want to be collecting data from
- sample population is the population you are collecting data from
- sample population is usually subset of target population
- sample population is useful when the target population is too large
- sometimes it is not the same as the sample population
 - o e.g., when informing factory workers that their productivity is being observed, they'll act differently

Simple random sample: from entire population

Stratified random sample: from a sub-population (1 from each row)

Convenience sample: not entirely random; what is easy to obtain (first row)

Dot plot: quantifying increments and representing them by dots Number of Snakes in a Zoo



Mean: average

Median: middle value; if length of set is even, average of (n+1)/2 and n/2; if length of set is odd, (n+1)/2

Mode: common number

Unimodal: 1 peak **Bimodal**: 2 peaks

Multimodal: more than 2 peaks

Graphs can also be **symmetric** or **asymmetric**, which is when the top half of the boxplot looks similar to the bottom half.

Left skew: mostly on right side **Right skew**: mostly on left side Graphs can also be **unskewed**.

Outliers:

- values that must be mistakes or abstract exceptions
- $> 1.5 \times$ forth spread (see below) beyond closest quartile
- **extreme outlier** is $> 3 \times$ forth spread

Each data set is split up into 4 quartiles.

Q1: median of bottom half (includes middle number if odd length)

Q2:

Q3: median of top half (includes middle number if odd length)

The Five-Number Summary:

- 1. Minimum
- 2. Q1
- 3. Q2
- 4. Q3
- 5. Maximum

The range, minimum, and maximum can include outliers

range: max - min

Variance

Variance: distribution of range

N is target population size

 x_i are the values

n is sample population size

Trimmed mean: mean calculated by trimming away a given percentage of elements (relative to number of elements) from the top and bottom. If the percentage gives a non-discrete number of elements, you have to calculate multiple trimmed means and find the mean of the 2 trimmed means

Population mean: expected outcome of mean of <u>target population</u>, i.e. average given a theoretically <u>infinite</u> amount of measurements; a.k.a. true mean, expected value

$$\mu = \frac{1}{N} \sum_{i=1}^{N} x_i$$

Sample mean: average given finite number of inputs; an estimate of the population mean

$$\overline{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

Sample median: \tilde{x}

Sample variance: $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \overline{y})^2$

Population variance: $\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2$

Spread: interquartile range

Standard Deviation

s.d.

- Average distance from the mean
- Larger s.d. means more spread
- i.e. when all values are the same, s.d. = 0
- Square root of variance = $\sqrt{s^2}$

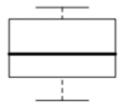
Degrees of freedom: n-1

Another measure of spread is **interquartile range** or **forth spread**. $(Q_3 - Q_1)$

Whiskers: minimum and maximum points of the range that does not include outliers

Boxplot:

- Top and bottom lines are whiskers
- Box surrounds forth spread
- Middle line is median
- Can be vertical or horizontal
- Outliers are still placed on boxplots, using circles (o) or stars (*)
- (a.k.a. Boxplot-and-whisker plot)



Chapter 2

This is similar to the logic course <u>SFWR ENG 2FA3</u>. Probability is between 0 and 1

Sample space: all possible outcomes

The size of the sample space is: outcomes events.

N: number of outcomes for an event

N(A): number of outcomes in sample space, A

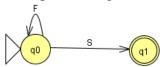
Relative frequency probability: events that occur frequently, such as rolling dice or buying lottery tickets

Relative frequency of a value = $\frac{\text{occurrences of value}}{\text{observations in data set}}$

Personal probability: events that cannot be repeated or non-random events with unknown quantities that is <u>based on belief of an individual</u>

Coherent: personal probability of one event does not contradict personal probability of another

Sometimes you can have an **infinite number of possible outcomes**. For example, if you are testing something until failure, you will repeat testing until success {S, FS, FFS, ...}



If there are a given number of outcomes, such as 1 through 6 for a dice, and a sample space, A, such as containing all odd outcomes, A', the **complement**, contains everything A does not, such as all even outcomes. Therefore, P(A) + P(A') = 1

Simple Event: Only one way to get each outcome

Compound Event: Multiple ways to get the same outcome

Replacement

Without replacement: e.g. if you are picking names out of a hat and you put the names back after each pick

With replacement: when you use each option only once

Mutually-Exclusive Events

Mutually exclusive (a.k.a. disjoint event): 2 outcomes cannot occur simultaneously; $A \cap B = \emptyset$; e.g. rolling a dice can either be 3 or 5–not both, whereas it being 3 or odd is not mutually exclusive

The **probability** is the sum of the probability of each individual event:

$$P(A_1 \cup A_2 \cup ... \cup A_k) = \sum_{i=1}^{i=k} P(A_i)$$

$$P(A) = \frac{N(A)}{N}$$

For ordered pairs, number of possible arrangements is: *N*!

Permutations are ordered sequences that are made up by *k* elements that are a subset of a set of *n* elements.

The notation for **number of permutations** is: $P_{k,n} = \frac{n!}{(n-k)!}$

Bayes's Theorem:
$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Non-mutually exclusive events

For non-mutually exclusive events, there can be overlap, so:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

For ordered pairs, number of possible arrangements for k events is: $\prod_{i=1}^{k} N(A_i)$

Unordered permutations are known as **combinations** (n choose k). They are denoted:

$$C_{k,n} = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

For unordered pairs, number of combinations is: $\frac{n!}{k!(n-k)!}$, where n is the number of objects and

k is the size of the group (pick k, 5, players for the team from n, 8 people. number of permutations?)

Dependent: you can't put it back **Independent**: you can put it back

Conditional probability: Probability of A given B: $P(A | B) = \frac{P(A \cap B)}{P(B)}$

Chapter 3

Random variables

rv

- function whose domain is the sample space and whose range is the set of real numbers, but is subject to random variations
- denoted by a capital letter, whereas its values have the same letter as the rv, but lower-case
- can either be continuous or discrete
- x is a particular value of a random variable

Bernoulli: binary output; can only be either a 0 or a 1

Probability Mass Function (pmf): a function that gives the probability that a <u>discrete random variable</u> is exactly equal to some value

Cumulative Distribution Function

CDF: add up all probabilities within a given range

$$F(x) = P(X \le x)$$

$$= \int_{-\infty}^{x} f(y) dy$$

$$= \begin{cases} 0, & x < a \\ \frac{x - a}{b - a}, & a \le x \le b \\ 1, & x > b \end{cases}$$

$$F(x)$$

$$= \begin{cases} 0, & x < a \\ \frac{x - a}{b - a}, & x < b \end{cases}$$

Expected Value

- mean using probability of discrete rv's
- gives same result as population mean
- use if you're not given data, but given probability $E(X) = \mu_x$

$$= \sum_{x \in D} xp(x)$$

$$= \int_{-\infty}^{\infty} xf(x) dx$$

- <u>Variance</u>: $V(X) = \sum_{x \in D} (x \mu)^2 p(x) = E[(X \mu)^2]$
- General Expectation formula: $E(x) = \sum xP(x)$

Variance of **CDF**

$$Var(X) = E[(X - \mu)^{2}]$$

$$= \sigma^{2}$$

$$= \int_{-\infty}^{\infty} (x - \mu)^{2} f(x) dx$$

$$= \int x^{2} f(x) dx - \mu^{2}$$

Binomial experiment

- 1. fixed trial
- 2. 2 outcomes-success or failure
- 3. Trials are independent (without replacement)
- 4. Probability of each outcome is the same for each trial

- If the sample size is at most 5% of the population size, the experiment can be analyzed as though it were a binomial experiment (<u>without replacement</u>).
- *n*: repetitions of trials
- p = P(success in single trial)
- q = P(fail in single trial)
- x: total number of successes

•
$$b(x;n,p) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0..n \\ 0, & \text{else} \end{cases}$$

Hypergeometric (H.D.): same as binomial, but dependent (with replacement)

- *N*: number of items in population
- *M*: number of successes in population
- *n*: number of items in sample
- x: number of successes in sample

•
$$P(X = x) = h(x(n, M, N)) = \frac{\binom{M}{x} \binom{N - M}{n - x}}{\binom{N}{n}}$$

• $E(X) = n \cdot \frac{M}{N}$ (same as binomial)

•
$$V(X) = \left(\frac{N-n}{N-1}\right) \cdot n \cdot \frac{M}{N} \cdot \left(1 - \frac{M}{N}\right)$$

Negative Binomial Distribution:

- n is fixed in *binomial*, whereas *here*, n is random
- trials repeated until success we want
- \bullet r is the number of successes you want
- If r = 1, this is known as a **geometric distribution**

Poisson distribution:

- discrete pdf
- number of occurrences of an event in a given interval, given average rate and time (independent), since last event
- $p(x;\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$
- *x*: probability that *x* things will happen
- λ (or μ): average occurrences given population (multiply average rate by population)
- mean = variance = λ , so <u>S.D.</u> = $\sqrt{\lambda}$
- α expected number of events during unit interval
- t time interval length

$$\bullet \quad P_k(t) = \frac{e^{-\alpha t} \cdot (\alpha t)^k}{k!}$$

Chapter 4

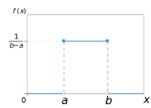
Probability Density Function

PDF: a function that gives the probability that a <u>continuous</u> random variable is exactly equal to some value, such that: $P[a \le X \le b] = \int_a^b f(x) dx$

Area under whole curve = 1

Uniform Distribution: if a continuous random variable, X, has a pdf, f(x; a, b):

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & \text{else} \end{cases}$$



Note: a and b do not represent the entire range of the <u>PDF</u>. Just look at the f(x) formula above!

To get pdf from cdf, take the derivative of the cdf.

$$F'(x) = f(x)$$

Percentile: percentage of data below you; relative to other data in the range

- p: percentile
- η : percentile function
- $p = F(\eta(p)) = \int_{-\infty}^{\eta(p)} f(y) dy$

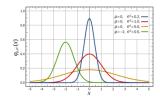
$$\mu_{x} = E(X)$$

$$E(X) = \alpha \beta$$

Normal Distribution

A.k.a. population normality symmetric; mean = median = mode

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty$$



Bell curve (a.k.a. Gaussian curve): normal curve, normal distribution; Central Limit Theory says the sampling distribution of sample means will be bell-shaped; s.d. = population s.d./ $\sqrt{\text{sample size}}$

Z-Tables

A.K.A. Standard Normal Cumulative Probability Table

Z-function: a standardized cdf that you use to predict data

$$\bullet \quad Z = \frac{X - \mu}{\sigma}$$

• It's horizontal units are s.d.'s

z_c: critical value

If you're given a probability, you find the value on the z-table and choose the values at the location. If you cannot find the value on the z-table, find the two closest ones and find the weighted average.

$$E = z_c \frac{\sigma}{\sqrt{n}}$$

Standardized Score: a.k.a. "z-score", observed value mean s.d.

 α -level is the area of the graph of a normal distribution curve $\alpha = P(Z \ge z_{\alpha})$

 Z_{α} : for the standard normal distribution

Empirical rule: you can identify that your data has normal distribution by using the rule that:

- 68% of data is within 1 s.d. from mean
- 95% of data is within 2 s.d. from mean
- 99.7% is within 3 s.d. from mean
- there are 3 s.d.'s from the mean

Exponential: time between events, whereas poisson is more the number of events

$$p(x) = \lambda e^{-\lambda x}$$

Chapter 5

 $p \leftarrow discrete$

 $f \leftarrow continuous$

$$p_x(x) = \sum_{y} p(x, y), p_y(y) = \sum_{x} p(x, y)$$

Mean of sum of joint pdf (discrete): $E(x+y) = \sum_{x,y} (x+y) p(x,y)$

Mean of sum of joint pmf (continuous): $E(x+y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x,y) \cdot f(x,y) dxdy$

Covariance: variance for multiple variables; $Cov = E(XY) - \mu_x \cdot \mu_y$

Independent and Identically Distributed (i.i.d.):

- form a simple, random sample of size n
- X_i's are independent r.v.'s
- X_i's all have same probability distribution

Multinomial distribution: represented by the pmf, $f(x_{1..k}; n, p_{1..k}) = \prod_{i=1}^{n} \frac{1}{x_i!} p_i^{x_i}$

Marginal pdf (continuous):
$$f_x(x) = \int_{-\infty}^{\infty} f(x, y) dy, -\infty < x < \infty$$
$$f_y(y) = \int_{-\infty}^{\infty} f(x, y) dx, -\infty < y < \infty$$

$$f_{y}(y) = \int_{-\infty}^{\infty} f(x, y) dx, -\infty < y < \infty$$

Conditional probability of joint pdf: $f(x|y) = \frac{f(x,y)}{f(y)}, -\infty < x < \infty$

Correlation coefficient: $\rho_{x,y} = \frac{\text{Cov}(x,y)}{\sigma_x \cdot \sigma_x}$

Chapter 6

 θ represents the parameter of interest

 $\hat{\theta} = \theta + \text{error of estimation}$

• a function of the sample, i.e. rv

Point estimate: mean from multiple estimate(s), using the standard error, where θ represents parameter of interest (e.g. μ or σ), where you estimate $\hat{\theta}$.

Bias of $\hat{\theta}$: $E(\hat{\theta}) - \theta$

Unbiased: $E(\hat{\theta}) = \theta$

Estimator: the formula

Should be unbiased (0 avg. error)

$$\hat{\sigma}^2 = S^2 = \frac{\sum (X_i - \bar{X})^2}{n - 1}$$

$$\hat{\sigma}(S^2) = \hat{\sigma}(S^2) = \hat{\sigma}($$

- Should have minimum variance (i.e. little spread)
- Summary for good estimators: Minimum Variance Unbiased Estimator (MVUE)
 - Unbiased is not always better than minimum variance

Estimate: value obtained from the formula after data has been inputted

What is point estimate for each θ :

- $\mu: \overline{x}$
- $\bullet \quad p: \hat{p} = \frac{x}{n}$

True value: mean of the population (instead of sample)

Trimmed means will result in **robust estimator**.

Standard error of an estimator, $\hat{\theta}$ is its standard deviation, $\sigma_{\hat{\theta}} = \sqrt{V\left(\hat{\theta}\right)}$

Estimated standard error: $\hat{\sigma}_{\hat{\theta}}$ or $s_{\hat{\theta}}$

Bootstrapping: Creating multiple samples from one sample with replacement

- Only works for independent, equally-distributed, random samples
- Not useful if small data set, lots of outliers (remove outliers first), dependence structures (data based on changing time, etc.)
- n* depends on computing capacity, type of problem, and complexity
- 1. Compute x^* , which is from x, sampled with replacement
- 2. Compute $\hat{\theta}^*$ from x^*
- 3. Estimate standard error, $\operatorname{Se}_{B}(\hat{\theta}) = \sqrt{\frac{\sum_{i=1}^{B} (\theta_{i}^{*} \overline{\theta}^{*})^{2}}{B-1}}$

Parametric bootstrap:

Point estimation: 2 main methods: a method of getting the population based on a guess by calculating standard error

- Method of moments
- Maximum Likelihood estimation

Minimum Variance Unbiased Estimator (MVUE)

Method of Moments

 k^{th} moment of f(x) is $E(X^k) = \frac{1}{n} \sum_{i=1}^n X_i^k$

$$\lambda = \frac{1}{\mathrm{E}(X^k)} \xrightarrow{\mathrm{E}(X^k) = \overline{X}} = \frac{1}{\overline{X}}$$

Recall: $E(X) = \mu = \alpha \beta$

Maximum Likelihood Estimation

(MLE)

Maximum value of the random sample that you have

 $\hat{\theta} = \max(P(X_i))$ is the value of X where you have the highest probability

More popular

Equating the derivative of the logarithm of the pmf to 0 gives maximizing value

Method of Moments Estimator (MME):

Joint pdf:

- Likelihood
- Independent
- $f(x_1,...x_n;\lambda) = (\lambda e^{-\lambda x_1})...(\lambda e^{-\lambda x_n}) = \lambda^n e^{-\lambda \sum x_i}$
- pdf governing occurrences of A & B, not just one (i.e. pdf of occurrences of multiple potential events), like for regular pdf's

$$n\ln(\lambda) - \lambda \sum x_i$$

 \hat{a} : variables with a hat means they are an estimate

$$\hat{\lambda} = 1/\bar{X}$$

e.g.)

$$\begin{split} \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^{2}}} e^{\frac{-(y_{i}-\mu)^{2}}{2\sigma^{2}}} &= \left(\frac{1}{\sqrt{2\pi\sigma^{2}}}\right)^{n} e^{\frac{-n(y_{i}-\mu)^{2}}{2\sigma^{2}}} \\ \lambda &= \frac{1}{\sqrt{2\pi\sigma^{2}}} \\ e^{-\lambda\sqrt{2\pi\sigma^{2}} \frac{n(y_{i}-\mu)^{2}}{2\sigma^{2}}} &= e^{-\lambda\sqrt{2\pi\sigma^{2}} \frac{n\pi(y_{i}-\mu)^{2}}{2\pi\sigma^{2}}} \\ &= e^{-\lambda\sqrt{2\pi\sigma^{2}} \frac{n\pi(y_{i}-\mu)^{2}}{\sqrt{2\pi\sigma^{2}}}} \\ &= e^{-\lambda\sum_{i=1}^{n} \frac{\pi(y_{i}-\mu)^{2}}{\sqrt{2\pi\sigma^{2}}}} \\ x_{i} &= \frac{\pi\left(y_{i}-\mu\right)^{2}}{\sqrt{2\pi\sigma^{2}}} \end{split}$$

Chapter 7

Most important chapter!!!

Confidence interval

Confidence level: measures reliability of confidence interval; most popular confidence levels: 90, 95, and 99%; the percent of all samples that will give correct results

Confidence interval: interval where certain where data is reliable

Precision is width of confidence interval

- First determine confidence level
- use the z tables to find
- In order for this to work:
 - o Population distribution is normal
 - o s.d. given
- Actual mean μ does not necessarily have to be in the interval even if the estimated mean is in it

Sample mean \pm 1.96 standard errors

$$P(CI) = 100(1-\alpha)\%$$

$$CI = \left(\overline{x} - z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}, \overline{x} + z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

$$CI = \left(\overline{x} - t_{\frac{\alpha}{2}, \frac{n-1}{200F}} \cdot \frac{s}{\sqrt{n}}, \overline{x} + t_{\frac{\alpha}{2}, n-1} \cdot \frac{s}{\sqrt{n}}\right)$$

Hypothesis:

Standard error: conversion of standard deviation (total population) to sample distribution

(sample population) =
$$\frac{\text{s.d.}}{\sqrt{n}}$$

Bound of the error is half the width

Sample size:
$$n = \left(2z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{w}\right)^2$$

A larger sample size gives a narrower confidence interval.

A smaller sample size gives a wider confidence interval.

Statistical Inference:

Sampling variability:

T-Table: Z-Table, but for s, instead of σ , but you can still use z if sample size > 40; uses 2 parameters: degrees of freedom and probability level

Chapter 8

The point of this to see if the error in the sample mean is low enough to make the sample valid/satisfactory.

Statistical hypothesis: assumption about a population characteristic; 2 types:

- Null Hypothesis
- Alternative Hypothesis
- Choose the hypothesis based on the level of significance
 - o for lower level, choose Type I / Null

o for higher level, choose Type II / Alternative

Null Hypothesis

• H_0 : $\mu = \mu_0$

$$0 z = \frac{\overline{x} - \mu_0}{\sigma} \sqrt{n}$$

- proof by contradiction
- assume it is the thing you think it isn't and prove that wrong
- think equality
- If you reject it, the evidence is **statistically significant**

Alternative Hypothesis

- H_A OR H_a OR H₁
 - o $\mu > \mu_0, z \ge z_\alpha$
 - o $\mu < \mu_0, z \leq z_\alpha$
 - \circ $\mu \neq \mu_0, \dots$
- specified range
- think >, <, or \neq
- if you only choose one inequality, it is called a **one-sided hypothesis test**
 - o **upper**: $\mu > H_a$
 - o **lower**: $\mu < H_a$

Errors

- **Type I**: say something is right when it's wrong
 - o **Level of significance** (α): P(Type I error)
 - o Proving null hypothesis true
 - o Since null hypothesis is a value, P has one value
- **Type II**: say something is wrong when it's right
 - o P(Type II error) = β
 - o Proving <u>alternative hypothesis</u> true
 - o Since alternative hypothesis is a range, P is a range

Case I

 σ given (not s), normal distribution

$$z = \frac{\overline{x} - \mu_0}{\sigma} \sqrt{n}$$

Case II

For large n (i.e. n > 40), s is close to σ

$$z = \frac{\overline{x} - \mu_0}{s} \sqrt{n}$$

Case III

normal dist, s given

Test statistic value:
$$t = \frac{\overline{x} - \mu_0}{s} \sqrt{n}$$

One tail: z_{α} Two tails: $z_{\alpha/2}$

If in rejection region, reject the null hypothesis.

P-Value

P-Value: observed level of significance

Level of significance (α): a percentage or decimal that represents the cut-off value

- If P-value is small enough, reject the null hypothesis and accept the alternative hypothesis
- If p-value is not small enough, don't reject the null hypothesis and there is not enough information to determine whether or not to accept the alternative hypothesis.

Chapter 9 - Test Statistics

Degrees of Freedom: number of samples 1. For normal populations with known variances, test statistic value: $z = \frac{\overline{x} - \overline{y} - \Delta_0}{\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}}$

 Δ_0 is usually 0

Null hypothesis: $\mu_1 - \mu_2 = \Delta_0$

Alternative Hypothesis: $\begin{cases} \mu - \mu_0 > \Delta_0 \\ \mu - \mu_0 < \Delta_0 \\ \mu - \mu_0 \neq \Delta_0 \end{cases}$

3 Cases (conditions stay the same as before):

- 1. Case I:
- 2. Case II:
- 3. Case III: *t* distribution

Round down to the nearest integer

Pooled *t* happens when $\sigma_1^2 = \sigma_2^2$

Margin of Error: $E = t_{\frac{a}{2}} \frac{S_d}{\sqrt{n}}$

f distribution:

- pdf distribution is too difficult, so we will work with tables
- Assumptions:
 - o 2 populations independent
 - o Simple random samples

o Normally distributed

• Test statistic for test hypothesis, given two variances is: $f = \frac{s_1^2}{s_2^2}$

• Demonstrates the difference between the two variances

Determines whether or not the rejection region is too high or not

• Inputs:

o Significance level, α

• Null Hypothesis: $\sigma_1^2 = \sigma_2^2$

• Alternative Hypothesis: $\sigma_1^2 < \sigma_2^2$

Chapter 10

Determine a line with the least variance

Deterministic Relationship: $y = \beta_0 + \beta_1 x$ (i.e. y = mx + b)

Regression Analysis:

Model equation: $y = \beta_0 + \beta_1 x + \varepsilon$

ε: measure of variation; error in data

Principle of least squares: gives minimum error

$$S_{xy} = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{n-1}$$

$$s_x^2 = \frac{\sum_{i=1}^n \left(x_i - \overline{x}\right)^2}{n-1}$$

$$b_{1} = \frac{s_{xy}}{s_{x}^{2}} = \frac{\sum_{i=1}^{n} (x_{i} - \overline{x})(y_{i} - \overline{y})}{\sum_{i=1}^{n} (x_{i} - \overline{x})^{2}} =$$

$$b_0 = \overline{y} - b_1 \overline{x}$$

Point Prediction: plugging in values of x into the regression equation

Residual: error; vertical deviation from estimated line $(y - y_0)$

Extrapolation: usually doesn't work, though

library (MASS)

summary() gives 5-number summary

Sum of Squares for Errors (SSE): $SSE = \sum (y_i - \hat{y})^2 = \sum y_i^2 - \hat{B}_0 \sum y_i - \hat{B}_1 \sum x_i y_i$

Chapter 11

(actually chapter 10, but week 11. Go back and rename the sections by chapter and not by week)

ANOVA:

Factor:

levels of the factor:

The number of populations being compared is I.

 $X_{i,j}$ represents the random variable for the j^{th} experiment for the i^{th} population

Hypothesis

$$H_0: \mu_1 = \mu_2 = \ldots = \mu_I$$

 H_a : at least 2 values of μ_I

$$E(X_{i,j}) = \mu_i$$

$$V(X_{i,j}) = \sigma$$