

SFWR ENG 3DX4 Summary

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Course: SFWR ENG 3DX4

Math objects made using [MathType](#); graphs made using [Winplot](#).

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Note: the following summaries may be useful:

- [SFWR ENG 2MX3](#)
- [ENGINEER 3N03](#)
- [TRON 3TA4](#)

I may review to clarify or correct, but mostly I will omit those things.

Introduction to Systems

Systems can be represented by **block diagrams** to make it easier to marginalize the different parts of the systems.

Laplace

Useful for...

Time begins when your signal begins

$$h(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

Initial conditions:

- $c(0)$

Time domain (t): variables are lower case, e.g. $f(t)$

Frequency domain (s): variables are upper case, e.g. $F(s)$

Transfer function:

When doing the inverse Laplace, it's useful to break your fractions up so that you can

Strictly Stable: it will eventually get back to the initial position

Marginally Stable:

Unstable: it will progressively get worse

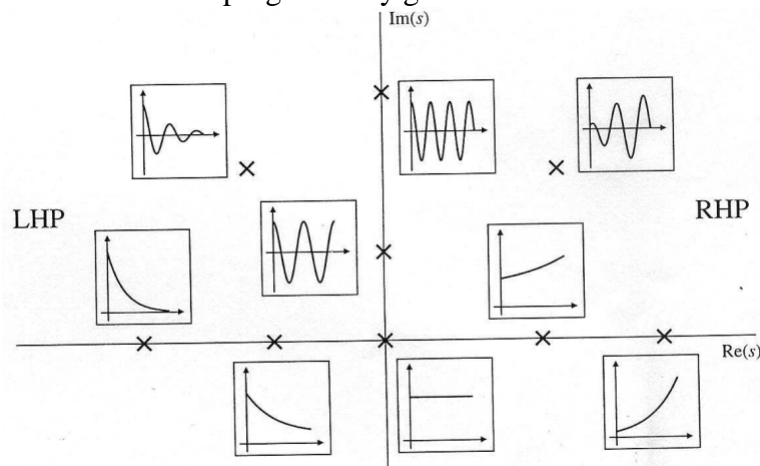


Figure 2.5 from Dorf and Bishop, *Modern Control Systems (10th Edition)*, Prentice-Hall, 2004.

Transfer Functions

Electrical

Component stuff

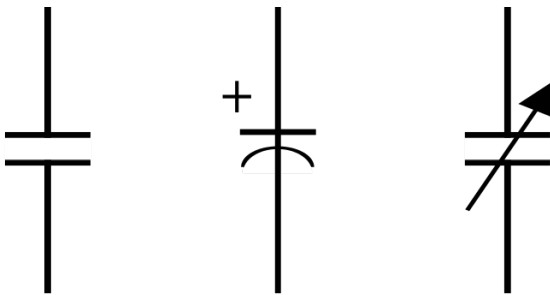
Impedance:

$$Z_R = R$$

$$Z_L = sL = j\omega L$$

$$Z_C = \frac{1}{sC} = \frac{-j}{\omega C}$$

Polarized capacitors: Z is positive when current is going from - to +, but negative from + to -



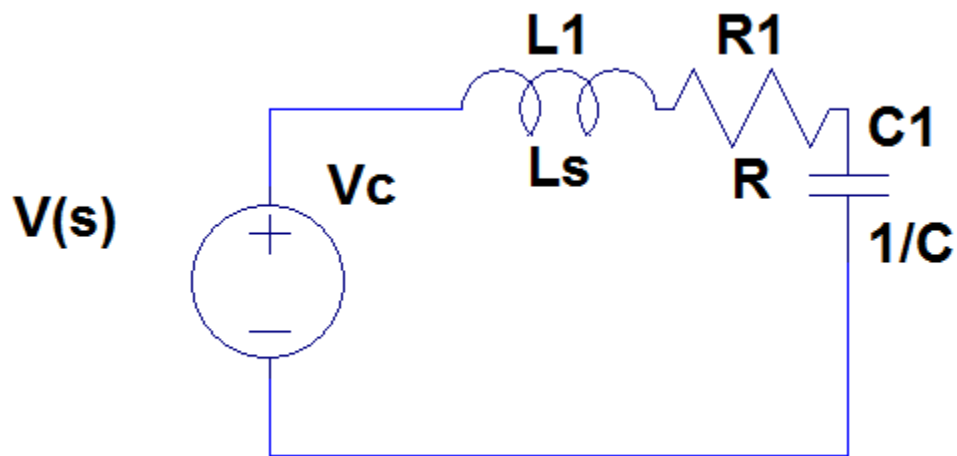
Fixed Capacitor Polarized Capacitor Variable Capacitor

admittance:

$$Y(s) = \frac{I(s)}{V(s)} = \frac{1}{R} = G$$

$$V_c(s) = I(s) \frac{1}{Cs}$$

$$I(s) = \frac{V(s)}{L_s + R + \frac{1}{Cs}}$$



Mesh Analysis

Add the voltages, where $V = IZ$

Cramer's Rule

$$x_1 = \frac{\det(A_1)}{\det(A)}, x_2 = \frac{\det(A_2)}{\det(A)}, \dots, x_n = \frac{\det(A_n)}{\det(A)}$$

$$V_c(s) = \overbrace{H(s)}^{\text{transfer function}} \frac{1}{C_s}$$

OP-Amps

Mechanical

Translational systems:

Rotational Systems:

Newton's Second Law of Motion: $\Sigma f = Ma$

$$Z_m(s) = \frac{F(s)}{X(x)}$$
$$f(t) = Ma(t)$$
$$= M \frac{d^2x}{dt^2}$$

Translational Systems

Spring

Spring is like a capacitor

Force displacement: $f(t) = Kx(t)$

Viscous Damper

Using viscous fluid to slow something down

Viscous Damper is like a resistor

Force displacement: $f(t) = f_v \frac{dx(t)}{dt}$

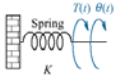
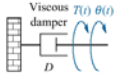
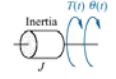
Mass

Mass is like an inductor

Force displacement: $f(t) = M \frac{d^2x(t)}{dt^2}$

Rotational Systems

Impedance: $Z_m(s) = \frac{T(s)}{\theta(s)}$

Component	Torque-angular velocity	Torque-angular displacement	Impedance $Z_m(s) = T(s)/\theta(s)$
	$T(t) = K \int_0^t \omega(\tau) d\tau$	$T(t) = K \theta(t)$	K
	$T(t) = D \omega(t)$	$T(t) = D \frac{d\theta(t)}{dt}$	Ds
	$T(t) = J \frac{d\omega(t)}{dt}$	$T(t) = J \frac{d^2\theta(t)}{dt^2}$	Js^2

Signals

Transducer: anything that converts energy to electrical energy

Transmitter: long distances

Unstable systems have ∞ steady state error

Steady-state error $[e_\infty]$:

$$e_\infty = \lim_{t \rightarrow \infty} e(t)$$

Final value theorem: finds steady state error

$$\lim_{x \rightarrow \infty} f(t) = \lim_{x \rightarrow 0} sF(s)$$

So $e_\infty = \lim_{x \rightarrow 0} sF(s)$ and you're given $F(s)$, so just multiply by s and find the limit.

Rise time $[T_r]$: time between 10% and 90% of final value (c_{final})

Peak time $[T_p]$: time it takes to get to highest peak (c_{max})

Settling time $[T_s]$: how long it takes to get to the steady state within $\pm 2\%$

Percent overshoot [%OS]: how much further is the peak from the final

$$\% \text{OS} = \frac{c_{\text{max}} - c_{\text{final}}}{c_{\text{final}}} \times 100\%$$

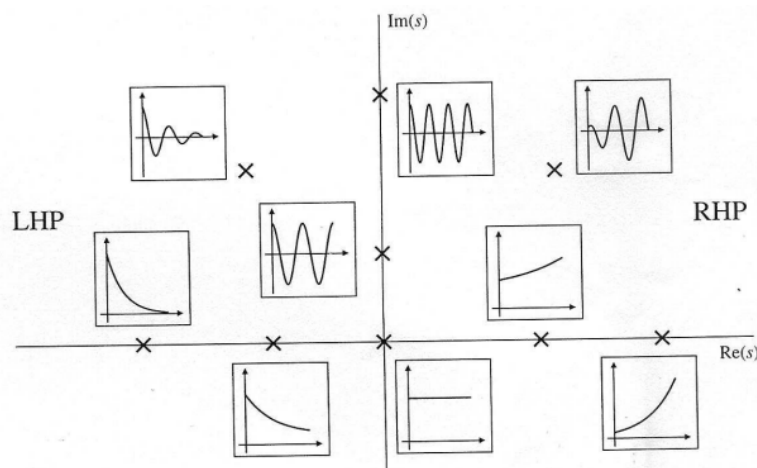
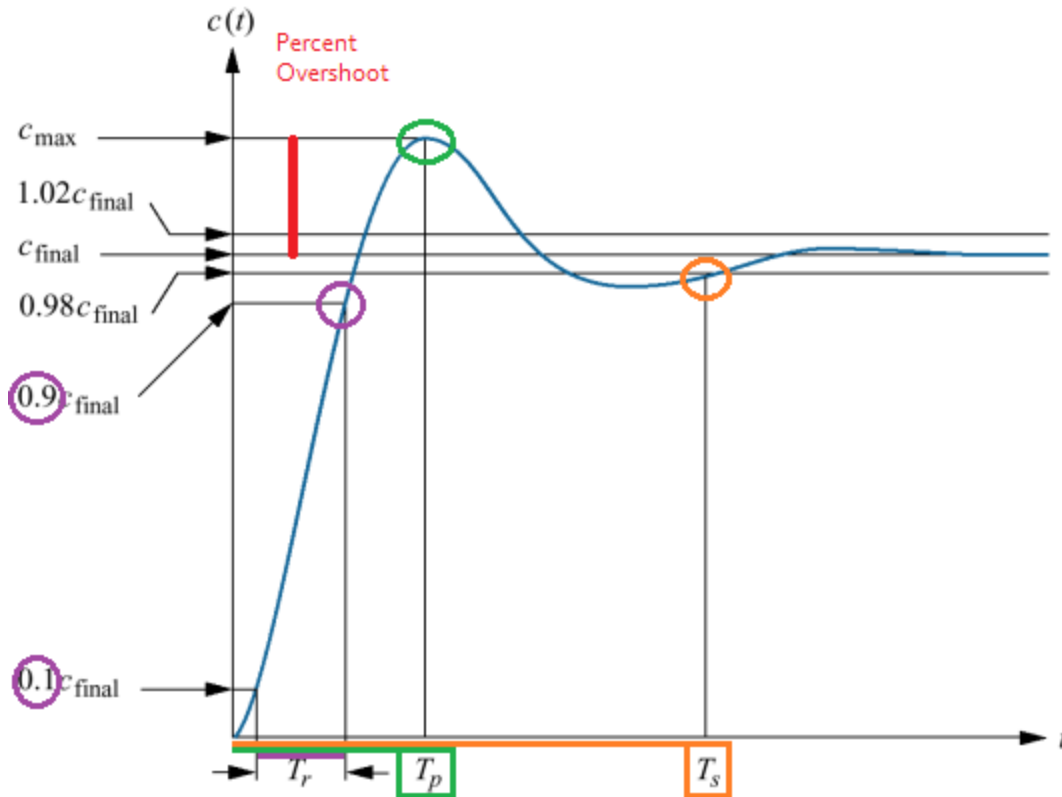


Figure 2.5 from Dorf and Bishop, *Modern Control Systems (10th Edition)*, Prentice-Hall, 2004.

Non-/Linear Systems

- Op Amps are linear
- If you don't have enough voltage, your motor magnets won't have enough power to switch poles, so they require a minimum voltage

You can't model non-linear systems, until you linearize it. To do this, we find the slope and approximate the equation of the line, using $y=mx+b$

Proportional-Integral-Derivative (PID):

If your gears are vibrating, your PID is probably too high

Block Diagrams

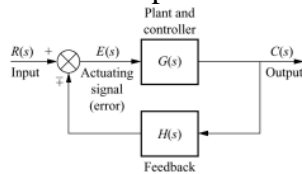
A way of representing a system

Summing junction: could be an X or +, but usually an X in this course

Cascade: subsystems in series are multiplied

Parallel: parallel subsystems have a *summing junction* at the end, so you just add everything together

Feedback: positive feedback is bad



Positive: $\frac{G(s)}{1 - G(s)H(s)}$

Negative: $\frac{G(s)}{1 + G(s)H(s)}$

Simplification:

State Space Equations

Yeah, you think you know them from 2MX3, but you don't really know them. Apparently the ABCD variables actually have names.

- **System Matrix [A]:**
- **Input Matrix [B]:**
- **Output Matrix [C]:**
- **Feedforward Matrix [D]:**

Transfer Function -> State Space

Phase Variable Approach:

The n state variables will consist of:

- y
- the derivatives of y