ECON 2B03 Summary

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Math objects made using [MathType](); graphs made using [Winplot]().

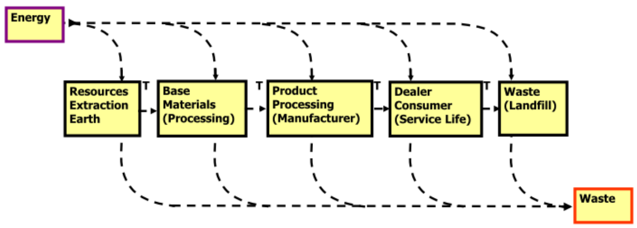
Please join GitHub and contribute to this document. There is a guide on how to do this on my GitHub.

Table of Contents

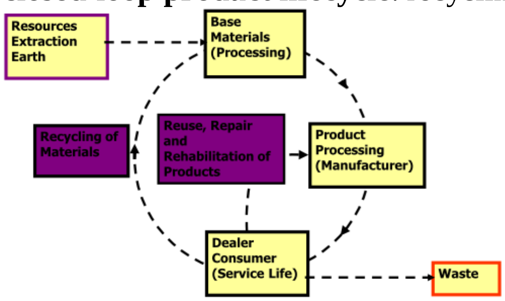
Week 1 – Sustainability

model: an simplification of reality that captures information useful and appropriate for a specific purpose

linear product lifecycle: energy in and out at every stage



closed-loop product lifecycle: recycling, re-use, energy only lost at consumer level



Ingenuity Gap: the gap between requirements and solutions, which is caused by an increasing complexity (?)

Triple-Bottom Line

Focuses on:

- **Social sustainability**: productive service to society
- **Environmental sustainability**: resources/land
- **Economic sustainability**: cost efficient

Seven Revolutions

1. Markets: compliance to competition
2. Values: hard to soft
3. Transparency: closed to open
4. Life-cycle Technology: product to function
    a. Companies responsible for entire product life-cycle
5. Partnership: subversion to symbiosis
    a. Companies cooperate
6. Time: wider to longer
    a.

Week 2

Cash-flow period: time over which you are calculating effective interest rate

Interest [I]: compensation for giving up the money

Nominal: doesn't account for inflation

Annual interest rate [r]: nominal interest rate over one year

Present worth [P]:

- The amount of money that is currently being dealt with (whether being loaned, or an annuity)
- Before initiating a time period exchange, the present worth is known as the **principle amount**
- Trying to bring all arrows on cash flow diagram to 0 (one period before the first payment)

Future Worth [F]: the future value of the time period exchange

Interest period [m]: interest compounds per year (not cashflow periods)

Cash flow period (cfp): or *payment period* is how long it is between your payments

Don't forget that there are 4 quarters in a year and 3 months in a quarter-year.

Nominal Interest rate per cfp [i]: interest for each interest period $i = i_s = \frac{r}{m}$

Number of compounds per cfp [k]: should never be a fraction

My way of calculating it is: $k = \dfrac{m}{\text{cfp's per year}}$

Effective Interest rate [i_e]: overall interest rate that takes compounding and payment periods into consideration

$i_e = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+i_s\right)^m - 1$

Effective interest per cash flow [$i_{e/k}$]: $i_{\frac{e}{k}} = \left(1+\frac{r}{m}\right)^k - 1$

Your effective interest rate should be close to nominal interest rate/cash-flow periods per year.

Methods of Interest Calculation

- Lump Sum
- Simple Interest
- Compound interest

Lump Sum

Lump sum: one payment at the end of the time period exchange covers all the funds borrowed, so there is only one interest calculation. The interest on a lump sum does not change over time, but simply the amount paid.

$I = Pi$

$F = P(1+i)$

Simple Interest

Simple interest: a method of calculating interest that is based off the time it takes to pay off the loan and the principle amount

$I = PiN; F = (1+iN)P$

Compound Interest

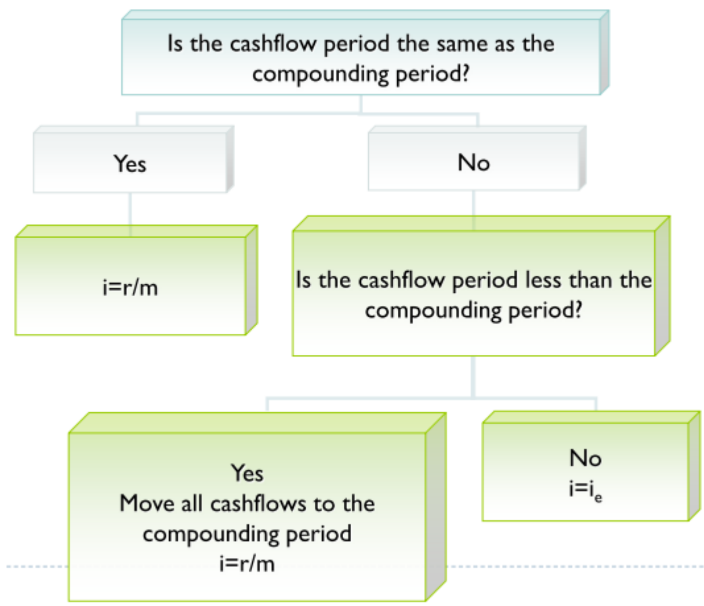
Compound interest: a method of calculating interest that charges interest on the principle as well as unpaid interest each “compound”

$$F = P(1+i)^N$$

$$P = \frac{F}{(1+i)^N}$$

$$I = P(1+i)^N - P$$

With compound interest comes a **compound period**, which is the amount of time, until interest begins to be charged on unpaid charges on top of previous unpaid interest. Compounding periods are assumed to have <u>equal length</u>.



Continuous Compound

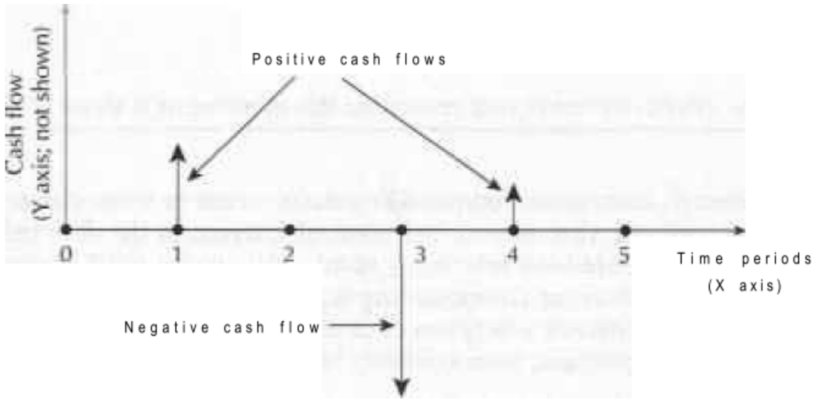
$$i_e = \lim_{m\to\infty}\left(1+\tfrac{r}{m}\right)^m - 1$$

$$\boxed{i_e = \mathrm{e}^r - 1}$$

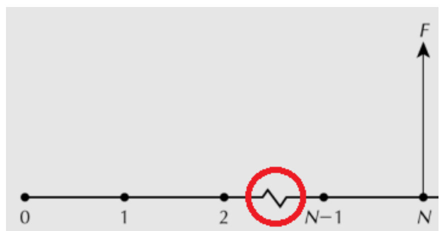
Cash Flow Diagrams

Cash flow diagrams are graphical representations of a system that aid in analysis of cash flows

Since each cash flow is paid as an impulse, instead of continuous outflow from an account, the cash flows are represented by arrows, which can be positive (up) or negative (down) on a chart where time is along the y-axis (represented horizontally) and the cash flow is represented along the x-axis (represented vertically)



Sometimes when there are repeated points, with the same value, we represent the area with repeated points with a *squiggle* on the cash flow diagram:



Linear Interpolation: using 2 different *i*'s: one that's bigger than the desired one and one that's smaller than the desired one

$$x^* = x_1 + \left(x_2 - x_1\right)\left[\frac{y^* - y_1}{y_2 - y_1}\right]$$

Sometimes you can approximate to i = r/m
actually, your x's are i's in this case (change that)
y's are your annuities
y_2 would be your given A at given N and a guessed (upper bound) x_2 and y_1 would be the A at the point in the table at your given N and x_1.

$$i^* = \lfloor i \rfloor + \left(\lceil i \rceil - \lfloor i \rfloor\right)\left[\frac{A^* -}{}\right]$$

Week 3

Equivalence

Equivalence: when the value of something is the same at the one period as at a different period, which is determined by the interest rate

- **Mathematical**: comparing future to present by a factor of *time*
- **Decisional**: comparing future to present by a factor of *periods*
- **Market**: (?)

Note: a payment at time 0 can be assumed to occur at the end of the previous period

Compound Interest Factors

A/B: A given B; A is unknown, B is known

Number of payments [N]: total periods − first payment + 1[interest begins from day one, payment does not];
Number of arrows on cash-flow diagram

Compound Amount Factor

$$\left(F/P,i,N\right)\boxed{=\left(1+i\right)^N}$$

$$F=P\left(1+i\right)^N \Rightarrow F=P\left(F/P,i,N\right)$$

Present Worth Factor

$$\left(P/F,i,N\right)=\frac{1}{\left(F/P,i,N\right)}\boxed{=\frac{1}{\left(1+i\right)^N}}$$

Sinking Fund Factor

$$\left(A/F,i,N\right)\boxed{=\frac{i}{\left(1+i\right)^N-1}}$$

Uniform Series Factor

$$\left(F/A,i,N\right)\boxed{=\frac{\left(1+i\right)^N-1}{i}}$$

Capital Recovery Factor

$$\left(A/P,i,N\right)=\left(A/F,i,N\right)\times\left(F/P,i,N\right)=\left(\frac{i}{\left(1+i\right)^N-1}\right)\left(1+i\right)^N\boxed{=\frac{i\left(1+i\right)^N}{\left(1+i\right)^N-1}}$$

$$A=\left(P-S\right)\left(A/F,i,N\right)$$

Salvage value [S]: value after process is complete (usually 0)

Series Present Worth Factor

$$\left(P/A,i,N\right)=\frac{1}{\left(A/P,i,N\right)}\boxed{=\frac{\left(1+i\right)^N-1}{i\left(1+i\right)^N}}$$

Week 4

Growth-Adjusted Interest Factors

Arithmetic Gradient Series Factor

Base payment of Annuity $[A']$

Nominal increase of an annuity $[G]$: also known as gradient

$(A/G, i, N)$

A': base annuity cost

$A_{\text{total}} = A' + G(A/G, i, N)$

$A/G = \frac{1}{i} - \frac{N}{(1+i)^N - 1}$

Cases	Meaning
A > 0, G > 0	Positive annuity, increasing cash flow
A > 0, G < 0	Positive annuity, decreasing cash flow
A < 0, G > 0	Negative cash flow, decreasing magnitude
A < 0, G < 0	Negative cash flow, increasing magnitude

Remember that the first compounding period has G = 0

Arithmetic Gradient Series Factor

Percentage Increase of an Annuity $[g]$

Growth-adjusted interest rate [i°]: $i° = \frac{1+i}{1+g} - 1$

$(A, g, i, N) = \frac{(A, i°, N)}{1+g}$

$(P/A, g, i, N) = \frac{(P/A, i°, N)}{1+g} = \frac{(1+i°)^N - 1}{i°(1+i°)^N}\left(\frac{1}{1+g}\right)$

Cases	Meaning	Procedure
i > g > 0	<u>Positive</u> *growth*, but <u>decreasing</u> *interest rate*, so i_0 is <u>positive</u>	Use tables or formulas
g > i > 0	<u>Positive</u> *growth*, but <u>increasing</u> *interest rate*, so i_0 is <u>negative</u>	Use formulas only
g = i > 0	*Growth* = interest rate, so i_0 is <u>zero</u>	$P = N\left[\frac{A}{1+g}\right]$
g < 0	<u>Negative</u> *growth*, so i_0 is <u>positive</u>	Use tables & formulas

Discrete Model: cash flows occur at the end of periods
Continuous Model: compound continuously over time

Long-lived projects: $P = \frac{A}{i}$

Amortization: number of years it would take to repay a mortgage loan in full for a given interest rate and payment schedule
Term:

Mortgage:
P = Total – Down
$A = P(A/P, 9)$

The default for car loans and mortgages is to convert the interest rate into monthly compounding from annual.

How much of a given payment, M, within an annuity is interest: (value of annuity) – (F_{N-1} – F_N)

Week 5

Bond

Bond: issuer of a bond takes a loan from the investor

- **purchase price**: amount "lent"
- **going interest rate**: interest rate of
- **par/face value**: total nominal amount the issuer will have given back to the investor
  - the difference between this and purchase price depends on the going interest rate ($\neq$ coupon rate)
- **coupon rate**: the annual percentage of the par value that the issuer pays the investor, similar to monthly mortgage payments
  - The default frequency for payments is semi-annually (i.e. every 6 months), so i = r/2
- **maturity date**: set end date of loan
- a type of fixed-income security, since you know how much you'll get back
- useful if fluctuating stock market

The coupon rate is stated for the year, but you need to cut that by how many times (multiply by face value/times received) you get the coupon, just as you need to cut down your nominal interest by number of compounds for any other interest

Comparison Methods

- **PW Method**: examine present worth of all project cash flows
- **AW method**: convert all cash flows to annuities

Payback Period: number of years it takes for an investment to be recouped = $\frac{\text{first cost}}{\text{annual benefits}}$

Assumptions:
1. Costs & benefits are always measurable by money
2. Future cash flows are known with certainty
3. Cash flows are unaffected by inflation/deflation
4. Sufficient funds are available
5. Taxes are not applicable
6. Down payments ≤ proceeding cash flows

Fixed rate: fixed as a percentage of par value
Floating rate: adjustable interest payments

Characteristics of Projects

Independent
benefits of choosing one project doesn't affect the other project, so it is possible to choose multiple projects

Mutually Exclusive
Choosing one makes it impossible to pick the others

Related but not mutually exclusive
Choosing one project will affect the benefit of another, but it is possible to do both

For example, building 1 train station vs 2 means that the benefit of project 1 will be decreased by that of the benefit of project 2, since some of the number of people who would go to station 1 would go to station 2, instead. This doesn't mean the railway company won't have additional increase from building the 2nd station

MARR
Minimum Acceptable Rate of Return (MARR): an interest rate that must be earned for a project for it to be worthwhile
- would have to be larger for tech companies, since they can't afford to stretch projects over longer periods of time
- if it is < 0, you are losing money

Investing: greatest present worth (PW)
Minimum cost problems: least PW

If you need to compare to figure out which is the best option, doing an annual worth will save time because you only have to calculate for one year.
1. Imagine cash flow diagram as annuities
2. Alternative with largest annuity is the best

Unequal Lives

When you have projects with different time periods, you need to consider the methods of comparing the projects.

- Repeated Lives: Lowest Common Multiple of the service lives
- Adopted Study period: specific time period (some may not be active for part of the time, but overall they're [hopefully] better), usually if projects cannot be repeated

Week 8 – IRR

IRR [i*]: Internal Rate of Return

- interest rate when Net Present Value (NPV) = 0
- $P_{in} = P_{out}$
- e.g. If $100 is invested today and it returns $110 in a year, IRR = 10%

Steps

1. Find the IRR (i.e. interest) for each case
2. Reject if IRR < MARR
3. Choose the one(s) with the highest IRR
4. If there are multiple highest, choose the one with the largest nominal gain.

NPV

NPV: Net Present Value (a.k.a. normal IRR question)

- Bring all cash flows to the present and add them together
- Include the signs for each of the cash flows
- Useful when comparing multiple independent projects
- Note: when not given an IRR, guess *i** = MARR

Here's the idea:

$$\begin{aligned}\text{NPV} &= \sum_{t=0}^{T} \frac{(R_t - D_t)}{(1+i^*)^t} \\ &= \sum_{t=0}^{T} (R_t - D_t)(P/F, i^*, t) \\ &= \sum_{t=0}^{T} (R_t - D_t)(P/F, \text{MARR}, t)\end{aligned}$$

Receipts [R]: outflows
Disbursements [D]: inflows

Steps

1. Put everything in the present, using i = i* = MARR
2. Choose the highest one.

Incremental Analysis

Incremental Rate of Return [ΔIRR]: calculating the optimal project when the projects are mutually exclusive given an investment amount
comparing = Higher cost alternative – lower cost alternative

ΔIRR $\geq$ MARR => choose higher cost
ΔIRR < MARR => choose lower cost
Note: reverse when borrowing

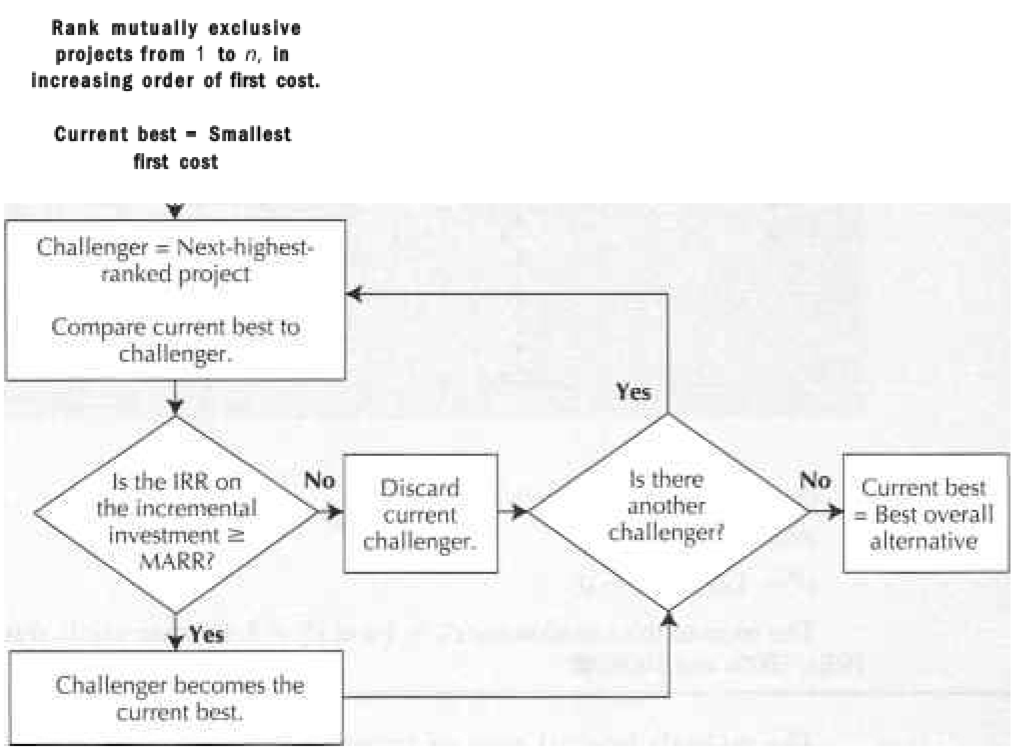
When calculating each ΔIRR:

- You are finding a weighted average
- The amount of the investment that is unused uses the MARR as an i
- The amount that is used uses
- $i_a^* = \dfrac{(\text{used})\, i_{\text{IRR},a}^* + (\text{unused})\, i_{\text{MARR}}}{\text{amount to invest}}$

Identify all options, including doing nothing (not picking any options)

Follow this flow chart:

Figure 5.5 Flowchart for Comparing Mutually Exclusive Alternatives



ERR

ERR (External Rate of Return) [i_{err}*]: the rate of return on a project where all cash flows that are not invested in the project are assumed to earn interest at a predetermined rate (such as the ΔIRR). Use this when you have multiple sign changes

today (F/P, MARR, N) − annuity rate + later (P/F, i_{err}*, N) = 0

Approximate ERR [i_{ea}*]: since ERR is too difficult, just use this to find an approximation
net receipts at MARR = net disbursements at i_{ea}*, find i_{ea}*

It's a good investment if i_{ea}* > MARR

Week 9

If you see illegal activity, document and report to superiors. If it poses a serious threat and they don't do anything then it's your call if you whistleblow. Either way document everything.

Week 10

Depreciation [D]: the loss in value of a capital asset, since most things lose value as soon as it's purchased
D(*n*) is the amount of depreciation in the n^{th} period

Some things **appreciate**, but most things don't.

- art
- old alcohol

Some follow an arc, where they depreciate, then appreciate

Market Value [MV(*n*)]: value an asset can be sold for in an open market

Purchase Value [*p*]: the amount paid at purchase
MV(purchase) = MV(0)

Book Value [BV(*n*)]: value of asset, with depreciation accounted for

Let *N* = useful life of assets in periods
Note: this represents the usefulness to the given project, not in general, i.e. the asset can still have a salvage value after *N* periods

Scrap Value: value of asset when sold at end of physical life (broken for parts, etc.)
Let [S] = Scrap Value and/or Salvage Value
S = BV(*N*)

Types of depreciation:

- Straight-line: BV decreases equal amount
- Declining balance: BV decreases equal proportion (i.e. percentage)

Straight-Line

<u>Idea</u>: BV decreases an equal amount

$D_{\text{total}} = D(N) = p - S$

$$D(n) = \frac{p - S}{N}$$

BV(n) = p – n × D(n)

Declining Balance

Idea: BV decreases by a percentage

Depreciation rate [d]

$d = 1 - \sqrt[N]{S / p}$

$D(n) = d \times BV(n-1)$

$BV(n) = p \times (1 - d)^n$

Replacement Decisions

Mutually exclusive options of determining what to do with existing/unnecessary/obsolete:

- Do nothing
- Upgrade/repair
- Get rid of it
- Get rid of it and replace it

Should you replace existing, unnecessary, obsolete, and/or damaged assets (machinery, software, etc.)?

Defender: existing asset
Challenger: replacement

Reasons:

- Reduced performance: extensive usage causes physical deterioration, reducing reliability and productivity
- New requirements
- Obsolescence and/or lack of support/spare parts

- Installation costs are not depreciable
- When installed, defender has cost advantage
- Sunk costs are irrelevant
    - i.e. even if you just spent a shit ton on repairing your car, you don't include that in your decision of whether to sell or not

O & M: Operating & Maintainance

Equivalent Annual Costs (EAC): annual costs of replacing the machine every N years

$EAC_{total} = EAC_{capital} + EAC_{OP\&M}$

$EAC_{capital}$ = capital recovery factor = $A = (P - S)(A / F, i, N)$

One-year principle: if the cost of keeping the defender one more year exceeds the EAC of the challenger at its economic life, then the defender should be replaced immediately

Replace if:

- Defender = Challenger
- Defender ≠ Challenger_0 ≠ Challenger_1 = $\text{Challenger}_{2\ldots N}$
- Defender ≠ Challengers (each challenger is unique)

Week 11

Capital: Revenue − Expenses

Undepreciated Capital Cost (UCC): remaining capital balance
UCC = Capital – $\text{CCA}_{\text{previous}}$

Capital Cost Allowance (CCA): maximum level of capital cost expense (depreciation) which a company can claim each year
CCA = UCC × CCA Rate

Half-year rule: only one-half of the value of the depreciable asset bought in the current year can be used for current year CCA
i.e. $\text{CCA}_{\text{base}} = \text{UCC}_{\text{base}}/2 \times \text{CCA Rate}$

Tax Amount = Tax Revenue – Tax Savings
= ((Revenue – Operating Expense) × Tax Rate) – ((CCA) × Tax Rate)
= (Revenue – Operating Expenses – CCA) × Tax Rate

Week 13

Real interest [*i'*]: interest accounting for inflation

$$i' = \frac{1+i}{1+f} - 1$$

Inflation rate [*f*]

Real Cash Flow [*R*]: cash flow accounting for inflation

$$R = \frac{C}{(1+f)^N}$$

C = current cash flow
N = # of years