

# SFWR ENG 3DX4 Summary

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Course: SFWR ENG 3DX4

*Math objects made using [MathType](#); graphs made using [Winplot](#).*

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Note: the following summaries may be useful:

- [SFWR ENG 2MX3](#)
- [ENGINEER 3N03](#)
- [TRON 3TA4](#)

I may review to clarify or correct, but mostly I will omit those things.

## Introduction to Systems

Systems can be represented by **block diagrams** to make it easier to marginalize the different parts of the systems.

## Laplace

Useful for...

Time begins when your signal begins

$$h(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

Initial conditions:

- $c(0)$

**Time domain** ( $t$ ): variables are lower case, e.g.  $f(t)$

**Frequency domain** ( $s$ ): variables are upper case, e.g.  $F(s)$

**Transfer function:**

When doing the inverse Laplace, it's useful to break your fractions up so that you can

**Strictly Stable:** it will eventually get back to the initial position

**Marginally Stable:**

**Unstable:** it will progressively get worse

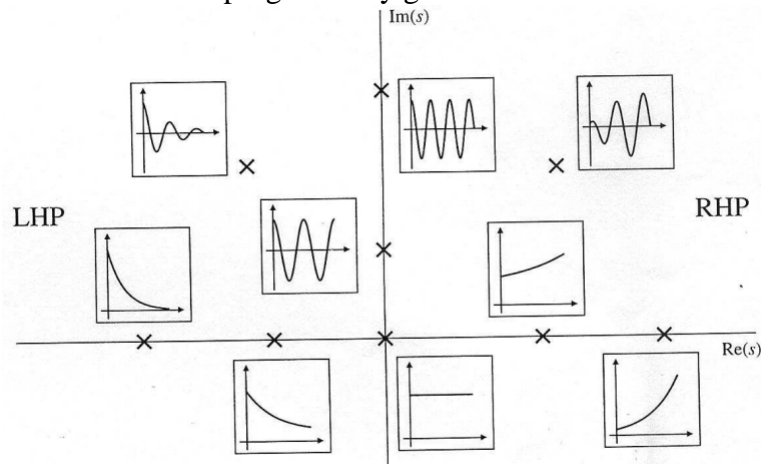


Figure 2.5 from Dorf and Bishop, *Modern Control Systems (10th Edition)*, Prentice-Hall, 2004.

## Transfer Functions

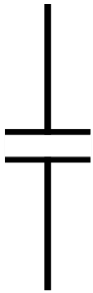
### Electrical

#### Component stuff

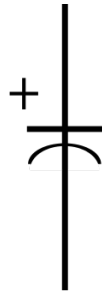
$$i = VR$$

$$i = C \frac{dv}{dt}$$

$$i = \frac{1}{L} \int_0^t v dt$$



Fixed Capacitor



Polarized Capacitor



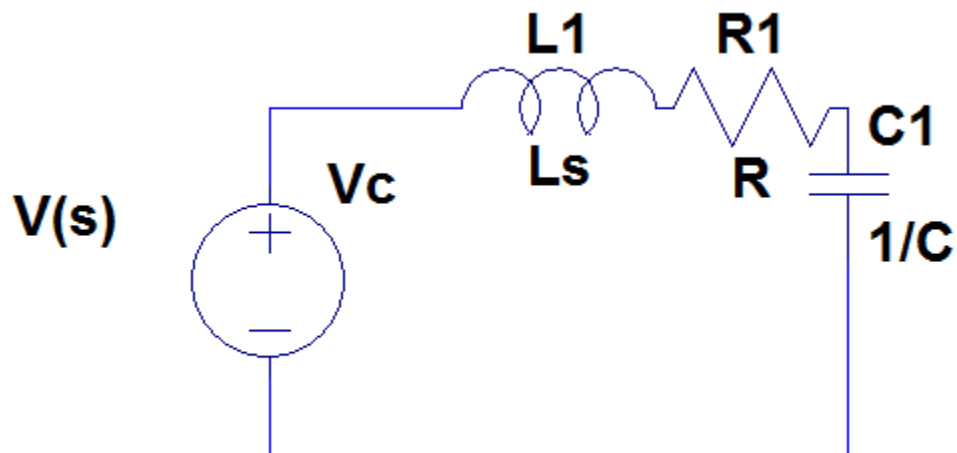
Variable Capacitor

**admittance:**

$$Y(s) = \frac{I(s)}{V(s)} = \frac{1}{R} = G$$

$$V_c(s) = I(s) \frac{1}{Cs}$$

$$I(s) = \frac{V(s)}{L_s + R + \frac{1}{Cs}}$$



#### Mesh Analysis

You cannot use Ohm's law to find the current through a voltage source, so represent the current by  $i_{\text{something}}$ , like  $i_x$ .

## Cramer's Rule

$$x_1 = \frac{\det(A_1)}{\det(A)}, x_2 = \frac{\det(A_2)}{\det(A)}, \dots, x_n = \frac{\det(A_n)}{\det(A)}$$

$$V_C(s) = \overbrace{H(s)}^{\text{transfer function}} \frac{1}{Cs}$$

## OP-Amps

## Mechanical

**Translational systems:**

**Rotational Systems:**

**Newton's Second Law of Motion:**  $\Sigma f = Ma$

$$Z_m(s) = \frac{F(s)}{X(s)}$$

$$f(t) = Ma(t)$$

$$= M \frac{d^2x}{dt^2}$$

## Translational Systems

### Spring

Spring is like a capacitor

**Force displacement:**  $f(t) = Kx(t)$

### Viscous Damper

Using viscous fluid to slow something down

Viscous Damper is like a resistor

**Force displacement:**  $f(t) = f_v \frac{dx(t)}{dt}$

### Mass

Mass is like an inductor

**Force displacement:**  $f(t) = M \frac{d^2x(t)}{dt^2}$

## Rotational Systems

**Transducer:** anything that converts energy to electrical energy

**Transmitter:** long distances

Unstable systems have  $\infty$  steady state error

**Steady-state error** [ $e_{\infty}$ ]:

$$e_{\infty} = \lim_{t \rightarrow \infty} e(t)$$

**Final value theorem:** finds steady state error

$$\lim_{x \rightarrow \infty} f(t) = \lim_{x \rightarrow 0} sF(s)$$

So  $e_{\infty} = \lim_{x \rightarrow 0} sF(s)$  and you're given  $F(s)$ , so just multiply by  $s$  and find the limit.

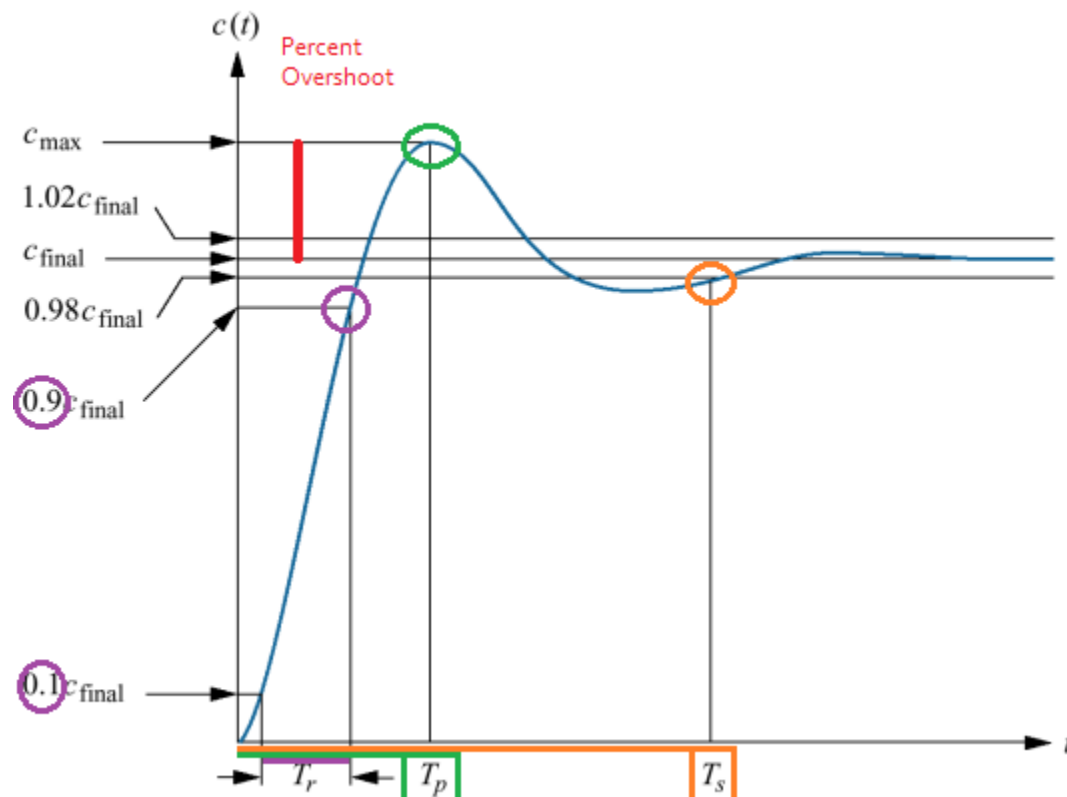
**Rise time** [ $T_r$ ]: time between 10% and 90% of final value ( $c_{\text{final}}$ )

**Peak time** [ $T_p$ ]: time it takes to get to highest peak ( $c_{\text{max}}$ )

**Settling time** [ $T_s$ ]: how long it takes to get to the steady state within  $\pm 2\%$

**Percent overshoot** [%OS]: how much further is the peak from the final

$$\% \text{OS} = \frac{c_{\text{max}} - c_{\text{final}}}{c_{\text{final}}} \times 100\%$$



## Non-/Linear Systems

- Op Amps are linear

- If you don't have enough voltage, your motor magnets won't have enough power to switch poles, so they require a minimum voltage

You can't model non-linear systems, until you linearize it. To do this, we find the slope and approximate the equation of the line, using  $y=mx+b$

### Proportional-Integral-Derivative (PID):

If your gears are vibrating, your PID is probably too high

## Block Diagrams

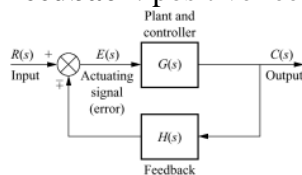
A way of representing a system

**Summing junction:** could be an X or +, but usually an X in this course

**Cascade:** subsystems in series are multiplied

**Parallel:** parallel subsystems have a *summing junction* at the end, so you just add everything together

**Feedback:** positive feedback is bad



Positive: 
$$\frac{G(s)}{1 - G(s)H(s)}$$

Negative: 
$$\frac{G(s)}{1 + G(s)H(s)}$$

Simplification:

## State Space Equations

Yeah, you think you know them from 2MX3, but you don't really know them. Apparently the ABCD variables actually have names.

- **System Matrix [A]:**
- **Input Matrix [B]:**
- **Output Matrix [C]:**
- **Feedforward Matrix [D]:**

## Transfer Function -> State Space

### Phase Variable Approach:

The  $n$  state variables will consist of:

- $y$
- the derivatives of  $y$