Stats 3Y03 Summary

Note: R might be on the final :$

Chapter 1

Categorical variable: qualitative variable, such as funny; limited number of options

- e.g. Blood type, Political party
- It can still be a number if the number doesn't describe a quantity
- **Ordinal**: Values that can be ordered, such as academic grade
- **Nominal**: Values that cannot be ordered, such as brand name

Types of variables

Numerical variable: quantitative variable, such as position

- **Continuous**: decimals
- **Discrete**: integer

Univariate Data: single variable

Bivariate Data: 2 variables (not required in this course)

Multivariate Data: more than 2 variables

Probability: average of population is from average of sample
Inferential statistics: average of sample is from average of population

Sampling Frame: list of things in a list that can be sampled

- telemarketers' sample frame is the people with a phone number in the phone book/phone archive of the company
- when doing a culture study of farms, the sample frame could even be a map

Enumerative study:

- identifiable goal
- well-defined, unchanging sample frame
- enumerate (explain, evaluate, describe) a condition that exists with the existing population

Analytic study:

- focused on improvement of the process which created the results and which will continue creating results in the future
- no well-defined sampling frame

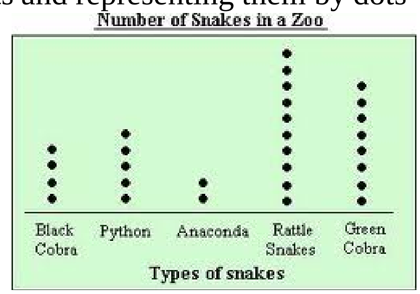
Target population

- population you want to be collecting data from
- **sample population** is the population you are collecting data from
- sample population is usually subset of target population
- sample population is useful when the target population is too large
- sometimes it is not the same as the sample population
    - e.g., when informing factory workers that their productivity is being observed, they'll act differently

Simple random sample: from entire population
Stratified random sample: from a sub-population (1 from each row)

Convenience sample: not entirely random; what is easy to obtain (first row)

Dot plot: quantifying increments and representing them by dots



Mean: average

Median: middle value; if length of set is even, average of (n+1)/2 and n/2; if length of set is odd, (n+1)/2

Mode: common number

Unimodal: 1 peak
Bimodal: 2 peaks
Multimodal: more than 2 peaks

Graphs can also be **symmetric** or **asymmetric**, which is when the top half of the boxplot looks similar to the bottom half.

Left skew: mostly on right side
Right skew: mostly on left side
Graphs can also be **unskewed**.

Outliers:

- values that must be mistakes or abstract exceptions
- > 1.5 × forth spread (see below) beyond closest quartile
- **extreme outlier** is > 3 × forth spread

Each data set is split up into 4 **quartiles**.
Q1: median of bottom half (includes middle number if odd length)
Q2:
Q3: median of top half (includes middle number if odd length)

The Five-Number Summary:

1. Minimum
2. Q1
3. Q2
4. Q3
5. Maximum

The range, minimum, and maximum can include outliers

range: max − min

Variance

Variance: distribution of range

N is target population size
x_i are the values
n is sample population size

Trimmed mean: mean calculated by trimming away a given percentage of elements (relative to number of elements) from the top and bottom. If the percentage gives a non-discrete number of elements, you have to calculate multiple trimmed means and find the mean of the 2 trimmed means

Population mean: expected outcome of mean of target population, i.e. average given a theoretically infinite amount of measurements; a.k.a. true mean, expected value

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Sample mean: average given finite number of inputs; an estimate of the population mean

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

Sample median: $\tilde{x}$

Sample variance: $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2$

Population variance: $\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}\left(x_i - \mu\right)^2$

Spread: interquartile range

Standard Deviation

s.d.

- Average distance from the mean
- Larger s.d. means more spread
- i.e. when all values are the same, s.d. = 0
- Square root of variance = $\sqrt{s^2}$

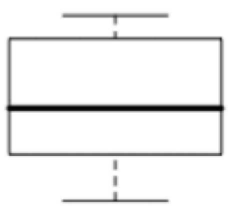
Degrees of freedom: n – 1

Another measure of spread is **interquartile range** or **forth spread**. ($Q_3 - Q_1$)

Whiskers: minimum and maximum points of the range that does not include outliers

Boxplot:

- Top and bottom lines are whiskers
- Box surrounds forth spread
- Middle line is median
- Can be vertical or horizontal
- Outliers are still placed on boxplots, using circles (o) or stars (*)
- (a.k.a. Boxplot-and-whisker plot)



Chapter 2

This is similar to the logic course SFWR ENG 2FA3.
Probability is between 0 and 1

Sample space: all possible outcomes

The size of the sample space is: $\text{outcomes}^{\text{events}}$.

N: number of outcomes for an event
N(A): number of outcomes in sample space, A

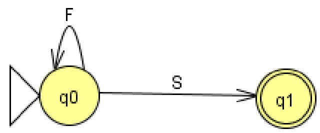
Relative frequency probability: events that occur frequently, such as rolling dice or buying lottery tickets

Relative frequency of a value $= \dfrac{\text{occurrences of value}}{\text{observations in data set}}$

Personal probability: events that cannot be repeated or non-random events with unknown quantities that is based on belief of an individual

Coherent: personal probability of one event does not contradict personal probability of another

Sometimes you can have an **infinite number of possible outcomes**. For example, if you are testing something until failure, you will repeat testing until success {S, FS, FFS, …}



If there are a given number of outcomes, such as 1 through 6 for a dice, and a sample space, A, such as containing all odd outcomes, A', the **complement**, contains everything A does not, such as all even outcomes. Therefore, P(A) + P(A') = 1

Simple Event: Only one way to get each outcome
Compound Event: Multiple ways to get the same outcome

Replacement

With replacement: e.g. if you are picking names out of a hat and you put the names back after each pick; independent
Without replacement: when you use each option only once; dependent

Mutually-Exclusive Events

Mutually exclusive (a.k.a. disjoint event): 2 outcomes cannot occur simultaneously; A ∩ B = Ø; e.g. rolling a dice can either be 3 or 5–not both, whereas it being 3 or odd is not mutually exclusive

The **probability** is the sum of the probability of each individual event:

$$P\left(A_1 \cup A_2 \cup \ldots \cup A_k\right) = \sum_{i=1}^{i=k} P\left(A_i\right)$$

$$P\left(A\right) = \frac{N\left(A\right)}{N}$$

For ordered pairs, number of possible arrangements is: $N!$

Permutations are ordered sequences that are made up by k elements that are a subset of a set of n elements.

The notation for **number of permutations** is: $P_{k,n} = \frac{n!}{(n-k)!}$

Bayes's Theorem: $P\left(A \mid B\right) = \frac{P\left(B \mid A\right) P\left(A\right)}{P\left(B\right)}$

Non-mutually exclusive events

For non-mutually exclusive events, there can be overlap, so:
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

For ordered pairs, number of possible arrangements for k events is: $\prod_{i=1}^{k} N(A_i)$

Unordered permutations are known as **combinations** (n choose k). They are denoted:
$C_{k,n} = \binom{n}{k} = \frac{n!}{k!(n-k)!}$

For unordered pairs, number of combinations is: $\frac{n!}{k!(n-k)!}$, where n is the number of objects and k is the size of the group (pick k, 5, players for the team from n, 8 people. number of permutations?)

Dependent: you can't put it back
Independent: you can put it back

Conditional probability: Probability of A given B: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Chapter 3

Random variables

rv

- function whose domain is the sample space and whose range is the set of real numbers, but is subject to random variations
- denoted by a capital letter, whereas its values have the same letter as the rv, but lower-case
- can either be continuous or discrete
- x is a particular value of a random variable

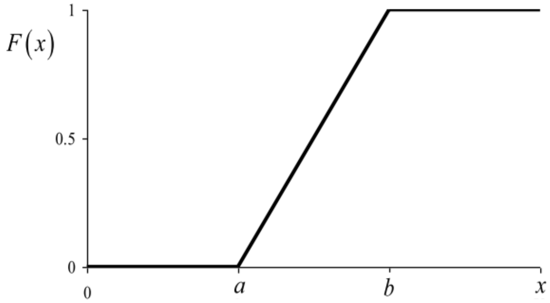
Bernoulli: binary output; can only be either a 0 or a 1

Probability Mass Function (pmf): a function that gives the probability that a discrete random variable is exactly equal to some value

Cumulative Distribution Function

CDF: add up all probabilities within a given range

$$
\begin{aligned}
F(x) &= P(X \le x) \\
&= \int_{-\infty}^{x} f(y)\,\mathrm{d}y \\
&= \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & x > b \end{cases}
\end{aligned}
$$



Expected Value

- mean using probability of discrete rv's
- gives same result as population mean
- use if you're not given data, but given probability
- $$\begin{aligned} \mathrm{E}(X) &= \mu_x \\ &= \sum_{x \in \mathrm{D}} x\mathrm{p}(x) \\ &= \int_{-\infty}^{\infty} x f(x)\,\mathrm{d}x \end{aligned}$$
- Variance: $V(X) = \sum_{x \in D} (x-\mu)^2\,\mathrm{p}(x) = \mathrm{E}\left[(X-\mu)^2\right]$
- **General Expectation formula**: $E(x) = \sum xP(x)$

Variance of CDF

$$
\begin{aligned}
\mathrm{Var}(X) &= \mathrm{E}\left[(X-\mu)^2\right] \\
&= \sigma^2 \\
&= \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,\mathrm{d}x \\
&= \int x^2 f(x)\,\mathrm{d}x - \mu^2
\end{aligned}
$$

Binomial experiment

1. fixed trial
2. 2 outcomes–success or failure
3. Trials are independent (with replacement)
4. Probability of each outcome is the same for each trial

- If the sample size is at most 5% of the population size, the experiment can be analyzed as though it were a binomial experiment (with replacement).
- n: repetitions of trials
- p = P(success in single trial)
- q = P(fail in single trial)
- x: total number of successes
- $b(x;n,p) = \begin{cases} \binom{n}{x} p^x \underbrace{(1-p)}_{q}^{n-x}, & x = 0..n \\ 0, & \text{else} \end{cases}$
- Note: the above notation can be read, where x is a variable in b and n and p are constants

Hypergeometric (H.D.): same as binomial, but dependent (without replacement)

- N: number of items in population
- M: number of successes in population
- n: number of items in sample
- x: number of successes in sample
- $P(X = x) = h(x;n,M,N) = \dfrac{\overbrace{\binom{M}{x}}^{p}\overbrace{\binom{N-M}{n-x}}^{q}}{\underbrace{\binom{N}{n}}_{\substack{\text{removes redundancy} \\ \text{since order isn't important}}}}$
- $E(X) = n \cdot \dfrac{M}{N}$ (same as binomial)
- $V(X) = \left(\dfrac{N-n}{N-1}\right) \cdot n \cdot \dfrac{M}{N} \cdot \left(1 - \dfrac{M}{N}\right)$

Negative Binomial Distribution:

- n is fixed in *binomial*, whereas *here*, n is random
- trials repeated until success we want
- r is the number of successes you want
- If r = 1, this is known as a **geometric distribution**

Poisson distribution:

- discrete pdf
- number of occurrences of an event in a given interval, given average rate and time (independent), since last event
- $p(x;\lambda) = \dfrac{e^{-\lambda}\lambda^x}{x!}$
- x: you are determining the probability that x things will happen
- λ (or μ): average occurrences given population (multiply average rate by population)

- mean = variance = λ, so S.D. = √λ
- α – expected number of events during unit interval
- t – time interval length
- $\lambda = \alpha t$
- $P_k(t) = \dfrac{\mathrm{e}^{-\alpha t} \cdot (\alpha t)^k}{k!}$

Exponential: time between events, whereas poisson is more the number of events; continuous distribution

Expected value: $\frac{1}{\lambda} = \mu$

$p(x) = \lambda e^{-\lambda x} = \frac{1}{\mu} e^{\frac{-x}{\mu}}$

For ranges, $p(a < x < b) = \int_a^b \frac{1}{\mu} e^{\frac{-x}{\mu}} \mathrm{d}x$

Chapter 4

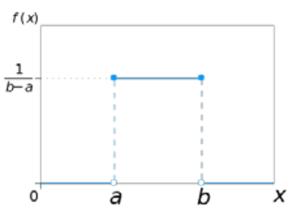
Probability Density Function

PDF: a function that gives the probability that a continuous random variable is exactly equal to some value, such that: $P[a \le X \le b] = \int_a^b f(x)\mathrm{d}x$

Area under whole curve = 1

Uniform Distribution: if a continuous random variable, *X*, has a pdf, f(x; a, b):

$$f(x) = \begin{cases} \dfrac{1}{b-a}, & a \le x \le b \\ 0, & \text{else} \end{cases}$$



Note: *a* and *b* do not represent the entire range of the PDF. Just look at the f(x) formula above!

To get pdf from cdf, take the derivative of the cdf.

$F'(x) = f(x)$

Percentile

Percentile: percentage of data below you; relative to other data in the range

- p: percentile
- η: percentile function
- $p = F(\eta(p)) = \int_{-\infty}^{\eta(p)} f(y)\mathrm{d}y$

$\mu_x = \mathrm{E}(X)$

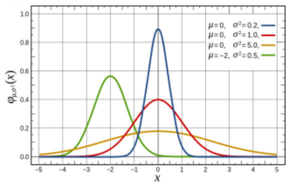
$\mathrm{E}(X) = \alpha\beta$

This can be used to determine the probability

Normal Distribution

A.k.a. population normality
symmetric; mean = median = mode

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$



Bell curve (a.k.a. Gaussian curve): normal curve, normal distribution; **Central Limit Theory** says the sampling distribution of sample means will be bell-shaped; s.d. = population s.d./√sample size

Z-Tables

A.K.A. Standard Normal Cumulative Probability Table
Z-function: a standardized cdf that you use to predict data

- z_c: critical value; this is also the area of the graph from 0 to c, where c is a point on the z-graph
- $z_{c<x} = \dfrac{x-\mu}{\sigma}$
- It's horizontal units are s.d.'s

If you're given a probability (or percentile), you find the value on the z-table, where the probability represents α and choose the values at the location. If you cannot find the value on the z-table, find the two closest ones and find the weighted average.

$$E(x) = z_c \frac{\sigma}{\sqrt{n}}$$

Standardized Score: a.k.a. "z-score" $\dfrac{\text{observed value mean}}{\text{s.d.}}$

α-level is the area of the graph of a normal distribution curve $\alpha = P(Z \geq z_\alpha)$

Z_α: for the standard normal distribution
When trying to find the α based on a z, make sure you round to the preferred sig figs

Empirical rule: you can identify that your data has normal distribution by using the rule that:

- 68% of data is within 1 s.d. from mean
- 95% of data is within 2 s.d. from mean
- 99.7% is within 3 s.d. from mean

- there are 3 s.d.'s from the mean

Chapter 5

p $\leftarrow$ discrete
f $\leftarrow$ continuous

$$p_x(x) = \sum_y p(x,y),\, p_y(y) = \sum_x p(x,y)$$

Mean of sum of joint pdf (discrete): $E(x+y) = \sum_{x,y} (x+y)\, p(x,y)$

Mean of sum of joint pmf (continuous): $E(x+y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x,y) \cdot f(x,y)\, \mathrm{d}x\mathrm{d}y$

Covariance: variance for multiple variables; $\mathrm{Cov} = E(XY) - \mu_x \cdot \mu_y$

Independent and Identically Distributed (IID):

- form a simple, random sample of size *n*
- X_i's are independent r.v.'s
- X_i's all have same probability distribution

Multinomial distribution: represented by the pmf, $f(x_{1..k}; n, p_{1..k}) = \prod_{i=1}^{n} \frac{i}{x_i!} p_i^{x_i}$

Marginal pdf (continuous): $f_x(x) = \int_{-\infty}^{\infty} f(x,y)\, \mathrm{d}y, -\infty < x < \infty$
$f_y(y) = \int_{-\infty}^{\infty} f(x,y)\, \mathrm{d}x, -\infty < y < \infty$

Conditional probability of joint pdf: $f(x \mid y) = \frac{f(x,y)}{f_y(y)}, -\infty < x < \infty$

Correlation coefficient: $\rho_{x,y} = \frac{\mathrm{Cov}(x,y)}{\sigma_x \cdot \sigma_y}$

Chapter 6

θ represents the parameter of interest
$\hat{a}$: variable with a hat means it is an estimate
$\hat{\theta} = \theta +$ error of estimation

- a function of the sample, i.e. rv

Point estimate: mean from multiple estimate(s), using the standard error, where θ represents parameter of interest (e.g. μ or σ), where you estimate $\hat{\theta}$.

Bias of $\hat{\theta}$: $\mathrm{E}(\hat{\theta}) - \theta$

Unbiased: $\mathrm{E}(\hat{\theta}) = \theta$

Estimator: the formula

- Should be unbiased (0 avg. error)
  - $\hat{\sigma}^2 = S^2 = \dfrac{\sum\left(X_i - \overline{X}\right)^2}{n-1}$
  - $\mathrm{E}\left(S^2\right) = \sigma^2$
- Should have minimum variance (i.e. little spread)
- Summary for good estimators: Minimum Variance Unbiased Estimator (MVUE)
  - Unbiased is not always better than minimum variance

Estimate: value obtained from the formula after data has been inputted

What is point estimate for each θ:

- $\mu : \overline{x}$
- Estimated chance of success $p : \hat{p} = \dfrac{\overline{x}}{n}$

True value: mean of the population (instead of sample)

Trimmed means will result in **robust estimator**.
Robust estimators are less affected by outliers

Standard error of an estimator, $\hat{\theta}$ is its standard deviation, $\sigma_{\hat{\theta}} = \sqrt{V\left(\hat{\theta}\right)}$

Estimated standard error: $\hat{\sigma}_{\hat{\theta}}$ or $s_{\hat{\theta}}$

Bootstrapping

Bootstrapping: fabricating multiple samples from one sample with replacement

- Only works for independent, equally-distributed, random samples
- Not useful if small data set, lots of outliers (remove outliers first), dependence structures (data based on changing time, etc.)
- n^* depends on computing capacity, type of problem, and complexity
- Computed bootstrap value is indicative of the accuracy of your sample. If it is higher than sample, sample is probably higher than actual; if lower than sample, sample is probably lower than actual

1. Compute x^*, which is from x, sampled with replacement
2. Compute $\hat{\theta}^*$ from x^*
3. Estimate standard error, $\mathrm{Se}_B\left(\hat{\theta}\right) = \sqrt{\dfrac{\sum_{i=1}^{B}\left(\theta_i^* - \overline{\theta}^*\right)^2}{B-1}}$

e.g. Bootstrapping

Given a sample of:

Given	0.5	1.5	2.5	3.5	4.5

Fabricated Range	0-1	1-2	2-3	3-4	4-5

Now that you've established a range, you use a random number generator to generate 5 new points.
I randomly generated: 4.5290, 0.6349, 4.5669, 3.1618, 0.4877
Now tally how many are within each range:

Quantity	2	0	0	1	2

Multiply this quantity by the initial value of the range, pretend that's the new point, and add it up:

μ_{boot} = 2×0.5 + 0×1.5 + 0×2.5 + 1×3.5 + 2×4.5
= 13.5/5
= 2.7

Whereas, the sample average was actually 2.5

Parametric bootstrap: note: parameter refers to the population

Point Estimation

Point estimation: 2 main methods: a method of inferring a value for a large population, θ, based on a small IID random sample, X, by calculating standard error

- Method of moments
- Maximum Likelihood estimation

Minimum Variance Unbiased Estimator (MVUE)
Estimators

Population mean, μ	Sample mean $\overline{x}$
Population s.d., σ	Sample s.d., s

Method of Moments

α and β are unknown parameters that yield the estimator

k^th sample moment of f(x) is $\mathrm{E}\left(X^k\right) = \frac{1}{n}\sum_{i=1}^{n} X_i^k$

Method of Moments Estimator (MME): $\lambda = \frac{1}{\mathrm{E}\left(X^k\right)} = \frac{1}{\overline{X}}$

Maximum Likelihood Estimation

Joint pdf: pdf governing occurrences of A & B, not just one (i.e. pdf of occurrences of multiple potential events), like for regular pdf's

Likelihood function: PMF = PDF = P(X;θ) = f(X;θ) [f ⇔ P]

$$P\left(X_{i\ldots n};\theta\right) = \prod_{i=1}^{n} P\left(X_i;\theta\right)$$

More popular, easier
Results in normal distribution

Maximum Likelihood Estimator: (MLE) $\hat{\theta} = \max\left(P\left(X_i\right)\right)$ is the random sample, X, with the highest probability of being an appropriate estimator for the population, θ

How to find:

1. Find $\ln\left(P\left(X_{i\ldots n};\theta\right)\right)$.
2. Find $\frac{\mathrm{d}}{\mathrm{d}\theta}\left[\ln\left(P\right)\right]$.
3. Equate to 0.
4. Solve for θ.

e.g.

e.g. for exponential distribution $f\left(x_1,\ldots x_n;\lambda\right) = \left(\lambda e^{-\lambda x_1}\right)\ldots\left(\lambda e^{-\lambda x_n}\right) = \lambda^n e^{-\lambda \overbrace{\sum x_i}^{*}}$ Note that at *, the product becomes a summation of the x_i values

Chapter 7

Most important chapter for the midterm!

Confidence interval

Confidence level: measures reliability of confidence interval; most popular confidence levels: 90, 95, and 99%; the percent of all samples that will give correct results, CL = P(CI)

Confidence interval: interval where certain where data is reliable

- Precision is width of confidence interval
- First determine confidence level
- use the z tables to find
- In order for this to work:
  - Population distribution is normal
  - s.d. given
- Actual mean μ does not necessarily have to be in the interval even if the estimated mean *is* in it

Sample mean ± 1.96 standard errors

$$P\left(CI\right) = 100\%\left(1-\alpha\right)$$

$$CI = \left(\bar{x} - z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

$$CI = \left(\bar{x} - t_{\frac{\alpha}{2},\underbrace{n-1}_{\text{DOF}}} \cdot \frac{s}{\sqrt{n}}, \bar{x} + t_{\frac{\alpha}{2},n-1} \cdot \frac{s}{\sqrt{n}}\right)$$

The bound of the error is half the width, i.e. if estimate is within 1% of the true percentage, the 1% represents the bound of the error, so the width is 0.01×2.

Sample size: $n = \left(2z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{w}\right)^2$ OR $n = \left(2z_{\frac{\alpha}{2}} \cdot \frac{1}{w}\right)^2 \hat{p}\hat{q}$

CI: $\left(\hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$

A larger sample size gives a narrower confidence interval.
A smaller sample size gives a wider confidence interval.

Standard error: conversion of standard deviation (total population) to sample distribution (sample population) = $\frac{\text{s.d.}}{\sqrt{n}}$

Statistical Inference: a method of inferring certain statistical characteristics of a population based off a smaller sample, where characteristics could include things, such as sample mean or sample portion

Sampling variability: a concept in statistical inference, where even though you are inferring from a sample, each sample's inferred population characteristics can vary from sample-to-sample; the smaller the standard error, the less the sampling variability; the larger the sample size, the smaller the standard error of the mean

T-Table: Z-Table, but for s, instead of σ, but you can still use z if sample size > 40; uses 2 parameters: degrees of freedom and probability level

Chapter 8

The point of this to see if the error in the sample mean is low enough to make the sample valid/satisfactory.

Statistical hypothesis: assumption about a population characteristic; 2 types:

- Null Hypothesis
- Alternative Hypothesis
- Choose the hypothesis based on the level of significance
  - for lower level, choose Type I / Null
  - for higher level, choose Type II / Alternative

Null Hypothesis

- H_0: $\mu = \mu_0$, where μ_0 is the given value of μ
- proof by contradiction
- assume it is the thing you think it isn't and prove that wrong
- think *equality*
- If you reject it, the evidence is **statistically significant**

Alternative Hypothesis

- H_A OR H_a OR H_1
  - $\mu > \mu_0$, $z \geq z_\alpha$

- $\mu < \mu_0$, $z \leq z_\alpha$
- $\mu \neq \mu_0$, …

- specified *range*
- think >, <, or ≠
- if you only choose one inequality, it is called a **one-sided hypothesis test**

Errors

- **Type I**: say something is right when it's wrong
  - **Level of significance** (α): P(Type I error)
  - Proving null hypothesis true
  - Since null hypothesis is a value, P has one value
- **Type II**: say something is wrong when it's right
  - P(Type II error) = β
  - Proving alternative hypothesis true
  - Since alternative hypothesis is a range, P is a range

Case I

σ given (not *s*), normal distribution

$$z = \frac{\overline{x} - \mu_0}{\sigma}\sqrt{n}$$

Case II

For large n (i.e. n > 40), s is close to σ

$$z = \frac{\overline{x} - \mu_0}{s}\sqrt{n}$$

Case III

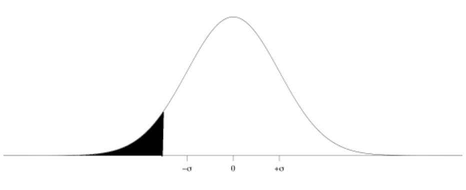
normal dist, *s* given

Test statistic value: $t = \dfrac{\overline{x} - \mu_0}{s}\sqrt{n}$

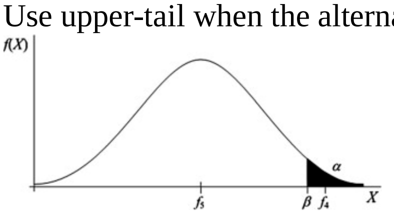
Hypothesis Test

One tail: z_α

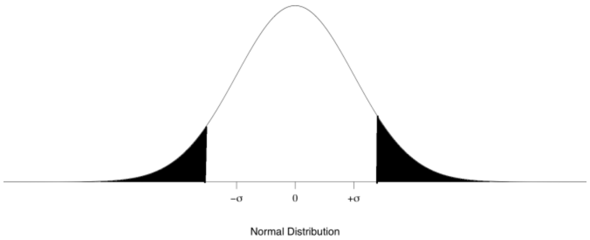
Use lower-tail when the alternative hypothesis is: $\mu < H_a$



Use upper-tail when the alternative hypothesis is: $\mu > H_a$



Two tails: $z_{\alpha/2}$
Use this when alternative hypothesis is $\mu \neq H_a$



The rejection region is the dark part of these graphs. If in rejection region, reject the null hypothesis.

P-Value

P-Value: observed level of significance

Level of significance (α): a percentage or decimal that represents the cut-off value

- If P-value < α , reject the null hypothesis and accept the alternative hypothesis
- If p-value > α , don't reject the null hypothesis and there is not enough information to determine whether or not to accept the alternative hypothesis
- It is different for each region
  - $\Phi(z_\alpha) = \alpha$
  - Upper tailed: P = $1 - P(z < z_c)$
  - Lower tailed: P = $P(z < z_c)$
  - Two-tailed: P = $2(1 - P(z < z_c))$

Chapter 9 – Test Statistics

Degrees of Freedom: number of samples − 1

For normal populations with known variances, test statistic value: $z = \dfrac{\overline{x} - \overline{y} - \Delta_0}{\sqrt{\dfrac{\sigma_1^2}{m} + \dfrac{\sigma_2^2}{n}}}$

Δ_0 is usually 0

Null hypothesis: $|\mu_1 - \mu_2| = \mu_D = \Delta_0$

Alternative Hypothesis: $\begin{cases} \mu_D > \Delta_0 \\ \mu_D < \Delta_0 \\ \mu_D \neq \Delta_0 \end{cases}$

3 Cases (conditions stay the same as before):
Note: n's must be the same, use $\overline{d} = |\overline{x}_1 - \overline{x}_2|$, instead of $\overline{x}$, use Δ_0 instead of μ; $\sigma_D = |\sigma_1 - \sigma_2|$, and $s_D = |s_1 - s_2|$

1. Case I

2. Case II
3. Case III

Round down to the nearest integer

Pooled *t* happens when $\sigma_1^2 = \sigma_2^2$

Margin of Error: $E = t_{\frac{\alpha}{2}} \frac{s_d}{\sqrt{n}}$

***f* distribution**:

- pdf distribution is too difficult, so we will work with tables
- Assumptions:
  - 2 populations independent
  - Simple random samples
  - Normally distributed
  - Test statistic for test hypothesis, given two variances is: $f = \frac{s_1^2}{s_2^2}$
- Demonstrates the difference between the two variances
- Determines whether or not the rejection region is too high or not
- Inputs:
  - Significance level, α
- Null Hypothesis: $\sigma_1^2 = \sigma_2^2$
- Alternative Hypothesis: $\sigma_1^2 < \sigma_2^2$

Chapter 12

Determine a line with the least variance

Deterministic Relationship: $y = \beta_0 + \beta_1 x$

one variable can be found in terms of the other variable

Linear: a first order polynomial example of a deterministic relationship (i.e. y = mx + b)

Statistical: non-deterministic; relies on probability

Regression Analysis: looks at correlations between two things by removing other variables

Model equation: $y = \beta_0 + \beta_1 x + \varepsilon$

ε: measure of variation; error in data

Principle of least squares: gives minimum error

$$s_{xy} = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{n-1}$$

$$s_x^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}$$

$$b_1 = \frac{s_{xy}}{s_x^2} = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2} =$$

$$\boxed{b_0 = \overline{y} - b_1\overline{x}}$$

Point Prediction: plugging in values of *x* into the regression equation
Residual: error; vertical deviation from estimated line ($y - y_0$)

Extrapolation: usually doesn't work, though

library (MASS)
summary() gives 5-number summary

Sum of Squares for Errors (SSE): $SSE = \sum (y_i - \hat{y})^2 = \sum y_i^2 - \hat{B}_0 \sum y_i - \hat{B}_1 \sum x_i y_i$

Chapter 10

ANOVA:

Factor:
levels of the factor:

The number of populations being compared is *I*.

$X_{i,j}$ represents the random variable for the j^{th} experiment for the i^{th} population

Hypothesis

$H_0 : \mu_1 = \mu_2 = \ldots = \mu_I$
$H_a :$ at least 2 values of μ_I

$E\left(X_{i,j}\right) = \mu_i$
$V\left(X_{i,j}\right) = \sigma$