Face Recognition with Eigenfaces

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Table of Contents

Chapter 1

Introduction

The Principal Component Analysis (PCA) was independently proposed by Karl Pearson (1901) and Harold Hotelling (1933) to turn a set of possibly correlated variables into a smaller set of uncorrelated variables. The idea is, that a high-dimensional dataset is often described by correlated variables and therefore only a few meaningful dimensions account for most of the information. The PCA method finds the directions with the greatest variance in the data, called principal components.

This project is to implement PCA by Python.

Chapter 2

Environment Description

- Window 10
- Virtual Studio 2015
- Python 2.7
  1. numpy
  2. matplotlib

Keep the floder '*train_data*', '*test_data*' and python file '*eigenfaces.py*' in the same floder and simply run

```
python eigenfaces.py
```

Chapter 3

Algorithmic Description

step 1 Let $X = \{x_1, x_2, \dots, x_n\}$ be the training set's matrix with $x_i \in \{[0, 255]\}^d$.

d is the size of the image. In this case, $d = 45045$.

step 2 Compute the mean

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

.

step 3 Compute the the Covariance Matrix

$$S = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)(x_i - \mu)^T$$

. The size of S in this case is 8×8.

step 4 Compute the eigenvalues λ_i and eigenvectors v_i of

$$Sv_i = \lambda_i v_i, i = 1, 2, \dots, n$$

step 5 Order the eigenvectors descending by their eigenvalue. The k principal components are the eigenvectors corresponding to the k largest eigenvalues.

In this case, I use $k = 5$.

step 6 The 5 principal components of the test set x are then given by:

$$y = W^T(x - \mu)$$

where $W = (v_1, v_2, \ldots, v_k)$.

step 1 For each test image x. Subtract mean face from it.

step 2 Compute the projection of it.

$$\Omega = W^T(x - \mu)$$

step 3 Reconstruct it from eigenfaces.

$$x' = W\Omega$$

step 4 Compute the Manhattan distance between x and x'. We use the threshold 1.4×10^{14} to identify non-face and face.

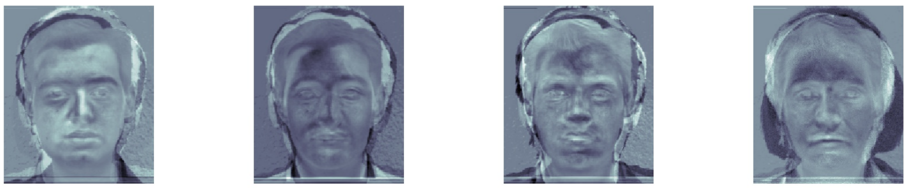
step 5 Finding the nearest neighbor between the projected training images and the projected test image, using Manhattan distance. We use the threshold 10^9 to identify unknown face and other faces.

Chapter 4

Result

The mean face.

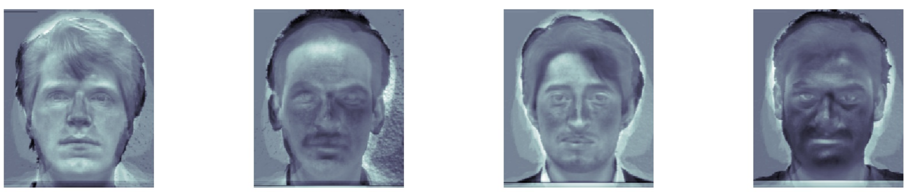
(a) Eigen Faces 0 (b) Eigen Faces 1 (c) Eigen Faces 2 (d) Eigen Faces 3



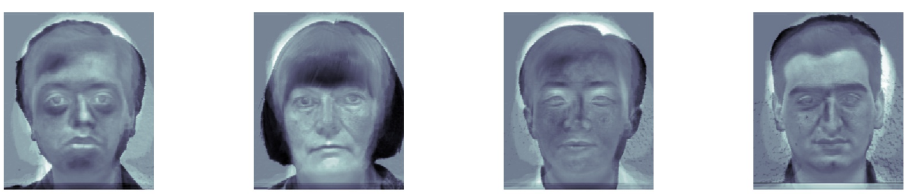
(e) Eigen Faces 0 (f) Eigen Faces 1 (g) Eigen Faces 2 (h) Eigen Faces 3

Figure 4.1: Eigen Faces

The eigen space.

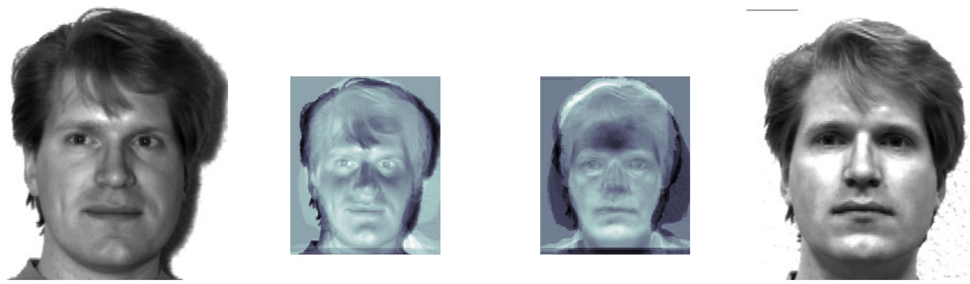


(a) Eigen Faces 0 (b) Eigen Faces 1 (c) Eigen Faces 2 (d) Eigen Faces 3



(e) Eigen Faces 0 (f) Eigen Faces 1 (g) Eigen Faces 2 (h) Eigen Faces 3

Figure 4.2: Train Face

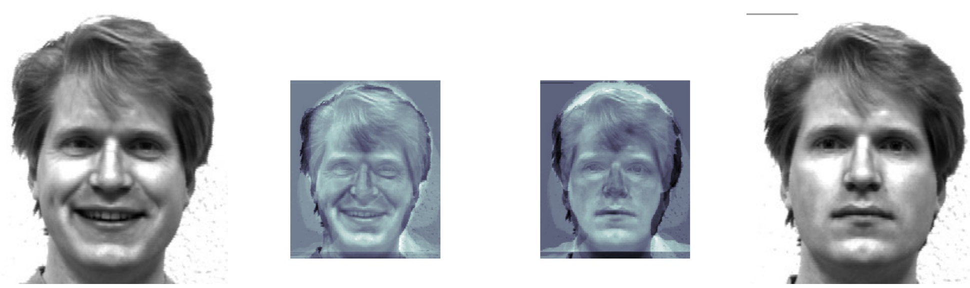


(a) test image (b) subtracting image (c) reconstructed image (d) result image

Figure 4.3: Label 1

The distance from the projected test image to its nearest neighbor is 188439677.034777.

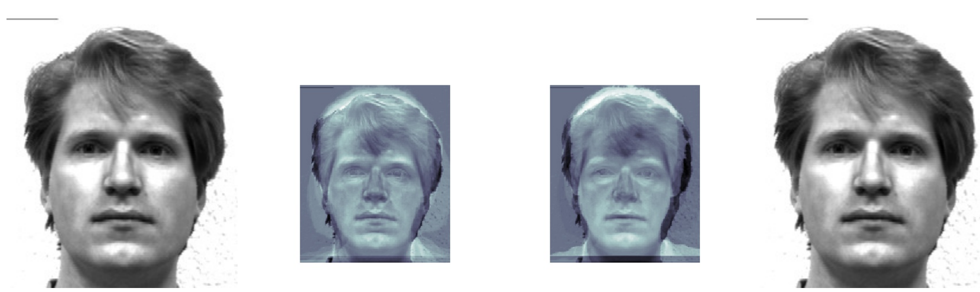
For the first face, if we use Euclidean Distance, it won't be detected correctly. If we use Manhattan Distance, it will be detected correctly.



(a) test image (b) subtracting image (c) reconstructed image (d) result image

Figure 4.4: Label 1

The distance from the projected test image to its nearest neighbor is 106821588.638217.



(a) test image (b) subtracting image (c) reconstructed image (d) result image

Figure 4.5: Label 1

The distance from the projected test image to its nearest neighbor is 0.0.

(a) test image (b) subtracting image (c) reconstructed image (d) result image

Figure 4.6: Label 2

The distance from the projected test image to its nearest neighbor is 0.0.

For 2nd face, the test image is also the train image, it should be correctly.

(a) test image (b) subtracting image (c) reconstructed image (d) result image

Figure 4.7: Label 3

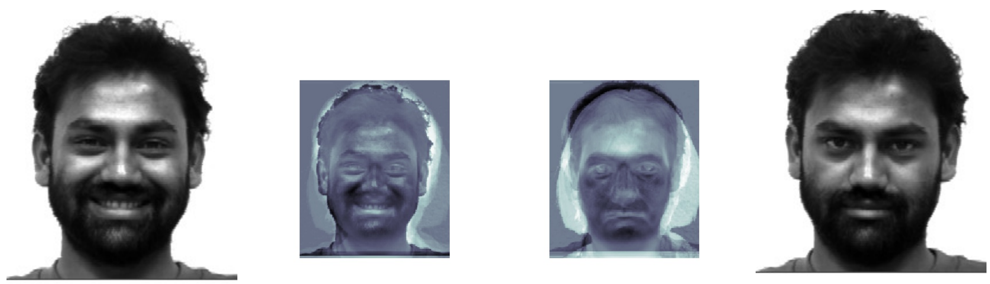
The distance from the projected test image to its nearest neighbor is 0.0.

For the 3rd face, the test image is also the train image, it should be correctly.

(a) test image (b) subtracting image (c) reconstructed image

Figure 4.8: Label 7

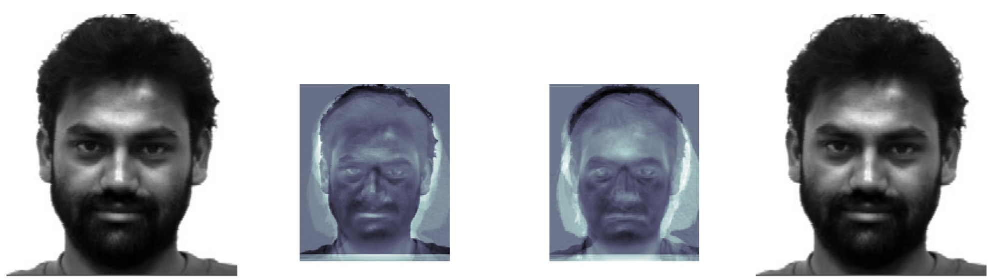
This image is detected as non-face.



(a) test image (b) subtracting image (c) reconstructed image (d) result image

Figure 4.9: Label 7

The distance from the projected test image to its nearest neighbor is 173336241.992190.



(a) test image (b) subtracting image (c) reconstructed image (d) result image

Figure 4.10: Label 7

The distance from the projected test image to its nearest neighbor is 0.0.

(a) test image (b) subtracting image (c) reconstructed image (d) result image

Figure 4.11: Label 10

The distance from the projected test image to its nearest neighbor is 0.0.

The test image is also the train image, it should be correctly.

(a) test image (b) subtracting image (c) reconstructed image (d) result image

Figure 4.12: Label 11

The distance from the projected test image to its nearest neighbor is 190962100.776104.

(a) test image (b) subtracting image (c) reconstructed image (d) result image

Figure 4.13: Label 11

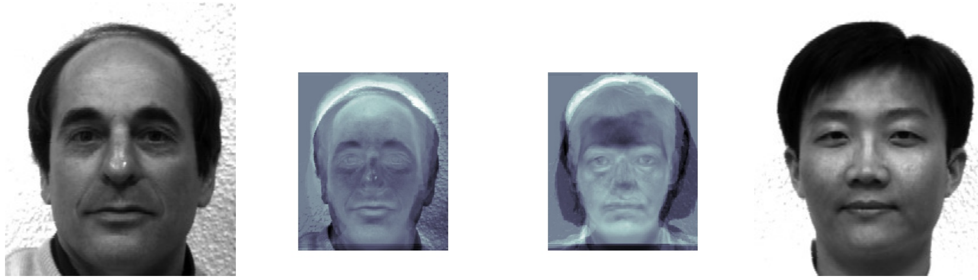
The distance from the projected test image to its nearest neighbor is 37711066.350580.

(a) test image (b) subtracting image (c) reconstructed image (d) result image

Figure 4.14: Label 11

The distance from the projected test image to its nearest neighbor is 0.0.

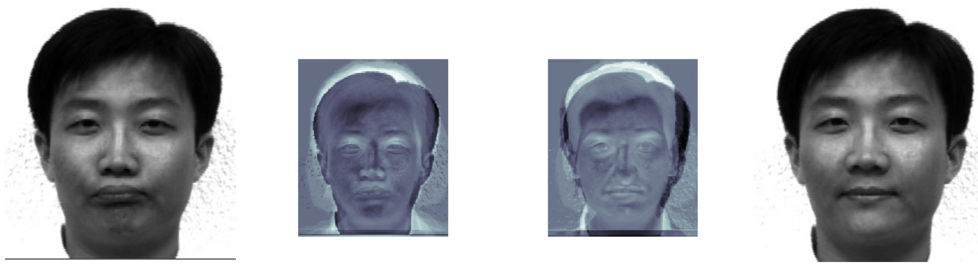
For the 11th face, all three test image are detected correctly.



(a) test image (b) subtracting image (c) reconstructed image (d) result image

Figure 4.15: Label 12

The distance from the projected test image to its nearest neighbor is 172771700.658602.

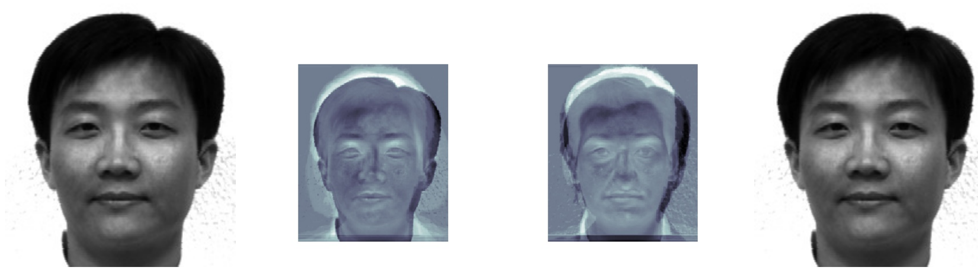


(a) test image (b) subtracting image (c) reconstructed image (d) result image

Figure 4.16: Label 14

The distance from the projected test image to its nearest neighbor is 38834390.761937.

The distance from the projected test image to its nearest neighbor is 44587580.401787.



(a) test image (b) subtracting image (c) reconstructed image (d) result image

Figure 4.18: Label 14

The distance from the projected test image to its nearest neighbor is 0.0.

(a) test image

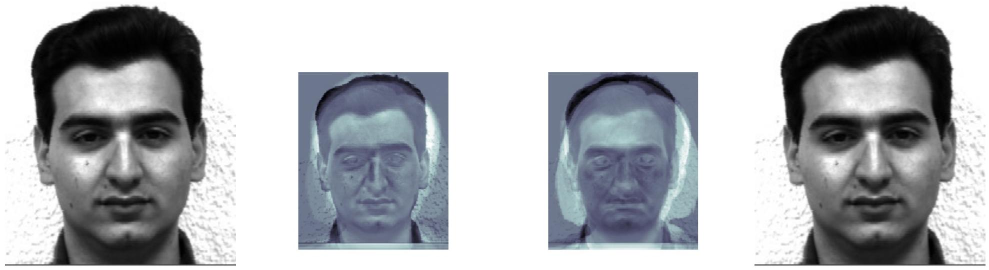
(b) subtracting image

(c) reconstructed image

(d) result image

Figure 4.17: Label 14

For the 14th face, all three test image are detected correctly.

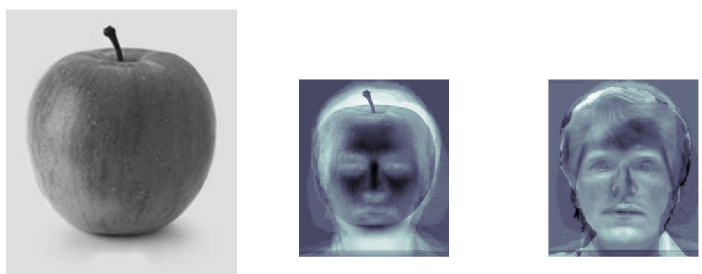


(a) test image (b) subtracting image (c) reconstructed image (d) result image

Figure 4.19: Label 15

The distance from the projected test image to its nearest neighbor is 0.0.

For the 15th face, the test image is also the train image, it should be correctly.



(a) test image (b) subtracting image (c) reconstructed image

Figure 4.20: Non-face

This image is detected as non-face.

Appendix A

Source Code

```
import os
import cv2
import sys
import shutil
import random
import numpy as np
import matplotlib.pyplot as plt



class Eigenfaces(object):

    # number of labels
    faces_count = 8
    # directory path to the AT&T faces
    faces_dir = '.'

    train_faces_count = 1
    test_faces_count = 1
    face_train_ids = [1, 2, 3, 7, 10, 11, 14, 15]

    # training images count
    l = train_faces_count * faces_count
    # number of columns of the image
    m = 195
    # number of rows of the image
    n = 231
```

```
# length of the column vector
mn = m * n

"""
Initializing the Eigenfaces model.
"""
def __init__(self, _faces_dir = '.', _threshold = 0.85):
    print '> Initializing started'

    self.faces_dir = _faces_dir
    self.threshold = _threshold
    self.training_ids = []

    L = np.empty(shape=(self.mn, self.l), dtype='float64')
    cur_img = 0

    for face_id in self.face_train_ids:
        training_ids = [1]
        self.training_ids.append(training_ids)
        for training_id in training_ids:
            path_to_img = os.path.join(self.faces_dir,
                    's' + str(face_id), str(training_id) + '.jpg')
            #print '> reading file: ' + path_to_img
            img = cv2.imread(path_to_img, 0)
            img_col = np.array(img, dtype='float64').flatten()
            L[:, cur_img] = img_col[:]
            cur_img += 1
    # get the mean of all images / over the rows of L
    self.mean_img_col = np.mean(L, axis=1)

    """
    # output the mean image
    fig, ax1 =  plt.subplots(ncols=1, nrows=1, figsize=(4, 4))
    ax1.set_axis_off()
    tmp = np.reshape(self.mean_img_col, (self.n, self.m))
    ax1.imshow(tmp, cmap="bone")
```

```
fig.savefig("image/mean.jpg")
"""

# subtract from all training images
for j in xrange(0, self.l):
    L[:, j] -= self.mean_img_col[:]

"""
print(L.shape)
for j in range(len(self.face_train_ids)):
    fig, ax1 = plt.subplots(ncols=1, nrows=1, figsize=(4, 4))
    ax1.set_axis_off()
    tmp = np.reshape(L[:, j], (self.n, self.m))
    ax1.imshow(tmp, cmap="bone")
    fig.savefig("image/train_"+str(self.face_train_ids[j])+".jpg")
"""

"""
Because L's columns represent the image vector,
we set C = L^T*L
"""
C = np.matrix(L.transpose()) * np.matrix(L)
#C /= L.shape[1]

"""
Eigenvectors/values of the covariance matrix.
And set them into decreasing order of values.
"""
self.evalues, self.evectors = np.linalg.eig(C)
sort_indices = self.evalues.argsort()[::-1]
self.evalues = self.evalues[sort_indices]
self.evectors = self.evectors[:, sort_indices]

"""
include only the first k evectors/values so
that they include approx.
```

```
        evalues_count = 0
        evalues_sum = sum(self.evalues[:])
        evalues_radio = 0.0
        for evalue in self.evalues:
            evalues_count += 1
            evalues_radio += evalue / evalues_sum

            if evalues_radio >= self.threshold:
                break
        """
        evalues_count = 5
        # truncate the number of eigenvectors/values to consider
        self.evalues = self.evalues[0:evalues_count]
        self.evectors = self.evectors[:, 0:evalues_count]

        """
        change eigenvectors from rows to columns
        left multiply to get the correct evectors
        find the norm of each eigenvector
        normalize all eigenvectors
        """
        self.evectors = L * self.evectors

        """
        # output the eigen face
        print(self.evectors.shape)
        for j in range(self.evectors.shape[1]):
            fig, ax1 = plt.subplots(ncols=1, nrows=1, figsize=(4, 4))
            ax1.set_axis_off()
            tmp = np.reshape(self.evectors[:, j], (self.n, self.m))
            ax1.imshow(tmp, cmap="bone")
            fig.savefig("image/"+str(j)+".jpg")
        """
        self.W = self.evectors.transpose() * L
        print '> Initializing ended'
```

```
"""
Classify an image to one of the eigenfaces.
"""
def classify(self, path_to_img):
    img = cv2.imread(path_to_img, 0)
    img_col = np.array(img, dtype='float64').flatten()
    img_col -= self.mean_img_col
    img_col = np.reshape(img_col, (self.mn, 1))
                                                                                 #
                                          from row vector to col vector

    S = self.evectors.transpose() * img_col
    """
    projecting the normalized probe onto the
    Eigenspace, to find out the weights
    """
    diff = self.W - S

                                          # finding the min ||W_j - S||
    norms = np.linalg.norm(diff, axis=0)

    closest_face_id = np.argmin(norms)

                                          # the id [0..240) of the
                                          minerror face to the sample
    return self.face_train_ids[(closest_face_id / self.train_faces_count
                                          )]

"""
Evaluate the model using the 4 test faces left
from every different face in the AT&T set.
"""
def validate(self):
    print '> Evaluating faces started'
    results_file = os.path.join('results', 'results.txt')
                                          # filename for writing the
```

```
                                            evaluating results in
f = open(results_file, 'w')

                                            # the actual file

test_count = self.test_faces_count * self.faces_count
                                                          # number of
                                            all AT&T test images/faces
test_correct = 0
for face_id in self.face_train_ids:
    for test_id in xrange(1, self.test_faces_count+1):
        # if (test_id in self.training_ids[face_id-1]) == False:
                                                         # we skip the
                                                  image if it is part
                                                 of the training set
        path_to_img = os.path.join(self.faces_dir,
                'sn' + str(face_id), str(test_id) + '.jpg')
                                                        # relative
                                                        path

        result_id = self.classify(path_to_img)
        result = (result_id == face_id)

        if result == True:
            test_correct += 1
            f.write('image: %s\nresult: correct, got %2d\n\n' % (
                                                    path_to_img,
                                                    result_id))
        else:
            f.write('image: %s\nresult: wrong, got %2d\n\n' %
                    (path_to_img, result_id))

print '> Evaluating faces ended'
self.accuracy = float(100. * test_correct / test_count)
print 'Correct: ' + str(self.accuracy) + '%'
f.write('Correct: %.2f\n' % (self.accuracy))
```

```
        f.close()
                                                    # closing the file

    """
    Evaluate the model for the small celebrity data set.
    Returning the top 5 matches within the AT&T set.
    Images should have the same size (92,112) and are
    located in the celebrity_dir folder.
    """
    def evaluate(self, celebrity_dir='.'):
        print '> Evaluating test data set matches started'
        # go through all the celebrity images in the folder
        flag = -1.0
        for img_name in os.listdir(celebrity_dir):
            path_to_img = os.path.join(celebrity_dir, img_name)
            name_noext = os.path.splitext(img_name)[0]
            """
            # read as a grayscale image
            # flatten the image
            # subract the mean column
            # from row vector to col vector
            """
            img = cv2.imread(path_to_img, 0)
            img_col = np.array(img, dtype='float64').flatten()
            img_col -= self.mean_img_col
            img_col = np.reshape(img_col, (self.mn, 1))
            """
            # projecting the normalized probe onto the
            # Eigenspace, to find out the weights
            """
            S = self.evectors.transpose() * img_col

            reconsturction = self.evectors * S
            """
            # output the reconstruction image
```

```
# and the origin image subtracts the mean image
print(S.shape)
print(self.W.shape)

fig, ax1 = plt.subplots(ncols=1, nrows=1, figsize=(4, 4))
ax1.set_axis_off()
tmp = np.reshape(reconsturction, (self.n, self.m))
#tmp = np.reshape(img_col, (self.n, self.m))
ax1.imshow(tmp, cmap="bone")
fig.savefig("image/"+name_noext+".jpg")
"""

diff = reconsturction - img_col
dis0 = np.linalg.norm(diff, ord=1)
if flag < 0:
    flag = dis0
    print flag

if dis0 <= flag:
    result_dir = 'results' #os.path.join('results', name_noext)
    # os.makedirs(result_dir)
    result_file = os.path.join(result_dir, 'results_no_face_' +
                                                    name_noext + '.txt')
    f = open(result_file, 'w')
    f.write('Not a face')
    continue

# finding the min ||W_j - S||
diff = self.W - S
norms = np.linalg.norm(diff, axis=0, ord=1)
mean_dis = np.mean(norms)
top_ids = [np.argmin(norms)]#np.argpartition(norms, 1)[:1]

# the image file name without extension
# path to the respective results folder
# make a results folder for the respective celebrity
```

```
# the file with the similarity value and id's
name_noext = os.path.splitext(img_name)[0]
result_dir = 'results' #os.path.join('results', name_noext)
# os.makedirs(result_dir)
result_file = os.path.join(result_dir, 'results_' + name_noext +
                                       '.txt')

f = open(result_file, 'w')

                                      # open the results file
                                      for writing

for top_id in top_ids:
    if norms[top_id] < 1000000000:
        face_id = (top_id / self.train_faces_count) + 1
                                                          #
                                              getting the
                                              face_id of one of
                                              the closest
                                              matches
        subface_id = self.training_ids[face_id-1][top_id % self.
                                              train_faces_count]
                                                   #
                                              getting the exact
                                              subimage from the
                                              face
        face_idx = self.face_train_ids[face_id-1]
        path_to_img = os.path.join(self.faces_dir,
                'S' + str(face_idx), str(subface_id) + '.jpg')

                                                      # relative
                                                       path to
                                                      the top5
                                                      face
```

```
                    shutil.copyfile(path_to_img,
                                                                # copy the top
                                                                face from source
                            os.path.join(result_dir, 'results_' + name_noext
                                                                        + '.jpg')
                                                                        )
                                                                        # to
                                                                        destination

                    f.write('id: %3d, score: %.6f\n' % (top_id, norms[top_id
                                                                ]))     # write
                                                                the id and its
                                                                score to the
                                                                results file
                else:
                    f.write("Unkown Face")
                    #print(name_noext, norms[top_id], mean_dis)
            f.close()
        print '> Evaluating test data set matches ended'



if __name__ == "__main__":
    TRAIN_DATA_DIR  = ".\\train_data"
    TEST_DATA_DIR = ".\\test_data"

    if not os.path.exists('results'):
        os.makedirs('results')
    else:
        shutil.rmtree('results')
        os.makedirs('results')

    efaces = Eigenfaces(str(TRAIN_DATA_DIR), _threshold=0.7)
    #efaces.validate()
```

```
# if we have third argument (celebrity folder)
efaces.evaluate(str(TEST_DATA_DIR))
```