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DELTA Pg No.

1. x hectare land is used for crop X and y hectare land is used for crop Y.

herbicide use

$$20x + 10y \le 800$$
$$2x + y \le 80$$

land use

$$x + y \le 50$$

maximize total profit

$$10{,}500\,x + 9000\,y$$
$$x \ge 0, \quad y \ge 0$$

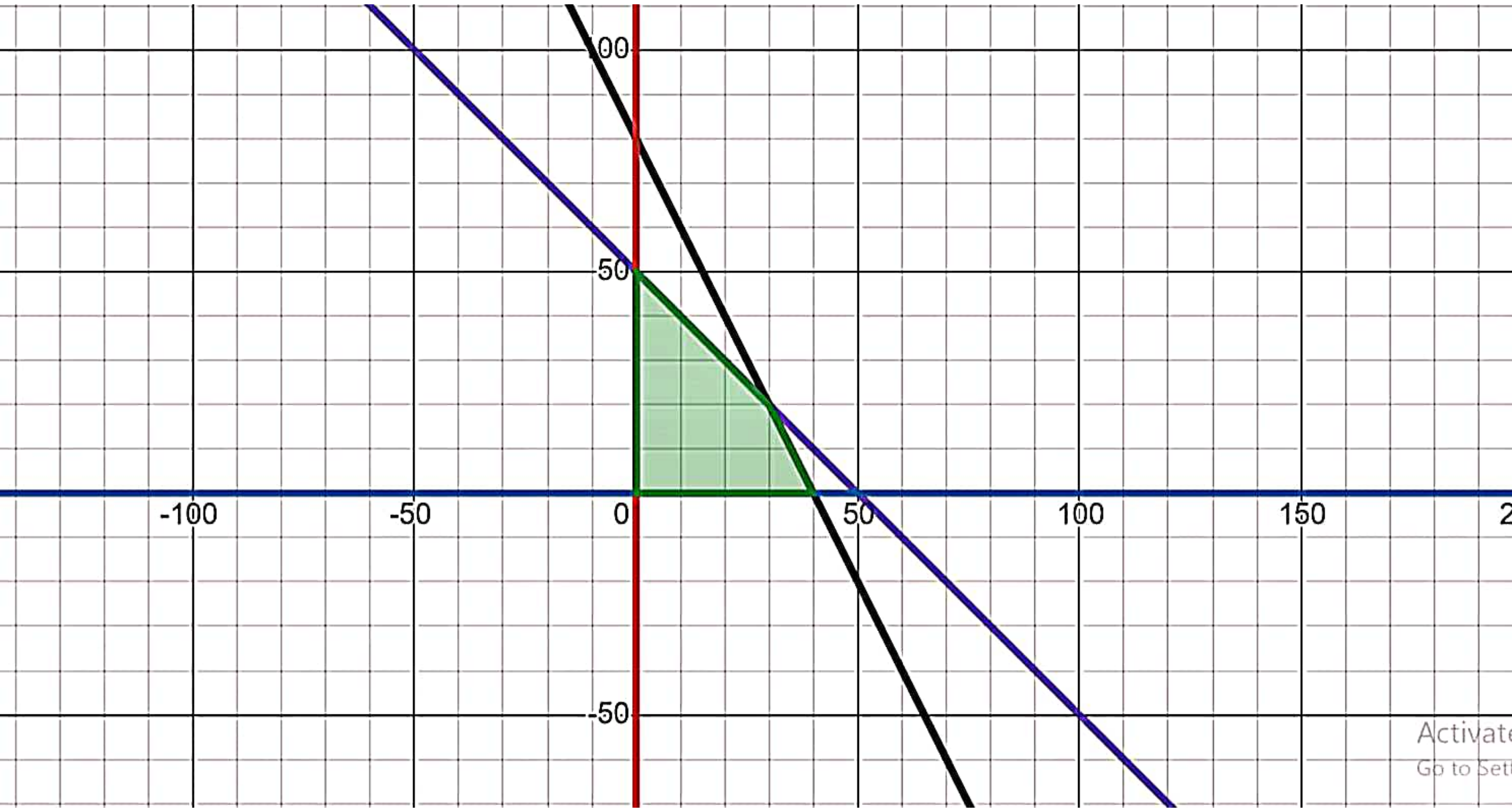
The LPP is

maximize $\quad z = 10500x + 9000y$

subject to

$$\begin{cases} 2x + y \le 80 \\ x + y \le 50 \\ x \ge 0, \; y \ge 0 \end{cases}$$

From the graph, we observe that the LPP has a feasible solution.



100
50
-100
-50
0
50
100
150
-50
Activate
Go to Setti

From the graph the corner points are:
(40, 0), (30, 20), (0, 50) and (0, 0)

The value of z at these points are
(40, 0) - 420,000
(30, 20) - 495,000 → max
(0, 50) - 450,000
(0, 0) - 0

hence $x = 30$ hectare and $y = 20$ hectare is the optimal solution and the maximum profit is Rs 495,000

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Q2.

ans) Given open half space $S = \{x : c^T x > z\}$

$c \in R^n, z \in R$

To Prove : S is convex

Proof: Let x_1 and x_2 be 2 points of S

Therefore, $c^T x_1 > z$ ——— (1)

$c^T x_2 > z$ ——— (2)

Now, $\forall \lambda \in [0,1]$

$$\Rightarrow c^T((1-\lambda)x_1 + \lambda x_2) = c^T[(1-\lambda)x_1] + c^T(\lambda x_2)$$

$$= (1-\lambda)(c^T x_1) + \lambda(c^T x_2)$$

$$> (1-\lambda)z + \lambda z \quad \left[\text{from eq}^n \text{ (1) and (2)}\right]$$

$$= z$$

Hence, $\forall x_1, x_2 \in S$ and $\lambda \in [0,1]$

$$\Rightarrow [\lambda x_2 + (1-\lambda)x_1] \in S$$

So, S is convex

PROBLEM 3 We will use Principal of Mathematical Induction to prove the following proposition:

P_k : $\bigcap_{i=1}^{k} S_i$ is convex, where S_i is a convex set.

BASE CASE $\equiv P_1$

P_1 holds trivially, since $\bigcap_{i=1}^{1} S_i = S_1$ is convex.

INDUCTION HYPOTHESIS : Let P_k hold for $k=n$, $n \in \mathbb{N}$.

INDUCTION STEP : We need to prove P_k holds for $k=n+1$.

Let $S' = \bigcap_{i=1}^{n+1} S_i$, where S_i is a convex set.

$= \bigcap_{i=1}^{n} S_i \cap S_{n+1}$

Let $S = \bigcap_{i=1}^{n} S_i$

$\Rightarrow S' = S \cap S_{n+1}$.

Let $x, y \in S$ and $x, y \in S_{n+1}$

(Hence, by definition, $x, y \in S'$)

For some $\lambda \in [0,1]$,

$\lambda x + (1-\lambda) y \in S$ and $\lambda x + (1-\lambda) y \in S_{n+1}$

☆ By virtue of induction hypothesis, S is a convex set.

$\Rightarrow \lambda x + (1-\lambda) y \in S \cap S_{n+1} = S'$

$\therefore S'$ is a convex set.

Hence, by PMI, P_k holds for all $n \in \mathbb{N}$.

4 Given: x is an extreme point of convex set C

To prove: x is on boundary of C.

Proof by contradiction

Suppose x is an interior point of C. i.e. $\exists\ \varepsilon > 0$ s.t. $B(x, \varepsilon) \subseteq C$

We take 2 distinct points z and y of C

$$y = x + \frac{\varepsilon e}{2\sqrt{n}}$$

$$z = x - \frac{\varepsilon e}{2\sqrt{n}}$$

$$e = (1, 1, \ldots, 1)^T \in \mathbb{R}^n$$

Now,

$$y = x + \frac{\varepsilon e}{2\sqrt{n}}$$

$$y - x = \frac{\varepsilon e}{2\sqrt{n}}$$

$$\|y - x\| = \left\| \frac{\varepsilon e}{2\sqrt{n}} \right\|$$

$$= \sqrt{\underbrace{\left(\frac{\varepsilon}{2\sqrt{n}}\right)^2 + \cdots \left(\frac{\varepsilon}{2\sqrt{n}}\right)^2}_{n \text{ times}}}$$

$$= \sqrt{\frac{n}{n}\left(\frac{\varepsilon}{2}\right)^2}$$

$$= \frac{\varepsilon}{2} < \varepsilon$$

hence $\|y - x\| < \varepsilon$

Similarly $\|z - x\| < \varepsilon$

we get $y, z \in B(x, \varepsilon)$

$y \neq z$ and $x = \frac{y+z}{2}$

$$= \left(1 - \frac{1}{2}\right) y + \frac{1}{2} z$$

$$= (1-\lambda) y + \lambda z$$

where $\lambda = \frac{1}{2}$

$\Rightarrow$ x can be written as a convex combination of 2 distinct points of C.

$\Rightarrow$ x is not an extreme point.
This leads to a contradiction

$\Rightarrow$ x is a boundary point of C.

Qs.

ans) Given system of eq^{n}s

$$2x_1 + 3x_2 - 2x_3 - 7x_4 = 1$$
$$x_1 + x_2 + x_3 + 3x_4 = 6$$
$$x_1 - x_2 + x_3 + 5x_4 = 4$$

Here, $n = 4$ and $m = 3$

In order to get the basic solutions, we need to set $(n-m) = 4-3 = 1$ variables to 0 and then calc. ∀ the rest.

So,

→ let $x_1 := 0$

then $B = \begin{bmatrix} 3 & -2 & -7 \\ 1 & 1 & 3 \\ -1 & 1 & 5 \end{bmatrix}$

$|B| = 8 \neq 0$

and $x_{B_1} = \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix}$

→ let $x_2 := 0$

then, $B = \begin{bmatrix} 2 & -2 & -7 \\ 1 & 1 & 3 \\ 1 & 1 & 5 \end{bmatrix}$

here $|B| = 8 \neq 0$

and, $x_{B_2} = \begin{pmatrix} 3 \\ 6 \\ -1 \end{pmatrix}$

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→ let $x_3 := 0$

then $$B = \begin{bmatrix} 2 & 3 & -7 \\ 1 & 1 & 3 \\ 1 & -1 & 5 \end{bmatrix}$$

here $|B| = 24 \neq 0$

and $$x_{B_3} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

→ let $x_4 := 0$

then $$B = \begin{bmatrix} 2 & 3 & -2 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

here $|B| = 8 \neq 0$

and $$x_{B_4} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

$\therefore$ Basic solutions are →

$[0 \ 3 \ -3 \ 2]$, $[3 \ 0 \ 6 \ -1]$, $[1 \ 2 \ 0 \ 1]$

and $[2 \ 1 \ 3 \ 0]$

Out of these, only 2 are Basic feasible solns

BFS → $[1 \ 2 \ 0 \ 1]$ and $[2 \ 1 \ 3 \ 0]$

Q6.

ans) We need to maximize $7x_1 + 6x_2$

s.t

$$\begin{cases} 3x_1 + x_2 \leq 120 \\ x_1 + 2x_2 \leq 160 \\ x_1 \leq 35 \\ x_1, x_2 \geq 0 \end{cases}$$

Our first step is to add slack variables in order to convert the above inequalities into equality.

So,

$$\max Z = 7x_1 + 6x_2$$

s.t.

$$3x_1 + x_2 + S_1 = 120$$
$$x_1 + 2x_2 + S_2 = 160$$
$$x_1 + S_3 = 35$$

and $x_1, x_2, S_1, S_2, S_3 \geq 0$

→ In our 1st iteration

B.V.	x_1	x_2	S_1	S_2	S_3	b_i	$\frac{b_i}{a_{ij}}$
S_1	3	1	1	0	0	120	40
S_2	1	2	0	1	0	160	160
S_3	1	0	0	0	1	35	35
$Z_j - C_j$	−7	−6	0	0	0	$0 = Z_B$	

Here, we see the negative minimum $Z_j - C_j = -7$

its col index → 1

$\therefore$, the entering variable is x_1

Also, their min ratio $= 35$
its row index $= 3$

$\therefore$, the leaving variable is S_3

Hence, pivot element $= 1$

→ Now, in our 2nd iteration, we'll first perform the following operations

$$R_1 \rightarrow R_1 - 3R_3$$
$$R_2 \rightarrow R_2 - R_3$$

So,

B.v	x_1	x_2	S_1	S_2	S_3	b_i	$\frac{b_i}{a_{ij}}$
S_1	0	1	1	0	-3	15	15
S_2	0	2	0	1	-1	125	62.5
x_1	1	0	0	0	1	35	∞
$Z_j - C_j$	0	-6	0	0	7	$245 = Z_B$	

Here, negative minimum is $Z_j - C_j = -6$
its col idx $= 2$

So, entering variable $= x_2$
Also,
min ratio $= 15$
row idx $= 1$
$\therefore$, leaving basis is S_1

$\therefore$, pivot element $= 1$

→ In 3^{rd} iteration

$$R_2 \rightarrow R_2 - 2R_1$$

So,

B.V	x_1	x_2	S_1	S_2	S_3	b_i	$\frac{b_i}{a_{ij}}$
x_2	0	1	1	0	-3	15	∞
S_2	0	0	-2	1	5	95	19
x_1	1	0	0	0	1	35	35
$Z_j - C_j$	0	0	6	0	-11	$335 = Z_B$	

Here neg. min $= -11$; col idx $= 5$.

∴, entering variable $= S_3$

Also,

min ratio $= 19$; row idx $= 2$

So, leaving basis variable is S_2

∴, pivot element $= 5$

→ In 4^{th} iteration

$$R_2 \rightarrow R_2/5$$

$$R_1 \rightarrow R_1 + 3R_2$$

$$R_3 \rightarrow R_3 - R_2$$

So,

(next page) →

B.V	x_1	x_2	S_1	S_2	S_3	b_i
x_2	0	1	$-1/5$	$3/5$	0	72
S_3	0	0	$-2/5$	$1/5$	1	19
x_1	1	0	$2/5$	$-1/5$	0	16
$z_j - c_j$	0	0	$8/5$	$11/5$	0	$544 = Z_B$

Since all $z_j - c_j \geq 0$, we have arrived at the optimal sol^n

So,

$x_1 = 16$

$x_2 = 72$

$S_3 = 19$

S_1 and S_2 are not found in the basis and hence their val is 0

Max $Z = 544$

7 maximize $z = 7x_1 + 6x_2$

s.t.

$3x_1 + x_2 \le 120$

$x_1 + 2x_2 \le 160$

$x_1 \le 35$

$\frac{7}{4}x_1 + x_2 \le 100$

$x_1, x_2 \ge 0$

Converting to standard form

$3x_1 + x_2 + s_1 = 120$

$x_1 + 2x_2 + s_2 = 160$

$x_1 + s_3 = 35$

$\frac{7}{4}x_1 + x_2 + s_4 = 100$

$x_1, x_2, s_1, s_2, s_3, s_4 \ge 0$

Iteration - 1

B.V.	z	x_1	x_2	s_1	s_2	s_3	s_4	b_i	$\frac{b_i}{a_{ij}}$
s_1	0	3	1	1	0	0	0	120	40
s_2	0	1	2	0	1	0	0	160	160
s_3	0	1	0	0	0	1	0	35	$\boxed{35}$
s_4	0	1.75	1	0	0	0	1	100	57.14
$z_j - c_j$	1	$\boxed{-7}$	-6	0	0	0	0	0	-

entering = x_1, leaving = s_3

Iteration - 2

B.V	z	x_1	x_2	s_1	s_2	s_3	s_4	b_i	$\frac{b_i}{a_{ij}}$
s_1	0	0	1	1	0	-3	0	15	15
s_2	0	0	2	0	1	-1	0	125	62.5
x_1	0	1	0	0	0	1	0	35	-
s_4	0	0	1	0	0	-1.75	1	38.75	38.75
$z_j - c_j$	1	0	-6	0	0	7	0	245	

entering $= x_2$, leaving $= s_1$

Iteration-3

B.V.	z	x_1	x_2	s_1	s_2	s_3	s_4	b_i	$\frac{b_i}{a_{ij}}$
x_2	0	0	1	1	0	-3	0	15	–
s_2	0	0	0	-2	1	5	0	95	19
x_1	0	1	0	0	0	1	0	35	35
s_4	0	0	0	-1	0	1.25	1	23.75	$\boxed{19}$
$Z_j - C_j$	1	0	0	6	0	$\boxed{-11}$	0	335	

entering = s_3, leaving = s_4

Iteration-4

B.V.	z	x_1	x_2	s_1	s_2	s_3	s_4	b_i	$\frac{b_i}{a_{ij}}$
x_2	0	0	1	-1.4	0	0	2.4	72	-
s_2	0	0	0	2	1	0	-4	0	$\boxed{0}$
x_1	0	1	0	0.8	0	0	-0.8	16	20
s_3	0	0	0	-0.8	0	1	0.8	19	-
$Z_j - C_j$	1	0	0	$\boxed{-2.8}$	0	0	8.8	544	

entering = s_1, leaving = s_2

Iteration -5

B.V.	z	x_1	x_2	s_1	s_2	s_3	s_4	b_i	$\frac{b_i}{a_{ij}}$
x_2	0	0	1	0	0.7	0	-0.4	72	
s_1	0	0	0	1	0.5	0	-2	0	
x_1	0	1	0	0	-0.4	0	0.8	16	
s_3	0	0	0	0	0.4	1	-0.8	19	
$z_j - c_j$	1	0	0	0	1.4	0	3.2	544	

all $z_j - c_j \geq 0$, we stop the iterations

$x_1 = 16$

$x_2 = 72$

$s_1 = 0$

$s_3 = 19$

$\max z = 544$

Thus the LPP has a feasible solution. It is bounded, an optimal solution exists which is degenerate