Assignment - 1

1. A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further, no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximise the total profit of the society? Formulate this as a LPP. Use graphical solution approach to conclude whether this LPP has a unbounded solution/ feasible solution/infeasible solution? If problem is feasible obtain the optimal solution to the corresponding LPP.

2. Prove that the open half space $S = \left\{ \boldsymbol{x} : \boldsymbol{c}^T \boldsymbol{x} > z \right\}$ is convex.

3. Arbitrary intersection of convex sets is convex! Is it true? Justify your answer.

4. Suppose C is a convex set. If x is an extreme point of C, then prove that x is on the boundary of C.

5. Find all the basic solutions corresponding to the system of equations

$$\begin{aligned} 2x_1 + 3x_2 - 2x_3 - 7x_4 &= 1 \\ x_1 + x_2 + x_3 + 3x_4 &= 6 \\ x_1 - x_2 + x_3 + 5x_4 &= 4. \end{aligned}$$

   Are all the solutions basic feasible solutions?

6. Solve the following LPP using simplex method in tabular form

$$\begin{aligned} &\text{maximize} 7x_1 + 6x_2 \\ &\text{s.t.} \\ &\begin{cases} 3x_1 + x_2 \leq 120 \\ x_1 + 2x_2 \leq 160 \\ x_1 \leq 35 \\ x_1, x_2 \geq 0 \end{cases} \end{aligned}$$

7. Conclude the nature of following LPP using simplex method in tabular form

$$\text{maximize} 7x_1 + 6x_2$$

s.t.

$$\begin{cases} 3x_1 + x_2 \leq 120 \\ x_1 + 2x_2 \leq 160 \\ x_1 \leq 35 \\ \frac{7}{4}x_1 + x_2 \leq 100 \\ x_1, x_2 \geq 0 \end{cases}$$