

We have  $k$  classes and  $k - 1$  frontiers (profiles) between classes :

$$b^1, b^2, \dots, b^h, \dots, b^{k-1}$$

Lets denote  $K = \{1, 2, \dots, k\}$  and  $K_{no\_end} = \{1, 2, \dots, k - 1\}$  and  $K_{no\_no\_end} = \{1, 2, \dots, k - 2\}$

$$\left\{ \begin{array}{ll} \text{Maximize } \alpha & \\ \text{subject to } \sum_{i \in N} c_{ij}^h + x_j + \epsilon = \lambda & \forall a_j \in A^{*h}, \forall h \in K_{no\_end} \\ \sum_{i \in N} c_{ij}^{h-1} = \lambda + y_j & \forall a_j \in A^{*h}, \forall h \in \{2, \dots, k\} \\ \alpha \leq x_j, \alpha \leq y_j & \forall a_j \in A^* \\ c_{ij}^l \leq w_i & \forall i \in N, \forall a_j \in A^{*h}, \forall h \in K, \forall l \in \{h, h - 1\} \cap K_{no\_end} \\ c_{ij}^l \leq \delta_{ij}^l & \forall i \in N, \forall a_j \in A^{*h}, \forall h \in K, \forall l \in \{h, h - 1\} \cap K_{no\_end} \\ c_{ij}^l \geq \delta_{ij}^l - 1 + w_i & \forall i \in N, \forall a_j \in A^{*h}, \forall h \in K, \forall l \in \{h, h - 1\} \cap K_{no\_end} \\ M\delta_{ij}^l + \epsilon \geq g_i(a_j) - b_i^l & \forall i \in N, \forall a_j \in A^{*h}, \forall h \in K, \forall l \in \{h, h - 1\} \cap K_{no\_end} \\ M(\delta_{ij}^l - 1) \leq g_i(a_j) - b_i^l & \forall i \in N, \forall a_j \in A^{*h}, \forall h \in K, \forall l \in \{h, h - 1\} \cap K_{no\_end} \\ \sum_{i \in N} w_i = 1, \quad \lambda \in [0.5, 1] & \\ w_i \in [0, 1] & \forall i \in N \\ c_{ij}^l, \delta_{ij}^l \in \{0, 1\} & \forall i \in N, \forall a_j \in A^{*h}, \forall h \in K, \forall l \in \{h, h - 1\} \cap K_{no\_end} \\ x_j, y_j \in \mathbb{R} & \forall a_j \in A^* \\ \alpha \in \mathbb{R} & \end{array} \right.$$

To have the ability of cancelling noise :

$$\left\{ \begin{array}{ll} \text{Maximize } \sum_{a_j \in A^*} \gamma_j & \\ \text{subject to } \sum_{i \in N} c_{ij}^h + \epsilon \leq \lambda + M(1 - \gamma_j) & \forall a_j \in A^{*h}, \forall h \in K_{no\_end} \\ \sum_{i \in N} c_{ij}^{h-1} \geq \lambda - M(1 - \gamma_j) & \forall a_j \in A^{*h}, \forall h \in \{2, \dots, k\} \\ c_{ij}^l \leq w_i & \forall i \in N, \forall a_j \in A^{*h}, \forall h \in K, \forall l \in \{h, h - 1\} \cap K_{no\_end} \\ c_{ij}^l \leq \delta_{ij}^l & \forall i \in N, \forall a_j \in A^{*h}, \forall h \in K, \forall l \in \{h, h - 1\} \cap K_{no\_end} \\ c_{ij}^l \geq \delta_{ij}^l - 1 + w_i & \forall i \in N, \forall a_j \in A^{*h}, \forall h \in K, \forall l \in \{h, h - 1\} \cap K_{no\_end} \\ M\delta_{ij}^l + \epsilon \geq g_i(a_j) - b_i^l & \forall i \in N, \forall a_j \in A^{*h}, \forall h \in K, \forall l \in \{h, h - 1\} \cap K_{no\_end} \\ M(\delta_{ij}^l - 1) \leq g_i(a_j) - b_i^l & \forall i \in N, \forall a_j \in A^{*h}, \forall h \in K, \forall l \in \{h, h - 1\} \cap K_{no\_end} \\ b_i^{h+1} \geq b_i^h & \forall i \in N, \forall h \in K_{no\_no\_end} \\ \sum_{i \in N} w_i = 1, \quad \lambda \in [0.5, 1] & \\ w_i \in [0, 1] & \forall i \in N \\ c_{ij}^l, \delta_{ij}^l \in \{0, 1\} & \forall i \in N, \forall a_j \in A^{*h}, \forall h \in K, \forall l \in \{h, h - 1\} \cap K_{no\_end} \\ x_j, y_j \in \mathbb{R} & \forall a_j \in A^* \end{array} \right.$$

