ILP representation

We have k classes and $k-1$ frontiers (profiles) between classes :

$$b^1, b^2, \dots, b^h, \dots, b^{k-1}$$

Lets denote $K = \{1, 2, \dots, p\}$ and $K_{no\_end} = \{1, 2, \dots, p-1\}$ *and* $K_{no\_no\_end} = \{1, 2, \dots, p-2\}$

$$\begin{cases}
Maximize \quad \alpha & \\
subject\ to \displaystyle\sum_{i \in N} c_{ij}^{h} + x_j + \epsilon = \lambda & \forall a_j \in C_h, \forall h \in K_{no_end} \\
\displaystyle\sum_{i \in N} c_{ij}^{h-1} = \lambda + y_j & \forall a_j \in C_h, \forall h \in \{2, \dots, k\} \\
\alpha \leq x_j, \alpha \leq y_j & \forall a_j \in C^* \\
c_{ij}^{l} \leq w_i & \forall i \in N, \forall a_j \in C_h, \forall h \in K, \forall l \in \{h, h-1\} \cap K_{no_end} \\
c_{ij}^{l} \leq \delta_{ij}^{l} & \forall i \in N, \forall a_j \in C_h, \forall h \in K, \forall l \in \{h, h-1\} \cap K_{no_end} \\
c_{ij}^{l} \geq \delta_{ij}^{l} - 1 + w_i & \forall i \in N, \forall a_j \in C_h, \forall h \in K, \forall l \in \{h, h-1\} \cap K_{no_end} \\
M\delta_{ij}^{l} + \epsilon \geq g_i(a_j) - b_i^{l} & \forall i \in N, \forall a_j \in C_h, \forall h \in K, \forall l \in \{h, h-1\} \cap K_{no_end} \\
M(\delta_{ij}^{l} - 1) \leq g_i(a_j) - b_i^{l} & \forall i \in N, \forall a_j \in C_h, \forall h \in K, \forall l \in \{h, h-1\} \cap K_{no_end} \\
\displaystyle\sum_{i \in N} w_i = 1, \quad \lambda \in [0.5, 1] & \\
w_i \in [0, 1] & \forall i \in N \\
c_{ij}^{l} \in [0, 1], \delta_{ij}^{l} \in \{0, 1\} & \forall i \in N, \forall a_j \in C_h, \forall h \in K, \forall l \in \{h, h-1\} \cap K_{no_end} \\
x_j, y_j \in \mathbb{R} & \forall a_j \in C^* \\
\alpha \in \mathbb{R} &
\end{cases}$$

To have the ability of noise cancelling :

$$
\begin{cases}
Maximize \quad \sum_{a_j \in C^*} \gamma_j & \\
subject\ to \sum_{i \in N} c_{ij}^h + \epsilon \leq \lambda + M(1-\gamma_j) & \forall a_j \in C_h, \forall h \in K_{no\_end} \\
\sum_{i \in N} c_{ij}^{h-1} \geq \lambda - M(1-\gamma_j) & \forall a_j \in C_h, \forall h \in \{2, \ldots, k\} \\
c_{ij}^l \leq w_i & \forall i \in N, \forall a_j \in C_h, \forall h \in K, \forall l \in \{h, h-1\} \cap K_{no\_end} \\
c_{ij}^l \leq \delta_{ij}^l & \forall i \in N, \forall a_j \in C_h, \forall h \in K, \forall l \in \{h, h-1\} \cap K_{no\_end} \\
c_{ij}^l \geq \delta_{ij}^l - 1 + w_i & \forall i \in N, \forall a_j \in C_h, \forall h \in K, \forall l \in \{h, h-1\} \cap K_{no\_end} \\
M\delta_{ij}^l + \epsilon \geq g_i(a_j) - b_i^l & \forall i \in N, \forall a_j \in C_h, \forall h \in K, \forall l \in \{h, h-1\} \cap K_{no\_end} \\
M(\delta_{ij}^l - 1) \leq g_i(a_j) - b_i^l & \forall i \in N, \forall a_j \in C_h, \forall h \in K, \forall l \in \{h, h-1\} \cap K_{no\_end} \\
b_i^{h+1} \geq b_i^h & \forall i \in N, \forall h \in K_{no\_no\_end} \\
\sum_{i \in N} w_i = 1, \quad \lambda \in [0.5, 1] & \\
w_i \in [0,1] & \forall i \in N \\
c_{ij}^l \in [0,1], \delta_{ij}^l \in \{0,1\} & \forall i \in N, \forall a_j \in C_h, \forall h \in K, \forall l \in \{h, h-1\} \cap K_{no\_end} \\
x_j, y_j \in \mathbb{R} & \forall a_j \in C^*
\end{cases}
$$