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CO_2 reconstructions from ice cores.

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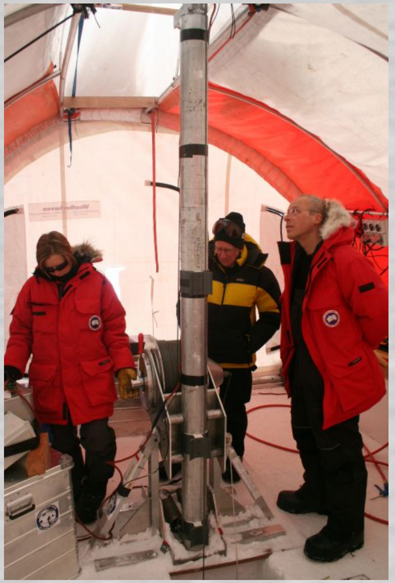
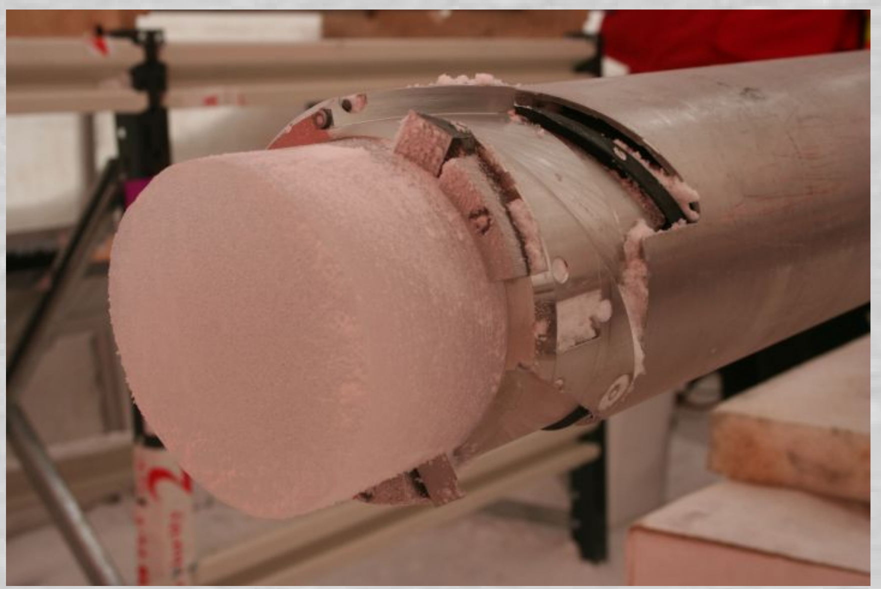
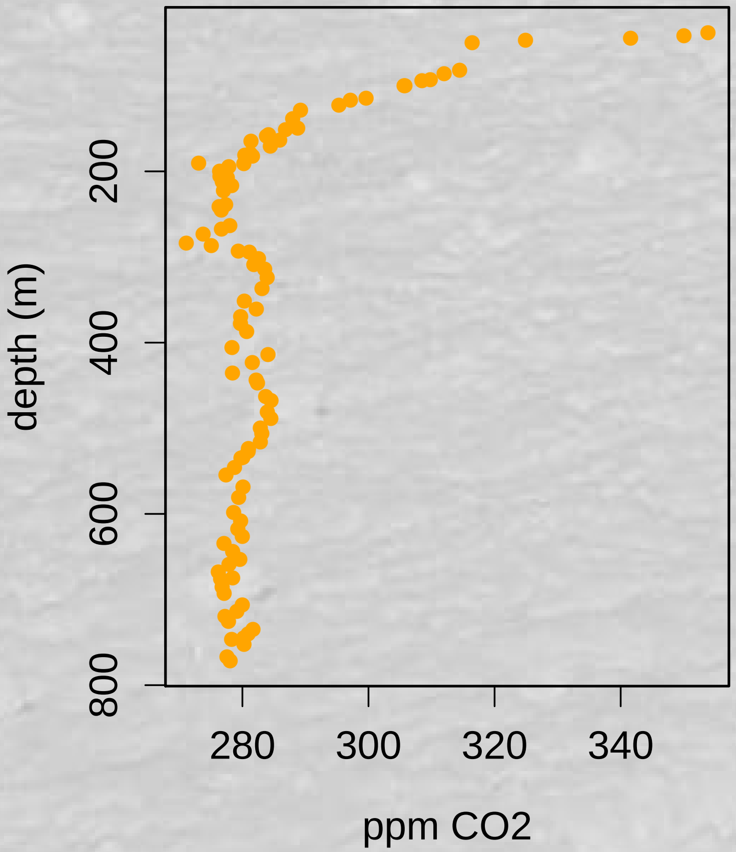




Ice core drill and glaciologists from the Australian Antarctic Division and Antarctic Climate and Ecosystems CRC, Law Dome, East Antarctica

Observations

CO_2 concentrations as a function depth.



Conceptually: Depth is related to the *time* air was trapped in the core.

$$y(\text{depth}) = \mathcal{F}(c(\text{time}))$$

$\mathcal{F}$ developed by Trudinger et al. (2013)

Inverse problem: Invert CO_2 concentrations by depth to concentrations by time.

$$c(\text{time}) = \mathcal{F}^{-1}(y(\text{depth})$$

Firn Ice Inverse problem harder for upper ice layers that have not completely consolidated.

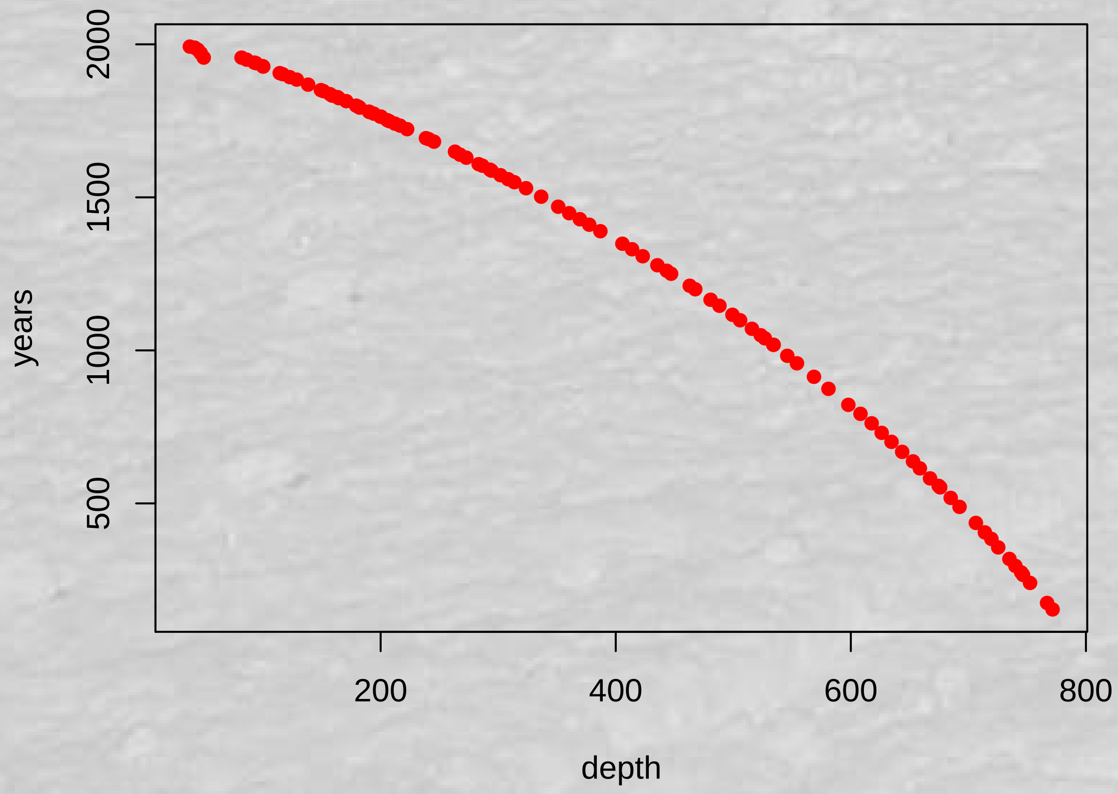
Parts of data set

```
load("EnvironmentalDataAnalytics/Data/CO2PaleoData.rda"
ls()
[1] "CO2Emissions" "cTime"        "depth"        "metaDataObs"
[5]      "weights"      "WObs"          "y"
```

- `CO2Emissions` : Annual human emissions of CO2 for the years in cTime
- `cTIme`: Years that correspond to the columns of the WObs matrix
- `depth`: Depth of ice core layer,
- `MetaDataObs`: a three column matrix first column depth of layer, second column average time (i.e. age of the ice in that layer), third column uncertainty (variance) of measurement.
- `WObs`: The W matrix mapping concentrations over time to concentrations at a given depth.
- `y`: CO2 concentrations at the depths.

Conversion of depth to average time

```
plot( metaDataObs[,1:2], xlab="depth", ylab="years", pch=16, col="red")
```



To find the "average time" of a layer one can use WObs%*%cTime

To transform the emission covariate: WObs%*%CO2Emissions

A Hierarchical (geophysical) model

The goal: Estimate a continuous geophysical process: $c(t)$ CO_2 atmospheric concentration at time t.

Data level:

$$\boldsymbol{y} = W\boldsymbol{c} + \boldsymbol{e}$$

$\boldsymbol{e} = e_1, e_2, ..., e_m$: measurement errors – assumed to be $N(0, \sigma^2)$

Note the role of the W matrix to map from the time series into the expected concentration at a depth.

Process level:

$$c_t = \alpha_1 + \alpha_2 t + \alpha_3 \text{Emissions}(t) + u_t$$

u_t may be correlated over time.

Questions

1. Generate posterior samples of the exponential curve – not just the parameters. Convert these curves from depth to time.

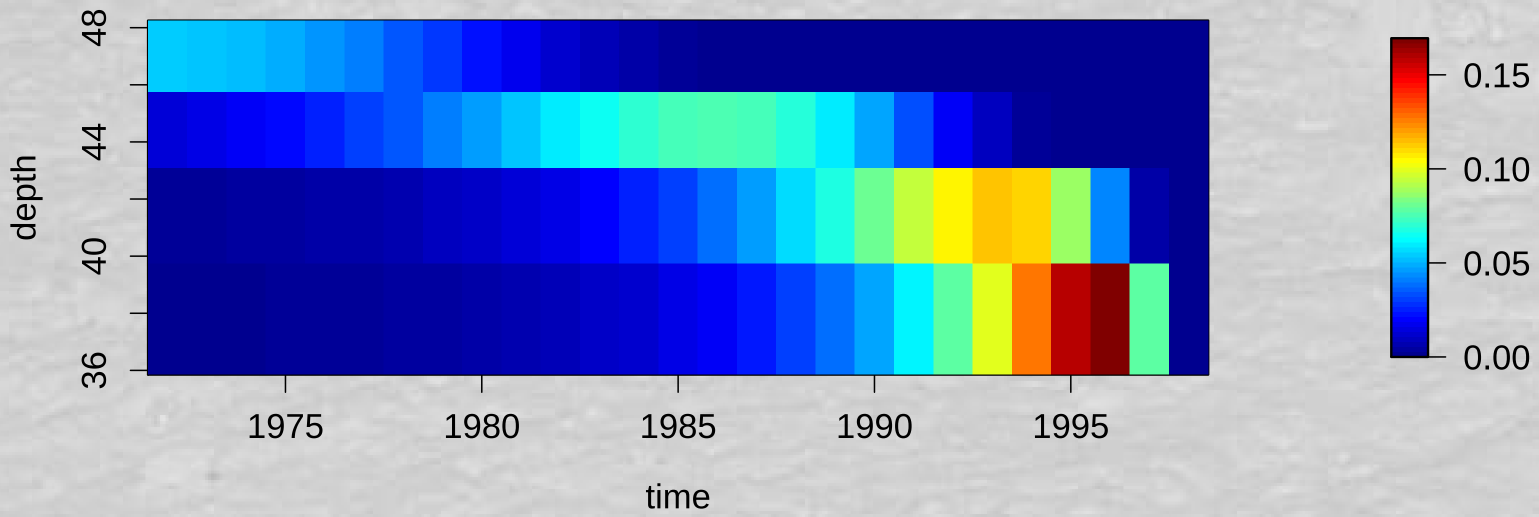
2. Use the emissions covariate in your model and include any other relevant regression functions

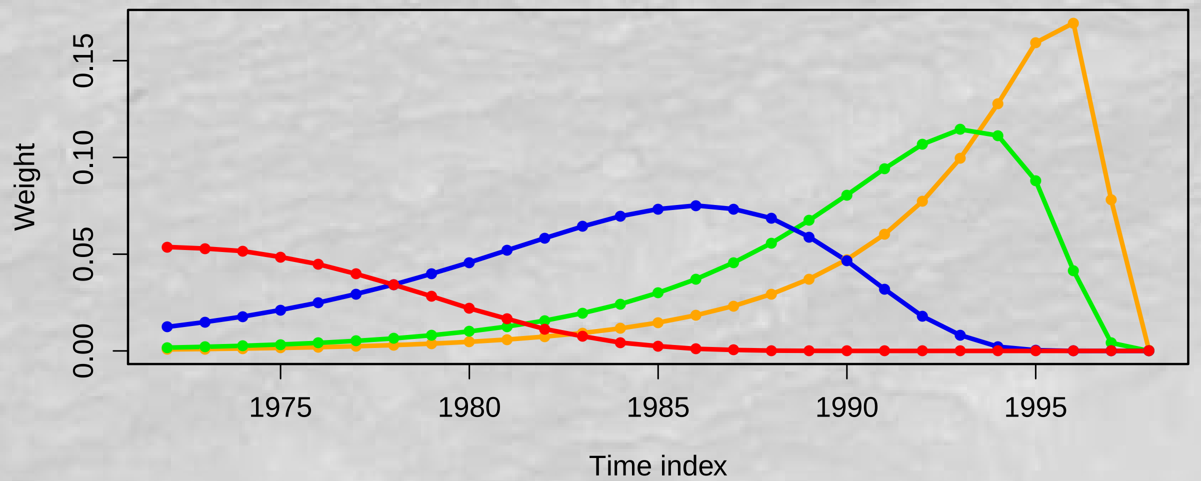
3. Model the correlation in the data using a covariance model for the concentrations over time and the W matrix relationship:

   If the concentrations over time have covariance matrix `Sigma` then the covariance matrix for the concentrations for the depths is `W%*% Sigma %*% t( W)` See the nimble example creating the dgp function to see how to build this into a BUGS/NIMBLE model.

What does W look like?

Most recent (shallower) ice core layers:





Rows are weights applied to concentrations over time.

What does W look like? cont.

Rows of W for most recent ice layers.