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This note outlines how to obtain relative wind and ground-referenced wind from the LAMS measurements.

Start in the reference frame of the LAMS, with wind components $\{u, v, w\}$ in that frame. Relative to LAMS, u is the longitudinal wind (toward the tail), v the lateral wind (toward the port wing), and w the vertical wind (toward the upward direction). The underlying coordinate system is $\{x, y, z\}$ with x in the forward direction, y in the starboard direction, and z downward. In this coordinate system, call Θ the angle from the x-axis and Φ the azimuthal rotation angle about the x-axis in the right-hand sense from the z axis. Then the sensing volumes of the three LAMS beams are approximately at $\{L_i \cos\Theta_i,\ -L_i \sin\Theta_i \sin\Phi_i,\ L_i sin \Theta_i \cos\Phi_i\}$. The configuration selected for testing in the September IDEAS flights is, approximately, $\{\Theta, \Phi\}$ = {{35, 180}, {35, -60}, {35, +60}} for the three beams (named here in the order {1,2,3}). The three measurements $\{a_1, a_2, a_3\}$ then are related to $\{u, v,$w} by the matrix of direction cosines, σ, each element of which gives the contribution of one component of $\{u, v, w\}$ to one component of the measurements $\{a_1, a_2, a_3\}$. Each direction cosine is the dot product between one of the unit vectors $\{\hat{\imath}, \hat{\jmath}, \hat{k}\}$ representing the rectilinear "LAMS" coordinate system and one of the unit vectors $\{\hat{b}_1, \hat{b}_2, \hat{b}_3\}$. Note that the sense of the relative wind is that positive wind is inward along the axis, so for example $\hat{\imath} \bullet \hat{b}_1$ is the positive quantity $L_1 \cos\Theta_1$ and represents the contribution of u to the inward velocity measured in beam #1. The complete set of direction cosines is then the matrix $\mathbf{S}$ giving the relationship between the rectilinear components $\{u, v, w\}$ and the measured inward airspeeds in the three beams $\{a_1, a_2, a_3\}$:

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \mathbf{S} \begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{1}$$

Evaluating these components gives

$$\mathbf{S} = \left(\begin{array}{ccc} \cos\Theta_1 & -\sin\Theta_1 \sin\Phi_1 & \sin\Theta_1 \cos\Phi_1 \\ \cos\Theta_2 & -\sin\Theta_2 \sin\Phi_2 & \sin\Theta_2 \cos\Phi_2 \\ \cos\Theta_3 & -\sin\Theta_3 \sin\Phi_3 & \sin\Theta_3 \cos\Phi_3 \end{array} \right) . \tag{2}$$

For $\Theta_1 = \Theta_2 = \Theta_3 = 35^\circ$ and $\{\Phi_i\}$={180, -60, 60}, the matrix becomes approximately

$$\mathbf{S} = \left(\begin{array}{ccc} 0.819 & 0 & -0.574 \\ 0.819 & 0.497 & 0.287 \\ 0.819 & -0.497 & 0.287 \end{array} \right) . \tag{3}$$

Inverting σ then gives the wind components relative to the LAMS coordinate system in terms of the measured components:

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \mathbf{S}^{-1} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \tag{4}$$

where

$$\mathbf{S^{-1}} = \begin{pmatrix} 0.4069249 & 0.40692490 & 0.4069249 \\ 0 & 1.0065795 & -1.0065795 \\ -1.1622979 & 0.5811489 & 0.5811489 \end{pmatrix} \tag{5}$$

Once the wind components relative to LAMS are obtained, the next step is to transform these to the aircraft reference frame for comparison to the radome-derived gust components. Two transformations are needed: (i) to correct for the difference in wind vectors at the LAMS and at the IRS that results from rotation of the aircraft and the offset of the LAMS sensing volumes from the location of the IRS; and (ii) to correct for differences in attitude angles between LAMS and the aircraft.

(i) Correction for rotation:

For the first, Bulletin 23 gives the required correction to the relative wind:

$$\vec{v}_a' = \vec{v}_a + \vec{\Omega}_p \times \vec{R} \tag{6}$$

where $\vec{\Omega}_p$ is the angular rotation rate or angular velocity of the aircraft[1] and $\vec{R}$ is the distance from the IRS to the wind sensor. This footnote[2] is what we discussed at our first meeting, but I now realize that it is unnecessarily complex. The vector $\vec{R}$ needed in (6) is only the vector from the IRS to the LAMS, without the additional extension to the three sample volumes, because the motion

[1] Bulletin 23 says this is the angular acceleration but later equations make it clear that rotation rates like $\dot{\phi}$ are used.

[2] In the case of LAMS, the instrument is mounted on the wing but the sensed wind is located at three locations ahead of the aircraft, so this transformation is not as straightforward as in Bulletin 23. The three components $\{a_i\}$ measured by LAMS need to be transformed separately, each with its own $\vec{R}_i = X_{\mathrm{LAMS}} + L_i \cos\Theta_i, Y_{\mathrm{LAMS}} - L_i \sin\Theta_i \sin\Phi_i, Z_{\mathrm{LAMS}} + L_i sin\Theta_i \cos\Phi_i$ given by the vector from the location of the IRS to the location of the volume sensed by the ith beam. In this formula, $\{X_{\mathrm{LAMS}}, Y_{\mathrm{LAMS}}, Z_{\mathrm{LAMS}}\}$ is the location of the LAMS relative to the IRS. These need to be determined accurately, but for the present purposes of this note I will make the guess that the coordinates of the LAMS are {-5., -8., +1} m; i.e., LAMS is behind, to port, and below the IRS. The distances from LAMS to the focal points $\{L_i\}$ will all be taken to be 30 m, pending better information.

The rotation vector of the aircraft has components given respectively by the rates of change of ROLL, PITCH, and THDG (heading), expressed of course in radians. Examination of some flight data led to these approximations: {4, 0,25 2.} degrees per second. These will be used here to estimate the importance of including these corrections, but for real data the derivatives of the measured attitude angles should be used.

The wind component sensed by the ith beam is then the dot product of the unit vector along the beam ($\hat{b}_i$) with the relative wind vector given by (6). For the ith measured component,

$$a_i = \sum_j S_{ij}(v_j + [\vec{\Omega}_p \times \vec{R}_i] \bullet \hat{m}_j) \tag{7}$$

where $\hat{m}_j$ is the appropriate unit vector from the set $\{\hat{\imath}, \hat{\jmath}, \hat{k}\}$. The transformation from $\mathbf{v}'$ to $\mathbf{a}$ can then be written

$$\mathbf{a} = \mathbf{S}\,\mathbf{v} + \mathbf{C} \tag{8}$$

where

$$C_i = \sum_j S_{ij}\,[\vec{\Omega}_p \times \vec{R}_i] \bullet \hat{m}_j) \tag{9}$$

are then the component contributions, arising from rotation, to the airspeeds sensed in the three beams.

of the LAMS determines the rotational contribution to the measurements. The other components introduced by the extension from LAMS to the sample volumes only produce extra contributions orthogonal to the beams, and these then vanish when the results are multiplied to **S** to get the measured components from the rectilinear components. I verified this by calculation, and also verified that the calculation produces the same results if, instead of (9), the following equation is used:

$$\mathbf{C} = \mathbf{S}\vec{\Omega} \times \vec{R} \tag{10}$$

or

$$\mathbf{a} = \mathbf{S}\,(\mathbf{v} + \vec{\Omega}_p \times \vec{R}) \ . \tag{11}$$

Then the solution is

$$\mathbf{v} = \mathbf{S}^{-1}\mathbf{a} - \vec{\Omega} \times \vec{R} \ . \tag{12}$$

Technically, the rotation vector should be transformed into the LAMS coordinate frame from the aircraft frame before calculating this, but because this is expected to be a small correction this step will be omitted.

<u>(ii) Correction for differences in orientation angles: (REVISED FROM EARLIER NOTE)</u>

The LAMS orientation is measured by a CMIGIT in the LAMS pod, while the aircraft orientation is measured by a fuselage-mounted IRS. Each measures heading, pitch, and roll. Transformation to the aircraft system involves transforming from the LAMS reference system to the aircraft reference system. The required transformations are described by three rotation matrices, defined in Bulletin 23 Eqs. 2.5 and 2.6:

$$T_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix}$$

$$T_2 = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$T_3 = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $\{\phi,\,\theta,\,\psi\}$ are {pitch, roll, heading.

Because the LAMS (CMIGITS unit) is not aligned perfectly with the aircraft reference frame as defined by the Honeywell IRU, the relative-wind components measured by LAMS should first be transformed from the CMIGITS (LAMS) coordinate frame (attitude angles $\{\phi', \theta', \psi'\}$) to an earth reference frame while following the proper order of transformations that corresponds to the definitions of the attitude angles. Then the resulting coordinates should be transformed to the Honeywell (aircraft) coordinate system. Specifially, the transformation is made in the following order:

1. Rotate by T_1 using the CMIGITS roll angle ϕ' to level the wings:
2. Rotate by T_2 using the CMIGITS pitch angle θ' to level the aircraft.
3. Rotate by T_3 using the CMIGITS heading angle ψ' to obtain components in a true-north reference frame. At this point, the relative-wind vector in an Earth-reference coordinate system is is $v_r = T_3(T_2(T_1 v))$
4. Rotate by T_3 using the negative of the aircraft heading angle ψ.
5. Rotate by T_2 using the negative of the aircraft pitch angle θ.
6. Rotate by T_1 using the negative of the aircraft roll angle ϕ.

This results in relative-wind components in the reference frame of the aircraft, so the results can be compared to standard measurements where the relative wind components are {TAS, TAS*tan(SSLIP), TAS*tan(ATTACK)}.

The wind components in a ground reference frame are already found at the end of step 3 so these should be saved for use in creating new Earth-relative winds. The procedure is relatively simple compared to routine 'gust.c' because all the work of rotation and transformation to Earth-reference coordinates has already been done, although the components are still in a coordinate system with $\{x, y, z\}$ {north, east, down}. For example, the equations to get variables {WD_LAMS, WS_LAMS, WI_LAMS) analogous to {WDC, WSC, WIC} are:

$$v_g = v_r + \begin{pmatrix} -\mathrm{VNSC} \\ -\mathrm{VEWC} \\ \mathrm{VSPD} \end{pmatrix}$$

$$\mathrm{WD_{LAMS}} = \arctan 2(\mathrm{v_{g,y}}, v_{g,x})$$

$$\mathrm{WS_{LAMS}} = \sqrt{(\mathrm{v_{g,x}^2} + \mathrm{v_{g,y}^2})}$$

$$\mathrm{WI_{LAMS}} = \mathrm{v_{g,z}}$$

A python routine that implements the preceding procedure is saved as LAMSprocessor.py in the directory ~cooperw/LAMS. See also the files LAMSprocessor.ipynb and LAMSnotebook.pdf, the former for implementation in ipython notebook format and the latter a screen capture of the output from that notebook.

Comments:

1. Re S:
   (a) u makes an equal contribution to all beams, as expected.
   (b) the (2,1) element, with e-17, should be zero except for machine-precision errors.
   (c) v makes equal and opposite contributions to beams 2 and 3.
   (d) w makes positive contributions to beams 2 and 3 but a negative contribution to beam 1.
2. Re Si (the inverse of S):
   (a) The longitudinal component of the wind, u or TAS, is recovered by dividing the average of the three measurements by cos 35°.
   (b) The lateral component (v) is almost the difference between beams 2 and 3. (The term with e-16 should be zero except for machine-precision errors.)
   (c) All three measurements must be used to obtain the vertical wind.
3. The 'Omega x R' terms give the contributions to the measured airspeeds from rotation, in umits of m/s. This rotation can produce corrections of about 0.5 m/s in the w component; although the correction is small it should not be neglected. Because the corrections are proportional to the lever arms for the rotations, this also indicates that knowledge of these distances to perhaps 10% is adequate.
4. Differences in orientation angles between the IRS and CMIGIT reference frames of about 1° have little effect on the measured u component but can change v and w by more than 1.5 m/s. This is because a small rotation moves the large u component to make a significant contribution to v and w but only has a cosine-of-rotation-angle effect on u. The relative orientation angles need to be known with low uncertainty if the lateral components of the wind are to be measured with low uncertainty. A rough estimate from this calculation is that we need to know the orientation angles to around 0.1° to determine the lateral wind components to 0.15 m/s.