Background

The variable GGALTC is described as the "Corrected GPS Altitude" in the headers, and Bulletin 9 says it is obtained by reference to another altitude, typically the inertial altitude. Comments in the code (altc.c) say that this routine creates a corrected altitude variable by combining the GPS altitude with the IRS altitude when needed. However, a check of the current code and dependence table shows that the reference is to the pressure altitude. The approach taken is to determine a for-all-time regression fit between pressure altitude and GPS altitude, then use that fit while the GPS is good to determine an offset between the GPS and pressure altitude. That offset is saved, and when GPS is lost that offset is used to adjust the pressure altitude to obtain a replacement for the GPS altitude. When the GPS returns, the value is gradually stepped back to the GPS value over several (typically 10) seconds to avoid an abrupt transition.

Reasons for considering a change:

1. Using pressure altitude in this way is clearly unjustified, especially given a specific regression between GPS and pressure altitude; any such relationship will be highly variable. Furthermore, it doesn't take into account the change in slope that occurs at the tropopause, which almost surely leads to significant errors at GV altitude. A simple linear fit cannot give a good representation of the conversion from pressure altitude to geometric altitude, even if the fit were adjusted for each flight (which it is not).

2. The routine is an ad hoc approach that isn't documented or justified, and it has arbitrary switches and branches. I don't see a reason for "walking" back to the right answer except to make plots obscure the fact that there is a gap in the measurements.

3. The alternate branch in the routines, to use "ALT" from the IRS, is subject to the same criticism. ALT is adjusted internally in the IRS to match the pressure altitude, not the GPS altitude. A check of GV PREDICT flights showed that the difference between GPS and IRS altitude is often 500 m or more, showing that ALT is also not a good representation of the geometric altitude.

4. Improvements can be achieved readily. Two options are explored in the remainder of this note.

5. It would be desirable to have the altitude from the GPS treated, to the extent possible, in the same or a similar way to the horizontal positions, with short-term changes obtained from the IRS but long-term stability achieved by reference to the GPS. The algorithms suggested below, particularly "B", achieve this.

Suggestions for consideration:

Valid reference altitudes to use for adjustment can be obtained in these alternate ways:

1. Integrate the vertical speed from the IRS to obtain the change in altitude. A baro-inertial loop in the IRS keeps this from undergoing unstable oscillations in accuracy but also tends to suppress some of the real altitude changes. However, this is not as compromised as the direct altitude output from the IRS and so should provide a better short-term signal than either PALT or ALT.

2. Integrate the hydrostatic equation, using the temperature measured along the flight path as well as the pressure, to obtain the new altitude. Unlike pressure altitude, this uses the actual temperature encountered instead of the temperature in the standard atmosphere, and so will be more accurate, although there will be some error arising from mixing horizontal and vertical gradients. The result will still need updating to the GPS altitude when available to remove these errors.

Two replacement algorithms I tried:

A. Feedback to VSPD:

1. Integrate VSPD from the IRS to obtain an altitude reference (here called ALTVC). Also calculate ALTH by integration of the hydrostatic equation, to use as a reference to use for adjustment of the bias in VSPD, as in step 3 below.

2. When there is valid GPS information, correct for the error in ALTVC via the application of an exponential filter referenced to the GPS information, and apply the same filter to ALTH to adjust it toward the GPS value.

3. Adjust the VSPD used by a "BIAS" term, determined from a PID feedback loop, but with the respective terms for the components proportional to the error, integrated error, and derivative of the error determined from average values over some short period of time to avoid noise. Use the resulting BIAS as a correction term to VSPD in subsequent steps for calculation of ALTVC from integration of VSPD. Then use (VSPD-Bias) for subsequent iterations in Step 1.

4. When GPS information is lost, continue integration to calculate new values of ALTV and ALTVC (including application of the PID feedback referenced to ALTH), but suppress updating of either ALTV or ALTH to the invalid GPS information.

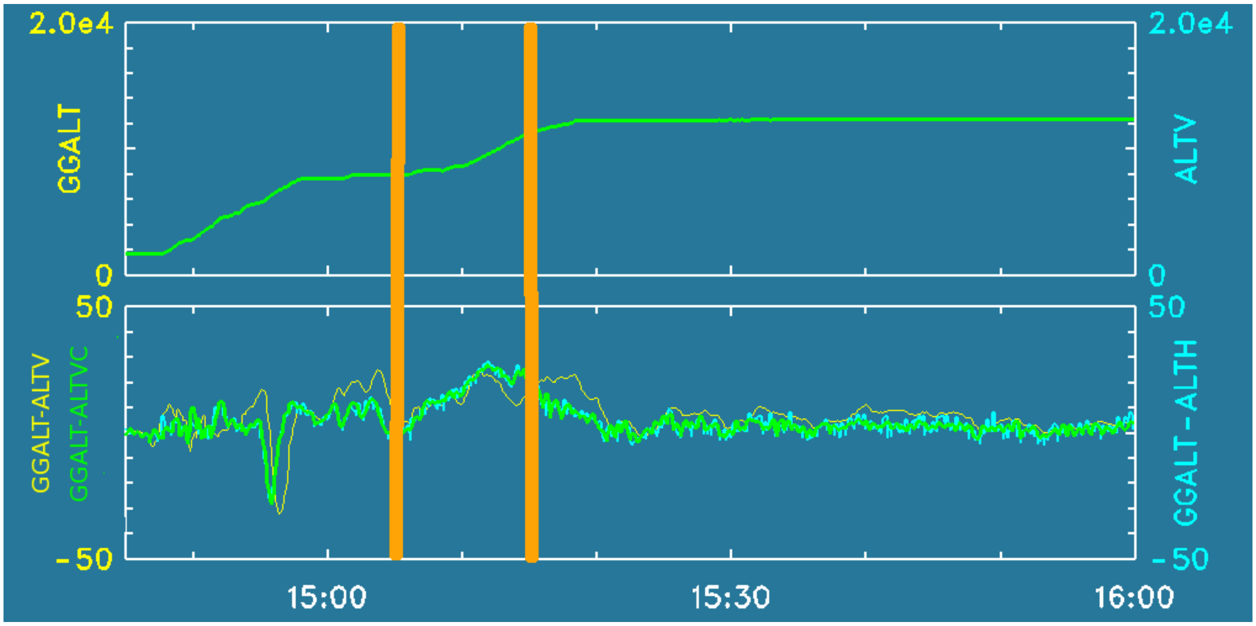
B. Low-pass filtering to determine offset (This is the same approach as applied to the horizontal wind via the complementary-filter approach, except for step 4.)

1. Integrate the VSPD output from the IRS, with PID feedback as above.
2. Filter the difference dH=GGALT-ALTV to obtain the mean offset between the GPS altitude and the altitude obtained by integration of the IRS velocity.
3. Add this offset to ALTV via an exponential filter.
4. When GPS is lost, switch to the reference altitude ALTH determined as in method A above, and continue use of this to update the mean offset.
5. When the GPS returns, exponentially update to the new offset by switching back to GGALT as the reference for the filter.

I also tested an exact analogy to the horizontal-wind treatment: Simply integrate VSPD and use exponential filtering to adjust the result to the GPS position. However, it is well known that the vertical integration is unstable and leads to larger errors than the horizontal integration, as explained in Lenschow's original documents on wind measurements. When I tried this, I found that the result of the integration without updating (as required when the GPS is absent) led to unacceptable errors in periods of 5-10 min, so I added step B.4 above.

Results

Before discussion the choices of filter time constants, feedback coefficients, coding, etc., I'll first show the results in this section. Figure 1 shows that both of the above methods worked well:



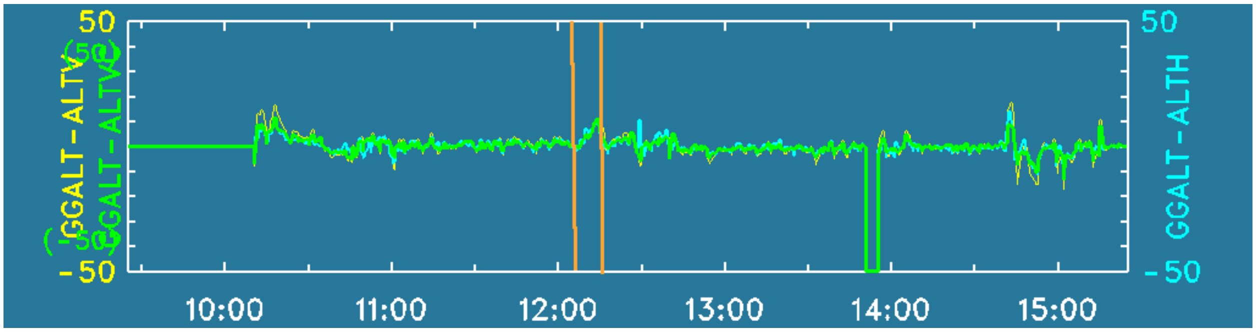
The data used for this plot come from a TREX flight, chosen because high-rate data are available. The top panel shows three values of the altitude (in m) plotted in yellow (GGALT), cyan (ALTV), and green (ALTH). With the line width used, all three overlap. To test performance through periods when the GPS is not available, GPS information was not used for updating during the period between the orange lines, as discussed in the next paragraph. GGALT is the usual altitude measurement from the GPS; ALTH is the altitude determined from integration of the hydrostatic equation with updating to the GPS altitude, and ALTV is the altitude determined from the IRS with updating to the GPS altitude. The bottom panel shows the difference (in m) between the GPS altitude and that determined according to Method B above (yellow thin line, GGALT-ALTV), by Method A (green thicker line, GGALT-ALTVC), and by integration of the hydrostatic equation with updating to the GPS altitude (cyan line, GGALT-ALTH). The standard deviations in these variables over the period of the plot are, respectively, 7.9 m, 7.8 m, and 6.65 m.

To test performance when the GPS information is not available, updating to the GPS altitude was suppressed for the 10-min period between the orange lines. The GPS altitude actually was available, as shown in the plot, so the resulting altitude can be compared to the actual altitude. A period was chosen when there is rapid change in the altitude during a climb and transition from slow to fast climb; results for other tested periods are similar. There is a small excursion in altitude for the test period relative to the reference altitude (GGALT), but it is not outside the limits of other variability observed while the GPS was valid (e.g., at about 1456) and is only about 25 m. The period of simulated drop-out, 10 min, is long compared to the periods of normal signal loss for the current GV GPS system, which is often available continuously and seldom is lost for more than a minute.

The good performance of both algorithms is in contrast to the performance without incorporation of information from ALTH, the altitude obtained by integration of the hydrostatic equation. Without this, the information obtained solely by integrating the IRS-provided variable VSPD causes the error to grow rapidly, exceeding 50 m by the end of the plotted period. It therefore is valuable to calculate ALTH and use it as a substitute reference altitude when GGALT is not available.

Despite the slightly better performance of ALTH during the simulated drop-out period, I don't think this is the best choice for a new variable, for two reasons. First, the resulting signal from integration of the hydrostatic equation is noisy, mostly because of high sensitivity to the measured pressure (typically 10-40 m per mb), so the resulting signal is noisy in comparison to altitude determined from either the GPS or IRS. Second, false signals can appear that are caused by temperature fluctuations (as might occur during passage through a cloud), so there is an advantage to use of one of the measurements that is based solely on position measurement, as provided by either the GPS or IRS.

Another example, from processing a PREDICT flight at low rate, is shown in the following figure. (The error scales are $\pm$50m.) Again, a simulated GPS drop-out was included between the times denoted by the orange lines. There is a true drop-out from 135036-135535, and throgh this gap the new variables look reasonable (but are not shown here).



My recommendation is Method B. This is parallel to the result for the horizontal position, except for the addition of reference to ALTH when the GPS is unavailable. The result is less noisy than the altitude obtained from Method A, as shown by the thin yellow line in the bottom panel of the above figure. Method B also performs better than Method A during the simulated drop-out period. The new variable incorporates the best of information from the GPS and IRS into a new variable that should be an improvement over either of the original measurements because it combines the short-term smoothness, resolution, and high-frequency response of the IRS with the long-term stability of the GPS.

Some additional lessons learned and useful follow-up steps:

1. The variable VSPD from the IRS is biased enough that its integral, following the procedures outlined above, leads to large departures from the GPS reference altitude unless that bias is removed. This is a problem that extends beyond the immediate topic of this note, because it means that the vertical speed of the aircraft has an error that will enter our vertical-wind calculation. The magnitude of this was significant during the climbs and descents of the PREDICT flights, leading to altitude errors of 120–200 m before correction for the VSPD bias. This is being investigated as part of a separate study of the vertical-wind measurement. My speculation is that the internal updating of VSPD by the IRS, set to match the standard atmosphere, caused VSPD to be in error in PREDICT because the tropical atmosphere was substantially warmer than the standard atmosphere, and this led to significant errors in VSPD.

2. Integration of the hydrostatic equation along the flight path produces a usable estimate of the true geometric altitude, although with some potential errors that arise from horizontal gradients in either temperature or pressure. It was necessary to use this to constrain the integration of VSPD during GPS gaps; otherwise the integration developed unacceptable errors in only 5-10 min, for the reason explained in comment #1.

3. The accuracy of the integration of the hydrostatic equation is sufficient to consider using this as an indicator of the accuracy of measurements of temperature. The relationship can be expressed as follows:

$$\frac{dp}{p} = -\frac{g}{RT}dz$$

$$\ln(\frac{p_2}{p_1}) = -\frac{g}{R}\int_{z_1}^{z_2}\frac{1}{T(z)}dz$$

If pressure can be measured to *relative* accuracy of about 0.5 mb, the pressure ratio can be determined (for an example layer from 300 to 200 mb) to about 0.5% accuracy, and the altitude difference can be measured better than this, so this constrains the temperature of the layer to about 0.4% accuracy or, for a typical temperature of about 225K, about 1K. The test is still better at low altitude. This may be worth pursuing in connection with the study of temperature measurement on the GV.

4. I determined the coefficients used for the PID feedback loop by trial and (mostly) error. (I did use the Ziegler–Nichols method as a starting point.) It may be helpful to get someone who understands how to tune a PID feedback loop to see if better coefficients can be found. Also, it may be preferable to implement a different feedback loop instead. Our standard baro-inertial loop is not a candidate because there is only one integration involved, not two, so only one place to inject feedback.

5. It will be important in processing this way to have resolved any time lags between the GPS and IRS measurements. Otherwise, in steep ascents or descents, there can be significant offsets that result just from the time delays. For example, in PREDICT there were occasions where descent rates exceeded 25 m/s, so an offset of as much as 25 m can result from a time difference of about 1 s.

6. There are many time constants in the algorithm described in detail in the next section. Those all interact and must be adjusted together to keep the calculation stable. For example, one might consider lowering the time constant “tau4” of the feedback loop that updates the integrated-VSPD altitude to the GPS altitude, in the last step. However, lowering this below about the setting given here will result in an unstable oscillation that produces very large errors, because the time constant used to determine the coefficients in the PID feedback loop will become comparable to the time constant of the final-filter adjustment to the GPS altitude. At this time, I haven’t explored the limits of stability very much, but I know that tuning these coefficients and time constants can easily cause the errors in the results to become unacceptable. The below values seem to work for the cases I studied, all pertaining to the GV in either PREDICT or TREX.

Details of the Algorithm

The following explains in detail how I implemented the “Method B” algorithm for testing:

For each time step:

- initialize at the time of first valid GPS signal

- initialize the variable ALTV to the initial value of GGALT. Also set the working variables altvc and alth to the same value. The variable altvc is used to determine the bias in the IRS measurement of vertical motion; althr is the result of integration of the hydrostatic equation.
- zero the 6-element array "zf" used for the Butterworth filter and the three PID coefficients {dap,dai,dad}, as well as the variables "bias", "daplast" and "dhf". Save the pressure as "plast"

• if pressure (PSXC) and temperature (ATX) are not MISSING_VALUE:

  - increment the hydrostatic-equation altitude:

$$\begin{aligned} \text{alth+} &= (R/g)(\{\text{ATX}\}+273.15)\ln\left(\frac{p_{last}}{\{\text{PSXC}\}}\right) \\ p_{last} &= \{\text{PSXC}\} \end{aligned}$$

• if IRS vertical aircraft velocity (VSPD) is greater than -100 [which avoids missing_value and other errors]:

  - increment "ALTV" and altvc using "VSPD" and bias (updated in later steps):

$$\text{altv+} = (\{\text{VSPD}\} - bias)/srate$$

  where "srate" is the sample rate, e.g. 25 for high-rate data.

  - if GGALT is also valid (i.e., GGALT>0.) then update the Butterworth-filtered value of the difference between GGALT and ALTV, via a call to the function "bwfltr(x,z)". This is the same function now in use for calculation of the variables VNSC and VEWC, and called there "filter()", *but with a different time constant.* The time constant is 200 s as implemented here; for high-rate data, the time constant must be changed to 200*srate. The function call is:
  dhf = bwfltr((GGALT-ALTV), zf)
  - else (i.e., GGALT is invalid) update dhf instead to the altitude obtained from integration of the hydrostatic equation:
  dhf = bwfltr((alth-ALTV), zf)
  - update the PID coefficients and the "bias". This does not require valid GGALT, only valid VSPD:

```
        static float tau2 = 300., tau3 = 20.;
        static float akp = 10., akt = 80.;
        dap += ((altvc-alth)-dap)/(tau3*srate);
        dai += (altvc-alth)/(tau2*srate);
    // srate not in equation for derivative because the
    // increment used to determine the derivative involves
    // delta-t between samples which is the inverse of srate
        dad += ((altvc-alth)-daplast)/tau3;
        daplast = altvc-alth;
        bias = akp*dap/tau3 + 2.*akp/akt*dai/tau2
             + akp*akt*dad/(5000.*tau3);
```

(This is not a pure PID feedback loop because the proportional and derivative components are each averaged using an exponential filter with time constant of 20 s, while the integral component is averaged over 300 s. This is done to avoid noise.)

- If GGALT is valid, then whether VSPD is valid or not, update both altvc and ALTH (but not ALTV) toward GGALT:

```
        static float tau = 60.;
        altvc += (GGALT-altvc)/(tau*srate);
        alth +=  (GGALT-alth) /(tau*srate)
```

- in all cases, update ALTV toward the offset dhf determined by low-pass filtering the difference between the reference altitude (GGALT when available, ALTH otherwise) and VSPD:

```
        static float tau4 = 100.;
        ALTV += dhf/(tau4*srate);
```

—— END OF THIS NOTE ——