Equivalent Potential Temperature

Al Cooper

RAF Algorithm Review

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What Is Equivalent Potential Temperature?

Rossby Form

- L_v and c_{pd} are kept constant.

$$\boxed{\Theta_p^{[Rossby]} = \Theta_d \exp\left\{\frac{L_v r}{c_{pd} T}\right\}}$$

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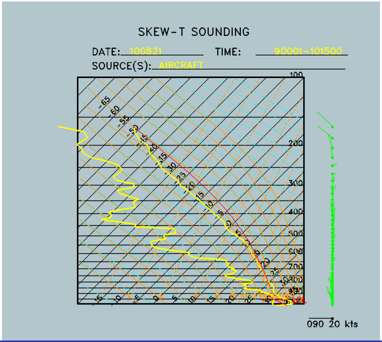

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"A thermodynamic quantity, with its natural logarithm proportional to the entropy of moist air, that is conserved in a reversible moist adiabatic process. "

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Equations

$$\boxed{\Theta_q = T\left(\frac{p_0}{p_d}\right)^{R_d/c_{pt}} \exp\left(\frac{L_v r}{c_{pt} T}\right) \qquad (1)}$$

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- Bolton: If Θ_D is the dry-air potential temperature at the LCL, e the vapor pressure in mb, T_K the air temperature in kelvin, T_L the temperature at the LCL in kelvin and r the mixing ratio

$$T_L = \frac{2840}{3.5\ln T_K - \ln e - 4.805} + 55$$

$$\Theta_p^{Bolton} = \Theta_D \exp\left\{\left(\frac{3.376}{T_L} - 0.00254\right) r(1 + 0.81 \times 10^{-3} r)\right\}$$

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1. Bolton offered a second formula in his 1980 paper that is more accurate.
2. Davies-Jones repeated the analysis and obtained still better fit coefficients investigated over a wider numerical range.
3. If we change to the Davies-Jones formula, it may be useful to change the variable name to "pseudo-adiabatic equivalent potential temperature" at the same time to remove the conflict with the AMS definition.

Three Associated Analyses

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  1. Integrate the exact equation for dT/dp_d from point 1 to point 2 to find T_2.
  2. Evaluate the equation for potential temperature at point 1, then invert it at point 2 to find T_2.

Example: Check Davies-Jones Equation

Method 1: Integration

1. Initialize: at $\{p_1, T_1\}$

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$$\Theta_{DL} = T \left(\frac{p_0}{p - e} \right)^{2/7} \left(\frac{T}{T_L} \right)^{0.28 \times 10^{-3} r}$$

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Results

Primed quantities result from numerical integration; unprimed from formula evaluation

T [°C]	p [hPa]	final p_2 [hPa]	$T_q[K]$	$T'_q[K]$	$T_p[K]$	$T'_p[K]$
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15	750	100	189.34	189.73	185.39	185.42
10	750	100	178.69	178.71	175.66	175.69
0	700	100	167.05	166.90	165.39	165.41
-10	600	100	163.54	163.44	162.68	162.70
25	850	300	222.59	222.69	222.27	222.29
10	750	300	239.43	239.46	238.52	238.51
0	700	300	226.46	226.59	225.82	225.84

Example: 1.19 K error in Θ_q, 0.03 K in Θ_p

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Largest error in Θ_p: 0.03 K

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Error in Θ_q larger when extended to low pressure

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   POTENTIALLY WORTH ATTENTION FOR Θ_q: Differences can exceed 1K but are more typically 0.1 K.

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FORMULA IS GOOD: Numerical tests showed that results were within about 0.05 K and in most cases were better than this limit, suggesting that little error is introduced by using the Bolton formula.

Recommendations

1. Change to (6.5) of Davies-Jones (2009), and change the variable name to "pseudo-adiabatic equivalent potential temperature". Continue to use (21) of Bolton (1980) to determine the saturation temperature T_L.

$$\Theta_p^{[DJ\}} = \Theta_{DL} \exp\left\{ \frac{(L_0^* - L_1^*(T_L - T_0) + K_2 r) r}{c_{pd} T_L} \right\}$$

$$\Theta_{DL} = T_k \left(\frac{1000}{p_d} \right)^{0.2854} \left(\frac{T_K}{T_L} \right)^{0.28 \times 10^{-3} r}$$

$$T_L = \frac{2840}{3.5 \ln T_K - \ln e - 4.805} + 55$$

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2. Add a new variable "wet-equivalent potential temperature" and use the standard equation for its evaluation. .

$$\Theta_q = T\left(\frac{p_0}{p_d}\right)^{R_d/c_{pt}} \exp\left(\frac{L_v r}{c_{pt} T}\right)$$

where $c_{pt} = c_{pd} + r_t c_w$ and r_{tot} is the total water mixing ratio, $r_{tot} = r + r_w$ where $r_w = \chi/\rho_d$ with χ the liquid water content and ρ_d the density of dry air: $\rho_d = (p - e)/(R_d T)$.