Angle of Attack

The first-order expression for the vertical wind w is

$$w = V\sin(\alpha - \phi) + w_p \tag{1}$$

where V is the true airspeed, α the angle of attack, ϕ the pitch, and w_p the vertical motion or rate-of-climb of the aircraft. The solution for the angle-of-attack is

$$\alpha = \phi + \arcsin\frac{w - w_p}{V} \tag{2}$$

If it is reasonable to assume that w is zero, or that it averages to zero, then

$$\alpha = \phi - \arcsin\frac{w_p}{V} \tag{3}$$

can be used to determine the angle-of-attack from the measurements of pitch, rate-of-climb, and true airspeed. Even in the presence of waves, fitting to this as functions of the radome measurements and other flight characteristics should average any real effects of vertical wind as long as the vertical wind over the flight segments used averages to zero.

Old Radome (before SAANGRIA-TEST, so prior to 2013)

The simplest result is

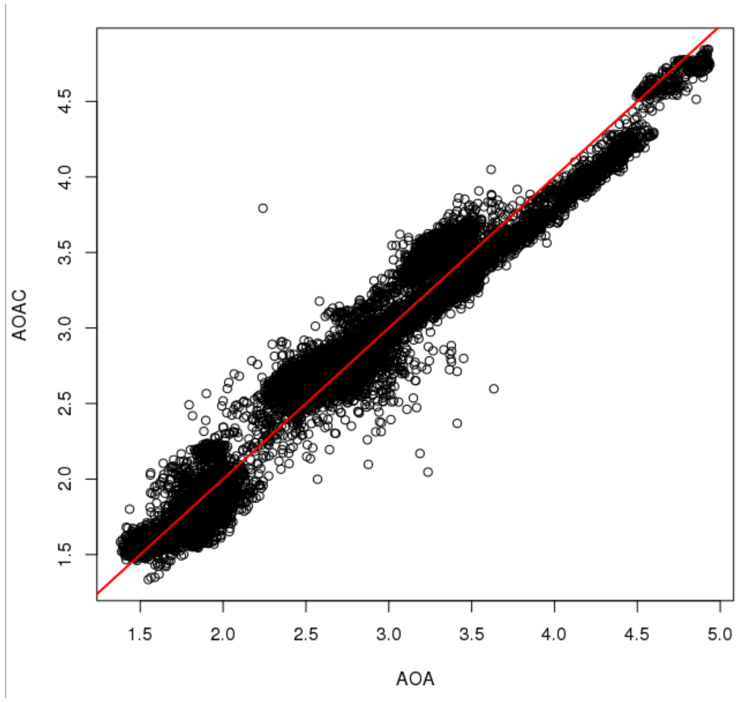
$$\alpha = a_0 + a_1\frac{ADIFR}{QCR} \tag{4}$$

where, for the radome in use prior to 2013, the coefficients (determined by fitting to the PREDICT ferry flight from BJC to St. Croix) are {5.529309, 20.427622}. Note that ADIFR is normally negative and a normal range for ADIFR/QCR is about (-0.1 – 0.0). The code in use for this radome has coefficients (5.44, 21.17}, with an additional adjustment for dependence on Mach number. I didn't find any significant dependence on MACH, MACH**2, or MACH**3.

A slightly improved result over (4) is

$$\alpha = a_0 + a_1\frac{ADIFR}{QCR} + a_2 VSPD \tag{5}$$

with coefficients {5.52103486, 20.39866922, 0.01964274}, as plotted in the following figure:



The standard error for (5) is 0.15°, vs 0.16° for (4).

New Radome (beginning in 2013):

Since 2013, beginning with the SAANGRIA test flights, a new radome was used. The calibration of this is evidently different from the old radome, so it is necessary to determine a new calibration for angle-of-attack. The best data for this appear to be from four flights, with maneuvers as shown in the following table (thanks, Allen S.):

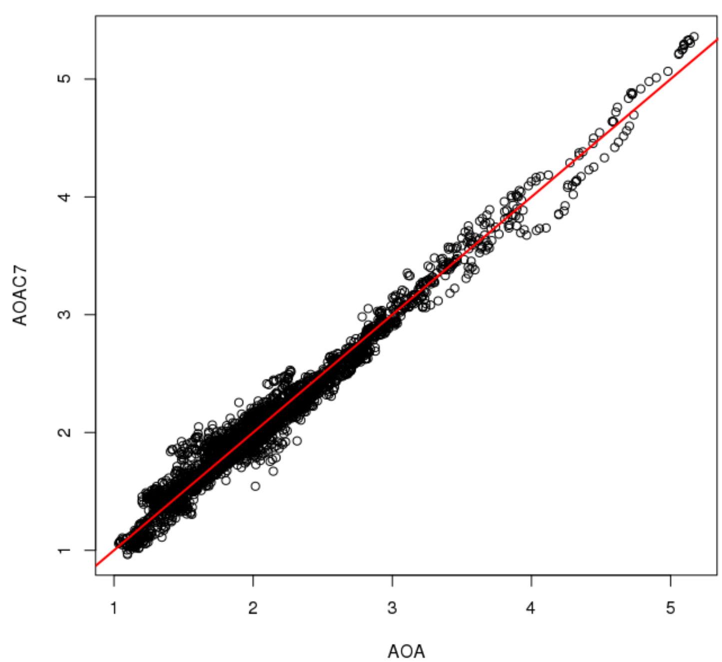
PROJECT	Date	Times	Nature of flight segments
SAANGRIA-TEST	130213 (rf01)	221300–221800	speed run at 25k ft
"	"	224050–224700	speed run, 35k ft
"	"	244500–245000	level segment at 12k ft
SPRITES-II	130806 (rf05)	55100–55500	pitch maneuvers
"	130812 (rf06)	63600–64200	speed run
"	"	82500–83000	"
"	"	83200–83650	"
"	130813 (rf07)	85900–90500	"
"	"	90800–91400	"

The best fits arise from using only the data for these time segments, but it is also possible to use all the data from these flights (excluding cases where the low airspeed indicates that flaps might have been deployed and also excluding cases where there might be questionable data). The latter has the advantage that a large part of the flight envelope is covered, including climbs and descents, but flight transitions in power settings or climb rate might be expected to increase the scatter in the measurements. Therefore both approaches are shown here.

The measurements from all the flight segments in the above table were used with the following fit:[1]

$$\alpha' = c_0 + \frac{ADIFR}{QCF}(c_1 + c_2 M) \tag{6}$$

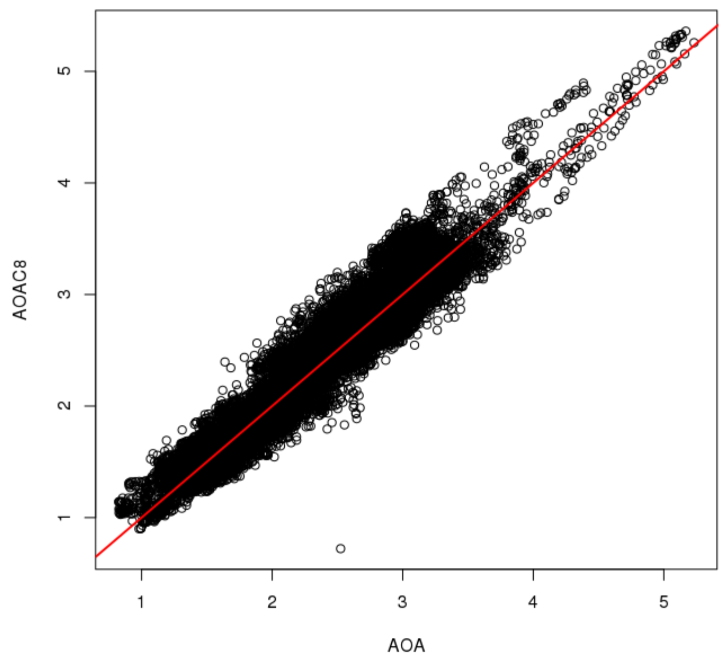
where M is the Mach number. The best-fit coefficients are {4.604, 18.67, 6.49}, and the standard error vs α determined from (2) is 0.11°. Adding additional terms dependent on pressure, true airspeed, density, higher powers of Mach number, QCF, QCR, PSF, VSPD, etc., could only reduce this standard error by less than 10%, so the added complexity and danger of invalid extrapolation seemed reasons to keep the simpler form above. The following figure shows the angle-of-attack calculated from (6), labeled AOAC7, vs that measured (assuming $w = 0$) using (3). The red line in that figure indicates 1:1 prediction. The figure is consistent with the low standard error and indicates that there is no systematic departure from the fit, at least for these data.



If the calculation is repeated for all the data from the four flights in the table above, without restraining the data to the periods of maneuvers, the resulting fit coefficients applicable to (6) are

[1] I have used QCF instead of QCR because QCF is more reliable while QCR more often has problems caused by plugging of the port by ice or water freezing in the lines.

{4.533, 19.787, 3.856} and the resulting standard error is about 0.14°. The same standard error applies to the difference between the angle-of-attack determined from the fit to the maneuvers, and both plots look like the graph below:



Part of the reason that the scatterplot looks broader than the one based on the maneuvers, despite having only 30% higher standard error, is that there are many more points in the second plot so the concentration of points along the red line is harder to detect. Nevertheless, there is enough scatter, especially at high angle of attack, that it is worth exploring if the scatter can be reduced by including additional terms in the fits. As before, more complex fits only reduce the standard error by <10%, but they do reduce the apparent scatter at high angle-of-attack slightly.

Measurements at flight speeds less than 130 m/s have been excluded here to avoid periods of possible flap deployment. If measurements at slow speed and high angle of attack are desired, perhaps for boundary-layer measurements, a different approach may produce better results. One might be to do a running fit to (4) based on exponentially decaying formulas for the coefficients, so that the mean vertical wind will stay near zero. The approach would be as follows:

1. For a fit of the form $y = y_0 + bx$, the best-fit values for the coefficients are:

$$b = \frac{\overline{xy} - \overline{x}\,\overline{y}}{\overline{x^2} - \overline{x}^2} \tag{7}$$

$$y_0 = \overline{y} - b\overline{x} \tag{8}$$

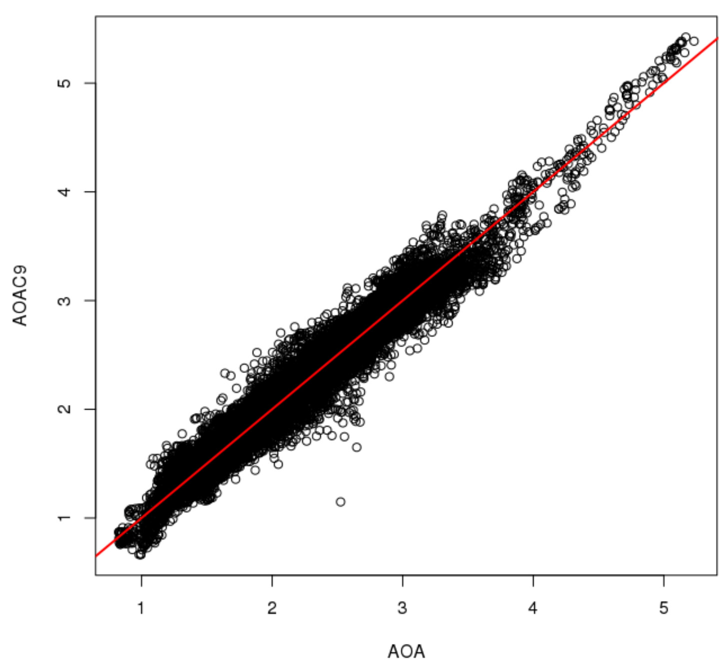
2. Initialize the sums to realistic values to avoid problems when the updating starts. I found that these values worked well: $\overline{x} = -0.0811$, $\overline{y} = 2.705$, $\overline{x^2} = 0.00684$, $\overline{xy} = -0.214$.

3. Exponentially updating values for the mean values, with time constant τ of perhaps 300–900 s, can be accumulated using formulas of the following form, for each of the four mean values needed:

$$\overline{xy}' \leftarrow \overline{xy} + (xy_i - \overline{xy})/\tau \tag{9}$$

4. Use the values of y_0 and b so obtained to calculate angle-of-attack (y) from the ratio $ADIFR/QCF$ (x).

For example, with a time constant of 300 s, the predicted angle of attack vs that expected from (3) is shown in the following figure:



The standard error in the prediction is reduced to about 0.10° with this procedure. This approach therefore would reduce the offset in vertical wind, at the expense of falsely reducing the amplitude of real vertical-wind features that extend over a significant part of the updating time period (here 5 min). It would compensate for problems like disturbance of flow near the radome ports by accumulation of ice or insect debris, but might mask real problems with the measuring system by removing them as part of the updating. Perhaps the best way to use an updated sensitivity like this would be to include another variable, rather than replacing the standard angle-of-attack with this measurement.

The new radome is quite different from the old one, offset by about -1.3 deg and showing different dependences, a Mach number dependence as the dominant effect vs a dependence on vertical speed of the aircraft as the dominant effect with the old radome. It is still possible to find the value of ADIFR/QCR that would have been measured by the old radome, in terms of the measurement from the new radome, by equating the formulas for the two radomes and solving for the old value in terms of the new value, If the simplest formulas are used,

$$\alpha = a_0 + a_1 \left(\frac{ADIFR}{QCF}\right)_{old} = c_0' + \left(\frac{ADIFR}{QCF}\right)_{new} (c_1' + c_2' M) \tag{10}$$

or

$$\left(\frac{ADIFR}{QCF}\right)_{old} = \frac{1}{a_1}\left(c_0' - a_0 + \left(\frac{ADIFR}{QCF}\right)_{new} (c_1' + c_2' M)\right) \tag{11}$$

This would then give the value to use in the old determination of PCORS from LAMS, pending repeat of the calibration with LAMS and the new radome.

Recommendations, New Radome

1. For standard angle-of-attack (AKRD), use (6) with the coefficients listed following that equation. The Mach number used for this should be calculated from QCF and PSF, the measurements before correction for 'PCORS', and ADIFR/QCF should be calculated similarly using the original measurements.

2. A new variable for angle-of-attack can be calculated using formulas (7–9) and would be used for a new vertical-wind variable. The name and identification for this variable need to highlight that it is the result of a running update of the sensitivity coefficients for the radome. This new variable needs further study before it should be implemented for general use, but this approach should provide good stability of the baseline for the vertical wind.

3. We need yaw maneuvers and reverse-heading maneuvers to calibrate the sideslip sensitivity in the same way.

4. When the LAMS-based PCORS are implemented for the new radome, the variable to use in place of ADIFR/QCR in the equation for Δp (see separate memo and paper draft) is the revised value given by (11), with the following coefficients: $\{a_0, a_1\}$={5.5293, 20.4276} and $\{c_0, c_1, c_2\}$={4.533, 19.787, 3.856}. The Mach number M entering (11) should be calculated from the raw measurements QCF and PSF.

QCR

Old Radome (before 2013)

To duplicate QCW, the best estimate of QCF from the PCORS, based on measurements from PREDICT FF01:

1. QCR should be corrected by the same difference needed to correct QCF (because both arise from errors in the static source): QCRP = QCR - PC with PC given by the PCOR formula for the GV. This however doesn't take into account that the pitot-tube value QCF is relatively insensitive to angle of attack while the radome port used for QCR is known to be sensitive to angle of attack.
2. The expression $QCW \sim d_0 + QCRP(d_1 + d_2 QCRP + d_3 \frac{ADIFR}{QCR}$ was then used to find the best-fit coefficients $\{d_i\}$. QCW is QCF-PC or the corrected value for QCF. This correction then takes into account possible dependence of QCR on angle of attack and also possible differences in calibration or in flow-distortion effects between QCR and QCF. This leads to the following new variable based on QCR:

$$QCRP = QCR - PC$$
$$QCRCC = d_0 + QCRP(d_1 + d_2 QCRP + d_3 \frac{ADIFR}{QCR} \tag{12}$$

3. The result was that the mean difference between QCW and QCRCC is 0., with standard deviation 0.25 hPa. This new variable could then serve as a backup QC measurement, with about 0.25 hPa uncertainty.

Best-fit coefficients are {1.052, 9.750e-01, -5.58e-05, -1.817e-01}, all indicated to be significant. However, omitting the quadratic term in QCRP makes little difference in the results (changing the standard deviation from 0.25 to 0.26), so this fit may be preferable: {1.860, 0.96164, 0., -0.1783}.

New Radome (beginning in 2013)

1. In the above equation, determine 'PC' using the substitution (11) – necessary because there are no calibration data with the new radome and LAMS.
2. Then find QCRP using this revised PC.
3. Use QCRP in (12) and use (11) to substitute values measured using the new radome for the values that would have been measured by the old radome.