Calibrations Based On A Laser Air Motion Sensing System

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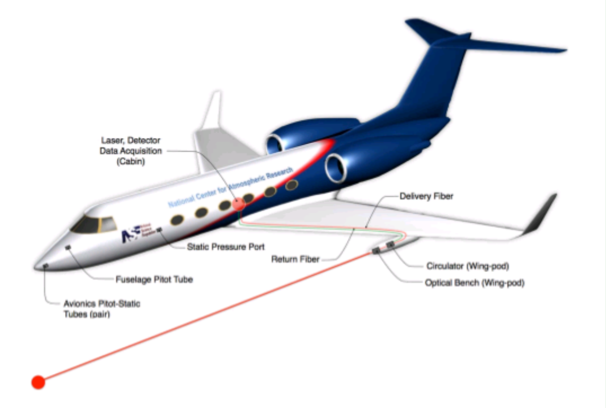
SOME RECENT ALGORITHM CHANGES

Part of a Comprehensive Review of Data Processing

1. Altitude Calculations
   (a) Pressure altitude
   (b) Combining GPS and IRS measurements
2. Wind Processing
   (a) Complementary-Filter Combination, GPS and IRS
   (b) Calibration using a laser air motion sensing system
3. "King" liquid water content
4. Pressure corrections via a laser air motion sensor
5. Changes related to humidity
   (a) The Murphy-Koop equation for equilibrium vapor pressure
   (b) Correcting chilled-mirror hygrometers for the pressure in the housing
   (c) Humidity corrections to airspeed
   (d) Dew point inversion from vapor pressure
6. Studies of equivalent potential temperature (esp.the equation of Davies-Jones, 2009)

THE LASER AIR MOTION SENSING SYSTEM



Spuler, S. M., D. Richter, M. P. Spowart, and K. Rieken, 2011: Optical fiber-based laser remote sensor for airborne measurement of wind velocity and turbulence. Applied Optics, 50, 842-851.

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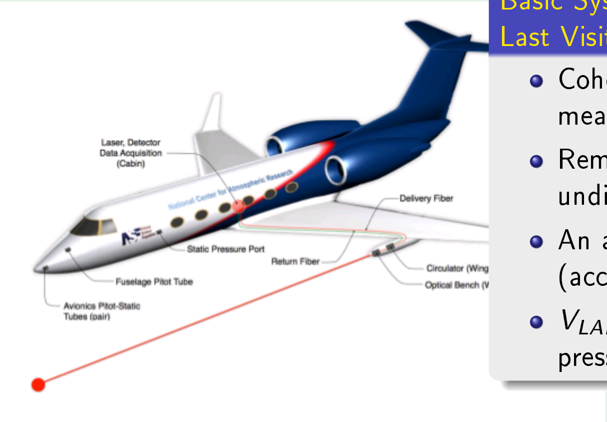

Basic System Described On Last Visit:

- Coherent Doppler measurement
- Remote sensing in undisturbed air
- An absolute measurement (accuracy$\lesssim$0.1 m/s)
- $V_{LAMS} \Rightarrow q$, dynamic pressure

BASIS FOR THE APPROACH

Steps:

1. $p_t = p + q$ is accurate

p_t

- p_t=total pressure, p=static, q =dynamic
- pitot tube insensitive to flow angle (<0.1%)
- orientation removes normal offset
- redundant measurements ⇒consistency
- still, an assumption

BASIS FOR THE APPROACH

Steps:

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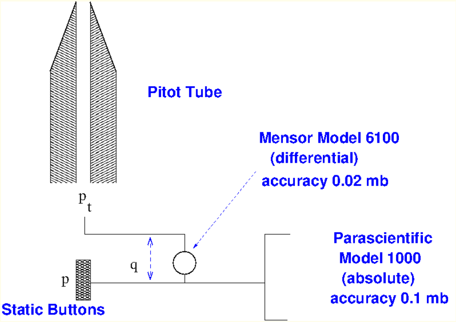
2. Errors in p and q arise from error at static sources



Calibrating q then calibrates p

BASIS FOR THE APPROACH

Steps:

1. $p_t = p + q$ is accurate
2. Errors in p and q arise from error at static sources
3. Find Δq required to match LAMS; hence Δp

What q is required to match LAMS?

- TAS: depends on q, p, T
- relatively insensitive to T (iterate...)
- find Δq s.th. $q_m + \Delta q$, $p_m - \Delta q$, T gives v_{LAMS}

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4. Refinements for accuracy

Adjustments

- correct because p enters the prediction of Δp
- calculate humidity influences on R_a, C_p, C_v, $\gamma = c_p/c_v$
- correct for offset angle of LAMS
- recalculate T using corrected pressures

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5. Δp is a function of measured quantities like p_m, q_m, α_m

Fit Correction to p and q given by LAMS:

- use second-by-second prediction of Δq from LAMS
- try fits like $(\Delta p/p) = a_0 + a_1 q + a_2 (ADIFR/QCR)$
- find $\Delta p(measurements)$

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6. Flight maneuvers: checks and to calibrate T

Maneuvers for testing results

- reverse-heading maneuvers
- climbs and descents to calibrate temperature via integration of the hydrostatic equation

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7. Use LAMS with the above results to measure T directly.

LAMS-based measurement of temperature

- LAMS provides TAS=v
- p and q determine Mach number $M = v/v_s$
- $v_s = \sqrt{\gamma R_a T}$ so measured M and $v \Rightarrow T$ without reference to a temperature sensor

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1. Calibration of dynamic pressure, hence true airspeed, hence longitudinal component of wind

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3. Calibration of temperature via accurate measurements of pressure + GPS

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Results:

1. Calibration of dynamic pressure, hence true airspeed, hence longitudinal component of wind
2. Calibration of pressure
3. Calibration of temperature via accurate measurements of pressure + GPS
4. Provide new independent temperature measurement that *should* work in cloud

THE MEASUREMENT OF TOTAL PRESSURE, C-130

Basic Measurements, C-130

static pressure PSFD, PSFRD

- absolute sensors
- $\pm$0.1 mb (Parascientific Model 1000)

dynamic pressure QCF, QCRF

- differential sensors, total vs static
- $\pm$0.05 mb

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pitot tube performance

- typical sensitivity <1% at angles up to 10°, 0.2% up to 5°.
- mean orientation chosen along expected flow direction, not centerline

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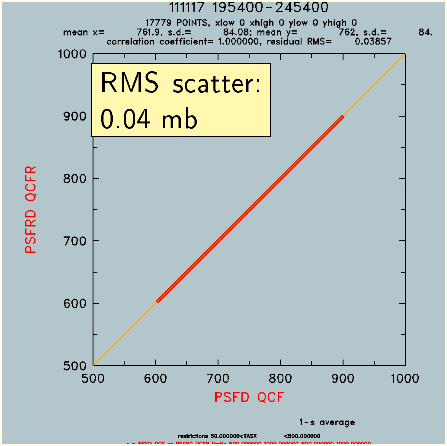
Skip Equations

$$\mathrm{PSFRD+QCFR}=a_1(\mathrm{PSFD+QCF})+a_2$$
$$a_1=1.0000,\ a_2=0.10\ \mathrm{mb}$$



THE EQUATIONS

Energy conservation in compressible flow: $\frac{v^2}{2} + c_p T = \text{constant}$
For adiabatic compression to stagnation $(v = 0)$,

$$M^2 = \frac{v^2}{\gamma R_a T} = \left\{ \left(\frac{2c_v}{R_d} \right) \left[\left(\frac{p+q}{p} \right)^{R_a/c_p} - 1 \right] \right\}$$

$$q = p \left\{ \left(\frac{v^2}{2c_p T} + 1 \right)^{c_p/R_a} - 1 \right\} = p\chi$$

where the last equality defines $\chi(v, T)$. Write in terms of measured quantities $p_m = p - \Delta p$ and $q_m = q - \Delta q$ and unknown $\Delta p = -\Delta q$:

$$\boxed{\Delta p = \frac{q_m - p_m \chi}{1 + \chi}}$$

HOW TO ADDRESS NEED FOR TEMPERATURE

Need Temperature:

Have v from LAMS

$$\chi(v,T)=\left\{\left(\frac{v^2}{2c_pT}+1\right)^{c_p/R_a}-1\right\}$$

- Not very sensitive: Fractional error in T is small
- Use available-processed T as the first approximation
- Then, iterate both in calculation and in calibration

Determining "PCORR" Function

$$\boxed{\Delta p=\frac{q_m-p_m\chi}{1+\chi}}$$

- Airspeed from LAMS gives second-by-second estimates of Δp
- Can fit those values to get Δp as function of other measurements

REFINEMENTS FOR ACCURACY

Moist Air Corrections

- humidity matters at this level of precision
- Use R_a, c_p, γ adjusted for humidity

Pointing Angle Corrections

- $v_l = v\cos(\theta)$ where v_l is the speed measured by LAMS
- $\cos\theta \simeq \cos(\theta_1+\alpha)\cos(\theta_2-\beta)$
  - θ_1 is the pointing angle above the longitudinal axis=0.1°
  - θ_2 is the pointing angle to starboard of the longitudinal axis=-0.2°
  - if $\alpha=-4^\circ$,$\cos\theta \simeq 0.9976$ and at 130 m/s $\delta v = 0.3$ m/s.
- Therefore, use $v = v_l/\cos(\theta)$ in the preceding equation

FITS TO Δp

Candidate Fit Parameters

1. p: Effects on Δp probably scale with pressure, so fitting functions of the form $\frac{\Delta p}{p} = f(...)$ seems appropriate and provided improved fits
2. q or q/p
3. α or ADIFR/QCR - no significant dependence found
4. β or BDIFR/QCR - no significant dependence found, either β or $|\beta|$
5. M

PREFERRED FIT, C-130

Best Option?

$$\frac{\Delta p}{\mathrm{PSFD}} = a_0 + a_1 \frac{\mathrm{ADIFR}}{\mathrm{QCR}} + a_2 \frac{\mathrm{QCF}}{\mathrm{PSFD}}$$

Fit procedure:

1. Determine a_1 by fit to pitch maneuvers
2. Keep a_1 constant and fit for a_0 and a_2 using data from all times when LAMS provides a measurement of v.
3. Obtain coefficients by separate fits for {PSFD, QCF} and {PSFRD, QCFR}

RMS error vs LAMS measurements: 0.3 mb, 1-Hz measurements.

- (Different sample volumes)
- Mean correction: uncertainty $<$ 0.01 mb ($>$10,000 measurements)

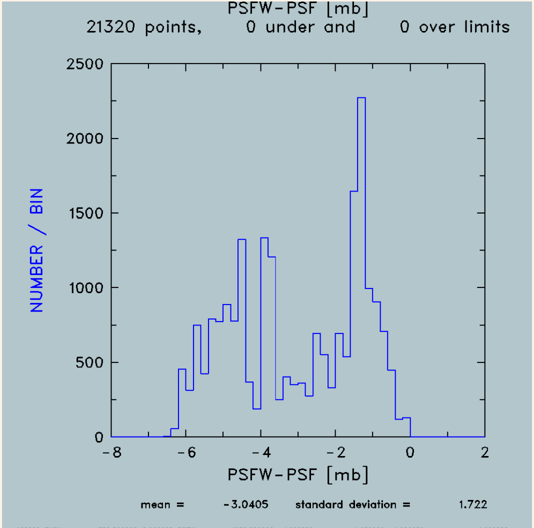
FIT FOR GV DATA

Representation in terms of Mach number:

$$\frac{\Delta p}{p} = a_0 + a_1 \frac{q}{p} + a_2 M + a_3 M^2 + a_4 M^3 \tag{1}$$

- no significant dependence on angle of attack, sideslip, or abs(sideslip)
- all three terms in M were significant
- resulting RMS of fit to LAMS values: corresponds to RMS in TAS of about 0.3 m/s (but some contribution arises from resolution of LAMS measurement and different locations sensed by LAMS vs the pitot tube)

MAGNITUDE OF THE CORRECTION



mean difference, corrected-measured: -3.0$\pm$1.7 mb

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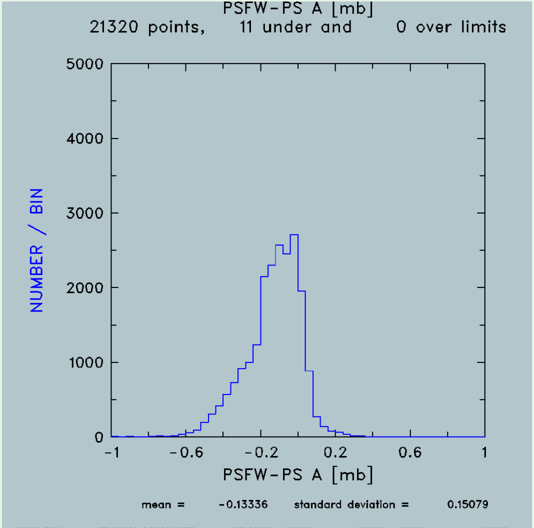


mean difference, corrected-avionics: -0.13±0.15 mb

RVSM requirement: ca. 25 m or typically 1 mb

REVERSE-HEADING FLIGHT LEGS

GV Flight Legs from PREDICT:

longitudinal wind: Should reverse sign for reverse-track leg.
For $\theta =$heading,

$$\mathrm{V}_{\mathrm{Long}} = V_{GPS}\cos(\theta_G - \theta) - V_{LAMS}$$

- 18 reverse-heading legs used, at various altitudes
- result: $\Delta = +0.4 \pm 0.2$m m/s, indicating a measurement +0.2 m/s too high
- However, temperature enters this and the evaluation of the values of Δp and Δq, so this should be re-visited after the temperature calibration is considered.

CALIBRATING TEMPERATURE USING LAMS

Integrating the Hydrostatic Equation

- Assume that the absolute pressure is accurate as calibrated
- GPS provides acurate measurement of geometric altitude
- Consider Integration of the hydrostatic equation between two pressure levels:
  - Predicted altitude change depends on the "mean" temperature.
  - Comparison to a similar "mean" of the measurements checks the accuracy of the temperature measurement.

Skip Equations

EQUATIONS USED

Three Sums Are Needed: S_1, S_2, S_3:

$$\delta p_i = -\frac{g\, p_i}{R_a\, T_{a,i}}\delta z_i \quad (\mathrm{too\,noisy,\,second-by-second})$$

$$S_1 = \sum_i \frac{R_{a,i}}{g_i} \ln\left(\frac{p_i}{p_{i-1}}\right)$$

$$S_2 = \sum_i (z_i - z_{i-1})$$

$$S_3 = \sum_i \frac{z_i - z_{i-1}}{T_{m,i}}$$

Then compare prediction (T_p) to observed (T_m)

$T_p = -S_1/S_2$ and $\overline{T}_m = S_2/S_3$, weighted appropriately

RESULTS FOR MEAN TEMPERATURES BETWEEN LAYERS

Flight Segment	T_p	$\overline{T}_m$	ΔT
RF05, 205800–211100	-10.98	-10.37	-0.5
RF07, 212510–213300	-6.36	-5.89	-0.47
RF07, 212510–212900	2.27	2.42	-0.15
RF07, 212900–213300	-12.85	-12.15	-0.70
RF08, 214500–215300	-0.9	-0.5	-0.4
RF08, 233700–234130	-6.5	-6.3	-0.4
RF08, 234500–235000	-9.4	-8.8	-0.6
RF08, 235600–240100	-9.5	-8.4	-1.1
mean offset[a], $T_p - \overline{T}_m$			-0.55

[a] excluding the first listed value for RF07 because the next two break this climb segment into two segments

APPROACH TO TEMPERATURE CALIBRATION/CHECK:

"HIPPO" Dataset: Numerous climbs and descents, "pole-to-pole"

- Assume a polynomial correction to T_t expressed in Celsius::

$$T_t = f(T_m) = T_m + a_0 + a_1 T_m + a_2 T_m^2$$
$$\chi^2 = (\Delta h - \Delta z_{GPS})^2$$

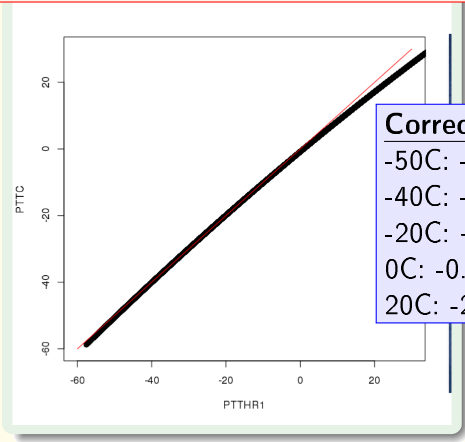
  where Δh is the height difference predicted from the hydrostatic equation:

$$\Delta h = -\frac{R_a}{g} \ln \frac{p_i}{p_{i-1}} (f(T_m) + T_0)$$

- Find the coefficients that minimize this χ^2 over the 25 flights and >300 climbs or descents from HIPPO circuits 4 and 5.

RESULT FOR TEMPERATURE:

$$T_t = T_m - 0.817 - 0.075\,T_m - 0.00141\,T_m^2$$



Corrections:

-50C: -0.60C

-40C: -0.08C

-20C: +0.12C

0C: -0.82C

20C: -2.88C

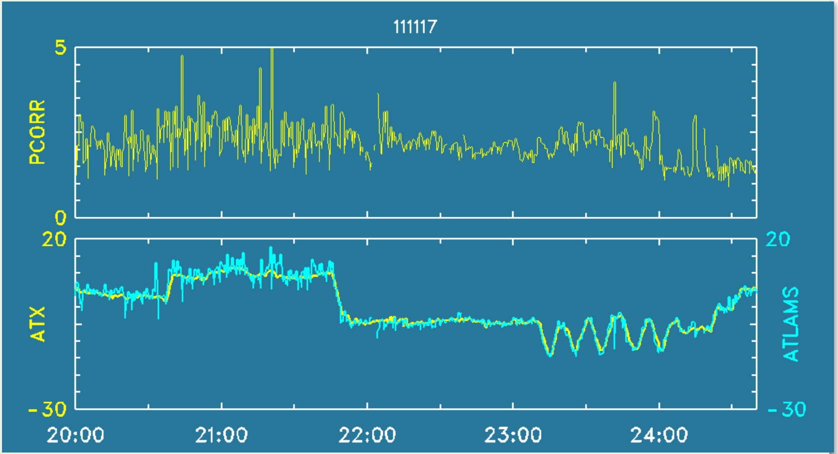
DETERMINING THE MACH NUMBER

$$M^2 = \left\{ \left(\frac{2c_v}{R_a} \right) \left[\left(\frac{p+q}{\mathrm{p}} \right)^{R_a/c_p} - 1 \right] \right\}$$

- LAMS provides a correction Δp to be added to p and subtracted from q (affecting only the denominator).
- Measured temperature is not needed (except indirectly as it enters fitting to find Δp).
- Once calibrated, the above equation for M^2 can be used to find the temperature, independent of a temperature probe, using only pressure measurements and v determined by LAMS:

$$T_{LAMS} = v^2/(\gamma R_a M^2)$$

AN EXAMPLE OF T FROM LAMS



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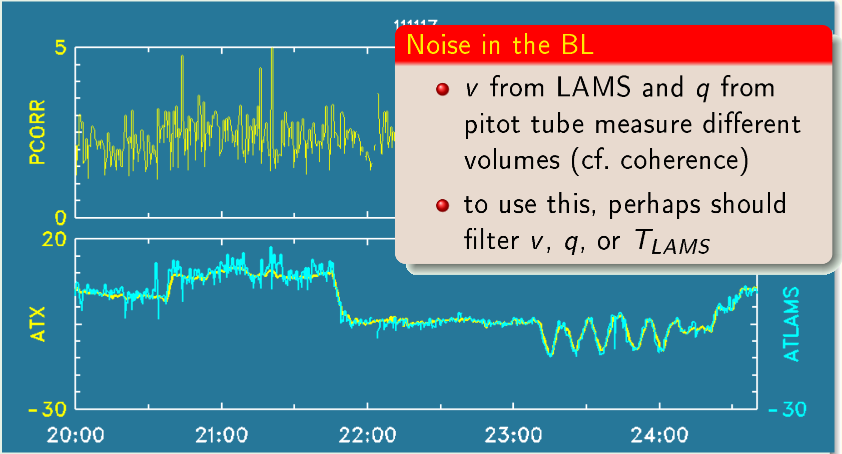

Noise in the BL

- v from LAMS and q from pitot tube measure different volumes (cf. coherence)
- to use this, perhaps should filter v, q, or T_{LAMS}

CONCLUSIONS (GV)

Key Results:

1. LAMS provides a direct calibration of airspeed.
   - → Longitudinal wind after correction:accuracy <0.2 m/s
2. This calibrates dynamic pressure:
   - → q should be increased by about 3.0 mb on average.
3. If total pressure is accurately measured, this calibrates pressure:
   - → p should be decreased by about 3.0 mb on average.
4. Accurate pressure supports integration of the hydrostatic equation to calibrate temperature:
   - → Results indicate that temperature is accurate to within about 1°C for T<0°C.
5. It is possible to obtain a new temperature measurement solely from p, q, and v_{LAMS}