

Homework 7: Undecidability

CSE 30151 Spring 2017

Due 2017/04/20 at 11:55pm

Instructions

- Create a PDF file (or files) containing your solutions.
- Please name your PDF file(s) as follows:
 - If you're making a complete submission, please name your PDF file `netid-hw7.pdf`, where `netid` is replaced with your NetID.
 - If you're submitting some problems now and want to submit other problems later, name your PDF file `netid-hw7-123.pdf`, where `123` is replaced with the problems you are submitting at this time.
- Submit your PDF file in Sakai. Don't forget to click Submit!

Problems

In all of the following problems, any Turing machines that you write can be written as high-level descriptions.

1. Bounds checking

- (a) [Problem 5.14] Prove that it is undecidable whether a Turing machine M , on input w , ever attempts to move its head past the left end of the tape.
- (b) Prove that it is decidable whether a Turing machine M , on input w , ever attempts to move its head past the right end of the input string w .

2. **The Power of 10** is a set of rules for writing mission-critical code developed at JPL.¹ Among other things, the rules have the following properties:

- If a TM complies with the rules, then it is a decider, that is, it always halts on every input.²
- It is decidable whether a TM complies with the rules.

In this problem, we'll show that any such set of rules will be incomplete in the sense that there is a language L_2 that is decidable, yet no TM that decides L_2 complies with the rules.

Consider the language

$$L_2 = \{\langle M \rangle \mid M \text{ is a TM that complies with the rules and rejects } \langle M \rangle\}.$$

- (a) Prove that L_2 is decidable.
- (b) Prove that any TM that decides L_2 must not comply with the rules.

3. **A preview of Mire.** In this problem, we'll prove that it is undecidable, given two strings u and v and a finite transducer T , whether u can be transformed into v by applying T zero or more times.

For example, if T is the transducer defined by the regular transduction expression $(0|1)^*(: 0)$, which appends a 0 to a string, then we can get from $x = 101$ to $y = 101000$ by three applications of T . But we can't get from $x = 101$ to $y = 10101$.

- (a) Given a TM M , show how to construct a finite transducer T (or a regular transduction expression is fine too) that simulates one step of M . More formally, we want T to transform uqv into $u'q'v'$ if and only if M 's configuration uqv yields configuration $u'q'v'$ in one step.³
- (b) Show how to convert any TM M into an equivalent TM M' that has a single accepting configuration, namely q_{accept} , the configuration with the head at the left end of a blank tape. (This is not difficult, but is needed for the next part.)
- (c) Prove that it is undecidable, given strings x and y and a finite transducer T , whether x can be transformed into y by applying T zero or more times.

¹<http://bit.ly/powof10>

²The rules also allow for a TM that never halts on any input; this is the desired behavior for, e.g., a daemon. But let's ignore this case.

³Please see Sipser, pages 168–169, for an explanation of configurations and the *yields* relation.