

Homework 3: Regular expressions and non-regular languages

CSE 30151 Spring 2017

Due Friday, 2017/02/24 at 11:55pm

Instructions

- Create a PDF file (or files) containing your solutions.
- Please name your PDF file(s) as follows:
 - If you're making a complete submission, please name your PDF file `netid-hw3.pdf`, where `netid` is replaced with your NetID.
 - If you're submitting some problems now and want to submit other problems later, name your PDF file `netid-hw3-123.pdf`, where `123` is replaced with the problems you are submitting at this time.
- Submit your PDF file in Sakai. Don't forget to click Submit!

Problems

Each problem is worth 10 points.

1. **Regular expressions vs. Unix regular expressions.** Regular expressions and Unix regular expressions have some superficial differences, but also some deeper ones that affect the class of languages recognized.
 - (a) Unix regular expressions do not have \emptyset . Prove that a regular expression without \emptyset cannot describe the empty language, and conversely, if L is a nonempty regular language, it can be described by a regular expression without \emptyset .
 - (b) Unix regular expressions have *backreferences*.¹ Give an example of a Unix regular expression² that uses backreferences to describe a nonregular language, and prove that this language is not regular. You may

¹<http://www.regular-expressions.info/backref.html>

²Let's define this to mean POSIX Basic Regular Expressions, but we aren't picky about this.
<http://www.regular-expressions.info/posix.html>

not simply cite one of the non-regularity results proved in the book's Examples 1.73–77; please write out a full proof.

2. Binary addition.

- (a) [Problem 1.32] Let

$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\},$$

that is, an alphabet of eight symbols, each of which is a 3-tuple of binary digits. Thus, a string over Σ_3 gives three rows of binary digits. Show that the following is regular:

$$B = \{w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top two rows}\}.$$

Hint: Since it's easier to think about addition from right to left, design an automaton for B^R first, then convert it into an automaton for B .

- (b) [Problem 1.53] Let $\Sigma = \{0, 1, +, =\}$, and prove that the following is not regular:

$$ADD = \{x=y+z \mid x, y, z \text{ are binary natural numbers, and } x = y + z \text{ is true}\}.$$

3. Two similar but different languages [Problem 1.49].

- (a) Let $B = \{1^k y \mid y \in \{0, 1\}^* \text{ and } y \text{ contains at least } k \text{ 1s, for } k \geq 1\}$. Show that B is a regular language. Hint: Try out some strings to see what does and doesn't belong to B , in order to find another simpler way of thinking about B .
- (b) Let $C = \{1^k y \mid y \in \{0, 1\}^* \text{ and } y \text{ contains at most } k \text{ 1s, for } k \geq 1\}$. Prove that C is not a regular language.