③ a) letrec fun = λl . listcase l of (

        nil,

        $\lambda h . \lambda t$. listcase t of (

           $h :: nil$,

           $\lambda h' . \lambda t'$. $h :: fun \left((h + h') :: t' \right)$

)

b) letrec $l \equiv \lambda n . \lambda step . n :: l \, (n + step)$

      fun $\equiv \dots$

in fun $(l \, 1 \, 4)$

Questão 4

① $\langle exp \rangle ::= empty \mid node \ \langle exp \rangle \ \langle exp \rangle$
$\mid rosecase \ \langle exp \rangle \ of \ (\langle exp \rangle, \langle exp \rangle)$
$\mid \ldots$

$\langle type \rangle ::= RTree \ \langle type \rangle \mid \ldots$

$$\frac{}{T \vdash empty : RTree \ \theta} \qquad \frac{T \vdash e_1 : \theta \quad T \vdash e_2 : [RTree \ \theta]}{T \vdash node \ e_1 \ e_2 : RTree \ \theta}$$

$$\frac{T \vdash e : RTree \ \theta \quad T \vdash e_1 : \theta' \quad T \vdash e_2 : \theta \to [RTree \ \theta] \to \theta'}{T \vdash rosecase \ e \ of \ (e_1, e_2) \vdash \theta'}$$

② $\langle cfm \rangle ::= empty \mid node \ \langle cfm \rangle \ \langle cfm \rangle \mid \ldots$

$$\left(\frac{}{empty \Rightarrow empty} \right) \qquad \frac{e \Rightarrow z \quad e' \Rightarrow z'}{node \ e \ e' \Rightarrow node \ z \ z'} \qquad \frac{e \Rightarrow empty \quad e_1 \Rightarrow \delta}{rosecase \ e \ of \ (e_1, e_2) \Rightarrow z}$$

$$\frac{e \Rightarrow node \ z \ z' \quad e_2 \ z \ z' \Rightarrow z''}{rosecase \ e \ of \ (e_1, e_2) \Rightarrow z''}$$

③ $letrec \ sumRT \equiv \lambda rt. \ rosecase \ rt \ of$
$\qquad (0,$
$\qquad\qquad \lambda x. \lambda lrt. \ x + sum \ (map \ sumRT \ lrt))$
$\quad map \equiv \lambda f. \lambda l. \ listcase \ l \ of$
$\qquad\qquad (nil, \ \lambda h.\lambda t. \ f \ h :: map \ f \ t)$
$\quad sum \equiv \lambda l. \ listcase \ l \ of$
$\qquad\qquad (0, \ \lambda h.\lambda t. \ h + sum \ t)$
$in \ \ldots$

(4) $\quad RTree\ \theta \doteq 1 + \theta \times dist\ (RTree\ \theta)$

$empty \doteq @1\langle\rangle$

$node \doteq \lambda x.\lambda y.\ @2\ \langle x,y\rangle$

$rstcase\ e\ of\ (e_1, e_2) \doteq sumcase\ e\ of\ (\lambda\langle\rangle.\ e_1,\ \lambda\langle x,y\rangle.\ e_2\ x\ y)$