1º teste SLP
2021/22

Universidade do Minho

(a) ______________________ Unidade Curricular ______________________

Curso ______________________ Docente ______________________

ALUNO (b) ______________________ em __/__/____

QUESTÃO 1

$P \equiv \{x:=5\ ;\ z:=x*2\};\ \text{if } x \leq y \text{ then } \{x:=x+y\ ;\ z:=y+x\} \text{ else } x=0$

$s\,y = 10$

1

$$
\begin{aligned}
\langle P, s\rangle &\overset{1}{\Rightarrow} \langle z:=x*2\ ;\ \text{if}\ldots,\ s[x\to 5]\rangle \\
&\overset{2}{\Rightarrow} \langle \text{if}\ldots,\ s[x\to 5, z\to 10]\rangle \\
&\overset{3}{\Rightarrow} \langle x:=x+y\ ;\ z:=y+x,\ s[x\to 5, z\to 10]\rangle \\
&\Rightarrow \langle z:=y+x,\ s[x\to 15, z\to 10]\rangle \\
&\Rightarrow s[x\to 15, z\to 25]
\end{aligned}
$$

$$
\dfrac{\dfrac{}{\langle x:=5, s\rangle \Rightarrow s[x\to 5]}\ (ass_{sos})}{\langle x:=5;\ z:=x*2, s\rangle \Rightarrow \langle z:=x*2, s[x\to 5]\rangle}\ (comp^{2}_{sos})\ (comp^{1}_{sos})
$$

$$\ldots \overset{1}{\Rightarrow} \ldots$$

$$
\dfrac{\dfrac{}{\langle z:=x*2, s[x\to 5]\rangle \Rightarrow s[x\to 5, z\to 10]}\ (ass)}{}\ (comp^{2})
$$

$$\ldots \overset{2}{\Rightarrow} \ldots$$

$$
\dfrac{}{\ldots \overset{3}{\Rightarrow} \ldots}\ (if^{tt}_{sos}) \qquad \text{pq. } [\![x \leq y]\!]\, s[x\to 5, z\to 10] = 5 \leq 10 = tt
$$

2 $\langle P, s_0\rangle \to s_0[x\to 0, z\to 10]$ $(comp_{ns})$

1. $\langle x:=5;\ z:=x*2, s_0\rangle \to s_0[x\to 5, z\to 10]$ $(comp_{ns})$
   1. $\langle x:=5, s_0\rangle \to s_0[x\to 5]$ (ass_{ns})
   2. $\langle z:=x*2, s_0[x\to 5]\rangle \to s_0[x\to 5, z\to 10]$ (ass_{ns})
2. $\langle \text{if } x \leq y \text{ the} \ldots \text{ else } x:=0,\ s_0[x\to 5, z\to 10]\rangle \to s_0[x\to 0, z\to 10]$ (if^{ff}_{ns})
   1. $\langle x:=0, s_0[x\to 5, z\to 10]\rangle \to s_0[x\to 0, z\to 10]$

(3) $\{y<5\}\ P\ \{x=0\}$ (comp)

1. $\{y<5\}\ x:=5;\ z:=x*2\ \{y<5 \land x=5\}$ (comp)
    1. $\{y<5\}\ x:=5\ \{y<5 \land x=5\}$ pq. $y<5 \Rightarrow y<5 \land 5=5$ (cons)
        1. $\{y<5 \land 5=5\}\ x:=5\ \{y<5 \land x=5\}$ (ass)
    2. $\{y<5 \land x=5\}\ z:=x*2\ \{y<5 \land x=5\}$ (ass)

2. $\{y<5 \land x=5\}$ if $x \leq y$ then ... else $x:=0\ \{x=0\}$ (if)
    1. $\{y<5 \land x=5 \land \neg(x \leq y)\}\ x:=0\ \{x=0\}$ (cons)
        pq. $y<5 \land x=5 \land x>y \rightarrow 0=0$ pois $0=0$ é True
        1. $\{0=0\}\ x:=0\ \{x=0\}$ (ass)

    2. $\{y<5 \land x=5 \land x \leq y\}\ x:=x+y;\ z:=y+x\ \{x=0\}$ (comp)
        1. $\{y<5 \land x=5 \land x \leq y\}\ x:=x+y\ \{x=0\}$ (cons)
            pq. $y<5 \land x=5 \land x \leq 5 \rightarrow x+y=0$ pois Falso $\rightarrow x+y=0$
        2. $\{x=0\}\ z:=y+x\ \{x=0\}$ (ass)

(4)

$$S_{ds}[\![P]\!]s = S_{ds}[\![\text{if} \dots]\!]\left(S_{ds}[\![x:=5;\ z:=x*2]\!]s\right)$$

$$= \text{Cond}\left([\![x \leq y]\!],\ S_{ds}[\![z:=x+y]\!] \circ S_{ds}[\![x:=x+y]\!],\ S_{ds}[\![x:=0]\!]\right)\left(S_{ds}(z:=x*2)\, s[x \to 5]\right)$$

$$= \text{Cond}(\ \dots,\ \dots,\ \dots\ )\, s[x \to 5, z \to 10]$$

$$= \begin{cases} \left([\![z:=x+y]\!] \circ [\![x=x+y]\!]\right) s[x \to 5, z \to 10] & \text{se } [\![x \leq y]\!]s[x \to 5, z \to 10] = tt \\ [\![x:=0]\!]s[x \to 5, z \to 10] & \text{se } [\![x \leq y]\!]s[x \to 5, z \to 10] = ff \end{cases}$$

$$= \begin{cases} [\![z:=x+y]\!]\, s[x \to 5+sy, z \to 10] & \text{se } 5 \leq sy \\ s[x \to 0, z \to 10] & \text{se } 5 > sy \end{cases}$$

$$= \begin{cases} s[x \to 5+sy, z \to 5+2*sy] & \text{se } 5 \leq sy \\ s[x \to 0, z \to 10] & \text{se } 5 > sy \end{cases}$$

QUESTÃO 2

①

$$[\text{if}^1_{ns}] \quad \frac{\langle c_1, s\rangle \to s'}{\langle \text{if } b_1 \text{ then } c_1 \text{ elseif } b_2 \text{ then } c_2 \text{ else } c_3, s\rangle \to s'} \quad \text{se } [\![b_1]\!]_s = tt$$

$$[\text{if}^2_{ns}] \quad \frac{\langle c_2, s\rangle \to s'}{\langle \text{if } b_1 \text{ then } c_1 \text{ elseif } b_2 \text{ then } c_2 \text{ else } c_3, s\rangle \to s'} \quad \text{se } [\![b_1]\!]_s = ff \text{ e } [\![b_2]\!]_s = tt$$

$$[\text{if}^3_{ns}] \quad \frac{\langle c_3, s\rangle \to s'}{\langle \text{if } \dots, s\rangle \to s'} \quad \text{se } [\![b_1]\!]_s = ff \text{ e } [\![b_2]\!]_s = ff$$

②

$$\frac{\{\phi \wedge b_1\}\ \overset{①}{c_1}\ \{\psi\} \qquad \{\phi \wedge \neg b_1 \wedge b_2\}\ \overset{②}{c_2}\ \{\psi\} \qquad \{\phi \wedge \neg b_1 \wedge \neg b_2\}\ \overset{③}{c_3}\ \{\psi\}}{\{\phi\}\ \underbrace{\text{if } b_1 \text{ then } c_1 \text{ elseif } b_2 \text{ then } c_2 \text{ else } c_3}_{P}\ \{\psi\}}$$

se $[\![\phi]\!]_s = tt$ e $\langle P, s\rangle \to s'$ então pode acontecer um dos seguintes casos

1. $[\![b_1]\!]_s = tt$ e $\langle c_1, s\rangle \to s'$ então temos $[\![\phi \wedge b_1]\!]_s = tt$, logo por $\models ①$, $[\![\psi]\!]_s = tt$
2. $[\![b_1]\!]_s = ff$ e $[\![b_2]\!]_s = tt$ e $\langle c_2, s\rangle \to s'$ então $[\![\phi \wedge \neg b_1 \wedge b_2]\!]_s = tt$ $\models ②$, $[\![\psi]\!]_s = tt$
3. $[\![b_1]\!]_s = ff$ e $[\![b_2]\!]_s = ff$ e $\langle c_3, s\rangle \to s'$ então $[\![\phi \wedge \neg b_1 \wedge \neg b_2]\!]_s = tt$, $\models ③$, $[\![\psi]\!]_s = tt$

Portanto $\models \{\phi\}\ \text{if } b_1 \text{ then } c_1 \text{ elseif } b_2 \text{ then } c_2 \text{ else } c_3\ \{\psi\}$

③

$$\mathcal{CS}[\![\text{if } b_1 \text{ then } c_1 \text{ elseif } b_2 \text{ then } c_2 \text{ else } c_3]\!] =$$
$$\mathcal{CB}[\![b_1]\!] : \text{BRANCH}\left(\mathcal{CS}[\![c_1]\!], \mathcal{CB}[\![b_2]\!] : \text{BRANCH}(\mathcal{CS}[\![c_2]\!], \mathcal{CS}[\![c_3]\!])\right)$$

QUESTÃO 3

① Asgn(skip) = $\emptyset$
Asgn(x:=a) = {x}
Asgn(C1;C2) = Asgn(C1) ∪ Asgn(C2)
Asgn(if b then C1 else C2) = Asgn(C1) ∪ Asgn(C2)
Asgn(while b do C) = Asgn(C)

② se <C, s> → s' e x∈Asgn(C) então sx = s'x ?

Prova por indução na derivação de <C,s> → s'. caso a última regra da derivação seja

[skip] <skip, s> → s   logo sx = sx ✓

[ass] <y:=a, s> → s[y→⟦a⟧s] e Asgn(y:=a) = {y}

se x ≠ y temos sx = s[y→⟦a⟧s] x ✓

[comp] se <C1;C2, s> → s' ① e x ∉ Asgn(C1;C2) ②

De ① temos <C1, s> → s'' ③ e <C2, s''> → s' ④ para algum s''

De ② temos x ∉ Asgn(C1) ⑤ e x ∉ Asgn(C2) ⑥

De 3+5, por HI, temos sx = s''x
De 4+5, por HI, temos s''x = s'x
logo sx = s'x ✓