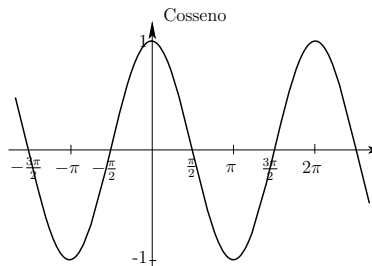
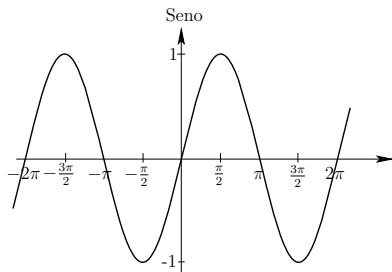


Funções Especiais

Maria Joana Torres

2022/23

Gráficos das funções trigonométricas



Tangente

$$\operatorname{tg} : \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi : k \in \mathbb{Z} \right\} \longrightarrow \mathbb{R} \quad \text{tal que} \quad \operatorname{tg} x = \frac{\operatorname{sen} x}{\operatorname{cos} x}$$

Cotangente

$$\operatorname{cotg} : \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\} \longrightarrow \mathbb{R} \quad \text{tal que} \quad \operatorname{cotg} x = \frac{\operatorname{cos} x}{\operatorname{sen} x}$$

Secante

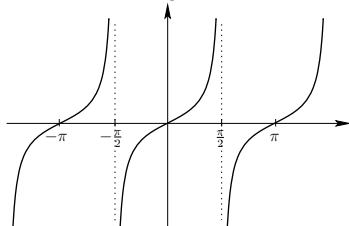
$$\operatorname{sec} : \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi : k \in \mathbb{Z} \right\} \longrightarrow \mathbb{R} \quad \text{tal que} \quad \operatorname{sec} x = \frac{1}{\operatorname{cos} x}$$

Cossecante

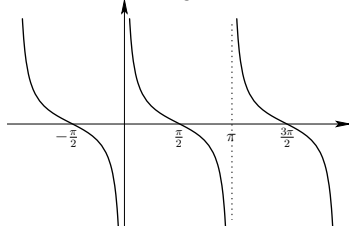
$$\operatorname{cosec} : \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\} \longrightarrow \mathbb{R} \quad \text{tal que} \quad \operatorname{cosec} x = \frac{1}{\operatorname{sen} x}$$

Gráficos das funções trigonométricas

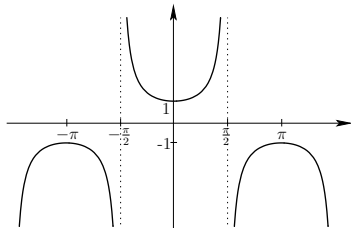
Tangente



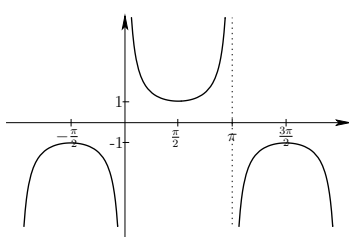
Cotangente



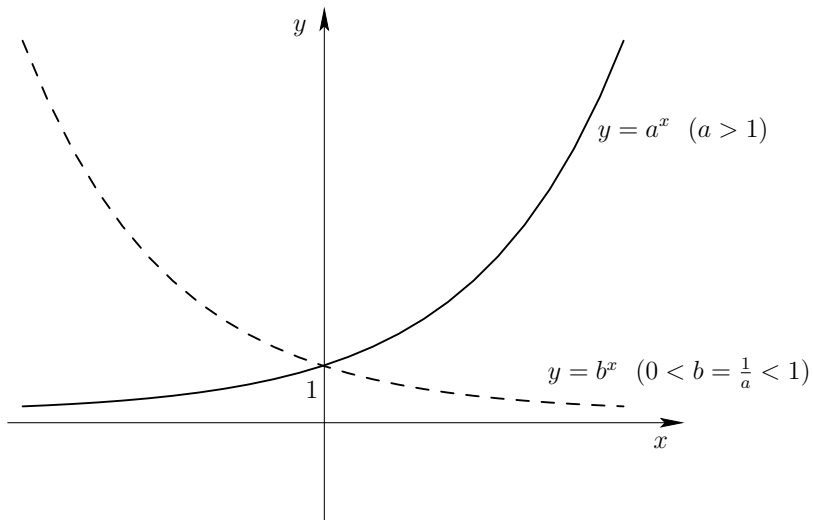
Secante

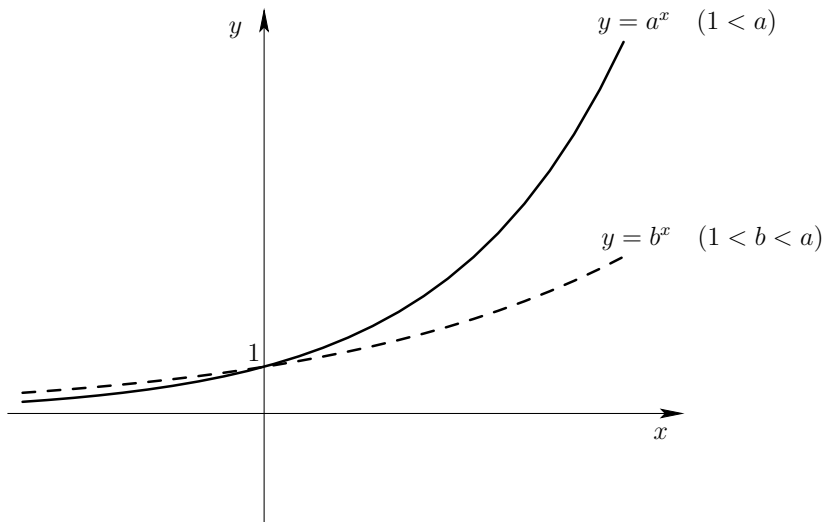


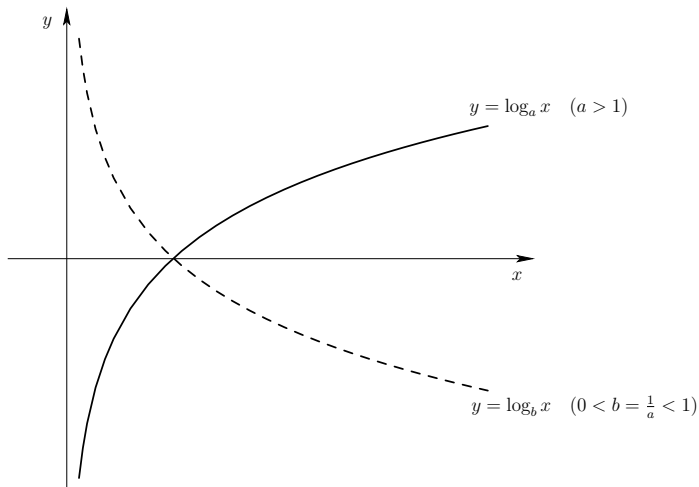
Cossecante

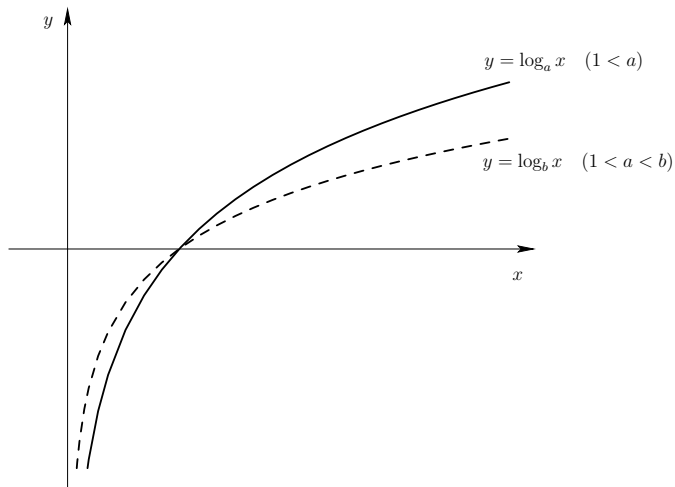


1. $\forall a \in \mathbb{R} \quad \sin^2 a + \cos^2 a = 1;$
2. $\forall a \in \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{tg}^2 a = \sec^2 a;$
3. $\forall a \in \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{cotg}^2 a = \operatorname{cosec}^2 a;$
4. $\forall a \in \mathbb{R} \quad \sin(-a) = -\sin a \quad (\text{a função seno é ímpar});$
5. $\forall a \in \mathbb{R} \quad \cos(-a) = \cos a \quad (\text{a função cosseno é par});$
6. $\forall a \in \mathbb{R} \quad \cos(\frac{\pi}{2} - a) = \sin a \quad \text{e} \quad \sin(\frac{\pi}{2} - a) = \cos a;$
7. $\forall a \in \mathbb{R} \quad \sin(a + 2\pi) = \sin a \quad (\text{a função seno tem período } 2\pi);$
8. $\forall a \in \mathbb{R} \quad \cos(a + 2\pi) = \cos a \quad (\text{a função cosseno tem período } 2\pi);$
9. $\forall a, b \in \mathbb{R} \quad \sin(a + b) = \sin a \cos b + \sin b \cos a;$
10. $\forall a, b \in \mathbb{R} \quad \cos(a + b) = \cos a \cos b - \sin b \sin a;$
11. $\forall a, b \in \mathbb{R} \quad \cos a - \cos b = -2 \sin \frac{a-b}{2} \sin \frac{a+b}{2};$
12. $\forall a, b \in \mathbb{R} \quad \sin a - \sin b = 2 \sin \frac{a-b}{2} \cos \frac{a+b}{2}.$









Seno hiperbólico

$$\begin{aligned}\text{sh} : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{e^x - e^{-x}}{2}\end{aligned}$$

Cosseno hiperbólico

$$\begin{aligned}\text{ch} : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{e^x + e^{-x}}{2}\end{aligned}$$

Tangente hiperbólica

$$\begin{aligned}\text{th} : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{\text{sh } x}{\text{ch } x}\end{aligned}$$

Cotangente hiperbólica

$$\begin{aligned}\text{coth} : \mathbb{R} \setminus \{0\} &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{1}{\text{th } x}\end{aligned}$$

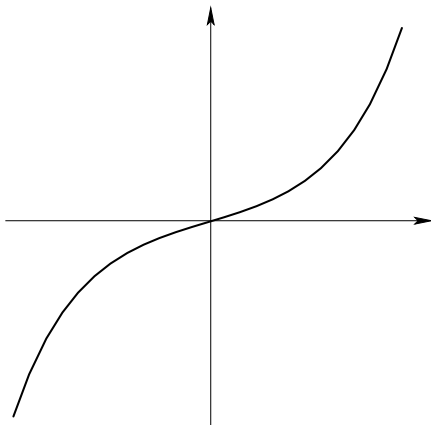
Secante hiperbólica

$$\begin{aligned}\text{sech} : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{1}{\text{ch } x}\end{aligned}$$

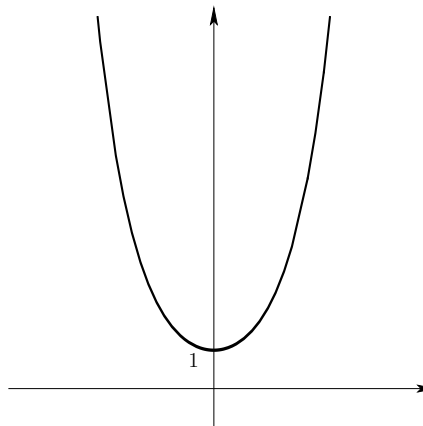
Cossecante hiperbólica

$$\begin{aligned}\text{cosech} : \mathbb{R} \setminus \{0\} &\longrightarrow \mathbb{R} \\ x &\longmapsto \frac{1}{\text{sh } x}\end{aligned}$$

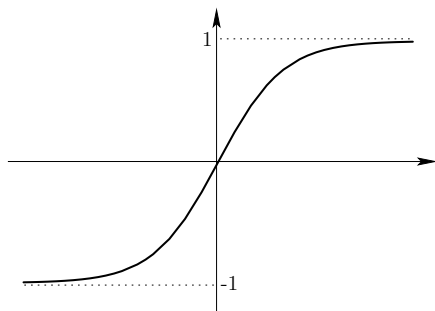
Seno hiperbólico



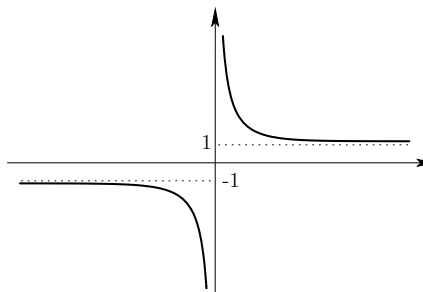
Cosseno hiperbólico



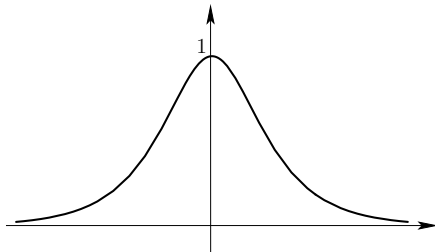
Tangente hiperbólica



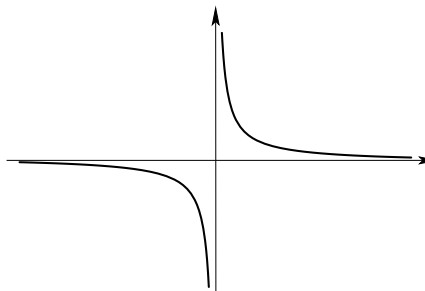
Cotangente hiperbólica



Secante hiperbólica



Cossecante hiperbólica



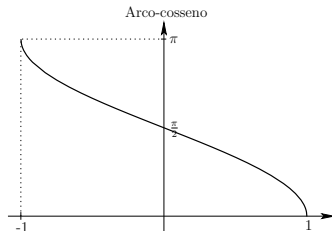
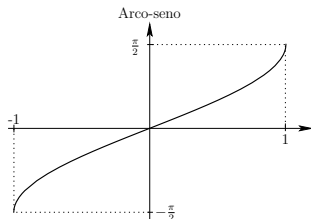
1. $\forall a \in \mathbb{R} \quad \operatorname{ch}^2 a - \operatorname{sh}^2 a = 1;$
2. $\forall a \in \mathbb{R} \quad \operatorname{th}^2 a + \operatorname{sech}^2 a = 1;$
3. $\forall a \in \mathbb{R} \setminus \{0\} \quad \operatorname{coth}^2 a - \operatorname{cosech}^2 a = 1;$
4. $\forall a \in \mathbb{R} \quad \operatorname{sh}(-a) = -\operatorname{sh} a \quad (\text{a função seno hiperbólico é ímpar});$
5. $\forall a \in \mathbb{R} \quad \operatorname{ch}(-a) = \operatorname{ch} a \quad (\text{a função cosseno hiperbólico é par});$
6. $\forall a, b \in \mathbb{R} \quad \operatorname{sh}(a+b) = \operatorname{sh} a \operatorname{ch} b + \operatorname{sh} b \operatorname{ch} a;$
7. $\forall a, b \in \mathbb{R} \quad \operatorname{ch}(a+b) = \operatorname{ch} a \operatorname{ch} b + \operatorname{sh} b \operatorname{sh} a;$
8. $\forall n \in \mathbb{N} \quad \forall a \in \mathbb{R} \quad (\operatorname{ch} a + \operatorname{sh} a)^n = \operatorname{ch}(na) + \operatorname{sh}(na).$

Arco-seno

$$\begin{aligned}\arcsen : [-1, 1] &\longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ x &\longmapsto \left(\sin|_{[-\frac{\pi}{2}, \frac{\pi}{2}]}\right)^{-1}(x)\end{aligned}$$

Arco-cosseno

$$\begin{aligned}\arccos : [-1, 1] &\longrightarrow [0, \pi] \\ x &\longmapsto \left(\cos|_{[0, \pi]}\right)^{-1}(x)\end{aligned}$$

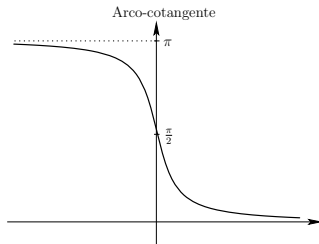
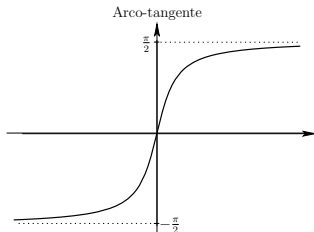


Arco-tangente

$$\begin{aligned}\operatorname{arctg} : \mathbb{R} &\longrightarrow \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[\\ x &\longmapsto \left(\operatorname{tg} \mid_{\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[}\right)^{-1}(x)\end{aligned}$$

Arco-cotangente

$$\begin{aligned}\operatorname{arccotg} : \mathbb{R} &\longrightarrow]0, \pi[\\ x &\longmapsto \left(\operatorname{cotg} \mid_{]0, \pi[}\right)^{-1}(x)\end{aligned}$$

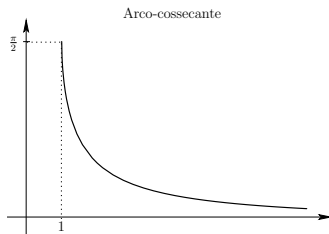
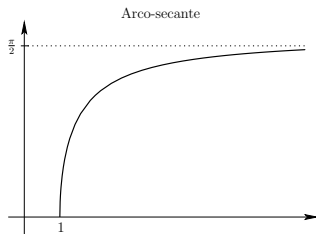


Arco-secante

$$\begin{aligned} \operatorname{arcsec} : [1, +\infty[&\longrightarrow [0, \frac{\pi}{2}[\\ x &\longmapsto \left(\sec|_{[0, \frac{\pi}{2}[} \right)^{-1}(x) \end{aligned}$$

Arco-cossecante

$$\begin{aligned} \operatorname{arccosec} : [1, +\infty[&\longrightarrow]0, \frac{\pi}{2}] \\ x &\longmapsto \left(\operatorname{cosec}|_{]0, \frac{\pi}{2}]} \right)^{-1}(x) \end{aligned}$$



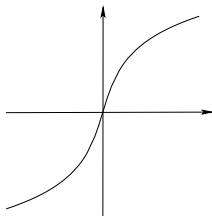
Argumento do seno hiperbólico

$$\begin{aligned}\operatorname{argsh} : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto (\operatorname{sh})^{-1}(x)\end{aligned}$$

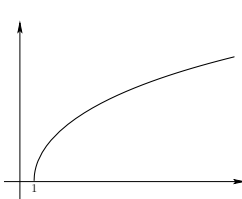
Argumento do cosseno hiperbólico

$$\begin{aligned}\operatorname{argch} : [1, +\infty[&\longrightarrow \mathbb{R}_0^+ \\ x &\longmapsto \left(\operatorname{ch}|_{\mathbb{R}_0^+}\right)^{-1}(x)\end{aligned}$$

Argumento do seno hiperbólico



Argumento do cosseno hiperbólico



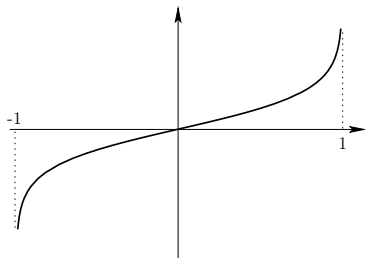
Argumento da tangente hiperbólica

$$\begin{array}{ccc} \operatorname{argth} :]-1, 1[& \longrightarrow & \mathbb{R} \\ x & \longmapsto & \operatorname{th}^{-1}(x) \end{array}$$

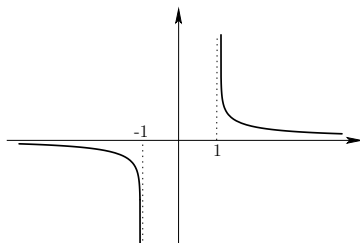
Argumento da cotangente hiperbólica

$$\begin{array}{ccc} \operatorname{argcoth} : \mathbb{R} \setminus [-1, 1] & \longrightarrow & \mathbb{R} \setminus \{0\} \\ x & \longmapsto & \operatorname{coth}^{-1}(x) \end{array}$$

Argumento da tangente hiperbólica



Argumento da cotangente hiperbólica



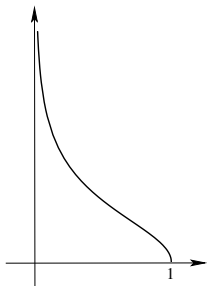
Argumento da secante hiperbólica

$$\begin{aligned}\operatorname{argsech} :]0, 1] &\longrightarrow \mathbb{R}_0^+ \\ x &\longmapsto \left(\sec|_{\mathbb{R}_0^+} \right)^{-1}(x)\end{aligned}$$

Argumento da cossecante hiperbólica

$$\begin{aligned}\operatorname{argcosech} : \mathbb{R} \setminus \{0\} &\longrightarrow \mathbb{R} \setminus \{0\} \\ x &\longmapsto \operatorname{cosech}^{-1}(x)\end{aligned}$$

Argumento da secante hiperbólica



Argumento da cossecante hiperbólica

