Soluções da Folha Prática 4

1. —

2. (a) —

   (b) $F_{M_1}(c) = \begin{cases} 0 & se \quad c < 2 \\ 1/3 & se \quad 2 \leq c < 3 \\ 1 & se \quad c \geq 3 \end{cases}$, $F_{M_2}(c) = \begin{cases} 0 & se \quad c < 3 \\ 1 & se \quad c \geq 3 \end{cases}$. M_2 é quase certa.

3. (a) $(\Omega, \mathcal{P}(\Omega), P)$ em que $\Omega = \{(x_1, x_2) : x_i \in \{C_a, C_o\}, i \in \{1, 2\}\}$ e P é a medida de probabilidade de Laplace, i.e.,

$$\begin{array}{rcl} P : \mathcal{P}(\Omega) & \to & [0,1] \\ A & \to & P(A) = \frac{\#A}{4} \end{array}$$

   (b) i. X e Y não são iguais.

   (b) ii. $F_X(c) = F_Y(c) = \begin{cases} 0 & se \quad c < 0 \\ 1/4 & se \quad 0 \leq c < 1 \\ 3/4 & se \quad 1 \leq c < 2 \\ 1 & se \quad c \geq 2 \end{cases}$

4. —

5. (a). $f(a) = \begin{cases} 6/36 & se \quad a = 0 \vee a = 3 \\ 10/36 & se \quad a = 1 \\ 8/36 & se \quad a = 2 \\ 4/36 & se \quad a = 4 \\ 2/36 & se \quad a = 5 \\ 0 & se \quad c.c. \end{cases}$, $F(c) = \begin{cases} 0 & se \quad c < 0 \\ 6/36 & se \quad 0 \leq c < 1 \\ 16/36 & se \quad 1 \leq c < 2 \\ 24/36 & se \quad 2 \leq c < 3 \\ 30/36 & se \quad 3 \leq c < 4 \\ 34/36 & se \quad 4 \leq c < 5 \\ 1 & se \quad c \geq 5 \end{cases}$

   (b) $f(a) = \begin{cases} 1/36 & se \quad a = 1 \\ 3/36 & se \quad a = 2 \\ 5/36 & se \quad a = 3 \\ 7/36 & se \quad a = 4 \\ 9/36 & se \quad a = 5 \\ 11/36 & se \quad a = 6 \\ 0 & se \quad c.c. \end{cases}$, $F(c) = \begin{cases} 0 & se \quad c < 1 \\ 1/36 & se \quad 1 \leq c < 2 \\ 4/36 & se \quad 2 \leq c < 3 \\ 9/36 & se \quad 3 \leq c < 4 \\ 16/36 & se \quad 4 \leq c < 5 \\ 25/36 & se \quad 5 \leq c < 6 \\ 1 & se \quad c \geq 6 \end{cases}$

   (c) $f(a) = \begin{cases} 11/36 & se \quad a = 1 \\ 9/36 & se \quad a = 2 \\ 7/36 & se \quad a = 3 \\ 5/36 & se \quad a = 4 \\ 3/36 & se \quad a = 5 \\ 1/36 & se \quad a = 6 \\ 0 & se \quad c.c. \end{cases}$, $F(c) = \begin{cases} 0 & se \quad c < 1 \\ 11/36 & se \quad 1 \leq c < 2 \\ 20/36 & se \quad 2 \leq c < 3 \\ 27/36 & se \quad 3 \leq c < 4 \\ 32/36 & se \quad 4 \leq c < 5 \\ 35/36 & se \quad 5 \leq c < 6 \\ 1 & se \quad c \geq 6 \end{cases}$

   (d) $f(a) = \begin{cases} 1/4 & se \quad a \in \{0, 2\} \\ 1/2 & se \quad a = 1 \\ 0 & se \quad c.c. \end{cases}$, $F(c) = \begin{cases} 0 & se \quad c < 0 \\ 1/4 & se \quad 0 \leq c < 1 \\ 3/4 & se \quad 1 \leq c < 2 \\ 1 & se \quad c \geq 2 \end{cases}$

   (e) Igual à alínea (d)

(f) $f(a) = \begin{cases} 1/36 & se \quad a \in \{2,12\} \\ 2/36 & se \quad a \in \{3,11\} \\ 3/36 & se \quad a \in \{4,10\} \\ 4/36 & se \quad a \in \{5,9\} \\ 5/36 & se \quad a \in \{6,8\} \\ 6/36 & se \quad a = 7 \\ 0 & se \quad c.c. \end{cases}$, $F(c) = \begin{cases} 0 & se \quad c < 2 \\ 1/36 & se \quad 2 \le c < 3 \\ 3/36 & se \quad 3 \le c < 4 \\ 6/36 & se \quad 4 \le c < 5 \\ 10/36 & se \quad 5 \le c < 6 \\ 15/36 & se \quad 6 \le c < 7 \\ 21/36 & se \quad 7 \le c < 8 \\ 26/36 & se \quad 8 \le c < 9 \\ 30/36 & se \quad 9 \le c < 10 \\ 33/36 & se \quad 10 \le c < 11 \\ 35/36 & se \quad 11 \le c < 12 \\ 1 & se \quad c \ge 12 \end{cases}$

6. (a) i. $\binom{20}{7}(\frac{1}{2})^{20}$; ii. $\sum_{k=0}^{9}\binom{20}{k}(\frac{1}{2})^{20}$; iii. $\sum_{k=15}^{20}\binom{20}{k}(\frac{1}{2})^{20}$; iv. $\sum_{k=4}^{15}\binom{20}{k}(\frac{1}{2})^{20}$.
   No R: i. `dbinom(7, 20, 1/2)`; ii. `pbinom(9, 20, 1/2)`; iii. `1-pbinom(14, 20, 1/2)`; iv. `pbinom(15, 20, 1/2)-pbinom(3, 20, 1/2)`
   (b) $\sum_{k=6}^{10}\binom{10}{k}(0.2)^k(0.8)^{10-k}$; $\sum_{k=0}^{5}\binom{10}{k}(0.2)^k(0.8)^{10-k}$.
   No R: `1-pbinom(5, 10, 0.2)`; `pbinom(5, 10, 0.2)`
   (c) $\binom{6}{6}(\frac{5}{13})^6$; $\binom{6}{0}(\frac{8}{13})^6$. No R: `dbinom(6, 6, 5/13)`; `dbinom(0, 6, 5/13)`
7. 0; $\frac{\binom{8}{6}\binom{5}{0}}{\binom{13}{6}}$. No R: `dhyper(6, 5, 8, 6)`; `dhyper(0, 5, 8, 6)`
8. (a) $\frac{2}{3}$
   (b) i. $\binom{10}{1}\frac{2}{3}(\frac{1}{3})^9$; ii. $\sum_{k=0}^{7}\binom{10}{k}(\frac{2}{3})^k(\frac{1}{3})^{10-k}$; iii. $\sum_{k=3}^{7}\binom{10}{k}(\frac{2}{3})^k(\frac{1}{3})^{10-k}$; iv. $\frac{\sum_{k=1}^{9}\binom{10}{k}(\frac{2}{3})^k(\frac{1}{3})^{10-k}}{\sum_{k=0}^{9}\binom{10}{k}(\frac{2}{3})^k(\frac{1}{3})^{10-k}}$
   No R: i. `dbinom(1, 10, 2/3)`; ii. `pbinom(7, 10, 2/3)`;
   iii. `pbinom(7, 10, 2/3)-pbinom(2, 10, 2/3)`;
   iv. `(pbinom(9, 10, 2/3)-pbinom(0, 10, 2/3))/pbinom(9, 10, 2/3)`
   (c) $\frac{\binom{10}{1}(1/2)^{10}(4/5)}{\binom{10}{1}(1/2)^{10}(4/5)+\binom{10}{1}(1/3)(2/3)^9(1/5)}$
   No R: `dbinom(1,10,1/2)*4/5 /( dbinom(1,10,1/2)*4/5 + dbinom(1,10,1/3)*1/5 )`
9. (a) $\binom{6}{3}(\frac{1}{2})^6$. No R: `dbinom(3, 6, 1/2)`.
   (b) $\binom{6}{2}(\frac{1}{10})^2(\frac{9}{10})^4$; $\sum_{k=0}^{2}\binom{6}{k}(\frac{1}{10})^k(\frac{9}{10})^{6-k}$; $\sum_{k=2}^{6}\binom{6}{k}(\frac{1}{10})^k(\frac{9}{10})^{6-k}$.
   No R: `dbinom(2, 6, 1/10)`; `pbinom(2, 6, 1/10)`; `1-pbinom(1, 6, 1/10)`.
10. $n = 22$
11. (a) $\frac{\binom{25}{7}\binom{24}{0}}{\binom{49}{7}}$. No R: `dhyper(7, 25, 24, 7)`.
    (b) $\frac{\binom{9}{3}\binom{40}{4}}{\binom{49}{7}}$. No R: `dhyper(3, 9, 40, 7)`.
    (c) $\sum_{k=5}^{7}\frac{\binom{9}{k}\binom{40}{7-k}}{\binom{49}{7}}$. No R: `1-phyper(4, 9, 40, 7)`.
12. (a) $a = 0$, $b = \frac{3}{4}$, $d = 1$, $f(a) = \begin{cases} 1/2 & se \quad a = 0 \\ 1/4 & se \quad a \in \{1,2\} \\ 0 & se \quad c.c. \end{cases}$. (b) $\frac{1}{2}$; $\frac{1}{2}$; $\binom{6}{3}(\frac{1}{4})^3(\frac{3}{4})^3$