Ficha de Trabalho 6

1. $$\int_1^2\int_0^4 (x^2+xy)\,dy\,dx = \int_1^2 \left[x^2y+\frac{1}{2}xy^2\right]_{y=1}^{y=4} dx$$

$$= \int_1^2 \left[(4x^2+8x)-\left(x^2+\frac{1}{2}x\right)\right] dx$$

$$= \int_1^2 \left[3x^2+8x-\frac{1}{2}x\right] dx = \int_1^2 \left[3x^2+\frac{15}{2}x\right] dx$$

$$= \left[x^3+\frac{15}{4}x^2\right]_1^2 = (8+15) - \left(1+\frac{15}{4}\right)$$

$$= 23 - \frac{19}{4} = \frac{73}{4}$$

2. (a) $$\iint_D f(x,y)\,dA$$

Para verticalmente simples, $0 \leq x \leq 3$ $\quad$ (a_1)
$1 \leq y \leq x+1$

Para horizontalmente simples, $1 \leq y \leq 4$ $\quad$ (b_1)
$y-1 \leq x \leq 3$

(a_1) $$\iint_D f(x,y)\,dA = \int_0^3\int_1^{x+1} f(x,y)\,dy\,dx$$

(b_1) $$\iint_D f(x,y)\,dA$$

$$= \int_1^4\int_{y-1}^3 f(x,y)\,dx\,dy$$

(b) $\text{volume}(S) = \iint_D f(x,y)\, dA$

$$= \int_0^3 \int_1^{x+1} 2\, dy\, dx$$

$$= \int_0^3 \Big[2y \Big]_{y=1}^{y=x+1} dx$$

$$= \int_0^3 [\, 2(x+1) - 2\,]\, dx = \int_0^3 2x\, dx$$

$$= \Big[x^2 \Big]_0^3 = 9$$

3. $$\int_0^1 \int_0^3 \int_0^2 f(x,y,z)\, dx\, dy\, dz = \int_0^1 \int_0^3 \int_0^2 xyz\, dx\, dy\, dz$$

$$= \int_0^1 \int_0^3 \Big[\frac{1}{2} x^2 y z \Big]_0^2 dy\, dz = \int_0^1 \int_0^3 2yz\, dy\, dz$$

$$= \int_0^1 \Big[y^2 z \Big]_0^3 dz = \int_0^1 9z\, dz = \Big[\frac{9}{2} z^2 \Big]_0^1 = \frac{9}{2}$$

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