Universidade do Minho
Escola de Ciências
DMat

Cálculo
LCC
2019/2020

Soluções da Folha 7 - Algumas funções importantes

Exercício 1

a) 1.

b) $-\sqrt{3}-1$.

Exercício 2

a) $\operatorname{sen}\alpha = -4/5$ e $\operatorname{tg}\alpha = 4/3$.

b) $\operatorname{sen}\alpha = -\dfrac{2\sqrt{5}}{5}$ e $\cos\alpha = \dfrac{\sqrt{5}}{5}$.

Exercício 3

a) $1/2$;
b) $\pi/4$;
c) $\pi/4$;
d) $-1/2$;
e) 1;
f) $-\pi/6$;
g) $\pi/6$;
h) $\pi/3$;
i) 0;
j) $\sqrt{3}/2$;
k) $\sqrt{2}/2$;
l) $-\sqrt{2}/2$;
m) $7/25$;
n) -1;
o) $-\pi/4$.

Exercício 4

a) $\dfrac{11\,\pi}{4}$;

b) $\dfrac{3\sqrt{3}}{16}$.

Exercício 5

a) $\mathrm{Dom}(f)=\left[-\frac{1}{2},\frac{1}{2}\right]$; $\mathrm{Im}(f)=\left[\frac{3\pi}{4},\frac{5\pi}{4}\right]$;
b) $\mathrm{Dom}(g)=[-1,1]\setminus\{0\}$; $\mathrm{Im}(g)=\left]-\infty,-\frac{2}{\pi}\right]\cup\left[\frac{2}{\pi},+\infty\right[$;
c) $\mathrm{Dom}(h)=\left[-\frac{1}{2},\frac{1}{2}\right]$; $\mathrm{Im}(h)=[0,\pi]$;
d) $\mathrm{Dom}(j)=\left[-\frac{1}{3},\frac{1}{3}\right]$; $\mathrm{Im}(j)=[0,\sqrt{\pi}]$.

Exercício 6

a) $\pi/2$.
b) $\mathrm{Dom}(t)=\mathbb{R}\setminus\{-1\}$; $\mathrm{Im}(t)=\left]-\frac{\pi}{4},\frac{3\pi}{4}\right[\setminus\{\frac{\pi}{4}\}$;
c) $]-\infty,-2[\cup]-1,+\infty[$;
d)

$$\begin{array}{rccl} t^{-1}: & \left]-\frac{\pi}{4},\frac{3\pi}{4}\right[\setminus\{\frac{\pi}{4}\} & \to & \mathbb{R}\setminus\{-1\} \\ & x & \longmapsto & t^{-1}(x)=\operatorname{cotg}(x-\pi/4)-1 \end{array}.$$

Exercício 7

a) $\frac{4\pi}{3}$.

b) $\text{Dom}(g)=]-\infty,-1]\cup[1,+\infty[$; $\text{Im}(g)=\left[-\frac{2\pi}{3},\frac{4\pi}{3}\right]\setminus\{\frac{\pi}{3}\}$;

c) $]-\infty,-1]\cup[2,+\infty[$;

d)
$$\begin{array}{rcl} g^{-1}:\left[-\frac{2\pi}{3},\frac{4\pi}{3}\right]\setminus\{\frac{\pi}{3}\} & \to &]-\infty,-1]\cup[1,+\infty[\\ x & \longmapsto & g^{-1}(x)=\dfrac{1}{\text{sen}\,(x/2-\pi/6)} \end{array}.$$

Exercício 8

a) f é contínua em $\mathbb{R}\setminus\{-1\}$.

b) $\text{Im}(f)=\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

c) $\lim\limits_{x\to-\infty} f(x)=0$; não existe $\lim\limits_{x\to+\infty} f(x)$.

Exercício 9

a) $k=2/\pi$.

b) $\lim\limits_{x\to-\infty} f(x)=0$; $\lim\limits_{x\to+\infty} f(x)=0$.

Exercício 10

a) $x=1/2$; b) $x=0$; c) $x=-\frac{1}{4}\ln 2$; d) $x=3/2$.

Exercício 11

Exercício 12

a) $+\infty$; c) 1; e) 1; g) $-\infty$.

b) $-\infty$; d) -1; f) $+\infty$;

Exercício 13