Exercício 5.1 Estabeleça as seguintes igualdades, válidas em $\mathbb{R}$:

a) $\cos^2 x = \dfrac{\cos 2x + 1}{2}$;

b) $\operatorname{sen}^2 x = \dfrac{1 - \cos 2x}{2}$;

c) $\cos 3x = 4\cos^3 x - 3\cos x$.

Exercício 5.2 Calcule:

a) $\operatorname{arcsen}\left(\operatorname{sen}\frac{5\pi}{4}\right)$;

b) $\operatorname{sen}\left(\operatorname{arcsen}(-\frac{1}{2})\right)$;

c) $\operatorname{sen}(\operatorname{arcsen} 1 + \pi)$;

d) $\operatorname{arcsen}\left(\operatorname{sen}\left(-\frac{\pi}{6}\right)\right)$;

e) $\operatorname{arcsen}\left(\operatorname{sen}\frac{23\pi}{6}\right)$;

f) $\cos\left(\arccos\frac{1}{8}\right)$;

g) $\arccos\left(\cos\left(-\frac{\pi}{3}\right)\right)$;

h) $\operatorname{arctg}\left(\operatorname{tg}\frac{9\pi}{4}\right)$;

i) $\operatorname{arctg}(\operatorname{tg}\pi)$;

j) $\operatorname{tg}(\operatorname{arctg}(-1))$;

k) $\operatorname{tg}(\operatorname{arccotg} 3)$;

l) $\operatorname{arctg}(\operatorname{cotg}\frac{\pi}{5})$.

Exercício 5.3 Deduza as seguintes igualdades em domínios que deverá especificar:

a) $\operatorname{sen}(\arccos x) = \sqrt{1 - x^2}$;

b) $\operatorname{tg}(\arccos x) = \dfrac{\sqrt{1-x^2}}{x}$;

c) $\cos(\operatorname{arcsen} x) = \sqrt{1 - x^2}$;

d) $\operatorname{tg}(\operatorname{arcsen} x) = \dfrac{x}{\sqrt{1-x^2}}$;

e) $\operatorname{sen}(\operatorname{arctg} x) = \dfrac{x}{\sqrt{1+x^2}}$;

f) $\cos(\operatorname{arctg} x) = \dfrac{1}{\sqrt{1+x^2}}$.

Exercício 5.4 Calcule:

a) $\operatorname{arcsen}\left(-\frac{\sqrt{2}}{2}\right)$;

b) $\operatorname{cotg}\left(\operatorname{arcsen}\left(-\frac{4}{5}\right)\right)$;

c) $\cos\left(\operatorname{arcsen}\frac{1}{2} - \arccos\frac{3}{5}\right)$;

d) $\operatorname{sen}(\pi - \operatorname{arcsen} 1)$;

e) $\operatorname{sen}\left(\frac{\pi}{2} + \arccos\frac{\sqrt{3}}{2}\right)$;

f) $\operatorname{sen}(\operatorname{arctg}(-1))$;

g) $\operatorname{sen}\left(\arccos\frac{\sqrt{2}}{2}\right)$;

h) $\cos\left(-2\operatorname{arcsen}\left(-\frac{3}{5}\right)\right)$;

i) $\operatorname{tg}\left(-\operatorname{arcsen}\frac{\sqrt{2}}{2}\right)$;

j) $\operatorname{arctg}\left(-2 + \operatorname{tg}\frac{5\pi}{4}\right)$;

k) $\operatorname{arcsen}\left(\operatorname{sen}\frac{\pi}{2}\right) + 2\arccos\left(-\frac{\sqrt{2}}{2}\right)$;

l) $\cos^2\left(\frac{1}{2}\arccos\frac{1}{3}\right) - \operatorname{sen}^2\left(\frac{1}{2}\arccos\frac{1}{3}\right)$;

m) $\operatorname{tg}^2\left(\operatorname{arcsen}\frac{3}{5}\right) - \operatorname{cotg}^2\left(\arccos\frac{4}{5}\right)$.

Exercício 5.5 Considere a função $g(x) = \frac{\pi}{3} + 2\operatorname{arcsen}\frac{1}{x}$.

a) Calcule $g(1) + g(-2)$.

b) Determine o domínio e o contradomínio de g.

c) Determine o conjunto de soluções da inequação $g(x) \leq \frac{2\pi}{3}$.

d) Caraterize a função inversa de g.

Exercício 5.6 Seja $f:\mathbb{R}\longrightarrow\mathbb{R}$ a função definida por

$$f(x)=\begin{cases}0 & \text{se } x\leq -1,\\ \operatorname{arcsen} x & \text{se } -1<x<1,\\ \frac{\pi}{2}\ \operatorname{sen}\left(\frac{\pi}{2}x\right) & \text{se } x\geq 1.\end{cases}$$

a) Estude a continuidade da função f.

b) Indique o contradomínio de f.

c) Determine, caso existam, $\lim\limits_{x\to-\infty} f(x)$ e $\lim\limits_{x\to+\infty} f(x)$.

Exercício 5.7 Seja $f:\mathbb{R}\longrightarrow\mathbb{R}$ a função definida por

$$f(x)=\begin{cases}k\ \operatorname{arctg}\frac{1}{x} & \text{se } x>0,\\ \frac{1}{x^2+1} & \text{se } x\leq 0.\end{cases}$$

a) Determine k de modo que f seja contínua.

b) Calcule $\lim\limits_{x\to-\infty} f(x)$ e $\lim\limits_{x\to+\infty} f(x)$.

Exercício 5.8 Resolva as seguintes equações:

a) $e^x=e^{1-x}$;

b) $e^{2x}+2e^x-3=0$;

c) $e^{3x}-2e^{-x}=0$;

d) $\ln(x^2-1)+2\ln 2=\ln(4x-1)$.

Exercício 5.9 Recorde que $\operatorname{ch} x=\frac{e^x+e^{-x}}{2}$ e que $\operatorname{sh} x=\frac{e^x-e^{-x}}{2}$. Mostre que:

a) $\operatorname{ch}^2 x-\operatorname{sh}^2 x=1$;

b) $\operatorname{ch} x+\operatorname{sh} x=e^x$;

c) $\operatorname{sh}(-x)=-\operatorname{sh} x$;

d) $\operatorname{ch}(-x)=\operatorname{ch} x$;

e) $\operatorname{sh}(x+y)=\operatorname{sh} x\ \operatorname{ch} y+\operatorname{ch} x\ \operatorname{sh} y$;

f) $\operatorname{ch}(x+y)=\operatorname{ch} x\ \operatorname{ch} y+\operatorname{sh} x\ \operatorname{sh} y$;

g) $\operatorname{th}^2 x+\dfrac{1}{\operatorname{ch}^2 x}=1$;

h) $\coth^2 x-\dfrac{1}{\operatorname{sh}^2 x}=1$.

Exercício 5.10 Verifique que:

a) $\operatorname{argsh} x=\ln\left(x+\sqrt{x^2+1}\right),\quad x\in\mathbb{R}$;

b) $\operatorname{argch} x=\ln\left(x+\sqrt{x^2-1}\right),\quad x\in[1,+\infty[$;

c) $\operatorname{argth} x=\ln\sqrt{\frac{1+x}{1-x}},\quad x\in\,]-1,1[$;

d) $\operatorname{argcoth} x=\ln\sqrt{\frac{x+1}{x-1}},\quad x\in\mathbb{R}\backslash]-1,1[$.