Proposta de resolução do TG2

$\boxed{1}$ $$g(x,y)=\begin{cases}\dfrac{x^3}{2x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

a) $(x,y)\neq(0,0)$

$$\frac{\partial g}{\partial x}(x,y)=\frac{3x^2(2x^2+y^2)-x^3.4x}{(2x^2+y^2)^2}=\frac{6x^4-4x^4+3x^2y^2}{(2x^2+y^2)^2}=\frac{2x^4+3x^2y^2}{(2x^2+y^2)^2}$$

$(x,y)=(0,0)$

$$\frac{\partial g}{\partial x}(0,0)=\lim_{h\to0}\frac{g(h,0)-g(0,0)}{h}=\lim_{h\to0}\frac{\frac{h^3}{2h^2+0^2}-0}{h}=\lim_{h\to0}\frac{h^3}{2h^3}=\frac{1}{2}$$

$$\frac{\partial g}{\partial x}:\mathbb{R}^2\longrightarrow\mathbb{R}$$
$$(x,y)\longmapsto\begin{cases}\dfrac{2x^4+3x^2y^2}{(2x^2+y^2)^2}, & (x,y)\neq(0,0)\\ \dfrac{1}{2}, & (x,y)=(0,0)\end{cases}$$

b) $\vec{u}=(u_1,u_2)\in\mathbb{R}^2$

$$D_g((0,0);(u_1,u_2))=\lim_{h\to0}\frac{g(hu_1,hu_2)-g(0,0)}{h}=\lim_{h\to0}\frac{\frac{(hu_1)^3}{2(hu_1)^2+(hu_2)^2}-0}{h}$$
$$=\lim_{h\to0}\frac{h^3u_1^3}{h^3(2u_1^2+u_2^2)}=\frac{u_1^3}{2u_1^2+u_2^2}$$

c) A função g não é derivável em (0,0) porque a função

$$(u_1,u_2)\longmapsto D_g((0,0);(u_1,u_2))=\frac{u_1^3}{2u_1^2+u_2^2}$$

não é linear.

$\boxed{2}$ $f:D\subseteq\mathbb{R}^2\longrightarrow\mathbb{R}$ com $f(x,y)=\sqrt{x-y^2}=(x-y^2)^{1/2}$

a $$\frac{\partial f}{\partial x}(x,y)=\frac{1}{2}(x-y^2)^{-1/2}$$
$$\frac{\partial f}{\partial x}(x,y)=\frac{1}{2}(x-y^2)^{-1/2}.(-2y)$$

Sendo $\frac{\partial f}{\partial x}$ e $\frac{\partial f}{\partial y}$ funções contínuas numa bola de centro em (2,1), logo f é derivável em (2,1).

b) $f'(2,1): \mathbb{R}^2 \longrightarrow \mathbb{R}$

$$(u,v) \longmapsto \frac{\partial f}{\partial x}(2,1)\,u + \frac{\partial f}{\partial y}(2,1)\,v = \frac{u}{2} - v$$

$$\frac{\partial f}{\partial x}(2,1) = \frac{1}{2}\left(2-1^2\right)^{-1/2} = \frac{1}{2} \quad ; \quad \frac{\partial f}{\partial y}(2,1) = \frac{1}{2}(2-1)^{-1/2}(-2\cdot 1) = -1$$

c)

$$(x,y) = (2,1) + \lambda \nabla f(2,1), \quad \lambda \in \mathbb{R}$$

$$\Leftrightarrow (x,y) = (2,1) + \lambda\left(\frac{1}{2}, -1\right), \quad \lambda \in \mathbb{R}$$