Universidade do Minho
Departamento de Matemática

LCC **Análise**

Formulário 1 — 2019/2020

Regras de derivação

Sejam $u\colon I \longrightarrow \mathbb{R}$ e $v\colon I \longrightarrow \mathbb{R}$ funções deriváveis no intervalo I.

$(a)' = 0 \quad (a \in \mathbb{R})$

$(x)' = 1$

$(ax+b)' = a \quad (a, b \in \mathbb{R})$

$(u+v)' = u' + v'$

$(uv)' = u'v + u\,v'$

$\left(\frac{u}{v}\right)' = \frac{u'v - u\,v'}{v^2}$

$(u^n)' = n\,u^{n-1}u' \quad (n \in \mathbb{R})$

$\left(\sqrt[n]{u}\right)' = \frac{u'}{n\sqrt[n]{u^{n-1}}} \quad (n \in \mathbb{N})$

$(u \circ v)' = (u' \circ v)\, v'$

$(\operatorname{sen} u)' = u' \cos u$

$(\cos u)' = -u' \operatorname{sen} u$

$(\operatorname{tg} u)' = \frac{u'}{\cos^2 u}$

$(\operatorname{cotg} u)' = -\frac{u'}{\operatorname{sen}^2 u}$

$(\operatorname{arcsen} u)' = \frac{u'}{\sqrt{1-u^2}}$

$(\arccos u)' = -\frac{u'}{\sqrt{1-u^2}}$

$(\operatorname{arctg} u)' = \frac{u'}{1+u^2}$

$(\operatorname{arccotg} u)' = -\frac{u'}{1+u^2}$

$(e^u)' = u'e^u$

$(a^u)' = u'a^u \ln a \quad (a \in \mathbb{R}^+ \setminus \{1\})$

$(\ln u)' = \frac{u'}{u}$

$(\log_a u)' = \frac{u'}{u \ln a} \quad (a \in \mathbb{R}^+ \setminus \{1\})$

$(\operatorname{sh} u)' = u' \operatorname{ch} u$

$(\operatorname{ch} u)' = u' \operatorname{sh} u$

$(\operatorname{th} u)' = \frac{u'}{\operatorname{ch}^2 u}$

$(\coth u)' = -\frac{u'}{\operatorname{sh}^2 u}$

- Produto escalar, norma e produto externo em $\mathbb{R}^3$.
  Sendo $\vec{a} = (a_1, a_2, a_3)$ e $\vec{b} = (b_1, b_2, b_3)$,

$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3, \qquad \|\vec{a}\| = \sqrt{a \cdot a} = \sqrt{a_1^2 + a_2^2 + a_3^2}, \qquad \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}.$$

- Para $0 \leq \theta \leq \pi$, ângulo entre os vetores $\vec{a}$ e $\vec{b}$,

$$\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\| \cos\theta, \qquad \|\vec{a} \times \vec{b}\| = \|\vec{a}\|\|\vec{b}\| \operatorname{sen}\theta.$$

- Sejam $f\colon D \subset \mathbb{R}^n \longrightarrow \mathbb{R}$ e $\mathbf{a} = (a_1, \ldots, a_n) \in D$.
  - *Diferencial* de f em $\mathbf{a}$

$$df_{\mathbf{a}}(dx_1, \ldots, dx_n) = \frac{\partial f}{\partial x_1}(\mathbf{a})\, dx_1 + \ldots + \frac{\partial f}{\partial x_n}(\mathbf{a})\, dx_n$$

  - *Gradiente* de f em $\mathbf{a}$

$$\vec{\nabla} f(\mathbf{a}) = \left(\frac{\partial f}{\partial x_1}(\mathbf{a}), \ \ldots, \ \frac{\partial f}{\partial x_n}(\mathbf{a})\right)$$

  - *Derivada direcional* de f em $\mathbf{a}$ na direção de um vetor unitário $\vec{u} = (u_1, \ldots, u_n)$

$$D_{\vec{u}} f(\mathbf{a}) = \vec{\nabla} f(\mathbf{a}) \cdot \vec{u}$$

- Sejam $f\colon D \subset \mathbb{R}^2 \longrightarrow \mathbb{R}$, $\mathcal{C}$ a curva de nível $f(x,y)=k$, $k \in \mathbb{R}$, e $P=(a,b) \in \mathcal{C}$ com $\vec{\nabla} f(P) \neq \vec{0}$.

  - *Reta tangente* a $\mathcal{C}$ em P
$$\vec{\nabla} f(P) \cdot (x-a, y-b) = 0$$

- Sejam $g\colon D \subset \mathbb{R}^3 \longrightarrow \mathbb{R}$, $\mathcal{S}$ a surperfície $g(x,y,z)=k$, $k \in \mathbb{R}$, e $P=(a,b,c) \in \mathcal{S}$ com $\vec{\nabla} g(P) \neq \vec{0}$.

  - *Plano tangente* a $\mathcal{S}$ em P
$$\vec{\nabla} g(P) \cdot (x-a, y-b, z-c) = 0$$

  - *Reta normal* a $\mathcal{S}$ em P
$$(x,y,z) = P + \lambda \vec{\nabla} g(P), \qquad \lambda \in \mathbb{R}$$

- Regra da cadeia

  Supondo que u é uma função de n variáveis $x_1, x_2, \ldots, x_n$ e cada x_j é uma função de m variáveis $t_1, t_2, \ldots, t_m$, diferenciáveis, então, para cada $i = 1, \ldots, m$,
$$\frac{\partial u}{\partial t_i} = \frac{\partial u}{\partial x_1}\frac{\partial x_1}{\partial t_i} + \cdots + \frac{\partial u}{\partial x_n}\frac{\partial x_n}{\partial t_i}.$$
  Caso particular: $m = 1$
$$\frac{du}{dt} = \frac{\partial u}{\partial x_1}\frac{dx_1}{dt} + \cdots + \frac{\partial u}{\partial x_n}\frac{dx_n}{dt}.$$

- Derivada da função implícita

  Assumindo que $F(x_1, \ldots, x_n, y) = 0$ define implicitamente y como função de $x_1, x_2, \ldots, x_n$, então, para cada $i = 1, \ldots, n$,
$$\frac{\partial y}{\partial x_i}(x_1, \ldots, x_n) = -\frac{\dfrac{\partial F}{\partial x_i}(x_1, \ldots, x_n, y)}{\dfrac{\partial F}{\partial y}(x_1, \ldots, x_n, y)}, \quad \text{desde que } \frac{\partial F}{\partial y}(x_1, \ldots, x_n, y) \neq 0.$$

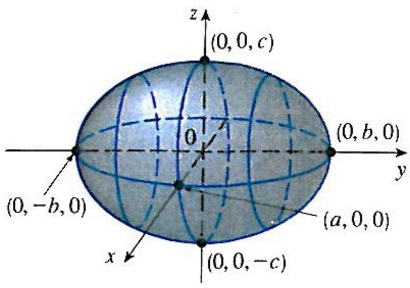
  Casos particulares:

  - $F(x,y) = 0$
$$\frac{dy}{dx}(x) = -\frac{\dfrac{\partial F}{\partial x}(x,y)}{\dfrac{\partial F}{\partial y}(x,y)}$$

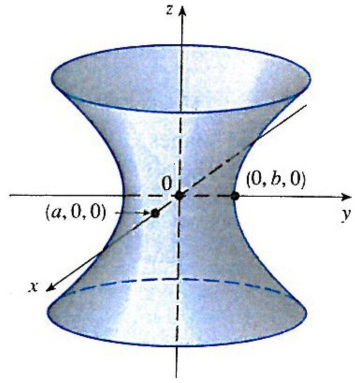
  - $F(x,y,z) = 0$
$$\frac{\partial z}{\partial x}(x,y) = -\frac{\dfrac{\partial F}{\partial x}(x,y,z)}{\dfrac{\partial F}{\partial z}(x,y,z)}; \qquad \frac{\partial z}{\partial y}(x,y) = -\frac{\dfrac{\partial F}{\partial y}(x,y,z)}{\dfrac{\partial F}{\partial z}(x,y,z)}$$

- Razões trigonométricas

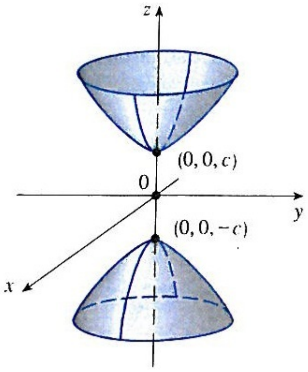
	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
sen	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tg	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$



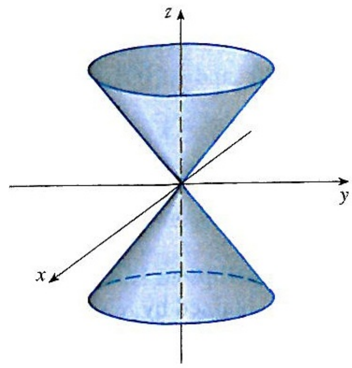
(a) Elipsóide $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$



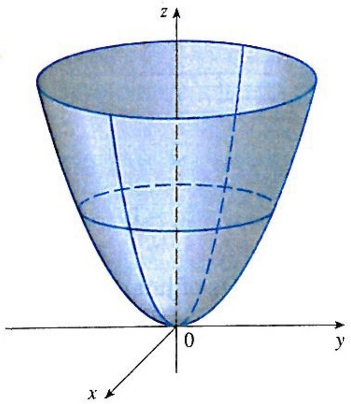
(b) Hiperbolóide de uma folha $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$



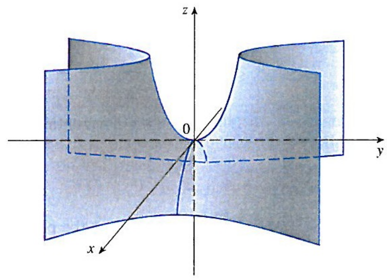
(c) Hiperbolóide de duas folhas $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$



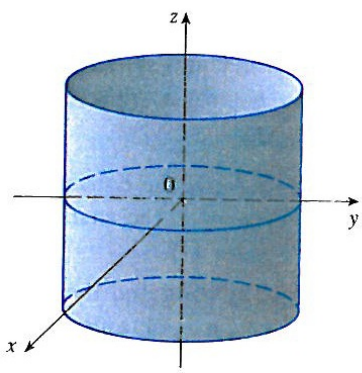
(d) Cone $\frac{z^2}{c^2}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$



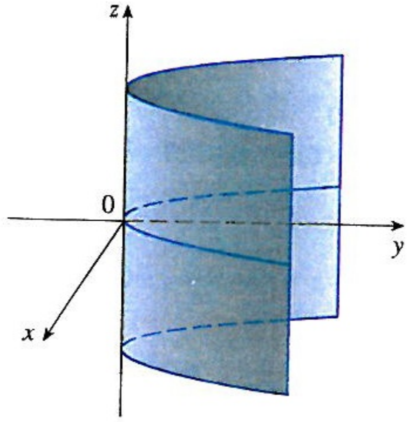
(e) Parabolóide elíptico $\frac{z}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2},\ c>0$



(f) Parabolóide hiperbólico $\frac{z}{c}=\frac{x^2}{a^2}-\frac{y^2}{b^2},\ c<0$



(g) Cilíndro elíptico $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



(h) Cilíndro parabólico $y=ax^2$

Figura 1: Superfícies quádricas