

[Ficha 3] 23/24

1.

$$A \xrightarrow{id} A, B \times e \xrightarrow{\text{swap}} e \times B$$

$$A \times (B \times e) \xrightarrow{id \times \text{swap}} A \times (e \times B)$$

$$D \times E \xrightarrow{\text{swap}} E \times D, A \times (B \times e) \xrightarrow{id \times \text{swap}} A \times (e \times B)$$

$$A \times (B \times e) \xrightarrow{\alpha} e \times D \quad \{ A \times (e \times B) = D \times E \}$$

Logo:  $A = D$  &  $E = e \times B$

Então:

$$\boxed{A \times (B \times e) \xrightarrow{\alpha} (e \times B) \times A} \leftarrow \text{Tipo Grid}$$

$$A \times (B \times e) \xrightarrow{\alpha} (e \times B) \times A$$

$$f \times (g \times h) \downarrow \qquad \qquad \downarrow (h \times g) \times f \\ j \times (e \times f) \xrightarrow{\alpha} (f \times e) \times j$$

$$\boxed{| ((h \times g) \times f) \cdot \alpha = \alpha \cdot (f \times (g \times h)) |} \leftarrow \text{Propriedade Nitwz}$$

2.

$$\alpha = \text{dup} \cdot \text{join}$$

$$A \xrightarrow{id} A, B \xrightarrow{id} B$$

$$A + B \xrightarrow{\text{join}} B \quad \{ A = B \}$$

$$c \xrightarrow{id} c, d \xrightarrow{id} d$$

$$e \xrightarrow{\text{dup}} e \times d \quad \{ e = d \}$$

$$\text{Logo: } A + A \xrightarrow{\text{join}} A$$

$$(\text{Logo, } c \xrightarrow{\text{dup}} c \times c)$$

$$A + A \xrightarrow{\text{join}} A, e \xrightarrow{\text{dup}} e \times e$$

$$A + A \xrightarrow{\alpha} e \times e \quad \{ A = e \}$$

$$(\text{Logo, } \boxed{A + A \xrightarrow{\alpha} A \times A}) \leftarrow \text{Tipo Grid}$$

$$A + A \xrightarrow{\alpha} A \times A$$

$$\begin{array}{c} f + f \\ \cancel{f + f} \\ \downarrow \\ B + B \end{array} \xrightarrow{\alpha} B \times B$$

Prop. Natur. da  $\alpha$

$$\boxed{| (f + f) \cdot \alpha = \alpha \cdot (f + f) |}$$

Nota: se pode escolher = propriedade gridis para join-dup, porque não há possivel compar essas duas funções.

$$\textcircled{3} \quad A \xrightarrow{!} 1, A \xrightarrow{!} 1$$

$$\underline{A+A \xrightarrow{!+!} 1+1, B+B \xrightarrow{\text{join}} B}$$

$$A+A \xrightarrow{\text{iso}} (1+1) \times B \quad \left\{ \begin{array}{l} A+A = B+B \\ A=B \end{array} \right.$$

Logo,  $\boxed{A+A \xrightarrow{\text{iso}} (1+1) \times A}$  Tyro Gerl ob iso

$$A+A \xrightarrow{\text{iso}} (1+1) \times A$$

$$\begin{matrix} f+f \\ \downarrow \\ S+B \end{matrix} \quad \begin{matrix} \downarrow \\ \text{id} \times f \\ \downarrow \\ (1+1) \times B \end{matrix} \quad \boxed{(\text{id} \times f) \cdot \text{iso} = \text{iso} \cdot (f+f)}$$

$$(\text{id} \times f) \cdot \text{iso} = \text{iso} \cdot (f+f)$$

$$\hookrightarrow (\text{id} \times f) \cdot \langle !+!, \text{join} \rangle = \langle !+!, \text{join} \rangle \cdot (f+f) \quad \{ \text{Def iso} \}$$

$$\hookrightarrow \langle !+!, f \cdot \text{join} \rangle = \langle !+!, \text{join} \rangle \cdot (f+f) \quad \{ \text{11} \}$$

$$\hookrightarrow \begin{cases} \pi_1 \cdot \langle !+!, \text{join} \rangle \cdot (f+f) = !+! \\ \pi_2 \cdot \langle !+!, \text{join} \rangle \cdot (f+f) = f \cdot \text{join} \end{cases} \quad \{ \text{64} \}$$

$$\hookrightarrow \begin{cases} (!+!) \cdot (f+f) = !+! \\ \text{join} \cdot (f+f) = f \cdot \text{join} \end{cases} \quad \{ \text{7(x2)} \}$$

$$\hookrightarrow \begin{cases} (! \cdot f) + (! \cdot f) = !+! \\ \text{join} \cdot (f+f) = f \cdot \text{join} \end{cases} \quad \{ \text{25, Natural-join} \}$$

$$\hookrightarrow !+! = !+! \quad \{ \text{3(x2)} \}$$

$$\hookrightarrow \text{TRUE} =$$

$$\text{iso} = \langle !+!, \text{join} \rangle$$

$$\Rightarrow \text{iso} = \langle [i_1 \cdot !, i_2 \cdot !], [\text{id}, \text{id}] \rangle \quad \{ \text{21} \}$$

$$\hookrightarrow \text{iso} = [\langle i_1 \cdot !, \text{id} \rangle, \langle i_2 \cdot !, \text{id} \rangle] \quad \{ \text{284} \}$$

$$\hookrightarrow \begin{cases} \text{iso} \cdot i_1 = \langle i_1 \cdot !, \text{id} \rangle \\ \text{iso} \cdot i_2 = \langle i_2 \cdot !, \text{id} \rangle \end{cases} \quad \{ \text{17} \}$$

$$\hookrightarrow \begin{cases} \text{iso} (\text{Left } a) = (\text{Left } (), a) \\ \text{iso} (\text{Right } a) = (\text{Right } (), a) \end{cases} \quad \{ \text{72(x2), 77(x2), 73(x2), 74(x2)} \}$$

$$4. \quad \begin{cases} \nabla \cdot i_1 = \text{id} \\ \nabla \cdot i_2 = \text{id} \end{cases} \Rightarrow \nabla = [\text{id}, \text{id}]$$

$A \xrightarrow{\text{id}} A, B \xrightarrow{\text{id}} B$

$A+B \xrightarrow{\nabla} A \quad \{A=B\}$

Tipps Sessel  $\nabla$

Log.,  $A+A \xrightarrow{\nabla} A$

$$\begin{array}{ccc} A+A & \xrightarrow{\nabla} & A \\ f+f & \downarrow & \downarrow f \\ B+B & \xrightarrow{\nabla} & B \end{array} \quad \boxed{f \cdot \nabla = \nabla \cdot (f+f)} \quad \text{Prop. Natural } \nabla$$

$$5. \quad \begin{array}{ccc} A+(e \times B) & \xrightarrow{\alpha} & A+B \\ f+g \times h & \downarrow & \downarrow f+h \\ D+(E \times F) & \xrightarrow{\alpha} & D+F \end{array} \quad \begin{array}{ccc} A & \xrightarrow{i_1} & A+B & \xleftarrow{i_2} & B \\ & \uparrow \text{id} & \uparrow \text{id} + \overline{\text{id}}_2 & \uparrow \overline{\text{id}}_2 & \uparrow \overline{\text{id}}_2 \\ A & \xrightarrow{i_1} & A+(e \times B) & \xleftarrow{i_2} & (e \times e) \end{array}$$

$\alpha = \text{id} + \overline{\text{id}}_2$

$$(f+h) \cdot \alpha = \alpha \cdot (f+(g \times h))$$

$$\Leftrightarrow (f+h) \cdot (\text{id} + \overline{\text{id}}_2) = (\text{id} + \overline{\text{id}}_2) \cdot (f+(g \times h))$$

$$\Leftrightarrow (f \cdot \text{id}) + (h \cdot \overline{\text{id}}_2) = (\text{id} \cdot f) + (\overline{\text{id}}_2 \cdot (g \times h))$$

$$\Leftrightarrow f + (h \cdot \overline{\text{id}}_2) = f + (h \cdot \overline{\text{id}}_2)$$

True

$$6. \quad A \times (B+E) \xrightarrow{\text{distr}} (A \times B) + (A \times E)$$

$$\begin{array}{ccc} f \times (g+h) & \downarrow & (f \times g) + (f \times h) \\ D \times (E+F) & \xrightarrow{\text{distr}} & (D \times E) + (D \times F) \end{array}$$

$((f \times g) + (f \times h)) \cdot \text{distr} = \text{distr} \cdot (f \times (g+h))$

$$h \cdot \text{distr} \cdot (g \times (\text{id} + \alpha)) = K$$

$$\Leftrightarrow h \cdot ((g \times \text{id}) + (g \times \alpha)) \cdot \text{distr} = K \quad \{ \text{Natural distr} \}$$

$$\Leftrightarrow h \cdot ((g \times \text{id}) + (g \times \alpha)) = K \cdot \text{undistr} \quad \{ \text{zur} \}$$

$$7. \quad ap \cdot (\bar{f} \times id) = f$$

$$\Leftrightarrow \forall a, b \mid (ap \cdot (\bar{f} \times id))(a, b) = f(a, b)$$

{ 724}

$$\Leftrightarrow \forall a, b \mid ap((\bar{f} \times id)(a, b)) = f(a, b)$$

{ 731}

$$\Leftrightarrow \forall a, b \mid ap(\bar{f} a, id b) = f(a, b)$$

{ 731}

$$\Leftrightarrow \forall a, b \mid ap(\bar{f} a, b) = f(a, b)$$

{ 741}

$$\Leftrightarrow \forall a, b \mid \bar{f} a \ b = f(a, b)$$

{ 2f ap}

$$\Leftrightarrow \forall a, b \mid \text{urry } f \ a \ b = f(a, b)$$

8. Substituindo  $f$  por  $\hat{g}$  em Fg, temos:

$$ap \cdot (\hat{g} \times id) = \hat{g}$$

$$\Leftrightarrow ap \cdot (g \times id) = \hat{g}$$

{ ury-urery = id }

$$\Leftrightarrow \forall a, b \mid (ap \cdot (g \times id))(a, b) = \hat{g}(a, b)$$

{ 724 }

$$\Leftrightarrow \forall a, b \mid ap(g a, b) = \hat{g}(a, b)$$

{ 73, 781 }

$$\Leftrightarrow \forall a, b \mid g a \ b = \hat{g}(a, b)$$

{ 2f cp }

$$\Leftrightarrow \forall a, b \mid \text{urry } g (a, b) = g a \ b$$

9.

Point ::  $a \rightarrow a \rightarrow a \rightarrow \text{Point A}$

$$\text{Point } \in ((\text{Point A})^A)^A$$

$$\text{urry Point } \in ((\text{Point A})^A)^{A \times A}$$

$$\text{urry (urry Point)} \in (\text{Point A})^{(A \times A) \times A}$$

$$\text{in} = \text{urry (urry Point)}$$