Computer Vision Methods Applicable to Forensic Science

Fernanda A. Andaló, and Siome Goldenstein
Institute of Computing, University of Campinas – UNICAMP
Campinas, SP – Brazil
Email: {feandalo,siome}@ic.unicamp.br

***Abstract*—Crimes in our society, increasing in volume and sophistication, have determined the need for knowledge and use of scientific methods to their prevention and investigation. This work presents three Computer Vision methods that can be applied in forensic investigations. In the first method, a new vanishing point detector facilitates the process of making measurements in a single 2D image, and is used to estimate the height of a person in an image, important measurement to corroborate pieces of evidence. In the second method, multiview stereo techniques are used to obtain a three-dimensional model from photographs taken from a footwear impression, evidence commonly found in crime scenes. In the third method, the need for reconstructing shredded images is explored. A photograph can be shredded in order to hide information and it is up to the field of Computer Forensics its automatic reconstruction. Experimental results are provided, showing the effectiveness of the proposed methods when compared to relevant work in the literature.**

***Keywords*-vanishing point; footwear impression; image puzzle; computer forensics; computer vision;**

I. Introduction

Forensic Science is the application of knowledge from several branches of science to answer questions relevant to a legal system. Due to the evolution of criminal activities, more specialized disciplines have been involved, such as Computer Science, Engineering and Economics.

The research field that unites the fields of Forensic Science and Computer Science is called *Computer Forensics* and encompasses the study of research methods, driven by hypothesis, of a specific problem, through the use of computers and computational methods [1].

Computer Forensics, however, requires joint efforts from forensic and computer scientists. Several collaborations are possible between Computer Science and Forensic Science [2], [3]. In several works, the area of *Computer Vision* provides useful knowledge to tackle forensic problems, which includes methods for acquiring, processing, analyzing, and understanding images from the real world. Computer Vision and Forensic Science are a powerful combination for the recovery and analysis of evidence. And the practical use of such methods tends to increase as more scientific evaluation becomes available.

The goal of this work[1] is the proposition of Computer Vision methods that can assist traditional Forensic Science procedures. The methods are associated with three main forensic topics:

- Photogrammetry: When photographing a certain region of space, the three-dimensional (3D) points are projected to points in the two-dimensional (2D) image, which causes the loss of depth information. Rests with Photogrammetry the study and the proposition of methods to recover such information, allowing geometric measurements in the image plane.
- 3D reconstruction of impressions: Experts in different localities must collaborate to answer questions pertinent to an investigation and thus they need to share existing physical pieces of evidence. Such collaboration requires the transport of such pieces between several locations. This is an expensive process and fragile pieces can be damaged. It is up to this area of study to propose methods for scanning these pieces of evidence or new methodologies for capturing the 3D shapes directly at the crime scene.
- Reconstruction of fragmented documents and images: It is very common in forensic investigations that examiners depend on the quality of preservation of a document or image, for handwriting analysis or content identification. In some cases, however, these documents and images could have been damaged, torn or obliterated. The reconstruction process when they have been torn, for example, can be done manually, which suggests a tedious and time consuming work. It is up to this area of study to propose methods for the automatic reconstruction of such fragmented documents or images, improving the reconstruction accuracy and efficiency when compared to a manual process.

Considering the three previous forensic topics, the contributions of this work are:

1) A new effective vanishing point detector applicable to a single 2D image, and its validation in comparison to other relevant methods, regarding the orthogonality error of the vanishing points and the focal length error [4]. The vanishing point detector is inserted in a Photogrammetry framework in order to facilitate height measurements in a single 2D image within a small error range [5].
2) A pipeline for the 3D reconstruction of footwear im-

[1] This work relates to the Ph.D. thesis "Métodos de Visão Computacional aplicáveis à Ciência Forense", by Fernanda A. Andaló.

pressions from photographs taken around the evidence in different angles. The pipeline is compared to two methods currently used in practice – casting and 3D scanning [6].
3) A new and state-of-the-art method for automatic reconstruction of images from a collection of small rectangular tiles, based on quadratic programming. The method is compared to the previous state of the art, showing its several advantages [7].

In the following sections, we briefly explain each one of these contributions. For a more detailed explanation and thorough experimental analysis, please refer to [8].

II. Measuring heights of objects in an image

By analyzing certain image properties – occlusion, gradient, texture, vergence, etc. – we can infer 3D geometry data of the depicted scenes. The process of extracting geometric properties from images, such as heights, areas, and angles, is denominated *photogrammetry*. Photogrammetry methods are widely used in forensic investigations, where measurements in images can provide useful information about the curse of events and size of objects in crime scenes [9].

To perform photogrammetry tasks, previous works showed that vanishing points and vanishing lines are important characteristics to be analyzed [10]. The contribution published in [4] introduces a novel method to automatically detect vanishing points in images, based on a geometric approach, in which all points are estimated in a single image. The solution is based on the clustering of line segments in the image plane, which obviates the need for *a priori* camera calibration.

Our method for vanishing point detection is divided in three steps:

1) Extraction of line segments on the image plane.
2) Clustering of line segments converging to the same vanishing point (repeated until convergence):
   2.1. Selection of seeds.
   2.2. Grouping of segments based on the seeds and on the distance among intersection points and their corresponding lines on the projective space.
3) Detection of a vanishing point to each final cluster.

Fig. 1 illustrates the three steps to estimate the vanishing points, numerated as shown above.

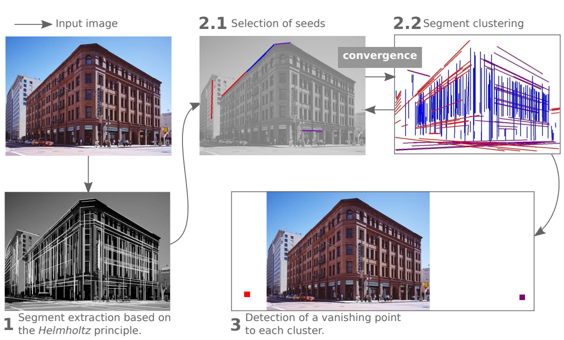


Fig. 1. Steps of the proposed vanishing point detector.

Our vanishing point detector considers line segments as primitives. To detect these primitives, we use a method based on the Helmholtz Principle [11] to construct a set $\mathcal{S} = \{s_1, \ldots, s_{|S|}\}$ of segments. The intersection point between the lines corresponding to segments s_i and s_j is represented by $w_{(l_i,l_j)}$.

The goal of the line segment clustering is to assign a cluster for each one of the line segments in $\mathcal{S}$. To achieve this goal, the clustering process is subdivided in two steps: selection of seeds and assignment.

We select the $2M$ lines with highest corresponding segment quality and randomly choose pairs as seeds for the M clusters. We denote d_{1_i} and d_{2_i} the two seeds of cluster i.

Each segment $s \in \mathcal{S}$ has to be assigned to a cluster. The assignment step is based on the distance between the lines corresponding to the segments and the pseudo-centroids $c_i = w_{(d_{1_i},d_{2_i})}, i = 1, \ldots, M$. The segment s_i is assigned to the cluster with the closest pseudo-centroid.

The selection of the new seeds d_{1_i} and d_{2_i}, for each cluster $i = 1, \ldots, M$, is such that they minimize the error to the lines that would pass through the real corresponding vanishing point. The seed d_{1_i} minimizes the distance to the mean line of cluster i and d_{2_i} is chosen so that the new pseudo-centroid c_i minimizes the distance to some key intersection points.

The vanishing point v_i for the cluster i is the intersection point that is the closest one to all lines in the cluster.

Fig. 2 illustrates a few obtained results for the detection of vanishing points.

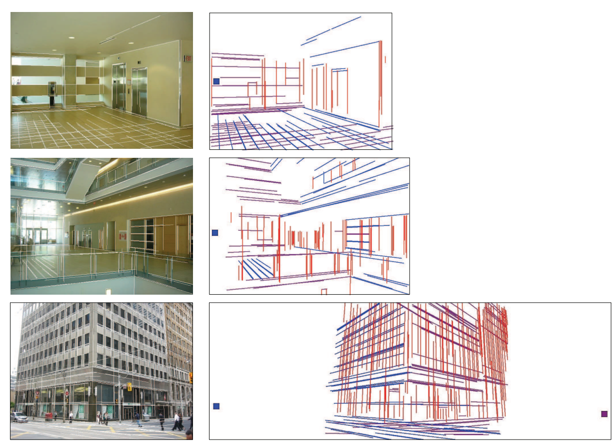


Fig. 2. The first column shows the input image and all detected segments. The second column shows the line clustering result and the detected finite vanishing points. Parallel lines with the same color are associated with a vanishing point at infinity; the other lines are associated with finite vanishing points.

For validation purposes, we compared the obtained results to those of other relevant methods [12], [13], [14]. The first experiment consisted in computing the *orthogonality error* of the detected vanishing points: how much the more orthogonal vanishing points deviate from the real orthogonality on the image plane. The second experiment consisted in computing the *focal length error*: how much the focal length computed from the more orthogonal vanishing points deviates from the expected focal length. In both experiments, the York Urban database [12] was used and our detector provided significantly superior results.

Besides being more accurate, our detector has some advantages: it is invariant to translation and rotation, the accuracy of the points is not limited, it detects finite and infinite vanishing points seamlessly, and it does not require camera calibration.

To compute heights in images, we considered a photogrammetry framework proposed in [10]. This framework considers that vanishing points have been previously detected, and on this detection depends its success. Our detected vanishing points were then inserted into the framework [5].

For measuring unknown heights, the framework also needs the height of a reference object to compute absolute values. Experiments showed that with the use of the photogrammetry framework, it is possible to calculate heights of objects in the image plane, within a small interval of ± 0.41cm. Fig. 3 shows one of the obtained results.

Fig. 3. The first image contains the segment clustering result. The second image contains the height measurement results, where t_x and b_x delimits the measured person; b_r and t_r delimits the reference object.

III. 3D Reconstructing footwear impressions from photographs

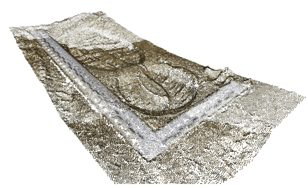
Crime investigation depends on the collection and analysis of various types of evidence. Impression evidence, such as footprints, tire tracks, and tool marks, are an important and common source of physical evidence that can be used to corroborate or refute information provided by witnesses or suspects. According to a study conducted in Switzerland, shoe prints can be found in approximately 35% of all crime scenes [15].

Shoe prints can indicate whether a person was walking or running, was carrying something heavy or was unfamiliar with the area or unsure of the terrain [16]. They can provide additional information about the wearer, such as weight, height, and wear patterns that can be compared with a suspect's shoes. The location of the impressions at the scene can also often help in the reconstruction of the crime [15].

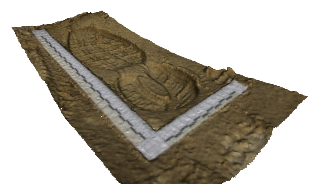
In recent years, the standard method for capturing these 3D impressions is by casting using materials such as dental stone [15] or plaster [16]. Another ever growing system to collect evidence is 3D scanning. The contribution published in [6] introduces an alternative solution that consists of a pipeline through which it is possible to reconstruct the impression three-dimensionally, using only digital photographs. Our new insight for footwear print capturing uses a Computer Vision method, *multiview stereo*, and has some advantages over the current solutions. Multiview stereo methods require photographs of an object at different camera viewpoints. They compute correspondences between image pairs and a depth estimation for each camera viewpoint. By combining these estimates, multiview stereo methods can provide a final 3D model of the object. A similar approach has been applied successfully to recover dinosaur footprints [17].

The pipeline is formed by three previously proposed methods, which together reconstruct a complete 3D model from a collection of photographs taken at different angles around the impression (Fig. 4a). The first step is to retrieve the set of camera parameters and the location of 3D keypoints in each image, using *Bundler* [18]. The second step is to generate a dense point cloud using the method *Patch-based Multiview Stereo* (PVMS) [19] (Fig. 4b). The last step is to reconstruct the surface using the method *Smooth Signed Distance* (SSD) [20] (Fig. 4c).

(a) Examples of input images



(b) 3D Point cloud



(c) Reconstructed 3D model

Fig. 4. Footwear impression 3D reconstruction.

The comparison of this pipeline with the current methods reveal its advantages: it does not require special materials, just the digital camera already used in the crime scene documentation; the time acquisition is negligible; it preserves the evidence, not requiring contact with the surface; there are no restrictive scenarios, as long as pictures can be taken around the evidence; and the accuracy is comparable to 3D scanning. Table I summarizes the three discussed methodologies: casting, 3D scanning, and multiview stereo.

The validation experiments consisted in comparing the 3D model generated by the pipeline with the model generated by 3D scanning, using *Haursdoff distance* computed by *Metro tool* [22]. We concluded that the models are very similar, considering the simplicity of proposed pipeline, with a mean difference of only 0.072 mm. Fig. 5 shows one of the obtained results.

TABLE I
COMPARISON BETWEEN FOOTWEAR PRINTS RECOVERY.

	Casting	3D Scanning	Multiview stereo
Special materials	Dental stone or other quality material	3D scanner	A camera. It can be the same camera already used in crime scene investigations
Time for acquisition	$\sim$ 30min [21]	$\sim$ 15min [21]	$\sim$ 5min
Post acquisition work	Transportation to the analysis location	Depends on the scanner. The model can be ready to be analyzed or it may need to be processed	Pipeline presented in this work
Intrusiveness	It may destroy the evidence	None	None
Good scenarios	The material cannot be too soft	Not all materials are suitable for being scanned	There is no limitation, as long as the examiner can take pictures at different viewpoints

(a) Model generated by 3D scanning.

(b) Model generated by our pipeline.

(c) Model generated by casting.

Fig. 5. 3D models of a footwear impression.

IV. RECONSTRUCTING FRAGMENTED IMAGES

The traditional Jigsaw puzzle is a problem classified as *NP*-complete when the affinity between the tiles is uncertain [23]. However, several scientific challenges such as the reconstruction of documents from shredded paper [24], and reassembling broken archeological artifacts from fragments [25], can be reformulated as 2D or 3D jigsaw puzzle problems.

The contribution published in [8] focus on the problem of reconstructing images from identically shaped rectangular tiles placed without repetition within a regular rectangular grid of known dimensions. Contrary to what occurs in traditional jigsaw puzzles, here the tile shape does not provide any information, making the problem even more challenging.

To solve the described problem, it is necessary to deal with two difficulties. The first one is the combinatorial nature of the problem: the number of possible solutions increases exponentially as a function of the amount of tiles. The second difficulty is the global nature of the problem: no exact compatibility is known to date, based only on local similarity between the tiles' borders.

The proposed method, *PSQP – Puzzle Solving by Quadratic Programming* – is based in maximizing a global matching function which calculates the overall compatibility of a certain tile permutation.

First consider an image partitioned into a regular 2D grid of size $N_{cols} \times N_{rows}$, forming N tiles $t_1, \ldots, t_N$, of identical dimensions. Now consider an empty grid of the same size as the previous one with N locations labeled $1, \ldots, N$. The problem is to determine a one-to-one correspondence between the N tiles and the N locations, optimal with respect to the following global matching function of a permutation π:

$$\varepsilon(\pi) = \sum_{(i,j)\in E_H} C_{H_{\pi(i)\pi(j)}} + \sum_{(i,j)\in E_V} C_{V_{\pi(i)\pi(j)}}, \qquad (1)$$

where $e = (i, j)$ represents the neighboring locations i and j; E_H and E_V are the sets that contain the horizontal and vertical neighboring locations, respectively; $\pi(i)$ can be regarded as a 1-1 mapping which assigns the tile $t_{\pi(i)}$ to the location i; and $C_{H_{i,j}}$ and $C_{V_{i,j}}$ are two local matching compatibilities of assigning tiles t_i and t_j to horizontal and vertical neighboring locations, respectively.

Our goal is to maximize this function over all the permutations π of N elements. Since this is a hard combinatorial optimization problem, we first extend the domain of the global matching function to the set of doubly stochastic matrices, and we reformulate the problem as a constrained continuous optimization problem, which we solve using numerical methods.

By representing π as a permutation matrix P, we can reformulate the global function as

$$\varepsilon(P) = \sum_{(i,j)\in E_H} (P^\top C_H P)_{ij} + \sum_{(i,j)\in E_V} (P^\top C_V P)_{ij}, \tag{2}$$

where a generic term $(P^\top CP)_{ij}$, corresponding to $e = (i, j)$, is the element (ij) of the square matrix $(P^\top CP)$.

If we represent the columns of the $N \times N$ matrix P as a vector p of dimension N^2, we get

$$\varepsilon(P) = \sum_{(i,j)\in E_H} p_i^\top C_H p_j + \sum_{(i,j)\in E_V} p_i^\top C_V p_j. \tag{3}$$

We can reformulate this equation in the canonical form $p^\top Ap$, where A is a symmetric $N^2 \times N^2$ matrix, representing the Hessian of $\varepsilon(P)$. By extending the domain of $\varepsilon(P)$ to all the doubly stochastic matrices, the problem reduces to solving the following quadratic optimization problem

$$\begin{aligned} &\text{Maximize } f(p) = p^\top Ap, \\ &\text{subject to } P\mathbb{1} = \mathbb{1}, P^\top \mathbb{1} = \mathbb{1}, \text{ and } p_{ij} \geq 0, \end{aligned} \tag{4}$$

where $\mathbb{1}$ is a column vector of size N with all elements equal to one. We use a constrained gradient ascent algorithm, with gradient projection [26], to search for local maxima of this problem.

To compare *PSQP* to the recently proposed method by Pomeranz et al. [27], we use the same database of 20 images provided by [28], where each puzzle consists of 432 tiles of 28×28 pixels. We also consider two performance metrics presented in [28]: **Direct comparison** – the obtained permutation is compared directly to the ground-truth permutation; and **Neighbor comparison** – the reconstruction accuracy is the average fraction of correct neighboring tiles.

The average performance is 96.0% under Direct comparison and 95.6% under Neighbor comparison. The reported accuracy for the method of Pomeranz et al. [27] is 91% and 94%.

The *PSQP* method improves the state of the art, since it has several advantages not found in other methods [28], [27]: it is fully automatic, requiring no information *a priori*; it is deterministic; it has higher accuracy; it can solve problems with rectangular tiles, not just square ones; it has a good tradeoff between accuracy and runtime.

Fig. 6 shows some image puzzles in which *PSQP* is more accurate according to both metrics, and Fig. 7 illustrates a problem with 3300 tiles.

V. Conclusion

This Ph.D. thesis explored Computer Vision methods that can be applied to traditional problems in Forensic Science. This study is justified because although there is a certain degree of automation in traditional forensic methods, a large part of what is used in practice involves skill and art, rather than science [29]. This work has resulted in publications [4], [5], [6], [8] and has raised community interest, being covered by *Jornal da Unicamp*[2] and by a local radio program *CBN Total, CBN Campinas*.

Fig. 6. Image puzzles with 432 tiles of 28×28 pixels each. For each sub-image, the upper left is the original image, the upper right is the initial configuration of the puzzle for *PSQP*, the lower left is the final result for *PSQP*, and the lower right is Pomeranz et al.'s result.

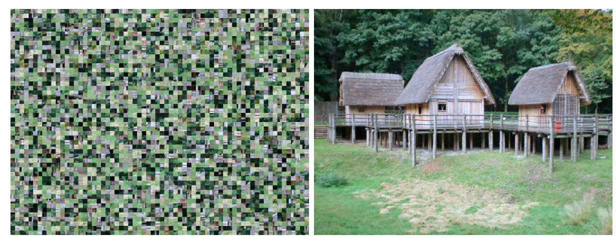


Fig. 7. Puzzle with 3300 tiles. Solution with 100% accuracy.

This work has contributed to three different forensic problems: photogrammetry, 3D reconstruction of impressions, and reconstruction of fragmented images. More specifically, the contributions are:

1) A method to the detection of vanishing points in single images, and their application in a photogrammetry framework [4], [5]. Its application in Forensic Science can be useful because it allows the measurement of heights of people and objects in images of anthropic environments, like the images generated by CCTV cameras – *Closed-circuit television*.
2) A pipeline for reconstructing three-dimensionally footwear impressions, based on a Computer Vision technique, *multiview stereo*, which had not been considered for forensic applications before [6]. The pipeline was compared to the main methods used in practice, casting and 3D scanning, presenting several advantages.
3) A new quadratic formulation called *PSQP – Puzzle Solving by Quadratic Programming* – to the resolution

[2] http://www.unicamp.br/unicamp/ju/553/do-computadorpara-ciencia-forense

of image puzzles [8]. The method is the current state-of-the-art in the resolution of such problems, because it provides higher accuracy when compared to other recent methods [28], [27].

There are several planned extensions to the three methods. Some of them are already being studied or implemented. They are described below.

- It is crucial that all three methods are tested with data from forensic investigations. Usually forensic data is confidential, but we seek partnerships so that our methods can be tested in real-world situations. For the first method, for example, we need CCTV images pertinent to an investigation along with relevant information.
- The 3D model generated by the pipeline proposed for the 3D reconstruction of footwear impressions is as faithful as possible to the physical impression. Any method for reconstructing impressions should capture as much detail as possible. Often, however, the impression is found incomplete or with noise caused by the very action that generated the impression, by objects that may be on the ground, or the material itself being stepped on. In this case, it is interesting to perform a reconstruction of the damaged parts, offering ideas on how the same impression would be like if complete and noiseless.
- The *PSQP* considers that all tiles are informed as input. However, in real scenarios, not all tiles are always available. An extension is needed to address this new challenge.
- The *PSQP* considers that, for a problem instance, all input tiles belong to a single puzzle. In real scenarios, however, tiles of different puzzles can be mixed together. An extension must be studied to yield solutions for every puzzle involved.
- In *PSQP*, tiles are provided with their correct rotation. In a real scenario, the puzzle tiles are shuffled and probably are not originally in their correct rotation. In this case, it is up to the assembler to choose the best rotation for each tile so that it fits in the general solution. The *PSQP* should be extended to cover this challenge.

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