Robustness Metrics for Interdependent Infrastructures

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Goal

Develop infrastructure robustness measures that account for the

- structure,
- dynamics, and
- physical properties

of interdependent infrastructures.

Robustness Questions

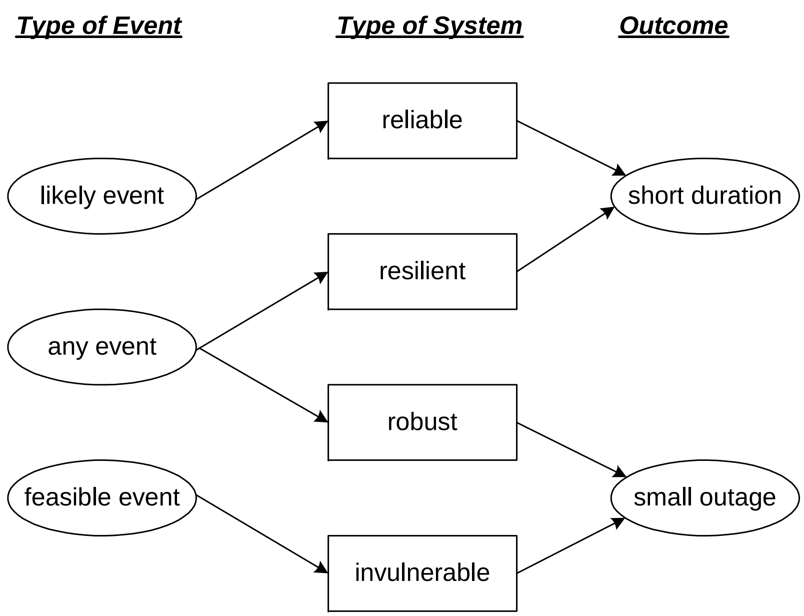
- closeness to limits
  - How close is the system to its consumption, production, and/or capacity limit?
  - How much can production limits and/or capacity limits be reduced and still serve consumption?
  - How much can consumption be increased given the production and capacity limits?
  - How much production or capacity must be lost to cause some consumption site to lose service?
  - How much production or capacity must be lost to cause all consumption sites to lose service?
  - Where can additional production capability or line capacity be added to the network to make
  - it more robust?
- breadth of support
  - In what localized area does a production site supply consumption sites?
  - In what localized area does a consumption site draw on production sites?
  - Are a few transmission lines or production sites responsible for most of the system capacity or are many?
- degradation
  - How much does random degradation of the system affect its ability to serve consumption?
  - How much does an attack on one consumption site affect others?
  - What is the structure of minimum cuts and how are they correlated?
  - How many pieces can the graph be broken into and still have the consumption satisfied (or
  - *x*% of it satisfied)?
- temporality
  - How much time does it take to restore the system after contingencies occur?

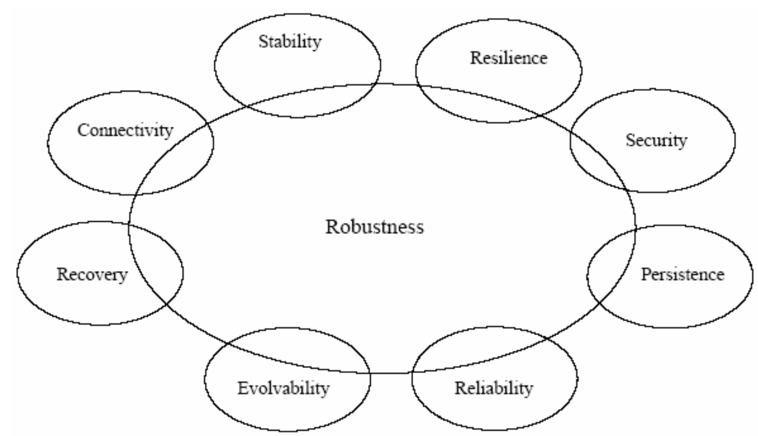
Multi-Pronged Approach to Robustness

- taxonomy of concepts
  - definitions for “robustness”
  - relationship between robustness and allied concepts
- metric and algorithm development
- analysis
  - traditional graph theory
  - “small world” parameters
  - statistical
- applications

Taxonomy of Robustness Concepts

- We are developing precise definitions of robustness and related concepts.
- We are mapping the interrelationships between various metrics.





metric	likelihood of event	size of event	duration of outage	severity of outage
reliability	likely	small	short	minor
robustness	any	any	any	minor
fragility	any	any	any	severe
resilience	any	small	short	minor
performance	any	any	any	any
invulnerability	rare	any	long	severe

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Metrics Definition & Algorithms

- connectivity metrics
  - cut sets
    - minimum number of edges that must be removed to separate a consumer *t* from a producer *s*
    - minimum number of edges that must be removed to separate a consumer *t* from all producers
    - minimum number of edges that must be removed to separate all of the consumers from all of the producers
  - connectedness
    - strength
    - toughness
    - independence
    - scattering number
  - partitions
    - largest number of subgraphs where demand is still satisfied

Metrics Definition & Algorithms (cont'd)

- structure of Gomory-Hu cut trees

- flow metrics
  - performance indices

$$I_\delta(f,\bar{f}) = \left(\frac{1}{|E|}\sum_{e\in E}\left|\frac{f(e)}{\bar{f}(e)}\right|^\delta\right)^{1/\delta}$$

  - how much consumption can be increased and still satisfied

$$\Delta x = \max_{\Delta c \geq 0}\left\{\|\Delta c\| \,\middle|\, (f', p', c'') = F(V, E, \bar{f}, \bar{p}, c' + \Delta c) \text{ provided } \begin{array}{c} c'' = c' + \Delta c \\ c' + \Delta c \leq \bar{c} \end{array}\right\}$$

  - how much production or capacity limits can be decreased and still satisfy consumption
- area of influence
  - minimum or average area supplied by a producer divided by production capacity

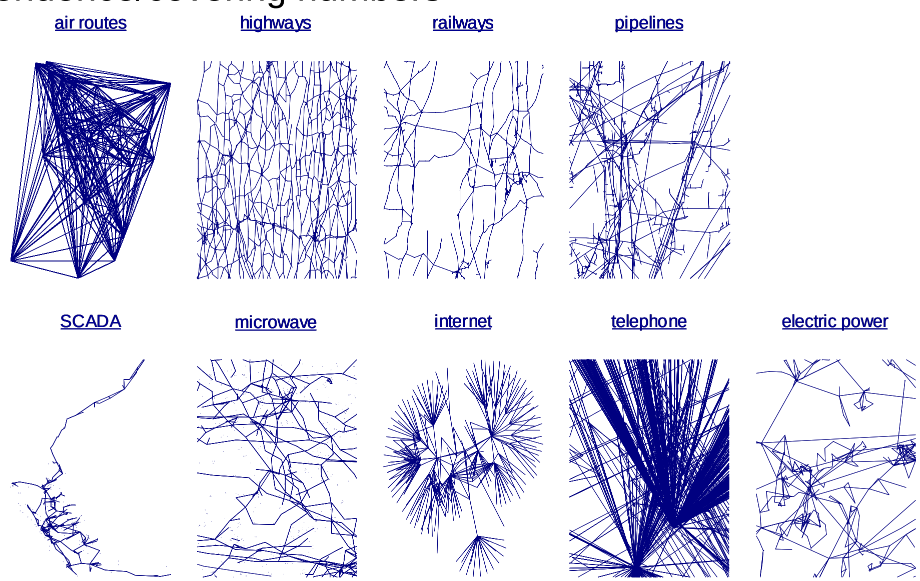
Metrics Definition & Algorithms (cont'd)

- redundancy
  - betweenness
    - measure of the number of shortest paths an edge is on
    - quantifies redundancy of edges
  - shortest cycles
- temporality
  - minimal cuts with a least a specified restoration time

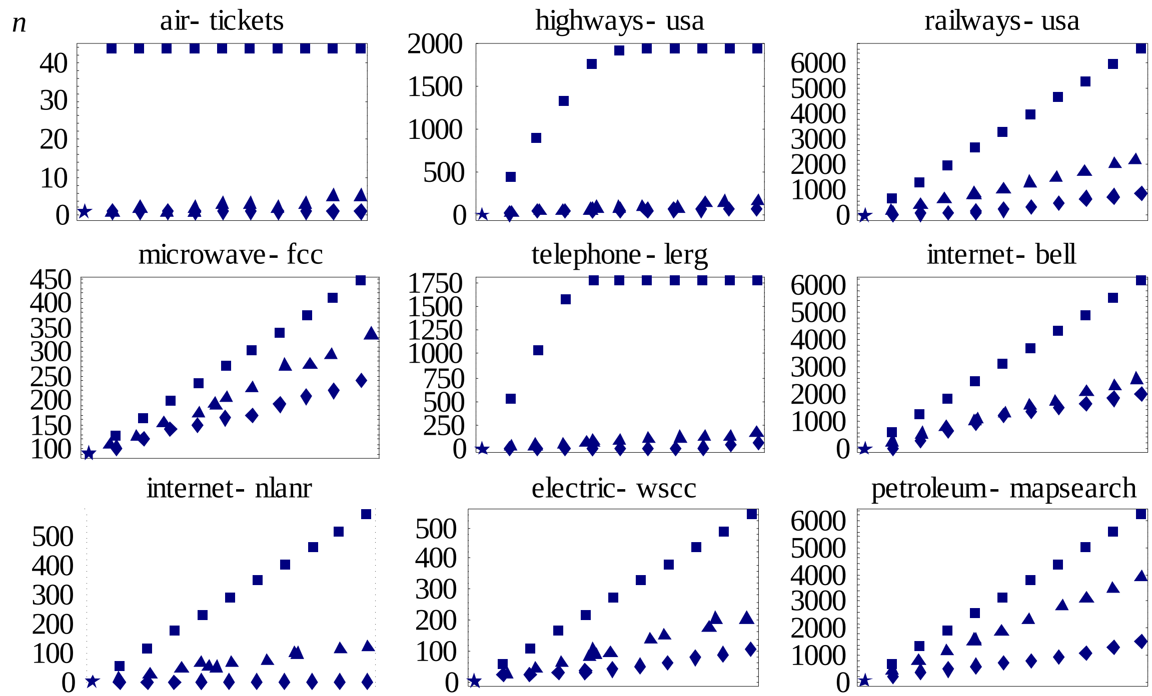
$$r(T_0, s, t) = \min_{S \subseteq E} \left\{ \sum_{e \in S} w(e) \,\middle|\, \sum_{e \in S} T(e) \geq T_0 \text{ and there is no path from } s \text{ to } t \text{ in } (V, E - S) \right\}$$

Analysis: Traditional Graph Theory

- infrastructure graph base
  - 14 network types: airport, highways, railways, roads, waterways, intermodal transport, utility communications, microwave backbones, telephone, internet, electric power transmission, petroleum transmission, natural gas transmission, military logistics
  - 44 networks
  - $\sim 10^6$ vertices and $> 10^6$ edges
- whole-graph metrics
  - order, size, number of components
  - vertex chromatic/domination/independence/covering numbers
  - radius, diameter, girth
  - minimum/maximum degree of a vertex
  - vertex/edge connectivity
- vertex metrics
  - articulation points
  - eccentricity
  - shortest path tree
  - shortest cycle
  - minimum cuts between pairs
- edge metrics
  - bridges



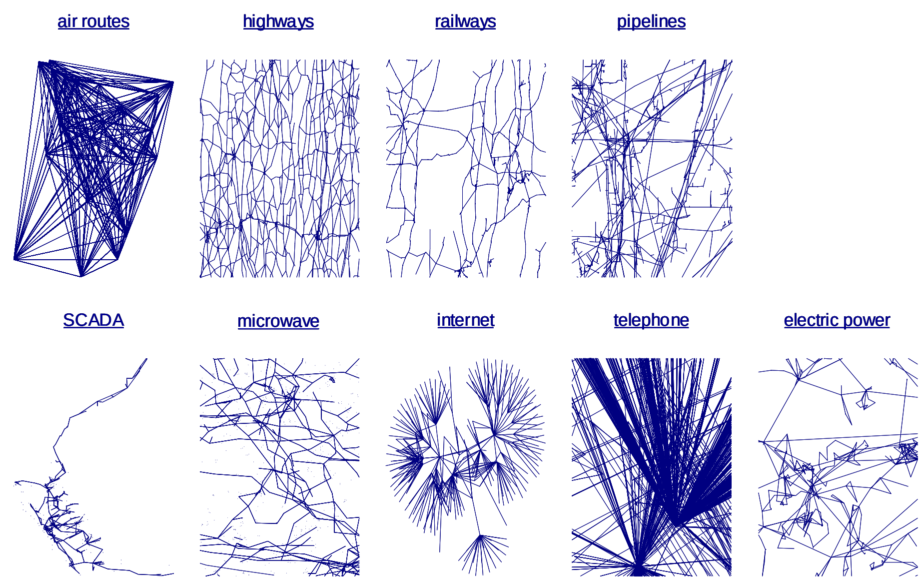
Fragmentation of Degraded Networks



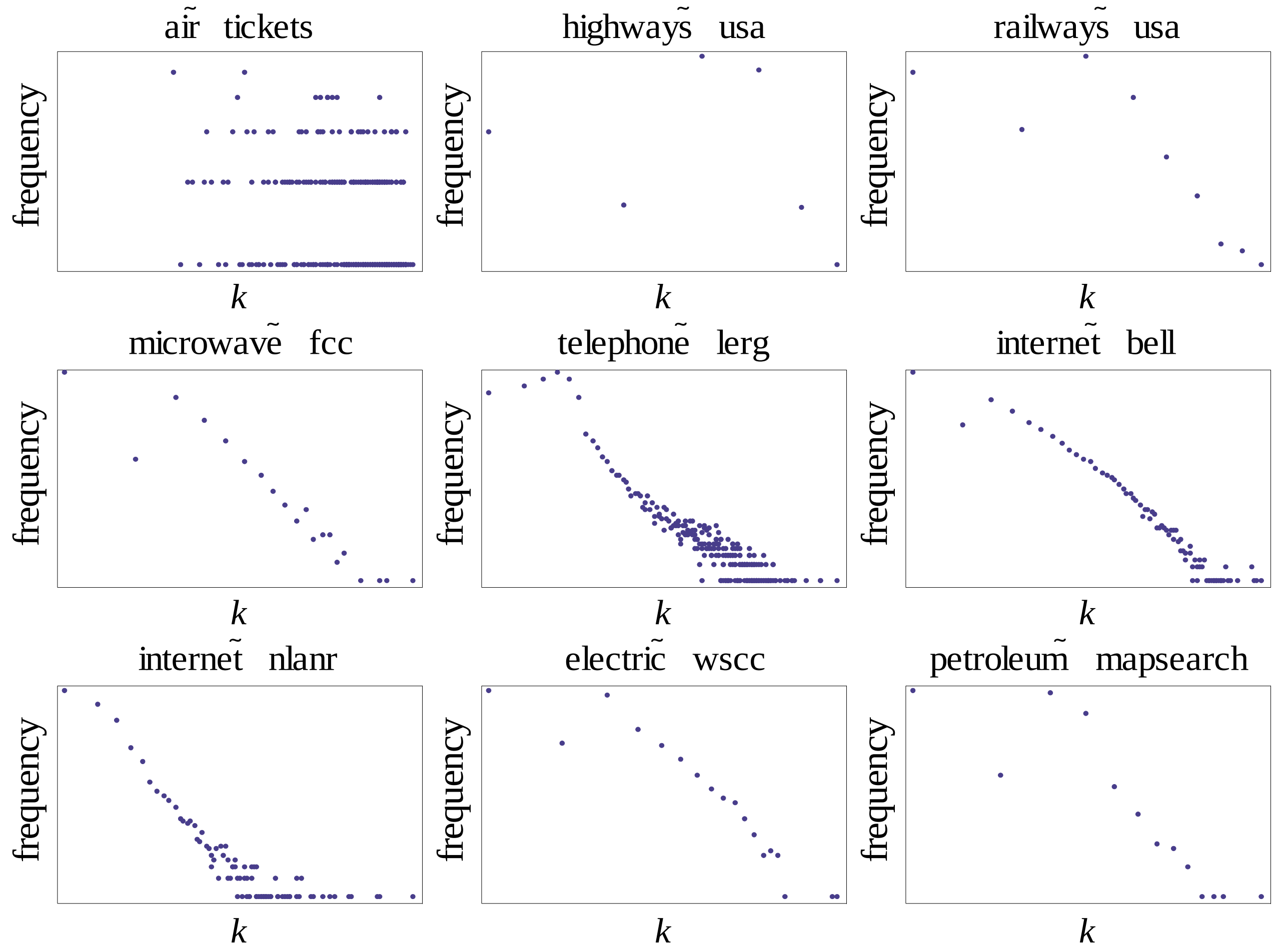
Measurements of the number of graph components (vertical axis) in degraded networks as a fraction of edges deleted (horizontal axis on a scale from 0% to 5%): the star represents the intact network, the triangles represent deletion at random, the diamond represents deletion by degree, and the box represents deletion of bridges.

Analysis: “Small World” Parameters

- infrastructure graph base
  - 14 network types: airport, highways, railways, roads, waterways, intermodal transport, utility communications, microwave backbones, telephone, internet, electric power transmission, petroleum transmission, natural gas transmission, military logistics
  - 44 networks
  - ~ 10^6 vertices and > 10^6 edges
- whole-graph metrics
  - mean degree
  - characteristic path length
- vertices metrics
  - clustering coefficient
  - contractions
- edges metrics
  - shortcuts
- Many of the traditional metrics and “small world” metrics directly apply to robustness, criticality, and interdependency analyses.



Scaling Behavior of Degree Distribution

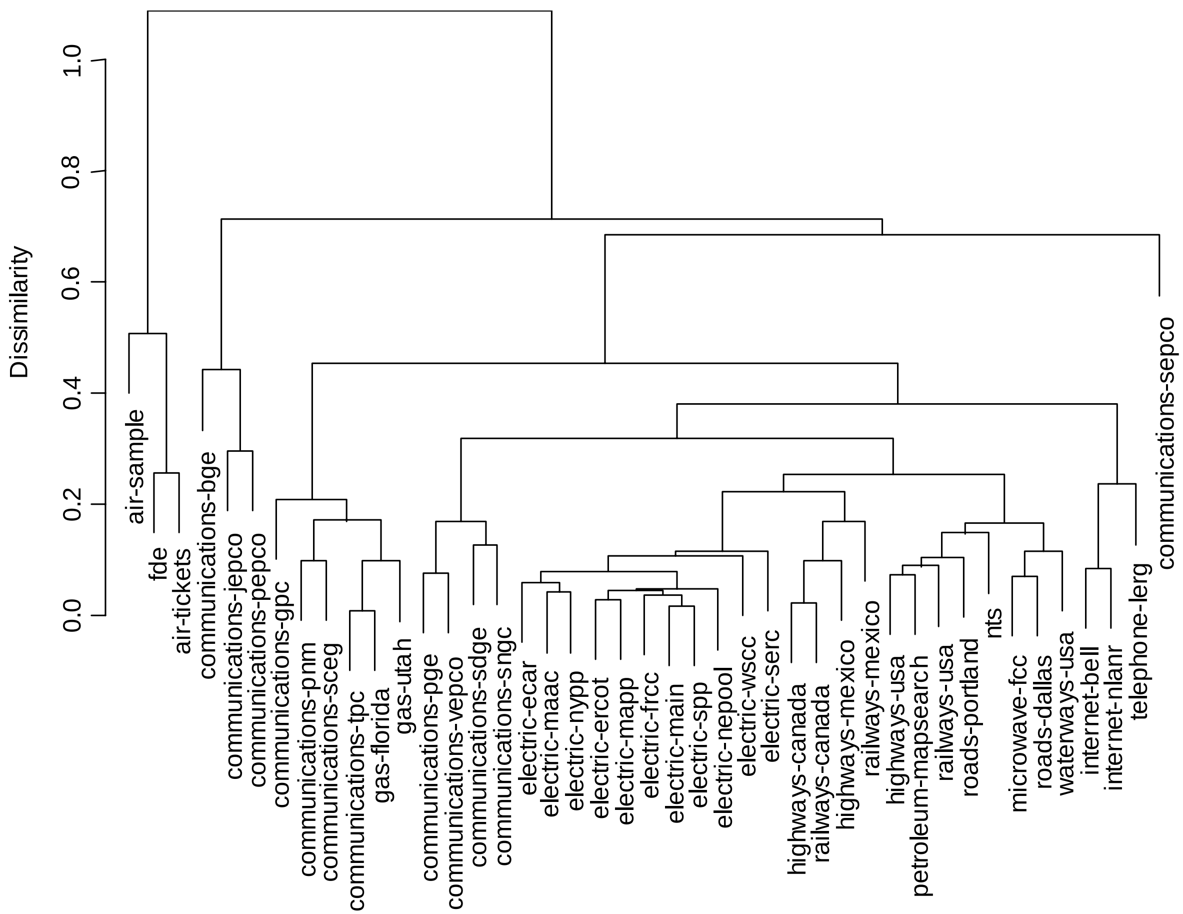


log-log scale

Analysis: Statistical Techniques & Applications

- Correlation matrices computations allowed us to identify redundancies between metrics.
- Principal component analysis allowed us to identify a small set of metrics accounting for most of the variation between networks.
- Regression analysis allowed us to identify predictors of hard-to-compute metrics in terms of easy-to-compute ones.
- Clustering, classification, and multinomial logit analyses allowed us to identify similarities between infrastructure networks.
- Data depth (convex null) computations allowed us to detect anomalous elements in the infrastructure networks.

Entropy-Based Clustering



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Applications

- identifying anomalous portions of networks
  - highlights possible problems in source data
  - finds atypical areas that may warrant special attention
- identifying critical components
  - constructs sets of contingencies for detailed simulation
  - eliminates uninteresting cases from large scale contingency screening simulations
- measuring network degradation
  - guides heuristic searches for finding worst case scenarios
- assesses impact of system modification
  - useful for system optimization