Day 3 AM: Sequences and Sets

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Math Camp 2021

Day 3: Sequences and Sets

Sequences

A sequence is an ordered list of numbers. They can be infinite or finite, but all are *countable*.

Lingo

We refer to the elements by their position in the sequence – the third element would be x_3. We can talk about the entire sequence as being generated by some equation or formula and represent it accordingly. So, if we take each element to the third, our sequence would be $\{1, 2^3, 3^3, \ldots\}$ and we could reference the sequence as $\{i^3\}_{i=1}^{\infty}$

Series

A series is the *sum* of a sequence. (Book reasoning unhelpful here – we care because you'll be adding probabilities in class).

Summation

We may have a large or otherwise complicated series of numbers to add. For example, suppose we wanted to add the numbers from 1-10. We could write the list out, (1,2, 3, ... , 10) or we could use the summation operator:

$$\sum_{n=1}^{10} n$$

SETS! & Matrices!

Sets: Numbers

Recall from yesterday:

- N: Natural numbers $\{(0), 1, 2, 3, ...\}$
- Z: Integers (negative and positive including zero) $\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$
- Q: Rational numbers (q for quotient, rational numbers, e.g. expressed as a fraction)
- R: Real numbers (positive, negative, zero, integers, fractions/rational); any point on the number line
- I: Imaginary numbers ($i = \sqrt{-1}$)
- C: Complex numbers ($a + bi$)

Notes: Subscript: Z_+ only the positive or Q_- negative elements of the set Superscript: N^2 dimensions of the space

Sets: displaying sets

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integers such that x is equal to its own square" The only number that satisfies this is $\{1\}$

Sets, details

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- Finite (finite number of elements) or infinite (no limit)
- Countable or uncountable (elements can be counted or not (e.g $\{0, 1\}$ is countable)

Sets, details

We care about sets and how elements are contained within them, and how the sets are shaped. We'll use this information in probability. It's also helpful when thinking about the possible responses and individuals who may fall in your dataset.

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Open and closed sets matter because this affects how we think about the contents of sets – what we call the 'elements'.

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- Universal set (the universe: all elements)

Sets: Union and Intersection

Suppose $A = \{4, hat\}$ and $B = \{hat, 7\}$

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Intersection ($\cap$)
Intersection is the collection of elements present in both sets (AND): $A \cap B = \{hat\}$ (Def: $A \cap B = X : X \in A$ and $X \in B$)
Sometimes the intersection is empty.

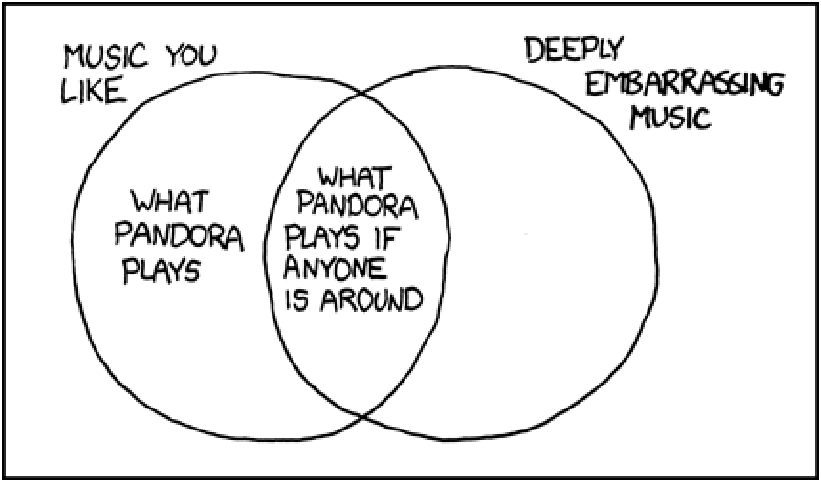

Figure 1: "Source: XKCD"

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Example: How to partition countries? Could do by continents, population size, etc – but pay attention that, say Turkey, isn't assigned to two regions. Or that you don't forget Malta!

Sets: cont'd

Difference Difference between A and B, $A \backslash B$ ("A difference B") is the set containing all the elements of A that are not also in B. $x \in A$ but $x \notin B$

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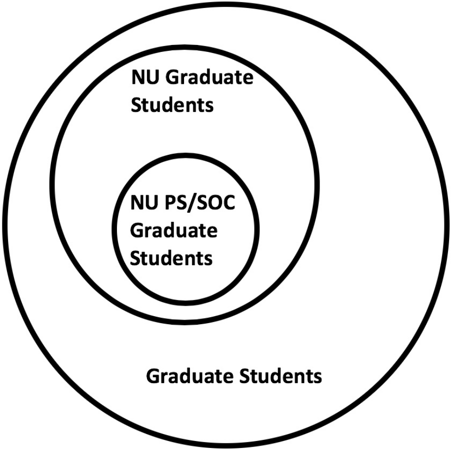
Complement Complement A' or A^c contains the elements that are not contained in A. $x \in A^c$ if $x \notin A$

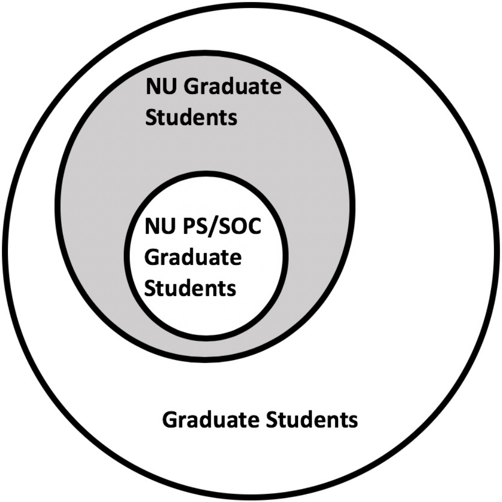
Sets: cont'd

Find NU GS$\setminus$PS/SOC Grad Students



Sets: cont'd

NU GS$\setminus$PS/SOC Grad Students: AKA, where are the NU grad students who aren't in PS/SOC



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  - AxB where $A = \{apple, banana, kiwi\}$ and $B = \{2, 4\}$. $AxB =$\{ \{apple, 2\}, \{apple, 4\}, \{banana,2\}, \{banana,4\}, \{kiwi, 2\}, \{kiwi, 4\} \}

Sets: Review

- Unions: OR, $\cup$
- Intersections: AND, $\cap$
- Ordered/Unordered
- Complements (not inside) c, written A^c, for example.
- Subsets and proper subsets (contained within): $\subset, \subseteq$
- Cardinality (number of elements)

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- Does order matter?
- How would it affect the sample space? Would we have more or fewer possible hands?
- Now, you can swap one card. How many ways could you do it? Does it matter what cards you have?

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- Now, you can swap one card. How many ways could you do it? Does it matter what cards you have? You could trade in any one of your 4 cards. In this scenario, it does not matter

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Suppose you have a deck of ten cards, numbered 1-10, and are dealt four cards. Answer the following questions, labelling them with the appropriate terms we've discussed so far:

- Finally, what if I told you that all 4 of my cards were even numbered. Would you be surprised? How would you know whether to be surprised?

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- Finally, what if I told you that all 4 of my cards were even numbered. Would you be surprised? How would you know whether to be surprised? We will discuss this, and many other interesting things!, in class! soon!