

## Day 4: Algebra Review

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Math Camp 2021

# Questions?

Concluding CH 1 of Moore & Siegel – moving to Ch 2

## Day 4: Concept Agenda

- ▶ Algebraic Properties
- ▶ Factoring
- ▶ Inequalities and other operations
- ▶ Combinatorics
- ▶ Functions and Relations
- ▶ Exponents and Logarithms
- ▶ PLUS! Practice on all these items and a bit to tie it together.

# Algebra Review

## Properties

- ▶ **Associative property:**  $(a + b) + c = a + (b + c)$  and  $(a * b) * c = a * (b * c)$
- ▶ **Commutative property**  $a + b = b + a$  and  $a * b = b * a$
- ▶ **Distributive property**  $a(b + c) = ab + ac$
- ▶ **Identity property**  $x + 0 = x$  and  $x * 1 = x$
- ▶ **Inverse property**  $-x + x = 0$ . Multiplicative inverse exists, but not for all numbers  $x^{-1} * x = 1$

# Factoring

We may need to break down different functions.

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2.  $m^2 + 3m + 2 =$

3.  $x^2 + 5x + 6 =$

4.  $25 - x^2 =$

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# Inequalities

## Relations: Intervals & Inequalities

Interval notation can be used to express ranges of numbers :

$[a, b]$      $a \leq x \leq b$     Square brackets include end points (closed interval)

$(a, b)$      $a < x < b$     Parenthesis mean exclude end points (open interval)

$\{a, b\}$     Typically used for sets – not inequalities/intervals

## Relations

**Graph the following:**

►  $4 < x$

►  $y > 12$

►  $3 < z < 7$

►  $(3, 9)$

►  $[-7, 2)$

## Solving Inequalities

Solve like regular equations but FLIP inequality when multiplying by negative values.

EX:

$$\blacktriangleright 3x < 4x + 2$$



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Try:

- ▶  $-6(x + 8) < 12$

# Absolute Value

Solve for TWO possibilities: quantity is positive or negative. EX:

$$|x - 3| > 4$$

- ▶ Quantity is positive: drop bars, solve like usual:

$$x - 3 > 4, x > 7$$

- ▶ Quantity is negative: then, really have  $x - 3 < -4$  Solve to find  $x < -1$

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Solution:  $x < -1$  OR  $x > 7$  Try:  $|2x - 3| > 9$

# Factorial

The factorial, !, multiplies a number by each subsequent number down to one.

For example,  $4! = 4 * 3 * 2 * 1$

You can also divide and multiply factorials:  $\frac{3!}{4!} = \frac{3*2*1}{4*3*2*1} = \frac{1}{4}$

**CANNOT ADD THE NUMBERS!! (e.g.  $6!3! \neq 9!$ )**

## Combinatorics: Combining elements

We can use factorials to help us understand ways of combining elements: e.g. suppose you are forming a committee of 3 people from a group of 5. How many ways can we do that?

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Then we can have the following committees: ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, CDE, for a total of ten configurations.



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We could also calculate them using the binomial function:

$$\binom{5}{3} = \frac{5!}{3!2!} = \frac{5*4}{2} = 10$$

Try  $\binom{4}{2}$  with population A,B,C,D. Using the general formula:

$$\binom{N}{k} = \frac{N!}{k!(N-k)!}$$

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You may have different populations and different treatments – how many different groups would you need to test the possible combinations?

# Functions and Relations

Relations allow comparison of variables and expressions – some may be more or less specific in how they assign or specify these relationships between the *range* ( $y$ ) and *domain* ( $x$ ). Suppose we have a domain as follows { apple, kiwi, lime } and a range of { 4, 5 }.

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(Exactly one  $y$  per  $x$ , but  $y$  can be assigned to multiple  $x$ )

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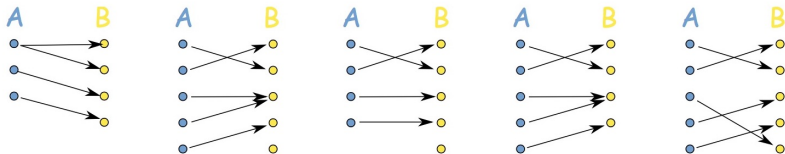


Figure 1:

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Functions can only assign one  $x$  to one  $y$  (and not one  $x$  to multiple  $y$  values) – otherwise it is a relation.

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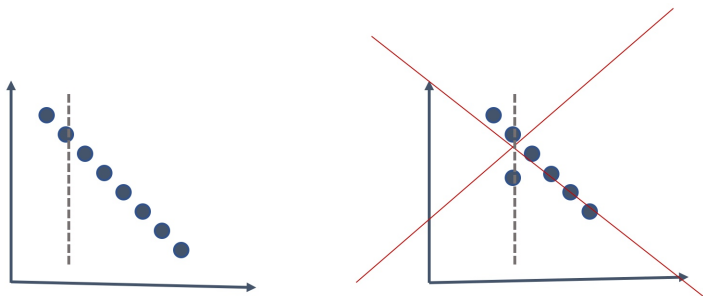


Figure 2: “Vertical line test”

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You can also combine functions, e.g.  $f(x) = x$  and  $g(x) = x^3$  can be combined to produce  $f \circ g(x) = f(g(x))$ .

# Function Terms

Table 3.1 from book (pg 49)

Term	Meaning
Identity function	Elements in domain are mapped to identical elements in codomain
Inverse function	Function that when composed with original function returns identity function
Surjective (onto)	Every value in codomain produced by value in domain
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Why do we care?

# Function Terms

We care about whether a function is surjective, injective, or bijective because we will know if we can trace back what produced what we have.

<https://www.mathsisfun.com/sets/injective-surjective-bijective.html>

# Function Terms

- ▶ We can see that both functions are surjective – each ‘covers’ the codomains
- ▶ In the case of  $g(x)$ , we can see that it is injective, while  $f(x)$  is not
- ▶ Thus,  $f(x)$  is not bijective but  $g(x)$  is (both surjective and injective) – thus, it is invertible (to be defined)

# Composite Functions

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Composite functions,  $f \circ g(x) = f(g(x))$  and  $g \circ f(x) = g(f(x))$

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- ▶  $f \circ g(x) = 4x^2 + 1$
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We can see that we only get one  $y$  value for each  $x$  and that, depending on the domain/codomain, the function is surjective as well, making this function bijective and have a legitimate inverse.

# Monotonic Function Terms

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Increasing	Function increases on subset of domain
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Strictly increasing	Function always increases on subset of domain
Strictly decreasing	Function always decreases on subset of domain
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(Strict) monotonicity	Order preservation; function (strictly) increasing or decreasing over domain

Table 3.2 from book (pg 51)

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Table 3.2 from book (pg 51)

This is useful for math land but also for theory building: how are  $x$  and  $y$  related? Does more  $x$  ALWAYS mean more  $y$  (function is monotonically increasing, etc)?

# Linear Equation vs Linear Function

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We have the **intercept** (where the line crosses the y axis) and the **slope** (unit increase in iv related to dv). This is the 'plain vanilla' version.

The **linear function** is much more expansive: includes multiple variables, exponents, and logs (logarithms).

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**Try:**  $xz^2(x^3z^{-2})^3 = (xz^2(x^9z^{-6})) = x^{10}z^{-4}$

# Exponents, Exponentials, Exponential functions

## Exponents

Exponents are where you take a variable to some power – e.g.  $x^a$  where  $x$  is a variable and  $a$  is a constant. Typically, we focus on the numerical portion of the exponent—calling it ‘the exponent’.

## Exponential

An exponential is the reverse of the exponent – here it is a number to the power of a variable, e.g.  $a^x$ . To get the  $x$  ‘down’, we need to use logarithms (aka logs).

## Exponential Function

The exponential function has a particular base,  $e$ , (where  $e$  is Euler’s  $e$  and is approx 2.72.)

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**We care because we might see these as utility functions or use higher order values as a way to represent the relationship between two variables in a linear regression** Know what these concepts are—will be relevant later.

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- ▶  $y = \log(z) \leftrightarrow 10^y = z$
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- ▶  $\log(1) = 0$

Exponents in log are different from what you might expect:

- ▶  $\log(x^2) = 2(\log(x))$
- ▶  $\log(x/y) = \log(x) - \log(y)$  provided  $(x, y > 0)$

Logs help weigh smaller values more heavily; adding units not linear—less meaningful for larger values ( $\log(100) = 2$ ,  $\log(1000) = 3$ ).



# Logs Practice

Simplify the following

▶  $\log(x^4)$

▶  $\log(xy)$

▶  $\ln(e^3)$

▶  $\ln(1)$

▶  $\log(3) + \log(7)$

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Simplify the following

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- ▶  $\log(3) + \log(7) = \log(21)$

## More Practice

- ▶  $2e^{6x} = 18$
- ▶  $e^{x^2} = 1$
- ▶  $2^x = e^5$
- ▶  $2^{x-2} = 5$
- ▶  $\ln(x^2) = 5$
- ▶  $\ln(x^3) - \ln(x) = \ln(16)$

When would I ever actually see a log function??



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<https://thesocietypages.org/graphicsociology/2010/12/07/1247/>

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