

Day 3: Sequences and Sets

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Math Camp 2022

Day 3: Concept Agenda

- ▶ Data Aggregation
 - ▶ Types of data elements
 - ▶ Sets and Subsets → will lead into Day 5's discussion on matrices
 - ▶ Notating different data aggregates and their relevant operations
 - ▶ How sets and other data aggregates translate into sample spaces

Sequences and Series

Sequences

A sequence is an ordered list of numbers. They can be infinite or finite, but all are *countable*.

Lingo

We refer to the elements by their position in the sequence – the third element would be x_3 . We can talk about the entire sequence as being generated by some equation or formula and represent it accordingly. So, if we take each element to the third, our sequence would be $\{1, 2^3, 3^3, \dots\}$ and we could reference the sequence as $\{i^3\}_{i=1}^{\infty}$

Sequences and Series

Series

A series is the *sum* of a sequence. (Book reasoning unhelpful here – we care because you'll be adding probabilities in class).

Summation

We may have a large or otherwise complicated series of numbers to add. For example, suppose we wanted to add the numbers from 1-10. We could write the list out, (1,2, 3, ... , 10) or we could use the summation operator:

$$\sum_{n=1}^{10} n$$

Sets: Numbers

Recall from yesterday:

- ▶ N: Natural numbers $\{(0), 1, 2, 3, \dots\}$
- ▶ Z: Integers (negative and positive including zero)
 $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- ▶ Q: Rational numbers (q for quotient, rational numbers, e.g. expressed as a fraction)
- ▶ R: Real numbers (positive, negative, zero, integers, fractions/rational); any point on the number line
- ▶ I: Imaginary numbers ($i = \sqrt{-1}$)
- ▶ C: Complex numbers ($a + bi$)

Notes: Subscript: Z_+ only the positive or Q_- negative elements of the set
Superscript: N^2 dimensions of the space

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integers such that x is equal to its own square” The only number that satisfies this is $\{1\}$

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- ▶ Countable or uncountable (elements can be counted or not (e.g $\{0, 1\}$ is countable)

Sets, details

We care about sets and how elements are contained within them, and how the sets are shaped. We'll use this information in probability. It's also helpful when thinking about the possible responses and individuals who may fall in your dataset.

Open

Open sets—essentially the boundary is a little fuzzy. This is the set version of open brackets $()$. The more technical definition has to do with an 'epsilon ball' where, you can always nudge a little closer to the boundary of the set without actually reaching it.

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Open and closed sets matter because this affects how we think about the contents of sets – what we call the 'elements'.

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- ▶ Universal set (the universe: all elements)

Sets: Union and Intersection

Suppose $A = \{4, \textit{hat}\}$ and $B = \{\textit{hat}, 7\}$

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Union is the combination of elements in either set (OR):

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Intersection (\cap)

Intersection is the collection of elements present in both sets

(AND): $A \cap B = \{\textit{hat}\}$ (Def: $A \cap B = X : X \in A \text{ and } X \in B$)

Sometimes the intersection is empty.

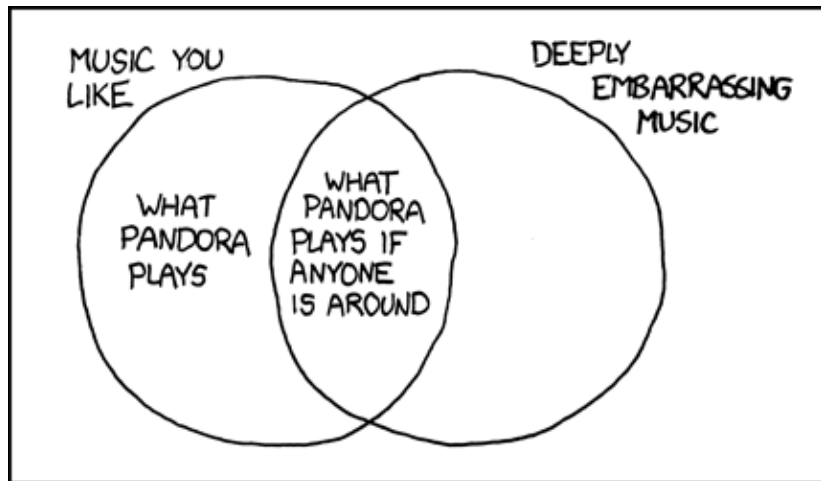


Figure 1: "Source: XKCD"

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Example: How to partition countries? Could do by continents, population size, etc – but pay attention that, say Turkey, isn't assigned to two regions. Or that you don't forget Malta!

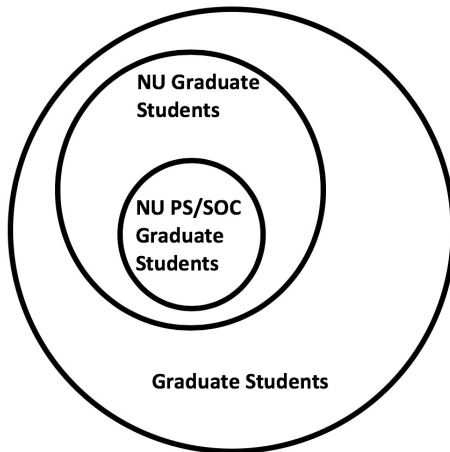
Difference Difference between A and B , $A \setminus B$ (“ A difference B ”) is the set containing all the elements of A that are not also in B . $x \in A$ but $x \notin B$

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Complement Complement A' or A^c contains the elements that are not contained in A. $x \in A^c$ if $x \notin A$

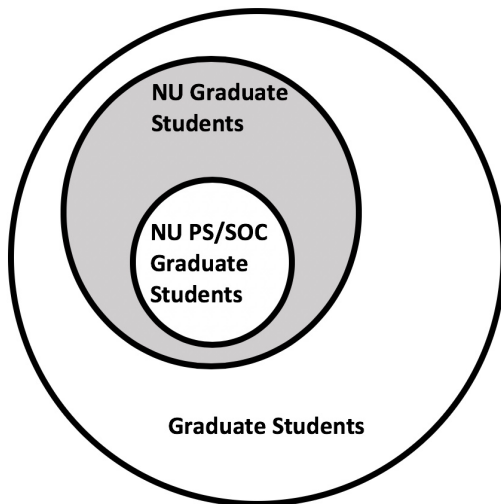
Sets: cont'd

Find $\text{NU GS} \setminus \text{PS/SOC Grad Students}$



Sets: cont'd

NU GS\PS/SOC Grad Students: AKA, where are the NU grad students who aren't in PS/SOC



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- ▶ Cartesian product: set containing all possible ordered pairs (a,b) where a is from set A and b is from set B for any two sets.
 - ▶ $A \times B$ where $A = \{apple, banana, kiwi\}$ and $B = \{2, 4\}$. $A \times B = \{\{apple, 2\}, \{apple, 4\}, \{banana, 2\}, \{banana, 4\}, \{kiwi, 2\}, \{kiwi, 4\}\}$

Sets: Review

- ▶ Unions: OR, \cup
- ▶ Intersections: AND, \cap
- ▶ Ordered/Unordered
- ▶ Complements (not inside) c , written A^c , for example.
- ▶ Subsets and proper subsets (contained within): \subset, \subseteq
- ▶ Cardinality (number of elements)

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- ▶ Does order matter?
- ▶ How would it affect the sample space? Would we have more or fewer possible hands?
- ▶ Now, you can swap one card. How many ways could you do it? Does it matter what cards you have?

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- Finally, what if I told you that all 4 of my cards were even numbered. Would you be surprised? How would you know whether to be surprised? We will discuss this, and many other interesting things!, in class! soon!