Day 1: Notation and Logic

Jean Clipperton

Math Camp 2021

- Going to jump around a little today
- PS 490 – R workshop to help smooth the way with this course
- Will post today's slides by end of day

Now, to the good stuff!

Overview of Math Camp

- TeX or Rmd are great for homework–Michelle will walk you through soon
- http://detexify.kirelabs.org/classify.html
- Canvas for symbol cheat sheet
- OH: we'll build in time at the end of class sessions in addition to during sessions. Please take advantage of this!
- Email: jean.clipperton@northwestern.edu, MichelleBuenoVasquez2024@u.northwestern.edu
- Will LOOSELY follow recommended book – may jump around a bit
- Homeworks not graded but you will submit them

Math (P)refresher: Review Topics

- Broad Overview
- Sets
- Algebra review
- Inequalities
- Combinatorics
- Summation
- Derivatives
- Integrals

Broad Overview

From the Top: Variables and Constants

Theories

Theories are how we frame statements (hypotheses, propositions) about the world, using concepts.

Concepts

Concepts are the ideas, aspects to be measured

Example

Actors and Laws

Theory: More *political actors* involved in a process hampers *legislative productivity* because it's harder to reach an agreement

Need to define how you count an actor and measure productivity

From the Top: Variables and Constants

Broad Overview

Theories

Theories are how we frame statements (hypotheses, propositions) about the world, using concepts.

Concepts

Concepts are the ideas, aspects to be measured

Example

Actors and Laws

Theory: More *political actors* involved in a process hampers *legislative productivity* because it's harder to reach an agreement

Need to define how you count an actor and measure productivity

Sets are collections of elements. Can relate variables to sets.

Abstract Thinking and Sets

Sets are collections of elements. Can relate variables to sets.

Example

Suppose we are measuring voting –what is the variable and what are our possible values?

Variable: number of times voted on a series of bills.

Variable: actors in support of a bill.

Abstract Thinking and Sets

Sets are collections of elements. Can relate variables to sets.

Example

Suppose we are measuring voting –what is the variable and what are our possible values?

Variable: number of times voted on a series of bills.
           Set $\{3, 4, 0\}$.
 Variable: actors in support of a bill.

Abstract Thinking and Sets

Sets are collections of elements. Can relate variables to sets.

Example

Suppose we are measuring voting –what is the variable and what are our possible values?

Variable: number of times voted on a series of bills.
Set $\{3, 4, 0\}$.
Variable: actors in support of a bill.
Set $\{A, O, P\}$.

We can think about these sets' possible elements as follows

We can think about these sets' possible elements as follows

- N: Natural numbers $\{(0), 1, 2, 3, ...\}$

We can think about these sets' possible elements as follows

- ▶ N: Natural numbers $\{(0), 1, 2, 3, ...\}$
- ▶ Z: Integers (negative and positive including zero)
  $\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$

Abstract Thinking and Sets

We can think about these sets' possible elements as follows

- N: Natural numbers $\{(0), 1, 2, 3, ...\}$
- Z: Integers (negative and positive including zero) $\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$
- Q: Rational numbers (q for quotient, rational numbers, e.g. expressed as a fraction)

Abstract Thinking and Sets

We can think about these sets' possible elements as follows

- N: Natural numbers $\{(0), 1, 2, 3, ...\}$
- Z: Integers (negative and positive including zero)
  $\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$
- Q: Rational numbers (q for quotient, rational numbers, e.g. expressed as a fraction)
- R: Real numbers (positive, negative, zero, integers, fractions/rational); any point on the number line

Abstract Thinking and Sets

We can think about these sets' possible elements as follows

- ▶ N: Natural numbers $\{(0), 1, 2, 3, ...\}$
- ▶ Z: Integers (negative and positive including zero) $\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$
- ▶ Q: Rational numbers (q for quotient, rational numbers, e.g. expressed as a fraction)
- ▶ R: Real numbers (positive, negative, zero, integers, fractions/rational); any point on the number line
- ▶ I: Imaginary numbers ($i = \sqrt{-1}$)

Abstract Thinking and Sets

We can think about these sets' possible elements as follows

- N: Natural numbers $\{(0), 1, 2, 3, ...\}$
- Z: Integers (negative and positive including zero) $\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$
- Q: Rational numbers (q for quotient, rational numbers, e.g. expressed as a fraction)
- R: Real numbers (positive, negative, zero, integers, fractions/rational); any point on the number line
- I: Imaginary numbers ($i = \sqrt{-1}$)
- C: Complex numbers ($a + bi$)

Abstract Thinking and Sets

We can think about these sets' possible elements as follows

- N: Natural numbers $\{(0), 1, 2, 3, ...\}$
- Z: Integers (negative and positive including zero) $\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$
- Q: Rational numbers (q for quotient, rational numbers, e.g. expressed as a fraction)
- R: Real numbers (positive, negative, zero, integers, fractions/rational); any point on the number line
- I: Imaginary numbers ($i = \sqrt{-1}$)
- C: Complex numbers ($a + bi$)

Abstract Thinking and Sets

We can think about these sets' possible elements as follows

- N: Natural numbers $\{(0), 1, 2, 3, ...\}$
- Z: Integers (negative and positive including zero) $\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$
- Q: Rational numbers (q for quotient, rational numbers, e.g. expressed as a fraction)
- R: Real numbers (positive, negative, zero, integers, fractions/rational); any point on the number line
- I: Imaginary numbers ($i = \sqrt{-1}$)
- C: Complex numbers ($a + bi$)

Notes:
Subscript: Z_+ only the positive or Q_- negative elements of the set Superscript: N^2 dimensions of the space

How we measure concepts (and turn them into variables):

How we measure concepts (and turn them into variables):

- ▶ Nominal (categorical): no mathematical relationship between the variables
- ▶ Ordinal: categorical variable with set relationship (can compare items to one another)
- ▶ Interval: distance between numerical values has meaning (e.g. 0,1, 2 – 2 is two greater than 0)
- ▶ Ratio (interval-ratio): distance between numerical values has meaning AND zero is meaningful

How we measure concepts (and turn them into variables):

How we measure concepts (and turn them into variables):

▶ Nominal (categorical): no mathematical relationship between the variables **Eye color**

▶ Ordinal: categorical variable with set relationship (can compare items to one another) **Age: above/under 18**

▶ Interval: distance between numerical values has meaning (e.g. 0,1, 2 − 2 is two greater than 0) **Approval from 1 to 5**

▶ Ratio (interval-ratio): distance between numerical values has meaning AND zero is meaningful **Number of years of grad school**

- Solution set: set of all solutions to an equation
- Sample space: set that contains all the values a variable can take
- Subsets: groupings that fall within other sets

Set operations

Can combine sets by looking at the *difference*, *complement*, *intersection*, *union*, and *partition* of sets.

Set operations

Can combine sets by looking at the *difference*, *complement*, *intersection*, *union*, and *partition* of sets.

Example

Consider the set of all men and all women. The intersection is what is in both sets ('and') while the union ('or') is what is the collection of the two. We'll go into this vocabulary more tomorrow.

Notation Refresher

- There exists, $\exists$ for all, $\forall$
- Union $\cup$, Intersection $\cap$
- Excluding $\notin$, $\neg$
- Empty set $\emptyset$
- Element $\in$
- Equivalent $\equiv$
- Such that (s.t) or $|$, e.g. $\{x|x > 7\}$
- Subset $\subset, \subseteq$ (these function roughly like the less than/less than equal to, but for sets)

It's all Greek to me!

You'll want to have a basic familiarity with Greek letters as they'll come up from time to time.

$$
\begin{array}{cccc}
\alpha & \theta & o & \tau \\
\beta & \vartheta & \pi & \upsilon \\
\gamma & \gamma & \varpi & \phi \\
\delta & \kappa & \rho & \varphi \\
\epsilon & \lambda & \varrho & \chi \\
\varepsilon & \mu & \sigma & \psi \\
\zeta & \nu & \varsigma & \omega \\
\eta & \xi & & \\
\Gamma & \Lambda & \Sigma & \Psi \\
\Delta & \Xi & \Upsilon & \Omega \\
\Theta & \Pi & \Phi &
\end{array}
$$

It's all Greek to me!

You'll want to have a basic familiarity with Greek letters as they'll come up from time to time.

α	θ	o	τ
β	ϑ	π	υ
γ	γ	ϖ	ϕ
δ	κ	ρ	φ
ϵ	λ	ϱ	χ
ε	μ	σ	ψ
ζ	ν	ς	ω
η	ξ		
Γ	Λ	Σ	Ψ
Δ	Ξ	Υ	Ω
Θ	Π	Φ	

Some commonly used letters include δ (integrals), Δ (difference/change), β (coefficients), μ (mean), σ (standard deviation), λ (eigenvalues (linear algebra)), ϵ (error)

Check in – all good?

What do I need to know?

We will use terms regarding proofs and you'll want to have a familiarity with these terms, but we won't be doing logic worksheets and/or testing you on these terms.