Day 4 AM: Algebra Review

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Math Camp 2021

Questions?

Concluding CH 1 of Moore & Siegel – moving to Ch 2

Algebra Review

Properties

- **Associative property**: $(a + b) + c = a + (b + c)$ and $(a * b) * c = a * (b * c)$
- **Commutative property** $a + b = b + a$ and $a * b = b * a$
- **Distributive property** $a(b + c) = ab + ac$
- **Identity property** $x + 0 = x$ and $x * 1 = x$
- **Inverse property** $-x + x = 0$. Multiplicative inverse exists, but not for all numbers $x^{-1} * x = 1$

Factoring

We may need to break down different functions.

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2. $m^2 + 3m + 2 =$
3. $x^2 + 5x + 6 =$
4. $25 - x^2 =$

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2. $2x^2 + 7x + 3$
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3. $6y^2 + y - 2$ $(2y - 1)(3y + 2)$

Inequalities

Relations: Intervals & Inequalities

Interval notation can be used to express ranges of numbers :

$[a, b]$	a $\leq x \leq b$	Square brackets include end points (closed interval)
(a,b)	$a < x < b$	Parenthesis mean exclude end points (open interval)
$\{$a,b$\}$		Typically used for sets – not inequalities/intervals

Relations

Graph the following:

- $4 < x$
- $y > 12$
- $3 < z < 7$
- $(3, 9)$
- $[\text{-}7, 2)$

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Solve like regular equations but FLIP inequality when multiplying by negative values.

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Try:

- -6$(x + 8) < 12$

Absolute Value

Solve for TWO possibilities: quantity is positive or negative. EX: $|x-3|>4$

- Quantity is positive: drop bars, solve like usual: $x-3>4, x>7$
- Quantity is negative: then, really have $x-3<$-4 Solve to find $x<$-1

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Solution: $x<-1$ OR $x>7$ Try: $|2x-3|>9$

Factorial

The factorial, !, multiplies a number by each subsequent number down to one.

For example, $4! = 4 * 3 * 2 * 1$

You can also divide and multiply factorials: $\frac{3!}{4!} = \frac{3*2*1}{4*3*2*1} = \frac{1}{4}$

CANNOT ADD THE NUMBERS!! (e.g. $6!3! \neq 9!$**)**

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We can use factorials to help us understand ways of combining elements: e.g. suppose you are forming a committee of 3 people from a group of 5. How many ways can we do that?

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We could also calculate them using the binomial function: $\binom{5}{3} = \frac{5!}{3!2!} = \frac{5*4}{2} = 10$

Try $\binom{4}{2}$ with population A,B,C,D. Using the general formula: $\binom{N}{k} = \frac{N!}{k!(N-k)!}$

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You may have different populations and different treatments – how many different groups would you need to test the possible combinations?

Functions and Relations

Relations allow comparison of variables and expressions – some may be more or less specific in how they assign or specify these relationships between the *range (y)* and *domain (x)*. Suppose we have a domain as follows $\{$ apple, kiwi, lime $\}$ and a range of $\{$ 4, 5$\}$.

Functions and Relations

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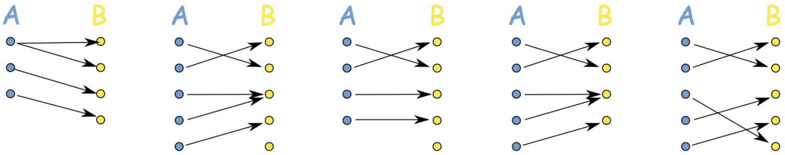

Figure 1:

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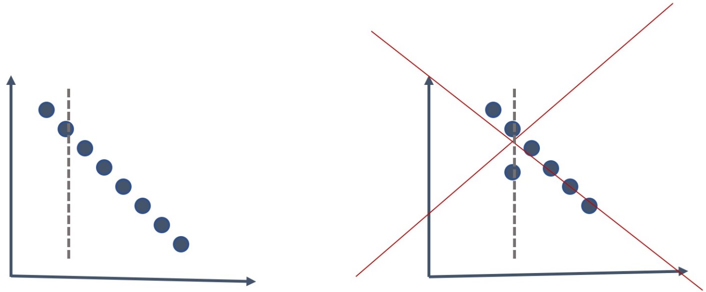

Figure 2: “Vertical line test”

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You can also combine functions, e.g. $f(x) = x$ and $g(x) = x^3$ can be combined to produce $f \circ g(x) = f(g(x))$.

Function Terms

Table 3.1 from book (pg 49)

Term	Meaning
Identity function	Elements in domain are mapped to identical elements in codomain
Inverse function	Function that when composed with original function returns identity function
Surjective (onto)	Every value in codomain produced by value in domain
Injective (one-to-one)	Each value in range comes from only one value in domain
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Why do we care?

Function Terms

We care about whether a function is surjective, injective, or bijective because we will know if we can trace back what produced what we have.

https://www.mathsisfun.com/sets/injective-surjective-bijective.html

Function Terms

- We can see that both functions are surjective – each ‘covers’ the codomains
- In the case of g(x), we can see that it is injective, while f(x) is not
- Thus, f(x) is not bijective but g(x) is (both surjective and injective) – thus, it is invertible (to be defined)

Composite Functions

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- $f \circ g(x) = 4x^2 + 1$
- $g \circ f(x) = 2x^2 + 2$

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We can see that we only get one y value for each x and that, depending on the domain/codomain, the function is surjective as well, making this function bijective and have a legitimate inverse.

Monotonic Function Terms

Term	Meaning
Increasing	Function increases on subset of domain
Decreasing	Function decreases on subset of domain
Strictly increasing	Function always increases on subset of domain
Strictly decreasing	Function always decreases on subset of domain
Weakly increasing	Function does not decrease on subset of domain
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(Strict) monotonicity	Order preservation; function (strictly) increasing or decreasing over domain

Table 3.2 from book (pg 51)

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Table 3.2 from book (pg 51)

This is useful for math land but also for theory building: how are x and y related? Does more x ALWAYS mean more y (function is monotonically increasing, etc)?

Linear Equation vs Linear Function

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We have the **intercept** (where the line crosses the y axis) and the **slope** (unit increase in iv related to dv). This is the 'plain vanilla' version.

The **linear function** is much more expansive: includes multiple variables, exponents, and logs (logarithms).

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Exponents, Exponentials, Exponential functions

Exponents

Exponents are where you take a variable to some power – e.g. x^a where x is a variable and a is a constant. Typically, we focus on the numerical portion of the exponent–calling it 'the exponent'.

Exponential

An exponential is the reverse of the exponent – here it is a number to the power of a variable, e.g. a^x. To get the x 'down', we need to use logarithms (aka logs).

Exponential Function

The exponential function has a particular base, e, (where e is Euler's e and is approx 2.72.)

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We care because we might see these as utility functions or use higher order values as a way to represent the relationship between two variables in a linear regression Know what these concepts are–will be relevant later.

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Logarithms (typically base 10 ($log(x)$) or base e ($ln(e)$), but any base is possible, e.g. $log_{8675309}x$ (Bases aside from e and 10 will be specified).

- $y = log(z) \leftrightarrow 10^y = z$
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- $log(1) = 0$

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Exponents in log are different from what you might expect:

- $log(x^2) = 2(log(x))$
- $log(x/y) = log(x) - log(y)$ provided $(x, y > 0)$

Logs help weigh smaller values more heavily; adding units not linear–less meaningful for larger values
($log(100) = 2$, $log(1000) = 3$)

Logs Practice

Simplify the following

- $log(x^4)$
- $log(xy)$
- $ln(e^3)$
- $ln(1)$
- $log(3) + log(7)$

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- $ln(e^3) = 3$
- $ln(1) = 0$
- $log(3) + log(7) = log(21)$

More Practice

- $2e^{6x} = 18$
- $e^{x^2} = 1$
- $2^x = e^5$
- $2^{x-2} = 5$
- $ln(x^2) = 5$
- $ln(x^3) - ln(x) = ln(16)$

When would I ever actually *see* a log function??

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https://thesocietypages.org/graphicsociology/2010/12/07/1247/

Tying it all together

Consider the example in Moore and Siegel (pg 72): predicting voting probability of an individual in a national education. Variables: education, partisan id, income, age, closeness. *How can we categorize these variables?*

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The probability is given by this function: how can we interpret the following? $p_v = f(ed, p, i, a, c)$ *The probability is a function of the variables given above.* Can further specify as a direct linear function: $p_v = \alpha + \beta_1 ed + \beta_2 p + \beta_3 i + \beta_4 a + \beta_5 c$ But this is one of many options (how would we apply the quadratic or linear functions?)

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Limits

The sums on the previous page had *limits*: you can add elements and get an answer. We say that these series **converge** while those series that just keep getting bigger and bigger and bigger (or smaller and smaller and smaller) **diverge**.

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We can talk in the same way about sequences $lim_{i \to \infty} x_i = L$.

Limits

Limits are also useful in calculus – when we take a derivative, we are essentially asking: “What is the slope of the line at this infinitesimally tiny part of the line“?

A second way to ask this is to look at what happens to the slope as as the distance between points approaches zero.

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A second way to ask this is to look at what happens to the slope as as the distance between points approaches zero. However, to actually calculate the derivative, we need to first be sure that the point is differentiable. Again, we will use limits. (useful in PS 405 and Soc 401-1)

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Continuity

Continuous function: draw without picking up a pencil

REPLACE IMAGE HERE

Limits

For us, you should be able to:

- Plug a value in to the limit and see what you get out (check for dividing by zero).
- Recognize an indeterminate limit: $\frac{0}{0}$ or $\frac{\infty}{\infty}$
- Compare powers in an indeterminate limit to find what happens: Same degree: limit goes to whatever fraction you get when you divide the top and bottom first terms

We don't go beyond here for our calculations, however the book has a nice explanation on limits and the bounds of limits and l'Hopital's rule. This is nifty, but more firepower than we need.

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Ans: $6/9$, simplifies to $2/3$ $\qquad$ simplifies to x-2; limit is 2.