

Day 7: Calculus Fundamentals, Derivatives II

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Math Camp 2022

Fundamentals of Derivatives: Review

Yesterday we went over the basics of derivatives, what they are conceptually, and how to solve for them.

Let's rehash these problems from yesterday to warm up:

- ▶ $f(x) = 5$
- ▶ $f(x) = 3x - 7$
- ▶ $f(x) = 3x^2$
- ▶ $f(x) = \frac{x^2}{x}$
- ▶ $f(s) = s^{-2}$
- ▶ $f(y) = y(y + 7)(y - 3)$
- ▶ $f(z) = \frac{z^2 - 5z - 6}{z + 1}$

Fundamentals of Derivatives: Review

Yesterday we went over the basics of derivatives, what they are conceptually, and how to solve for them.

Let's rehash these problems from yesterday to warm up:

- ▶ $f(x) = 5, f'(x) = 0$
- ▶ $f(x) = 3x - 7, f'(x) = 3$
- ▶ $f(x) = 3x^2, f'(x) = 6x$
- ▶ $f(x) = \frac{x^2}{x}, f'(x) = 1$
- ▶ $f(s) = s^{-2}, f'(s) = -2s^{-3}$ (not continuous at $s = 0$)
- ▶ $f(y) = y(y + 7)(y - 3), f'(y) = 3y^2 + 8y - 21$
- ▶ $f(z) = \frac{z^2 - 5z - 6}{z + 1}, f'(z) = 1$ (not continuous at $z = -1$)

Fundamentals of Derivatives: Review

Moving on from these basic rules, we will now cover higher order derivatives, partial derivatives, and more advanced (but still basic) rules of derivatives, such as the product rule, quotient rule, and chain rule.

Higher Order Derivatives

Second derivatives (n^{th} derivatives): take a derivative a second (n^{th}) time

Rate of change of rate of change (velocity vs acceleration)

► $f(x) = x^5 + 3x^3 + 2x + 8$

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- ▶ $f'(x) = 5x^4 + 9x^2 + 2$

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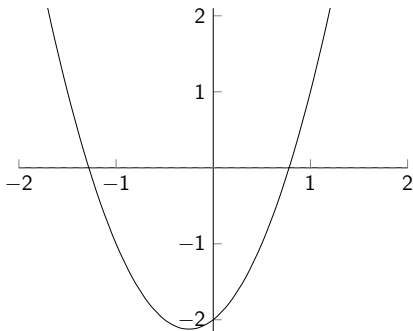
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- ▶ $f(x) = x^5 + 3x^3 + 2x + 8$
- ▶ $f'(x) = 5x^4 + 9x^2 + 2$
- ▶ $f''(x) = 20x^3 + 18x$

Critical Points

Critical points occur where the derivative is zero. We can find them by graphing (ocular method) or plugging in values after calculating the derivative.



Critical Points

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Try:

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Sometimes there are multiple critical points

- ▶ How many?
- ▶ How to find?

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(up to) Three critical points. $f'(x) = 4x^3 - 16x = 4x(x^2 - 4)$.

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Zeroes at -2, 0, 2

Partial Derivatives

Similar to 'regular' derivative; treat additional variable(s) as constants. Written as ∂_x or $\frac{\partial f}{\partial x}(x, \dots)$

THIS IS IMPORTANT FOR INTERACTION TERMS!

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$$f(x, z) = 7xz + 4x^2 + z \quad \partial_x = 7z + 8x$$

$$f(x, y) = x + 4y \quad \partial_x = 1$$

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$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \epsilon \quad \partial_X = \beta_1 + \beta_3 Z$$

How is that different from just β_1 or just β_3 ?

The effect is $\partial_X = \beta_1 + \beta_3 Z$. Now, suppose Z can be 0 or 1.

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Understanding Interactions

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \varepsilon$$

Hypothesis H_1 : An increase in X is associated with an increase in Y when condition Z is met, but not when condition Z is absent.

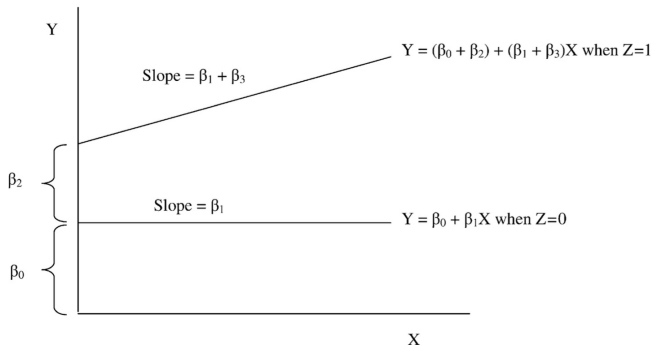


Fig. 1 A graphical illustration of an interaction model consistent with hypothesis H_1 .

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- ▶ Product Rule: $f(x) * g(x)$
- ▶ Quotient Rule: $\frac{f(x)}{g(x)}$
- ▶ Chain Rule: $f(g(x))$
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- ▶ Quotient Rule: $\frac{f(x)}{g(x)}$, $\frac{x^4+3x}{x^2}$
- ▶ Chain Rule: $f(g(x))$, $(x^2 + 1)^3$ (composition!)
- ▶ Other: eg, exponentials: e^x , $\ln(x)$

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To take the derivative using our previous approach, we first multiply:
 $3x^2 + 4x + 6x + 8 = 3x^2 + 10x + 8$. Then, just take the derivative of each term: $f'(x) = 6x + 10$.

WHY DO WITH THE PRODUCT RULE??

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 $(6x + 3)(x^3 + 2x^2 + x + 2) + (3x^2 + 4x + 1)(3x^2 + 3x + 4)$. This is
a mess – but you have your answer at least (and much easier than
doing it the long way)!

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Practice: $\frac{x-4}{x+5} \cdot \frac{3x^3}{x+2}$

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Practice: $\frac{x-4}{x+5} \cdot \frac{3x^3}{x+2}$ **Ans:** $\frac{9}{(x+5)^2} \cdot \frac{6x^3 + 18x^2}{(x+2)^2}$

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Ans $4(4x + 8)(2x^2 + 8x)^3$

$6(9 - 2x)(9x - x^2)^5 = (54 - 12x)(9x - 2x^2)^5$

Exponentials: e and \ln

You can take the derivative of continuous functions – including those with a log and/or e in them. The rules are a little hard, but once you learn them, it's not too bad:

$$f(x) = e^x \quad f'(x) = e^x \quad (\text{a favorite of mine})$$

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$$\text{TRY: } \ln(x^2), e^{2x} \quad \text{ANS: } \frac{2}{x} \text{ and } 2e^{2x}$$

Additional Resources

- ▶ Daniel Kleitman's Calculus for Beginners and Artists:
www-math.mit.edu/~djk/calculus_beginners
- ▶ Dan Sloughter (online textbook): math.furman.edu/~dcs/book
- ▶ Calc refresher (Harvey Mudd Calc Tutorial):
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Table 6.1: List of Rules of Differentiation

Sum rule	$(f(x) + g(x))' = f'(x) + g'(x)$
Difference rule	$(f(x) - g(x))' = f'(x) - g'(x)$
Multiply by constant rule	$f'(ax) = af'(x)$
Product rule	$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$
Quotient rule	$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
Chain rule	$(g(f(x)))' = g'(f(x))f'(x)$
Inverse function rule	$(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}$
Constant rule	$(a)' = 0$
Power rule	$(x^n)' = nx^{n-1}$
Exponential rule 1	$(e^x)' = e^x$
Exponential rule 2	$(a^x)' = a^x(\ln(a))$
Logarithm rule 1	$(\ln(x))' = \frac{1}{x}$
Logarithm rule 2	$(\log_a(x))' = \frac{1}{x(\ln(a))}$
Trigonometric rules	$(\sin(x))' = \cos(x)$ $(\cos(x))' = -\sin(x)$ $(\tan(x))' = 1 + \tan^2(x)$
Piecewise rules	Treat each piece separately

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However, there comes a time where we have the rate of change but we need to know what the original is. Next, we'll cover integrals – undoing derivatives!