PS and Sociology Math Prefresher

Math Camp

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Misc Info

- Going to jump around a little today
- PS 490 – R workshop to help smooth the way with this course
- Will post today's slides by end of day

Now, to the good stuff!

Overview of this class

- TeX is great for homework–Michelle will walk you through soon
- http://detexify.kirelabs.org/classify.html
- Canvas for symbol cheat sheet
- OH: we'll build in time at the end of class sessions in addition to during sessions. Please take advantage of this!
- Email: jean.clipperton@northwestern.edu, MichelleBuenoVasquez2024@u.northwestern.edu
- Will LOOSELY follow recommended book – may jump around a bit
- Homeworks not graded but you will submit them

Math (P)refresher

- Broad Overview
- Algebra review
- Inequalities
- Combinatorics
- Summation
- Sets
- Derivatives
- Integrals

From the Top: Variables and Constants

Theories

Theories are how we frame statements (hypotheses, propositions) about the world, using concepts.

Concepts

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Actors and Laws

Theory: More *political actors* involved in a process hampers *legislative productivity* because it's harder to reach an agreement

Need to define how you count an actor and measure productivity

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Variables are how we try to capture concepts and test hypotheses (measurement applies to variables). Have *independent*(x) and *dependent*(y) variables. They (may) take on different values in a dataset.

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Perhaps actors who have veto power and number of laws (vs bills, circulars, etc) passed in one legislative cycle (include or exclude carryovers from previous year?)

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Set $\{$ A, O, P $\}$.

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Notes:
Subscript: Z_+ only the positive or Q_- negative elements of the set Superscript: N^2 dimensions of the space

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- Nominal (categorical): no mathematical relationship between the variables
- Ordinal: categorical variable with set relationship (can compare items to one another)
- Interval: distance between numerical values has meaning (e.g. 0,1, 2 – 2 is two greater than 0)
- Ratio (interval-ratio): distance between numerical values has meaning AND zero is meaningful

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- Nominal (categorical): no mathematical relationship between the variables **Eye color**
- Ordinal: categorical variable with set relationship (can compare items to one another) **Age: above/under 18**
- Interval: distance between numerical values has meaning (e.g. 0,1, 2 – 2 is two greater than 0) **Approval from 1 to 5**
- Ratio (interval-ratio): distance between numerical values has meaning AND zero is meaningful **Number of years of grad school**

Variables and Sets: Putting things together

- Solution set: set of all solutions to an equation
- Sample space: set that contains all the values a variable can take
- Subsets: groupings that fall within other sets

Variables and Sets: Putting things together

Set operations

Can combine sets by looking at the *difference*, *complement*, *intersection*, *union*, and *partition* of sets.

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Example

Consider the set of all men and all women. The intersection is what is in both sets ('and') while the union ('or') is what is the collection of the two. We'll go into this vocabulary more tomorrow.

Notation Refresher

- There exists, $\exists$ for all, $\forall$
- Union $\cup$, Intersection $\cap$
- Excluding $\notin$, $\neg$
- Empty set $\emptyset$
- Element $\in$
- Equivalent $\equiv$
- Such that (s.t) or $|$, e.g. $\{x|x > 7\}$
- Subset $\subset, \subseteq$ (these function roughly like the less than/less than equal to, but for sets)

It's all Greek to me!

You'll want to have a basic familiarity with Greek letters as they'll come up from time to time.

α	θ	o	τ
β	ϑ	π	υ
γ	γ	ϖ	ϕ
δ	κ	ρ	φ
ϵ	λ	ϱ	χ
ε	μ	σ	ψ
ζ	ν	ς	ω
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Some commonly used letters include δ (integrals), Δ (difference/change), β (coefficients), μ (mean), σ (standard deviation), λ (eigenvalues (linear algebra)), ϵ (error)

Check in – all good?

Logic

What do I need to know?

We will use terms regarding proofs and you'll want to have a familiarity with these terms, but we won't be doing logic worksheets and/or testing you on these terms.

Logic & Proofs

- Assumptions: taken to be true
- Proposition: statement thought to be true given the assumptions
- Theorem: proven proposition
- Lemma: theorem something of little interest
- Corollary: a type of proposition that follows directly from the proff of another proposition and does not require further proof

Necessary and Sufficient

Consider an outcome D with three possible input variables, A,B,C.

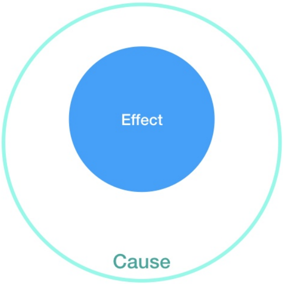
Sufficient

Something that occurs *also* when our outcome variable D occurs. Consider it as an if statement: D is true if A and B are true.

Necessary

Something that occurs *always* when our outcome variable occurs. Every time D is true, A and B are true. Consider it as an only if statement: D is true only if A and B are.

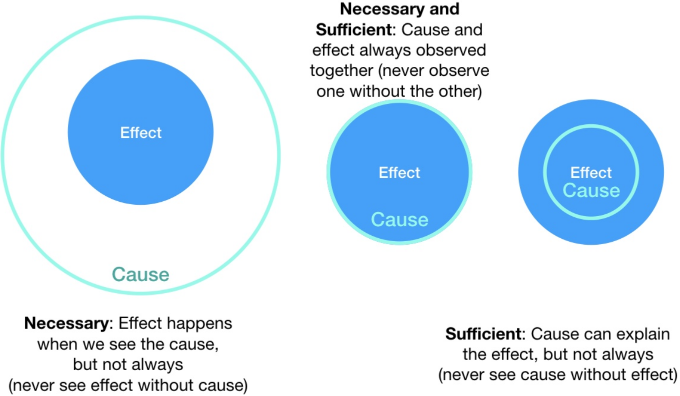
Necessary vs Sufficient



Necessary: Effect happens when we see the cause, but not always (never see effect without cause)

Sufficient: Cause can explain the effect, but not always (never see cause without effect)

Necessary vs Sufficient



Necessary and Sufficient

Example Suppose we're wondering if it will rain today. We notice the following:

Temperature	Pressure	Rain
L	H	N
L	M	N
H	M	N
H	L	Y

Recall that *necessary* means that something **ALWAYS** occurs when our outcome (rain) occurs while *sufficient* only doesn't happen when the outcome doesn't happen.

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Logic

Consider the following elements: A,B,C that may be associated with an outcome, D. Suppose we're considering that A and B imply D ($A \wedge B \rightarrow D$).

Converse

The converse changes a necessary statement to a sufficient one, or vice versa. (not always logically true). Ex: Converse of $A \wedge B \rightarrow D$ is $D \rightarrow A \wedge B$ (trade places of elements; not always true).

Contrapositive

The contrapositive flips arrow *and* negates items. Ex: Contrapositive of $A \wedge B \rightarrow D$ is $\neg A \vee \neg B \leftarrow \neg D$. (Always true)

Fun Fact: The contrapositive is the negation of the converse of a statement.

Proofs: Direct proofs

We won't do a lot of proofs here, but it's helpful to understand how they work.

Direct proofs demonstrate the statement deductively by one of the following methods.

- General (deductive) proof: typically done using definitions, etc. Showing how the outcome logically follows building on rules and assumptions.
- Proof by exhaustion: Break up the outcome into sub cases and show for each case (done often in game theory for possible values)
- Proof by construction: These proofs demonstrate existence (is there a square that is the sum of two squares?).
- Proof by induction: Start small and show it is true for any number (e.g. start with a small n, n=1, then expand to n+1)

Proofs: Indirect Proofs

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Indirect proofs show that something must be true because there is no logically possible alternative. They are typically demonstrated through the following methods.

- Proof by counterexample: using a counterexample (x implies y, yet we observe y without x...x cannot imply y (aka x not *necessary* for y)).
- Proof by contradiction: assume that the statement is false and try to prove it wrong, eventually demonstrating that a contradiction emerges. Thus, the statement cannot be false.

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Questions? Concluding CH 1 of Moore & Siegel – moving to Ch 2

Algebra Review

Properties

- **Associative property**: $(a+b)+c = a+(b+c)$ and $(a*b)*c = a*(b*c)$
- **Commutative property** $a+b = b+a$ and $a*b = b*a$
- **Distributive property** $a(b+c) = ab+ac$
- **Identity property** $x+0 = x$ and $x*1 = x$
- **Inverse property** $-x+x = 0$. Multiplicative inverse exists, but not for all numbers $x^{-1}*x = 1$

Factoring

We may need to break down different functions.

1. $x^2 + 3x - 4 =$
2. $m^2 + 3m + 2 =$
3. $x^2 + 5x + 6 =$
4. $25 - x^2 =$

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2. $2x^2 + 7x + 3$ $(2x + 1)(x + 3)$
3. $6y^2 + y - 2$ $(2y - 1)(3y + 2)$

Relations: Intervals & Inequalities

Interval notation can be used to express ranges of numbers :

$[a, b]$	a $\leq x \leq b$	Square brackets include end points (closed interval)
(a,b)	$a < x < b$	Parenthesis mean exclude end points (open interval)
{a,b}		Typically used for sets – not inequalities/intervals

Relations: Inequalities

Graph the following:

- $4 < x$
- $y > 12$
- $3 < z < 7$
- $(3, 9)$
- $[-7, 2)$

Solving Inequalities

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Try:

- -6$(x + 8) < 12$

Absolute Value

Solve for TWO possibilities: quantity is positive or negative.
EX: $|x-3| > 4$

- Quantity is positive: drop bars, solve like usual: $x-3 > 4, x > 7$
- Quantity is negative: then, really have $x-3 <$-4 Solve to find $x <$-1

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Solution: $x < -1$ OR $x > 7$
Try: $|2x-3| > 9$

Factorial

The factorial, !, multiplies a number by each subsequent number down to one.
For example, $4! = 4 * 3 * 2 * 1$
You can also divide and multiply factorials: $\frac{3!}{4!} = \frac{3*2*1}{4*3*2*1} = \frac{1}{4}$

CANNOT ADD THE NUMBERS!! (e.g. $6!3! \neq 9!$**)**

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Say these people are named A, B, C, D, and E.

Then we can have the following committees:
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ADE, BCD, BCE, BDE, CDE,
for a total of ten configurations.

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We could also calculate them using the binomial function:
$\binom{5}{3} = \frac{5!}{3!2!} = \frac{5*4}{2} = 10$

Try $\binom{4}{2}$ with population A,B,C,D. Using the general formula:
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WHY DOES THIS MATTER AT ALL????

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You may have different populations and different treatments – how many different groups would you need to test the possible combinations?

Functions and Relations

Relations allow comparison of variables and expressions – some may be more or less specific in how they assign or specify these relationships between the *range (y)* and *domain (x)*. Suppose we have a domain as follows { apple, kiwi, lime } and a range of { 4, 5}.

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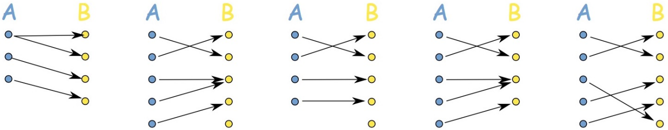

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Functions can only assign one x to one y (and not one x to multiple y values) – otherwise it is a relation.

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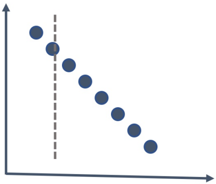
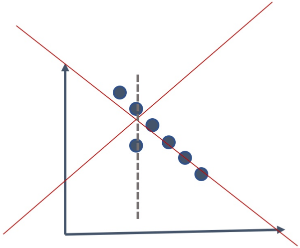
Functions can only assign one x to one y (and not one x to multiple y values) – otherwise it is a relation. Can use the vertical line test to check:





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Functions specify the relationship between x and y, e.g. $y = a + bx$: we often talk about *mapping* something onto something else.

Functions map from the domain to the codomain. Note that there is a distinction between the codomain and the range: codomain are possible values (e.g. natural numbers) and range are only the values reached/obtained. (sometimes called the image).

You can also combine functions, e.g. $f(x) = x$ and $g(x) = x^3$ can be combined to produce $f \circ g(x) = f(g(x))$.

Function Terms

Table 3.1 from book (pg 49)

Term	Meaning
Identity function	Elements in domain are mapped to identical elements in codomain
Inverse function	Function that when composed with original function returns identity function
Surjective (onto)	Every value in codomain produced by value in domain
Injective (one-to-one)	Each value in range comes from only one value in domain
Bijective (invertible)	Both surjective and injective; function has an inverse

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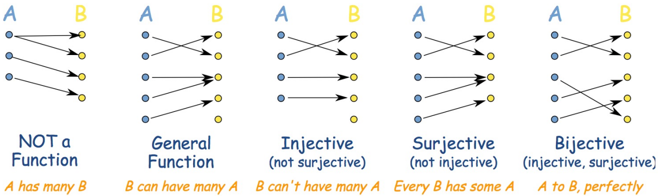
Why do we care?

Function Terms

We care about whether a function is surjective, injective, or bijective because we will know if we can trace back what produced what we have.



https://www.mathsisfun.com/sets/injective-surjective-bijective.html

Function Terms

- We can see that both functions are surjective – each ‘covers’ the codomains
- In the case of g(x), we can see that it is injective, while f(x) is not
- Thus, f(x) is not bijective but g(x) is (both surjective and injective) – thus, it is invertible (to be defined)

Composite Functions

Inner/Outer Functions: We begin at these two functions $f(x) = x^5$ and $g(x) = x^2 + 5x + 1$.. You essentially substitute the second function as 'x'.

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- Consider $f(x) = x^5$ and $g(x) = x^2 + 5x + 1$. Combine them to create $f \circ g(x) = f(g(x)) = (x^2 + 5x + 1)^5$

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- Similarly, $f(x) = x^{0.5}$ and $g(x) = x(5x - 1)$ produce $f \circ g(x) = f(g(x)) = \sqrt{5x^2 - x}$

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- $f \circ g(x) = 4x^2 + 1$
- $g \circ f(x) = 2x^2 + 2$

Function Terms: Invertible functions

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In this case, we get $f(g(x)) = 3 * (x/3) = x$, where x is the identity function. We can see that we only get one y value for each x and that, depending on the domain/codomain, the function is surjective as well, making this function bijective and have a legitimate inverse.

Monotonic Function Terms

Term	Meaning
Increasing	Function increases on subset of domain
Decreasing	Function decreases on subset of domain
Strictly increasing	Function always increases on subset of domain
Strictly decreasing	Function always decreases on subset of domain
Weakly increasing	Function does not decrease on subset of domain
Weakly decreasing	Function does not increase on subset of domain
(Strict) monotonicity	Order preservation; function (strictly) increasing or decreasing over domain

Table 3.2 from book (pg 51)

Monotonic Function Terms

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Increasing	Function increases on subset of domain
Decreasing	Function decreases on subset of domain
Strictly increasing	Function always increases on subset of domain
Strictly decreasing	Function always decreases on subset of domain
Weakly increasing	Function does not decrease on subset of domain
Weakly decreasing	Function does not increase on subset of domain
(Strict) monotonicity	Order preservation; function (strictly) increasing or decreasing over domain

Table 3.2 from book (pg 51)

This is useful for math land but also for theory building: how are x and y related? Does more x ALWAYS mean more y (function is monotonically increasing, etc)?

Linear Equation vs Linear Function

Your beloved friend: $y = a + bx$ (AKA $y = mx + b$ AKA affine function).

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We have the **intercept** (where the line crosses the y axis) and the **slope** (unit increase in iv related to dv). This is the 'plain vanilla' version.

The **linear function** is much more expansive: includes multiple variables, exponents, and logs (logarithms).

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- Multiplying a number by itself: $x * x * x * x = x^4$

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Exponents, Exponentials, Exponential functions

Exponents

Exponents are where you take a variable to some power – e.g. x^a where x is a variable and a is a constant. Typically, we focus on the numerical portion of the exponent–calling it 'the exponent'.

Exponential

An exponential is the reverse of the exponent – here it is a number to the power of a variable, e.g. a^x. To get the x 'down', we need to use logarithms (aka logs).

Exponential Function

The exponential function has a particular base, e, (where e is Euler's e and is approx 2.72.)

Functions: Quadratic Functions and Polynomials

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We care because we might see these as utility functions or use higher order values as a way to represent the relationship between two variables in a linear regression Know what these concepts are–will be relevant later.

Logs and other functions

Logs are the inverses of exponents: the power to which you raise the base, e.g. 10, to produce a given value, e.g.z

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- Logarithms (typically base 10 ($log(x)$) or base e ($ln(e)$), but any base is possible, e.g. $log_{8675309}x$ (Bases aside from e and 10 will be specified).
  - $y = log(z) \leftrightarrow 10^y = z$
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Exponents in log are different from what you might expect:

- $log(x^2) = 2(log(x))$
- $log(x/y) = log(x) - log(y)$ provided $(x, y > 0)$

Logs help weigh smaller values more heavily; adding units not linear–less meaningful for larger values ($log(100) = 2, log(1000) = 3$).

Logs practice

Simplify the following

- $log(x^4)$
- $log(xy)$
- $ln(e^3)$
- $ln(1)$
- $log(3) + log(7)$

Logs practice

Simplify the following

- $log(x^4) = 4log(x)$
- $log(xy)$
- $ln(e^3)$
- $ln(1)$
- $log(3) + log(7)$

Logs practice

Simplify the following

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- $log(x^4) = 4log(x)$
- $log(xy) = log(x) + log(y)$
- $ln(e^3) = 3$
- $ln(1) = 0$
- $log(3) + log(7) = log(21)$

When would I ever actually *see* a log function??

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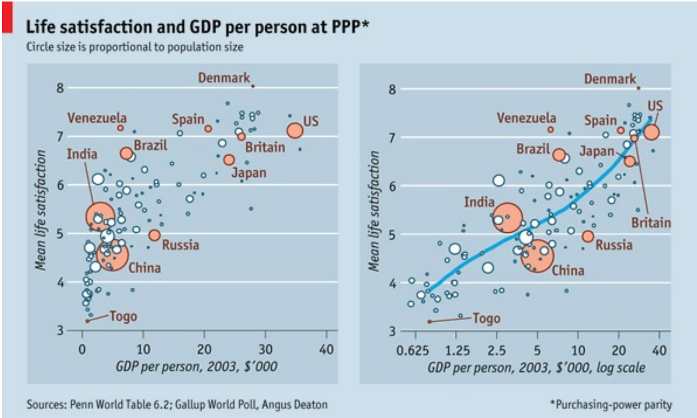

https://thesocietypages.org/graphicsociology/2010/12/07/1247/

Tying it all together

Consider the example in Moore and Siegel (pg 72): predicting voting probability of an individual in a national education. Variables: education, partisan id, income, age, closeness. *How can we categorize these variables?*

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Can further specify as a direct linear function:
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Can further specify as a direct linear function:
$p_v = \alpha + \beta_1 ed + \beta_2 p + \beta_3 i + \beta_4 a + \beta_5 c$ But this is one of many options (how would we apply the quadratic or linear functions?)

Summary, pt 1

- Handout on operators, series and products (Canvas)
- PEMDAS
- Rules:
  - Associative $(a+b)+c=a+(b+c)$ and $(a*b)*c=a*(b*c)$
  - Commutative $a+b=b+a$ and $a*b=b*a$
  - Distributive $a(b+c)=ab+ac$
  - Identity $x+0=x$ and $x*1=x$
  - Inverse $\exists -x| -x+x=0$ BUT NOT ALWAYS for other modes (e.g.$x^{-1}*x=1$ for real and rational numbers but not integers)

Summary, pt 2

- Broad Overview
- Algebra + Inequalities
- Functions (linear, exponents, exponentials (log))
- Relations

Assignment 1: GOOD LUCK!

Work on this, bring it to the pm session. I'll be in my office (021 Scott Hall) around 12:45 until 1:20 to answer questions.
Slides posted after class.

WRITE UP ON SEPARATE SHEET, SHOWING ALL WORK–DUE TOMORROW AM

Meet back here at 1:30 (will split from here...FYI Soc – will need ID to get to the library) Soc: Session times?

PM Goals:

This afternoon, we will cover:

- Intro to $\LaTeX$
- Intro to matrices
- Divide into groups and go to pm sessions

LaTeX

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```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
\begin{frame}
\frametitle{LaTeX}
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\includegraphics[width=1.05\textwidth]{Latex_screen}

 \end{frame}


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

LaTeX: Document Startup

Eventually you will have a document setup you like – you will just copy/paste that into all future new documents.

LaTeX: FYIs

- Math needs to be encased in dollar symbols ($)

LaTeX: Trouble Shooting

- Try hitting enter a bunch of times and look for the 'ugly' output
- Find missing ‘ends’
- Look at lists

LaTeX: Advantages & Disadvantages

Pros	**Cons**
Adaptable	Annoying to learn
Easy to change entire document format	~~Terrible~~ Pretty OK for co-authoring
Simple to export/import using statistical software	Troubleshooting not intuitive
Offers a 'signal' about your background/training	Not common in Sociology
R markdown also relies on LaTeX aspects	

LaTeX: Getting Started

Starting out with TEX, LATEX, and friends

Do you want to begin working with the TEX typesetting system? Most people start out by downloading free versions of the needed software, and a tutorial. This page gets you to the most popular choices.

Step one: Get a distribution

You first need a collection of the software. Such a collection is called a distribution, and comes with TEX, LATEX, BIBTEX, and everything else that will help you to perform TEX's magic on your computer. Each distribution also comes with programs specific to your computer platform, so make your choice from the list below.

Windows:
: The most popular choice here is the MiKTEX distribution, which lets you easily manage TEX packages. Many people advise beginners to get the proTEXt bundling of MiKTEX, which lets you install by using a .pdf file with links so you can read about your options and then click on the right one. And it includes other components that help you work with your TEX system.

Unix-type systems, including GNU/Linux:
: The best choice here is TEX Live, which contains many packages and programs. It is freely available over the Internet or on disc; see the web page for details. Note that most Unix systems have TEX as an installation option so you might already have it or be able to easily get it using your system administration package management tool: RPM, or DEB, or whatever.

Macintosh:
: Get the MacTEX distribution, which is TEX Live with some Mac-specific goodies.

LaTeX: Getting Started