Day 6: Calculus Fundamentals, Derivatives

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Math Camp 2021

Day 6: Concept Agenda

- Review of different algebraiac operations and their functional behavior
- Derivatives: concept and how-to
- Fundamental Derivative Rules

Exponents, Exponentials, Exponential functions

Recall from earlier this week:

Exponents

Exponents are where you take a variable to some power – e.g. x^a where x is a variable and a is a constant. Typically, we focus on the numerical portion of the exponent–calling it 'the exponent'.

Exponential

An exponential is the reverse of the exponent – here it is a number to the power of a variable, e.g. a^x. To get the x 'down', we need to use logarithms (aka logs).

Exponential Function

The exponential function has a particular base, e, (where e is Euler's e and is approx 2.72.)

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We care because we might see these as utility functions or use higher order values as a way to represent the relationship between two variables in a linear regression.

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  - $y = log(z) \leftrightarrow 10^y = z$
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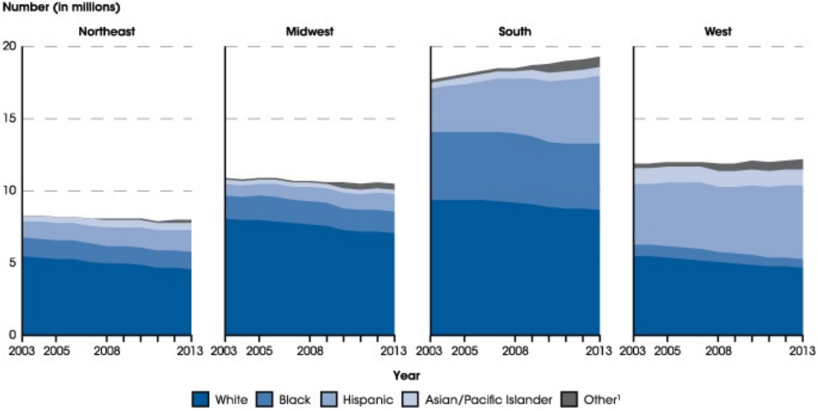
Exponents in log are different from what you might expect:

- $log(x^2) = 2(log(x))$
- $log(x/y) = log(x) - log(y)$ provided $(x, y > 0)$

Logs help weigh smaller values more heavily; adding units not linear–less meaningful for larger values ($log(100) = 2, log(1000) = 3$).

Example: Education Enrollment in the US

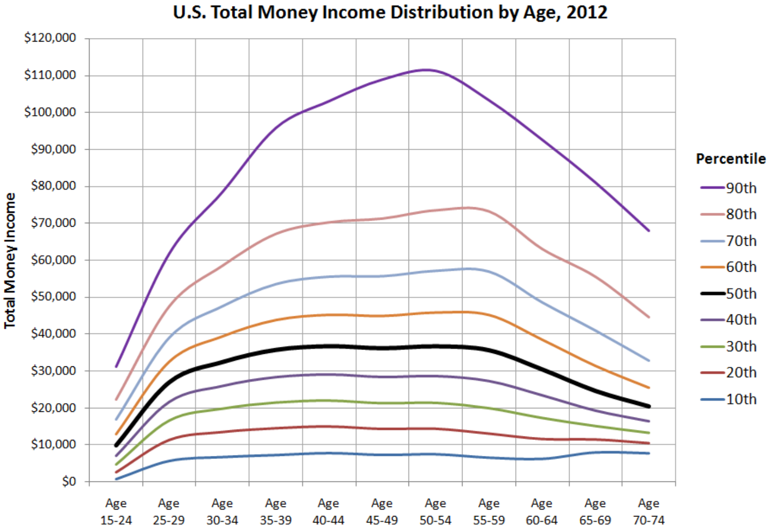
What trends in education can we surmise from these graphs?



Source:http://nces.ed.gov/programs/coe/indicator_cge.asp

Income by age

At what point is your income increasing the fastest? When do earnings slow down? When do they peak?



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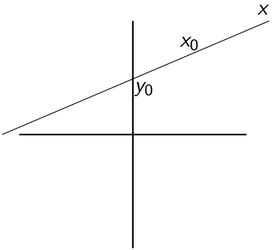
These rates may be between two points (AKA discrete change) or the rate may be at a particular point (AKA instantaneous change). We get at this by calculating the **derivative**, which we denote by $\frac{dy}{dx}$ or $f'(x)$. Both work and both mean the same.

Derivatives: Discrete Change

Slope: rate of change between two points.

$$y = a + bx = y_0 + b(x - x_0), \text{ intercept } y_0 \text{ or } a.$$



Recall that $b = \frac{\Delta y}{\Delta x}$. Find the formulas for the following linear functions whose graph

1. Has slope 2 and y-intercept $(0, 3)$
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4. Goes through the points $(2, -4)$ and $(0, 3)$

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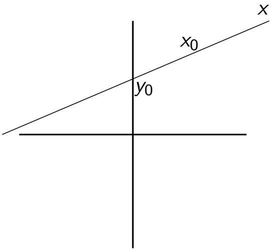

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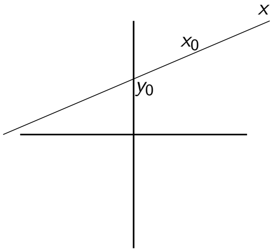

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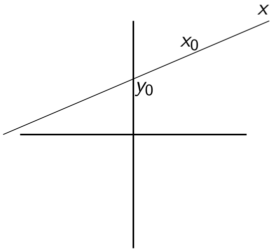

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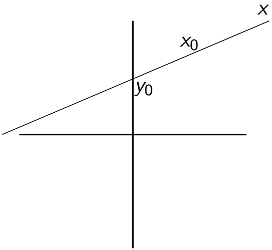

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4. Goes through the points $(2, -4)$ and $(0, 3)$ $y = \frac{7}{-2}x + 3$

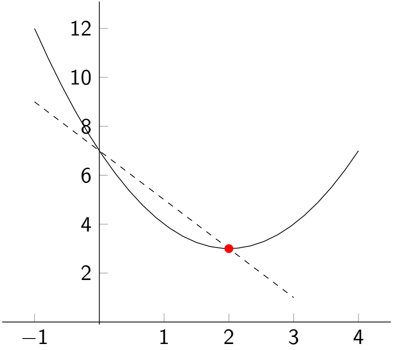
Change: Review

Derivatives allow us to focus upon rate of change.

- Notation: $f'(x)$ or $\frac{dy}{dx}$
- Discrete change: time between two points
- First difference: difference between the value at time$=t-1$ to time$=t$
- Instantaneous change: rate of change at a particular *moment*

Instantaneous Change & Limits

As the interval of change gets smaller, we approach a measure of instantaneous change



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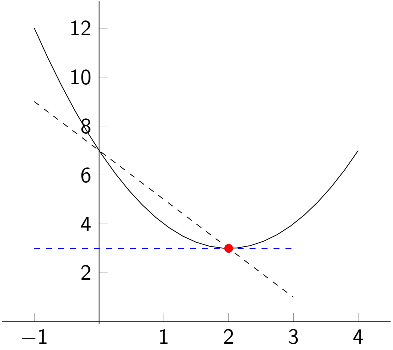

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Secants and Tangents

Secant: slope between two points (intersects two points on a curve)
Tangent: touches the curve at any given point

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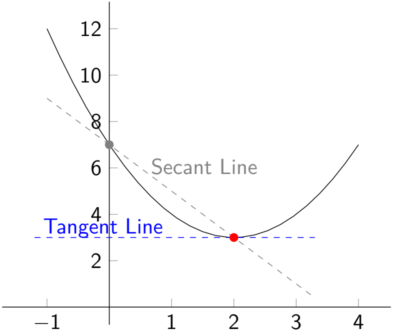

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To calculate the derivative, begin with the secant formula (discrete change between points), reducing the difference to some arbitrarily small value, h. As h goes to zero, we go from discrete (secant) to instantaneous (tangent).

$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

Example: $3x$

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Note: we're using composition here! Hello, day 1!

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OK great – we know derivatives tell us about rates of change. So what? Why does this matter?

They give us the information about the function's *rate of change* which again matters because we can know more about the relationship between x and y – e.g. more x is always, sometimes or never associated with more y.

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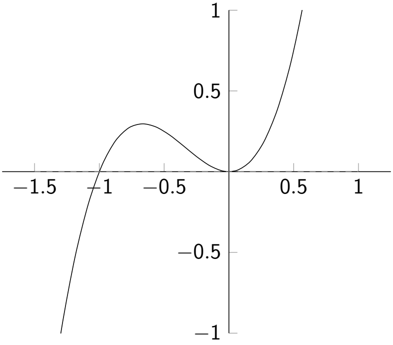
- Zero? max or min

Derivative of a function: max and min

Where are the maxima and/or minima of the function?



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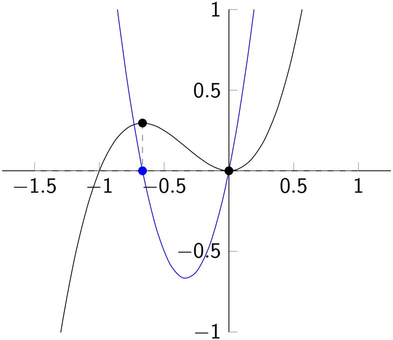


Behaving Badly

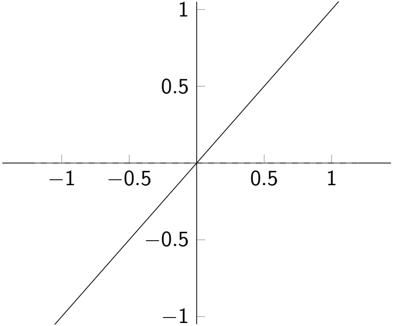
As we saw before: the function must be *continuous* on the *interval* to be differentiable

Some functions not differentiable or not differentiable at a certain point–not "well behaved" functions

Continuity

Continuous function: draw without picking up a pencil

YES



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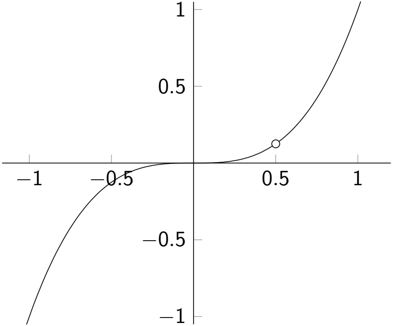

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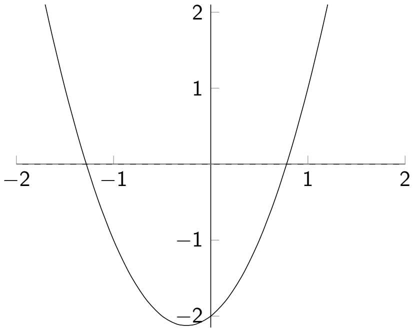

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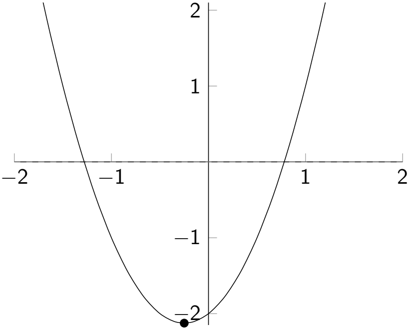

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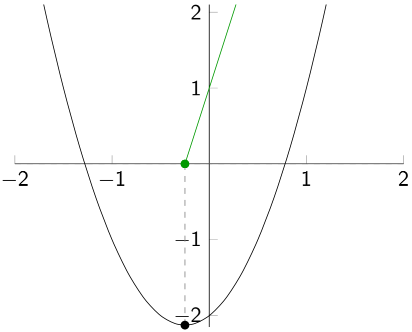

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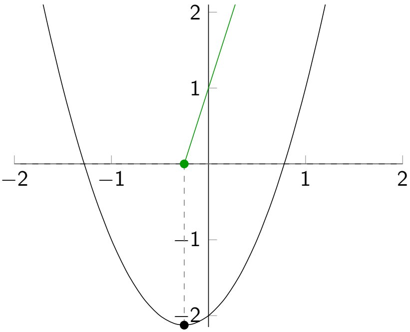

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Think about it: do you want to do $4x^3 + 3x - 2$ using $\frac{f(x_0+h)-f(x_0)}{h}$?

Derivative Rules

We refer to derivative of $f(x)$ as $f`(x)$ below with constant k:

1. $f(k * x) = k * f(x), f'(k * x) = k * f'(x)$
2. $f(x) = k$ has derivative $f'(x) = 0$
3. $f(x) = x^n, f'(x) = n * x^{n-1}$
4. $[f(x) + g(x)]' = f'(x) + g'(x)$
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1. $f(k * x) = k * f(x), f'(k * x) = k * f'(x)$
   - $f(x) = 3x,\ f'(x) = 3$
2. $f(x) = k$ has derivative $f'(x) = 0$
   - $f(x) = 4,$
3. $f(x) = x^n, f'(x) = n * x^{n-1}$

4. $[f(x) + g(x)]' = f'(x) + g'(x)$

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   - $f(x) = 3x, g(x) = 7,$
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NOTE: $[f(x) * g(x)]! = f'(x) * g'(x)$ Ex: $(3x * 10x)'! = 30$

Derivatives Two Ways

We can check these handy formulas work as they should. Let's try.
Find the derivative of $f(x) = \frac{1}{x}$

Formal Definition:
$f'(x) = lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h}$

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Practice Problems

Find where functions are continuous and find derivatives

- $f(x) = 5$
- $f(x) = 3x - 7$
- $f(x) = 3x^2$
- $f(x) = \frac{x^2}{x}$
- $f(s) = s^{-2}$
- $f(y) = y(y+7)(y-3)$
- $f(z) = \frac{z^2-5z-6}{z+1}$

Practice Problems

Find where functions are continuous and find derivatives

- $f(x) = 5$,$f'(x) = 0$
- $f(x) = 3x - 7$,$f'(x) = 3$
- $f(x) = 3x^2$,$f'(x) = 6x$
- $f(x) = \frac{x^2}{x}$,$f'(x) = 1$
- $f(s) = s^{-2}$,$f'(s) = -2s^{-3}$ (not continuous at $s = 0$
- $f(y) = y(y + 7)(y - 3)$,$f'(y) = 3y^2 + 8y - 21$
- $f(z) = \frac{z^2-5z-6}{z+1}$,$f'(z) = 1$ (not continuous at $z = -1$)