Day 7: Calculus Fundamentals, Derivatives II

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Math Camp 2022

Day 7: Concept Agenda

- More Derivatives!
  - Higher order derivatives
  - Inflection points– minima and maxima
  - Partial derivatives– what to do when there are $2+$ variables
  - Advanced rules of derivatives: product, quotient, chain rules and more

Fundamentals of Derivatives: Review

Yesterday we went over the basics of derivatives, what they are conceptually, and how to solve for them.

Let's rehash these problems from yesterday to warm up:

- $f(x) = 5$
- $f(x) = 3x - 7$
- $f(x) = 3x^2$
- $f(x) = \frac{x^2}{x}$
- $f(s) = s^{-2}$
- $f(y) = y(y+7)(y-3)$
- $f(z) = \frac{z^2-5z-6}{z+1}$

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- $f(x) = 5$,$f'(x) = 0$
- $f(x) = 3x - 7$,$f'(x) = 3$
- $f(x) = 3x^2$,$f'(x) = 6x$
- $f(x) = \frac{x^2}{x}$,$f'(x) = 1$
- $f(s) = s^{-2}$,$f'(s) = -2s^{-3}$ (not continuous at $s = 0$
- $f(y) = y(y+7)(y-3)$,$f'(y) = 3y^2 + 8y - 21$
- $f(z) = \frac{z^2-5z-6}{z+1}$,$f'(z) = 1$ (not continuous at $z = -1$)

Fundamentals of Derivatives: Review

Moving on from these basic rules, we will now cover higher order deriavtives, partial derivatives, and more advanced (but still basic) rules of derivatives, such as the product rule, quotient rule, and chain rule.

Higher Order Derivatives

Second derivatives (n^{th} derivatives): take a derivative a second (n^{th}) time

Rate of change of rate of change (velocity vs acceleration)

- $f(x) = x^5 + 3x^3 + 2x + 8$

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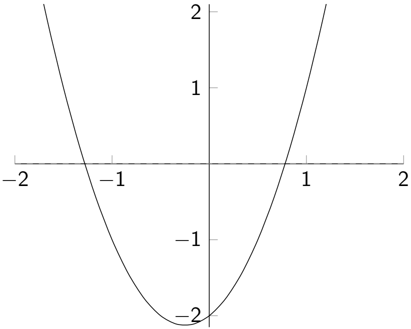
- $f''(x) = 20x^3 + 18x$

Critical Points

Critical points occur where the derivative is zero. We can find them by graphing (ocular method) or plugging in values after calculating the derivative.



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Critical Points, pt. 2

Sometimes there are multiple critical points

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- How to find?

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Zeroes at -2, 0, 2

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Similar to ‘regular’ derivative; treat additional variable(s) as constants. Written as ∂_x or $\frac{\partial f}{\partial x}(x, ...)$

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$f(x,z) = 7xz + 4x^2 + z$ $\partial_x = 7z + 8x$

$f(x,y) = x + 4y$ $\partial_x = 1$

Brambor, Clark, and Golder (05)

Partial derivatives show how rate of change moves with another variable. What is expected change of Y in relation to X?

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \epsilon$$

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How is that different from just β_1 or just β_3?

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Understanding Interactions

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \varepsilon$$

Hypothesis H_1: An increase in X is associated with an increase in Y when condition Z is met, but not when condition Z is absent.

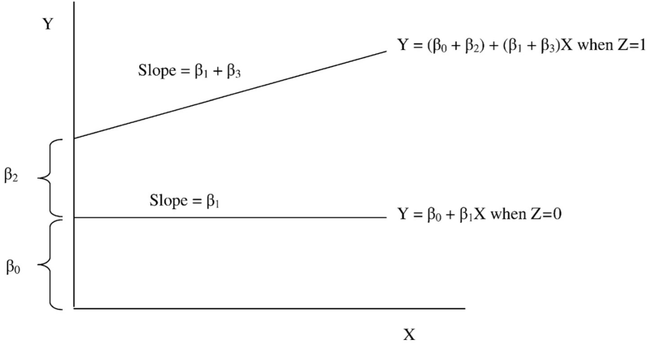

Fig. 1 A graphical illustration of an interaction model consistent with hypothesis H_1.

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- Product Rule: $f(x) * g(x)$
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- Chain Rule: $f(g(x))$
- Other: eg, exponentials: e^x, $ln(x)$

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- Quotient Rule: $\frac{f(x)}{g(x)}$, $\frac{x^4+3x}{x^2}$
- Chain Rule: $f(g(x))$, $(x^2+1)^3$ (composition!)
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To take the derivative using our previous approach, we first multiply: $3x^2 + 4x + 6x + 8 = 3x^2 + 10x + 8$. Then, just take the derivative of each term: $f'(x) = 6x + 10$.

WHY DO WITH THE PRODUCT RULE??

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$(6x + 3)(x^3 + 2x^2 + x + 2) + (3x^2 + 4x + 1)(3x^2 + 3x + 4)$. This is a mess – but you have your answer at least (and much easier than doing it the long way)!

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Practice: $\frac{x-4}{x+5}$ $\frac{3x^3}{x+2}$ **Ans:** $\frac{9}{(x+5)^2}$ $\frac{6x^3+18x^2}{(x+2)^2}$

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Try: $f(x) = (2x^2 + 8x)^4$ $f(x) = (9x - x^2)^6$
Ans $4(4x+8)(2x^2+8x)^3$
$6(9-2x)(9x-x^2)^5 = (54-12x)(9x-2x^2)^5$

Exponentials: e and ln

You can take the derivative of continuous functions – including those with a log and/or e in them. The rules are a little hard, but once you learn them, it's not too bad:

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TRY: $ln(x^2)$, e^{2x} **ANS:** $\frac{2}{x}$ **and** $2e^{2x}$

Additional Resources

- Daniel Kleitman's Calculus for Beginners and Artists: www-math.mit.edu/~djk/calculus_beginners
- Dan Sloughter (online textbook): math.furman.edu/~dcs/book
- Calc refresher (Harvey Mudd Calc Tutorial): www.math.hmc.edu/calculus/tutorials/

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Table 6.1: List of Rules of Differentiation

Sum rule	$(f(x)+g(x))' = f'(x)+g'(x)$
Difference rule	$(f(x)-g(x))' = f'(x)-g'(x)$
Multiply by constant rule	$f'(ax) = af'(x)$
Product rule	$(f(x)g(x))' = f'(x)g(x)+f(x)g'(x)$
Quotient rule	$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$
Chain rule	$(g(f(x)))' = g'(f(x))f'(x)$
Inverse function rule	$(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}$
Constant rule	$(a)' = 0$
Power rule	$(x^n)' = nx^{n-1}$
Exponential rule 1	$(e^x)' = e^x$
Exponential rule 2	$(a^x)' = a^x(\ln(a))$
Logarithm rule 1	$(\ln(x))' = \frac{1}{x}$
Logarithm rule 2	$(\log_a(x))' = \frac{1}{x(\ln(a))}$
Trigonometric rules	$(\sin(x))' = \cos(x)$
	$(\cos(x))' = -\sin(x)$
	$(\tan(x))' = 1+\tan^2(x)$
Piecewise rules	Treat each piece separately

Derivatives in Review

We can use derivatives to calculate rates of change and we're concerned how functions behave. We have a series of rules that can help us get there, even if we have multiple variables.

Table 6.1: List of Rules of Differentiation

Sum rule	$(f(x)+g(x))' = f'(x)+g'(x)$
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However, there comes a time where we have the rate of change but we need to know what the original is. Next, we'll cover integrals – undoing derivatives!