PS and Sociology Math Prefresher

Math Camp

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September 16, 2021

Agenda

- Exponents + logs (review)

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- Derivatives: intro

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- Derivatives: intro
- Derivatives FUN EXCITING RULES (chain rule! quotient rule!)

Exponents, Exponentials, Exponential functions

Recall from earlier this week:

Exponents

Exponents are where you take a variable to some power – e.g. x^a where x is a variable and a is a constant. Typically, we focus on the numerical portion of the exponent–calling it 'the exponent'.

Exponential

An exponential is the reverse of the exponent – here it is a number to the power of a variable, e.g. a^x. To get the x 'down', we need to use logarithms (aka logs).

Exponential Function

The exponential function has a particular base, e, (where e is Euler's e and is approx 2.72.)

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We care because we might see these as utility functions or use higher order values as a way to represent the relationship between two variables in a linear regression.

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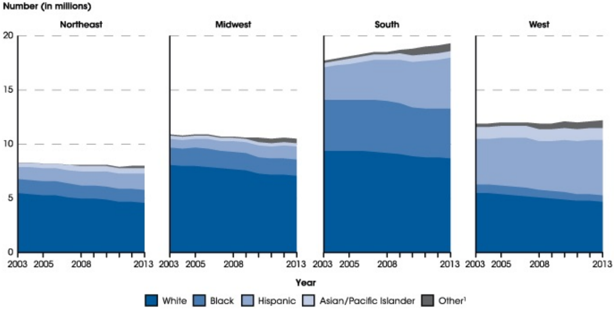
Exponents in log are different from what you might expect:

- $log(x^2) = 2(log(x))$
- $log(x/y) = log(x) - log(y)$ provided $(x, y > 0)$

Logs help weigh smaller values more heavily; adding units not linear–less meaningful for larger values ($log(100) = 2, log(1000) = 3$).

Education Enrollment in the US

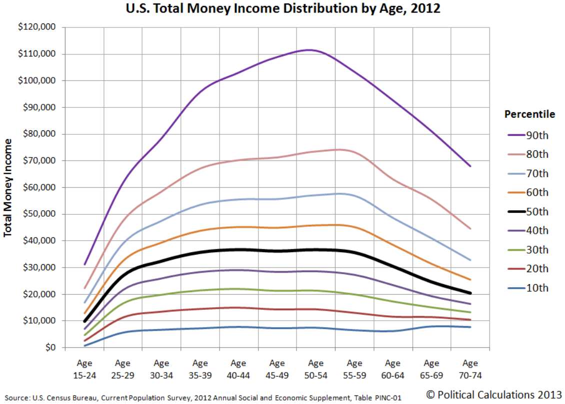
What trends in education can we surmise from these graphs?



Source: **NCES** `http://nces.ed.gov/programs/coe/indicator_cge.asp`

Income by age

At what point is your income increasing the fastest? When do earnings slow down? When do they peak?



Source: Political Calculations

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Derivatives

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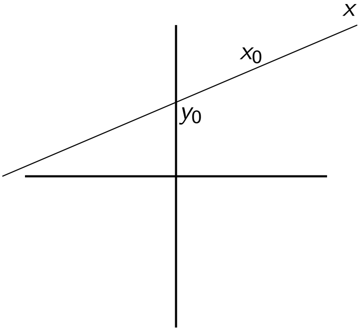
These rates may be between two points (AKA discrete change) or the rate may be at a particular point (AKA instantaneous change). We get at this by calculating the **derivative**, which we denote by $\frac{dy}{dx}$ or $f'(x)$. Both work and both mean the same.

Derivatives: Discrete Change

Slope: rate of change between two points.

$y = a + bx = y_0 + b(x - x_0)$, intercept y_0 or a.



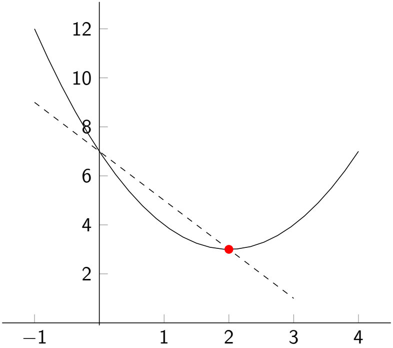
Change: Review

Derivatives allow us to focus upon rate of change.

- Notation: $f'(x)$ or $\frac{dy}{dx}$
- Discrete change: time between two points
- First difference: difference between the value at time=$t-1$ to time=t
- Instantaneous change: rate of change at a particular *moment*

Instantaneous Change & Limits

As the interval of change gets smaller, we approach a measure of instantaneous change



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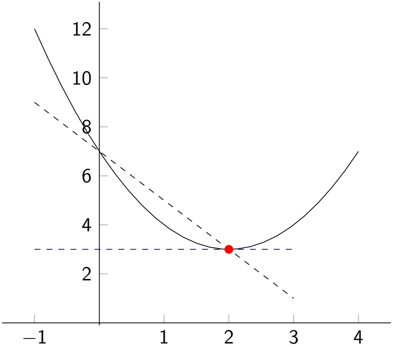

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Secants and Tangents

Secant: slope between two points (intersects two points on a curve)
Tangent: touches the curve at any given point
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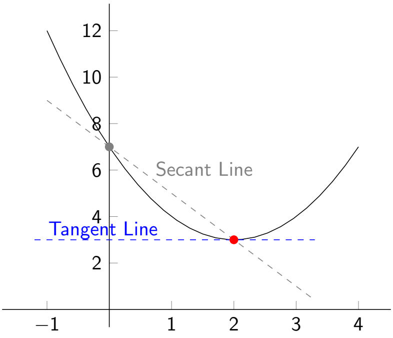

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To calculate the derivative, begin with the secant formula (discrete change between points), reducing the difference to some arbitrarily small value, h. As h goes to zero, we go from discrete (secant) to instantaneous (tangent).

$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

Example: $3x$

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Note: we're using composition here! Hello, day 1!

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OK great – we know derivatives tell us about rates of change. So what? Why does this matter?

They give us the information about the function's *rate of change* which again matters because we can know more about the relationship between x and y – e.g. more x is always, sometimes or never associated with more y.

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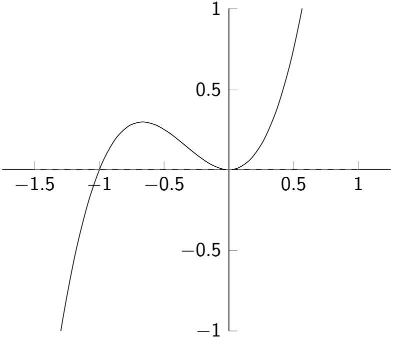
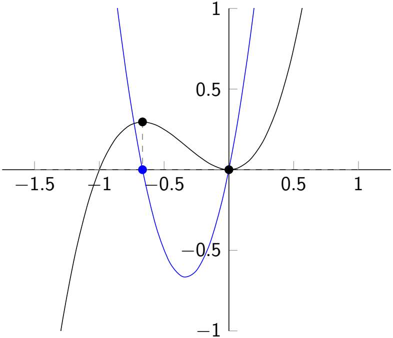
- Zero? max or min

Derivative of a function: max and min

Where are the maxima and/or minima of the function?



Derivative of a function: max and min

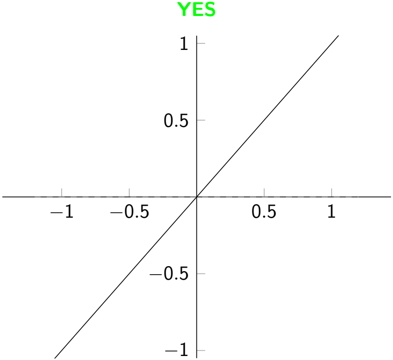


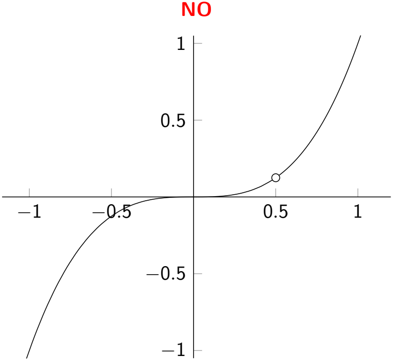
Behaving Badly

As we saw before: the function must be *continuous* on the *interval* to be differentiable

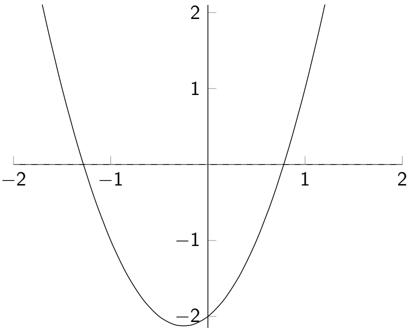
Some functions not differentiable or not differentiable at a certain point–not “well behaved” functions

Continuous function: draw without picking up a pencil





Derivative of this function



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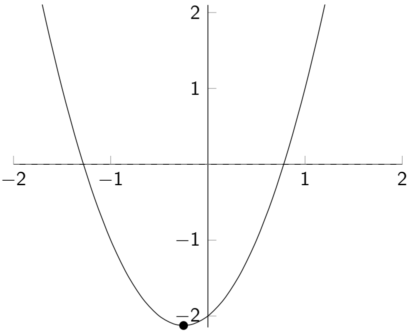

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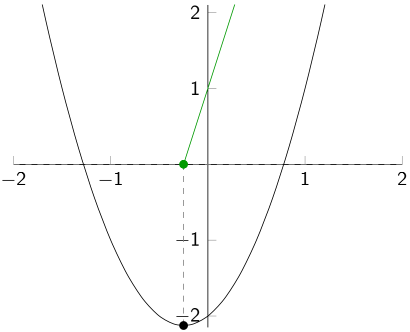

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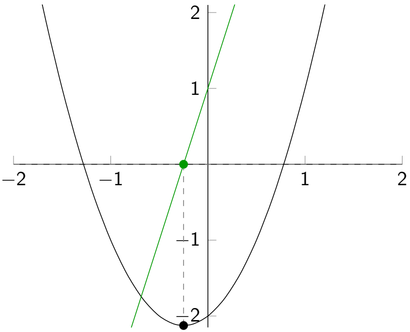

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Think about it: do you want to do $4x^3 + 3x - 2$ using $\frac{f(x_0+h)-f(x_0)}{h}$?

Derivative Rules

We refer to derivative of $f(x)$ as $f'(x)$ below with constant k:

1. $f(k * x) = k * f(x), f'(k * x) = k * f'(x)$
2. $f(x) = k$ has derivative $f'(x) = 0$
3. $f(x) = x^n, f'(x) = n * x^{n-1}$
4. $[f(x) + g(x)]' = f'(x) + g'(x)$
5. $[f(x) - g(x)]' = f'(x) - g'(x)$

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NOTE: $[f(x) * g(x)]! = f'(x) * g'(x)$ Ex: $(3x * 10x)'! = 30$

Derivatives Two Ways

We can check these handy formulas work as they should. Let's try. Find the derivative of $f(x) = \frac{1}{x}$

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Practice Problems

Find where functions are continuous and find derivatives

- $f(x) = 5$
- $f(x) = 3x - 7$
- $f(x) = 3x^2$
- $f(x) = \frac{x^2}{x}$
- $f(s) = s^{-2}$
- $f(y) = y(y+7)(y-3)$
- $f(z) = \frac{z^2-5z-6}{z+1}$

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- $f(x) = 3x - 7$,$f'(x) = 3$
- $f(x) = 3x^2$,$f'(x) = 6x$
- $f(x) = \frac{x^2}{x}$,$f'(x) = 1$
- $f(s) = s^{-2}$,$f'(s) = -2s^{-3}$ (not continuous at $s = 0$
- $f(y) = y(y+7)(y-3)$,$f'(y) = 3y^2 + 8y - 21$
- $f(z) = \frac{z^2-5z-6}{z+1}$,$f'(z) = 1$ (not continuous at $z = -1$)

Higher Order Derivatives

Second derivatives (n^{th} derivatives): take a derivative a second (n^{th}) time
Rate of change of rate of change (velocity vs acceleration)

- $f(x) = x^5 + 3x^3 + 2x + 8$

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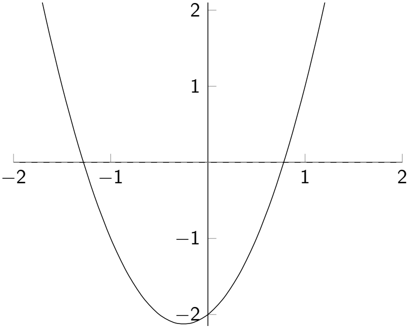
- $f''(x) = 20x^3 + 18x$

Critical Points

Critical points occur where the derivative is zero. We can find them by graphing (ocular method) or plugging in values after calculating the derivative.



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Sometimes there are multiple critical points

- How many?

- How to find?

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(up to) Three critical points.
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Zeroes at -2, 0, 2

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Similar to 'regular' derivative; treat additional variable(s) as constants. Written as ∂_x or $\frac{\partial f}{\partial x}(x, ...)$

THIS IS IMPORTANT FOR INTERACTION TERMS!

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$f(x, z) = 7xz + 4x^2 + z$ $\partial_x = 7z + 8x$
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Brambor, Clark, and Golder (05)

Partial derivatives show how rate of change moves with another variable. What is expected change of Y in relation to X?

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \epsilon$$

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How is that different from just β_1 or just β_3?

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Understanding Interactions

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \varepsilon$$

Hypothesis H_1: An increase in X is associated with an increase in Y when condition Z is met, but not when condition Z is absent.

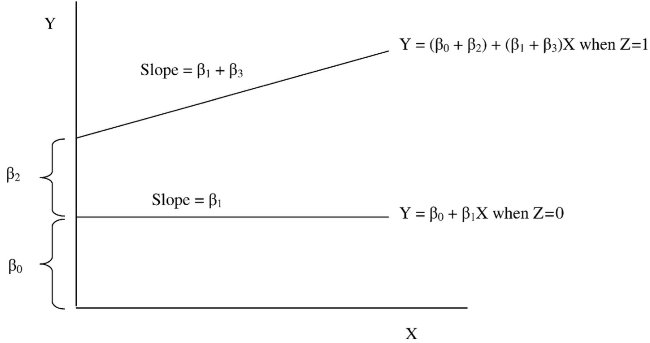

Fig. 1 A graphical illustration of an interaction model consistent with hypothesis H_1.

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- Product Rule: $f(x) * g(x)$
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- Quotient Rule: $\frac{f(x)}{g(x)}$, $\frac{x^4+3x}{x^2}$
- Chain Rule: $f(g(x))$, $(x^2+1)^3$ (composition!)
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$f'(x) * g(x) + g'(x)f(x) = 3(x + 2) + 1(3x + 4)$ Simplify to get: $3x + 6 + 3x + 4 = 6x + 10$.

To take the derivative using our previous approach, we first multiply: $3x^2 + 4x + 6x + 8 = 3x^2 + 10x + 8$. Then, just take the derivative of each term: $f'(x) = 6x + 10$.

WHY DO WITH THE PRODUCT RULE??

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$f'(x) = 6x + 3$ and $g'(x) = 3x^2 + 4x + 1$.

We can substitute this into the formula: $f'(x) * g(x) + g'(x)f(x)$

Product Rule

Suppose that instead you had $(3x^2 + 3x + 4)(x^3 + 2x^2 + x + 2)$. Now the product rule is looking a little nicer!

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Ans: $\frac{9}{(x+5)^2}$ $\frac{6x^3+18x^2}{(x+2)^2}$

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Ans $4(4x+8)(2x^2+8x)^3$
$6(9-2x)(9x-x^2)^5 = (54-12x)(9x-2x^2)^5$

Exponentials: e and ln

You can take the derivative of continuous functions – including those with a log and/or e in them. The rules are a little hard, but once you learn them, it's not too bad:

$f(x) = e^x \quad f'(x) = e^x$ (a favorite of mine)
$f(x) = e^{g(x)} \quad f'(x) = e^{g(x)} * g'(x)$
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TRY: $ln(x^2)$, e^{2x} **ANS:** $\frac{2}{x}$ **and** $2e^{2x}$

Additional Resources

- Daniel Kleitman's Calculus for Beginners and Artists: `www-math.mit.edu/~djk/calculus_beginners`
- Dan Sloughter (online textbook): `math.furman.edu/~dcs/book`
- Calc refresher (Harvey Mudd Calc Tutorial): `www.math.hmc.edu/calculus/tutorials/`

Derivatives in Review

We can use derivatives to calculate rates of change and we're concerned how functions behave. We have a series of rules that can help us get there, even if we have multiple variables.

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Table 6.1: List of Rules of Differentiation

Sum rule	$(f(x)+g(x))' = f'(x)+g'(x)$
Difference rule	$(f(x)-g(x))' = f'(x)-g'(x)$
Multiply by constant rule	$f'(ax) = af'(x)$
Product rule	$(f(x)g(x))' = f'(x)g(x)+f(x)g'(x)$
Quotient rule	$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$
Chain rule	$(g(f(x))' = g'(f(x))f'(x)$
Inverse function rule	$(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}$
Constant rule	$(a)' = 0$
Power rule	$(x^n)' = nx^{n-1}$
Exponential rule 1	$(e^x)' = e^x$
Exponential rule 2	$(a^x)' = a^x(\ln(a))$
Logarithm rule 1	$(\ln(x))' = \frac{1}{x}$
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Trigonometric rules	$(\sin(x))' = \cos(x)$
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Piecewise rules	Treat each piece separately

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