Digital Signal Processing EE 351M
Prof. Haris Vikalo HW #7
March 22, 2018 Due: 03/29/18

Homework Set #7

1. (5 points) [Programming exercise.] Consider an LTI system with a transfer function

$$H(z) = \frac{.98\sin(\pi/24)z}{z^2 - 1.96\cos(\pi/24) + .9604}.$$

Its impulse response is given by

$$h[n] = .98^n \sin(\pi n/24)u[n].$$

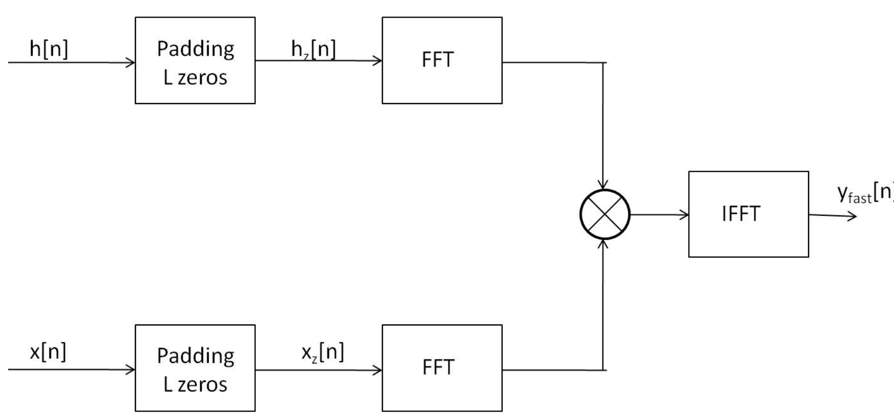
Suppose that this system has the following signal as input:

$$x[n] = n^2(.99)^n \cos(\pi n/48)u[n].$$

Since both $h[n]$ and $x[n]$ decay to zero, we can approximate them as L-point signals (i.e., keep their first L samples) when L is sufficiently large. For this problem, let us choose $L = 512$ (so, $x[n]$ and $h[n]$ are truncated and represented by their respective samples for $0 \leq n \leq 511$).

(a) (1 point) Directly compute the convolution of $x[n]$ and $h[n]$ and plot the resulting $y_{dir}[n]$. In Python, you may want to use 'np.convolve(x,h)'; you will also need to import numpy, scipy and matplotlib.pyplot.

(b) (3 points) Implement the convolution of $x[n]$ and $h[n]$ using the following scheme:



To compute the 1024-point FFT of $h[n]$ in Python, you could use 'np.fft.fft(y)' (same for $x[n]$). IFFT is called in a similar way ('np.fft.ifft(Y)'). Plot the resulting $y_{fast}[n]$ on the same graph as $y_{dir}[n]$.

(c) (1 point) A careful examination of the complexity of the direct method in (a) reveals that the number of required operations is $n_{dir} = 2L^2$. On the other hand, for the implementation (b), the number of required operations is $n_{fast} = 12L\log_2(2L) + 8L + 4$. Plot n_{dir} and n_{fast} (on the same graph) for $1 \leq L \leq 1000$, and comment on the result.

2. (3 points) Draw a complete data flow graph for an 8-point decimation-in-frequency FFT algorithm. Please label the constant terms and input/output indices clearly. How many complex multiplications and additions are needed in this 8-point algorithm? How does your answer change if you do not count trivial multiplications (i.e., multiplications with $W_N^0 = 1$)?

3. (3 points) Assume that a 4096-point DFT is computed using the radix-2 decimation-in-frequency algorithm.

   (a) How many stages of butterflies are needed?

   (b) Draw a butterfly used in the 5th stage (counting from input) and determine all coefficients (twiddle factors) used in this stage.

   (c) How many complex multiplications are needed for the entire 4096-point FFT? How many real multiplications are required?

4. (3 points) Suppose that we have a number of eight-point decimation-in-time FFT chips. How could these chips be used to compute a 24-point DFT? We are looking for an explicit expression for $X[k]$, $k = 0, 1, \ldots, 23$.

5. (4 points) Assuming that a 24-point DFT is computed using the prime factor algorithm,

   (a) Determine the input and output mapping tables.

   (b) If the input sequence is

$$x[n] = \begin{cases} 1, & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

   carry out the 24-point DFT of $x[n]$ step-by-step using the prime factor algorithm.