Identify Sustainable Credit Gap

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1 INTRODUCTION

- To overcome model uncertainty in using credit gap as an early warning indicator (EWI) of systemic financial crises, we propose using model averaging of different credit gap measurements. The method is based on Bayesian Model Average Raftery (1995)
- Area under the curve of operating characteristic (AUROC or AUC) has been widely used as a criterion to determine the performance of a EWI. But it has received some criticism regarding the lower left area of the curve representing low predictive ability of the indicator.
- Borio and Drehmann (2009) and Beltran et al (2021) proposed a policy loss function constraining the relevance of the curve measurement to just a portion where Type II error rate is less than 1/3 or at least 2/3 of the crises are predicted.
- Detken (2014) proposed using partial standardized area under the curve (psAUC) as an alternative measurement of the performance of an EWI.

Our contribution: - Compare different credit gap measurements' performance as EWIs using a new criterion - partial standarized AUC (psAUC) contraining Type II error < 1/3. - Overcome model uncertainty by implementing model averaging. We incoporated psAUC values in the model selection and weighting process, instead of AUC or BIC values. - For ease of policy implication, we propose a single credit gap measurement from weighted averaging other popularly studied credit gap measurements.

2 LITERATURE REVIEW

Beltran (2021) - measured and the performance of BIS Basel credit gap, Structural Time Series model (STM) gap, Moving average (MA) gap, Hamilton filter gap, and optimized the smoothing parameters ρ in those filters to minimize policy loss function.

$$L_{\theta,\rho} = \alpha TypeI(\theta) + (1-\alpha)TypeII(\theta)|TPR \ge 2/3$$

• θ is the optimized threshold that minizes loss function.

Galán (2019) proposed rolling sample of 15 and 20 years when creating one sided cycle.

Drehmann (2021) created Hamilton filter in a panel setting with fixed coefficients on independent variables across countries.

3 DATA DESCRIPTION

Our sample periods include quarterly data from 1970:Q4 to 2017:Q4 across 43 countries. The sample periods were chosen based on the availability of systemic crisis data and credit to gdp data. The main source of teh data comes from the Bank of International Settlement (BIS). The total credit data is measured as percentage of GDP.

Regarding data for dating systemic crises, there is no consensus single source of database. We follow the literature and use the crisis dates reported in the European Systemic Risk Board crisis data set (Lo Duca et al. 2017) and Laeven and Valencia (2018).

While the credit data is available until 2021:Q3, our systemic crisis data stopped at 2017:Q4. So we will limit our analysis till 2017:Q4. We will also omit periods of countries with shorter measurement of credit.

Insert descriptive statistics table here

4 EMPIRICAL MODEL

4.1 Credit gaps decompositions

$$100 * \frac{Credit}{GDP} = y_t = \tau_{yt} + c_{yt} \tag{1}$$

We created 90 candidate one-sided credit gap measurements based on the literature. Once a country has more than 15 years of credit measurement available, we start storing its one-sided credit gap values onward. Our decomposition methods include one sided Hodrick-Prescott filter, Hamilton filter (panel and non-panel setting), Moving Average, Structural Time-series model, linear, quadratic and polynomial decompositions, all are decomposed with full sample, rolling 15 years and 20 years rolling window settings.

4.2 Early Warning Indicator - Logistic regression:

$$****pre.crisis_{ti} \sim credit.gap_{tij} \tag{2}$$

- *i* is country indicator. *j* is credit gap filter type
- where $pre.crisis_{it} = 1$ or 0
- The pre-crisis indicator is set to 1 when t is between 5-12 quarters before a systemic crisis.

- We discard measurements between 1-4 quarters before a crisis, periods during a crisis and post-crisis periods identified in Lo Duca et al. (2017) and Laeven and Valencia (2018).
 - The indicator is set to 0 at other periods.
 - pre-crisis periods of imported crises identified in the dataset are also set to 0. However, we still discard measurements of periods during and post-crisis for imported crises.

4.3 AUROC

Each logistic regression with a different gap measurement yields a Area Under Curve (AUC) of receiver operating characteristic value. There is an underlying assumption that the higher the AUC value is the better overall performance of a credit gap is as an EWI.

• However, the AUC value received some criticism regarding the area on its lower left corner, where the predictive power of the threshold (TPR) is low.

$$AUC = \int_0^1 TPRd(FPR)$$

A ROC curve in the EWI setting represents True Positive Rate (TPR) and False Positive Rate (FPR) of different credit gap thresholds indicating a pre-crisis period. The thresholds are determined by the logistic regression predicted probability values.

4.4 partial standardised AUROC (psAUROC)

To overcome the issue of unnecessary information included in the full AUC. An approach to estimate partial AUC was proposed.

Detken (2014) on partial standardized AUC:

"Instead of considering only the full AUROC (e.g. Drehmann and Juselius, 2014), this paper also presents a partial standardised AUROC (psAUROC) that cuts off the area associated with a preference parameter of $\theta < 0.5$."...

"While the psAUROC has been used extensively in the area of medical statistics to assess the performance of a classifier only in specific regions of the ROC curve (e.g. McClish, 1989 and Jiang et al., 1996), it is a new approach in the literature evaluating EWMs"...

"The results reported in this paper show that the psAUROC can reveal useful additional information as long as the partial area does not become too restricted."

4.5 pAUROC (or pAUC)

Beltran (2021) constrainted the policy loss function to TPR $\geq 2/3$ or Type II error rate < 1/3. They then estimated the policy loss function value at different points on the ROC curve by assigning different policy preferences α .

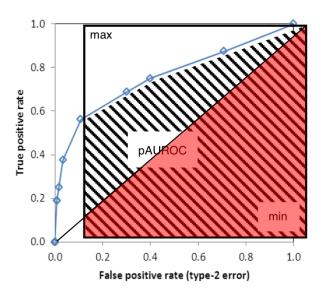
 \Rightarrow In this paper, we propose to restrict the consideration of the ROC curve to TPR $\geq 2/3$, then estimate the psAUC of the restricted ROC curve region instead. *Our notation of Type I and Type II error follows Beltran (2021) which deviated from previous literature.

$$pAUROC = \int_{\frac{2}{3}}^{1} TNRd(TPR) = \int_{\frac{2}{3}}^{1} specificity d(sensitivity)$$
 (3)

- TNR = 1- FPR
- FPR = Type I error rate, FNR = Type II error rate

4.6 standardize psAUROC - Detken (2014)

Figure B2.2: Procedure to standardise the pAUROC measure



$$psAUROC = \frac{1}{2} \left[1 + \frac{pAUROC - min}{max - min} \right]$$
 (4)

5 Model selection and averaging

5.1 Variable selection

5.1.1 Comparing performance of individual credit gaps

Using partial area under the curve (psAUC) values

5.1.2 Test for gaps combination performance

Using Markov Chain Monte Carlo Model Comparison (MC^3) developed by Madigan and York (1995). The method assigns posterior probability for different credit gaps being selected in most likely models/combinations. Babecky (2014) used this MC^3 method to identify potential variables in EWI models.

$$Model_k$$
: $pre.crisis_{ti} \sim \sum_j \beta_j * credit.gap_{tij}$

5.1.3 Variable selection

We selected 29 credit gap measurements based on these 2 criteria.

5.2 Variable selection (top 23 gaps ranked by psAUC)

Cycles	BIC	AIC	AUC	psAUC	c.Threshold	Type.I	Type.II	Policy.Loss.Function
null	0.0000	0.0000	0.5000	0.5000	NA	1.0000	0.0000	1.0000
c.bn6.r20	-108.0679	-114.4506	0.7048	0.6379	0.6581	0.3962	0.3019	0.2481
c.hamilton28.panel	-149.8518	-156.2346	0.7107	0.6359	9.7674	0.3912	0.3066	0.2470
c.hamilton13.panelr20	-150.2442	-156.6269	0.7036	0.6333	5.9895	0.4261	0.2547	0.2464
c.hamilton24.panel	-134.4093	-140.7920	0.6991	0.6322	7.1794	0.4383	0.2689	0.2644
c.hamilton20.panelr20	-151.5617	-157.9445	0.7048	0.6313	7.9350	0.4321	0.3066	0.2807
c.ma	-120.8108	-127.1936	0.6922	0.6313	5.7813	0.3989	0.3160	0.2590
c.hamilton20.panelr15	-135.3713	-141.7540	0.6985	0.6312	7.5244	0.4616	0.2689	0.2854
c.hamilton13.panelr15	-126.2968	-132.6796	0.6924	0.6311	6.5289	0.4297	0.2830	0.2647
c.hamilton28.panelr20	-164.6015	-170.9842	0.7158	0.6302	10.8558	0.3948	0.2925	0.2414
c.hamilton24.panelr20	-155.8638	-162.2466	0.7096	0.6301	9.1672	0.4251	0.2830	0.2608
c.hamilton24.panelr15	-143.2235	-149.6062	0.7033	0.6299	10.4963	0.3984	0.3160	0.2586
c.hamilton20.panel	-126.8625	-133.2452	0.6907	0.6288	5.6212	0.4686	0.2830	0.2997
c.hamilton28.panelr15	-154.4533	-160.8361	0.7091	0.6270	11.5510	0.3854	0.2972	0.2369
c.hamilton13.panel	-133.9347	-140.3175	0.6922	0.6250	4.9769	0.4285	0.2877	0.2664
c.bn2.r20	-109.3128	-115.6955	0.6963	0.6218	0.2776	0.4080	0.3255	0.2724
c.linear	-135.4069	-141.7896	0.6879	0.6204	3.9989	0.4616	0.2925	0.2986
c.bn2	-135.9914	-142.3741	0.6842	0.6165	0.1864	0.4530	0.3113	0.3021
c.bn6	-132.7915	-139.1742	0.6835	0.6113	0.4710	0.4371	0.2830	0.2712
c.bn6.r15	-54.9953	-61.3781	0.6756	0.6070	0.5680	0.4179	0.3255	0.2806
c.bn2.r15	-83.9469	-90.3297	0.6749	0.6047	0.1349	0.4761	0.3302	0.3357
c.poly4.r20	3.5738	-2.8090	0.5772	0.6011	0.1651	0.4980	0.3302	0.3570
BIS Basel gap	-121.5910	-127.9738	0.6733	0.5960	3.0578	0.4441	0.3255	0.3032
c.bn4	-169.1186	-175.5014	0.6892	0.5943	1.2840	0.3837	0.3255	0.2532
c.bn4.r15	-89.6147	-95.9975	0.6669	0.5929	0.4435	0.4792	0.2925	0.3152
c.bn5.r20	-99.7674	-106.1501	0.6744	0.5928	0.5016	0.4234	0.3302	0.2883

5.3 Variable Selection (MC3)

Variable	Pr(B!=0)	Variable	Pr(B!=0)
Intercept	1	c.bn2	0.3721
c.hamilton28_panel	0.999968	c.bn3_r10	0.2478
c.bn7_r15	0.93097125	c.hp400k	0.2309
c.poly3	0.915792	c.hamilton24_r20	0.2175
c.hamilton13_panel	0.733247	c.hamilton13_r20	0.1958
c.hamilton13_r10	0.70601225	c.bn2_r10	0.1916
c.hamilton28	0.65141375	c.stm	0.1894
c.hamilton13	0.6438865	c.hamilton28_r20	0.1799
c.bn3_r15	0.514728	c.hp_r20	0.1794
c.hp3k	0.4888055	c.hp	0.1761
c.hp3k_r20	0.4887615	c.bn8_r20	0.1561
c.poly6	0.39826475	c.linear_r20	0.1526
		c.hp221k	0.1440

6 Model Averaging

7 Model averaging

7.1 Bayesian Model Averging

The Bayesian Model Average method is formalized in Raftery (1995) to account for model uncertainty.

7.1.1 Model posterior probability

equation (33): Model k posterior probability (weight):

$$P(M_k|D) = \frac{P(D|M_k)P(M_k)}{\sum_{l=1}^{K} P(D|M_l)P(M_l)} \approx \frac{exp(-\frac{1}{2}BIC_k)}{\sum_{l=1}^{K} exp(-\frac{1}{2}BIC_l)}$$
(5)

- Where $P(M_k)$ is model prior probability and can be ignored if all models are assumed equal prior weights.
- $P(D|M_k)$ is marginal likehood. And $P(D|M_k) \propto exp(-\frac{1}{2}BIC_k)$
- In which $BIC_k = 2log(Bayesfactor_{sk}) = \chi_{sk}^2 df_k log(n)$. s indicates the saturated model.

7.2 Model posterior probability

- $BIC_k = 2log(Bayesfactor_{sk}) = \chi_{sk}^2 df_k log(n)$
- χ_{sk}^2 is the deviance of model K from the saturated model

 - $\chi_{sk}^2 = 2(ll(Ms) ll(Mk))$ ll(Mk) is the log-likelihood of model Mk given data D

Alternate deviance measurement 7.2.1

We propose using psAUC instead of log-likelihood in the measurement of deviance. Hence, an alternative BIC value can be estimated at:

$$BIC_{alt,k} = 2log(Bayesfactor_{alt,sk})$$
 (6)

$$= 2(1000 * (psAUC_s - psAUC_k)) - df_k log(n)$$
(7)

- We scaled the psAUC value by 1000 since 0 < psAUC < 1. Also, by design, $psAUC_s = 1$.

7.3 Posterior distribution of coefficients of interest:

 β_i is the coefficient of credit gap j (c_i) in a logistic regression model k against pre-crisis indicator. When considering a particular β_1 :

$$p(\beta_1|D,\beta_1 \neq 0) = \sum_{A_1} p(\beta_1|D,M_k) p'(M_k|D)$$

- where $p'(M_k|D) = p(M_k|D)/pr[\beta_1 \neq 0|D]$ and $pr[\beta_1 \neq 0|D] = \sum_{A_1} p(M_k|D)$
 - this is the probability that β_1 is in the averaged model
 - $A_1 = \{M_k : k = 1, ..., K; \beta_1 \neq 0\}$, is the set of models that includes β_1

7.4 **Approximation of Bayesian point estimate:**

$$\hat{\beta}_1 = E[\beta_1 | D, \beta_1 \neq 0] = \sum_{A_1} \hat{\beta}_1(k) p'(M_k | D)$$
(8)

$$SD^{2}[\beta_{1}|D,\beta_{1}\neq0]=[\sum_{A_{1}}[se_{1}^{2}(k)+]+\hat{\beta}_{1}(k)]p'(M_{k}|D)-E[\beta_{1}|D,\beta_{1}\neq0]^{2}$$

• Where $\hat{\beta}_1(k)$ and $se_1^2(k)$ are respectively the MLE and standard error of β_1 under the model *M_k*. (Leamer 1978, p.118; Raftery 1993a)

8 Weighted credit gap creation

8.1 Weighted averaged credit gap - motivation

GLM binomial estimation:

$$\widehat{pre.crisis_{ti}} = \widehat{probability_{ti}} = \frac{1}{1 + exp(-(a + \sum_{i} \hat{\beta}_{i}c_{tij}))}$$

• With
$$\hat{\beta}_j = E[\beta_j | D, \beta_j \neq 0] = \sum_{A_j} \hat{\beta}_j(k) p'(M_k | D)$$

 \Rightarrow We propose a single weighted credit gap \hat{c}_{ti} that satisfies:

$$\frac{1}{1 + exp(-(a + \hat{\boldsymbol{\beta}}\hat{c}_{ti}))} = \frac{1}{1 + exp(-(a + \sum_{i} \hat{\boldsymbol{\beta}}_{i}c_{tij}))}$$

OR

$$\sum_{j} \hat{\beta}_{j} c_{tij} = \hat{\beta} \hat{c}_{ti} \tag{9}$$

8.2 Weighted averaged credit gap - creation

$$\sum_{j} \hat{\beta}_{j} c_{tij} = \hat{\beta} \hat{c}_{ti}$$

We then propose $\hat{\beta} = \sum_j \hat{\beta}_j$ Therefore,

$$\hat{c}_{ti} = \frac{\sum_{j} (\hat{\beta}_{j} c_{tij})}{\sum_{j} \hat{\beta}_{j}} = \sum_{j} w_{j} c_{tij}$$
(10)

The weight of each candidate credit gap j is $w_j = \frac{\hat{\beta}_j}{\sum_i \hat{\beta}_i}$

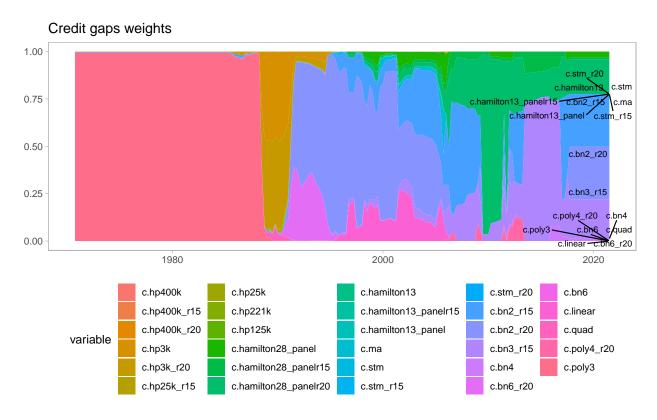
8.3 One-sided crisis weighted averaged credit gap

- The weight of each candidate credit gap j is $w_j = \frac{\hat{\beta}_j}{\sum_i \hat{\beta}_i}$
- We save the weights w_j at every incremental period t of available data to create a one-sided weight vector w_{tj} .
- \Rightarrow To create one-sided crisis weighted averaged credit gap for each country $i(\hat{c}_{ti})$, we compute:

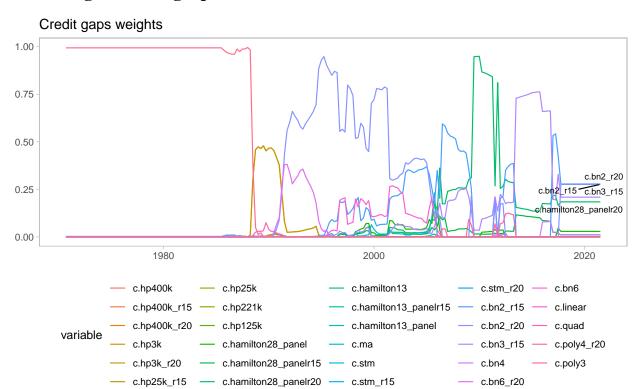
$$\hat{c}_{ti,one-sided} = \sum_{j} w_{tj} * c_{tij} \tag{11}$$

8.4 Weights stacked graph

Weights are restricted to be positive to ensure stability



8.5 Weights series graph



9 Empirical Results

9.1 Comparing performance of weighted gap - Full Sample

Cycles	BIC	AIC	AUC	psAUC	c.Threshold	Type.I	Type.II	Policy.Loss.Function
null	0.0000	0.0000	0.5000	0.5000	NA	1.0000	0.0000	1.0000
1.sided weighted.cycle	-127.5308	-133.9135	0.7182	0.6454	2.8892	0.3532	0.3255	0.2307
c.bn6.r20	-108.0679	-114.4506	0.7048	0.6379	0.6581	0.3962	0.3019	0.2481
c.hamilton28.panel	-149.8518	-156.2346	0.7107	0.6359	9.7674	0.3912	0.3066	0.2470
c.ma	-120.8108	-127.1936	0.6922	0.6313	5.7813	0.3989	0.3160	0.2590
c.hamilton13.panelr15	-126.2968	-132.6796	0.6924	0.6311	6.5289	0.4297	0.2830	0.2647
c.hamilton28.panelr20	-164.6015	-170.9842	0.7158	0.6302	10.8558	0.3948	0.2925	0.2414
c.hamilton28.panelr15	-154.4533	-160.8361	0.7091	0.6270	11.5510	0.3854	0.2972	0.2369
c.hamilton13.panel	-133.9347	-140.3175	0.6922	0.6250	4.9769	0.4285	0.2877	0.2664
c.bn2.r20	-109.3128	-115.6955	0.6963	0.6218	0.2776	0.4080	0.3255	0.2724
c.linear	-135.4069	-141.7896	0.6879	0.6204	3.9989	0.4616	0.2925	0.2986
c.bn6	-132.7915	-139.1742	0.6835	0.6113	0.4710	0.4371	0.2830	0.2712
c.bn2.r15	-83.9469	-90.3297	0.6749	0.6047	0.1349	0.4761	0.3302	0.3357
c.poly4.r20	3.5738	-2.8090	0.5772	0.6011	0.1651	0.4980	0.3302	0.3570
BIS Basel gap	-121.5910	-127.9738	0.6733	0.5960	3.0578	0.4441	0.3255	0.3032
c.bn4	-169.1186	-175.5014	0.6892	0.5943	1.2840	0.3837	0.3255	0.2532
c.stm.r15	-79.5531	-85.9358	0.6575	0.5924	2.0027	0.4778	0.3160	0.3281
c.hp125k	-92.2897	-98.6725	0.6562	0.5924	2.5216	0.4547	0.3302	0.3158
c.hp221k	-106.8842	-113.2670	0.6656	0.5921	2.6641	0.4561	0.3160	0.3079
c.hp400k.r15	-67.1228	-73.5055	0.6472	0.5912	2.6223	0.4592	0.3255	0.3168
c.stm	-89.2228	-95.6055	0.6523	0.5903	2.2064	0.4684	0.3302	0.3284
c.bn3.r15	-144.4817	-150.8645	0.6687	0.5882	0.1862	0.4780	0.3302	0.3375
c.hp400k.r20	-88.8450	-95.2277	0.6545	0.5871	2.8130	0.4494	0.3302	0.3110
c.stm.r20	-87.2179	-93.6006	0.6482	0.5859	1.9362	0.4826	0.3302	0.3419
c.hp25k.r15	-55.8805	-62.2632	0.6275	0.5812	1.1403	0.5032	0.3066	0.3473
c.hp25k	-56.0388	-62.4215	0.6274	0.5782	1.2839	0.4970	0.3160	0.3469
c.hamilton13	-23.7688	-30.1516	0.6075	0.5782	1.6938	0.5006	0.3160	0.3505
c.quad	-85.1640	-91.5468	0.6622	0.5774	0.2765	0.5162	0.3160	0.3664
c.poly3	5.5099	-0.8728	0.5551	0.5597	-0.5367	0.5860	0.2689	0.4156
c.hp3k	-24.5546	-30.9374	0.5979	0.5572	-0.0472	0.5706	0.3255	0.4315
c.hp3k.r20	-24.5252	-30.9080	0.5978	0.5571	-0.0420	0.5701	0.3255	0.4309

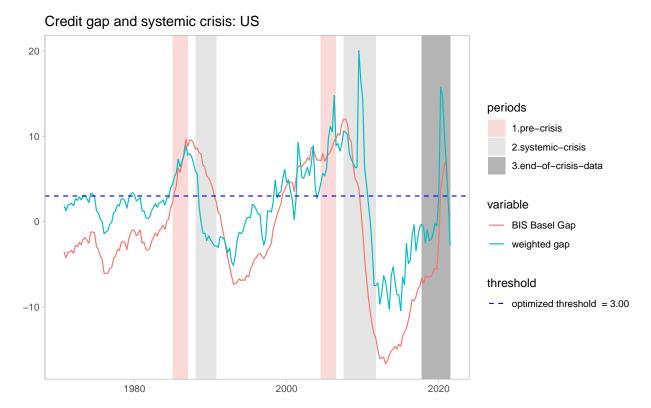
9.2 Comparing performance of weighted gap as an EWI - AE

Cycles	BIC	AIC	AUC	psAUC	c.Threshold	Type.I	Type.II	Policy.Loss.Function
null 1.sided weighted.cycle	0.0000 - 128.8749	0.0000 -134.9430	0.5000 0.7545	0.5000 0.6889	NA 3.5486	1.0000 0.3177	0.0000 0.2841	1.0000 0.1817
c.hamilton13.panelr15	-116.9260	-13 4.94 30 -122.9941	0.7282	0.6846	8.8000	0.3177	0.3011	0.2110
c.hamilton28.panelr15	-141.5772	-147.6453	0.7491	0.6825	11.5510	0.3834	0.2216	0.1961
c.hamilton28.panelr20	-147.4322	-153.5003	0.7514	0.6811	13.1072	0.3489	0.2898	0.2057
c.bn6.r20	-90.1022	-96.1703	0.7338	0.6751	0.6581	0.4126	0.2273	0.2219
c.hamilton28.panel	-129.1730	-135.2411	0.7382	0.6740	9.7674	0.3977	0.2386	0.2151
c.hamilton13.panel	-122.0054	-128.0735	0.7208	0.6684	7.2247	0.3509	0.3239	0.2280
c.ma	-97.4868	-103.5549	0.7133	0.6677	6.0375	0.4073	0.2727	0.2403
c.bn6	-104.1404	-110.2085	0.7042	0.6426	0.8256	0.4040	0.2955	0.2505
c.hp125k	-92.8331	-98.9012	0.6977	0.6397	3.6972	0.3788	0.3239	0.2484
BIS Basel gap	-113.9826	-120.0507	0.7124	0.6390	3.9706	0.3847	0.3011	0.2387
c.hp221k	-104.1190	-110.1871	0.7070	0.6388	3.8869	0.3755	0.3011	0.2317
c.stm.r15	-86.6063	-92.6744	0.6989	0.6382	4.0784	0.3486	0.3295	0.2301
c.stm	-89.9073	-95.9754	0.6929	0.6357	3.3071	0.3924	0.3295	0.2626
c.hp400k.r15	-73.5902	-79.6582	0.6869	0.6357	4.2355	0.3496	0.3125	0.2199
c.linear	-111.9570	-118.0251	0.7069	0.6356	3.9964	0.4620	0.2443	0.2732
c.hp400k.r20	-93.8576	-99.9257	0.6991	0.6340	4.3393	0.3622	0.3295	0.2398
c.bn2.r20	-84.0800	-90.1481	0.6955	0.6297	0.2773	0.4216	0.3182	0.2790
c.stm.r20	-87.6992	-93.7673	0.6879	0.6282	3.1055	0.4073	0.3239	0.2708
c.bn4	-119.0600	-125.1280	0.7038	0.6272	1.8875	0.3575	0.3295	0.2364
c.hp25k.r15	-54.6351	-60.7032	0.6561	0.6138	1.6927	0.4547	0.2784	0.2843
c.hp25k	-55.5895	-61.6576	0.6569	0.6117	1.7847	0.4458	0.2898	0.2827
c.poly4.r20	2.3864	-3.6817	0.5901	0.6113	0.7143	0.4494	0.3239	0.3069
c.quad	-88.1553	-94.2234	0.6998	0.6099	2.4569	0.4017	0.3295	0.2699
c.bn2.r15	-62.5255	-68.5936	0.6683	0.6070	0.0956	0.5075	0.3011	0.3482
c.hamilton13	-29.9493	-36.0174	0.6334	0.6028	3.3153	0.4398	0.3239	0.2983
c.bn3.r15	-93.9600	-100.0281	0.6534	0.5836	0.1563	0.4896	0.3239	0.3445
c.hp3k	-20.4943	-26.5624	0.6092	0.5725	0.1378	0.5446	0.3068	0.3907
c.hp3k.r20	-20.4682	-26.5363	0.6091	0.5724	0.1378	0.5446	0.3068	0.3907
c.poly3	-2.3248	-8.3929	0.5905	0.5723	0.2767	0.5317	0.3295	0.3913

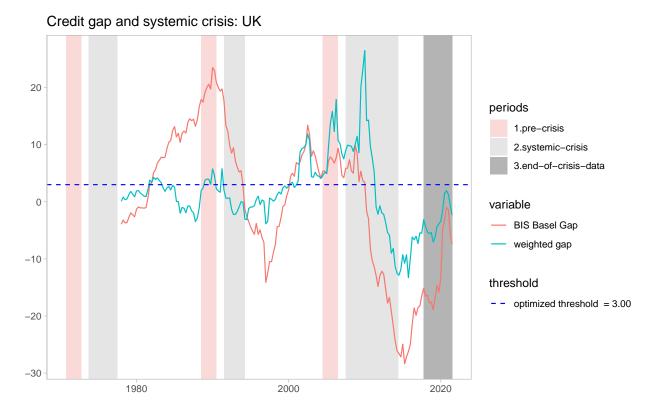
9.3 Comparing performance of weighted gap as an EWI - EME

Cycles	BIC	AIC	AUC	psAUC	c.Threshold	Type.I	Type.II	Policy.Loss.Function
null	0.0000	0.0000	0.5000	0.5000	NA	1.0000	0.0000	1.0000
c.bn3.r15	-46.2774	-51.3507	0.7365	0.6308	0.6244	0.3059	0.3333	0.2047
c.poly3	5.3862	0.3129	0.5737	0.6046	1.8089	0.5280	0.3056	0.3721
c.bn2.r15	-13.2062	-18.2795	0.6879	0.5879	0.2952	0.3566	0.3333	0.2383
c.poly4.r20	7.0732	1.9999	0.5040	0.5816	-0.9609	0.5962	0.3333	0.4665
1.sided weighted.cycle	6.2094	1.1361	0.5325	0.5811	-1.0639	0.6827	0.1111	0.4784
c.linear	-9.3676	-14.4409	0.5787	0.5774	-0.9783	0.6294	0.2222	0.4455
c.bn2.r20	-16.6411	-21.7144	0.6760	0.5751	0.1470	0.4510	0.3056	0.2968
c.hamilton13	6.5749	1.5016	0.5468	0.5710	3.6354	0.6206	0.3056	0.4785
c.ma	-11.6401	-16.7133	0.5572	0.5457	-0.2250	0.7220	0.1667	0.5491
c.hamilton28.panel	-7.8687	-12.9420	0.5392	0.5384	-1.7750	0.6958	0.2778	0.5613
c.quad	6.1997	1.1264	0.4654	0.5334	-6.4882	0.7456	0.1944	0.5938
c.hamilton13.panelr15	-1.6660	-6.7393	0.5087	0.5274	-1.1064	0.7002	0.3333	0.6014
c.hp25k.r15	3.6420	-1.4313	0.5018	0.5265	-3.5672	0.7850	0.1111	0.6285
c.hp25k	3.9466	-1.1267	0.4975	0.5247	-3.7339	0.7893	0.1111	0.6354
c.hp3k	3.3678	-1.7054	0.5276	0.5235	-1.1119	0.7019	0.3333	0.6038
c.hp3k.r20	3.3703	-1.7030	0.5276	0.5235	-1.1125	0.7028	0.3333	0.6050
c.hamilton13.panel	-1.6294	-6.7027	0.5166	0.5222	-2.9398	0.7500	0.2778	0.6397
BIS Basel gap	-0.7015	-5.7748	0.4928	0.5217	-5.3969	0.7920	0.1389	0.6465
c.hamilton28.panelr20	-4.9986	-10.0719	0.5123	0.5213	-1.5578	0.6932	0.3333	0.5916
c.hamilton28.panelr15	-3.3914	-8.4647	0.4987	0.5162	-1.8326	0.7220	0.3333	0.6324
c.hp400k.r15	5.0603	-0.0129	0.4777	0.5147	-5.7358	0.8121	0.1111	0.6718
c.stm.r15	4.7567	-0.3166	0.4780	0.5129	-5.6472	0.8191	0.0833	0.6778
c.hp221k	1.5902	-3.4831	0.4787	0.5123	-5.6416	0.8121	0.1111	0.6718
c.stm.r20	3.1397	-1.9336	0.4768	0.5095	-5.3727	0.8226	0.0833	0.6835
c.hp125k	3.0774	-1.9958	0.4740	0.5094	-5.8918	0.8226	0.0833	0.6835
c.stm	3.1932	-1.8801	0.4757	0.5087	-5.5221	0.8260	0.0833	0.6893
c.hp400k.r20	4.1478	-0.9255	0.4596	0.5081	-6.2903	0.8182	0.1389	0.6887
c.bn4	-42.1713	-47.2446	0.5479	0.4984	-0.5515	0.7509	0.2778	0.6410
c.bn6.r20	-9.5340	-14.6072	0.5296	0.4965	-0.5188	0.7002	0.2778	0.5674
c.bn6	-19.2453	-24.3186	0.5352	0.4911	-0.5146	0.7220	0.2778	0.5985

9.4 Plot weighted gap against BIS gap



9.5 Plot weighted gap against BIS gap



10 Comparison with other decomposition methods

11 Conclusion

APPENDIX