The class of all **quasigroups** is the variety $\mathcal{Q}$ with three binary operations symbolized by $\cdot$, $/$, and $\backslash$ axiomatized at the left. A **Mal'cev variety** $\mathcal{M}$ is one that has available term operation $p(x,y,z)$ satisfying the equations at right.

$$\begin{aligned}\mathcal{Q} \models (x \cdot y)/y &\approx x && \text{(Q1)}\\ (x/y) \cdot y &\approx x && \text{(Q2)}\\ y\backslash(y \cdot x) &\approx x && \text{(Q3)}\\ y \cdot (y\backslash x) &\approx x && \text{(Q4)}\end{aligned}$$

$$\begin{aligned}\mathcal{M} \models p(x,x,y) &\approx y\\ p(x,y,y) &\approx x\end{aligned}$$

Proposition 1. *Quasigroups are Mal'cev.*

Proof. Consider the term

$$p(x,y,z) := (x/(y\backslash y)) \cdot (y\backslash z)$$

Using (Q2) with $y\backslash y$ in place of y we get,

$$\mathcal{Q} \models p(x,y,y) = (x/(y\backslash y)) \cdot (y\backslash y) \approx x$$

To get $\mathcal{Q} \models p(x,x,y) \approx y$ it is useful to spin off a small observation:

Lemma 1. $\mathcal{Q} \models x/(x\backslash x) \approx x$

Proof. Using (Q4) with $y = x$,

$$\mathcal{Q} \models x \cdot (x\backslash x) \approx x \tag{1}$$

Substituting the left side of equation (1) in for x in the "numerator" of the left side of the equation we wish to prove results in a term to which (Q1) applies:

$$\mathcal{Q} \models x/(x\backslash x) \approx (x \cdot (x\backslash x))/(x\backslash x) \approx x$$

□

Thus we finish proof of Proposition 1:

$$\begin{aligned}\mathcal{Q} \models p(x,x,y) &= (x/(x\backslash x)) \cdot (x\backslash y)\\ &\approx x \cdot (x\backslash y) && \text{Lemma (1)}\\ &\approx x && \text{(Q4)}\end{aligned}$$

□

Remark 1. Equation (Q3) was not used in the proof of the foregoing.