

Course 3: Language Modeling



You

Explain how ChatGPT works as if I was a 5 years old child.



ChatGPT

Alright, imagine you have a really smart friend who knows a lot of things. That friend is like a big, magical library with lots and lots of books. Now, this friend is called ChatGPT. When you ask ChatGPT a question or tell it something, it looks into all those books in its library (which are actually a huge amount of information it learned) and tries to give you the best answer or response it can find.

So, you can think of ChatGPT as a helpful buddy who loves to chat and share what it knows with you!



How does it really work?

What is Language Modeling?

Definition

- A sequence of tokens (w_1, w_2, \dots, w_n)
- For a position i , a language model (**LM**) predicts

$$P(w_i \mid (w_j)_{j \neq i}) \in \Delta^V$$

- In words: a LM predicts the probability of a token given its context

Example

I went to the ??? yesterday

$P(\text{park} \mid \text{I went to the ??? yesterday}) = 0.1$

$P(\text{zoo} \mid \text{I went to the ??? yesterday}) = 0.07$

...

$P(\text{under} \mid \text{I went to the ??? yesterday}) = 0$

Why is it hard?

- **Large vocabularies:** 170,000 English words
- **Lots of possible contexts:**
 - For V possible tokens, there are V^L contexts of size L (in theory)
- **Inherent uncertainty:** not obvious even for humans

Basic approach - Unigram

- Learn the *non-contextual* probability (=frequency) of each token:

$$P(w_i | (w_j)_{j \neq i}) = f$$

Example

*chart against operations at influence the surface plays crown a inaro
the three @ but the court lewis on hand american of seamen mu role
due roger executives*

Include context - Bigram

- Predict based on the last token only:

$$P(w_i | (w_j)_{j \neq i}) = P_{\theta}(w_i | w_{i-1})$$

- (MLE): Measure next token frequency

Example

*the antiquamen lost to dios nominated former is carved stone oak
were problematic, 1910. his willingness to receive this may have been
seen anything*

Include more context - n-gram

- Predict based on the n last tokens only:

$$P(w_i | (w_j)_{j \neq i}) = P_{\theta}(w_i | w_{i-n} \dots w_{i-1})$$

- (MLE): Measure occurrences of tokens after $w_{i-n} \dots w_{i-1}$

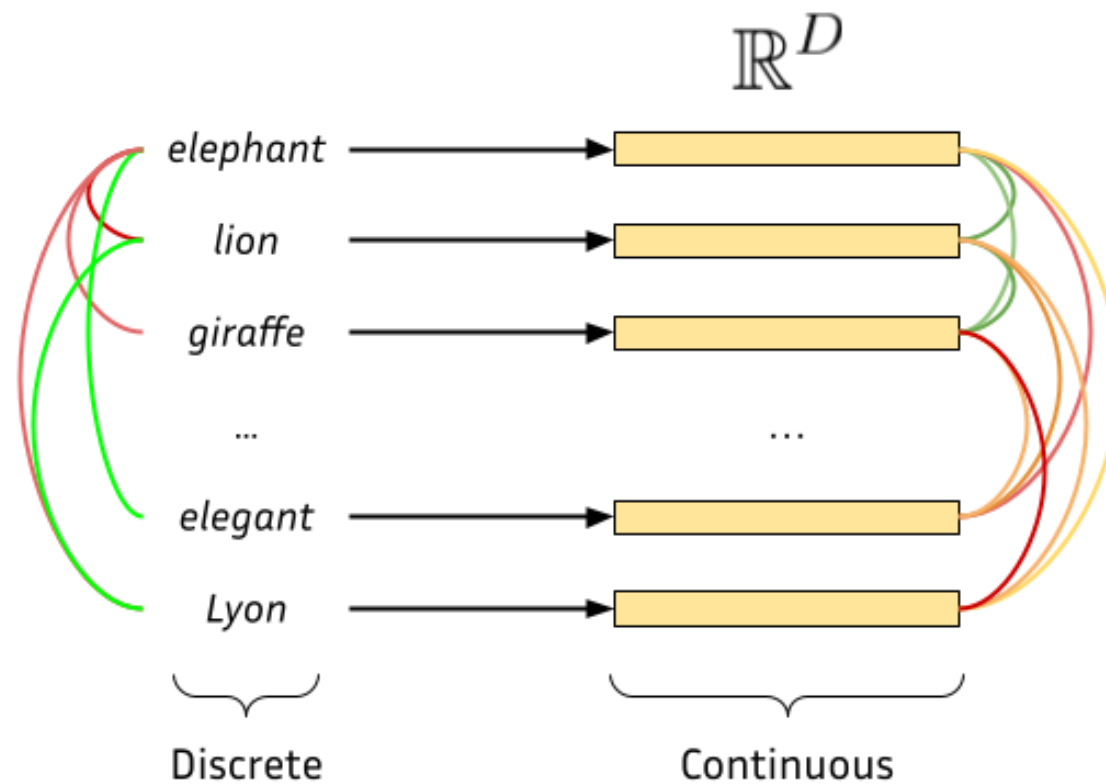
Example (n=4)

*eva gauthier performed large amounts of contemporary french music
across the united states marshals service traveled to frankfurt,
germany and took custody of the matthews*

Statistical n-grams: pro/cons

- Strengths:
 - Easy to train
 - Easy to interpret
 - Fast inference
- Limitations:
 - Very limited context
 - **Unable to extrapolate** : can only model what it has seen

The embedding paradigm



LM with RNNs



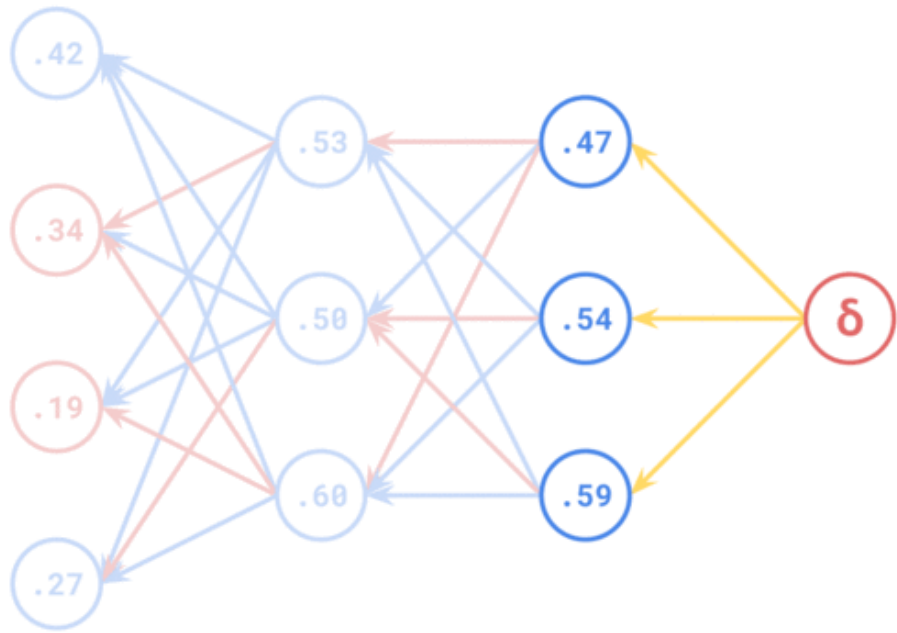
LM with RNNs - Training

- θ : parameters of the RNN
- (w_1, \dots, w_n) : training sequence
- Cross-entropy loss \mathcal{L}_{ce} :

$$\mathcal{L}_{ce}(w, \theta) = - \sum_{i=2}^n 1_{w_i} \cdot \log P_{\theta}(w_i | w_{i-1}, h_{i-1})$$

- Train via back-propagation + SGD

Reminder - Back-propagation



$$\delta_3 \cdot H_2^T = \Delta W_3$$

The equation shows the calculation of the weight update ΔW_3 for the weights connecting Layer 2 to Layer 3. It is the product of the error gradient δ_3 (represented by a red square) and the transpose of the hidden layer output H_2^T (represented by three blue squares). The result is the weight update ΔW_3 (represented by three red squares).

Reminder - Stochastic Gradient Descent

- **Goal** : Minimize a loss function $\mathcal{L}(X, \theta)$ for given data X with respect to model parameters θ
- **Method** :
 - Split X in smaller parts x^i (called mini-batches)
 - Compute $\mathcal{L}(x^i, \theta)$ (forward) and $\nabla_{\theta}\mathcal{L}(x^i, \theta)$ (back-prop)
 - Update: $\theta \leftarrow \theta - \eta \nabla_{\theta}\mathcal{L}(x^i, \theta)$ ($\eta \ll 1$, learning rate)

LM with RNNs: Generation



RNNs: pro/cons

- Strengths
 - Still relatively fast to train
 - ... and for inference ($O(L)$)
 - **Can extrapolate** (works with continuous features)
- Limitations
 - **Context dilution** when information is far away

Extending RNNs: BiLSTMs

- LSTM: improves context capacity
- Read the sequence in both directions



Transformers

Information flow - RNN

How many steps between source of info and current position?

- *What is the previous word?* $\Rightarrow O(L)$
- *What is the subject of verb X ?* $\Rightarrow O(L)$
- *What are the other occurrences of current word?* $\Rightarrow O(L^2)$
- ...

Information flow - Transformers

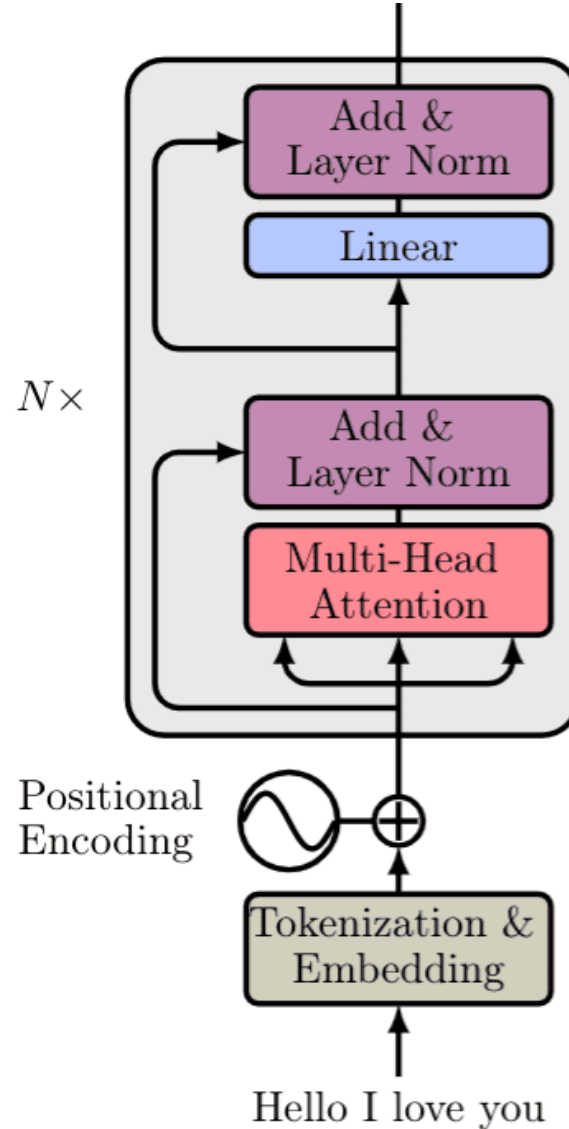
How many steps between source of info and current position?

- *What is the previous word?* $\Rightarrow O(1)$
- *What is the subject of verb X ?* $\Rightarrow O(1)$
- *What are the other occurrences of current word?* $\Rightarrow O(1)$
- ... $\Rightarrow O(1)$

Outside Transformers

- A Transformer network T_θ
- Input: Sequence of vectors $(e_1, \dots, e_n) \in \mathbb{R}^D$
- Output: Sequence of vectors $(h_1, \dots, h_n) \in \mathbb{R}^D$
- Each h_i may depend on the whole input sequence (e_1, \dots, e_n)

Inside Transformers



Inside Transformers : Embeddings

Before going in the network:

- Given an input token sequence (w_1, \dots, w_n)
- We retrieve token embeddings $(e_w(w_1), \dots, e_w(w_n)) \in \mathbb{R}^D$
- We retrieve position embeddings $(e_p(1), \dots, e_p(n)) \in \mathbb{R}^D$
- We compute input embeddings: $e_i = e_w(w_i) + e_p(i)$

Inside Transformers : Self-attention



Inside Transformers : Q and K

=> Model interactions between tokens:



Inside Transformers : Q and K

- Each row of QK^T is then normalized using softmax
- Interpretable patterns:



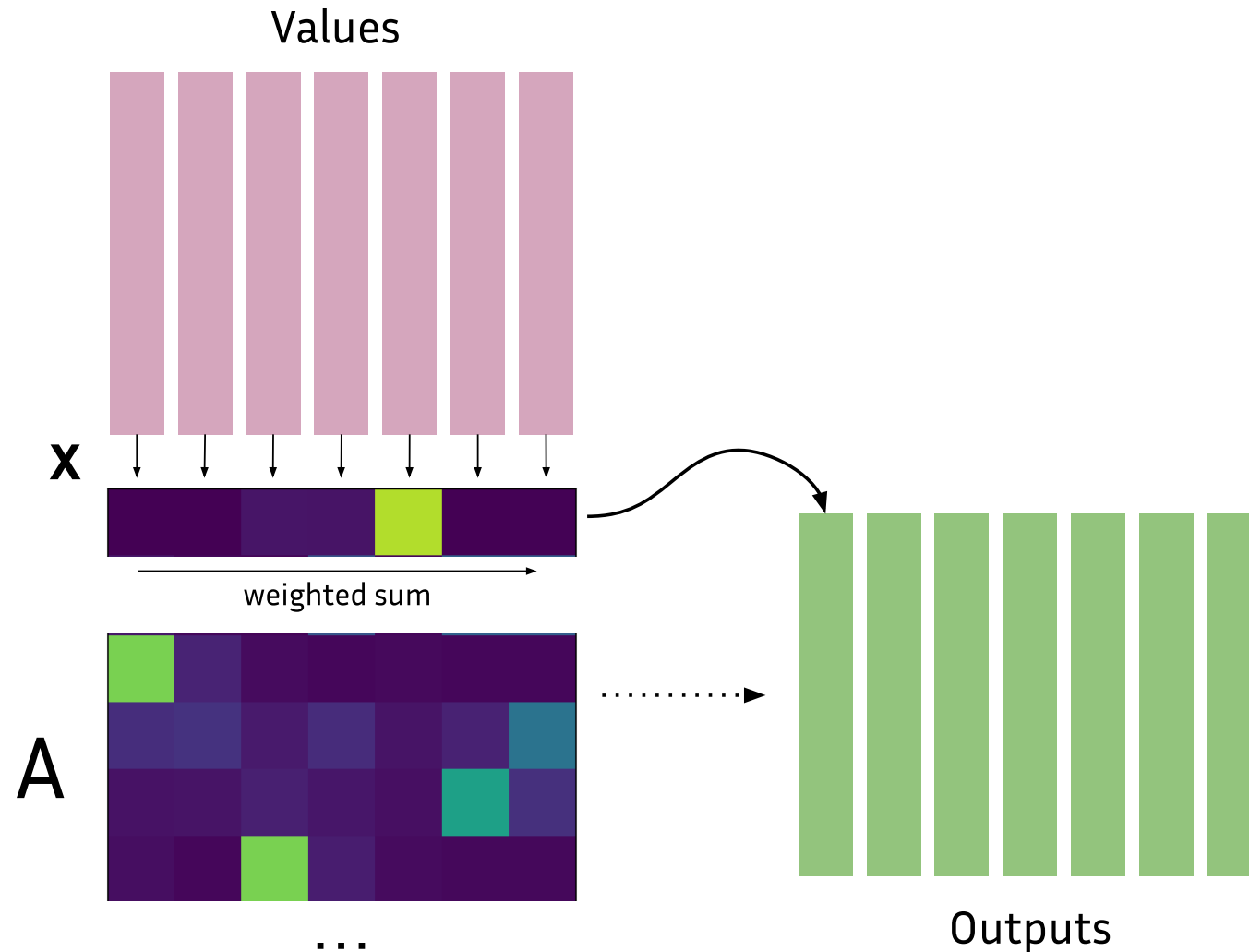
Inside Transformers : Q and K

- Formally:

$$A_{i,j} = \frac{1}{\sqrt{d_h}} \cdot \frac{e^{(QK^T)_{i,j}}}{\sum_k e^{(QK^T)_{i,k}}}$$

where d_h is the hidden dimension of the model

Inside Transformers : A and V



Inside Transformers : Self-attention summary

- Inputs are mapped to Queries, Keys and Values
- Queries and Keys are used to measure interaction (A)
- Interaction weights are used to "select" relevant Values combinations
- **Complexity: $O(L^2)$**



Inside Transformers : Multi-head attention

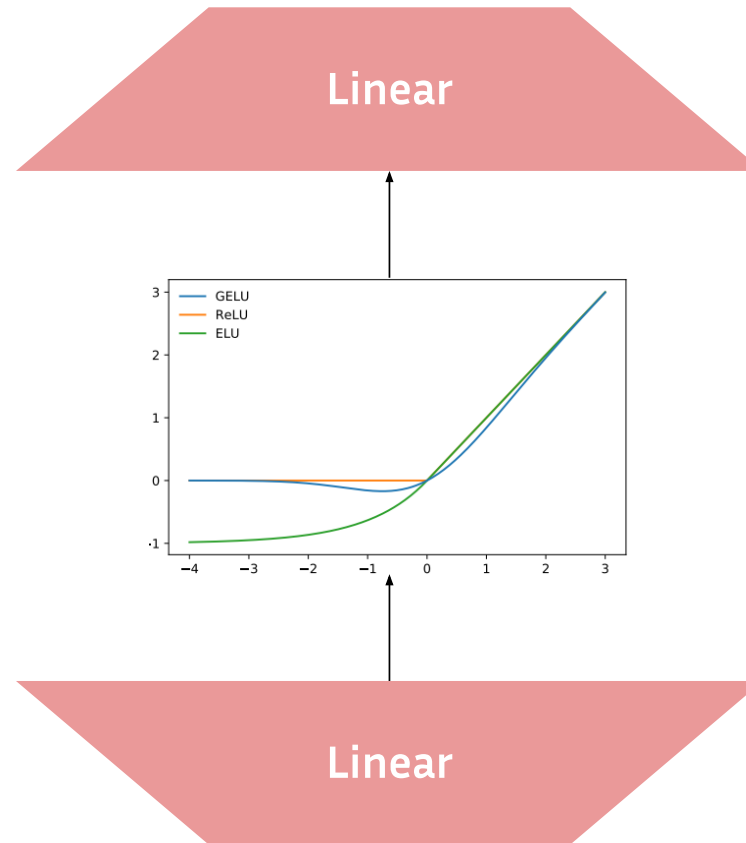


Inside Transformers : LayerNorm

- Avoids gradient explosion



Inside Transformers : Output layer



Modern flavors : Relative Positional Embeddings

- Encode position at attention-level:

$$(\Omega Q K^T)_{i,j} = \langle \omega_i(Q_i), \omega_j(K_j) \rangle + \beta_{i,j}$$

- Rotary Positional Embeddings (RoPE, Su et al. 2023)
 - ω_i is a rotation of angle $i\theta$; no β
- Linear Biases (ALiBi, Press et al. 2022)
 - $\beta_{i,j} = m \cdot (i - j)$ with $m \in \mathbb{R}$

Modern flavors : RMSNorm

- Replaces LayerNorm
- Re-scaling is all you need

$$RMSNorm_g(a_i) = \frac{a_i}{\sqrt{\frac{1}{N} \sum_{j=1}^N a_j^2}} g_i$$

Modern flavors : Grouped-Query Attention



Figure 2: Overview of grouped-query method. Multi-head attention has H query, key, and value heads. Multi-query attention shares single key and value heads across all query heads. Grouped-query attention instead shares single key and value heads for each *group* of query heads, interpolating between multi-head and multi-query attention.

Encoder Models

Masked Language Models



BERT (Devlin et al., 2018)

- Pre-trained on 128B tokens from Wikipedia + BooksCorpus
- Additional Next Sentence Prediction (NSP) loss
- Two versions:
 - BERT-base (110M parameters)
 - BERT-large (350M parameters)
- **Cost:** ~1000 GPU hours

RoBERTa (Liu et al., 2019)

- Pre-trained on ~~128B~~ **2T** tokens from web data (BERT x10)
- **No more** Next Sentence Prediction (NSP) loss
- Two versions:
 - RoBERTa-base (110M parameters)
 - RoBERTa-large (350M parameters)
- Better results in downstream tasks
- **Cost:** ~25000 GPU hours

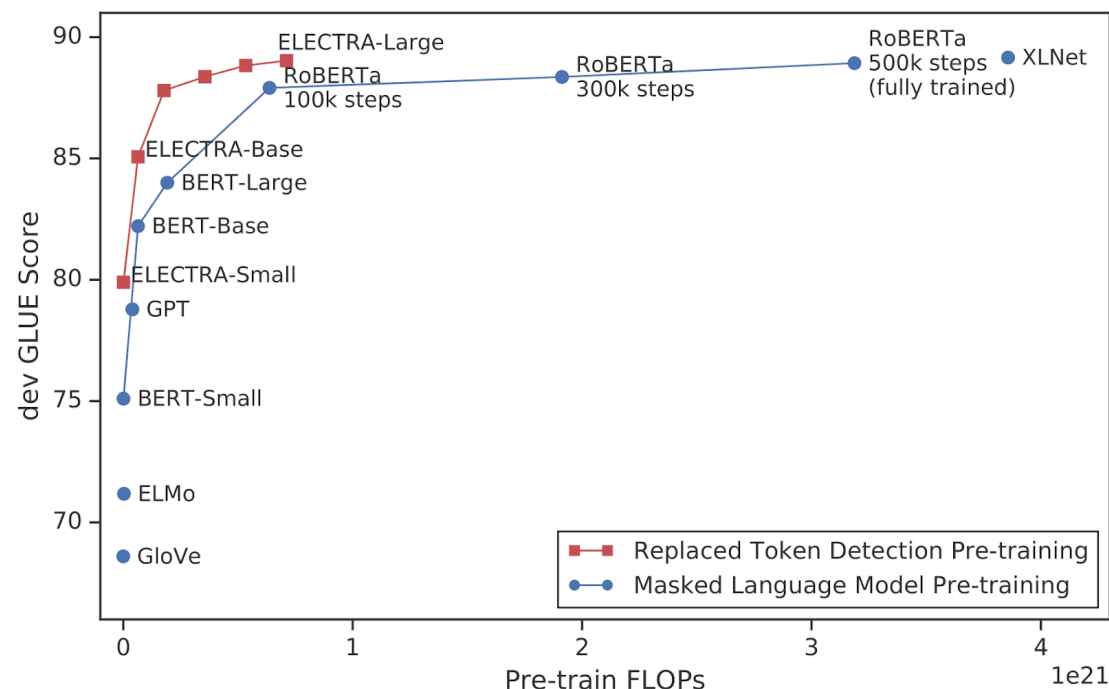
Multilingual BERT (mBERT)

- Pre-trained on 128B tokens from multilingual Wikipedia
- 104 languages
- One version:
 - mBERT-base (179M parameters)
- **Cost:** *unknown*

XLM-RoBERTa (Conneau et al., 2019)

- Pre-trained on **63T** tokens from CommonCrawl
- 100 languages
- Two versions:
 - XLM-RoBERTa-base (279M parameters)
 - XLM-RoBERTa-large (561M parameters)
- **Cost:** ~75000 GPU hours

ELECTRA (Clark et al., 2020)



ELECTRA (Clark et al., 2020)

- Pre-trained on **63T** tokens from CommonCrawl
- 100 languages
- Three versions:
 - ELECTRA-small (14M parameters)
 - ELECTRA-base (110M parameters)
 - ELECTRA-large (350M parameters)
- Really better than BERT/RoBERTa
- **Cost:** =BERT

Encoders: Fine-tuning



Encoders: Classical applications

- Natural Language Inference (NLI)
 - *I like cake! / Cake is bad* => ~~same~~|~~neutral~~|**opposite**
- Text classification (+ clustering)
 - *I'm so glad to be here!* => joy
- Named Entity Recognition (NER)
 - *I voted for Obama!* => (Obama, pos:3, class:PER)
- and many others...

Decoders

Decoders - Motivation

- Models that are designed to **generate text**
- Next-word predictors:

$$P(w_i \mid (w_j)_{j \neq i}) = P_\theta(w_i \mid w_1 \dots w_{i-1})$$

- **Problem:** How do we impede self-attention to consider future tokens?

Decoders - Attention mask



Attention

\times



Attention mask

$=$



Causal attention

- Each attention input can only attend to previous positions

Decoders - Causal LM pre-training

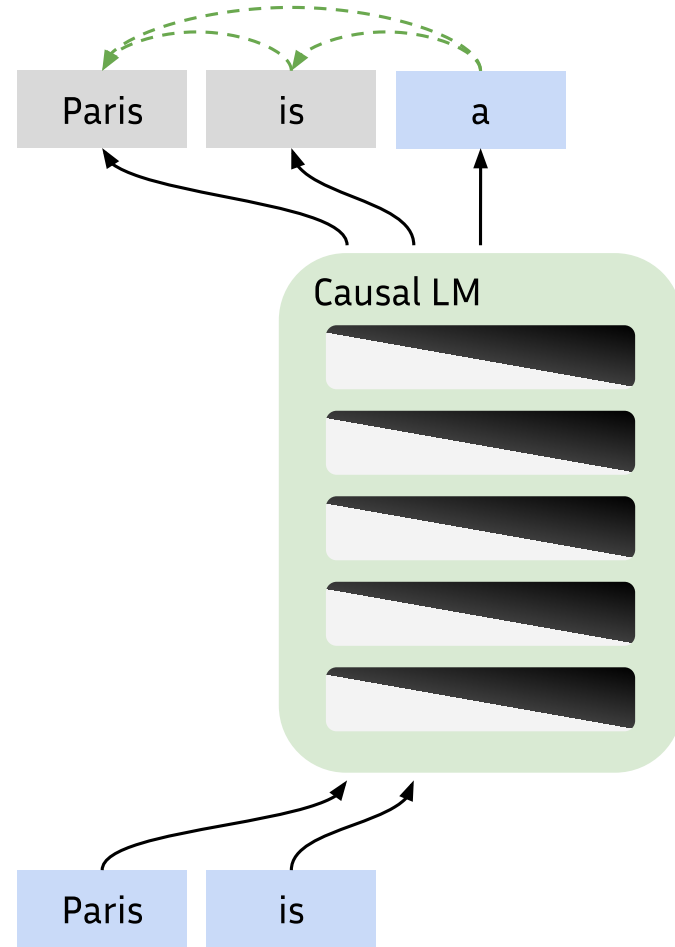
- Teacher-forcing



Decoders - Causal LM inference (greedy)



Decoders - Causal LM inference (greedy)



Decoders - Refining inference

- What we have : a good model for $P_{\theta}(w_i|w_1 \dots w_{i-1})$
- What we want at inference:

$$W^* = \text{\textcolor{red}{\argmax}}_{n, w_i \dots w_n} P_{\theta}(w_i \dots w_n | w_1 \dots w_{i-1})$$

- For a given completion length n , there are $|V|^n$ possibilities
 - e.g.: 19 new tokens with a vocab of 30000 tokens > #atoms in Ω
- We need approximations

Decoders - Greedy inference

- Keep best word at each step and start again:

$$W^* = \backslash \text{argmax}_{n, w_{i+1} \dots w_n} P_{\theta}(w_{i+1} \dots w_n | w_1 \dots w_{i-1} w_i^*)$$

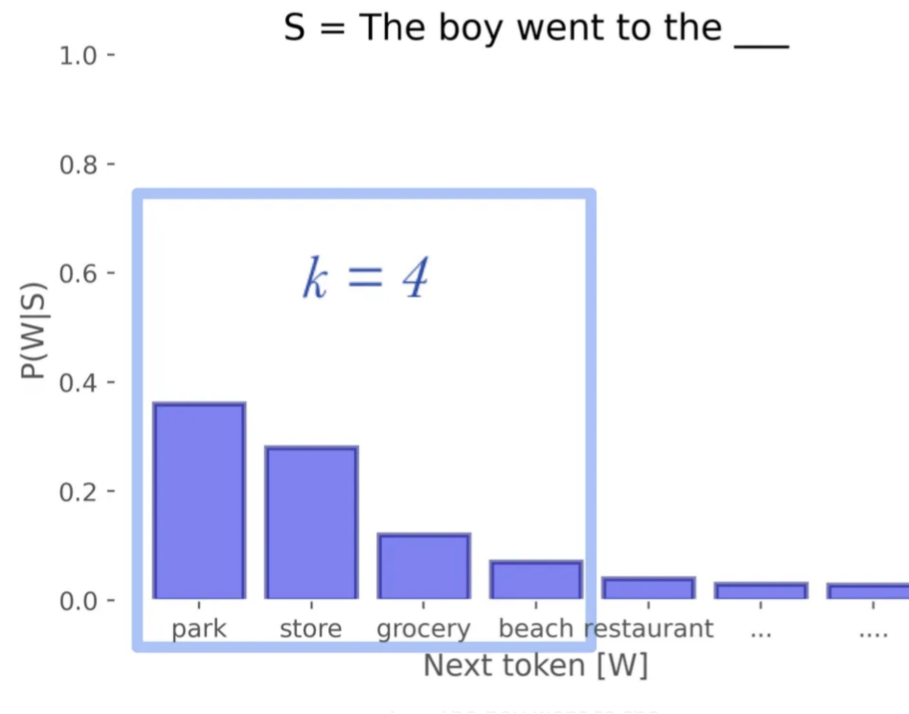
where $w_i^* = \backslash \text{argmax}_{w_i} P_{\theta}(w_i | w_1 \dots w_{i-1})$

Decoders - Beam search

- Keep best k chains of tokens at each step:
 - Take k best w_i and compute $P_\theta(w_{i+1} | \dots w_i)$ for each
 - Take k best w_{i+1} in each sub-case (now we have $k \times k$ (w_i, w_{i+1}) pairs to consider)
 - Consider only the k more likely (w_i, w_{i+1}) pairs
 - Compute $P_\theta(w_{i+2} | \dots w_i w_{i+1})$ for the k candidates
 - and so on...

Decoders - Top-k sampling

- Randomly sample among top- k tokens based on P_θ



Decoders - Top-p (=Nucleus) sampling

- Randomly sample based on P_θ up to $p\%$



Decoders - Generation Temperature

- Alter the softmax function:

$$\text{softmax}_\tau(x) = \frac{e^{\frac{x_i}{\tau}}}{\sum_j e^{\frac{x_j}{\tau}}}$$



Decoders - Inference speed

- For greedy decoding without prefix:
 - n passes with sequences of length n
 - Each pass is $O(n^2)$
 - Complexity: $O(n^3)$
- Other decoding are more costly
- Ways to go faster?

Decoders - Query-Key caching

$(Q * K^T) * V$ computation process with caching

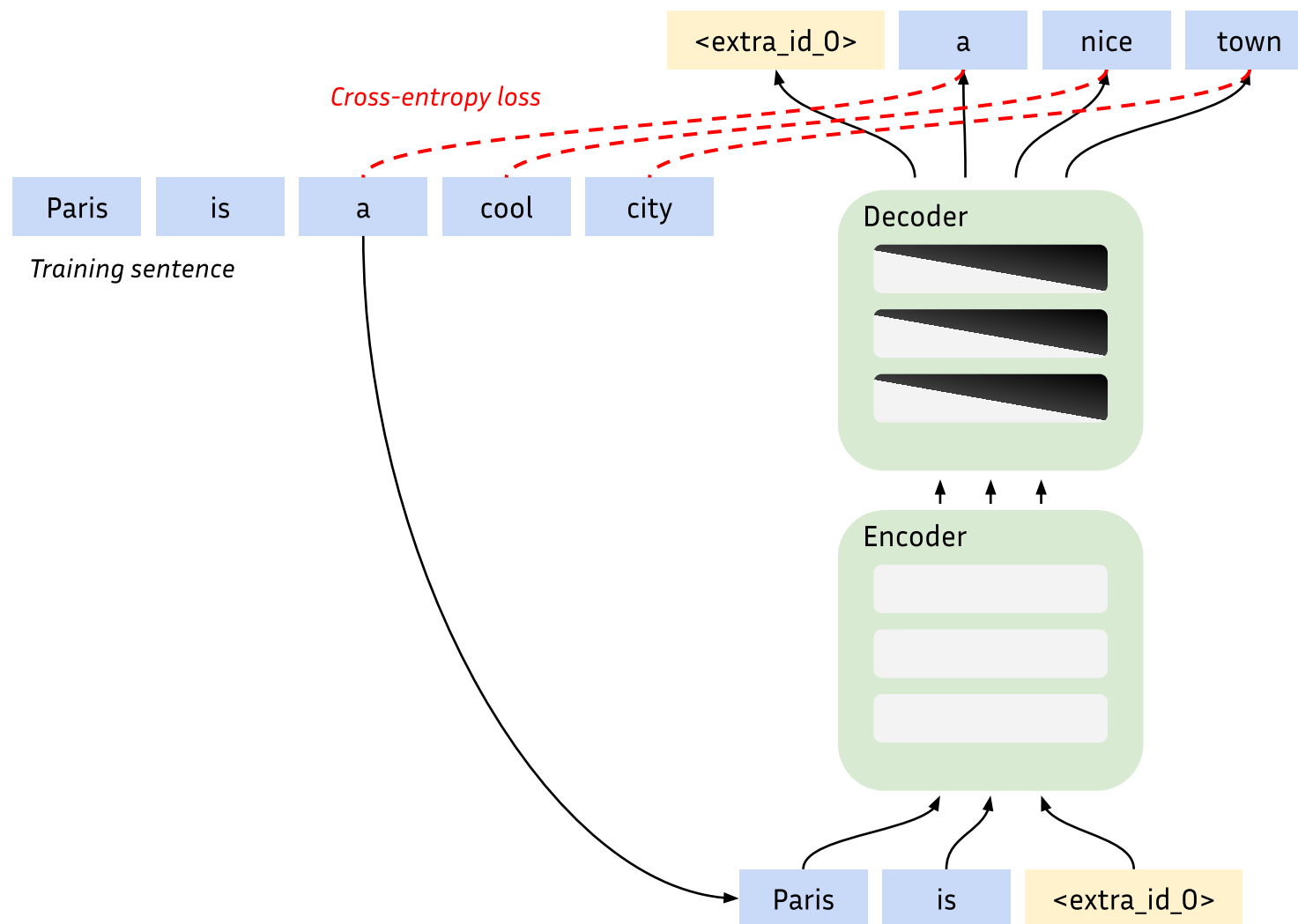


Decoders - Speculative decoding

- Generate γ tokens using P_ϕ where $|\phi| \ll |\theta|$ (smaller model)
- Forward $w_i \dots w_{i+\gamma}$ in teacher-forcing mode and predict $w_{i+\gamma+1}$ with the bigger model
- Compare P_θ and P_ϕ and only keep tokens where they **don't differ too much**

Encoder-Decoder models

T5 pre-training



All models can do everything

- Encoders are mostly used to get contextual embeddings
 - They can also generate : $T_{enc}(\text{"I love [MASK]"})$
- Decoders are mostly used for language generation
 - They can also give contextual embeddings : $T_{dec}(\text{"I love music!"})$
 - Or solve any task using prompts:
 - "What is the emotion in this tweet? Tweet: '...' Answer:"
- Encoders-decoders are used for language in-filling

Evaluating models

- A useful evaluation metric: ***Perplexity***
- Defined as:

$$ppl(T_{\theta}; w_1 \dots w_n) = \sqrt[n]{\frac{1}{P_{\theta}(w_1 \dots w_n)}}$$

- Other metrics: accuracy, f1-score, ...

Zero-shot evaluation

- Never-seen problems/data
- Example: *"What is the capital of Italy? Answer:"*
 - Open-ended: Let the model continue the sentence and check exact match
 - Ranking: Get next-word likelihood for *"Rome"*, *"Paris"*, *"London"*, and check if *"Rome"* is best
 - Perplexity: Compute perplexity of *"Rome"* and compare with other models

Few-shot evaluation / In-context learning

- Never-seen problems/data
- Example: *"Paris is the capital of France. London is the capital of the UK. Rome is the capital of"*
- Chain-of-Thought (CoT) examples:
 - Normal: *"(2+3)x5=25. What's (3+4)x2?"*
 - CoT: *"To solve (2+3)x5, we first compute (2+3) = 5 and then multiply (2+3)x5=5x5=25. What's (3+4)x2?"*

Open-sourced evaluation

- Generative models are evaluated on benchmarks
- Example (LLM Leaderboard from HuggingFace):

T	Model 	Average 	ARC 	HellaSwag 	MMLU 	TruthfulQA 	Winogrande 	GSM8K 
	Owen/Owen-72B 	73.6	65.19	85.94	77.37	60.19	82.48	70.43
	chargoddard/Yi-34B-Llama 	70.95	64.59	85.63	76.31	55.6	82.79	60.8
	01-ai/Yi-34B-200K 	70.81	65.36	85.58	76.06	53.64	82.56	61.64
	01-ai/Yi-34B 	69.42	64.59	85.69	76.35	56.23	83.03	50.64
	deepseek-ai/deepseek-llm-67b-base 	69.38	65.44	87.1	71.78	51.08	84.14	56.71
	mistralai/Mixtral-8x7B-v0.1 	68.42	66.04	86.49	71.82	46.78	81.93	57.47
	meta-llama/Llama-2-70b-hf 	67.87	67.32	87.33	69.83	44.92	83.74	54.06
	tiiuae/falcon-180B 	67.85	69.45	88.86	70.5	45.47	86.9	45.94

Lab session