

# Course 3: Language Modeling



**You**

Explain how ChatGPT works as if I was a 5 years old child.



**ChatGPT**

Alright, imagine you have a really smart friend who knows a lot of things. That friend is like a big, magical library with lots and lots of books. Now, this friend is called ChatGPT. When you ask ChatGPT a question or tell it something, it looks into all those books in its library (which are actually a huge amount of information it learned) and tries to give you the best answer or response it can find.

So, you can think of ChatGPT as a helpful buddy who loves to chat and share what it knows with you!



## How does it really work?

# What is Language Modeling?

# Definition

- A sequence of tokens  $(w_1, w_2, \dots, w_n)$
- For a position  $i$ , a language model (**LM**) predicts

$$P(w_i \mid (w_j)_{j \neq i}) \in \Delta^V$$

- In words: a LM predicts the probability of a token given its context

# Example

*I went to the ??? yesterday*

$P(\text{park} \mid \text{I went to the ??? yesterday}) = 0.1$

$P(\text{zoo} \mid \text{I went to the ??? yesterday}) = 0.07$

...

$P(\text{under} \mid \text{I went to the ??? yesterday}) = 0$

# Why is it hard?

- **Large vocabularies:** 170,000 English words
- **Lots of possible contexts:**
  - For  $V$  possible tokens, there are  $V^L$  contexts of size  $L$  (in theory)
- **Inherent uncertainty:** not obvious even for humans

# Basic approach - Unigram

- Learn the *non-contextual* probability (=frequency) of each token:

$$P(w_i | (w_j)_{j \neq i}) = f$$

## Example

*chart against operations at influence the surface plays crown a inaro  
the three @ but the court lewis on hand american of seamen mu role  
due roger executives*

# Include context - Bigram

- Predict based on the last token only:

$$P(w_i | (w_j)_{j \neq i}) = P_{\theta}(w_i | w_{i-1})$$

- (MLE): Measure next token frequency

## Example

*the antiquamen lost to dios nominated former is carved stone oak  
were problematic, 1910. his willingness to receive this may have been  
seen anything*



# Include more context - n-gram

- Predict based on the  $n$  last tokens only:

$$P(w_i | (w_j)_{j \neq i}) = P_{\theta}(w_i | w_{i-n} \dots w_{i-1})$$

- (MLE): Measure occurrences of tokens after  $w_{i-n} \dots w_{i-1}$

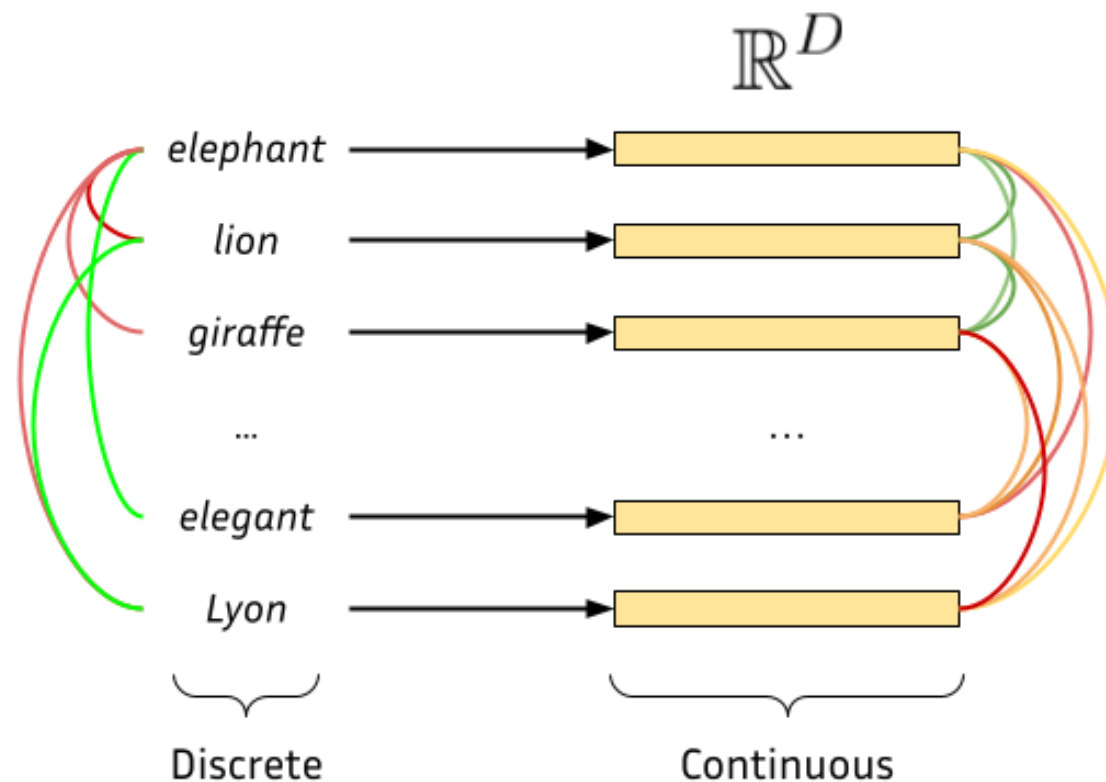
## Example (n=4)

*eva gauthier performed large amounts of contemporary french music  
across the united states marshals service traveled to frankfurt,  
germany and took custody of the matthews*

# Statistical n-grams: pro/cons

- Strengths:
  - Easy to train
  - Easy to interpret
  - Fast inference
- Limitations:
  - Very limited context
  - **Unable to extrapolate** : can only model what it has seen

# The embedding paradigm



# LM with RNNs



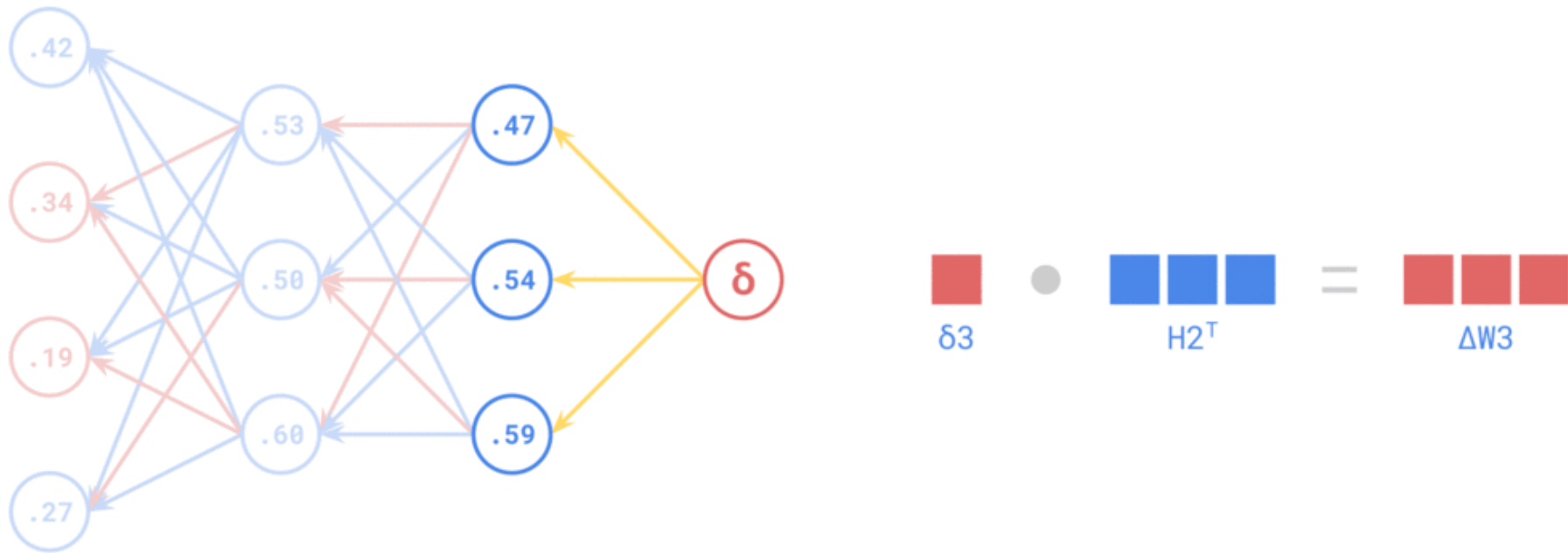
# LM with RNNs - Training

- $\theta$ : parameters of the RNN
- $(w_1, \dots, w_n)$ : training sequence
- Cross-entropy loss  $\mathcal{L}_{ce}$ :

$$\mathcal{L}_{ce}(w, \theta) = - \sum_{i=2}^n 1_{w_i} \cdot \log P_{\theta}(w_i | w_{i-1}, h_{i-1})$$

- Train via back-propagation + SGD

# Reminder - Back-propagation



# Reminder - Stochastic Gradient Descent

- **Goal** : Minimize a loss function  $\mathcal{L}(X, \theta)$  for given data  $X$  with respect to model parameters  $\theta$
- **Method** :
  - Split  $X$  in smaller parts  $x^i$  (called mini-batches)
  - Compute  $\mathcal{L}(x^i, \theta)$  (forward) and  $\nabla_{\theta}\mathcal{L}(x^i, \theta)$  (back-prop)
  - Update:  $\theta \leftarrow \theta - \eta \nabla_{\theta}\mathcal{L}(x^i, \theta)$  ( $\eta \ll 1$ , learning rate)

# LM with RNNs: Generation





# RNNs: pro/cons

- Strengths
  - Still relatively fast to train
  - ... and for inference ( $O(L)$ )
  - **Can extrapolate** (works with continuous features)
- Limitations
  - **Context dilution** when information is far away

# Extending RNNs: BiLSTMs

- LSTM: improves context capacity
- Read the sequence in both directions



# Transformers

# Information flow - RNN

How many steps between source of info and current position?

- *What is the previous word?*  $\Rightarrow O(L)$
- *What is the subject of verb  $X$ ?*  $\Rightarrow O(L)$
- *What are the other occurrences of current word?*  $\Rightarrow O(L^2)$
- ...

# Information flow - Transformers

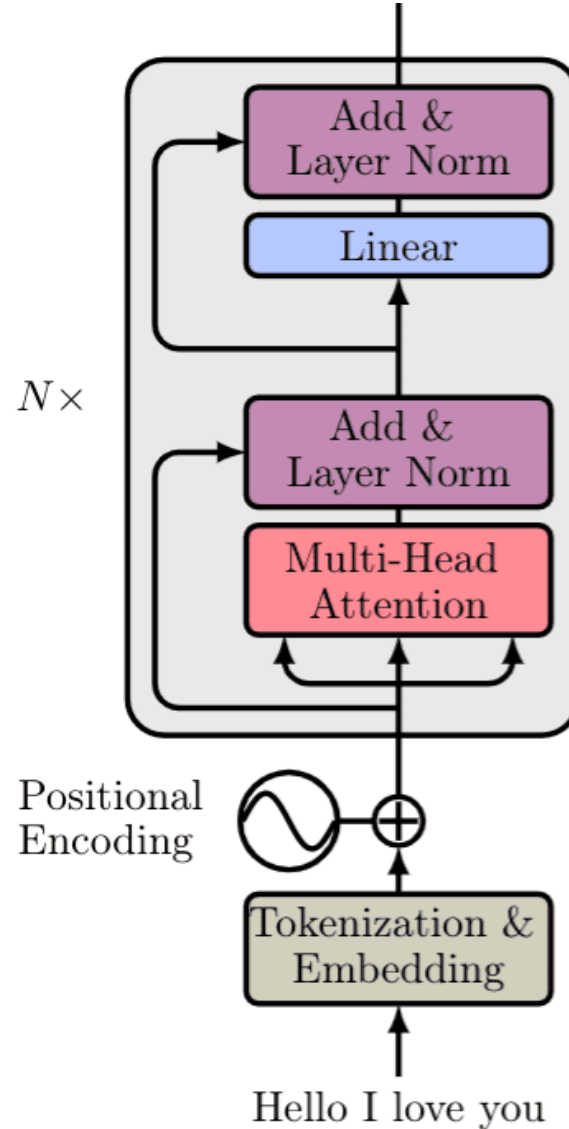
How many steps between source of info and current position?

- *What is the previous word?*  $\Rightarrow O(1)$
- *What is the subject of verb  $X$ ?*  $\Rightarrow O(1)$
- *What are the other occurrences of current word?*  $\Rightarrow O(1)$
- ...  $\Rightarrow O(1)$

# Outside Transformers

- A Transformer network  $T_\theta$
- Input: Sequence of vectors  $(e_1, \dots, e_n) \in \mathbb{R}^D$
- Output: Sequence of vectors  $(h_1, \dots, h_n) \in \mathbb{R}^D$
- Each  $h_i$  may depend on the whole input sequence  $(e_1, \dots, e_n)$

# Inside Transformers



# Inside Transformers : Embeddings

Before going in the network:

- Given an input token sequence  $(w_1, \dots, w_n)$
- We retrieve token embeddings  $(e_w(w_1), \dots, e_w(w_n)) \in \mathbb{R}^D$
- We retrieve position embeddings  $(e_p(1), \dots, e_p(n)) \in \mathbb{R}^D$
- We compute input embeddings:  $e_i = e_w(w_i) + e_p(i)$



# Inside Transformers : Self-attention



# Inside Transformers : Q and K

=> Model interactions between tokens:



# Inside Transformers : Q and K

- Each row of  $QK^T$  is then normalized using softmax
- Interpretable patterns:



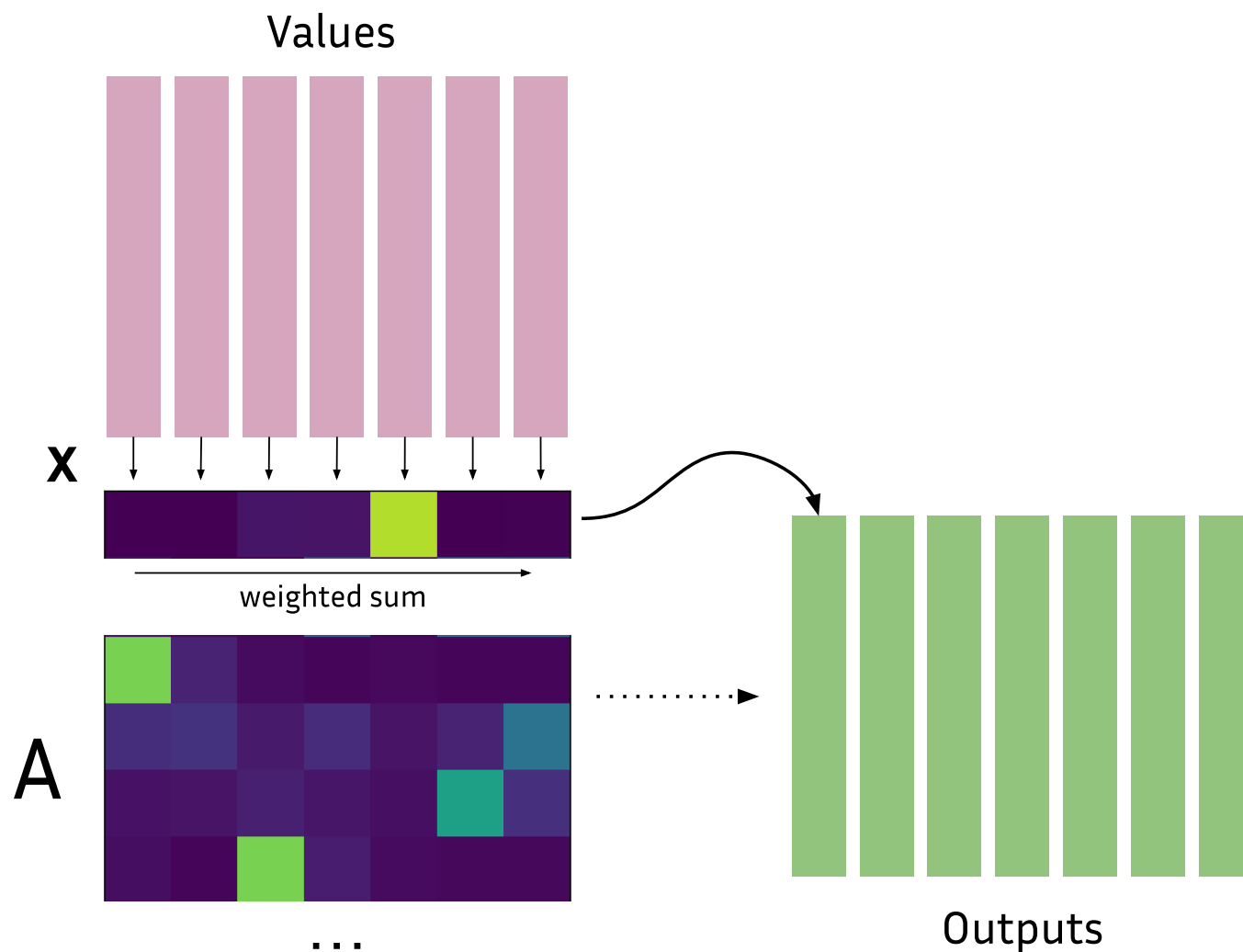
# Inside Transformers : Q and K

- Formally:

$$A_{i,j} = \frac{1}{\sqrt{d_h}} \cdot \frac{e^{(QK^T)_{i,j}}}{\sum_k e^{(QK^T)_{i,k}}}$$

where  $d_h$  is the hidden dimension of the model

# Inside Transformers : A and V



# Inside Transformers : Self-attention summary

- Inputs are mapped to Queries, Keys and Values
- Queries and Keys are used to measure interaction (A)
- Interaction weights are used to "select" relevant Values combinations
- **Complexity:  $O(L^2)$**



# Inside Transformers : Multi-head attention



# Inside Transformers : LayerNorm

- Avoids gradient explosion





# Inside Transformers : Output layer



# Modern flavors : Relative Positional Embeddings

- Encode position at attention-level:

$$(\Omega Q K^T)_{i,j} = \langle \omega_i(Q_i), \omega_j(K_j) \rangle + \beta_{i,j}$$

- Rotary Positional Embeddings (RoPE, Su et al. 2023)
  - $\omega_i$  is a rotation of angle  $i\theta$ ; no  $\beta$
- Linear Biases (ALiBi, Press et al. 2022)
  - $\beta_{i,j} = m \cdot (i - j)$  with  $m \in \mathbb{R}$

# Modern flavors : RMSNorm

- Replaces LayerNorm
- Re-scaling is all you need

$$RMSNorm_g(a_i) = \frac{a_i}{\sqrt{\frac{1}{N} \sum_{j=1}^N a_j^2}} g_i$$

# Modern flavors : Grouped-Query Attention



Figure 2: Overview of grouped-query method. Multi-head attention has  $H$  query, key, and value heads. Multi-query attention shares single key and value heads across all query heads. Grouped-query attention instead shares single key and value heads for each *group* of query heads, interpolating between multi-head and multi-query attention.

# Encoder Models

# Masked Language Models



# BERT (Devlin et al., 2018)

- Pre-trained on 128B tokens from Wikipedia + BooksCorpus
- Additional Next Sentence Prediction (NSP) loss
- Two versions:
  - BERT-base (110M parameters)
  - BERT-large (350M parameters)
- **Cost:** ~1000 GPU hours

# RoBERTa (Liu et al., 2019)

- Pre-trained on ~~128B~~ **2T** tokens from web data (BERT x10)
- **No more** Next Sentence Prediction (NSP) loss
- Two versions:
  - RoBERTa-base (110M parameters)
  - RoBERTa-large (350M parameters)
- Better results in downstream tasks
- **Cost:** ~25000 GPU hours



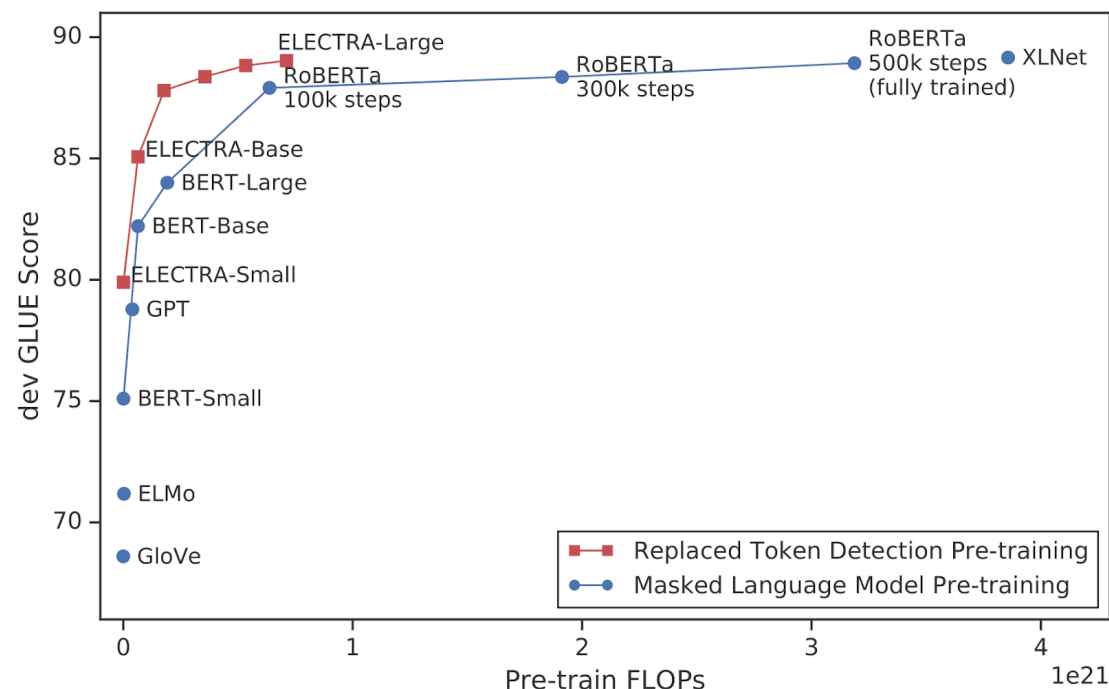
# Multilingual BERT (mBERT)

- Pre-trained on 128B tokens from multilingual Wikipedia
- 104 languages
- One version:
  - mBERT-base (179M parameters)
- **Cost:** *unknown*

# XLM-RoBERTa (Conneau et al., 2019)

- Pre-trained on **63T** tokens from CommonCrawl
- 100 languages
- Two versions:
  - XLM-RoBERTa-base (279M parameters)
  - XLM-RoBERTa-large (561M parameters)
- **Cost:** ~75000 GPU hours

# ELECTRA (Clark et al., 2020)



# ELECTRA (Clark et al., 2020)

- Pre-trained on **63T** tokens from CommonCrawl
- 100 languages
- Three versions:
  - ELECTRA-small (14M parameters)
  - ELECTRA-base (110M parameters)
  - ELECTRA-large (350M parameters)
- Really better than BERT/RoBERTa
- **Cost:** =BERT

# Encoders: Fine-tuning



# Encoders: Classical applications

- Natural Language Inference (NLI)
  - *I like cake! / Cake is bad* => ~~same~~~~neutral~~**opposite**
- Text classification (+ clustering)
  - *I'm so glad to be here!* => joy
- Named Entity Recognition (NER)
  - *I voted for Obama!* => (Obama, pos:3, class:PER)
- and many others...

# Decoders

# Decoders - Motivation

- Models that are designed to **generate text**
- Next-word predictors:

$$P(w_i \mid (w_j)_{j \neq i}) = P_\theta(w_i \mid w_1 \dots w_{i-1})$$

- **Problem:** How do we impede self-attention to consider future tokens?



# Decoders - Attention mask



*Attention*

$\times$



*Attention mask*

$=$



*Causal attention*

- Each attention input can only attend to previous positions

# Decoders - Causal LM pre-training

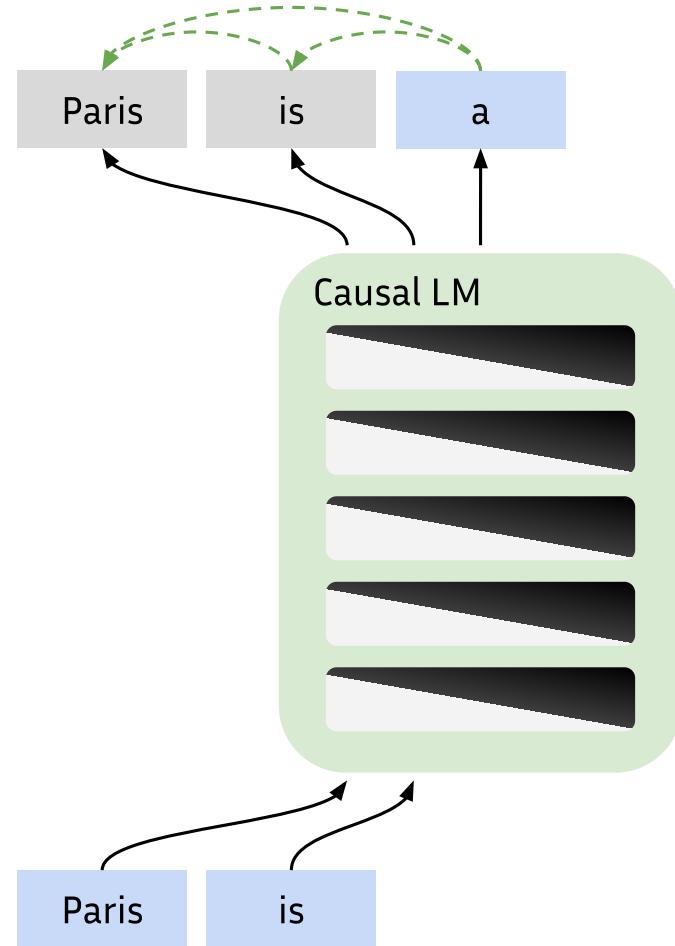
- Teacher-forcing



# Decoders - Causal LM inference (greedy)



# Decoders - Causal LM inference (greedy)



# Decoders - Refining inference

- What we have : a good model for  $P_\theta(w_i | w_1 \dots w_{i-1})$
- What we want at inference:

$$W^* = \operatorname{\textcolor{red}{argmax}}_{n, w_i \dots w_n} P_\theta(w_i \dots w_n | w_1 \dots w_{i-1})$$

- For a given completion length  $n$ , there are  $|V|^n$  possibilities
  - e.g.: 19 new tokens with a vocab of 30000 tokens > #atoms in  $\Omega$
- We need approximations

# Decoders - Greedy inference

- Keep best word at each step and start again:

$$W^* = \backslash \text{argmax}_{n, w_{i+1} \dots w_n} P_{\theta}(w_{i+1} \dots w_n | w_1 \dots w_{i-1} w_i^*)$$

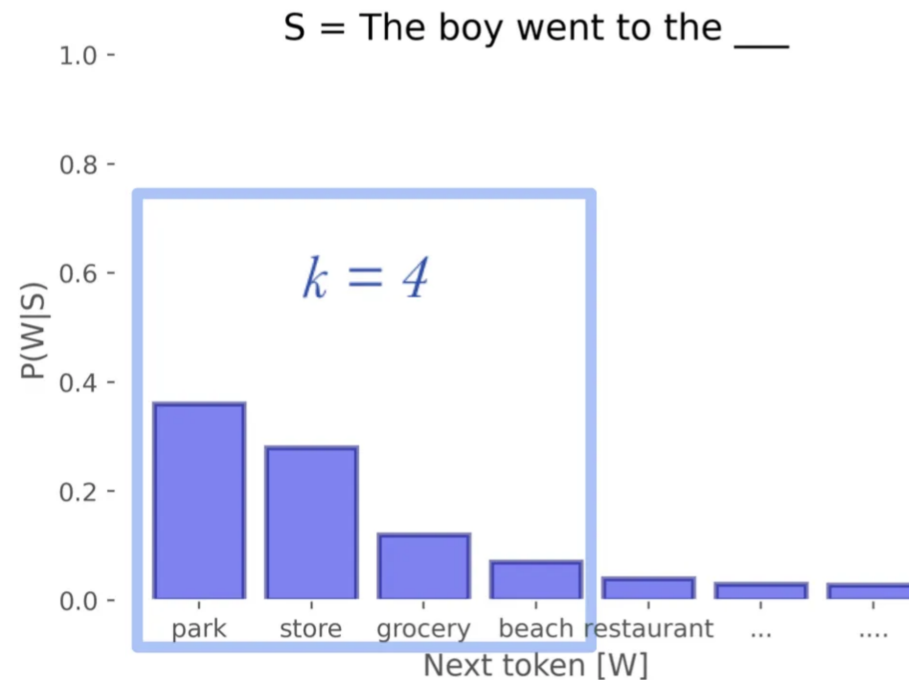
where  $w_i^* = \backslash \text{argmax}_{w_i} P_{\theta}(w_i | w_1 \dots w_{i-1})$

# Decoders - Beam search

- Keep best  $k$  chains of tokens at each step:
  - Take  $k$  best  $w_i$  and compute  $P_\theta(w_{i+1} | \dots w_i)$  for each
  - Take  $k$  best  $w_{i+1}$  in each sub-case (now we have  $k \times k$   $(w_i, w_{i+1})$  pairs to consider)
  - Consider only the  $k$  more likely  $(w_i, w_{i+1})$  pairs
  - Compute  $P_\theta(w_{i+2} | \dots w_i w_{i+1})$  for the  $k$  candidates
  - and so on...

# Decoders - Top-k sampling

- Randomly sample among top- $k$  tokens based on  $P_\theta$





# Decoders - Top-p (=Nucleus) sampling

- Randomly sample based on  $P_\theta$  up to  $p\%$



# Decoders - Generation Temperature

- Alter the softmax function:

$$\text{softmax}_\tau(x) = \frac{e^{\frac{x_i}{\tau}}}{\sum_j e^{\frac{x_j}{\tau}}}$$



# Decoders - Inference speed

- For greedy decoding without prefix:
  - $n$  passes with sequences of length  $n$
  - Each pass is  $O(n^2)$
  - Complexity:  $O(n^3)$
- Other decoding are more costly
- Ways to go faster?

# Decoders - Query-Key caching

$(Q * K^T) * V$  computation process with caching

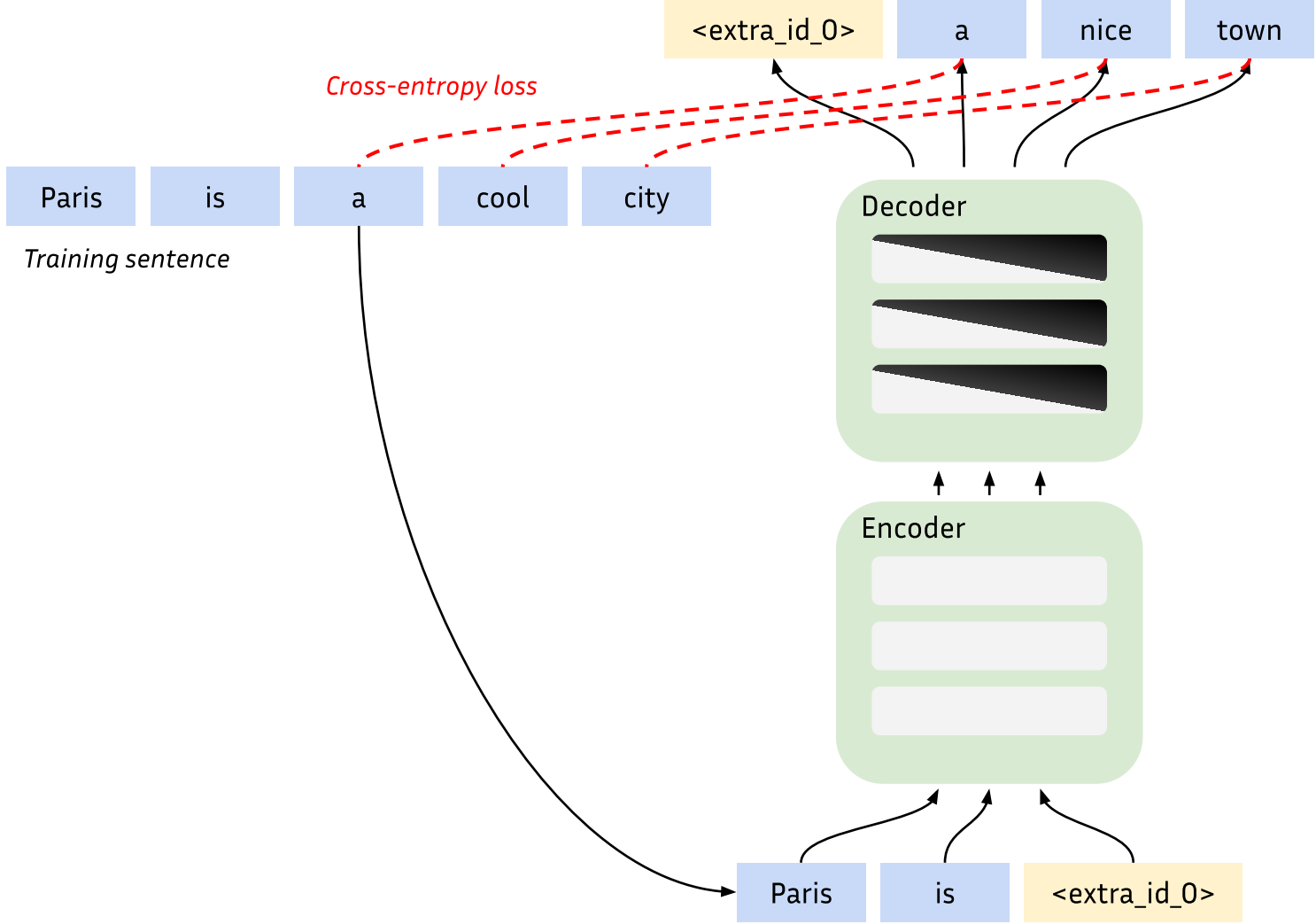


# Decoders - Speculative decoding

- Generate  $\gamma$  tokens using  $P_\phi$  where  $|\phi| \ll |\theta|$  (smaller model)
- Forward  $w_i \dots w_{i+\gamma}$  in teacher-forcing mode and predict  $w_{i+\gamma+1}$  with the bigger model
- Compare  $P_\theta$  and  $P_\phi$  and only keep tokens where they **don't differ too much**

# Encoder-Decoder models

# T5 pre-training



# All models can do everything

- Encoders are mostly used to get contextual embeddings
  - They can also generate :  $T_{enc}(\text{"I love [MASK]"})$
- Decoders are mostly used for language generation
  - They can also give contextual embeddings :  $T_{dec}(\text{"I love music!"})$
  - Or solve any task using prompts:
    - "What is the emotion in this tweet? Tweet: '...' Answer:"
- Encoders-decoders are used for language in-filling



# Evaluating models

- A useful evaluation metric: ***Perplexity***
- Defined as:

$$ppl(T_\theta; w_1 \dots w_n) = \sqrt[n]{\frac{1}{P_\theta(w_1 \dots w_n)}}$$

- Other metrics: accuracy, f1-score, ...

# Zero-shot evaluation

- Never-seen problems/data
- Example: *"What is the capital of Italy? Answer:"*
  - Open-ended: Let the model continue the sentence and check exact match
  - Ranking: Get next-word likelihood for *"Rome"*, *"Paris"*, *"London"*, and check if *"Rome"* is best
  - Perplexity: Compute perplexity of *"Rome"* and compare with other models

# Few-shot evaluation / In-context learning

- Never-seen problems/data
- Example: *"Paris is the capital of France. London is the capital of the UK. Rome is the capital of"*
- Chain-of-Thought (CoT) examples:
  - Normal: *"(2+3)x5=25. What's (3+4)x2?"*
  - CoT: *"To solve (2+3)x5, we first compute (2+3) = 5 and then multiply (2+3)x5=5x5=25. What's (3+4)x2?"*

# Open-sourced evaluation

- Generative models are evaluated on benchmarks
- Example (LLM Leaderboard from HuggingFace):

| T   | Model    | Average  | ARC  | HellaSwag  | MMLU  | TruthfulQA  | Winogrande  | GSM8K  |
|---|---|---|---|---|--|--|--|---|
|    | <a href="#">Owen/Owen-72B</a>                      | 73.6  | 65.19   | 85.94   | 77.37  | 60.19  | 82.48  | 70.43   |
|    | <a href="#">chargoddard/Yi-34B-Llama</a>           | 70.95   | 64.59   | 85.63   | 76.31  | 55.6   | 82.79  | 60.8  |
|    | <a href="#">01-ai/Yi-34B-200K</a>                  | 70.81   | 65.36   | 85.58   | 76.06  | 53.64  | 82.56  | 61.64   |
|    | <a href="#">01-ai/Yi-34B</a>                       | 69.42   | 64.59   | 85.69   | 76.35  | 56.23  | 83.03  | 50.64   |
|    | <a href="#">deepseek-ai/deepseek-llm-67b-base</a>  | 69.38   | 65.44   | 87.1  | 71.78  | 51.08  | 84.14  | 56.71   |
|    | <a href="#">mistralai/Mixtral-8x7B-v0.1</a>        | 68.42   | 66.04   | 86.49   | 71.82  | 46.78  | 81.93  | 57.47   |
|  | <a href="#">meta-llama/Llama-2-70b-hf</a>        | 67.87   | 67.32   | 87.33   | 69.83  | 44.92  | 83.74  | 54.06   |
|  | <a href="#">tiiuae/falcon-180B</a>               | 67.85   | 69.45   | 88.86   | 70.5   | 45.47  | 86.9   | 45.94   |

# Lab session