Overview of Channel Routing Techniques

I. General

Routing is a process used to predict the temporal and spatial variations of a flood hydrograph as it moves through a river reach or reservoir. The effects of storage and flow resistance, within a river reach, are reflected by changes in hydrograph shape and timing as the flood wave moves from upstream to downstream. Figure 1 shows the major changes that occur to a discharge hydrograph as a flood wave moves down a stream.

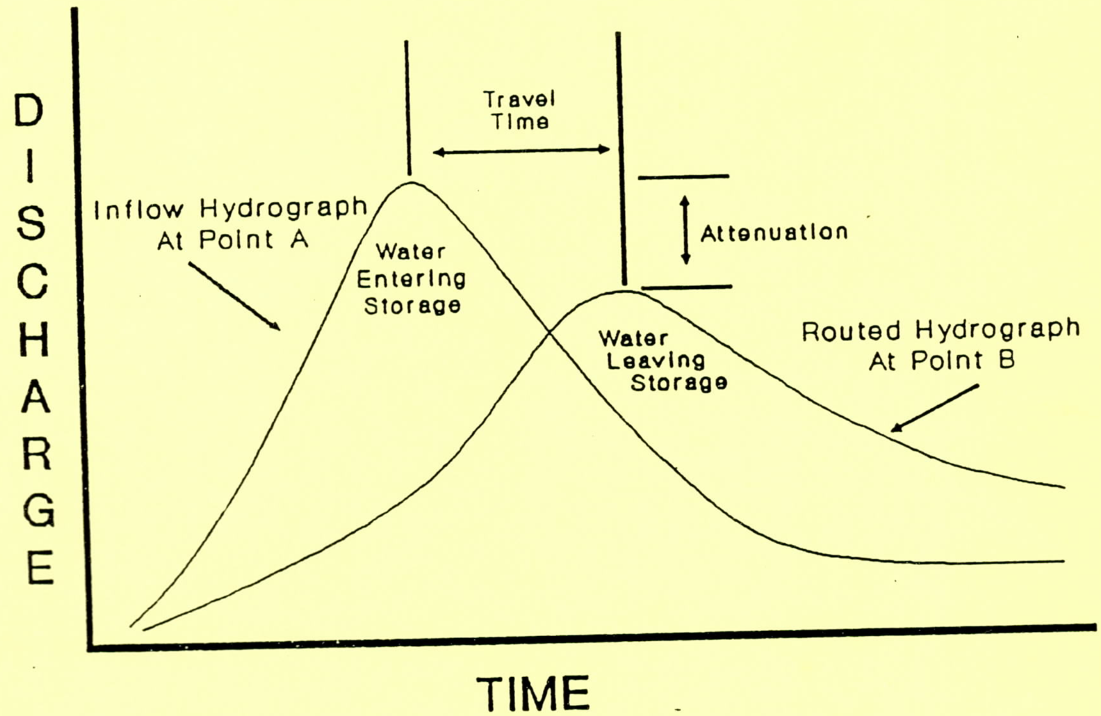

FIGURE 1
Discharge Hydrograph Routing Effects

In general, routing techniques may be classified into two categories: (1) hydraulic routing; and (2) hydrologic routing. Hydraulic routing techniques are based on the solution of the partial differential equations of unsteady open channel flow. These equations are often referred to as the St. Venant equations or the dynamic wave equations. Hydrologic routing employs the continuity equation and either an analytical or an emperical relationship between storage within the reach and discharge at the outlet.

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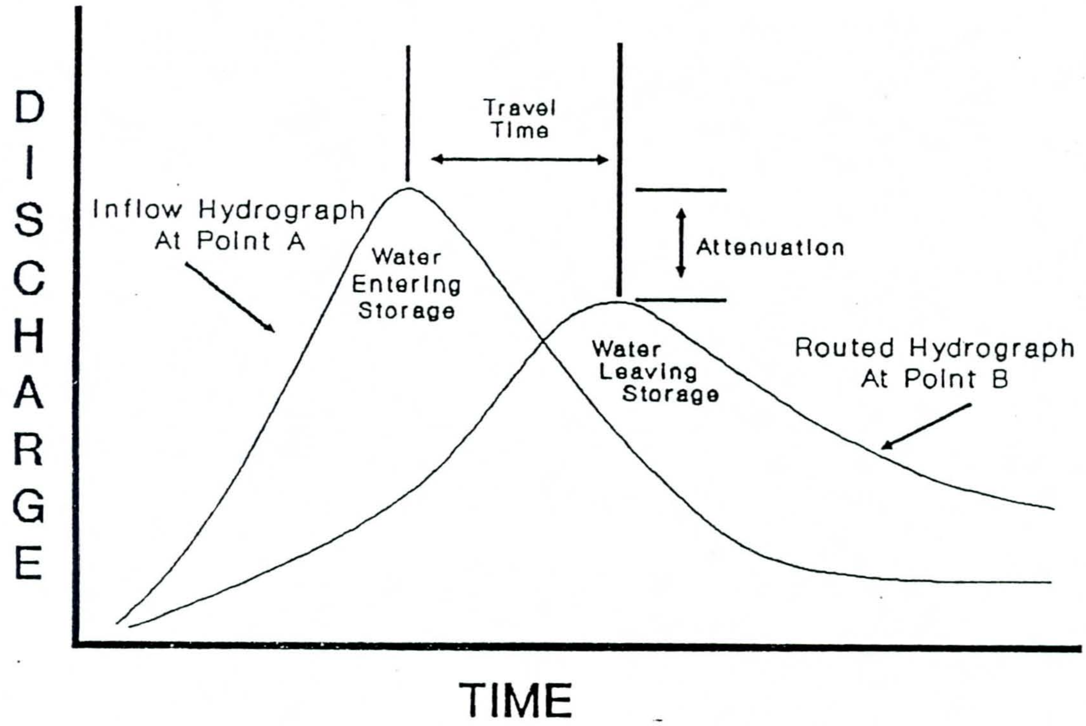

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Flood forecasting, reservoir and channel design, flood plain studies, and watershed simulations generally utilize some form of routing. Typically, in watershed simulation studies, hydrologic routing is utilized on a reach-by-reach basis from upstream to downstream. For example, it is often necessary to obtain a discharge hydrograph at a point downstream from a location where a hydrograph has been observed or computed. For such purposes, the upstream hydrograph is routed through the reach with a hydrologic routing technique that predicts changes in hydrograph shape and timing. Local flows are then added at the downstream location to obtain the total flow hydrograph. This type of approach is adequate as long as there are no significant backwater effects, or discontinuities in the water surface due to jumps or bores. When there are downstream controls that will have an effect on the routing process through an upstream reach, the channel configuration should be treated as one continuous system. This can only be accomplished with a hydraulic routing technique that can incorporate backwater effects as well as internal boundary conditions, such as those associated with culverts, bridges, and weirs.

This chapter describes several different routing techniques, hydraulic and hydrologic. Assumptions, limitations, and data requirements are discussed for each. The bases for selection of a particular routing technique are reviewed, and general calibration methodologies are presented. This chapter is limited to discussions on one dimensional flow routing techniques in the context of flood-runoff analysis. The focus of this chapter is on discharge (flow) rather than stage (water surface elevation). Detailed presentation of routing techniques and applications focused on stage calculations can be found in EM 1110-2-9020 (River Hydraulics EM).

II. Hydraulic Routing Techniques

a. The Equations of Motion. The equations that describe one-dimensional unsteady flow in open channels, the Saint Venant equations, consist of a continuity equation, (1), and a momentum equation, (2). The solution of these equations defines the propagation of a flood wave with respect to distance along the channel and time.

$$A\frac{\partial V}{\partial x} + VB\frac{\partial y}{\partial x} + B\frac{\partial y}{\partial t} = q \qquad (1)$$

$$S_f = S_o - \frac{\partial y}{\partial x} - \frac{V}{g}\frac{\partial V}{\partial x} - \frac{1}{g}\frac{\partial V}{\partial t} \qquad (2)$$

Where:
- A = cross-sectional flow area
- V = average velocity of water
- x = distance along channel
- B = water surface width
- y = depth of water
- t = time
- q = lateral inflow per unit length of channel

S_f = friction slope
S_o = channel bed slope
g = gravitational acceleration

Solved together with the proper boundary conditions, equations (1) and (2) are known as the complete dynamic wave equations. The meaning of the various terms in the dynamic wave equations are as follows (Henderson, 1966):

Continuity Equation

$A\frac{\partial V}{\partial x}$ = *Prism storage*

$VB\frac{\partial y}{\partial x}$ = *Wedge storage*

$B\frac{\partial y}{\partial t}$ = *Rate of rise*

q = *Lateral inflow per unit length*

Momentum Equation

S_f = *Friction slope (frictional forces)*

S_o = *Bed slope (gravitational effects)*

$\frac{\partial y}{\partial x}$ = *Pressure differential*

$\frac{V}{g}\frac{\partial V}{\partial x}$ = *Convective acceleration*

$\frac{1}{g}\frac{\partial V}{\partial t}$ = *Local acceleration*

The *dynamic wave equations* are considered to be the most accurate and comprehensive solution to one dimensional unsteady flow problems in open channels. Nonetheless, these equations are based on specific assumptions, and therefore have limitations. The assumptions used in deriving the dynamic wave equations are as follows:

(1) Velocity is constant and the water surface is horizontal <u>across</u> any channel section.

(2) All flows are gradually varied with hydrostatic pressure prevailing at all points in the flow, such that vertical accelerations can be neglected.

(3) No lateral secondary circulation occurs.

(4) Channel boundaries are treated as fixed, therefore no erosion or deposition occurs.

(5) Water is of uniform density, and resistance to flow can be described by empirical formulas, such as Manning's and Chezy's equation.

The dynamic wave equations can be applied to a wide range of one dimensional flow problems, such as: dam break flood wave routing; forecasting water surface elevations and velocities in a river system during a flood; evaluating flow conditions due to tidal fluctuations; and routing flows through irrigation and canal systems. Solution of the full equations is normally accomplished with an explicit or implicit finite difference technique. The equations are solved for incremental times (Δt) and incremental distances (Δx) along the waterway.

b. Approximations of the Full Equations. Depending on the relative importance of the various terms of the momentum equation (2), the equation can be simplified for various applications. Approximations to the full dynamic wave equations are created by combining the continuity equation with various simplifications of the momentum equation. The most common approximations of the momentum equation are:

$$S_f = S_o - \frac{\partial y}{\partial x} - \frac{V \partial V}{g \partial x} - \frac{1}{g}\frac{\partial V}{\partial t} \tag{3}$$

Steady Uniform Flow
Kinematic Wave Approx.

Steady Nonuniform Flow
Diffusion Wave Approximation

Steady Nonuniform Flow
Quasi-Steady Dynamic Wave Approximation

Unsteady Nonuniform Flow
Full Dynamic Wave Equation

The use of approximations to the full equations for unsteady flow can be justified when specific terms in the momentum equation are small in comparison to the bed slope. This is best illustrated by an example taken from Henderson's book Open Channel Flow, 1966. Henderson computed values for each of the terms on the right hand side of the momentum equation for a steep alluvial stream:

Term:	S_o	$\frac{\partial y}{\partial x}$	$\frac{V \partial V}{g \partial x}$	$\frac{1}{g}\frac{\partial V}{\partial t}$
Magnitude (ft/mi):	26	.5	.12-.25	.05

These figures relate to a very fast rising hydrograph in which the flow increased from 10,000 cfs to 150,000 cfs and decreased again to 10,000 cfs within 24 hours. Even in this case, where changes in depth and velocity with respect to distance and time are relatively large, the last three terms are still small in comparison to the bed slope. For this type of flow situation, an approximation of the full equations would be appropriate.

(1) Kinematic Wave Approximation. Kinematic flow occurs when gravitational and frictional forces achieve a balance. In reality, a true balance between gravitational and frictional forces never occurs. However, there are flow situations in which gravitational and frictional forces approach an equilibrium. For such conditions, changes in depth and velocity with respect to time and distance are small in magnitude when compared to the bed slope of the channel. Therefore, the terms to the right of the bed slope in equation (3) are assumed to be negligible. This

assumption reduces the momentum equation to the following:

$$S_f = S_o \tag{4}$$

Equation 4 essentially states that the momentum of the flow can be approximated with a uniform flow assumption as described by Manning's or Chezy's equation. Manning's equation can be written in the following form:

$$Q = \alpha A^m \tag{5}$$

where α and m are related to flow geometry and surface roughness. Since the momentum equation has been reduced to a simple functional relationship between area and discharge, the movement of a flood wave is described solely by the continuity equation. Writing the continuity equation in the following form:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q \tag{6}$$

Then by combining equations (5) and (6), the governing kinematic wave equation is obtained as:

$$\frac{\partial A}{\partial t} + \alpha m A^{(m-1)} \frac{\alpha A}{\alpha x} = q \tag{7}$$

Because of the steady uniform flow assumptions, the kinematic wave equations do not allow for hydrograph diffusion, but just simple translation of the hydrograph in time. The kinematic wave equations are usually solved by explicit or implicit finite difference techniques. Any attenuation of the peak flow that is computed using the kinematic wave equations is due to errors inherent in the finite difference solution scheme.

The application of the kinematic wave equation is limited to flow conditions that do not demonstate appreciable hydrograph attenuation. In general, the kinematic wave approximation works best when applied to steep (10 ft/mi or greater), well defined channels, where the flood wave is gradually varied. The kinematic wave approach is often applied in urban areas because the routing reaches are generally short and well defined (i.e. circular pipes, concrete lined channels, etc...). The kinematic wave equations cannot handle backwater effects since with a kinematic model flow disturbances can only propagate in the downstream direction. All of the terms in the momentum equation that are used to describe the propagation of the flood wave upstream (backwater effects) have been excluded.

(2) Diffusion Wave Approximation. Another common approximation of the full dynamic wave equations is the diffusion wave analogy. The diffusion wave model utilizes the continuity equation (1) and the following simplified form of the momentum equation:

$$S_f = S_o - \frac{\partial y}{\partial x} \tag{8}$$

The diffusion wave model is a significant improvement over the kinematic wave model because of the inclusion of the pressure differential term in equation (8). This term allows the diffusion model to describe the attenuation (diffusion effect) of the flood wave. It also allows the specification of a boundary condition at the downstream extremity of the routing reach to account for backwater effects. It does not use the inertial terms (last two terms) from equation (2) and, therefore, is limited to slow to moderately rising flood waves (Fread, 1982). However, most natural flood waves can be described with the diffusion form of the equations.

(3) Quasi-Steady Dynamic Wave Approximation. The third simplification of the full dynamic wave equations is the quasi-steady dynamic wave approximation. This model utilizes the continuity equation (9.1) and the following simplification of the momentum equation:

$$S_f = S_o - \frac{\partial y}{\partial x} - \frac{V}{g}\frac{\partial V}{\partial x} \tag{9}$$

In general this simplification of the dynamic wave equations is not used in flood routing. This form of the momentum equation is more commonly used in steady flow water surface profile computations. In the case of flood routing, the last two terms on the momentum equation are often opposite in sign and tend to counteract each other. By including the convective acceleration term and not the local acceleration term, an error is introduced. This error is of greater magnitude than the error that results when both terms are excluded, as in the diffusion wave model. For steady flow water surface profiles, the last term of the momentum equation (changes in velocity with respect to time) is assumed to be zero. However, changes in velocity with respect to distance are still very important in the calculation of steady flow water surface profiles.

c. *Data Requirements.* In general, the data requirements of the various hydraulic routing techniques are virtually the same. However, the amount of detail that is required for each type of data will vary depending upon the routing technique being used and the situation it is being applied too. The basic data requirements for hydraulic routing techniques are the following:

(1) Flow data (hydrographs);

(2) Channel cross sections and reach lengths;

(3) Roughness coefficients; and

(4) Initial and boundary conditions.

Flow data consist of discharge hydrographs from upstream locations as well as lateral inflow and tributary flow for all points along the stream. Channel cross sections are typically surveyed sections that are perpendicular to the flow lines. Key issues in selecting cross sections are the accuracy of the surveyed data and the spacing of the sections along the stream. If the routing procedure is utilized to predict stages, then the accuracy of the cross section dimensions will have a direct effect on the prediction of the stage. If the cross sections are only used to route discharge hydrographs, then it is only important to ensure that the cross section is an adequate representation of the discharge versus flow area of the section. Simplified cross-sectional shapes, such as 8-point cross sections or trapezoids and rectangles, are often used to fit the discharge versus flow area of a more detailed section. Cross section spacing affects the level of detail of the results as well as the accuracy of the numerical solution to the routing equations. Detailed discussions on cross section spacing can be found in the reference by HEC (1986).

Roughness coefficients for hydraulic routing models are typically in the form of Manning's n values. Manning's coefficients have a direct impact on the travel time and amount of diffusion that will occur when routing a flood hydrograph through a channel reach. Roughness coefficients will also have a direct impact on predicted stages.

All hydraulic models require that initial and boundary conditions be established before the routing can commence. Initial conditions are simply stated as the conditions at all points in the stream at the beginning of the simulation. Initial conditions are established by specifying a baseflow within the channel at the start of the simulation. Channel depths and velocities can be calculated through steady state backwater computations or a normal depth equation (e.g. Manning's equation). Boundary conditions are known relationships between discharge and time, and/or discharge and stage. Hydraulic routing computations require the specification of upstream, downstream, and internal boundary conditions in order to solve the equations. The upstream boundary condition is the discharge (or stage) versus time relationship of the hydrograph to be routed through the reach. Downstream boundary conditions are usually established with a steady-state rating curve (discharge versus depth relationship) or through normal depth calculations (Manning's equation). Internal boundary conditions consist of lateral inflow or tributary flow hydrographs, as well as depth versus discharge relationships for hydraulic structures within the river reach.

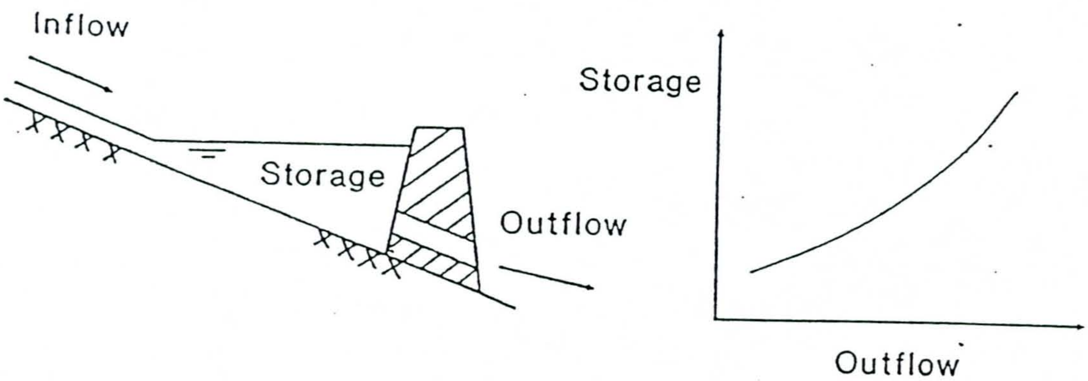
III. Hydrologic Routing Techniques

Hydrologic routing employs the use of the continuity equation and either an analytical or an emperical relationship between storage within the reach and discharge at the outlet. In its simplest form, the continuity equation can be written as inflow minus outflow equals the rate of change of storage within the reach:

$$I - O = \frac{dS}{dt} \tag{10}$$

Where: I = The average inflow to the reach during dt
O = The average outflow from the reach during dt
S = Storage within the reach

a. *Modified Puls Reservoir Routing.* One of the simplest routing applications is the analysis of a flood wave that passes through an unregulated reservoir (Figure 2a). The inflow hydrograph is known, and it is desired to compute the outflow hydrograph from the reservoir. Assuming that all gate and spillway openings are fixed, a unique relationship between storage and outflow can be developed, as shown in figure 2b.



(a) (b)

FIGURE 2
Reservoir Storage Routing

The equation defining storage routing, based on the principle of conservation of mass, can be written in approximate form for a routing interval Δt. Assuming the subscripts "1" and "2" denote the beginning and end of the routing interval, the equation is written as follows:

$$\frac{O_1 + O_2}{2} = \frac{I_1 + I_2}{2} - \frac{S_2 - S_1}{\Delta t} \tag{11}$$

The known values in this equation are the inflow hydrograph, and the storage and discharge at the beginning of the routing interval. The unknown values are the storage and discharge at the end of the routing interval. With two unknowns (O_2 and S_2) remaining, another relationship is required to obtain a solution. The storage-outflow relationship is normally used as the second equation. How that relationship is derived is what distinguishes various storage routing methods.

For an uncontrolled reservoir, outflow and water in storage are both uniquely a function of lake elevation. The two functions can be combined to develop a storage-outflow relationship, as shown in figure 3. Elevation-discharge relationships can be derived directly from hydraulic equations. Elevation-storage relationships are derived through the use of topographic maps. Elevation-area relationships are computed first, then either average end-area or conic methods are used to compute volumes.

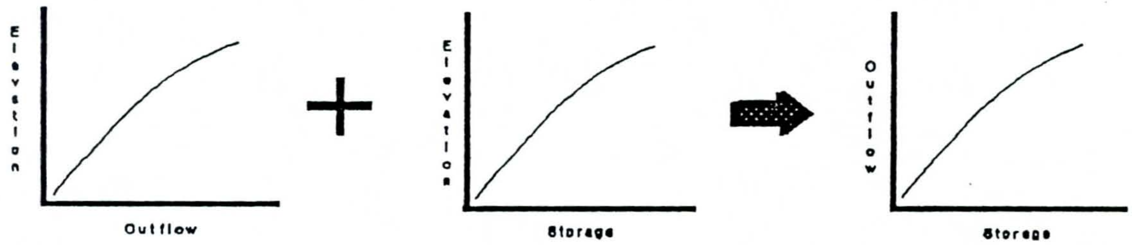


FIGURE 3
Reservoir Storage-Outflow Curve

The storage-outflow relationship provides the outflow for any storage level. Starting with an empty reservoir, the outflow capability would be a minimal. If the inflow is less than the outflow capability, the water would flow through. During a flood, the inflow increases and eventually exceeds the outflow capability. The difference between inflow and outflow produces a change in storage. In figure 4, the difference between the inflow and the outflow (on the rising side of the outflow hydrograph) represent the volume of water entering storage.

As water enters storage, the outflow capability increases because the pool level increases. Therefore, the outflow increases. This increasing outflow with increasing water in storage continues until the reservoir reaches a maximum level. This will occur the moment that the outflow equals the inflow, as shown in figure 4. Once the outflow becomes greater than the inflow, the storage level will start dropping. The difference between the outflow and the inflow hydrograph on the recession side reflects water withdrawn from storage.

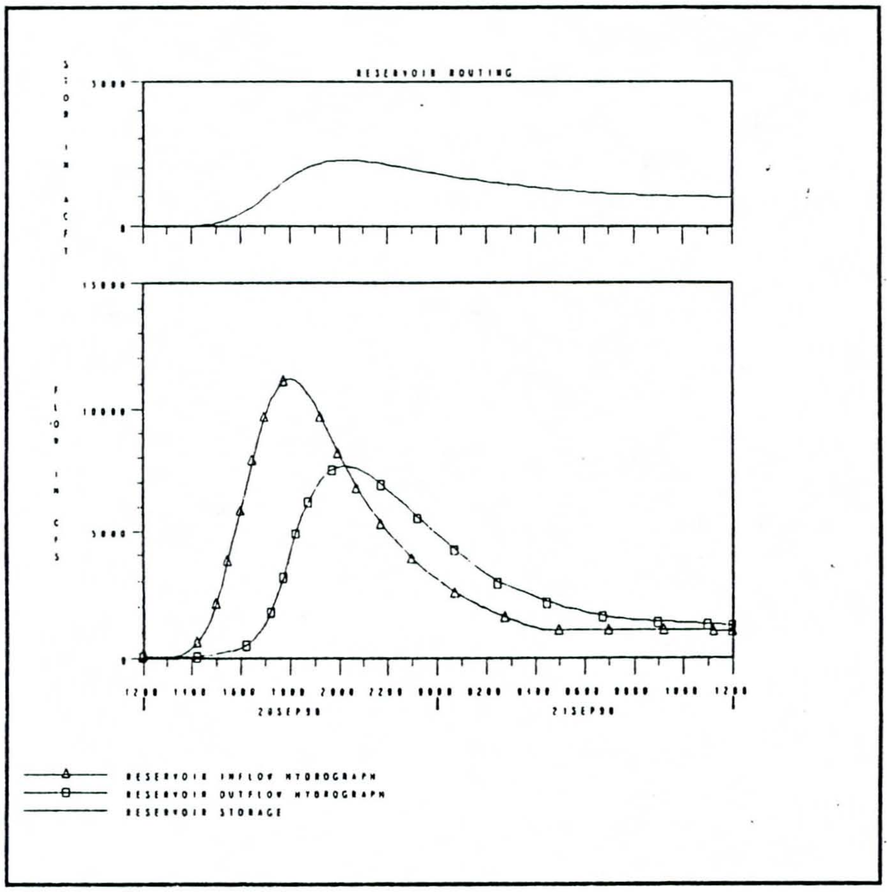


FIGURE 4
Reservoir Routing Example

The Modified Puls method applied to reservoirs consists of a repetitive solution of the continuity equation. It is assumed that the reservoir water surface remains horizontal, and therefore, outflow is a unique function of reservoir storage. The continuity equation (11) can be manipulated to get both of the unknown variables on the left hand side of the equation:

$$\left(\frac{S_2}{\Delta t} + \frac{O_2}{2}\right) = \left(\frac{S_1}{\Delta t} + \frac{O_1}{2}\right) - O_1 + \frac{I_1 + I_2}{2} \qquad (12)$$

Since I is known for all time steps, and O_1 and S_1 are known for the first time step, the right hand side of the equation can be calculated. The left hand side of the equation can be solved by trial and error. This is accomplished by assuming a value for either

S_2 or O_2, obtaining the corresponding value from the storage-outflow relationship, and then iterate until equation 12 is satisfied. Rather than resort to this iterative procedure, a value of Δt is selected and points on the storage-outflow curve are replotted as the "*Storage-Indication*" curve shown in figure 9.5. This graph allows for a direct determination of the outflow (O_2) once a value of storage-indication ($S_2/\Delta t + O_2/2$) has been calculated from equation 12 (Viessman, 1977). The numerical integration of equation 12 and figure 5 is illustrated as an example in table 1. The stepwise procedure for applying the Modified Puls method to reservoirs can be summarized as follows:

(1) Determine a composite discharge rating curve for all of the reservoir outlet structures.

(2) Determine the reservoir storage that corresponds with each elevation on the rating curve for reservoir outflow.

(3) Select a time step and construct a storage-indication vs. outflow curve $[(S/\Delta t) + (O/2)]$ vs. O.

(4) Route the inflow hydrograph through the reservoir based on equation 12 and the storage-indication curve.

(5) Compare the results with historical events to verify the model.

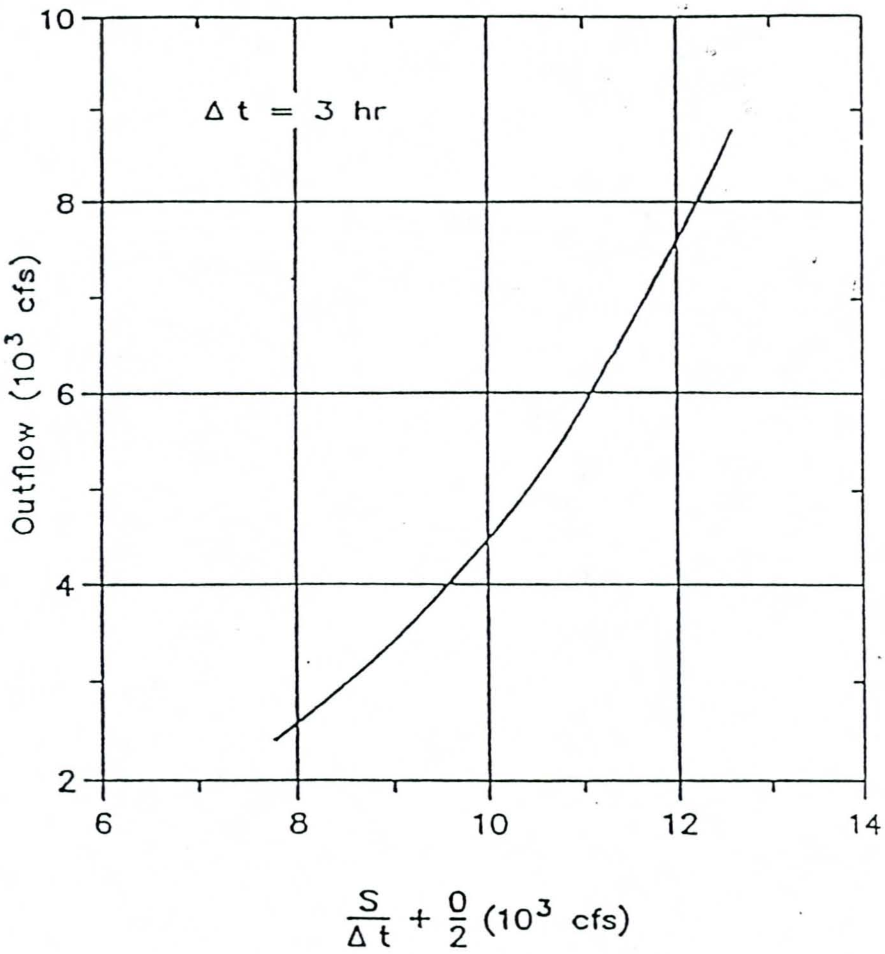


FIGURE 5
Storage-Indication Curve

b. Modified Puls Channel Routing. Routing in natural rivers is complicated by the fact that storage in a river reach is not a function of outflow alone. The water surface in a channel, during the passing of a flood wave, is not uniform. The storage and water surface slope within a river reach, for a given outflow, is greater during the rising stages of a flood wave than during the falling (figure 6). Therefore, the relationship between storage and discharge at the outlet of a channel is not a unique relationship, rather it is a looped relationship. An example storage-discharge function for a river is shown in figure 7.

TABLE 1
Storage Routing Calculation

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Time (hrs)	**Inflow (cfs)**	**Average Inflow (cfs)**	$\frac{S}{\Delta t} + \frac{O}{2}$ **(cfs)**	**Outflow (cfs)**	$\frac{S}{\Delta t}$ **(cfs)**	**S (Acre-ft)**
0	3000		8600	3000	7100	1760
		3130				
3	3260		8730	3150	7155	1774
		3445				
6	3630		9025	3400	7325	1816
		3825				
9	4020		9450	3850	7525	1866
		4250				
12	4480		9850	4300	7700	1909
.						
.						
etc.						

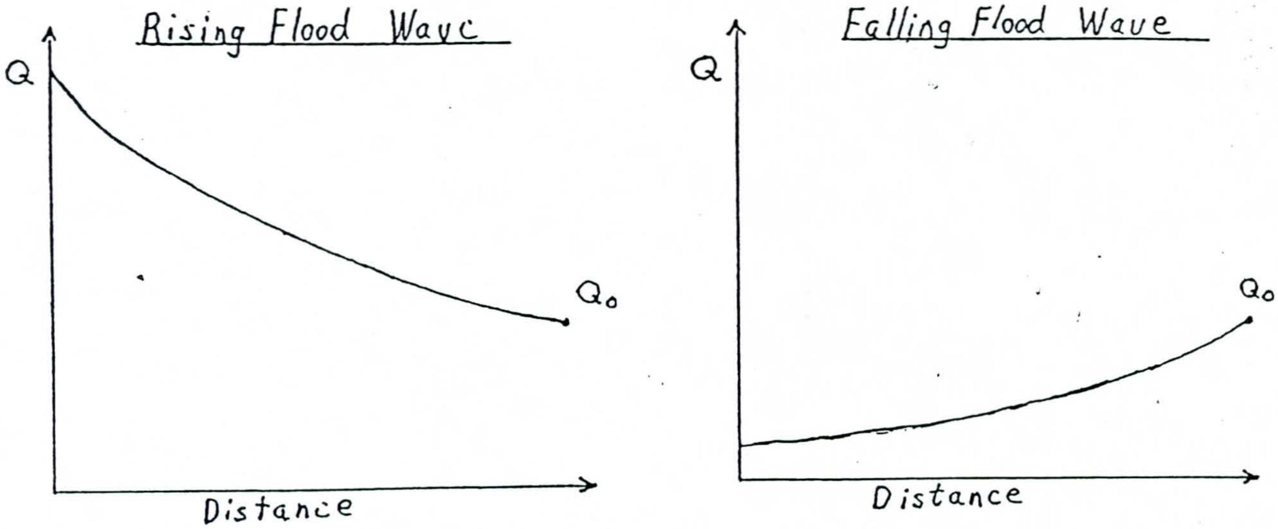


FIGURE 6
Rising and Falling Flood Wave

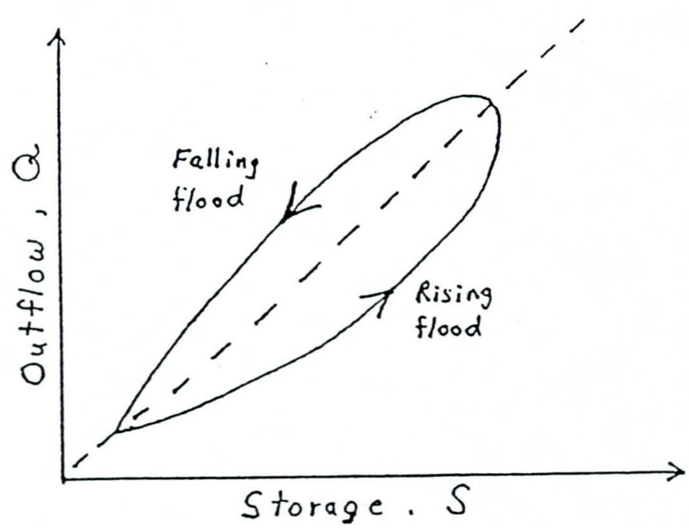


FIGURE 7
Looped Storage-Outflow Relationship for a River Reach

(1) Application of the Modified Puls Method to Rivers. In order to apply the Modified Puls method to a channel routing problem, the storage within the river reach is approximated with a series of *cascading reservoirs* (figure 8). Each reservoir is assumed to have a level pool, and therefore a unique storage-discharge relationship. The cascading reservoir approach is capable of approximating the looped storage-outflow effect when evaluating the river reach as a whole. The rising and falling flood wave is simulated with different storage levels in the cascade of reservoirs, thus producing a looped storage-outflow function for the total river reach. This is depicted graphically in figure 9.

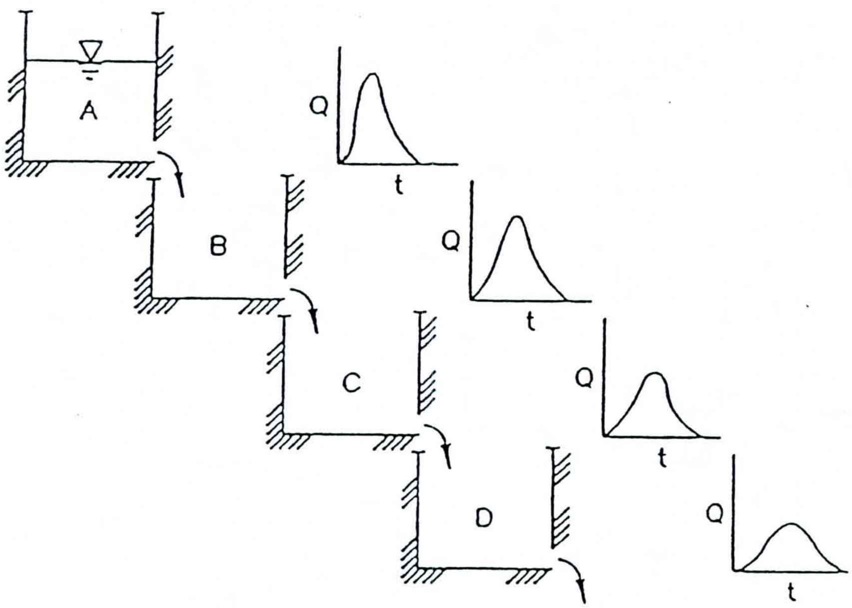


FIGURE 8
Cascade of Reservoirs, Depicting Storage Routing in a Channel

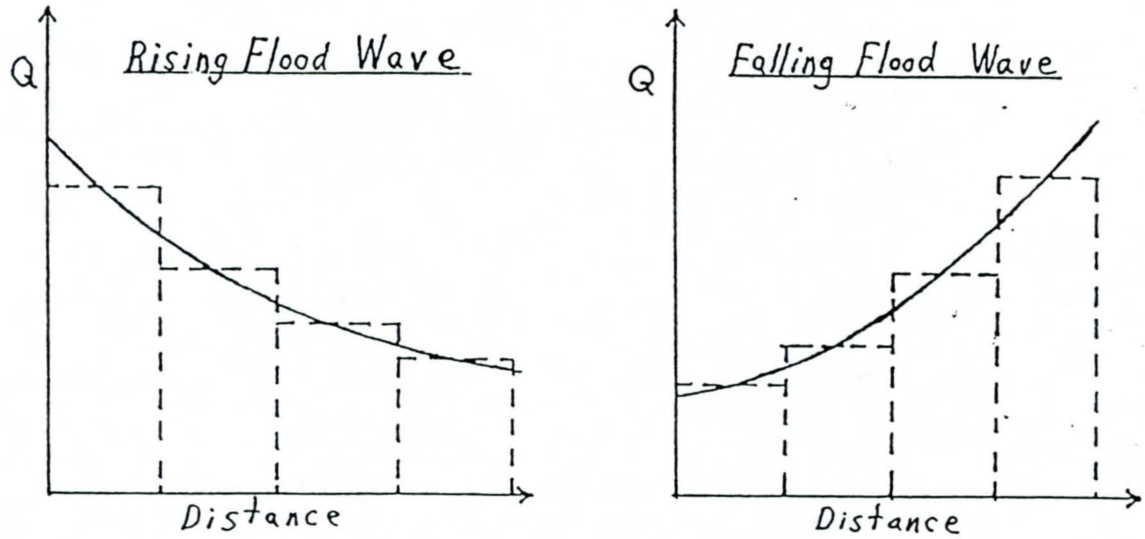


FIGURE 9
Modified Puls Approximation of the Rising and Falling Flood Wave

(2) Determination of the Storage-Outflow Relationship. Determining the storage-outflow relationship for a river reach is a critical part of the Modified Puls procedure. In river reaches, storage-outflow relationships can be determined from one of the following:

- steady-flow profile computations;
- observed water surface profiles;
- normal-depth calculations;
- observed inflow and outflow hydrographs; and
- optimization techniques applied to observed inflow and outflow hydrographs.

Steady-flow water surface profiles, computed over a range of discharges, can be used to determine storage outflow relationships in a river reach (figure 10). In this illustration, a known hydrograph at A is to be routed to location B. The storage-outflow relationship required for routing is determined by computing a series of water surface profiles, corresponding to a range of discharges. The range of discharges should encompass the range of flows that will be routed through the river reach. The storage volumes are computed by multiplying the cross-sectional area, under a

Normal depth associated with uniform flow does not exist in natural streams; however, the concept can be used to estimate water depth and storage in natural rivers if uniform-flow conditions can reasonably be assumed. With a typical cross section, Manning's equation is solved for a range of discharges, given appropriate "n" values and an estimated slope of the energy grade line. Under the assumption of uniform-flow conditions, the energy slope is considered equal to the average channel bed slope; therefore, this approach should not be applied in backwater areas.

Observed inflow and outflow hydrographs can be used to compute channel storage by an inverse process of flood routing. When both inflow and outflow are known, the change in storage can be computed, and from that a storage vs. outflow function can be developed. Tributary inflow, if any, must also be accounted for in this calculation. The total storage is computed from some base level storage at the beginning or end of the routing sequence.

Inflow and outflow hydrographs can also be used to compute routing criteria through a process of iteration in which an initial set of routing criteria is assumed, the inflow hydrograph is routed, and the results are evaluated. The process is repeated if necessary until a suitable fit of the routed and observed hydrograph is obtained.

(3) Determining the Number of Routing Steps. In reservoir routing, the Modified Puls method is applied with one routing step. This is under the assumption that the travel time through the reservoir is smaller than the computation interval Δt. In channel routing, the travel time through the river reach is often greater than the computation interval. When this occurs the channel must be broken down into smaller routing steps in order to simulate the flood-wave movement and changes in hydrograph shape. The number of steps (or reach lengths) affects the attenuation of the hydrograph and should be obtained by calibration. The maximum amount of attenuation will occur when the channel routing computation is done in one step. As the number of routing steps increases, the amount of attenuation decreases. An initial estimate of the number of routing steps (NSTPS) can be obtained by dividing the total travel time (K) for the reach by the computation interval Δt.

$$K = \frac{L}{V_w}$$

$$NSTPS = \frac{K}{\Delta t} \qquad (13)$$

Where:

K = total travel time through the reach
L = channel reach length
V_w = velocity of the flood wave
NSTPS = number of routing steps

The time interval Δt is usually determined by ensuring that there is a sufficient number of points on the rising side of the inflow hydrograph. A general rule of thumb is that the computation interval should be less than 1/5 of the time of rise (t_r) of the inflow hydrograph.

$$\Delta t \leq \frac{t_r}{5} \tag{14}$$

c. *Muskingum Method.* The Muskingum method was developed to directly accommodate the looped relationship between storage and outflow that exists in rivers. With the Muskingum method, storage within a reach is visualized in two parts: *Prism Storage* and *Wedge Storage*. Prism storage is essentially the storage under the steady-flow water surface profile. Wedge storage is the additional storage under the actual water surface profile. As shown in figure 11, during the rising stages of the flood wave the wedge storage is positive and added to the prism storage. During the falling stages of a flood wave the wedge storage is negative and subtracted from the prism storage.

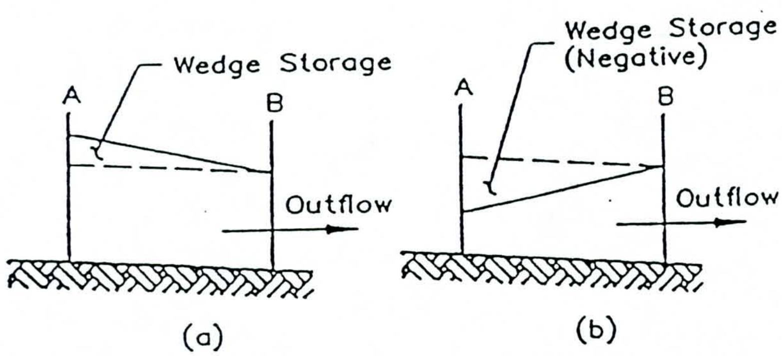


FIGURE 11
Muskingum Prism and Wedge Storage Concept

(1) Development of the Muskingum Routing Equation. Prism storage is computed as the outflow (O) times the travel time through the reach (K). Wedge storage is computed as the difference between inflow and outflow (I-O) times a weighting coefficient X and the travel time K. The coefficent K corresponds to the travel time of the flood wave through the reach. The parameter X is a dimensionless value expressing a weighting of the relative effects of inflow and outflow on the storage (S) within the reach. Thus, the Muskingum method defines the storage in the reach as a linear function of weighted inflow and outflow:

$$S = \text{prism storage} + \text{wedge storage}$$

$$S = KO + KX(I - O)$$

$$S = K\,[XI + (1 - X)O] \tag{15}$$

Where:
- S = Total storage in the routing reach
- O = Rate of outflow from the routing reach
- I = Rate of inflow to the routing reach
- K = Travel time of the flood wave through the reach
- X = Dimensionless weighting factor, ranging from 0.0 to 0.5

The quantity in the brackets of equation 15 is considered an expression of weighted discharge. When X=0.0, the equation reduces to S = KO, indicating that storage is only a function of outflow, which is equivalent to level-pool reservoir routing with storage as a linear function of outflow. When X=0.5, equal weight is given to inflow and outflow, and the condition is equivalent to a uniformly progressive wave that does not attenuate. Thus, "0.0" and "0.5" are limits on the value of X, and within this range the value of X determines the degree of attenuation of the flood wave as it passes through the routing reach. A value of "0.0" produces maximum attenuation, and "0.5" produces pure translation with no attenuation.

The Muskingum routing equation is obtained by combing equation 15 with the continuity equation (11), and solving for O_2.

$$O_2 = C_1 I_2 + C_2 I_1 + C_3 O_1 \tag{16}$$

The subscripts 1 and 2 in this equation indicate the beginning and end, respectively, of a time interval Δt. The routing coefficients C_1, C_2, and C_3 are defined in terms of Δt, K, and X.

$$C_1 = \frac{\Delta t - 2KX}{2K(1 - X) + \Delta t} \tag{17}$$

$$C_2 = \frac{\Delta t + 2KX}{2K(1-X) + \Delta t} \tag{18}$$

$$C_3 = \frac{2K(1-X) - \Delta t}{2K(1-X) + \Delta t} \tag{19}$$

Given an inflow hydrograph, a selected computation interval Δt, and estimates for the parameters K and X, the outflow hydrograph can be calculated.

(2) Determination of Muskingum K and X. In a gaged situation the Muskingum K and X parameters can be calculated from observed inflow and outflow hydrographs. The travel time, K, can be estimated as the interval between similar points on the inflow and outflow hydrographs. The travel time of the routing reach can be calculated as the elapsed time between centroid of areas of the two hydrographs, between the hydrograph peaks, or between midpoints of the rising limbs. After K has been estimated, a value for X can be obtained through trial and error. Assume a value for X, and then route the inflow hydrograph with these parameters. Compare the routed hydrograph with the observed outflow hydrograph. Make adjustments to X in order to obtain the desired fit. Adjustments to the original estimate of K may also be necessary to obtain the best overall fit between computed and observed hydrographs.

In an ungaged situation a value for K can be estimated as the travel time of the flood wave through the routing reach. The flood wave velocity (V_w) is greater than the average velocity at a given cross section, for a given discharge. The flood wave velocity can be estimated by a number of different techniques:

(a) Using Seddon's law, a flood wave velocity can be approximated from the discharge rating curve at a station whose cross section is representative of the routing reach. The slope of the discharge rating curve is equal to dQ/dy. The flood wave velocity, and therefore the travel time K, can be estimated as follows:

$$V_w = \frac{1}{B}\frac{dQ}{dy} \tag{20}$$

$$K = \frac{L}{V_w} \tag{21}$$

Where: V_w = flood wave velocity, ft/s
B = top width of the water surface
L = length of the routing reach, ft

(b) Another means of estimating flood wave velocity is to estimate the average velocity (V) and multiply it by a ratio. The average velocity can be calculated from Manning's equation with a representative discharge and cross section for the routing reach. For various channel shapes, the flood wave velocity has been found to be a direct ratio of the average velocity.

Channel Shape	Ratio V_w/V
Wide rectangular	1.67
Wide parabolic	1.44
Triangular	1.33

For natural channels, an average ratio of 1.5 is suggested. Once the wave speed has been estimated, the travel time (K) can be calculated with equation 21.

Estimating the Muskingum X parameter in an ungaged situation can be very difficult. X varies between 0.0 and 0.5, with 0.0 providing the maximum amount of hydrograph attenuation and 0.5 no attenuation. Experience has shown that for channels with mild slopes and flows that go out of bank, X will be closer to 0.0. For steeper streams, with well defined channels that do not have flows going out of bank, X will be closer to 0.5. Most natural channels lie somewhere in between these two limits, leaving a lot of room for "engineering judgement." One equation that can be used to estimate the Muskingum X coefficient in ungaged areas has been developed by Cunge (1969). This equation is taken from the Muskingum-Cunge channel routing method, which is described in section III.d. The equation is written as follows:

$$X = \frac{1}{2}\left(1 - \frac{Q_o}{BS_o c \Delta x}\right) \tag{22}$$

Where:
- Q_o = reference flow from the inflow hydrograph
- c = flood wave speed
- S_o = friction slope or bed slope
- B = top width of the flow area
- Δx = length of the routing subreach

The choice of which flow rate to use in this equation is not completely clear. Experience has shown that a reference flow based on average values (midway between the base flow and the peak flow) is in general the most suitable choice. Reference flows based on peak flow values tend to accelerate the wave much more than it would in nature, while the converse is true if base flow reference values are used (Ponce, 1983).

(3) Selecting the Number of Subreaches. The Muskingum equation has a constraint related to the relationship between the parameter K and the computation interval Δt. Ideally, the two should be equal, but Δt should not be less than 2KX to

avoid negative coefficients and instabilities in the routing procedure.

$$2KX < \Delta t \leq K \tag{23}$$

A long routing reach should be subdivided into subreaches so that the travel time through each subreach is approximately equal to the routing interval Δt. That is:

$$\textit{Number of subreaches} = \frac{K}{\Delta t}$$

This assumes that factors such as channel geometry and roughness have been taken into consideration in determining the length of the routing reach and the travel time K.

d. *Muskingum-Cunge Channel Routing.* The Muskingum-Cunge channel routing technique is a non-linear coefficient method that accounts for hydrograph diffusion based on physical channel properties and the inflowing hydrograph. The advantages of this method over other hydrologic techniques are: (1) the parameters of the model are more physically based; (2) the method has been shown to compare well against the full unsteady flow equations over a wide range of flow situations (Ponce, 1983); and (3) the solution is independent of the user specified computation interval. The major limitations of the Muskingum-Cunge technique are that: (1) it cannot account for backwater effects; and (2) the method begins to diverge from the full unsteady flow solution when very rapidly rising hydrographs are routed through flat channel sections.

(1) Development of Equations. The basic formulation of the equations is derived from the continuity equation (24) and the diffusion form of the momentum equation (25):

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_l \tag{24}$$

$$S_f = S_o - \frac{\partial Y}{\partial x} \tag{25}$$

By combining equations (24) and (25) and linearizing, the following convective diffusion equation is formulated (Miller and Cunge, 1975):

$$\frac{\partial Q}{\partial t} + c\frac{\partial Q}{\partial x} = \mu\frac{\partial^2 Q}{\partial x^2} + cq_L \tag{26}$$

Where:

Q	=	Discharge in cfs
A	=	Flow area in ft^2
t	=	Time in seconds
x	=	Distance along the channel in feet
Y	=	Depth of flow in feet
q_L	=	Lateral inflow per unit of channel length
S_f	=	Friction slope
S_o	=	Bed Slope
c	=	The wave celerity in the x direction as defined below.

The wave celerity (c) and the Hydraulic diffusivity (μ) are expressed as follows:

$$c = \frac{dQ}{dA} \tag{27}$$

$$\mu = \frac{Q}{2BS_o} \tag{28}$$

where B is the top width of the water surface. The convective diffusion equation (35) is the basis for the Muskingum-Cunge method.

In the original Muskingum formulation, with lateral inflow, the continuity equation (1) is discretized on the x-t plane (figure 12) to yield:

$$Q_{j+1}^{n+1} = C_1Q_j^n + C_2Q_j^{n+1} + C_3Q_{j+1}^n + C_4Q_L \tag{29}$$

It is assumed that the storage in the reach is expressed as the classical Muskingum storage:

$$S = K\,[XI + (1-X)O] \tag{30}$$

Where:

S	=	channel storage
K	=	cell travel time (seconds)
X	=	weighting factor
I	=	inflow
O	=	outflow

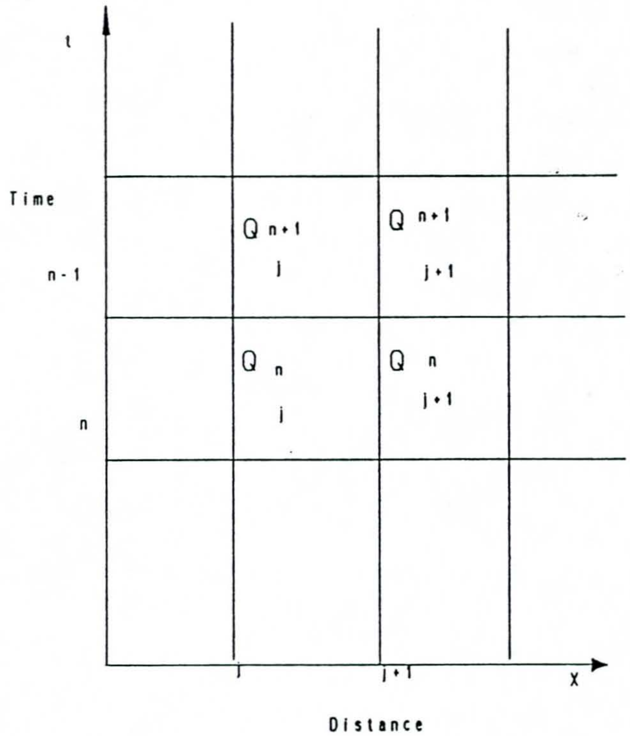


FIGURE 12
Discretization of the Continuity Equation on x-t Plane

Therefore, the coefficients can be expressed as follows:

$$C_1 = \frac{\frac{\Delta t}{K} + 2X}{\frac{\Delta t}{K} + 2(1-X)}$$

$$C_2 = \frac{\frac{\Delta t}{K} - 2X}{\frac{\Delta t}{K} + 2(1-X)}$$

$$C_3 = \frac{2(1-X) - \frac{\Delta t}{K}}{\frac{\Delta t}{K} + 2(1-X)}$$

$$C_4 = \frac{2(\frac{\Delta t}{K})}{\frac{\Delta t}{K} + 2(1-X)}$$

$$Q_L = q_L \Delta X$$

In the Muskingum equation the amount of diffusion is based on the value of X, which varies between 0.0 and 0.5. The Muskingum X parameter is not directly related to physical channel properties. The diffusion obtained with the Muskingum technique is a function of how the equation is solved, and is therefore considered numerical diffusion rather than physical. Cunge evaluated the diffusion that is produced in the Muskingum equation and analytically solved for the following diffusion coefficient:

$$\mu_n = c \; \Delta x \left(\frac{1}{2} - X \right) \tag{31}$$

In the Muskingum-Cunge formulation, the amount of diffusion is controlled by forcing the numerical diffusion to match the physical diffusion of the convective diffusion equation (26). This is accomplished by setting equations 28 and 31 equal to each other. The Muskingum-Cunge equation is therefore considered an approximation of the convective diffusion equation (26). As a result, the parameters K and X are expressed as follows (Cunge, 1969 and Ponce, 1978):

$$K = \frac{\Delta X}{c} \tag{32}$$

$$X = \frac{1}{2} \left(1 - \frac{Q}{B S_o c \Delta x} \right) \tag{33}$$

(2) Solution of the Equations. The method is non-linear in that the flow hydraulics (Q, B, c), and therefore the routing coefficients (C_1, C_2, C_3, and C_4) are re-calculated for every Δx distance step and Δt time step. An iterative four-point

averaging scheme is used to solve for c, B and Q. This process has been described in detail by Ponce (1986).

Values for Δt and Δx are chosen for accuracy and stability. First, Δt should be evaluated by looking at the following 3 criteria and selecting the smallest value: (1) the user defined computation interval; (2) the time of rise of the inflow hydrograph divided by 20 ($^{Tr}/_{20}$), and (3) the travel time through the channel reach.

Once Δt is chosen, Δx is defined as follows:

$$\Delta x = c\Delta t \tag{34}$$

but Δx must also meet the following criteria to preserve consistency in the method (Ponce, 1983):

$$\Delta x < \frac{1}{2}\left(c\Delta t + \frac{Q_o}{BS_oc}\right) \tag{35}$$

where Q_o is the reference flow and Q_B is the baseflow taken from the inflow hydrograph as:

$$Q_o = Q_B + 0.50\ (Q_{peak} - Q_B)$$

(3) Data Requirements. Data for the Muskingum-Cunge method consist of the following:

(a) Representative channel cross section;

(b) Reach length, L;

(c) Manning roughness coefficients, n (for main channel and overbanks); and

(d) Friction slope (S_f) or channel bed slope (S_o).

The method can be used with a simple cross section (i.e. trapezoid, rectangle, square, triangle, or circular pipe), or a more detailed cross section (i.e. cross sections with a left overbank, main channel, and a right overbank). The cross section is assumed to be representative of the entire routing reach. If this assumption is not adequate, the routing reach should be broken up into smaller subreaches with representative cross sections for each. Reach lengths are measured directly from topographic maps. Roughness coefficients (Manning's n) must be estimated for main channels as well as overbank areas. If information is available to estimate an approximate energy grade line slope (friction slope, S_f), that slope should be used instead of the bed slope. If no information is available to estimate the slope of the energy grade line, the channel bed slope should be used.

(4) Advantages and Limitations. The Muskingum-Cunge routing technique is considered to be a non-linear coefficient method that accounts for hydrograph diffusion based on physical channel properties and the inflowing hydrograph. The advantages of this method over other hydrologic techniques are: (1) the parameters of the model are physically based, and therefore this method will make for a good ungaged routing technique; (2) several studies have shown that the method compares very well with the full unsteady flow equations over a wide range of flow conditions (Ponce, 1983 and Brunner, 1989); and (3) the solution is independent of the user specified computation interval. The major limitations of the Muskingum-Cunge technique are that: (1) the method can not account for backwater effects; and (2) the method begins to diverge from the complete unsteady flow solution when very rapidly rising hydrographs are routed through flat channel sections (i.e. channel slopes less than 2 ft/mi). For hydrographs with longer rise times (T_r), the method can be used for channel reaches with slopes less than 2 ft/mi.

Selecting The Appropriate Routing Technique

I. Introduction

With such a wide range of hydraulic and hydrologic routing techniques, selecting the appropriate routing method for each specific problem is not clearly defined. However, certain thought processes and some general guidelines can be used to narrow the choices, and ultimately the selection of an appropriate method can be made.

Typically, in rainfall-runoff analyses, hydrologic routing procedures are utilized on a reach by reach basis from upstream to downstream. In general, the main goal of the rainfall-runoff study is to calculate discharge hydrographs at several locations in the watershed. In the absence of significant backwater effects, the hydrologic routing models offer the advantages of simplicity, ease of use, and computational efficiency. Also, the accuracy of hydrologic methods in calculating discharge hydrographs is normally well within the range of acceptable values. It should be remembered, however, that insignificant backwater effects alone do not always justify the use of a hydrologic method. There are many other factors that must be considered when deciding if a hydrologic model will be appropriate, or if it is necessary to use a more detailed hydraulic model.

The full unsteady flow equations have the capability to simulate the widest range of flow situations and channel characteristics. Hydraulic models, in general, are more physically based since they only have one parameter (the roughness coefficient) to estimate or calibrate. Roughness coefficients can be estimated with some degree of accuracy from inspection of the waterway, which makes the hydraulic methods more applicable to ungaged situations.

There are several factors that should be considered when evaluating which routing method is the most appropriate for a given situation. The following is a list of the major factors that should be considered in this selection process:

II. Factors To Consider When Selecting a Routing Method

a. Backwater Effects. Backwater effects can be produced by tidal fluctuations, significant tributary inflows, dams, bridges, culverts, and channel constrictions. A flood wave that is subjected to the influences of backwater will be attenuated and delayed in time. Of the hydrologic methods discussed previously, only the Modified Puls method is capable of incorporating the effects of backwater into the solution. This is accomplished by calculating a storage-discharge relationship that has the effects of backwater included in the relationship. Storage discharge relationships can be determined from steady flow water surface profile calculations; observed water surface profiles, normal depth calculations, and observed inflow and outflow hydrographs. All of these techniques, except the normal depth calculations, are capable of including the effects of backwater into the storage-discharge relationship. Of the hydraulic methods discussed in this chapter, only the Kinematic Wave technique is not capable of accounting for the influences of backwater on the flood wave. This is due to the fact that the Kinematic Wave equations are based on uniform flow assumptions and a normal depth downstream boundary condition.

b. Flood Plain Storage. When the flood hydrograph reaches a magnitude that is greater than the channels carrying capacity, water flows out into the overbank areas. Depending on the characteristics of the overbanks, the flow can be slowed greatly and often ponding of water can occur. The effects of the flood plains on the flood wave can be very significant. The factors that are important in evaluating to what extent the flood plain will impact the hydrograph are: (1) the width of the flood plain; (2) the slope of the flood plain in the lateral direction; and (3) the resistance to flow due to vegetation in the flood plain. In order to analyze the transition from main channel to overbank flows, the modeling technique must be able to account for varying conveyance between the main channel and the overbank areas. For one dimensional flow models, this is normally accomplished by calculating the hydraulic properties of the main channel and the overbank areas separately, then combining them to formulate a composite set of hydraulic relationships. This can be accomplished in all of the routing methods discussed previously except for the Muskingum method. The Muskingum method is a linear routing technique that uses coefficients to account for hydrograph timing and diffusion. These coefficients are usually held constant during the routing of a given flood wave. While these coefficients can be calibrated to match the peak flow and timing of a specific flood magnitude, they can not be used to model a range of floods that may remain in bank or go out of bank. When modeling floods through extremely flat and wide flood plains, the assumption of one dimensional flow in itself may be inadequate. For this flow condition, velocities in the lateral direction (across the flood plain) may be just as predominant as those in the longitudinal direction (down the channel). When this occurs, a two dimensional flow model would give a more accurate representation of the physical processes. This subject is beyond the scope of this chapter. For more information on this topic, the reader is referred to the River Hydraulics Engineering Manual (EM 1110-2-9020).

c. Channel Slope and Hydrograph Characteristics. The slope of the channel will not only effect the velocity of the flood wave, but it can also effect the amount of attenuation that will occur during the routing process. Steep channel slopes accelerate the flood wave, while mild channel slopes are prone to slower velocities and greater amounts of hydrograph attenuation. Of all the routing methods presented in this chapter, only the complete unsteady flow equations are capable of routing flood waves through channels that range from steep to extremely flat slopes. As the channel slopes become flatter, many of the methods begin to break down. For the simplified hydraulic methods, the terms in the momentum equation that were excluded become more important in magnitude as the channel slope is decreased. Because of this, the range of applicable channel slopes decreases with the number of terms excluded from the momentum equation. As a rule of thumb, the Kinematic wave equations should only be applied to relatively steep channels (10 ft/mi or greater). Since the Diffusion Wave approximation includes the pressure differential term in the momentum equation, it is applicable to a wider range of slopes than the Kinematic Wave equations. The Diffusion Wave technique can be used to route slow rising flood waves through extremely flat slopes. Although, rapidly rising flood waves should be limited to mild to steep channel slopes (approximately 1 ft/mi or greater). This is due to the fact that the acceleration terms in the momentum equation increase in magnitude as the time of rise of the inflowing hydrograph is decreased. Since the Diffusion Wave method does not include these acceleration terms, routing rapidly rising hydrographs through flat channel slopes can result in errors in the amount of diffusion that will occur. While "rules of thumb" for channel slopes can be established, it should be realized that it is the combination of channel slope and the time of rise of the inflow hydrograph together that will determine if a method is applicable or not.

Ponce (1978) established a numerical criteria for the applicability of hydraulic routing techniques. According to Ponce, the error due to the use of the Kinematic Wave model (error in hydrograph peak accumulated after an elapsed time equal to the hydrograph duration) is within 5 per cent, provided the following inequality is satisfied:

$$\frac{T S_o u_o}{d_o} \geq 171 \qquad (1)$$

Where:
- T = hydrograph duration in seconds
- S_o = friction slope or bed slope
- u_o = reference mean velocity
- d_o = reference flow depth

When applying equation 1 to check the validity of using the Kinematic Wave model, the reference values should correspond as closely as possible to the average flow conditions of the hydrograph to be routed.

The error due to the use of the Diffusion Wave model is within 5%, provided the following inequality is satisfied:

$$TS_o\left(\frac{g}{d_o}\right)^{1/2} \geq 30 \tag{2}$$

Where: g = acceleration of gravity

For instance, assume $S_o = 0.001$, $u_o = 3$ ft/s, and $d_o = 10$ ft. The Kinematic Wave model will apply for hydrographs of duration larger than 6.59 days. Likewise, the Diffusion Wave model will apply for hydrographs of duration larger than 0.19 days.

Of the hydrologic methods, the Muskingum-Cunge method is applicable to the widest range of channel slopes and inflowing hydrographs. This is due to the fact that the Muskingum-Cunge technique is an approximation of the Diffusion Wave equations, and therefore can be applied to channel slopes of a similar range in magnitude. The other hydrologic techniques all use an approximate relationship in place of the momentum equation. Through experience it has been shown that these techniques should not be applied to channels with slopes less than 2 ft/mi. However, if there is gaged data available, some of the parameters of the hydrologic methods can be calibrated to produce the desired attenuation effects that occur in very flat streams.

d. Flow Networks. In a dendritic stream system, if the tributary flows or the main channel flows do not cause significant backwater at the confluence of the two streams, any of the hydraulic or hydrologic routing methods can be applied. If significant backwater does occur at the confluence of two streams, then the hydraulic methods that can account for backwater (full unsteady flow and diffusion wave) should be applied. For full networks, where the flow divides and possibly changes direction during the event, only the full unsteady flow equations and the diffusion wave equations can be applied.

e. Subcritical and Supercritical Flow. During a flood event a stream may experience transitions between subcritical and supercritical flow regimes. If the supercritical flow reaches are long, or if it is important to calculate an accurate stage within the supercritical reach, the transitions between subcritical and supercritical flow should be treated as internal boundary conditions and the supercritical flow reach as a separate routing section. This is normally accomplished with hydraulic routing methods that have specific routines to handle supercritical flow. In general, none of the hydrologic methods have knowledge about the flow regime (supercritical or subcritical). This is due to the fact that the hydrologic methods are only concerned with flows and not stages. If the supercritical flow reaches are short, they will not have a noticeable impact on the discharge hydrograph. Therefore, when it is only important to calculate the discharge hydrograph, and not stages, hydrologic routing methods can be used for reaches with small sections of supercritical flow.

f. Calibrating to Observed Data. In general, if observed data are not available, the routing methods that are more physically based will have greater accuracy and will be easier to apply. When gaged data are available, all of the methods should be calibrated to match observed flows and/or stages as best as possible. The hydraulic methods, as well as the Muskingum-Cunge technique, are considered physically based in the sense that they only have one parameter (roughness coefficient) that must be estimated or calibrated. The other hydrologic methods may have more than one parameter to be estimated or calibrated. Many of these parameters, such as the Muskingum X and the number of subreaches (NSTPS), are not related directly to physical aspects of the channel and inflowing hydrograph. Because of this, these methods are generally not used in ungaged situations.

III. Summary and Conclusions

The final choice of an appropriate routing method is often influenced by other factors than those mentioned previously. Some of the other factors that should be considerd are: the required accuracy of the results; the type and availability of data; the type of information desired (flow hydrographs, stages, velocities, etc..); and the familiarity and experience of the user with a given method. The modeler must take all of these factors into consideration when selecting an appropriate routing technique for a specific problem. Table 1 contains a list of some of the factors discussed previously, along with some guidance as to which routing methods are appropriate and which are not. This table should be used as guidance in selecting an appropriate method for routing discharge hydrographs. By no means is this table all inclusive.

TABLE 1

Selecting the Appropriate Channel Routing Technique

Factors to consider in the selection of a routing technique.	Methods that are appropriate for this specific factor.	Methods that are not appropriate for this factor.
1. No observed hydrograph data available for calibration.	* Full Dynamic Wave * Diffusion Wave * Kinematic Wave * Muskingum-Cunge	* Modified Puls * Muskingum * Working R&D
2. Significant Backwater that will influence discharge hydrograph.	* Full Dynamic Wave * Diffusion Wave * Modified Puls * Working R&D	* Kinematic Wave * Muskingum * Muskingum-Cunge
3. Flood wave will go out of bank into the flood plains.	* All hydraulic and hydrologic methods that calculate hydraulic properties of main channel seperate from overbanks.	* Muskingum
4. Channel slope > 10 ft/mi and $\frac{TS_o u_o}{d_o} \geq 171$	* All methods presented	* None
5. Channel slopes from 10 to 2 ft/mi and $\frac{TS_o u_o}{d_o} < 171$	* Full Dynamic Wave * Diffusion Wave * Muskingum-Cunge * Modified Puls * Muskingum * Working R&D	* Kinematic Wave
6. Channel slope < 2 ft/mi and $TS_o(\frac{g}{d_o})^{1/2} \geq 30$	* Full Dynamic Wave * Diffusion Wave * Muskingum-Cunge	* Kinematic Wave * Modified Puls * Muskingum * Working R&D
7. Channel slope < 2 ft/mi and $TS_o(\frac{g}{d_o})^{1/2} < 30$	* Full Dynamic Wave	* All others

Appendix B
Comparison of Hydrologic and Hydraulic Routing Methods

The procedure for correctly selecting a hydraulic or hydrologic routing technique is often not clearly defined. This appendix serves as a supplement to Chapter 9 and as suggested guidance in selecting an appropriate method for routing discharge hydrographs. Comparisons between several hydraulic and hydrologic routing techniques were made for hypothetical river reaches and hydrograph data. Parameters for the various routing techniques need to be estimated from physical data. Consequently, examples of how to estimate parameters necessary for an ungaged situation are shown. Also presented are two numerical criteria for hydraulic routing methods. Finally, results of this study plus guide lines for selecting an appropriated routing technique are discussed.

B-1. General Testing Structure

The U.S. Army Corps of Engineers has written several computer programs to route discharge hydrographs through a section or reach of river. For this appendix, HEC-1 was used to route inflow hydrographs using Kinematic Wave, Muskingum, Muskingum-Cunge, and Modified Puls techniques. These approximate HEC-1 routing techniques are then compared to a more accurate one-dimensional full unsteady flow model called UNET.

The development of the inflow hydrographs was accomplished using HEC-1. A peak flow value of 20,135 cfs was arbitrarily selected. Using an upstream basin area of 50 mi^2, runoff was

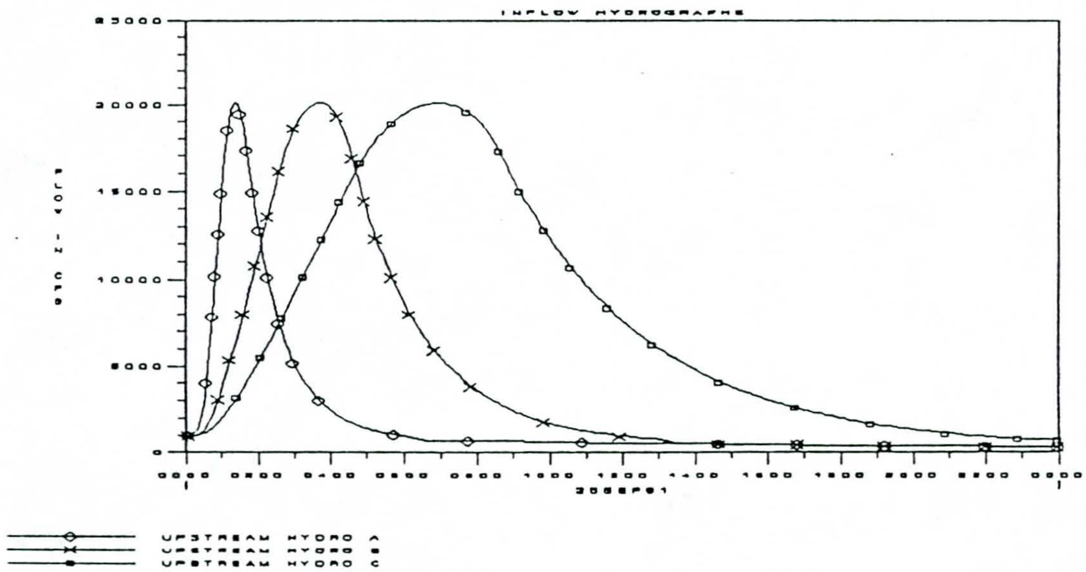


FIGURE B.1
Inflow Hydrographs

estimated using Clark's Method with a time of concentration of eight hours and a storage coefficient of four hours. Base flow at the start of the storm was 1000 cfs. Basin preçipitation and intensity were then adjusted until the desired hydrograph shape was achieved. Three hydrographs were developed with the following characteristics: Hydrograph A - rapidly rising with small volume, Hydrograph B - medium rise time with medium volume, Hydrograph C - slow rising with large volume (see Figure 1).

Three hypothetical cross sections were created. All cross sections have a base width of 50 feet and maintain a constant shape throughout the 20 mile test length. Section A is a simple trapezoid with sides always sloping upward at a 1:2 ratio. Section B begins like Section A for the first 10 vertical feet, then suddenly changes to a channel with short sloping overbanks that are 100 feet wide. Section C begins like Section B except its overbanks are 500 feet wide and are almost flat horizontally.

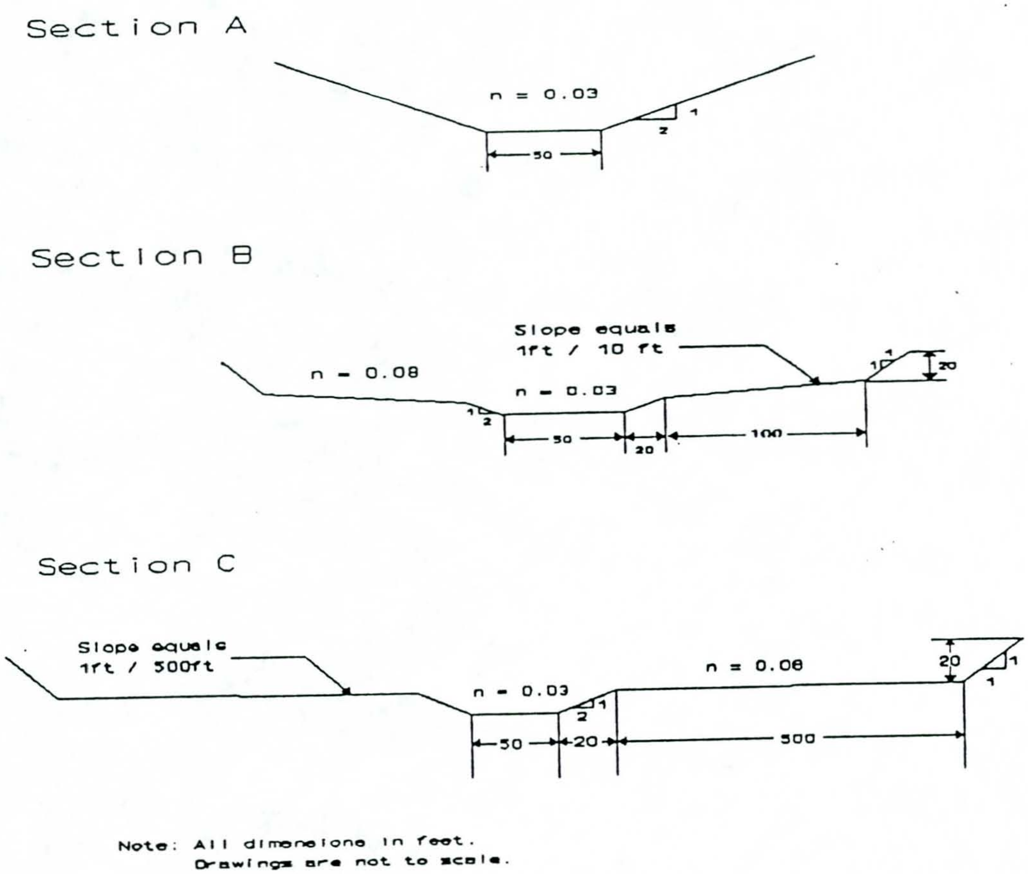


Figure 2
Cross Section Dimensions

During testing, sections A, B and C were subjected to inflow hydrographs A, B, and C in addition to longitudinal slopes of one, two, five and ten feet per mile. All 36 (3 sections X 4 slopes X 3 inflow hydrographs) possible combinations of inflow hydrographs, sections and slopes were used for Muskingum, one-dimensional unsteady flow (UNET), Muskingum-Cunge, and Modified Puls routing methods. For runs using the Kinematic Wave technique, only test runs using Section A were possible because HEC-1 only allows purely trapezoidal, square, or circular sections to be entered. A total of 156 runs were completed for this appendix study.

All river reaches were assumed to be ungaged. In order to route the inflow hydrographs using Muskingum and Modified Puls Methods, several parameters had to be estimated. For the Modified Puls method, the number of routing steps (NSTPS) had to be calculated. This is done by first approximating the total travel time (K).

$$K = \frac{L}{V_w} \tag{1}$$

Then NSTPS can be calculated with the following equation:

$$NSTPS = \frac{K}{\Delta t} \tag{2}$$

Where:

K	=	total travel time through the reach
L	=	channel reach length
V_w	=	velocity of the flood wave or celerity
Δt	=	computation interval
NSTPS	=	number of routing steps

For example, using Section A and a slop 1 ft/mile, the average normal depth velocity (V) can be determined using Chèzy's or Manning's equation. The wave celerity can then estimated using the approximate relation $V_w = 1.5$ V, which has a value of 8.34 ft/s in this example. The reach length was a constant 20 miles, or 105600 ft, which produced a K value of 12662 seconds. Since Δt was held constant at 5 minutes for all non-UNET runs, NSTPS has a value of 42.

When using the Muskingum method, two parameters must be estimated. The first is a total travel time through the reach, which is calculated in exactly the same manner as the Modified Puls method. The second term is a weighting coefficient, X. This coefficient can be estimated by the following equation:

$$X = \frac{1}{2}\left(1 - \frac{Q_0}{BS_0 c \Delta x}\right) \quad (3)$$

Where:

X	= Muskingum X; weighing coefficient
Q_0	= reference flow from the inflow hydrograph
c	= flood wave
S_0	= friction slope or bed slope
B	= top width of the flow area
Δx	= length of the routing subreach

Take for instance the same slope and section as used in the previous example. The length of the routing subreach approximately equals the celerity multiplied by the computation interval ($\Delta x = V_w \Delta t$ = 8.34 ft/s * 5 min * 60 sec/1 min or 2502 ft). A good estimation of the reference flow is the value midway between the base and peak flow or 10567.5 cfs [(20135-1000 / 2) + 1000]. Using a top width of 170 ft, X has a value of -6.64. This negative X value was then set to zero because X can only vary between 0.0 and 0.5.

B-2. Results

The output data generated from the 156 model runs completed for this appendix are quite voluminous. Therefore, an abridgement of results was necessary and only the most pertinent output is presented here. Foremost, it is relevant to examine the inflow hydrograph statistics and apply several widely used criteria for choosing the appropriate hydrograph routing method. Table 1 shows these statistics. The duration of inflow hydrographs A, B, and C are 6, 13.5, and 22.5 hours respectively. These numbers were determined visually, estimating the time of initial rise above base flow to the point where it returned again to the base flow. The mean reference velocities and depths for the inflow hydrographs were obtained using UNET results. The mean reference velocity was estimated to occur at 2/3 of the inflow hydrographs peak flow. Correspondingly, the reference depth was taken at the same instant in time.

As discussed in Chapter 9, Ponce (1978) established numerical criteria for the applicability of hydraulic routing techniques. One such criteria, for the Kinematic Wave model, is to satisfy the following inequality:

$$\frac{TS_0 u_0}{D_0} \geq 171 \quad (4)$$

Ponce also established the following numerical criteria for the Diffusion Wave model:

$$TS_0\left(\frac{g}{d_0}\right)^{\frac{1}{2}} \geq 30 \tag{5}$$

Where: T = hydrograph duration in seconds
S_0 = friction slope or bed slope (ft/ft)
u_0 = reference mean velocity (ft/s)
d_0 = reference flow depth (ft)
g = acceleration of gravity

The numerical values of these criteria for all inflow hydrographs are presented in Table 1. The criteria for the Diffusion Wave model was satisfied several times though the one for Kinematic Wave was never quite met. The implications of this observation will be discussed later in this appendix.

The two main components of the routed outflow hydrographs were their time to peak and peak flow. A compilation of these results are presented in Tables 2 to 5. Also provided are tables showing the percent change in peak flow and time to peak between the UNET calculated values and the corresponding values of the other methods. The percent change was computed by taking the difference between the alternative methods and the UNET calculated values, then dividing that difference by the UNET value. A positive number indicates a value greater than that calculated by UNET, whereas a negative number indicates the opposite.

Table B.1
Inflow Hydrograph Statistics

Hy/Sect/Slope	Slope (ft/ft)	Hy. Dur (hr)	Ref Vel. (ft/s)	Depth (ft)	T*S*V/D	T*S*(G/D)$^{0.5}$
A/A/1	0.000189	6.00	7.79	19.37	1.64	5.26
A/A/2	0.000379	6.00	8.28	18.59	3.65	10.77
A/A/5	0.000947	6.00	9.40	16.93	11.36	28.21
A/A/10	0.001894	6.00	11.09	15.06	30.13	59.82
A/B/1	0.000189	6.00	6.86	17.49	1.60	5.54
A/B/2	0.000379	6.00	7.30	16.95	3.53	11.28
A/B/5	0.000947	6.00	8.60	15.67	11.23	29.32
A/B/10	0.001894	6.00	10.00	14.66	27.91	60.63
A/C/1	0.000189	6.00	3.10	13.77	0.92	6.24
A/C/2	0.000379	6.00	3.25	13.60	1.96	12.60
A/C/5	0.000947	6.00	3.90	12.95	6.16	32.25
A/C/10	0.001894	6.00	4.90	12.32	16.27	66.14
B/A/1	0.000189	13.50	6.30	22.33	2.59	11.03
B/A/2	0.000379	13.50	6.90	20.91	6.08	22.86
B/A/5	0.000947	13.50	8.58	18.10	21.82	61.39
B/A/10	0.001894	13.50	10.66	15.51	63.26	132.63
B/B/1	0.000189	13.50	5.17	19.91	2.39	11.68
B/B/2	0.000379	13.50	5.23	19.82	4.86	23.48
B/B/5	0.000947	13.50	7.37	16.88	20.09	63.57
B/B/10	0.001894	13.50	9.85	14.76	61.43	135.96

Hy/Sect/Slope	Slope (ft/ft)	Hy. Dur (hr)	Ref. Vel (ft/s)	Depth (ft)	T*S*V/D	T*S*(G/D)$^{0.5}$
B/C/1	0.000189	13.50	2.34	15.07	1.43	13.43
B/C/2	0.000379	13.50	2.55	14.62	3.21	27.34
B/C/5	0.000947	13.50	3.06	13.83	10.18	70.23
B/C/10	0.001894	13.50	4.37	12.63	31.85	146.97
C/A/1	0.000189	22.50	5.71	23.96	3.65	17.75
C/A/2	0.000379	22.50	6.48	22.01	9.04	37.13
C/A/5	0.000947	22.50	8.37	18.44	34.82	101.36
C/A/10	0.001894	22.50	10.56	15.62	103.72	220.27
C/B/1	0.000189	22.50	4.29	21.79	3.01	18.61
C/B/2	0.000379	22.50	5.12	20.02	7.85	38.93
C/B/5	0.000947	22.50	7.17	17.11	32.14	105.23
C/B/10	0.001894	22.50	9.74	14.83	100.76	226.06
C/C/1	0.000189	22.50	2.05	15.79	1.99	21.86
C/C/2	0.000379	22.50	2.24	15.30	4.49	44.54
C/C/5	0.000947	22.50	3.15	13.72	17.61	117.51
C/C/10	0.001894	22.50	4.31	12.67	52.19	244.57
Notes:						
T=Hy. Dur; S=Slope; V=Ref Vel.; D=Depth of flow ; G=Gravity=32.2 ft/s^2						
Method Validity Checks: T*S*U/D > 171 (Kinematic); T*S*(G/D)^1/2 > 30 (Diffusion)						
Hydrograph duration is the time from starting rise above 1000 cfs to its return to 1000 cfs.						
Reference mean velocity calculated on rising limb of inflow hydrograph at 2/3 peak flow.						
Reference depth calculated on the rising limb of inflow hydrograph at 2/3 peak flow.						

Table B.2
Peak Flows (cfs)

Hy/Sect/Slope	Kwave	MCunge	MPuls	Musk	Unet
A/A/1	19966	5301	11182	14550	5188
A/A/2	19939	8289	12342	15202	7063
A/A/5	19976	13478	13732	15948	11265
A/A/10	19993	17188	14624	18534	15228
A/B/1		2972	7837	11341	4513
A/B/2		4880	9890	12498	6439
A/B/5		9613	12882	14100	10527
A/B/10		14436	15096	16782	13939
A/C/1		2138	1828	9680	2466
A/C/2		3083	5734	10197	3743
A/C/5		4787	5121	10752	6792
A/C/10		7214	6451	10949	10113
B/A/1	20106	11510	18748	19130	10872
B/A/2	20118	15925	19187	19336	13640
B/A/5	20117	19321	19490	19533	17949
B/A/10	20119	19919	19713	19970	19576
B/B/1		5638	14661	17561	8450
B/B/2		9526	16564	18250	11335
B/B/5		15894	18290	18965	16126
B/B/10		18905	19170	19705	18498

Hy/Sect/Slope	Kwave	MCunge	MPuls	Musk	Unet
B/C/1		3187	8675	14133	4524
B/C/2		5571	11577	16719	6985
B/C/5		11125	14568	17150	11713
B/C/10		15076	16428	17294	15693
C/A/1	20128	16591	19839	19897	15105
C/A/2	20130	19029	19928	19946	17542
C/A/5	20130	19954	19983	19993	19616
C/A/10	20131	20084	20035	20096	20014
C/B/1		9518	18758	19442	12618
C/B/2		14341	19345	19658	15516
C/B/5		18733	19699	19856	18736
C/B/10		19829	19902	20034	19686
C/C/1		6566	14756	17888	7775
C/C/2		10739	18041	19134	11345
C/C/5		16315	19428	19296	16491
C/C/10		18925	19861	19349	18203

Table B.3
Percent Change of Peak Flows

		+ value greater than Unet calculated value			
		- value less than Unet calculated value			
Hy/Sect/Slope	Unet Value (cfs)	Kwave (%)	MCunge (%)	MPuls (%)	Musk (%)
A/A/1	5188	285	2	116	180
A/A/2	7063	182	17	75	115
A/A/5	11265	77	20	22	42
A/A/10	15228	31	13	-4	22
A/B/1	4513		-34	74	151
A/B/2	6439		-24	54	94
A/B/5	10527		-9	22	34
A/B/10	13939		4	8	20
A/C/1	2466		-13	-26	293
A/C/2	3743		-18	53	172
A/C/5	6792		-30	-25	58
A/C/10	10113		-29	-36	8
B/A/1	10872	85	6	72	76
B/A/2	13640	47	17	41	42
B/A/5	17949	12	8	9	9
B/A/10	19576	3	2	1	2

Hy/Sect/Slope	Unet Value (cfs)	Kwave (%)	MCunge (%)	MPuls (%)	Musk (%)
B/B/1	8450		-33	74	108
B/B/2	11335		-16	46	61
B/B/5	16126		-1	13	18
B/B/10	18498		2	4	7
B/C/1	4524		-30	92	212
B/C/2	6985		-20	66	139
B/C/5	11713		-5	24	46
B/C/10	15693		-4	5	10
C/A/1	15105	33	10	31	32
C/A/2	17542	15	8	14	14
C/A/5	19616	3	2	2	2
C/A/10	20014	1	0	0	0
C/B/1	12618		-25	49	54
C/B/2	15516		-8	25	27
C/B/5	18736		0	5	6
C/B/10	19686		1	1	2
C/C/1	7775		-16	90	130
C/C/2	11345		-5	59	69
C/C/5	16491		-1	18	17
C/C/10	18203		4	9	6

Table B.4
Time to Peak (hours)

Hy/Sect/Slope	Kwave	MCunge	MPuls	Musk	Unet
A/A/1	4.08	5.58	6.33	4.92	5.58
A/A/2	3.50	4.83	5.08	4.33	5.25
A/A/5	3.00	3.67	4.00	3.58	4.33
A/A/10	2.67	3.00	3.42	3.08	3.50
A/B/1		7.92	10.75	11.50	7.42
A/B/2		7.17	8.17	8.42	6.92
A/B/5		5.50	5.67	5.67	5.42
A/B/10		4.33	4.33	4.17	4.42
A/C/1		14.50	23.08	18.33	12.00
A/C/2		12.58	20.42	15.75	17.67
A/C/5		8.42	13.92	13.50	12.58
A/C/10		7.42	9.67	12.75	9.50
B/A/1	6.42	6.92	7.67	7.17	7.17
B/A/2	5.83	6.25	6.67	6.50	6.83
B/A/5	5.25	5.42	5.83	5.75	6.08
B/A/10	4.92	5.00	5.25	5.33	5.50
B/B/1		10.75	11.42	13.50	9.67
B/B/2		9.92	9.50	10.50	9.08
B/B/5		7.83	7.83	7.75	7.75
B/B/10		6.67	6.67	6.33	6.67

Hy/Sect/Slope	Kwave	MCunge	MPuls	Musk	Unet
B/C/1		22.92	23.17	20.50	23.58
B/C/2		19.67	18.00	17.75	19.75
B/C/5		14.92	13.83	15.58	15.08
B/C/10		12.25	11.75	14.83	12.25
C/A/1	9.58	9.75	10.75	10.42	10.25
C/A/2	9.08	9.42	9.83	9.75	10.00
C/A/5	8.50	8.58	9.08	9.00	9.25
C/A/10	8.17	8.25	8.50	8.58	8.67
C/B/1		14.17	13.58	16.75	12.50
C/B/2		12.83	12.17	13.67	12.00
C/B/5		11.08	11.00	11.00	10.92
C/B/10		9.92	9.92	9.58	9.92
C/C/1		26.75	22.67	23.50	23.08
C/C/2		21.83	18.58	20.92	20.00
C/C/5		16.75	15.33	18.75	15.75
C/C/10		14.50	13.50	18.00	14.83

Table B.5
Percent Change of Time to Peak

		+ value greater than Unet calculated value - value less than Unet calculated value			
Hy/Sect/Slope	Unet Value (hours)	Kwave (%)	MCunge (%)	MPuls (%)	Musk (%)
A/A/1	5.58	-27	0	13	-12
A/A/2	5.25	-33	-8	-3	-17
A/A/5	4.33	-31	-15	-8	-17
A/A/10	3.50	-24	-14	-2	-12
A/B/1	7.42		7	45	55
A/B/2	6.92		4	18	22
A/B/5	5.42		1	5	5
A/B/10	4.42		-2	-2	-6
A/C/1	12.00		21	92	53
A/C/2	17.67		-29	16	-11
A/C/5	12.58		-33	11	7
A/C/10	9.50		-22	2	34
B/A/1	7.17	-11	-4	7	0
B/A/2	6.83	-15	-8	-2	-5
B/A/5	6.08	-14	-11	-4	-5
B/A/10	5.50	-11	-9	-5	-3

Hy/Sect/Slope	Unet Value (hours)	Kwave (%)	MCunge (%)	MPuls (%)	Musk (%)
B/B/1	9.67		11	18	40
B/B/2	9.08		9	5	16
B/B/5	7.75		1	1	0
B/B/10	6.67		0	0	-5
B/C/1	23.58		-3	-2	-13
B/C/2	19.75		0	-9	-10
B/C/5	15.08		-1	-8	3
B/C/10	12.25		0	-4	21
C/A/1	10.25	-7	-5	5	2
C/A/2	10.00	-9	-6	-2	-3
C/A/5	9.25	-8	-7	-2	-3
C/A/10	8.67	-6	-5	-2	-1
C/B/1	12.50		13	9	34
C/B/2	12.00		7	1	14
C/B/5	10.92		2	1	1
C/B/10	9.92		0	0	-3
C/C/1	23.08		16	-2	2
C/C/2	20.00		9	-7	5
C/C/5	15.75		6	-3	19
C/C/10	14.83		-2	-9	21

B-3. Discussion of Results

Ponce (1978) discussed several numerical criteria for hydrologic routing methods. Table 1 shows the numerical values that were calculated for Kinematic Wave and Diffusion Wave criteria. The Diffusion Wave criteria (equation 5) states that with a value greater than 30, the error of the model is within 5%. The values calculated from our test runs ranged from 5.26 to 244.57. Even though Muskingum-Cunge method is a hydrologic method, it is an approximation of the Diffusion Wave equations, and therefore can be applied to channels of a similar range in magnitude. For example, inflow hydrograph A, Section B and slope 1 ft/mile has a criteria value of 5.54. Examining Figure 3 there is a great deal of error between the UNET and the Muskingum-Cunge results. Furthermore, inflow hydrograph B, Section B and slope 5 ft/mile has a value of 63.57 and looking at Figure 4 the error between Muskingum-Cunge and UNET is within 5%. We found this criteria to be an accurate predictor of Muskingum-Cunge performance.

The numerical criteria for Kinematic Wave (equation 4) states that with a value greater than 171, the error in estimating the wave amplitude will be less than 5%. The values calculated from our test runs ranged from 0.92 to 103.72. Therefore, none of the model runs completed in this study satisfy this criteria. Several of the model runs did closely simulate the correct routed outflow hydrograph and are shown in Figures 5 (criteria value of 63.26, error of 2.8%), 6 (criteria value of 17.61, error of 2.6%), and 7 (criteria value of 52.19, error of 0.6%). Unfortunately, only simulation runs on Section A (the pure trapezoidal section) were possible in this study due to limitations in HEC-1. However, we found this criteria to be too conservative and would recommend lowering the inequalities criteria of 171.

Flood plain storage attenuates the peak flow and time to peak of the outflow hydrograph. This occurs as the water flows out of bank and experiences a large reduction in flow velocity. Due to this reduction in velocity, it can take large periods of time before that flow ultimately leaves the river reach. The percentage of flow that is trapped in flood storage is directly proportional to the size of the flood plane. This is evident by examining figure 8 (B/B/5 plot) and figure 9 (B/C/5 plot). The only parameter that changes between these two figures is the width of the overbank. The produced change in attenuation is due to the change in volume of water stored in these overbanks. Section C's overbanks are very broad and produce much slower velocities of flow in the out of bank region. This causes attenuation and a longer time to peak of the outflow hydrograph. Due to large eddies and significant lateral (normal to downstream) velocities in out of bank regions, further study with 2-dimensional unsteady flow models is recommended.

Increased longitudinal slope causes higher flow velocities. The inverse is also true, for as the slope decreases, so does the flow velocity. The hydrologic and hydraulic methods presented in Chapter 9 are adequate for river bed slope of 10 ft/mile or more. Unfortunately, all hydrologic methods tend to fail when faced with modeling slopes of less than 2 ft/mile. The most reliable of the hydrologic methods was Muskingum-Cunge, since it is partially derived from the momentum equation. Therefore, for reaches with 2 ft/mile of slope or less, where the Diffusion Wave criteria fails, only the Full Dynamic Wave method (UNET) should be applied.

The amount of time a flood wave takes to reach and decline from its peak, plus the amount of time the wave stays at its peak value, determines it volume. Since all of our hypothetical hydrographs reached the same peak flow, and only stayed at their peak flow for an instant in time, the time to peak determined the amount of inflow flood volume. Hydrograph A is rapidly rising and probably would only occur in the specific case of dam failure. Hydrographs B and C are slower in rising, and could occur from a rainfall event. The volume of the inflow hydrograph affects outflow hydrograph volume, time to peak, and attenuation. These effects are illustrated by comparing Figures 9 and 10. The only independent variable which changes between these two figures is the time to peak of the inflow hydrograph; which changes by 2.3 hrs. As a direct result, the outflow hydrograph peak flow changes by 5684 cfs and the time to peak by 1.58 hrs. Therefore, it can be concluded that the time to rise of the inflow hydrograph will greatly affect the routed outflow.

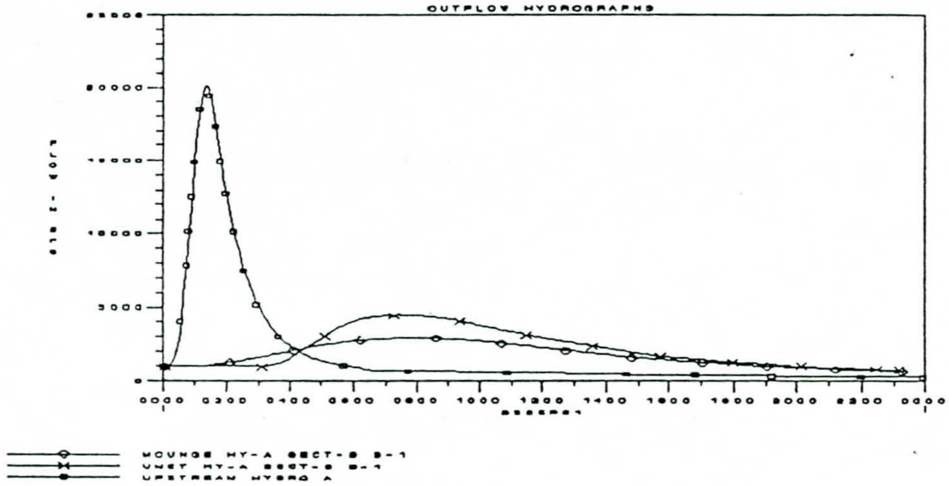


Figure 3
Outflow Hydrograph Comparison

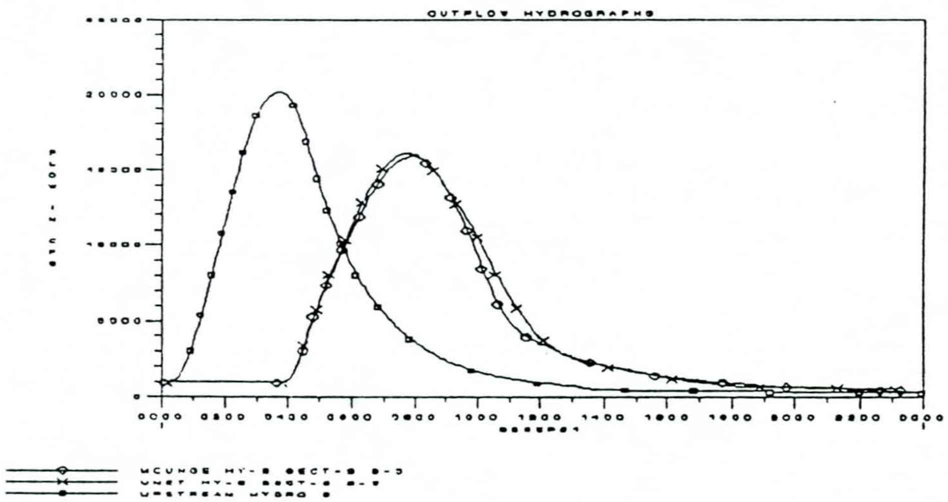


Figure 4
Outflow Hydrograph Comparison

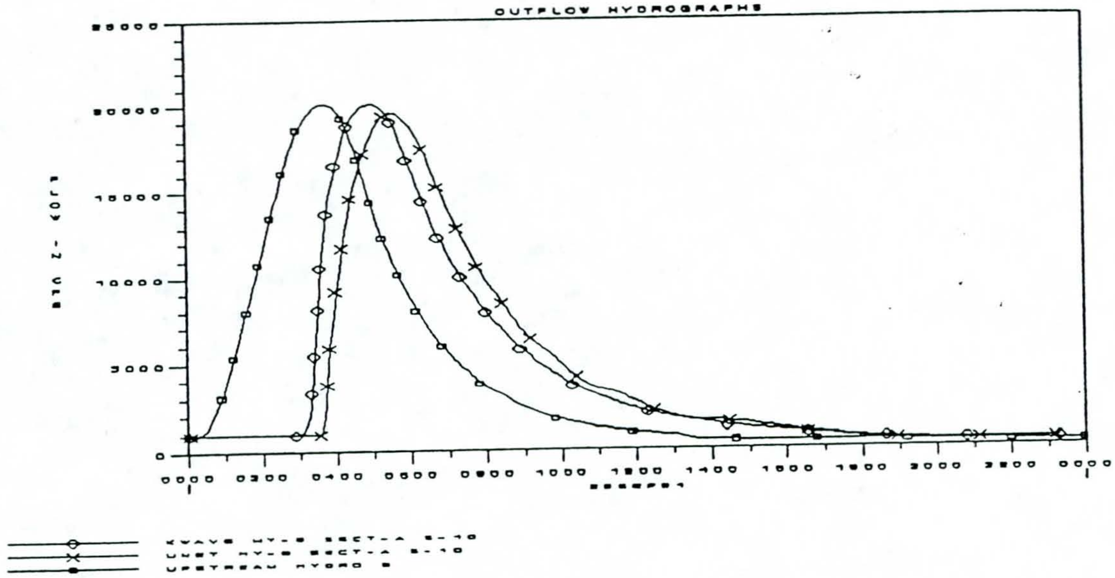


Figure 5
Comparison of Kinematic Wave

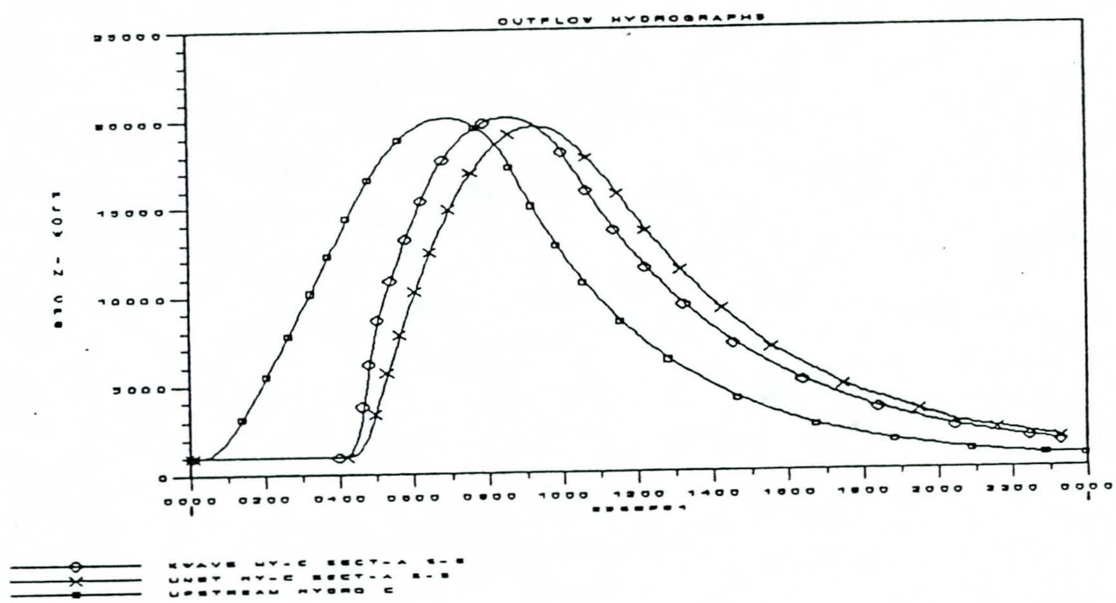


Figure 6
Comparison of Kinematic Wave

FLO-2D
DRAFT USERS
MANUAL

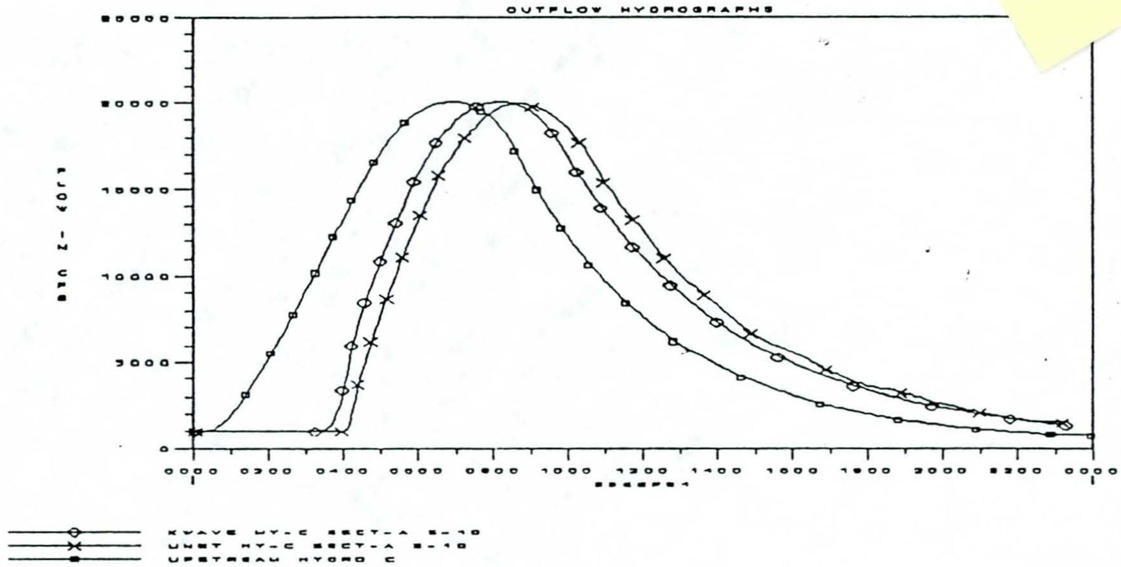


Figure 7
Comparison of Kinematic Wave

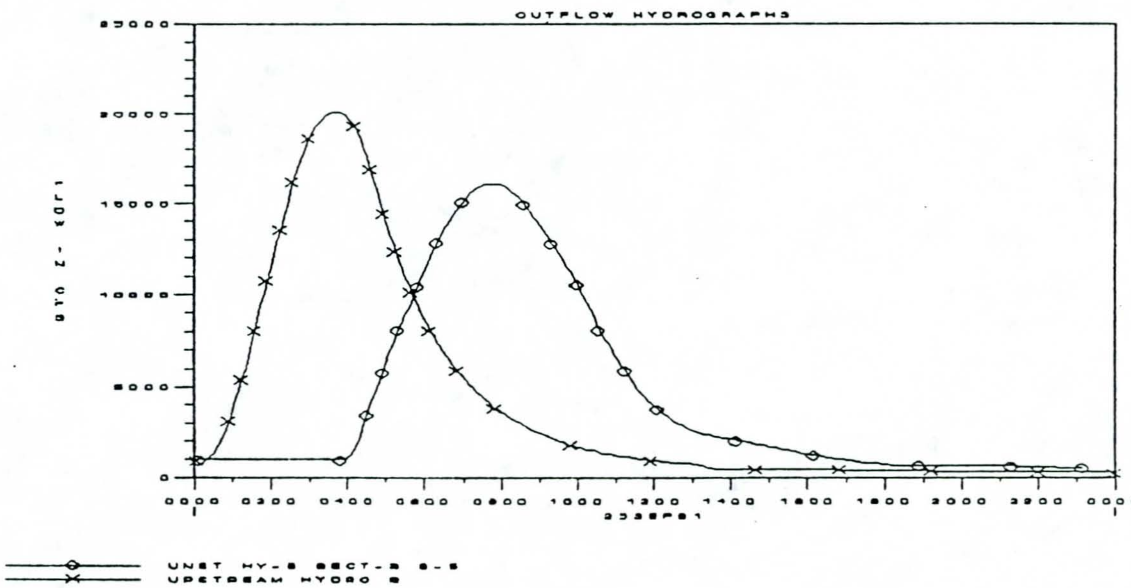


Figure 8
Effects of Flood Plain Storage

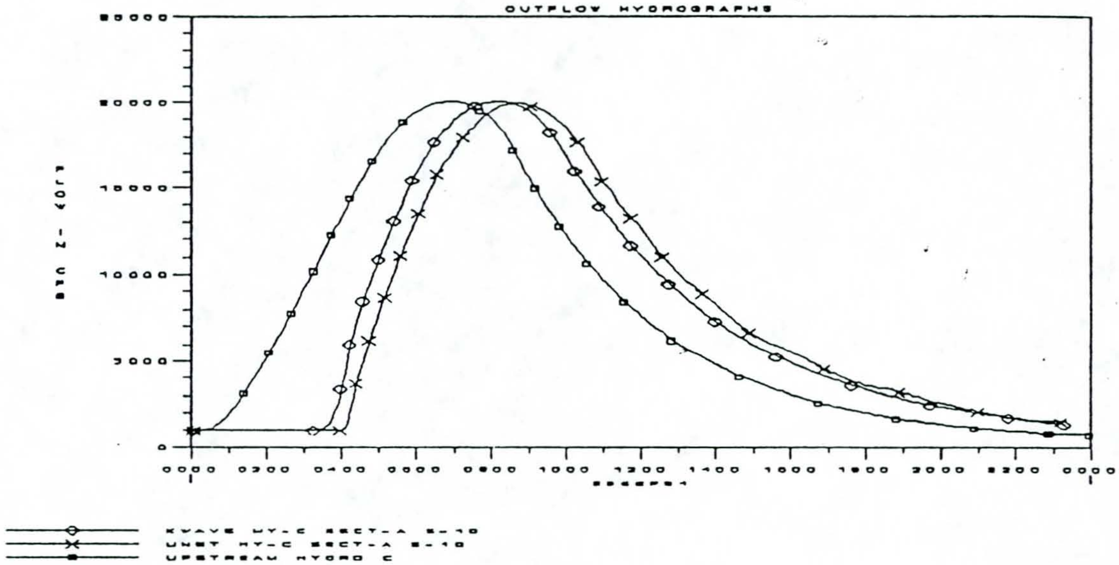


Figure 7
Comparison of Kinematic Wave

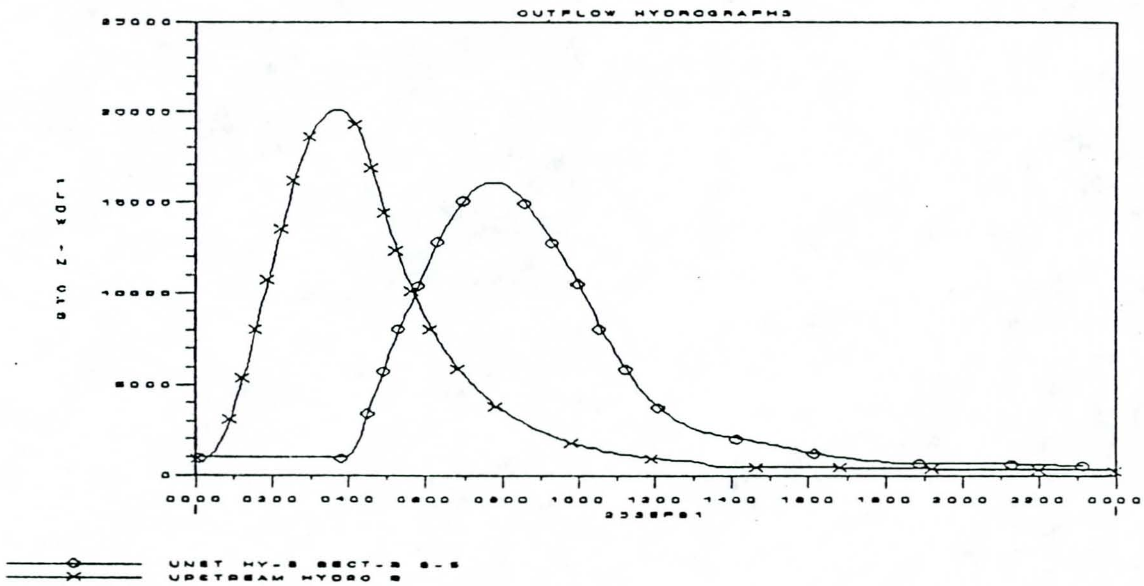


Figure 8
Effects of Flood Plain Storage

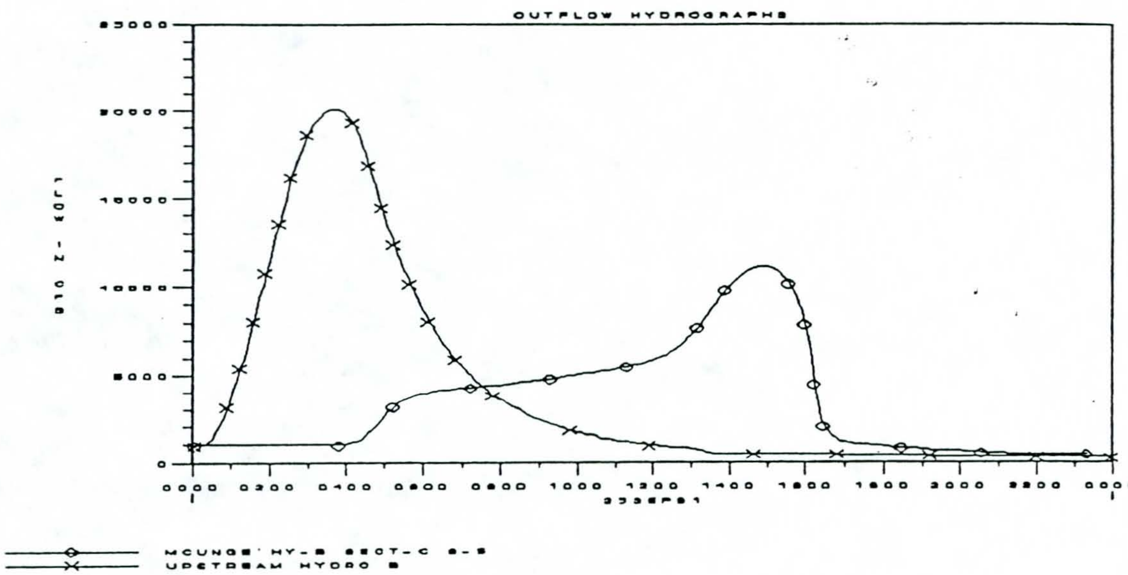


Figure 9
Effects of Flood Plain Storage

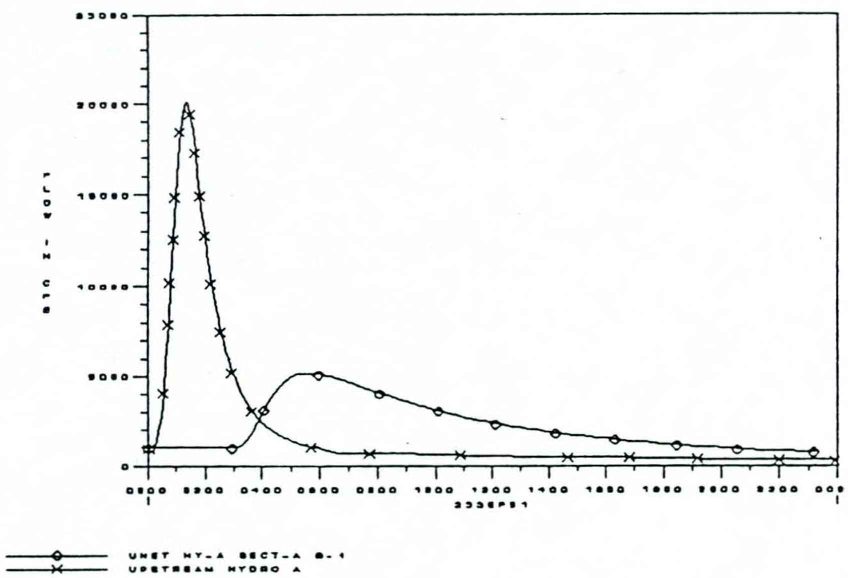


Figure 10
Effects of Time to Peak

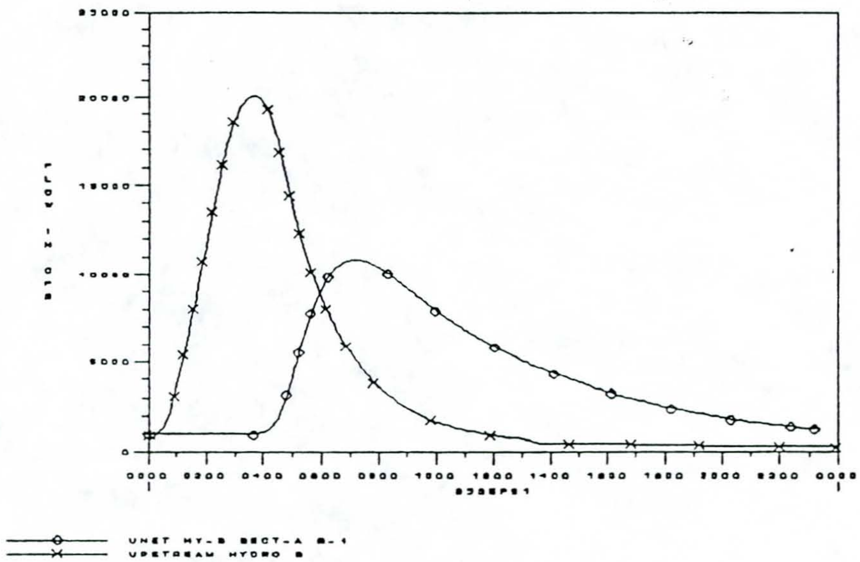


Figure 11
Effects of Time to Peak

B-4. Conclusion

This appendix serves as a supplement to Chapter 9 and as suggested guidance in selecting an appropriate method for routing discharge hydrographs. The general set up of this study has been presented, including the calculation of parameters needed when observed hydrograph data is unavailable. Numerical criteria for Kinematic Wave and Diffusion Wave methods were presented as a tool for predicting the applicability of these methods. It was concluded that the Diffusion Wave criteria is a good indicator of Muskingum-Cunge performance, but the Kinematic Wave criteria is too strict. Flood plain storage and longitudinal slope affect the outflow hydrographs. An increase in flood plain storage will attenuate the hydrograph as will a decrease in longitudinal slope. Lastly discussed were the effects of varying the inflow hydrographs time of rise. A general observation was that the longer the time of rise, the greater the outflow peak flow and the amount of flow to be routed.