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Modeling of rainfall time series and extremes using bounded random cascades and Levy-stable distributions

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Abstract. A new model for simulation of rainfall time series is proposed. It is shown that both the intensity and duration of individual rainfall events can be best modeled by a "fat-tailed" Levy-stable distribution. The temporal downscaling of individual events is produced by a new type of a bounded random cascade model. The proposed rainfall model is shown to successfully reproduce the statistical behavior of individual storms as well as, and in particular, the statistical behavior of annual maxima. In contrast, a model based on a gamma distribution for rainfall intensity substantially underestimates the absolute values of extreme events and does not correctly reproduce their scaling behavior. Similarly, a model based on self-similar random cascade (as opposed to the bounded cascade) substantially overestimates the extreme events.

1. Introduction

Rainfall time series exhibit considerable variability over a hierarchy of timescales: within storm, between storm, seasonal, interannual, interdecadal etc. For many hydrological applications we require models for the synthetic simulation of rainfall time series, explicitly incorporating the variability at all of these different timescales. Examples of such applications include the estimation of long-term water yield, investigation of the effects of climate variability or change on the reliability of water resources, and the estimation of design floods. It is very important, for example, for such rainfall time series models to accurately reproduce the statistical characteristics of individual storms and, at the same time, to correctly reproduce the characteristics of extreme rainfall as well, for example, relating to annual rainfall maxima, so that they are applicable for flood estimation purposes.

In a recent paper [*Menabde et al.*, 1999], we analyzed a data set from Melbourne, Australia, consisting of a 25 year sequence of 6 min rainfall intensities, logged electronically, and used these to construct intensity-duration-frequency (IDF) curves for extreme rainfalls. It was shown for this data set that the annual maximum (mean storm) rainfall intensity I_d over the storm duration d has a simple scaling property:

$$I_d \stackrel{d}{=} (d/D)^{-\eta} I_D. \tag{1}$$

The equality in (1) is understood in the sense of probability distributions and holds for the durations d and D in the range from 30 min to 24 hours. The equality (1) can also be written explicitly in terms of the cumulative distribution function (CDF):

$$F_d(i) = F_D\{(d/D)i\}. \tag{2}$$

A further simplification can be made if we assume [*Menabde et al.*, 1999] that the CDF of extreme events has the standardized functional form:

$$F_d(i) = F\left(\frac{i - \mu_d}{\sigma_d}\right), \tag{3}$$

where the location and scale parameters μ and σ will each have a scale dependence on the duration d:

$$\mu_d = (d/D)^{-\eta}\mu_D \tag{4}$$

$$\sigma_d = (d/D)^{-\eta}\sigma_D. \tag{5}$$

Substituting (3)–(5) into the definition of the return period

$$T = 1/1 - F \tag{6}$$

and inverting (6), a simple IDF relation can be derived as follows [*Menabde et al.*, 1999]:

$$i(D, T) = \frac{\mu + \sigma F^{-1}(1 - 1/T)}{D^{\eta}}, \tag{7}$$

where μ and σ are constants, independent of T.

The above results were obtained by selectively looking at extreme rainfall intensities in the data set. However, it is well known [e.g., *Lovejoy and Schertzer*, 1995; *Koutsoyiannis and Foufoula-Georgiou*, 1993; *Menabde et al.*, 1997a] that individual (i.e., nonextreme) storms also exhibit similar scaling behavior. Various types of models for temporal disaggregation of storm rainfall have been proposed in the literature. Most of these are based on some type of multiplicative random cascade as this type of model produces scaling fields in a simple and natural way.

In this paper we will introduce an improved model for the generation of continuous, multiyear rainfall time series based on a sequence of independent storm events, with randomly generated (1) arrival times, (2) storm durations, and (3) average storm rainfall intensities, the latter being disaggregated further to produce (4) random within-storm rainfall intensity patterns. The model we present is an advance over previous models in two respects: (1) for both storm duration and average storm rainfall intensity the model uses a special type of probability distribution function, called the Levy-stable distribution, with a longer, "fat" tail, and (2) it uses a special form of multiplicative random cascade model, called the bounded random cascade, which has been shown to produce better fits

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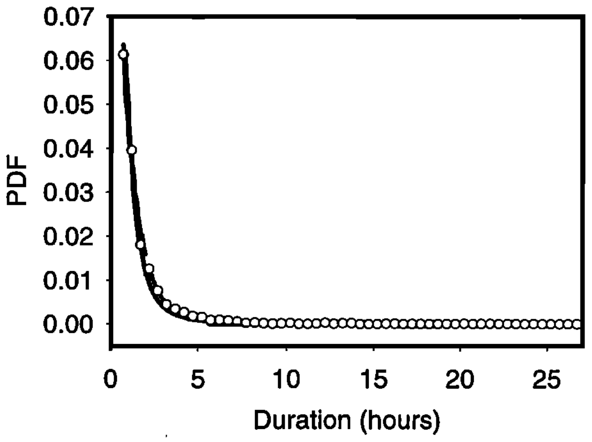

Figure 1. Probability density function (pdf) of storm duration fitted by stable (solid line) and exponential (dashed line) distributions.

to the scaling behavior exhibited by short-term rainfall observations [*Menabde et al.*, 1997a; *Menabde*, 1998]. It will be shown that as a consequence of these improvements, the model is able to accurately reproduce not only the statistical characteristics of individual storms but, more importantly, the properties of annual maximum rainfall (for various durations) as well, as described by (7) above.

2. Modeling of Storm Duration and Intensity: Levy-Stable Distributions

First, we separated the 6 min 25 year rainfall record from Melbourne into storm and interstorm periods using the criterion of a specified minimum dry period. This criterion is admittedly quite arbitrary, and we took it to be 1 hour. The selected data set showed that rainfall is more or less uniformly distributed throughout the year, with very little seasonal variability. This fact simplified our analysis, although our model can be easily extended to situations where strong seasonality is observed.

The common practice, which has considerable justification, is to assume that storm duration can be described by the exponential distribution and storm intensity (or depth) by either the gamma or the lognormal distribution. However, as we will see below, in our case these distributions provide very poor fits to the tail of the empirical distributions; that is, they grossly underestimate the probability of extreme events.

Therefore, in this paper, we propose to model the duration and intensity of individual storms by the so-called Levy-stable distribution [e.g., *Samorodnitsky and Taqqu*, 1994]. In the most general case this distribution is characterized by four parameters: α, the index of stability (or characteristic exponent), in the range (0, 2]; σ, the scale parameter, in the range $(0, \infty)$; β, the skewness parameter, in the range $[-1, 1]$; and μ, the shift parameter, in the range $(-\infty, \infty)$. Different notations with different meanings can be found for these in the literature, which may be quite confusing. We accept here the notations of *Samorodnitsky and Taqqu* [1994] and denote the Levy-stable random variable by $S_\alpha(\sigma, \beta, \mu)$. The scale and shift parameters, σ and μ, have simple interpretations, expressed by the following formula:

$$S_\alpha(\sigma, \beta, \mu) \stackrel{d}{=} \mu + \sigma S_\alpha(1, \beta, 0), \tag{8}$$

whereas the characteristic exponent α describes the behavior of the tail of the distribution.

A general closed-form analytical expression does not exist for the probability density function (pdf) of the Levy-stable distribution, except for some special cases. It can be shown [e.g., *Samorodnitsky and Taqqu*, 1994], however, that the Levy-stable pdf indeed has a hyperbolic (fat) tail, when $x \to \infty$ and $\alpha < 2$, i.e.,

$$p_x(x) \propto x^{-(\alpha+1)}. \tag{9}$$

The case $\alpha = 2$ is degenerate since, in that case, the Levy-stable distribution does not depend on β and, more importantly, it coincides with the normal distribution (thus no longer has a fat tail.). Furthermore, it can be easily inferred from (9) that moments of order equal to or greater than α do not exist; this means that the variance is not defined (except for the special case $\alpha = 2$). This makes many of the standard statistical estimation methods (such as the method of moments) inapplicable to the study of the Levy-stable distribution. For this reason, the parameter estimations reported in this paper have been carried out using the method of maximum likelihood using a software package called STABLE, which can be obtained from the World Wide Web at http://www.cas.american.edu/~mathstat/people/people.html. This software has been indispensable to us for the use of the Levy-stable distribution.

Figure 1 represents the empirical distribution of storm duration, fitted by the Levy-stable distribution using the maximum likelihood method, with the estimated parameters being $\alpha = 1.27$, $\sigma = 4.14$, $\beta = 1$, and $\mu = 15.36$. For a comparison, the empirical distribution was also fitted by an exponential distribution. At the coarse scale used in Figure 1 the exponential distribution appears to be no worse than the Levy-stable distribution. However, at a different finer scale, as shown in Figure 2, the exponential distribution provides a much poorer fit to the tail of the empirical distribution; in fact, for large intensities the discrepancy between the exponential and Levy-stable distributions can be several orders of magnitude.

Similarly, Figure 3 represents the empirical pdf of storm mean intensities, fitted by the Levy-stable distribution, with the parameters $\alpha = 1.46$, $\sigma = 0.64$, $\beta = 1$, and $\mu = 1.90$, again estimated by the maximum likelihood method. For a comparison, the same data were also fitted by the gamma distribution. At the coarse scale used in Figure 3, once again, both the

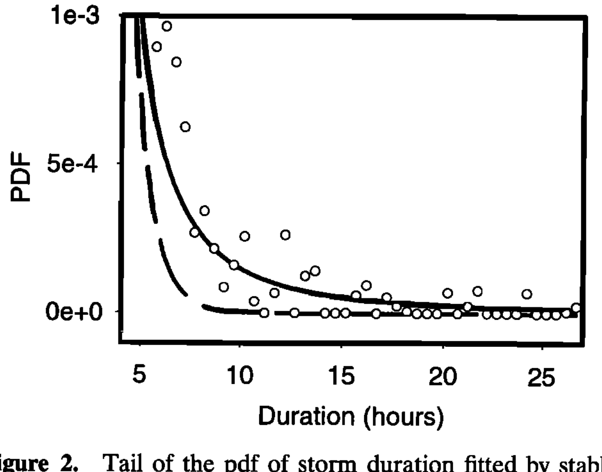

Figure 2. Tail of the pdf of storm duration fitted by stable (solid line) and exponential (dashed line) distributions.

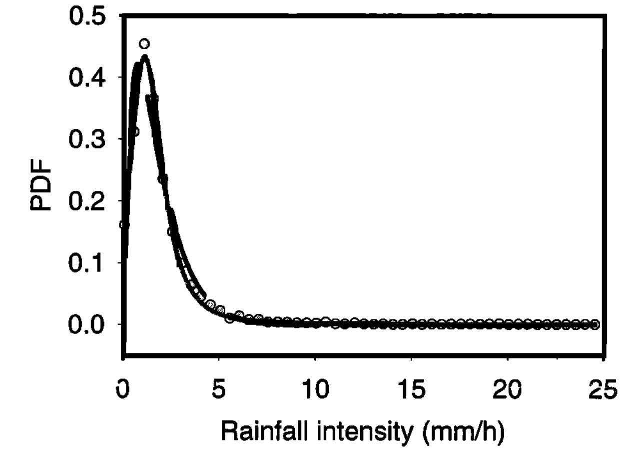

Figure 3. A pdf of storm intensity fitted by stable (solid line) and gamma (dashed line) distributions.

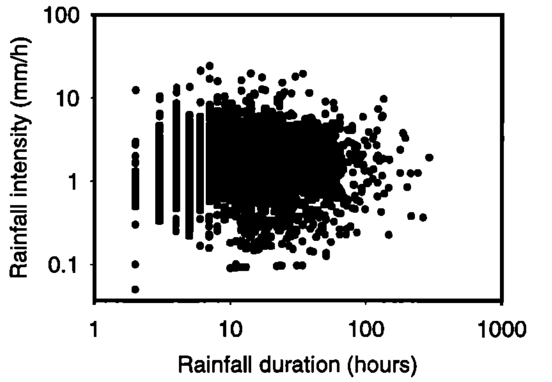

Figure 5. Rainfall intensity as a function of duration.

gamma and Levy-stable distributions appear to be equally good. However, at a finer scale, as shown in Figure 4, the Levy-stable distribution provides a much better fit to the tail of the data.

For this particular Melbourne data set the intensity and duration of individual events can be considered to be statistically independent as they did not exhibit any significant correlation (see Figure 5). Thus the duration and intensity of storm events can be sampled from the above two Levy-stable distributions independently. In addition, utilizing directly the empirical arrival time distribution of individual storms (in our case, without fitting a particular model to this distribution), we can then construct a simulation model of storm arrivals, durations, and average intensities, in a continuous manner and over many years. Details of such rainfall generation will be presented in a later section.

3. Within-Storm Temporal Patterns: Bounded Random Cascade Model

The next important step is to introduce an appropriate model for the temporal downscaling of individual storms, noting that in the model presented above we have only used an average rainfall intensity, constant over the duration. To analyze the statistical properties of within-storm temporal variability, we first calculated the ensemble-averaged power spectrum of all storms with durations longer than 6.4 hours (the 6.4 hours correspond to $2^6 = 64$ data points with 6 min resolution). The resulting power spectrum $P(f)$ is shown in Figure 6, and it is seen to be scaling, i.e.,

$$P(f) \propto f^{-\beta}, \tag{10}$$

with the exponent $\beta = 1.69$.

As we have demonstrated previously [*Menabde et al.*, 1997a, 1997b, *Menabde*, 1998], the fact that the power spectrum exponent β is greater than unity indicates that the rainfall time series belong to a class of (stochastically) self-affine rather than self-similar fields and that they can be conveniently modeled by the bounded random cascade model. To analyze the scaling properties of rainfall more directly for this case, it is convenient to use the technique of breakdown coefficients (BDC), which was developed by *Novikov* [1966, 1969] for application to the theory of turbulence and was adapted for application to rainfall modeling by *Menabde et al.* [1997b] and *Harris et al.* [1998].

The BDC method is applicable to both self-similar and self-affine fields and allows the difference in their scaling behaviors to be easily visualized. The BDC is defined as

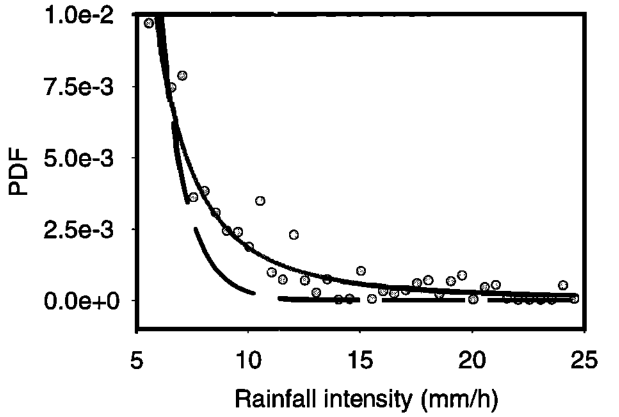

Figure 4. Tail of the pdf of storm intensity fitted by stable (solid line) and gamma (dashed line) distributions.

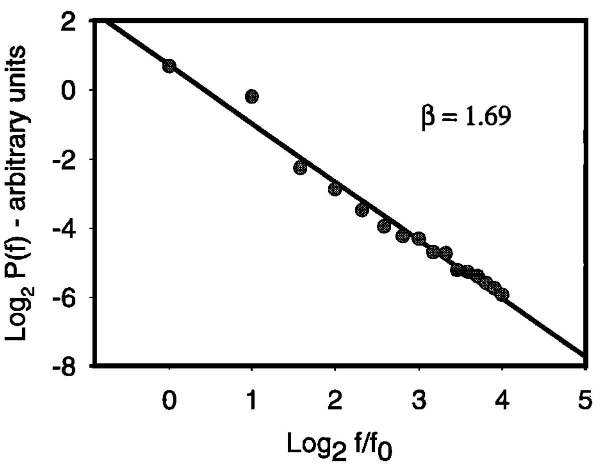

Figure 6. Ensemble-averaged power spectrum of rainfall time series, where $f_0 = 2.6 \times 10^{-5}$ Hz is the lowest frequency.

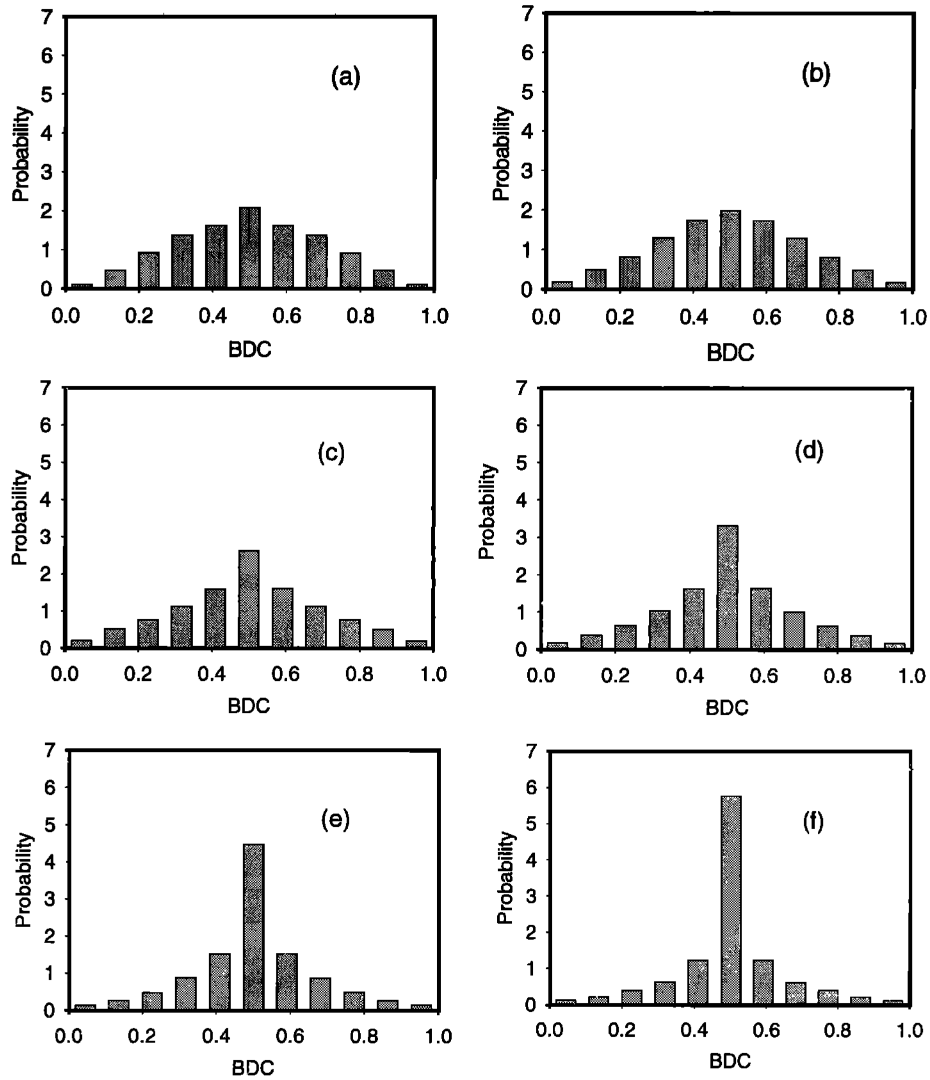

Figure 7. Histograms of BDC for different timescales: (a) $t = 64t_0$, $t_0 = 6$ min, (b) $t = 32t_0$, (c) $t = 16t_0$, (d) $t = 8t_0$, (e) $t = 4t_0$, and (f) $t = 2t_0$.

$$u(\tau, t) = \frac{R_\tau(t_1)}{R_t(t_2)} \qquad \tau < t, \tag{11}$$

where $R_\tau(t_1)$ and $R_t(t_2)$ are the rainfall totals accumulated over the durations τ and t and centered at t_1 and t_2, respectively. The interval τ defined above is assumed to be completely included in the interval t. The BDC constructed in this manner is clearly a random variable (in general, a function of the timescales τ and t and the locations t_1 and t_2) and ranges from a minimum of zero to a maximum of unity; using the available experimental data, we can estimate its pdf and the dependence of this pdf on the timescales durations τ and t. For self-similar random fields the pdf of $u(\tau, t)$ will depend only on the scale ratio τ/t and not on either of the two scales τ and t separately. However, previous analyses of many rainfall time series suggest that there is, indeed, a scale dependence; for example, the BDCs obtained from observed rainfall time series have been shown to exhibit a decreasing variance with decreasing timescale [*Menabde et al.*, 1997b; *Harris et al.*, 1998].

We carried out the above analyses with the Melbourne data and estimated the empirical pdfs of the BDCs for $t_n = 2^n t_0$, $\tau_n = t_n/2$, with $n = 1, 2, \ldots, 6$ and $t_0 = 6$ min. The estimated histograms of the BDCs are shown in Figure 7, noting again that the scales τ and t have been chosen that $\tau = t/2$ at each level of discretization n. We also estimated the cumulative distribution function (CDF) of BDCs for the same timescales. The results are shown in Figure 8. It can be seen that the variance of the BDCs is clearly decreasing with the decreasing timescale. All the pdfs, obtained at different timescales, were then fitted to the following symmetric (single parameter) beta distribution:

$$p_U(u) = \frac{1}{B(a)} u^{a-1}(1-u)^{a-1}, \tag{12}$$

where the beta function $B(a)$ is given by $B(a) = \int_0^1 x^{a-1}(1-x)^{a-1}\,dx$, with the sole parameter a changing with the timescale of observation t. The dependence of the parameter a on the timescale t can be parameterized by the following scaling law:

$$a(t) = a_0 t^{-H}, \tag{13}$$

which is shown in Figure 9, where $t = 2^n t_0$, $n = 1, 2, \ldots, 6$, and H and a_0 are constants estimated from data. Our choice of

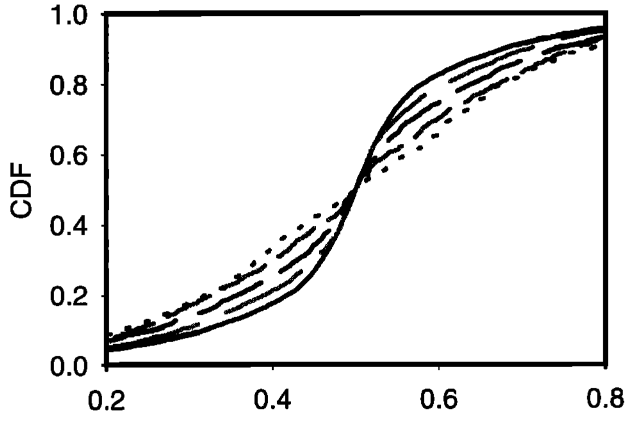

Figure 8. Cumulative distribution function (CDF) of breakdown coefficients (BDC) for the same timescales as in Figure 7.

modeling distribution is due to some nice analytical properties of the beta distribution, which will be demonstrated below and which make it very convenient for building a microcanonical random cascade.

This parameterization provides a natural way for the modeling of temporal disaggregation of individual storms by the bounded random cascade model. Indeed, if we can build a dyadic random cascade, with multiplicative factors on each step behaving as beta random variables with parameters given by (12) and (13), we can then expect such a model to correctly reproduce the observed rainfall (in a statistical sense). Such a model is introduced and tested in section 4. Also note that the model simplifies to the normal self-similar random cascade model if the exponent H in (13) is taken to be zero, allowing a to remain invariant with scale.

4. Synthetic Generation Rainfall Time Series

Simulation consists of the following steps. On the first step, we generate a rectangular pulse of rainfall corresponding to an individual event, with the duration and intensity each being generated by the Levy-stable random generators, with parameters estimated from the experimental data (Figures 1 and 3).

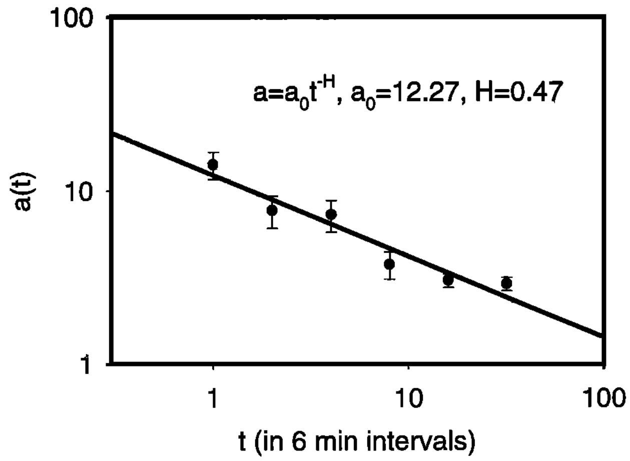

Figure 9. The modeling distribution parameter dependence on scale.

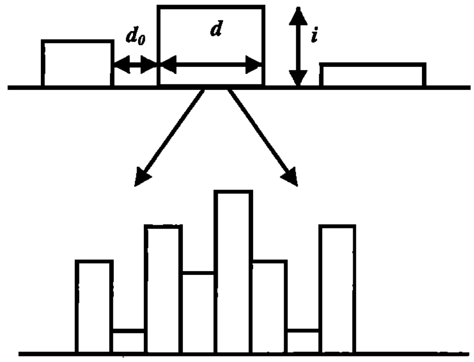

Figure 10. Schematic of the rainfall model.

We assume, following on from Figure 5, that there is no correlation between the intensity and duration of individual storms, although this kind of dependence can be easily incorporated into the present model if and when required.

For continuous modeling of rain storms, we need to specify the arrival times of each storm. In this paper, instead of arrival times, we sample the interstorm (dry) period between storms, denoted by d_0 (also see Figure 10), directly from the empirical distribution estimated from data, as shown in Figure 11. For the Melbourne data, neither the interarrival times nor the interstorm periods fitted any of the commonly assumed distributions [*Eagleson*, 1978; *Robinson and Sivapalan*, 1997], such as the exponential distribution. The choice of appropriate general distributions for these interarrival times or interstorm periods is left for further research and is not critical to the aims of this paper. Our use of interstorm (dry) periods (as opposed to interarrival times) has helped us to avoid the difficulty of defining a minimum dry period and the problem of possible overlap between individual storms (such as when the simulated storm duration is larger than simulated interarrival time). Also, as reported before, we assume that there is very little seasonal variability, which is sufficient for Melbourne. Once again, if and when required, any observed seasonal variability can be incorporated into the model in the same way as done by *Robinson and Sivapalan* [1997].

During the third step of simulation the rainfall depth for each storm is downscaled to a resolution of 6 min, as in the observed data, by the bounded random cascade model described before. The cascade model usually works as follows: we

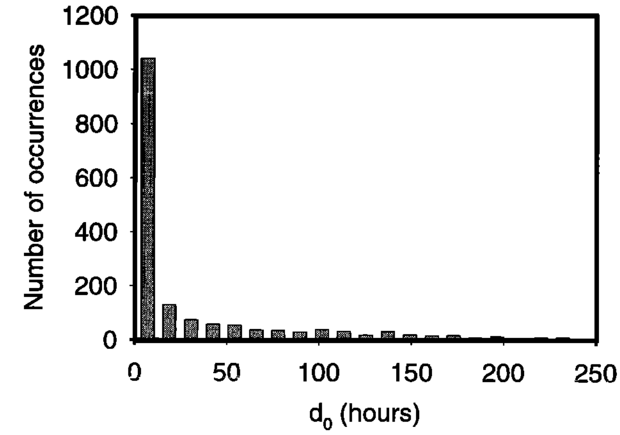

Figure 11. Histogram of dry periods between storms.

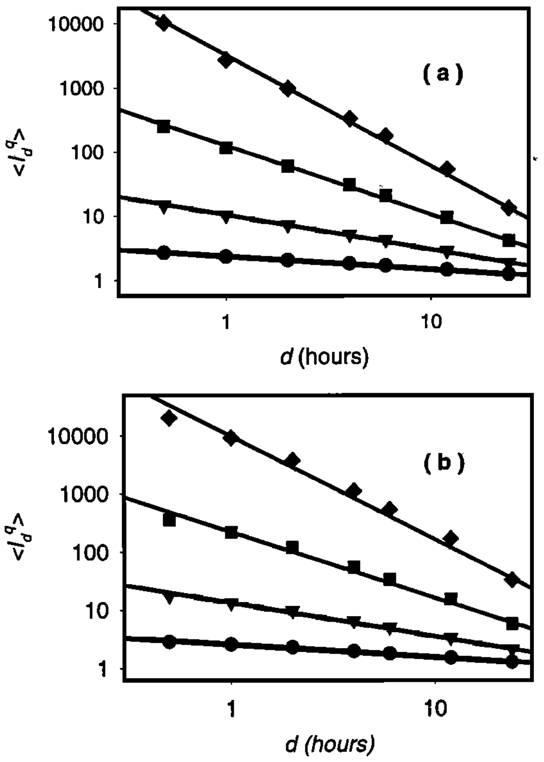

Figure 12. Scaling of the moments for the (a) real and (b) simulated data (circles, $q = 0.5$; triangles, $q = 1.0$; squares, $q = 2.0$; diamonds $q = 3.0$).

start with a homogeneous distribution of the rainfall field R over some time interval t, divide the initial interval into two halves, and assign to each of them the value $R_0 = Rw_1(0)$ and $R_1 = Rw_1(1)$, where $w_1(j)$ are different realizations of a random variable W_1 produced by some chosen generator with the probability density function $g_1(w)$. This procedure is repeated N times with the generator $g_n(w)$ at the nth step explicitly dependent on the number of steps leading to the discrete random field:

$$R_N(j_1, j_2, \ldots, j_n) = Rw_1(j_1)w_2(j_1, j_2) \ldots w_N(j_1, j_2, \ldots, j_N), \tag{14}$$

where R_N represents the piecewise constant function on the 2^N intervals of length $t_N = 2^{-N}t$ and the set of binary indexes j_1, $j_2, \cdots, j_N$ indicate 2^N different possible realizations of the random field. The same procedure can be used to generate self-similar fields, except that in this case, the random number generators remain invariant with the timescale or the step n. For the dyadic bounded random cascade model presented here the weights at any level n, $w_n(j_1, \cdots, j_{n-1})$ are, indeed, equivalent to the breakdown coefficients $u(\tau, t)$ with $\tau = t/2$ always and drawn from the beta distribution (12) with its sole parameter a estimated from the relationship (13).

There are two main types of cascade models, canonical, in which the random weights are conserved in a statistical sense only, i.e., $\langle w_n \rangle = 1$, and microcanonical, in which the weights explicitly satisfy the condition

$$w_n(j_1, \ldots, j_{n-1}, 0) + w_n(j_1, \ldots, j_{n-1}, 1) = 1 \tag{15}$$

on each step of the cascade. Canonical cascades are easier to construct as there are no restrictions on the type of random generator, except for the trivial normalization to unity, but it is more difficult to estimate their parameters. In the microcanonical case, the parameters of the distribution can be estimated directly from the pdfs of the BDCs obtained from the empirical data; the problem is to simulate a random variable satisfying condition (15) and having a closed-form analytical expression for pdf.

In this paper, we have overcome this problem by utilizing an interesting relationship between the beta and gamma distributions. It is well known that the ratio of two gamma-distributed random variables is distributed according to the beta distribution [*Mood et al.*, 1974]. Thus it can be shown that if X_1 and X_2 are two independent random variables belonging to the single-parameter gamma pdf,

$$p_x(x) = \frac{1}{\Gamma(a)} x^{a-1} \exp(-x), \tag{16}$$

where $\Gamma(a)$ is the gamma function, then $Y_1 = X_1/(X_1 + X_2)$, and $Y_2 = X_2/(X_1 + X_2)$ will be random variables distributed according to the beta pdf given by (12), with an identical parameter a [*Mood et al.*, 1974]; more importantly, Y_1 and Y_2 also automatically satisfy the condition (15). Thus the procedure to generate the weights $w_n(j, 0)$ and $w_n(j, 1)$ at each cascade level is simply to generate two gamma-distributed numbers x_1 and x_2 from (16) and then form the ratios $w_n(j, 0) = x_1/(x_1 + x_2)$ and $w_n(j, 1) = x_2/(x_1 + x_2)$. Of course, the parameter a will change as we cascade down the timescale, with the scale dependence of a given by the empirical formula (13).

Using the procedure described above, we simulated 500 years of rainfall time series with a 6 min resolution. We identified annual rainfall maxima I_d within each year for different durations (0.5–24 hours). The ensemble-averaged qth moments of the random variable I_d, denoted by $\langle I_d^q \rangle = E[I_d^q]$, are shown in Figure 12 together with the corresponding moments estimated from observed data. As we can see, the simulated time series reproduce the observed moments of extreme events and their scaling behavior very well indeed.

For a start, we assumed a general multiscaling relationship for the moments $\langle I_d^q \rangle$ with duration d of the following form,

$$\langle I_d^q \rangle \propto d^{-K(q)}, \tag{17}$$

and estimated the scaling exponents $K(q)$ from both the observed and simulated rainfall time series. Figure 13 presents the estimated scaling exponents $K(q)$ as a function of the exponent q. Both show linear behavior, with a scaling exponent $\eta = 0.651$ for the observed data and $\eta = 0.64$ for the simulated time series, indicating a simple scaling model more than adequately fits the data. Note that the exponent η (in the case of simple scaling) is the chief characteristic of the scaling behavior of annual rainfall maxima for the Melbourne data, and the model clearly reproduces it very well.

For a comparison, we simulated 500 years of rainfall time series using the same procedure and parameters as above, except that the storm intensities were this time drawn from a gamma distribution, with parameters again estimated by the maximum likelihood method, as shown in Figure 3. Everything else is exactly as before. Figure 14 represents the return period T (in years) of extreme events of 12 hour duration, estimated from both the observed and simulated rainfall time series. In this case we chose the larger duration of 12 hours since, at this level, any discrepancy between the observed and simulated time series is almost exclusively due to the distribution function of intensities and not at all due to any deficiency of the temporal disaggregation model. It is clear that for $T \geq 10$ years the model based on the gamma distribution underestimates the extreme rainfall intensity by as much as 50%, whereas the model based on the Levy-stable distribution provides a very good fit for all return periods. For the sake of brevity, we have presented this demonstration for just one duration (12 hours), but the tail of the distribution has an effect on the extremes for other durations as well.

We also carried out simulations of rainfall time series based on the use of a self-similar random cascade (as opposed to bounded) for the temporal (within storm) disaggregation of individual storms. This random cascade was constructed in the same way as the bounded one, except that for the generator (12) the parameter a was taken to be independent of scale and equal to the value we used previously on the first step of the bounded cascade. Figure 15 presents the dependence of the return period T of extreme events of 30 min duration. It can be seen that the self-similar cascade model substantially overestimates the intensities of extreme events for this duration. This is not surprising since it can be intuitively expected that the discrepancy between the bounded and self-similar cascades will increase with decreasing timescale; the two models should, on the other hand, produce identical results for large duration storms, provided we use probability distribution functions with a fat tail.

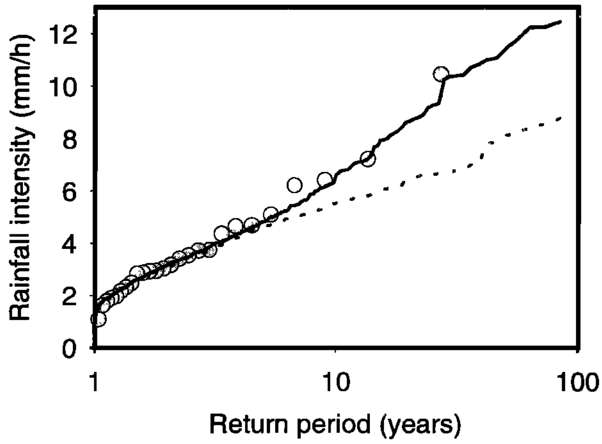

Figure 14. Return period for real data (circles), simulation based on stable model (solid line), and simulation based on gamma model (dashed line).

5. Discussion and Conclusions

We have addressed here two main questions about the modeling of rainfall time series: (1) how to model the duration and intensity of individual storms and (2) how to downscale the given storm to a desired resolution. We have argued that the empirically derived pdfs of both storm duration and intensity have hyperbolic tails, which cannot be captured by the exponential and gamma distributions, respectively, widely used in common practice. We showed that much better fits can be obtained by the Levy-stable distribution in both cases.

To derive an appropriate model for the downscaling of individual storms to simulate realistic within-storm patterns, we have proposed the use of the method of breakdown coefficients, which help us to easily visualize the scale dependence and self-similarity of observed rainfall intensities. On the basis of this analysis we introduced a new model for the temporal downscaling of rainfall. This model is a modified, microcanonical version of the bounded random cascade model, which was used for rainfall modeling in our previous work [*Menabde et al.*, 1997a; *Menabde*, 1998].

We simulated 500 years of rainfall time series with 6 min resolution, and the results were compared to the observed 25 year time series of rainfall with the same resolution. The new model has been shown to reproduce the observed scaling be-

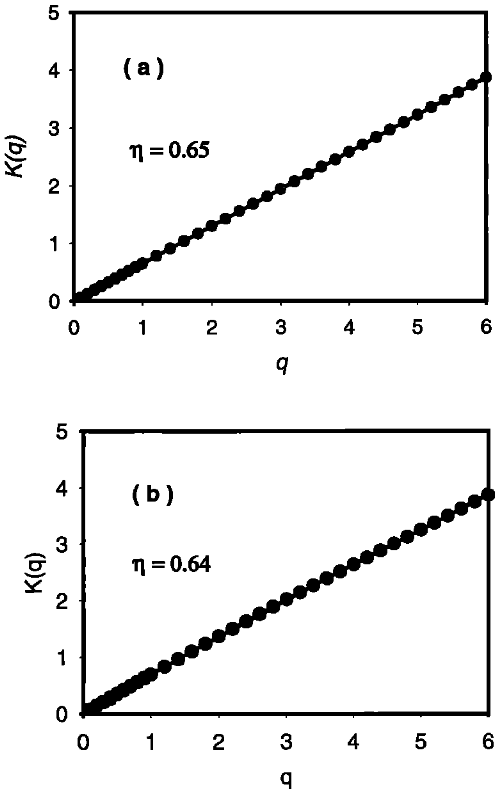

Figure 13. Scaling exponent for (a) real and (b) simulated data.

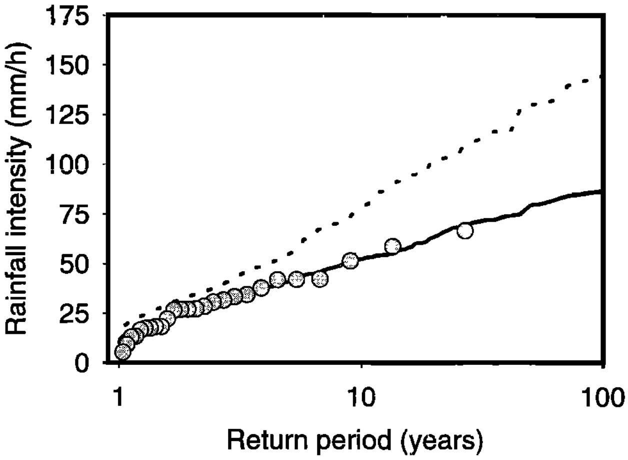

Figure 15. Return period for real data (circles), simulation based on the bounded cascade (solid line), and simulation based on the self-similar cascade (dashed line).

havior of extreme events very well, providing a much better fit to the observed annual maxima than traditional models.

The proposed model presents two major advantages to the analysis and modeling of rainfall time series: (1) while the model reproduces the extreme value statistics very well, the parameters of the model are and can be estimated from the parent (population) time series; thus the method is applicable even in regions with limited data to construct robust extreme value distributions, such as the IDF curves. (2) While the model presented in this paper was based on 6 min data, the model can be extended to other gauging locations in the neighborhood, with perhaps coarser-level rainfall gauges (e.g., hourly gauges), and to use these to construct the IDF curves since we can continue to utilize the scaling behavior exhibited in the fine-scale observational record. Future extensions could include naturally allowing for seasonal, interannual, and interdecadal variability in the observed rainfall record. More appropriate models of the interarrival times or the interstorm (dry) periods would also be useful for more compact and regionalizable models of rainfall time series.

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