Module 1, B-set 4: Frequency and Time

Not just for communications anymore!

November 3, 2016

Overview

This module has officially been about communication systems, but in fact, we've been dealing with ideas that are extremely general (as we discussed in class). This BSet is meant to wrap these ideas up a little bit by applying them to radically different systems than you've thought about in building the modem. The systems we'll look at are ones that you've already had some exposure to back in ModSim: a thermal system and a mechanical system. For both systems, we'll ask you both to do a bit of "modsim" style analysis (stock and flow/FBD, differential equation, maybe some `ode45`, and then we'll connect that stuff to the time/frequency ideas we've been dealing with during this module.

Learning Goals

By the end of this building block, you should be able to...

- Apply the FFT to different kind of data and interpret the results;
- Determine the transfer function of simple differential equation systems;
- Relate the behavior of the transfer function to the system's behavior in time; and
- Interpret the transfer function in order to suggest design improvements.

WARNING: You should look at this BSet TODAY so that you can get help on Friday.

By the time you're 4 or 5 hours into this, you should feel confident that you can complete the B-set in another 4-5 hours. If not, this is when you should **ask for help.** This means **talk to a colleague**, or **talk to a ninja**, or **track down an instructor**, or **send an email to an instructor**.

Solar House Design (Estimated Time 3.5hrs)

Back in ModSim you did a project relating to thermal systems. One common example we used was the solar house. We abstracted the house to a thermal mass, and imagined that the primary mechanisms by which the temperature inside the house might change included thermal conduction to/from the environment, and thermal gain from sun shining in the windows.

In this problem we'd like to resurrect some of these ideas, and extend them to connect to the ideas we've been working with in this module. In particular, we're interested in thinking about the house using *the signals and system abstraction*, in exploiting the idea of *working in the frequency domain* to gain some insight about the situation, and in trying to add control to the problem.

1. Developing a Time-Domain Model

   (a) Create a *stock and flow* diagram for the energy in the house. Your diagram should include flows due to solar energy and thermal conduction through the walls; it should include a stock of thermal energy in the house.

   (b) Let's define some notation and use it to come up with *differential equations*. Let C_H be the heat capacity of the house (J/Kelvin), and k_H be the net thermal conductivity of the walls (Watts/Kelvin). Thus the energy stored in the house (measured from U=0 at T=0C) is

   $$U = C_H T_H$$

   where T_H is the interior temperature.

   The power flow to the environment due to thermal conduction through the walls and roof is

   $$P_E = k_H(T_H - T_O)$$

   where T_H is the interior temperature and $T_O(t)$ is the exterior temperature. Finally, let $S(t)$ be a function of time that represents the solar power that enters the house.

   Given this notation, develop a differential equation that describes how the energy stored in the house, $U(t)$, varies in time.

   (c) Now we've represented this system using the stock-and-flow abstraction, and the differential equation abstraction. Can you also represent it using the sig-sys abstraction? What would you think of as "signals" here? What would you think of as the "system"? *Draw both a high-level conceptual block diagram to clarify how you are thinking about it, and a detailed block diagram that shows integrators/differentiators, adders, etc.*

2. Developing a Frequency Domain Model

   (a) Let's start with some data about the sun and the temperature in Boston. Load the provided file, `BostonTempData.mat`.

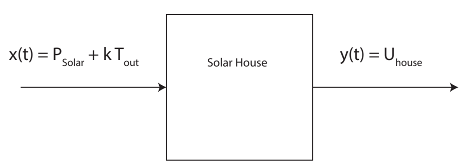
This workspace contains one year of data, taken every hour, at Boston Logan airport. The variables in the file are `hour`, which is simply a count of hours from the beginning of the year, `temp`, the measured temperature in Celsius, and `dni`, the direct normal incident solar power in Watts per meter squared.

Make a plot of both quantities versus time. Do they make sense at the yearly level? At the monthly level? At the daily level?

(b) Make a prediction. What do you expect the DFT of the temperature data to look like? What about the incident solar power? *Make a sketch of what you predict these graphs will look like.*

(c) Now take the FFT of both temperature and normally incident power (dni), and make plots of both versus frequency. Please include units, particularly on the frequency axis (no, you may not label the axis from $-\pi$ to π). Explain what you find – Were you right? Why are there more "spikes" in the dni data as opposed to the temperature data?

(d) Now let's think about the system in the frequency domain. Here's one way to frame the system in the sig-sys abstraction:



We'll call the interior energy the output signal $y(t)$, and the sum of the solar power and the exterior temperature times the thermal conductivity of the house the input signal $x(t)$.

Re-write the your differential equation in terms of x and y, and then use it and the CTFT to determine the *transfer function* $H(\omega)$ of the house. If you're totally lost here, look back at the RC circuit from BSet 2!

(e) Using your transfer function, predict what the temperature inside a passive solar house will be as a function of time over one year in Boston. Here are some numbers that you should use (if we were more cruel, we'd ask you to justify them; as it is we'll just take them as given...)

$$C_H = 10^7 J/K$$

$$k_H = 1500W/K$$

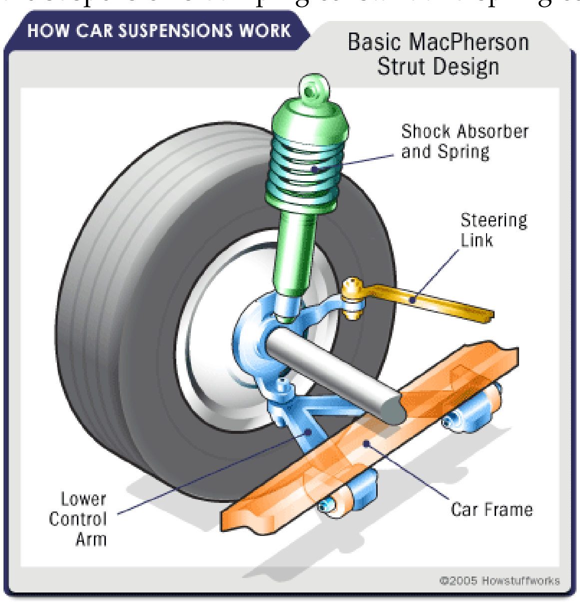
The following plots will be helpful:

   i. A plot of the input function versus time
   ii. A plot of the fft of the input function (versus frequency)
   iii. A plot of the transfer function versus frequency
   iv. A plot of the output fft versus frequency
   v. A plot of the interior temperature versus time.

(f) Based on your analysis, propose and justify an improvement to the solar house design.

Suspension Design (4.5 hours)

The basic idea of a car suspension is shown in the figure below: the wheels are attached to the chassis through a shock absorber and spring. In this question, we're going to ask you to choose appropriate values for the suspension's damping constant and spring constant.



3. Working in the time domain

   (a) There are obviously many options for how one might abstract a car and suspension: for example, you might think about the car as a rigid three-dimensional body attached to the four wheels via four springs and dampers, and then think of the four wheels as point masses attached to the ground via additional springs and dampers that represent the inflated tires. Or you might choose to think about the car's chassis as a single point mass, attached to the ground by a single spring and damper that represent the entire suspension system.

   The latter is a lot easier, so let's start with that abstraction...
   Using a point mass abstraction for the car chasssis, develop a free-

body diagram for the car at an instant when the height of the road is $r(t)$ (measured from sea level) and the height of the car's chassis is $x(t)$ (measured from the car's rest position when $r(t) = 0$).

(b) Using good old $F = ma$, develop a *differential equation* that relates the height of the car to the height of the road. Employ conventional models for springs and dampers:

$$F_s = -k(l - l_0)$$

$$F_d = -\beta \frac{dl}{dt}$$

where k is the spring constant, l is the length of the shock, l_0 is the rest length of the shock, and β is the damping constant. Be careful to think about the definition of zero for $x(t)$: it's defined in such a way that a lot of extra junk cancels out!

(c) Now re-write your model using block diagram formalism, at the level of integrators, differentiators, etc. Use $r(t)$ as your input and $x(t)$ as your output.

(d) Using `ode45`, simulate the vertical motion of the car (i.e., $x(t)$) over a road that is sinusoidally bumpy:

$$r(t) = A \cos 2\pi v t / L$$

where L is the period of the bumps on the road and v is the forward velocity of the car. For the sake of argument, let $A =$ 0.1 m, $L = 10$ m, $v = 10$ m/s, $k = 10^5$ N/m, $\beta = 10^3$ N-s/m, and $M = 10^3$ kg.

(e) Now explore the behavior of the car as you change v. Make plots that compare r to x as a function of time for different speeds. Particularly focus on speeds around between 5 and 30 m/sec (note that this is between about 10 mph and 60 mph). What do you observe? Can you give a physical interpretation of what you see?

4. Working in the frequency domain

(a) Let's think about the transfer function $H(\omega)$ of the car suspension for following definitions of the system:

i. Let's consider the input to be road position (i.e., $r(t)$), output of absolute car position (i.e.,$x(t)$). In this case, I claim that

$$H(\omega) = \frac{k + j\omega\beta}{k + j\omega\beta - m\omega^2}$$

Am I right? Why or why not?

ii. Find the transfer function for an input of road position (i.e., $r(t)$) and an output of car height above the road (i.e.,$x(t) - r(t)$).

iii. By looking at the expressions, interpret the two transfer functions. How will the system behave at slow speed? High speeds? Why? How does the transfer function in case (ii) differ from the transfer function for case (i)? Why?

(b) Now explore the behavior of the transfer function for different β values. Make a plot of the transfer function versus frequency for several different values of β: ranging from "small" to "large". What makes a value of β small or large mathematically?

(c) Compare simulations with ode45 with your transfer function plots. Do they tell you the same thing?

(d) What does each of these plots tell you about how the suspension will perform? What do they tell you about how to drive?

(e) Suggest an improvement to the suspension design based on the transfer function. In other words, how would you change β and/or k to get more desirable behavior? Show that your suggestion is an improvement.

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