QEA
PSET 1
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1

a

10,000 Hz $\Rightarrow$ 10 $\frac{\text{samples}}{\text{cycle}}$

5,000 Hz $\rightarrow$ 5 $\frac{\text{samples}}{\text{cycle}}$

b Increase sampling period:

- Less samples
- Lower resolution representation

c ~~We only s~~

i We sample at a much slower rate

ii 2000 Hz $\rightarrow$ 2 samples per cycle
1000 Hz $\rightarrow$ 1 sample per cycle

iii All samples have magnitude 0.

d 2000 Hz $\rightarrow$ samples at peaks and valleys
1000 Hz $\rightarrow$ samples at valleys only

(2)

(a) Both setups produce a CT sound of 440 Hz. This is because our sampling rate for a 2560 Hz waveform ~~[illegible]~~ is the same as a 440 Hz signal sampled at 3000 Hz.

(b) I would expect to hear 440 Hz and 2560 Hz. The ~~high saw~~ faster sampling rate should be able to capture both frequencies.

(3)

~~wb~~ ~~has a very few~~

wb has equalish ~~amplitudes~~ heights for a very wide band of frequencies

lf has high amplitudes for only low frequencies

~~wb~~ ~~is~~

wb sounds higher pitch than lf

(4) $Ae^{j\theta} \cdot Be^{j\phi} = ABe^{j(\theta+\phi)} = AB\cos(\theta+\phi) + jAB\sin(\theta+\phi)$

$|e^{j\theta}| = 1$

$e^{j\pi} = -1 + j0$

$e^{j2\pi} = 1 + j0$

5

$$\begin{pmatrix}2\\3\end{pmatrix} \qquad c_1 = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} \qquad c_2 = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix}$$

$$q = (c_1^T q)\, c_1 + (c_2^T q)\, c_2$$

$$\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}2\\3\end{pmatrix} c_1 + \begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}2\\3\end{pmatrix} c_2$$

$$\begin{pmatrix}2\\3\end{pmatrix} = \left(\frac{3}{\sqrt{2}} + \sqrt{2}\right) c_1 + \left(-\frac{3}{\sqrt{2}} + \sqrt{2}\right) c_2$$

$$\begin{pmatrix}2\\3\end{pmatrix} = \left(\frac{5}{\sqrt{2}}\right) c_1 + \left(-\frac{1}{\sqrt{2}}\right) c_2$$

$$\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$$

⑥ $N = 8, \; i = 0, \; k = 1$

$V_i^H \cdot V_k = 0$

Mathematica

⑦ Geometric summation formula

$$\sum_{k=0}^{N-1} (ar^k) = a\left(\frac{1-r^n}{1-r}\right)$$

$$\frac{1}{N}\sum_{l=0}^{N-1}\left(e^{j2\pi\frac{k-i}{N}}\right)^{l}$$

$$\frac{1}{N}\left(\frac{1-\left(e^{j2\pi\frac{k-i}{N}}\right)^{N}}{1-\left(e^{j2\pi\frac{k-i}{N}}\right)}\right)$$

$$\frac{1}{N}\left(\frac{1-e^{j2\pi(k-i)}}{1-e^{\left(j2\pi\frac{k-i}{N}\right)}}\right)$$

$$\frac{1}{N}\left(\frac{1-1}{1-\text{not }1}\right)$$

$$\frac{1}{N}(0) = 0$$

8

$$V_i = V_k$$

$$|V_i| = 1$$

$$|V_k| = 1$$

& $V_i \cdot V_k = |V_i||V_k| \cos(0)$

$$V_i \cdot V_k = 1$$

9

$$x = a_0 v_0 + a_1 v_1 + \dots + a_n v_n$$

$$v_i^H x = v_i^H (a_0 v_0 + a_1 v_1 + \dots + a_n v_n)$$

$$v_i^H x = a_i v_i^H v_i$$

$$v_i^H x = a_i$$

$$x = (v_0^H x) v_0 + (v_1^H x) v_1 + (v_2^H x) v_2 + \dots + (v_{N-1}^H x) v_{N-1}$$

(10) $X = \begin{pmatrix} \cos(\frac{\pi}{4}\cdot 0) \\ \cos(\frac{\pi}{4}\cdot 1) \\ \vdots \\ \cos(\frac{\pi}{4}\cdot 63) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}e^{j\frac{\pi}{4}\cdot 0} + \frac{1}{2}e^{-j\frac{\pi}{4}\cdot 0} \\ \frac{1}{2}e^{j\frac{\pi}{4}\cdot 1} + \frac{1}{2}e^{j\frac{\pi}{4}\cdot 1} \\ \vdots \\ \frac{1}{2}e^{j\frac{\pi}{4}\cdot 63} + \frac{1}{2}e^{-j\frac{\pi}{4}\cdot 63} \end{pmatrix}$

$$X = \underbrace{\frac{1}{2}\begin{pmatrix} e^{j\frac{\pi}{4}\cdot 0} \\ e^{j\frac{\pi}{4}\cdot 1} \\ \vdots \\ e^{j\frac{\pi}{4}\cdot 63} \end{pmatrix}} + \frac{1}{2}\begin{pmatrix} e^{-j\frac{\pi}{4}\cdot 0} \\ e^{-j\frac{\pi}{4}\cdot 1} \\ \vdots \\ e^{-j\frac{\pi}{4}\cdot 63} \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix} \vdots \\ e^{j\frac{2\pi}{64}\cdot 8\cdot 1} \\ e^{j\frac{2\pi}{64}\cdot 8\cdot 2} \\ \vdots \\ e^{j\frac{2\pi}{64}\cdot 8\cdot 63} \end{pmatrix}$$ if $i=8$

$i=56$

$4V_{56} = \begin{pmatrix} \vdots \\ e^{j\frac{2\pi}{4}\cdot 1} \\ \vdots \\ e^{j\frac{2\pi}{4}\cdot 63\cdot 56} \end{pmatrix}$

$4V_8$ $\qquad$ $4V_{56}$

$$Wx = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 4 \\ \vdots \\ 4 \\ \vdots \\ 0 \end{pmatrix}$$

8+1 = 9th entry

56+1 = 57th entry

(11) 64 point DFT of vector x

$$x_n = \left(\frac{\pi}{4}(n-1)\right)$$

$$x = \cos\left(pi/4 \cdot [0:1:63]\right)$$

Mathematica

(12) $X_{i+N} = X_i$

(a)

$$X_i = \frac{1}{\sqrt{N}} \sum_{k=0}^{N} x[k] e^{-j\frac{2\pi}{N} ik} \qquad (21)$$

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N} x[k] e^{j\frac{2\pi}{N} nk} \qquad (22)$$

$$X_i = \frac{1}{\sqrt{N}} \sum_{k=0}^{N} x[k] e^{-j\frac{2\pi}{N}(i+N)k}$$

$$= \frac{1}{\sqrt{N}} \sum_{k=0}^{N} x[k] e^{-j\frac{2\pi}{N}ki} \quad e^{-j\frac{2\pi}{N}kN}$$

$$= \frac{1}{\sqrt{N}} \sum_{k=0}^{N} x[k] e^{-j\frac{2\pi}{N}ki} \cdot 1 = X_i$$

(b)

(b) N is even

$X_k = X_{-1}$ period $= N$

$X_k = X_{-1+N}$

$k = N-1$

(c) $X_k = X_{-\frac{N}{2}}$

$X_k = X_{-\frac{N}{2}+N}$

$X_k = X_{\frac{N}{2}}$

$k = \frac{N}{2}$

5 - calculating linear weights

In[1]:=

```
c1 = (1/Sqrt[2]
      1/Sqrt[2])
c2 = ( 1/Sqrt[2]
      -1/Sqrt[2])
```

Out[1]= $$\left\{\left\{\frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}\right\}\right\}$$

Out[2]= $$\left\{\left\{\frac{1}{\sqrt{2}}\right\}, \left\{-\frac{1}{\sqrt{2}}\right\}\right\}$$

In[3]:=

```
q = (2
     3)
```

Out[3]= $$\{\{2\}, \{3\}\}$$

6 - orthonormal

In[4]:=

```
Vector[S_, Q_] := (1/Sqrt[S]) (Table[E^(I*2*Pi*Q/S (N-1)), {N, 0, S - 1}])
```

In[5]:=

```
i = 0;
k = 1;
vi = Vector[8, i]
vk = Vector[8, k]
```

Out[7]= $$\left\{\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right\}$$

Out[8]= $$\left\{\frac{e^{-\frac{i\pi}{4}}}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{e^{\frac{i\pi}{4}}}{2\sqrt{2}}, \frac{i}{2\sqrt{2}}, \frac{e^{\frac{3i\pi}{4}}}{2\sqrt{2}}, -\frac{1}{2\sqrt{2}}, \frac{e^{-\frac{3i\pi}{4}}}{2\sqrt{2}}, -\frac{i}{2\sqrt{2}}\right\}$$

In[9]:=

```
sum = (#1 + #2) &[vi, vk]
```

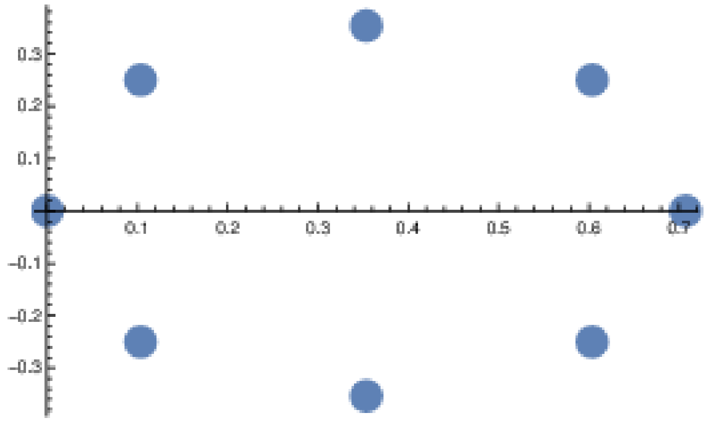
Out[9]= $$\left\{\frac{1}{2\sqrt{2}} + \frac{e^{-\frac{i\pi}{4}}}{2\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}} + \frac{e^{\frac{i\pi}{4}}}{2\sqrt{2}}, \frac{\frac{1}{2}+\frac{i}{2}}{\sqrt{2}}, \frac{1}{2\sqrt{2}} + \frac{e^{\frac{3i\pi}{4}}}{2\sqrt{2}}, 0, \frac{1}{2\sqrt{2}} + \frac{e^{-\frac{3i\pi}{4}}}{2\sqrt{2}}, \frac{\frac{1}{2}-\frac{i}{2}}{\sqrt{2}}\right\}$$

In[10]:=

```
data = Transpose@{Re[sum], Im[sum]}
```

Out[10]= $$\left\{\left\{\frac{1}{4}+\frac{1}{2\sqrt{2}}, -\frac{1}{4}\right\}, \left\{\frac{1}{\sqrt{2}}, 0\right\}, \left\{\frac{1}{4}+\frac{1}{2\sqrt{2}}, \frac{1}{4}\right\}, \left\{\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right\}, \left\{-\frac{1}{4}+\frac{1}{2\sqrt{2}}, \frac{1}{4}\right\}, \{0, 0\}, \left\{-\frac{1}{4}+\frac{1}{2\sqrt{2}}, -\frac{1}{4}\right\}, \left\{\frac{1}{2\sqrt{2}}, -\frac{1}{2\sqrt{2}}\right\}\right\}$$

In[11]:= `ListPlot[data, PlotStyle → PointSize[.05]]`

Out[11]=


In[12]:= `N@Total[sum]`

Out[12]= `2.82843 + 0. i`

In[13]:=
```
vi = Transpose[{vi}];
vk = Transpose[{vk}];
```

In[15]:= `Chop[N@ConjugateTranspose[vi].vk]`

Out[15]= `{{0}}`

8 - dotting a unit vector with itself is 1

In[16]:= `vector = (1 2 34 5 6)`

Out[16]= `{{1, 2, 34, 5, 6}}`

In[17]:= `vector = vector / Norm[vector]`

Out[17]= $\left\{\left\{\frac{1}{\sqrt{1222}}, \sqrt{\frac{2}{611}}, 17\sqrt{\frac{2}{611}}, \frac{5}{\sqrt{1222}}, 3\sqrt{\frac{2}{611}}\right\}\right\}$

In[18]:= `vector.Transpose@vector`

Out[18]= `{{1}}`

11 - calculate the 64 point dft of a cos of frequency pi/4 radians

In[19]:=
```
numberSamples = 64;
numberRange = Range[0, numberSamples - 1];
x = Cos[Pi / 4 * numberRange];
DFT = FourierMatrix[numberSamples];
```

In[23]:= `xDFT = DFT.x`

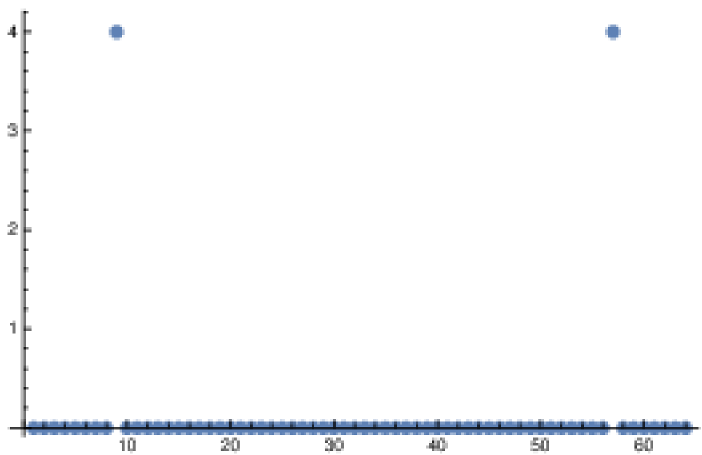
Out[23]=

$$\left\{0,\ \cdots 62 \cdots,\ \frac{e^{-\frac{i\pi}{32}}}{8\sqrt{2}}+\frac{e^{\frac{i\pi}{32}}}{8\sqrt{2}}-\frac{e^{-\frac{3i\pi}{32}}}{8\sqrt{2}}-\frac{e^{\frac{3i\pi}{32}}}{8\sqrt{2}}-\frac{1}{8}e^{-\frac{i\pi}{8}}-\frac{1}{8}e^{\frac{i\pi}{8}}-\frac{e^{-\frac{5i\pi}{32}}}{8\sqrt{2}}-\frac{e^{\frac{5i\pi}{32}}}{8\sqrt{2}}+\frac{e^{-\frac{7i\pi}{32}}}{8\sqrt{2}}+\frac{e^{\frac{7i\pi}{32}}}{8\sqrt{2}}+\frac{1}{8}e^{-\frac{i\pi}{4}}+\frac{1}{8}e^{\frac{i\pi}{4}}+\frac{e^{-\frac{9i\pi}{32}}}{8\sqrt{2}}+\frac{e^{\frac{9i\pi}{32}}}{8\sqrt{2}}-\frac{e^{-\frac{11i\pi}{32}}}{8\sqrt{2}}-\frac{e^{\frac{11i\pi}{32}}}{8\sqrt{2}}-\frac{1}{8}e^{-\frac{3i\pi}{8}}-\frac{1}{8}e^{\frac{3i\pi}{8}}-\frac{e^{-\frac{13i\pi}{32}}}{8\sqrt{2}}-\frac{e^{\frac{13i\pi}{32}}}{8\sqrt{2}}+\frac{e^{-\frac{15i\pi}{32}}}{8\sqrt{2}}+\frac{e^{\frac{15i\pi}{32}}}{8\sqrt{2}}+\frac{e^{-\frac{17i\pi}{32}}}{8\sqrt{2}}+\frac{e^{\frac{17i\pi}{32}}}{8\sqrt{2}}-\frac{e^{-\frac{19i\pi}{32}}}{8\sqrt{2}}-\frac{e^{\frac{19i\pi}{32}}}{8\sqrt{2}}-\frac{1}{8}e^{-\frac{5i\pi}{8}}-\frac{1}{8}e^{\frac{5i\pi}{8}}-\frac{e^{-\frac{21i\pi}{32}}}{8\sqrt{2}}-\frac{e^{\frac{21i\pi}{32}}}{8\sqrt{2}}+\frac{e^{-\frac{23i\pi}{32}}}{8\sqrt{2}}+\frac{e^{\frac{23i\pi}{32}}}{8\sqrt{2}}+\frac{1}{8}e^{-\frac{3i\pi}{4}}+\frac{1}{8}e^{\frac{3i\pi}{4}}+\frac{e^{-\frac{25i\pi}{32}}}{8\sqrt{2}}+\frac{e^{\frac{25i\pi}{32}}}{8\sqrt{2}}-\frac{e^{-\frac{27i\pi}{32}}}{8\sqrt{2}}-\frac{e^{\frac{27i\pi}{32}}}{8\sqrt{2}}-\frac{1}{8}e^{-\frac{7i\pi}{8}}-\frac{1}{8}e^{\frac{7i\pi}{8}}-\frac{e^{-\frac{29i\pi}{32}}}{8\sqrt{2}}-\frac{e^{\frac{29i\pi}{32}}}{8\sqrt{2}}+\frac{e^{-\frac{31i\pi}{32}}}{8\sqrt{2}}+\frac{e^{\frac{31i\pi}{32}}}{8\sqrt{2}}\right\}$$

large output | show less | show more | show all | set size limit...

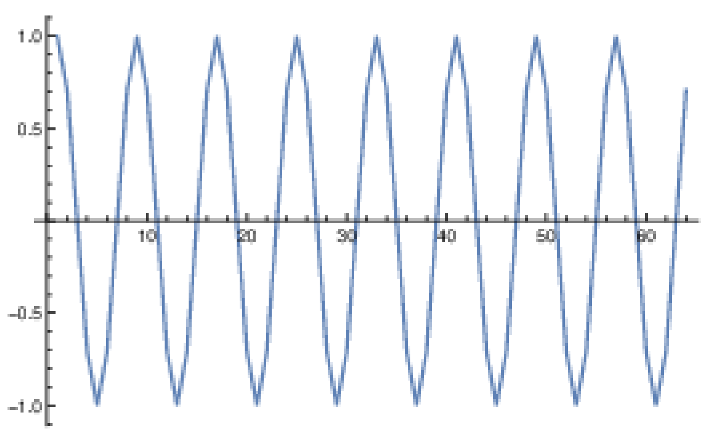
In[24]:= `ListPlot[xDFT, PlotStyle → PointSize[.02]]`

Out[24]=



In[25]:= `ListLinePlot[x]`

Out[25]=



In[26]:=

13 - low pass using fourier and inverse fourier

```
In[27]:= lowpassFourierFilter[data_] := Module[{fWeights, lWeights},
           fWeights = Fourier[data];
           lWeights = Join[fWeights[[ ;; Floor[Length[data] / 4]]],
             .1 * fWeights[[Floor[Length[data] / 4] + 1 ;;]]];
           Return[Re[Chop[InverseFourier[lWeights]]]]]
```

```
In[28]:= Fs = 8192
         data = Flatten[Transpose[Import[
              "/home/nathan/olin/fall2016/QEAFall2016Homework/bset1/matlab/handel.csv"]]];

Out[28]= 8192
```

highest frequency:

```
In[30]:= Fs / 2

Out[30]= 4096
```

lowest frequency

```
In[31]:= 0

Out[31]= 0
```

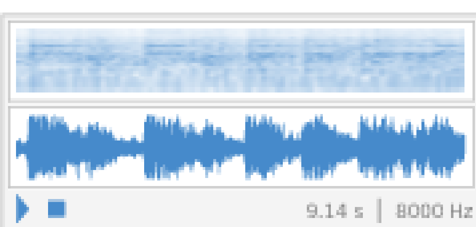
```
In[32]:= dataFiltered = lowpassFourierFilter[data]

Out[32]= {-0.00538123, -0.0177226, -0.025308, -0.0160855, 0.000450921, 0.0183174,
          0.0161275, -0.00682954, -0.026557, -0.0399389, -0.0572852, -0.08448,
          -0.106186, -0.0919217, -0.0421564, -0.0118159, -0.0191307, -0.0312334,
          ... 73 077 ..., 0.0838113, 0.0722977, 0.0473331, 0.0359092, 0.0480158,
          0.0165222, -0.0353874, -0.0828198, -0.103739, -0.0815486, -0.0313233,
          0.0222903, 0.0677911, 0.0880384, 0.0959882, 0.105543, 0.0756967, 0.0241307}

          large output | show less | show more | show all | set size limit...
```

```
In[33]:= ListPlay[dataFiltered]
```

Out[33]=



14 - filter hum

In[34]:=
```
guitarData = Import[
  "/home/nathan/olin/fall2016/QEAFall2016Homework/bset1/matlab/guitar1_60Hz_hum.wav",
  "Data"]
ListPlay[guitarData, SampleRate → 44 100]
```

Out[34]=
```
{-0.0154724, -0.0160522, -0.0166626, -0.0168457, -0.0168762,
 -0.0173645, -0.0177917, -0.0185242, -0.0195618, -0.0202026, -0.0209045,
 -0.0213623, -0.0210266, -0.0201416, -0.0189819, -0.0175171, -0.015625,
 -0.0146484, -0.0148621, -0.0149536, ... 83 751 ..., 0.0338145, 0.0344249,
 0.0414747, 0.0433668, 0.0404065, 0.0424207, 0.0491043, 0.0540483,
 0.0586261, 0.0653706, 0.0737632, 0.0828883, 0.084994, 0.0780969,
 0.073336, 0.0682699, 0.0531022, 0.0375683, 0.0326548, 0.0314951}
```

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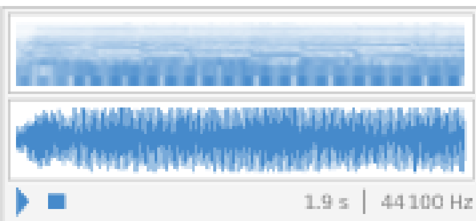
Out[35]=



In[36]:=
```
guitarDataFiltered = lowpassFourierFilter[guitarData]
ListPlay[guitarData, SampleRate → 44 100]
```

Out[36]=
```
{-0.00277813, -0.0103082, -0.0115646, -0.00897426, -0.00820755,
 -0.0101797, -0.0112135, -0.0102478, -0.0100633, -0.0116529, -0.0126471,
 -0.0118344, -0.0111206, -0.0115651, -0.0114154, -0.00973719, -0.00832838,
 -0.00851535, ... 83 755 ..., 0.0216298, 0.0225205, 0.0227415, 0.0237073,
 0.0260745, 0.029039, 0.0321897, 0.0359821, 0.040538, 0.0446006, 0.0458807,
 0.043966, 0.0404823, 0.0354255, 0.0286758, 0.0227108, 0.0181235, 0.0113857}
```

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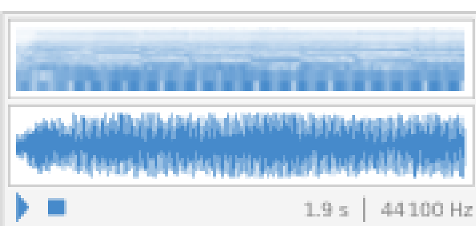
Out[37]=



In[38]:=
```
data = Re[Fourier[guitarDataFiltered]]
```

Out[38]=
```
{-0.192972, -0.0099005, 0.00241204, -0.00854094, -0.0119331, -0.011744,
 -0.0153276, -0.0213279, -0.0102596, -0.0172746, -0.00750142, -0.0010313,
 -0.0102605, -0.0190867, -0.00774538, -0.00828969, -0.0171675, 0.000548939,
 … 83 756 …, 0.000548939, -0.0171675, -0.00828969, -0.00774538, -0.0190867,
 -0.0102605, -0.0010313, -0.00750142, -0.0172746, -0.0102596, -0.0213279,
 -0.0153276, -0.011744, -0.0119331, -0.00854094, 0.00241204, -0.0099005}
```

large output | show less | show more | show all | set size limit...

In[39]:=
```
Length[data]
```

Out[39]= 83 791

In[40]:=
```
ListPlot[data[[Floor[83 791 / 2] ;;]], PlotRange → All]
```

Out[40]=