1

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

$$\mathcal{L}\left\{\frac{d}{dt}f(t)\right\} = sF(s) - f(0^-)$$

$$\mathcal{L}\left\{\frac{d^2}{dt^2}f(t)\right\} = s^2F(s) - sf(0^-) - f'(0^-)$$

$$sY(s) - y(0^-) + 2Y(s) = X(s)$$

$$y(0^-) = 1, \quad x(t) = u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{else} \end{cases}$$

$$sY(s) - 1 + 2Y(s) = \frac{1}{s}$$

$$(s+2)Y(s) = \frac{s+1}{s}$$

$$Y(s) = \frac{s+1}{s(s+2)}$$

$$Y(s) = \frac{1}{2s} + \frac{1}{2(2+s)}$$

$$y(t) = \frac{1}{2}u(t) + \frac{1}{2}e^{-2t} \quad \text{for } t \geq 0$$

(2)
(a)

$$m \frac{d^2}{dt} y(t) + c \frac{d}{dt} y(t) + k\, y(t) = c \frac{d}{dt} x(t) + k\, x(t)$$

$$x(t) = u(t)$$

$$y(0^-) = 0, \quad x(0^-) = 0, \quad y'(0^-) = 0$$

$$m s^2\, Y(s) + c\, Y(s) + k\, Y(s) = c\, X(s) + k\, X(s)$$

$$Y(s)\,(m s^2 + c s + k) = (cs + k)\, X(s),$$

$$Y(s) = \frac{cs + k}{m s^2 + c s + k} \quad \frac{1}{s}$$

(b) $$Y(s) = \frac{cs+k}{s(ms^2+cs+k)}$$

$$Y(s) = \frac{k}{a_1a_2s} + \frac{a_1c+k}{a(a_1-a_2)(s-a_1)} + \frac{a_2c+k}{a_2(a_2-a_1)(s-a_2)}$$

a_1 a_2 are roots of

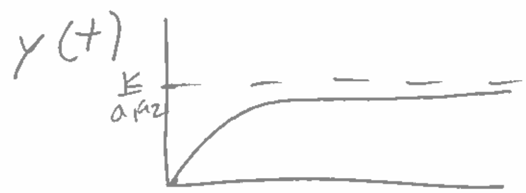
$$ms^2+cs+k$$

$$a_1 = \frac{-c+\sqrt{c^2-4mk}}{2m}$$

$$a_2 = \frac{-c-\sqrt{c^2-4mk}}{2m}$$

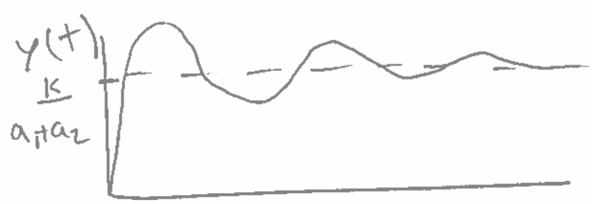
(c) $$y(t) = \frac{k}{a_1a_2}u(t) + \frac{a_1c+k}{a_1(a_1-a_2)}e^{a_1t} + \frac{a_2c+k}{a_2(a_2-a_1)}e^{a_2t}, \quad t\geq 0$$

$a_1, a_2 \in -\mathbb{R}$



$a_1, a_2 \in \mathbb{C}$

$e^{(\sigma+j\omega)t}$



QEA - Out of Class Assignment 4: Driven Systems

In[3]:=
```
SetDirectory@NotebookDirectory[];
<< "../General.m"
```

In[29]:= $\dfrac{s+1}{s\,(s+2)}$

Out[29]= $\dfrac{1+s}{s\,(2+s)}$

In[32]:= Apart$\left[\dfrac{1+s}{s\,(2+s)}, s\right]$

Out[32]= $\dfrac{1}{2\,s}+\dfrac{1}{2\,(2+s)}$

---

2)

In[10]:= `de = m y''[t] + c y[t] + k y[t] == c D[HeavisideTheta[t], t] + k HeavisideTheta[t]`

Out[10]= `c y[t] + k y[t] + m y''[t] == c DiracDelta[t] + k HeavisideTheta[t]`

In[12]:= `LaplaceTransform[de, t, s]`

Out[12]= c LaplaceTransform[y[t], t, s] + k LaplaceTransform[y[t], t, s] + m $\left(s^2\right.$ LaplaceTransform[y[t], t, s] − s y[0] − y′[0]$\left.\right)$ == c + $\dfrac{k}{s}$

In[62]:= params = <|a1 → $\dfrac{-c+\text{Sqrt}\left[c^2-4\,m\,k\right]}{2\,m}$, a2 → $\dfrac{-c-\text{Sqrt}\left[c^2-4\,m\,k\right]}{2\,m}$|>

y[t] = $\dfrac{k}{a1\,a2}$ HeavisideTheta[t] + $\dfrac{a1\,c\,+\,k}{a1\,(a1-a2)}\,e^{a1\,t}+\dfrac{a2\,c\,+\,k}{a2\,(a2-a1)}\,e^{a2\,t}$

Out[62]= $\langle\left|\,a1\to\dfrac{-c+\sqrt{c^2-4\,k\,m}}{2\,m},\ a2\to\dfrac{-c-\sqrt{c^2-4\,k\,m}}{2\,m}\right|\rangle$

Out[63]= $\dfrac{e^{a1\,t}\,(a1\,c+k)}{a1\,(a1-a2)}+\dfrac{e^{a2\,t}\,(a2\,c+k)}{a2\,(-a1+a2)}+\dfrac{k\,\text{HeavisideTheta}[t]}{a1\,a2}$

Large values of c seem to give the lowest oscillations after the bump in the road.

In[74]:=
```
Manipulate[
 constants = <|m → 10^3, c → cval, k → kval|>;
 equation = y[t] /. params /. constants,
 {cval, 1, 10^4}, {kval, 10^3, 10^5}]
```

Out[74]=

cval

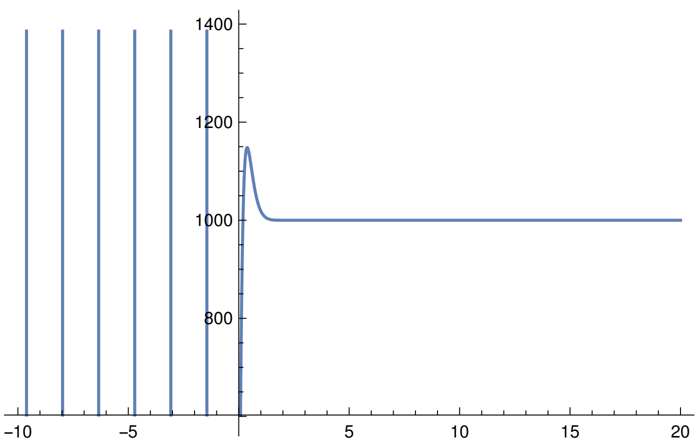
kval

28700.

$$(-500. + 1299.69\,i)\,e^{(-5.-1.92354\,i)\,t} - (500. + 1299.69\,i)\,e^{(-5.+1.92354\,i)\,t} + (1000. + 0.\,i)\,\text{HeavisideTheta}[t]$$

In[75]:=
```
Dynamic[Plot[equation, {t, -10, 20}]]
```

Out[75]=



Now let's make a1 and a2 the same. This happens when $c^2 - 4\,m\,k == 0$

In[78]:=
```
c^2 - 4 m k == 0 /. m → 10^3
```

Out[78]= $c^2 - 4000\,k == 0$

In[82]:=
```
Solve[c^2 - 4000 k == 0, {k}]
```

Out[82]= $\left\{\left\{k \to \frac{c^2}{4000}\right\}\right\}$

In[98]:=
```
value = 10;
constants = <|m → 10^3, c → value, k → value^2/4000|>;
equation2 = y[t] /. params /. constants
```

Power: Infinite expression $\frac{1}{0}$ encountered.

Power: Infinite expression $\frac{1}{0}$ encountered.

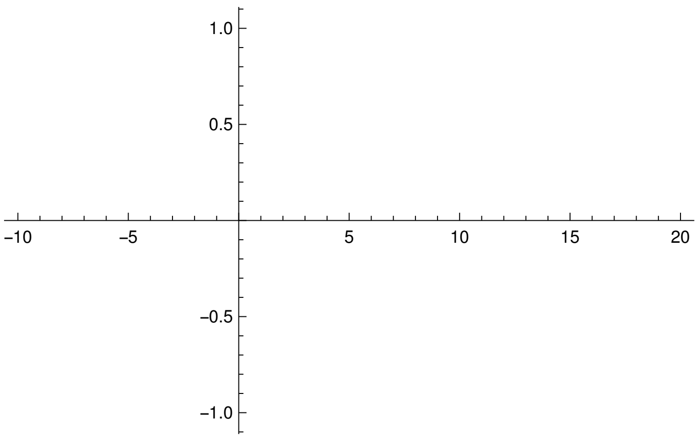
Infinity: Indeterminate expression ComplexInfinity + ComplexInfinity + 1000 HeavisideTheta[t] encountered.

Out[100]=
```
Indeterminate
```

It appears that a1 and a2 can’t be the same value as we get divide by zero errors

In[83]:=
```
Dynamic[Plot[equation2, {t, -10, 20}]]
```

Out[83]=



In[101]:=
```
exportNotebookPDF[]
```