Laplace Transforms

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(a) $u(t-a) \quad a \geq 0$

$$u(t-a) = \begin{cases} 1 & \text{if } t \geq a \\ 0 & \text{otherwise} \end{cases}$$

$$\mathcal{L}\{u(t)\} = \mathcal{L}\{u(t-0)\}, \quad u(t) = 1$$

$$\mathcal{L}\{1\} = \int_0^\infty 1 e^{-st}\, dt = -\frac{1}{-s} = \frac{1}{s}$$

(b) $$\mathcal{L}\{F(t-a)\, u(t-a)\} = e^{-as} F(s)$$

$$\int_0^\infty u(t-a) F(t-a) e^{-st}\, dt =$$

$$\int_a^\infty F(t-a) e^{-st}\, dt =$$

$u = t-a$
$t = u+a$
$dt = du$

$$\int_{a-a}^\infty e^{-s(u+a)} F(u)\, du =$$

$$\int_0^\infty e^{-su} e^{-sa} F(u)\, du$$

$$e^{-sa} \int_0^\infty e^{-su} F(u)\, du =$$

$$e^{-as} F(s) =$$

(c) $\mathcal{L}\{u(t-a)\} = \dfrac{e^{-as}}{s}$

$\mathcal{L}\{u(t-a)\cdot 1\}$

$e^{-as}\,\mathcal{L}\{1\} = \dfrac{e^{-as}}{s}$

$e^{-as}\,\dfrac{1}{s} = \dfrac{e^{-as}}{s}$

(a) (2)

$$Y(s) = \frac{s+1}{s^2+5s+4} = \frac{s+1}{(s+4)(s+1)}$$

Inverse
Laplace
Transforms

$$Y(s) = \frac{1}{s+4}$$

$$y(t) = e^{-4t}$$

(b) $$Y(s) = \frac{1}{s^2+2s+2} = \frac{1}{(s+1+i)(s+1-i)}$$

Don't understand how to partial
fraction this - so mathematica

$e^{-t}\sin(t)$