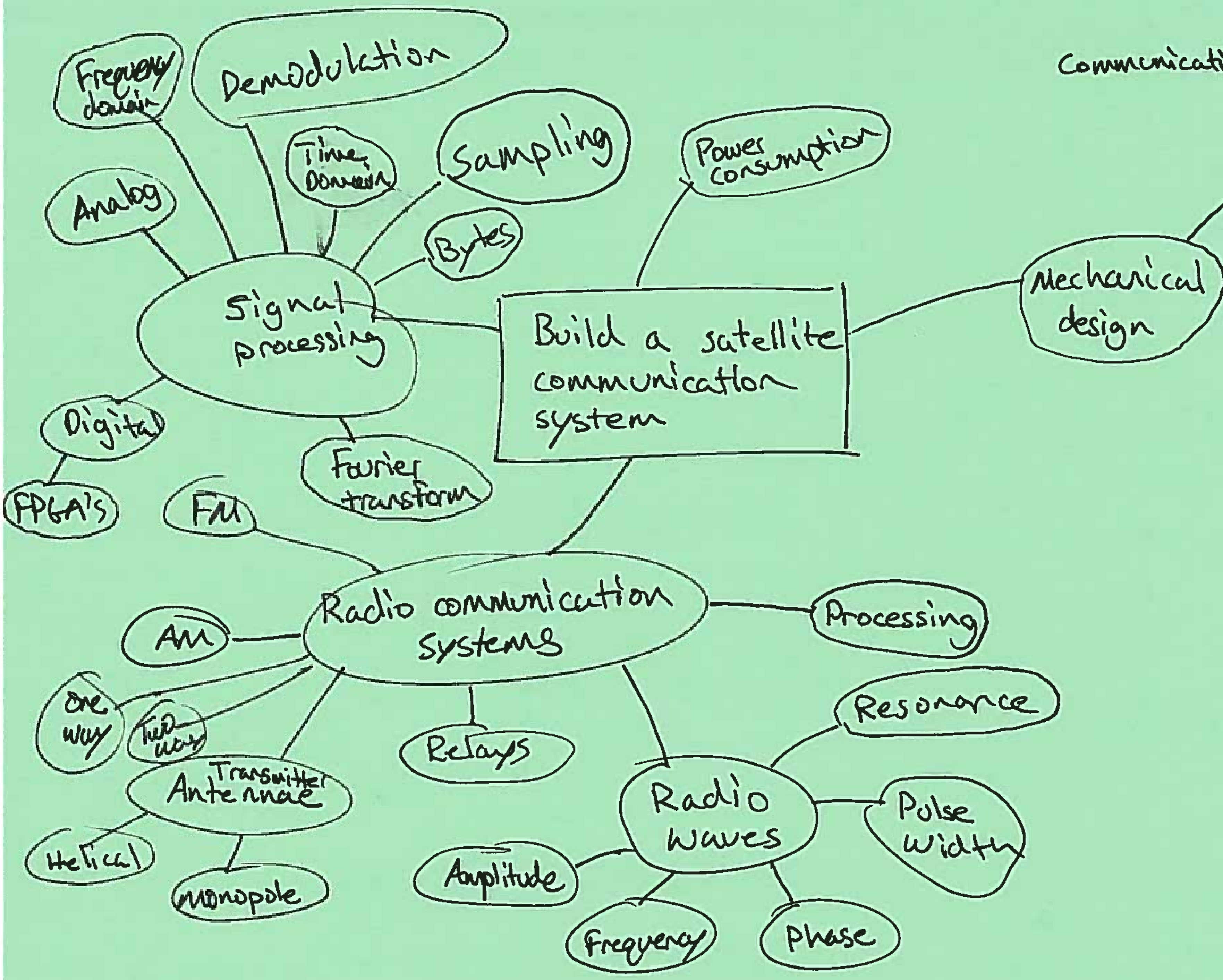
Nathan
BSET [redacted]0
Communications Breakdown



BSET #0
Communications
Breakdown

Signal processing

Time domain

Frequency domain

Sampling

Bytes

Sampling

Radio communications system

~~Frequency~~

Antennae

AM/FM

One way/Two way

~~Phase~~

Frequency, Amplitude, phase, pulse width, resonance

(1)

QEA BSET ~~#~~
Modsim
Questions

Flow in

2.1 m

45% precipitation = .945
55% aquifer = 1.155

Flow out

2.1

26% evaporation = .5460
74% seepage = 1.554

How long to dry out if precipitation 25% below normal

prec = .945 • .75 = .7087

$$\text{Flow in} - \text{Flow out} = \frac{-.2362\,m}{\text{year}}$$

$$12.9m - .2362m\,x = 0$$

$$-.2362\,x = -12.9$$

$$x = \frac{12.9}{.2362}$$

$$x = 54.6147$$

QEA
BSET D
MODSIM

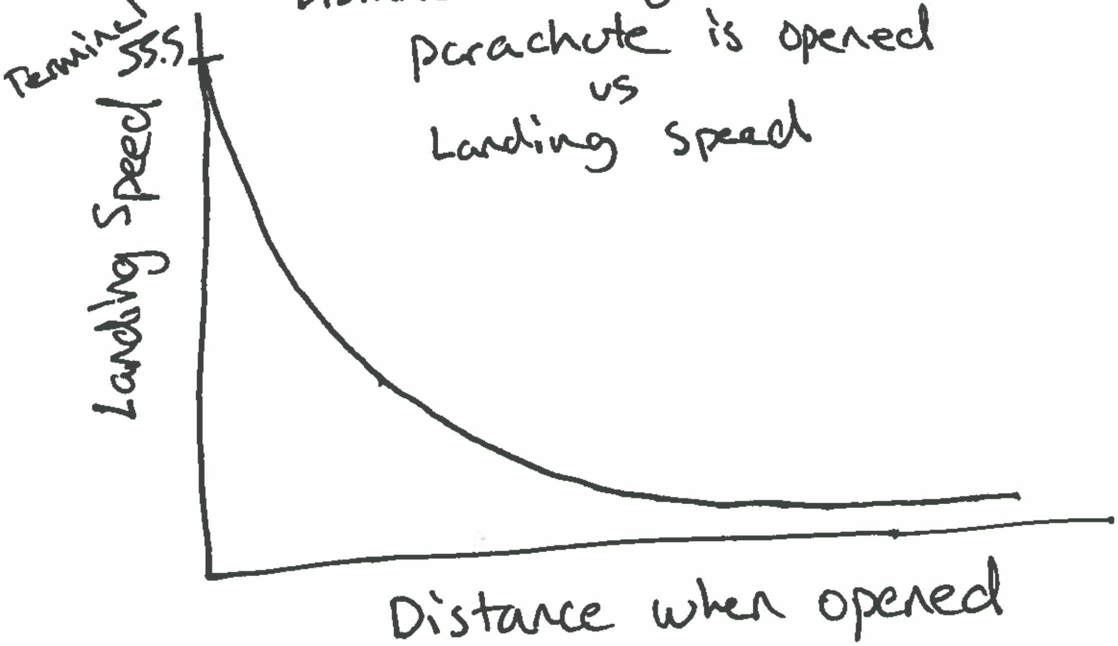
(2)

Top speed = terminal velocity

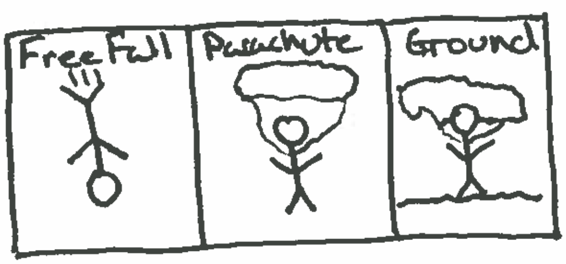
so, starting (HIGH) altitude
is not important

Terminal velocity for human = 55.5 m/s

Distance from ground when
parachute is opened
vs
Landing speed



Assume parachute is immediately ~~in full~~ opened



Position $\Rightarrow$

Velocity $\Rightarrow$ $\dot{x} = V$

Acceleration $\Rightarrow$ $\dot{V} =$ ~~gravity~~

$AirRes_{para} + AirRes_{person} - grav$

$\dot{x} = V$

$\dot{V} = \alpha V^2 + \beta V^2 - g$

BSET ~~#~~0
MODSIM
QUESTIONS

3

$$\dot{x} = v$$

$$\dot{v} = -\alpha x - \beta v^2$$

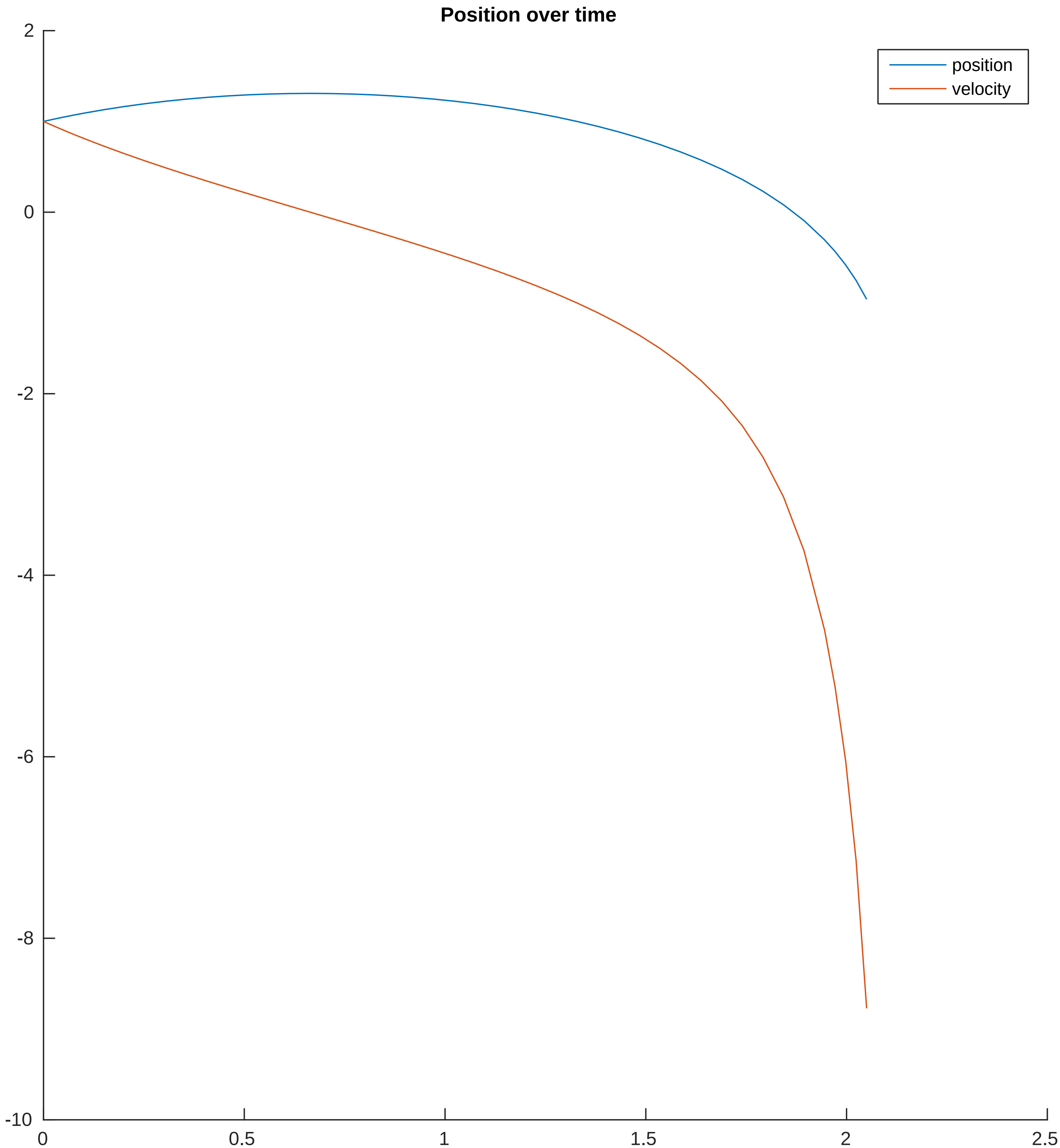
a

$\dot{v} = \frac{m}{s^2}$ $\alpha x = \frac{1}{s^2} m$ $\beta v^2 = \frac{1}{m} \frac{m^2}{s^2}$

$\boxed{\alpha = \frac{1}{s^2}}$ $\boxed{\beta = \frac{1}{m}}$

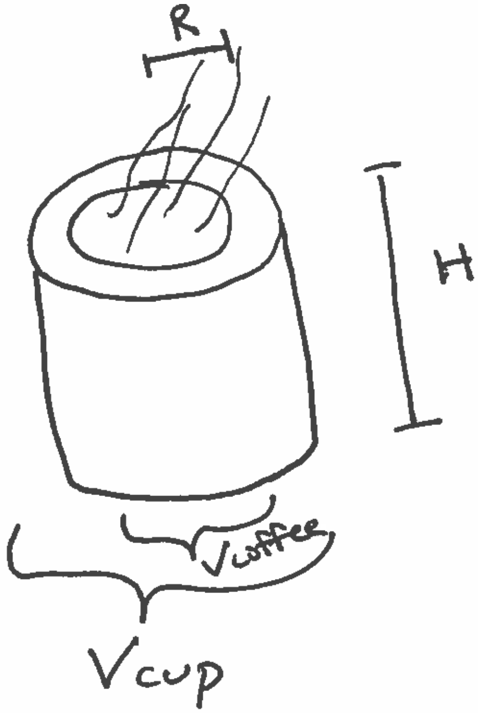
b ~~Code~~
~~Image~~
6

~~CODE : Modsim3.m~~
~~PLOT~~ : Modsim 3. pdf


Position over time
position
velocity
2
0
-2
-4
-6
-8
-10
0
0.5
1
1.5
2
2.5

4

QEA
MODSIM
QUESTIONS



Energy coffee = Temp Coffee · Specific Heat coffee

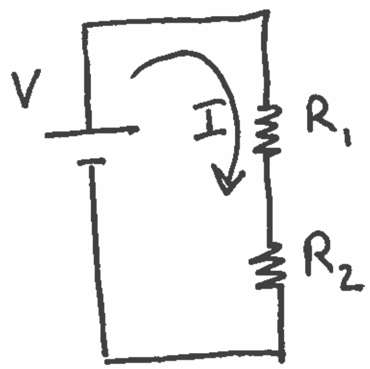
$$\frac{dE}{dt}\text{ coffee} = \text{Loss conduction}$$

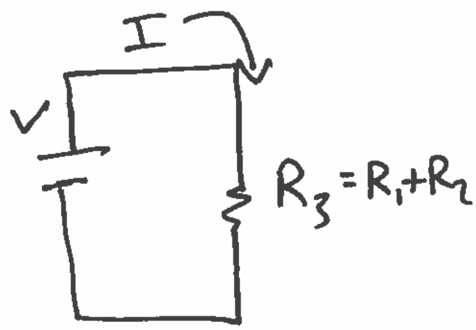
$$\frac{dE}{dt} = -\frac{\text{mugCond} \cdot \text{mugArea}}{\text{mugThickness}} \cdot (\text{TempCoffee} - \text{TempRoom})$$

QEA
ISIM
QUESTIONS

1

$V = 12v$
$R_1 = 10k\Omega$
$R_2 = 20k\Omega$



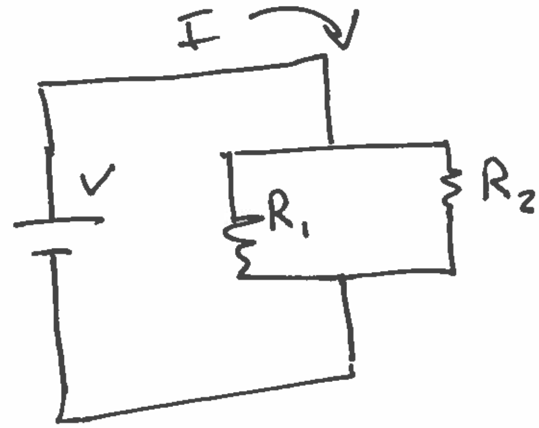


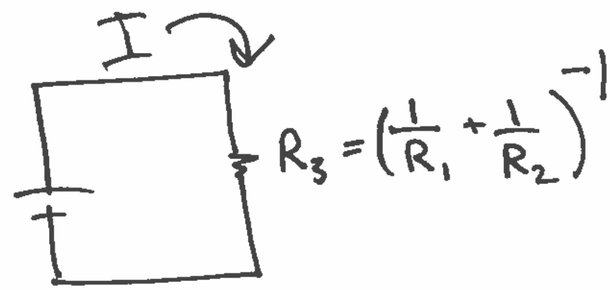
$R_3 = 30k\Omega$

$V = IR$

$12v = I\ 30k\Omega$

$I = .4\cancel{0}\,mA$





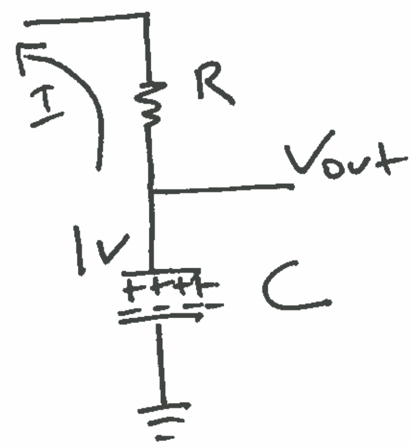
$$R_3 = \left(\frac{2}{20k} + \frac{1}{20k}\right)^{-1}$$
$$= \left(\frac{3}{20k}\right)^{-1}$$
$$= \frac{20k}{3}$$

$R_3 = 6.67k\Omega$

$12V = I\ 6.67k\Omega$

$I = 1.8mA$

② V_{in}



$I = I_R = I_c$

QEA-ISIM
BSET-0

$V_{in} = 0$

$V_{out_0} = 1$

$R = 10\,k\Omega$

$C = 1\,\mu F$

Resistor

$V_R = IR$

Capacitor

$I = C\dfrac{dV}{dt}$

$Q = CV$

$V_c = \dfrac{Q}{C}$

$I = \dfrac{-dQ}{dt}$

Kirchoff

$V_c - V_R = 0$

$\dfrac{Q}{C} - IR = 0$

$e^{t/RC} = e^{\ln\left(\frac{q}{Q_0}\right)}$

$e^{-t/RC} = \dfrac{q}{Q_0}$

$q = Q_0 e^{\frac{-t}{RC}}, \quad Q_0 = CV_0$

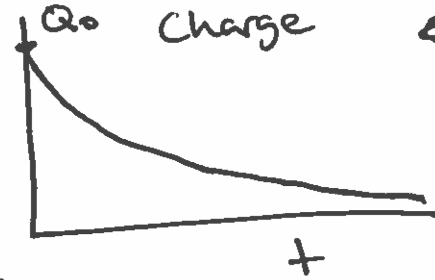
$$\frac{Q}{C} - \left(\frac{-dQ}{dt}\right)R = 0$$

$$\int_0^t \frac{-dt}{RC} = \int_{Q_0}^{q} \frac{dQ}{Q}$$

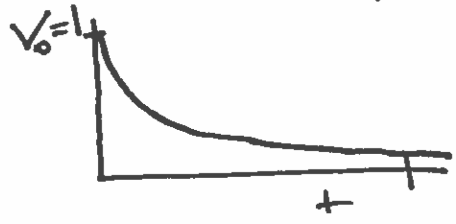
$$\frac{-t}{RC} = \ln(q) - \ln(Q_0)$$

$$\frac{-t}{RC} = \frac{\ln(q)}{\ln(Q_0)}$$

Charge



$V = V_0 e^{\frac{-t}{RC}}$



③

QEA ISIM QUESTIONS

$$\frac{(1+j)(3+j)(-2-j)}{(j)(3+4j)(5+j)}$$

Cartesian

$$\frac{(1+j)(3+j)(-2-j)}{(j)(3+4j)(5+j)}$$

$$\frac{(3+1j+3j-1)(-2-j)}{(3j-4)(5+j)}$$

$$\frac{(2+4j)(-2-j)}{(-4+3j)(5+j)}$$

$$\frac{(-4-2j-8j+4)}{(-20-4j+15j-3)}$$

$$\frac{-10j}{-23+11j}\left(\frac{11-23j}{11-23j}\right)$$

$$\frac{-230-110j}{650j} = \boxed{-\frac{11}{65}+\frac{23}{65}j}$$

$$= \boxed{-.169+.354j}$$

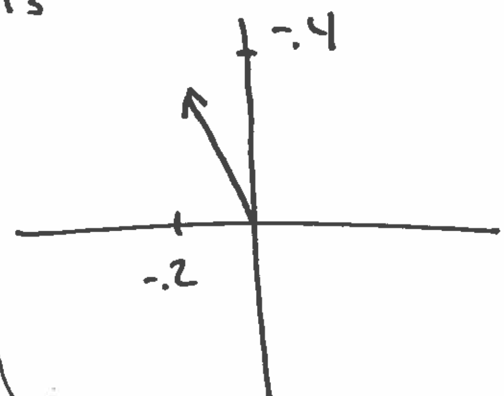
Polar

$$\left(\sqrt{2}\angle\frac{\pi}{4}\right)\left(\sqrt{10}\angle \text{ArcTan}\left(\tfrac{1}{3}\right)\right)$$

Arithmetic not worth my time

Mathematica!

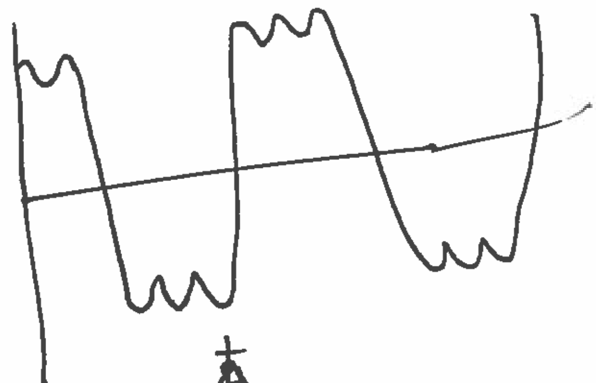
$$\sqrt{\frac{2}{13}}\, e^{j\left(\pi - \text{ArcTan}\left(\frac{23}{11}\right)\right)}$$



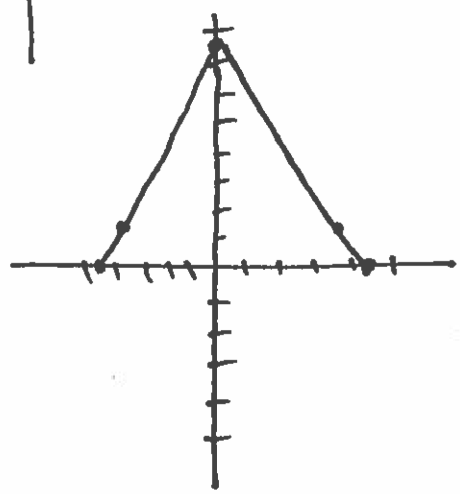
QEA
BSET 1
ISIM

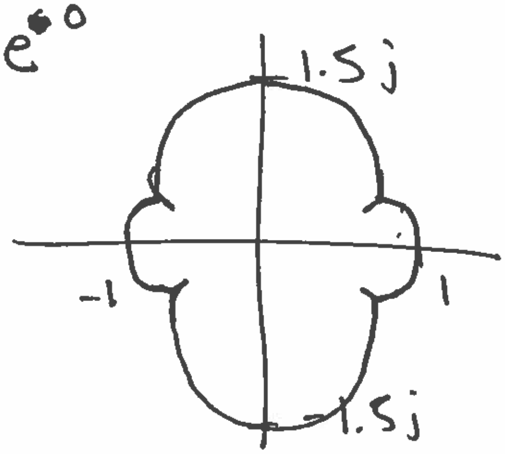
(4) $z(t) = e^{jt} - .3e^{j3t} + .2e^{j5t}$

t	$z(t)$	$z(t)$
0	$e^0 - .3e^0 + .2e^0$	$.9e^{j0}$
$\frac{\pi}{4}$	$e^{j\frac{\pi}{4}} - .3e^{j\frac{3\pi}{4}} + .2e^{j\frac{5\pi}{4}}$	$.8544e^{j.4206}$, $.78 + .353j$
$\frac{\pi}{2}$		



t	$z(t)$	$z(t)$
0	$.9 + 0j$	$.9e^{0j}$
$\frac{\pi}{4}$	$.77 + .35j$	$.855e^{j.427}$
$\frac{\pi}{2}$	$0 + 1.5j$	$1.5e^{j1.57}$
$3\frac{\pi}{4}$	$-.77 + .35j$	$.8544e^{2.715}$
π	$-.9 + 0j$	$-.9e^{0}$





Mathematica

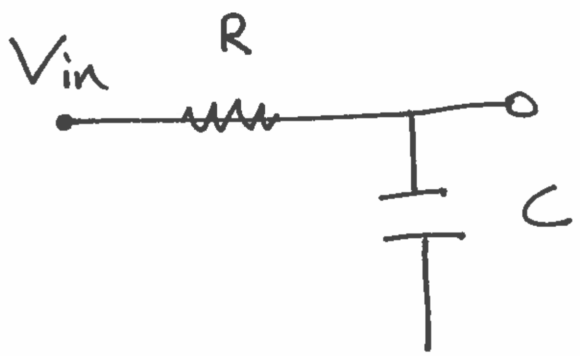
QEA
ISIM
QUESTIONS

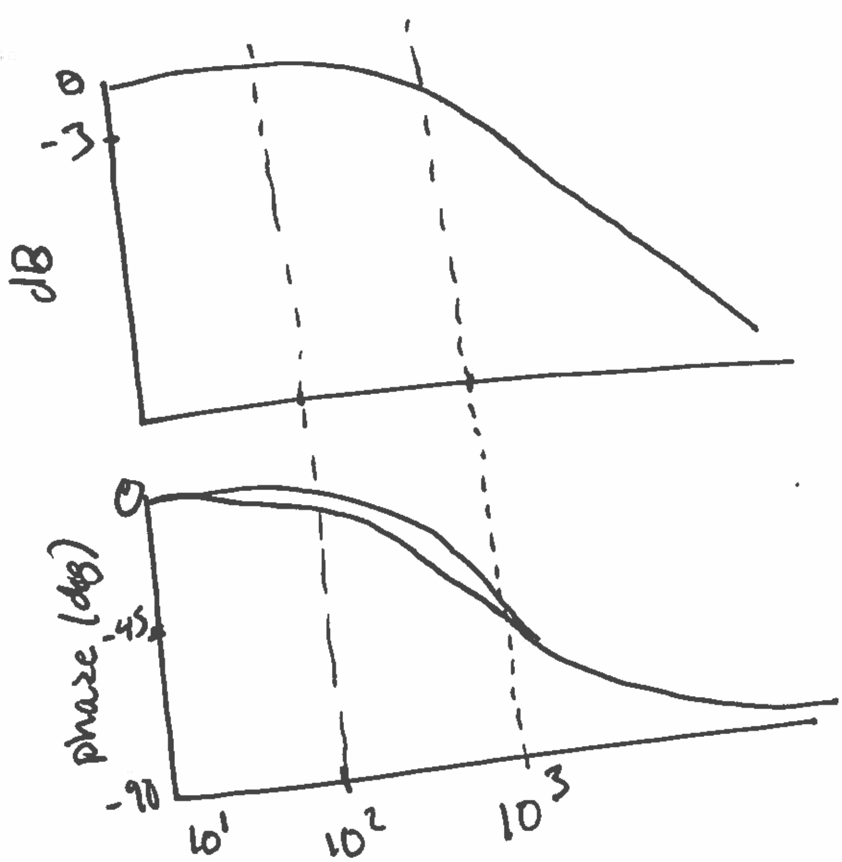
5 Low pass

$$W_c = \frac{1}{RC} = 1\,kHz$$

$R = 1000\,\Omega = 1\,k\Omega$

$C = \cancel{1000}\ 1\,\mu F$

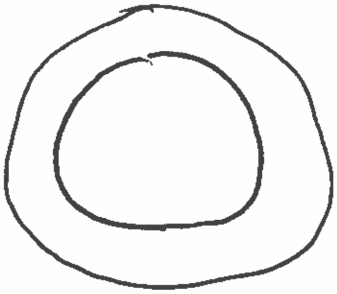


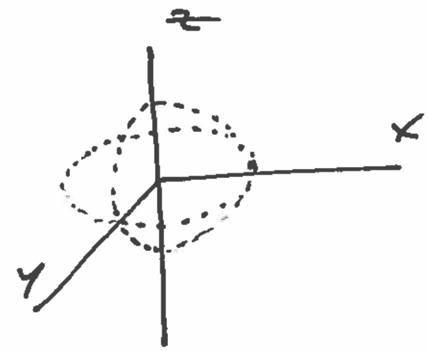


Integrate:

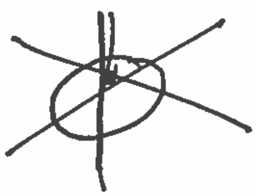
sphere, ~~Hemisphere, $\frac{1}{2}$ $\frac{1}{4}$ sphere, $\frac{1}{8}$ sphere~~

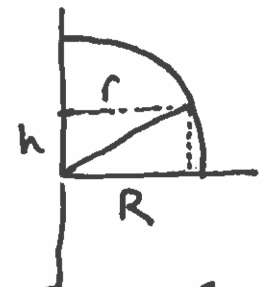
QEA
BSET ~~4~~0
QEA REVIEW





Add up layers of circles





At $z=0$ largest circle

$r^2 = x^2 + y^2$

$R^2 = h^2 + r^2$

$\sqrt{R^2 - h^2} = r$

A circle at z height h will have ~~area~~ radius $\sqrt{R^2-h^2}$

smallest $h = -R$
largest $h = R$
Area at height $h \Rightarrow A = \pi r^2 = \pi \sqrt{R^2-h^2}^{\,2} = \pi(R^2-h^2)$

$$\int_{-R}^{R} (\pi R^2 - \pi h^2)\, dh$$

$$\int_{-R}^{R} \pi R^2 dh - \int_{-R}^{R} \pi h^2 dh$$

$$\pi R^2 \int_{-R}^{R} dh - \pi \int_{-R}^{R} h^2 dh$$

$$\pi R^2 (R + R) - \pi \left. \frac{h^3}{3} \right|_{-R}^{R}$$

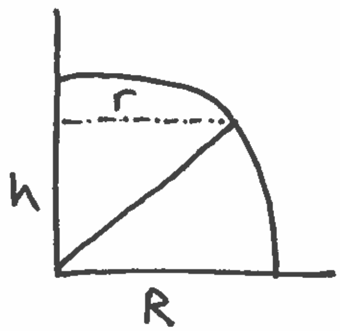
$$2\pi R^3 \quad -\pi\left(\frac{R^3}{3} - \frac{(-R)^3}{3}\right)$$

$$\frac{6}{3} \cancel{2} \pi R^3 \quad -2\pi \frac{R^3}{3}$$

$$\frac{4}{3}\pi R^3$$

integrate half sphere

QEA BSET 0
QEA REVIEW



Area of circle at height h

$r = \sqrt{R^2 + h^2}$

$A = \pi r^2 = \pi \sqrt{R^2+h^2}^2 = \pi (R^2 + h^2)$

$$\int_0^R \pi (R^2 + h^2)\, dh$$

$$\int_0^R \pi R^2 + h^2 \, dh$$

$$\int_0^R \pi R^2 dh + \int_0^R \pi h^2 \, dh$$

$$\pi R^2 \int_0^R dh + \pi \int_0^R h^2 \, dh$$

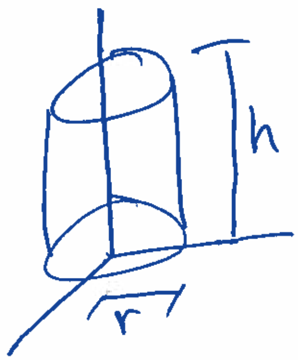
$$\pi R^2 (R) + \pi \left(\frac{R^3}{3} \right)$$

$$\frac{3}{3} \pi R^3 + \frac{\pi R^3}{3}$$

$\frac{4 \pi R^3}{3}$ Strange. Got volume of regular sphere

MORE INTEGRALS

$\theta \; R \; z$



$0 \le \theta \le 2\pi$

$0 \le R \le r$

$0 \le z \le h$

$$\int_0^{2\pi} \int_0^{r} \int_0^{h} r \, dz \, dR \, d\theta$$

COM

Limits of integration

Surfaces

Integration

Principal component analysis

QEA big ideas

Matrix transformations

Inverse matrices

Possible questions

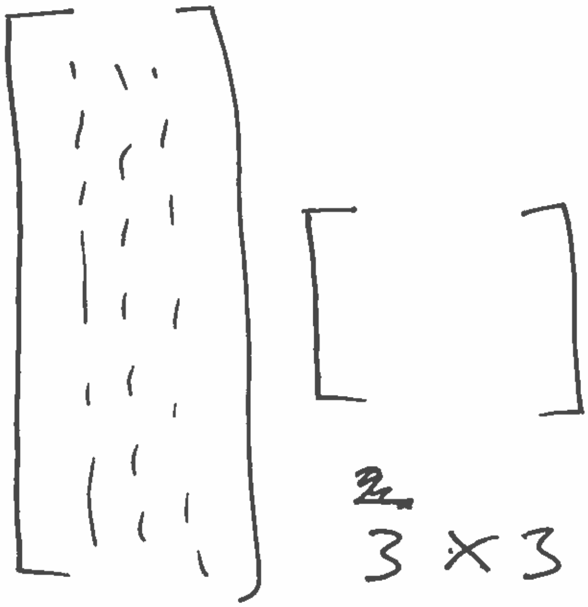
Integrate eighth of sphere

Animate polygon/points

Find COM of some two functions

Design some truss

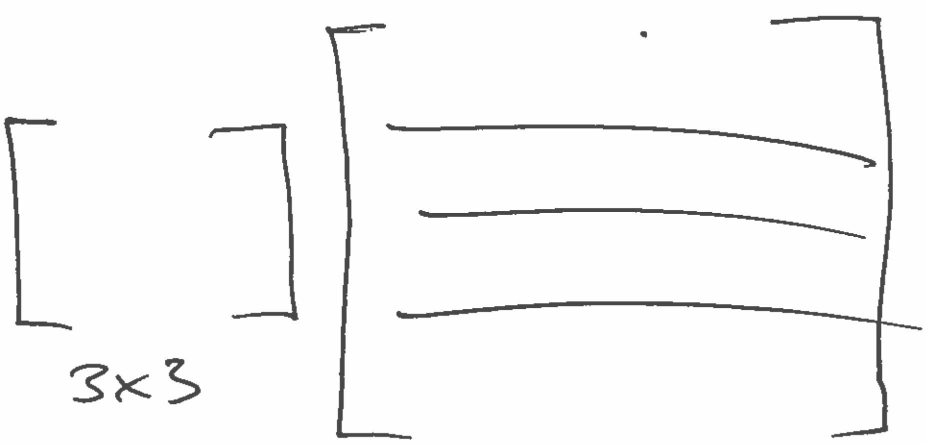
Animate points/polygon in mathematica to remember matrix transformations



3×3

$n \times 3$

$$n \times 3 \cdot 3 \times 3 \to n \times 3$$



3×3

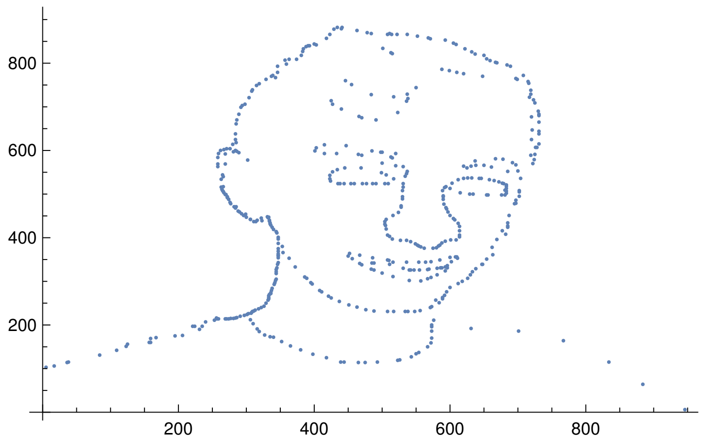
3×511

3×511

```
me = Import["/home/nathan/olin/fall2016/QEAFall2016Homework/bset1/matrixSelfie.jpg"];
```

```
me
```

```
coords = {{253.`, 211.`}, {240.`, 207.`}, {231.`, 190.`}, {206.`, 176.`},
   {195.`, 175.`}, {167.`, 171.`}, {159.`, 169.`}, {123.`, 151.`},
   {109.`, 142.`}, {38.`, 115.`}, {36.`, 114.`}, {17.`, 106.`}, {309.`, 231.`},
   {307.`, 227.`}, {306.`, 212.`}, {310.`, 203.`}, {316.`, 191.`},
   {319.`, 185.`}, {327.`, 177.`}, {335.`, 173.`}, {340.`, 172.`}, {352.`, 162.`},
   {365.`, 152.`}, {380.`, 143.`}, {398.`, 133.`}, {418.`, 125.`}, {439.`, 115.`},
   {444.`, 115.`}, {465.`, 114.`}, {475.`, 114.`}, {493.`, 115.`}, {523.`, 119.`},
   {526.`, 120.`}, {543.`, 127.`}, {550.`, 134.`}, {554.`, 137.`}, {567.`, 150.`},
   {572.`, 159.`}, {573.`, 170.`}, {573.`, 186.`}, {573.`, 196.`}, {573.`, 200.`},
   {648.`, 770.`}, {620.`, 776.`}, {610.`, 779.`}, {599.`, 783.`}, {588.`, 786.`},
   {515.`, 822.`}, {513.`, 824.`}, {501.`, 834.`}, {550.`, 744.`}, {537.`, 729.`},
   {517.`, 723.`}, {484.`, 728.`}, {455.`, 751.`}, {446.`, 760.`}, {538.`, 719.`},
   {536.`, 713.`}, {523.`, 687.`}, {491.`, 670.`}, {470.`, 675.`}, {466.`, 678.`},
   {440.`, 695.`}, {427.`, 706.`}, {425.`, 714.`}, {702.`, 508.`}, {702.`, 505.`},
   {697.`, 486.`}, {695.`, 478.`}, {687.`, 451.`}, {685.`, 427.`}, {685.`, 424.`},
   {682.`, 408.`}, {669.`, 396.`}, {662.`, 378.`}, {654.`, 351.`}, {648.`, 339.`},
   {638.`, 329.`}, {633.`, 322.`}, {630.`, 315.`}, {626.`, 307.`}, {618.`, 300.`},
   {612.`, 295.`}, {600.`, 286.`}, {595.`, 276.`}, {592.`, 268.`}, {589.`, 263.`},
```




```
ones = ConstantArray[{1}, Length[coords]];
coordsTemp = coords;
coords2 = Join[Transpose@coordsTemp, Transpose@ones];
```

```
Dimensions[coords2]
```

```
{3, 511}
```

```
Manipulate[
```

$$T = \begin{pmatrix} 1 & 0 & 500 * \mathrm{Sin}[\theta] \\ 0 & 1 & 500 * \mathrm{Cos}[\theta] \\ 0 & 0 & 1 \end{pmatrix};$$

$$R = \begin{pmatrix} \mathrm{Cos}[\theta] & \mathrm{Sin}[\theta] & 0 \\ -\mathrm{Sin}[\theta] & \mathrm{Cos}[\theta] & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

$$S = \begin{pmatrix} \mathrm{Sin}[\theta] + 1 & 0 & 0 \\ 0 & \mathrm{Sin}[\theta] & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

```
 data = Transpose[(S.R.T.coords2)[[{1, 2}]]];
 ListPlot[data, PlotRange → {{-2000, 2000}, {-2000, 2000}}], {θ, 0, 2 Pi}]
```

θ

```
ListPlot[Transpose[{{1., 0., 0.}, {0., 0., 0.}, {0., 0., 1.}}.coords2],
 PlotRange → {{-2000, 2000}, {-2000, 2000}}]
```

```
data
```

```
{{0.484759, 0.622509, 0.788992}, {-0.622509, 0.484759, 0.}, {0., 0., 1.}}.points2
```

①

QEA BSET 0
DIFFERENCE
EQUATIONS

$$X_{n+1} = 2X_n + 3$$

$$X_0 = X_0$$
$$X_1 = 2X_0 + 3$$
$$X_2 = 2(2X_0+3) + 3 = 4X_0 + 6$$
$$= 2^{\cancel{*}n}X_0 + 2^{n-1} \cdot 3$$

$$X_n = 2^n X_0 + 2^{n-1} \cdot 3$$

②

$$W_3 = \underbrace{\begin{pmatrix} 1 & 1 \\ \phi & 0 \end{pmatrix}}_{2\times 2} \underbrace{\begin{pmatrix} X_3 \\ X_2 \end{pmatrix}}_{2\times 1} = \underbrace{\begin{pmatrix} X_3+X_2 \\ X_3 \end{pmatrix}}_{2\times 1} = \begin{pmatrix} X_4 \\ X_3 \end{pmatrix}$$

$$A = \begin{pmatrix} \cancel{2}1 & 1 \\ 1 & 0 \end{pmatrix} \qquad \det(A-\lambda I) \neq 0$$

$$A - \lambda I = \begin{pmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{pmatrix} \cancel{=} 0$$

$$\det(A-\lambda I) = 0 = (1-\lambda)(-\lambda) - (1)(1)$$

$$\cancel{-\lambda + \lambda^2 2}$$

$$\lambda^2 - \lambda - 1 = 0$$

$$\cancel{\lambda} \quad \lambda = \frac{1 \pm \sqrt{1+4}}{2}$$

$$\lambda_1 = \frac{1+\sqrt{5}}{2} \qquad \lambda_2 = \frac{1-\sqrt{5}}{2}$$

For $\lambda = \frac{1+\sqrt{5}}{2}$,

$$\begin{bmatrix} 1-\left(\frac{1+\sqrt{5}}{2}\right) & 1 \\ 1 & -\left(\frac{1+\sqrt{5}}{2}\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1+\sqrt{5}}{2} & 1 \\ 1 & \frac{-1+\sqrt{5}}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$V_1 = \begin{pmatrix} \frac{1+\sqrt{5}}{2} \\ 1 \end{pmatrix}$$

$$V_2 = \begin{pmatrix} \frac{1-\sqrt{5}}{2} \\ 1 \end{pmatrix}$$

DIFFERENCE EQUATIONS

3 $w_n = c_1 \lambda_1^n v_1 + c_2 \lambda_2^n v_2$

4

DIFFERENCE EQUATIONS

$$w_n = c_1 \lambda_1^n v_1 + c_2 \lambda_2^n v_2$$

$$w_0 = \begin{pmatrix}1\\1\end{pmatrix} = c_1(\sim)^0 \begin{pmatrix}\frac{1+\sqrt{5}}{2}\\1\end{pmatrix} + c_2(\sim)^0 \begin{pmatrix}\frac{1-\sqrt{5}}{2}\\1\end{pmatrix}$$

$$\cancel{\begin{pmatrix}1\\1\end{pmatrix} = c_1}$$

$$1 = c_1 \frac{1+\sqrt{5}}{2} + c_2 \frac{1-\sqrt{5}}{2}$$

$$1 = c_1 + c_2$$

$$c_1 = \frac{1}{10}(5+\sqrt{5}), \qquad c_2 = \frac{1}{10}(5-\sqrt{5})$$

$$w_n = \frac{1}{10}(5+\sqrt{5})\left(\frac{1+\sqrt{5}}{2}\right)^n \begin{pmatrix}\frac{1+\sqrt{5}}{2}\\1\end{pmatrix} + \frac{1}{10}(5-\sqrt{5})\left(\frac{1-\sqrt{5}}{2}\right)^n \begin{pmatrix}\frac{1+\sqrt{5}}{2}\\1\end{pmatrix}$$

$x_n = \cancel{\frac{1}{10}(5+\sqrt{5})}$

5

$$x_n = \frac{1}{10}(5+\sqrt{5})\left(\frac{1+\sqrt{5}}{2}\right)^n(1) + \frac{1}{10}(5-\sqrt{5})\left(\frac{1-\sqrt{5}}{2}\right)^n(1)$$

$$x_{100} = 5.73 \times 10^{20}$$

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$\{\{1, 1\}, \{1, 0\}\}$

{{λ1, λ2}, {v1, v2}} = Eigensystem[A]

$$\left\{\left\{\frac{1}{2}\left(1+\sqrt{5}\right), \frac{1}{2}\left(1-\sqrt{5}\right)\right\}, \left\{\left\{\frac{1}{2}\left(1+\sqrt{5}\right), 1\right\}, \left\{\frac{1}{2}\left(1-\sqrt{5}\right), 1\right\}\right\}\right\}$$

Solve[1 == c1 * v1[[1]] + c2 * v2[[1]] && 1 == c1 * v1[[2]] + c2 * v2[[2]], {c1, c2}]

$$\left\{\left\{c1 \to \frac{1}{10}\left(5+\sqrt{5}\right), c2 \to \frac{1}{10}\left(5-\sqrt{5}\right)\right\}\right\}$$

$$\mathbf{c1 = \frac{1}{10}\left(5+\sqrt{5}\right)}$$

$$\mathbf{c2 = \frac{1}{10}\left(5-\sqrt{5}\right)}$$

$$\frac{1}{10}\left(5+\sqrt{5}\right)$$

$$\frac{1}{10}\left(5-\sqrt{5}\right)$$

w[n_] = c1 * λ1^n + c2 * λ2^n

$$\frac{1}{5} \times 2^{-1-n}\left(1-\sqrt{5}\right)^n\left(5-\sqrt{5}\right) + \frac{1}{5} \times 2^{-1-n}\left(1+\sqrt{5}\right)^n\left(5+\sqrt{5}\right)$$

N@w[100]

5.73148×10^{20}

In [1]:
```
%matplotlib inline
import matplotlib.pyplot as plt

from scipy import signal
import sounddevice as sd
import numpy as np
from scipy.io.wavfile import write
from time import sleep
```

Test recording / playing sounds

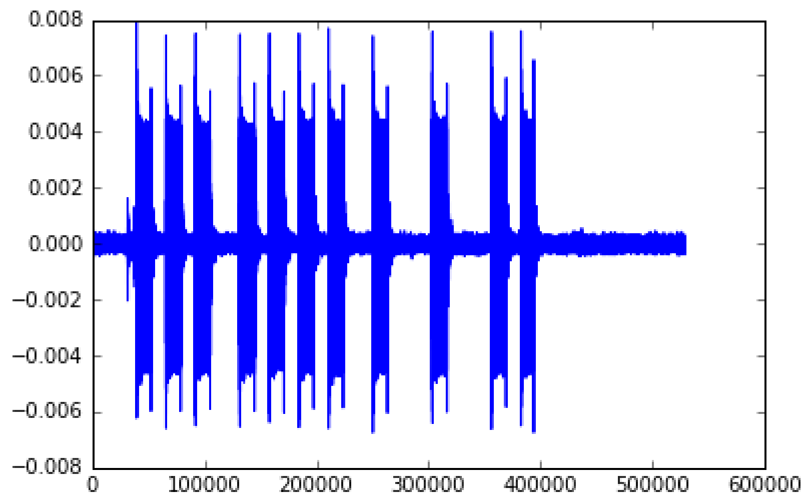
In [4]:
```
duration = 12
fs = 44100
myrecording = sd.rec(duration * fs, samplerate=fs, channels=1)
print("Done!")
```

```
Done!
```

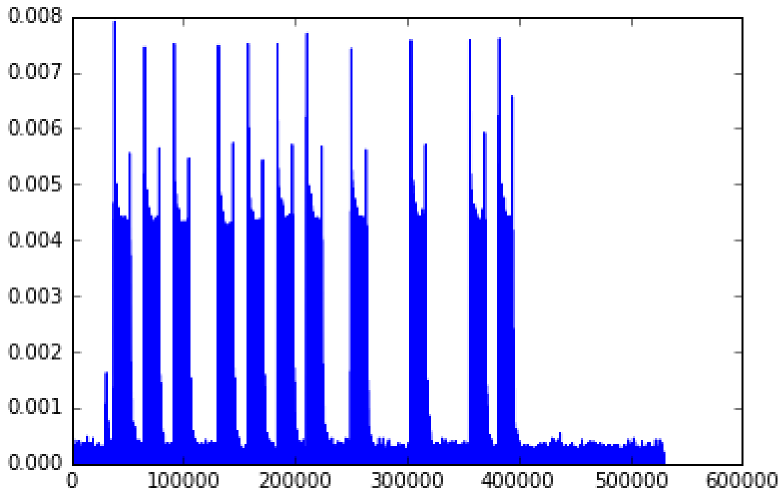
In [5]:
```
sd.play(myrecording, fs)
```

In [6]:
```
print(type(myrecording))
print(len(myrecording))
print(myrecording)
plt.plot(myrecording)
plt.show()
```

```
<class 'numpy.ndarray'>
529200
[[ -1.22070312e-04]
 [ -1.52587891e-04]
 [ -6.40869141e-04]
 ...,
 [  9.15527344e-05]
 [  2.13623047e-04]
 [  1.22070312e-04]]
```

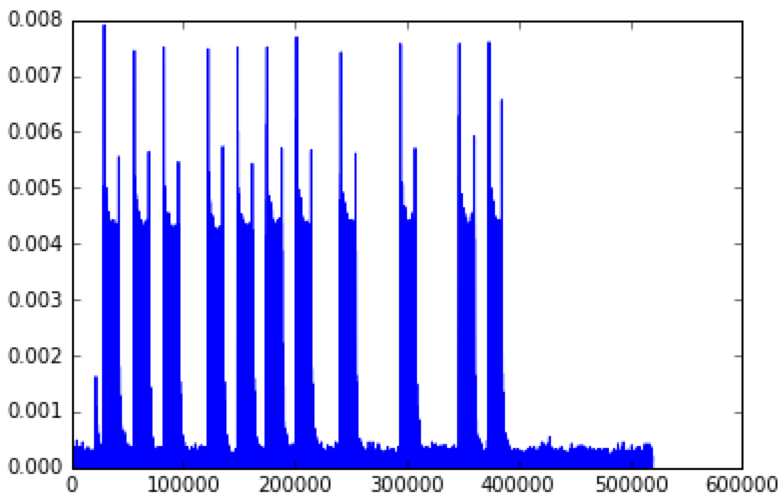


In [7]:
```
for i in range(myrecording.size):
    if myrecording[i] < 0:
        myrecording[i] = 0
plt.plot(myrecording)
plt.show()
```



In [46]:
```
def find_start(array):
    avg = []
    for i in range(array.size - 14700):
        avg.append(np.mean(array[i:i+14700]))

    try:
        return array[np.argmax(avg) % 14700:]
    except:
        print("no max")
        return array


test = find_start(myrecording)
plt.plot(test)
```

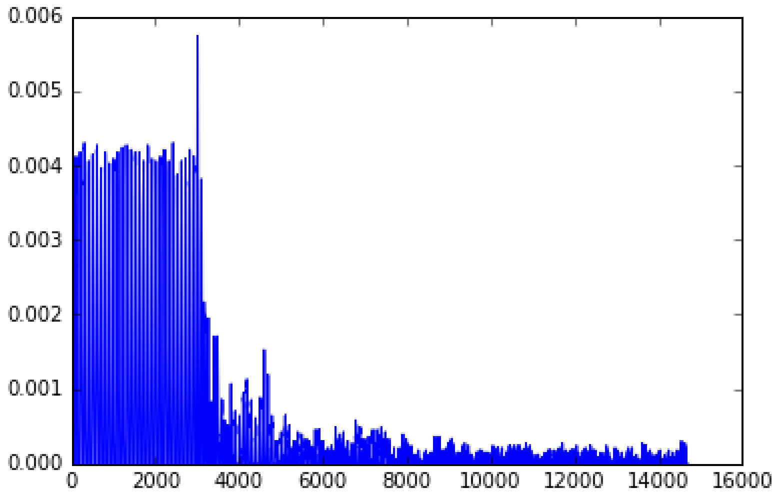
Out[46]:
```
[<matplotlib.lines.Line2D at 0x7fafbef3de48>]
```



In [60]:
```
print(test.size)
i=9
plt.plot(test[i*14700:(i+1)*14700])
np.mean(test[i*14700:(i+1)*14700])
```

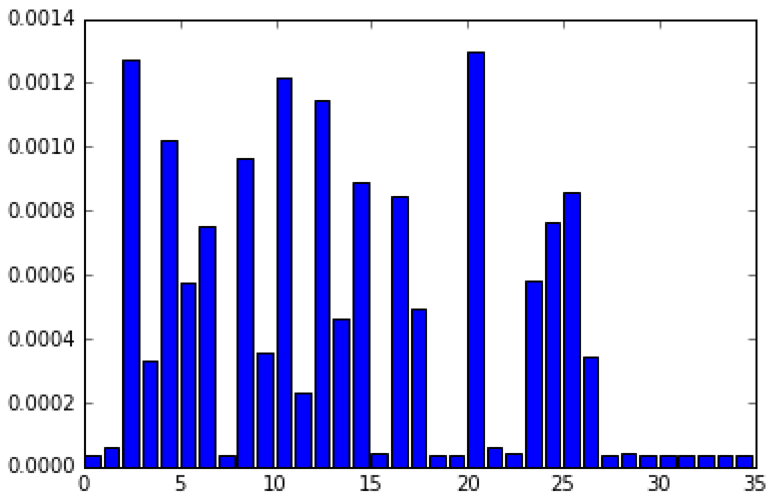
```
519850
```

Out[60]: 0.00035786323



In [61]:
```
avg = []
for i in range(int(test.size/14700)):
    avg.append(np.mean(test[i*14700:(i+1)*14700]))
plt.bar(range(len(avg)),avg)
# [1,1,0,1,1,1,1,1,0,1,0,0,1,0,0,1]
```

Out[61]: <Container object of 35 artists>



Test generating sounds

In [2]:
```
def zero_or_one(tone_array, i, one_tone, zero_tone):

    if i == 0:
        tone_array = np.concatenate((tone_array, zero_tone))
        tone_array = np.concatenate((tone_array, zero_tone))
        print("zero")
    else:
        tone_array = np.concatenate((tone_array, zero_tone))
        tone_array = np.concatenate((tone_array, one_tone))
        print("one")

    return tone_array
```

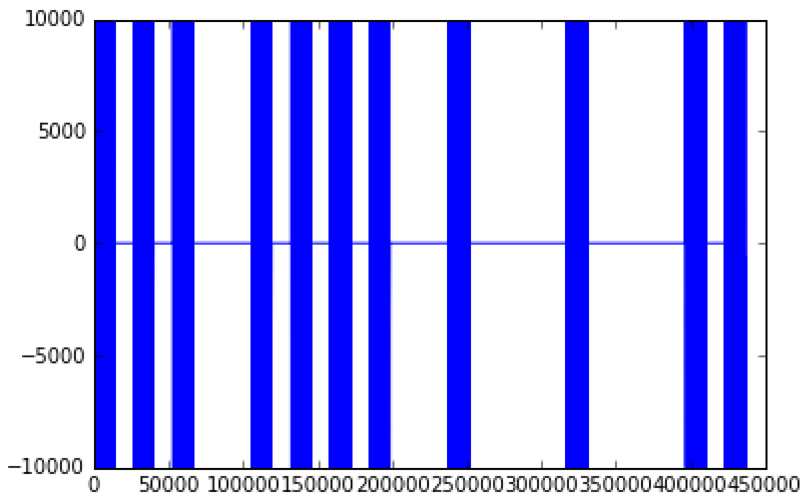
In [3]:
```
sd.default.samplerate = 44100

time = .3
frequency = 440

# Generate time of samples between 0 and time seconds
samples = np.arange(44100 * time) / 44100.0
# Recall that a sinusoidal wave of frequency f has formula w(t) = A*s
in(2*pi*f*t)

one_tone = 10000 * np.sin(2 * np.pi * frequency * samples)
zero_tone = samples * 0

# Convert it to wav format (16 bits)
```

In [4]:
```
#start with one
tone_array = one_tone

for i in [1,1,0,1,1,1,1,0,1,0,0,1,0,0,1]:
    tone_array = zero_or_one(tone_array, i, one_tone, zero_tone)

#end with one
tone_array = zero_or_one(tone_array, 1, one_tone, zero_tone)

tone_array = np.array(tone_array, dtype=np.int16)
```

```
one
one
zero
one
one
one
one
zero
one
zero
zero
one
zero
zero
one
one
```

In [15]:
```
sd.play(tone_array, blocking=True)
plt.plot(tone_array)
```

Out[15]: [<matplotlib.lines.Line2D at 0x7f8050967048>]



In [13]:

Out[13]: 3819.4160544217689

In [ ]:

In [ ]:
```
np.mean(myrecording)
```

In [ ]:
```
```