QEA - Differential Equations

In[1]:=
```
SetDirectory@NotebookDirectory[];
<< "../General.m"
```

Linear Equations Example 2

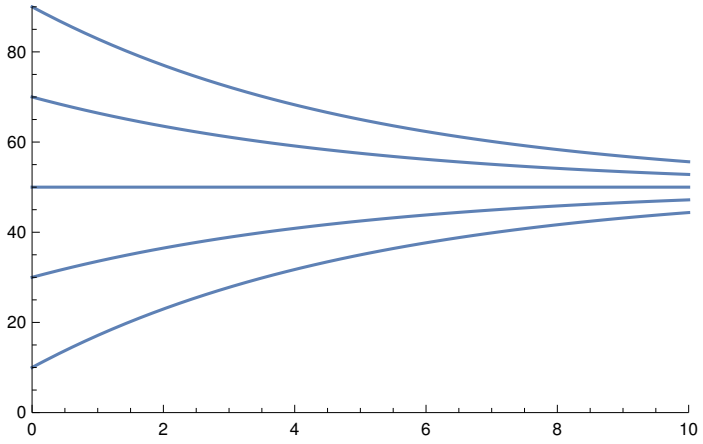
```
With[{context = "linearequationsexample2`"}, If[Context[] ≠ context, Begin[context]]];
Dynamic[Refresh[Context[], UpdateInterval → 1]]
```

```
Global`
```

```
With[{},
 sol = DSolveValue[{v'[t] == 9.8 - .196 v[t]}, v[t], t]
]
```

```
50. + ℯ^-0.196 t C[1]
```

```
Plot[Table[sol /. C[1] → v0, {v0, {Range[-40, 40, 20]}}],
 {t, 0, 10}, PlotRange → {{0, 10}, {0, 90}}]
```




```
With[{context = "linearequationsexample2`"},
 If[Context[] == context, End[], "Not in context"]]
```

```
linearequationsexample2`
```

Linear Equations Example 3

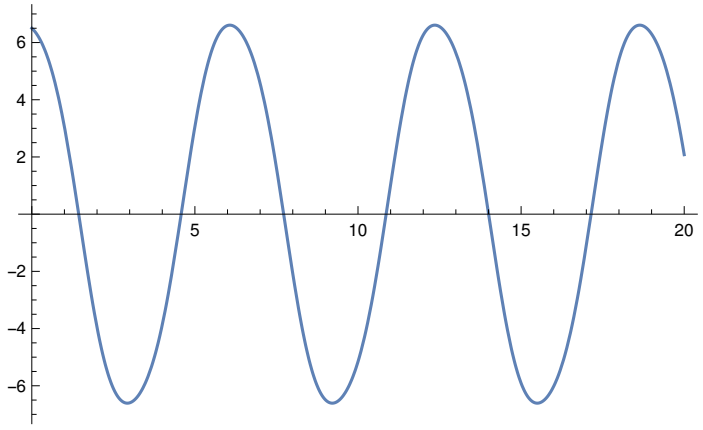
```
With[{context = "linearequationsexample3`"}, If[Context[] ≠ context, Begin[context]]];
Dynamic[Refresh[Context[], UpdateInterval → 1]]
```

```
Global`
```

```
With[{},
 sol = DSolveValue[
   {Cos[x] y'[x] + Sin[x] y[x] == 2 Cos[x]^3 Sin[x] - 1, y[π/4] == 3 Sqrt[2]}, y[x], x]
]
```

$$\frac{1}{2}\left(14\cos[x]-\cos[x]\cos[2x]-2\sin[x]\right)$$

```
Plot[sol, {x, 0, 20}]
```




```
With[{context = "linearequationsexample3`"},
 If[Context[] == context, End[], "Not in context"]]
```

```
linearequationsexample3`
```

Linear Equations Example 4

```
With[{context = "linearequationsexample4`"}, If[Context[] ≠ context, Begin[context]]];
Dynamic[Refresh[Context[], UpdateInterval → 1]]
```

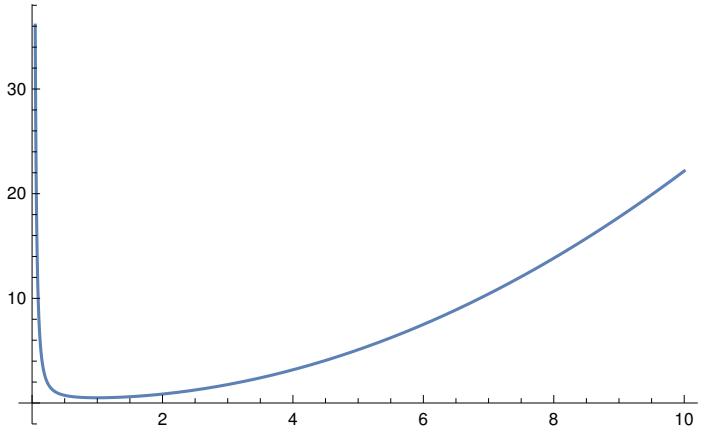
```
Global`
```

$$t\,y'+2y=t^2-t+1 \qquad y(1)=\frac{1}{2}$$

```
With[{
   de = {t y'[t] + 2 y[t] == t^2 - t + 1},
   initialConditions = {y[1] == 1/2}
  },
  sol = DSolveValue[Join[de, initialConditions], y[t], t]
]
```

$$\frac{1+6\,t^2-4\,t^3+3\,t^4}{12\,t^2}$$

```
Plot[sol, {t, 0, 10}]
```




```
With[{context = "linearequationsexample4`"},
 If[Context[] == context, End[], "Not in context"]]
```

```
linearequationsexample4`
```

Separable Equations Example 1

```
With[{context = "separableequationsexample1`"},
 If[Context[] ≠ context, Begin[context]]];
Dynamic[Refresh[Context[], UpdateInterval → 1]]
```

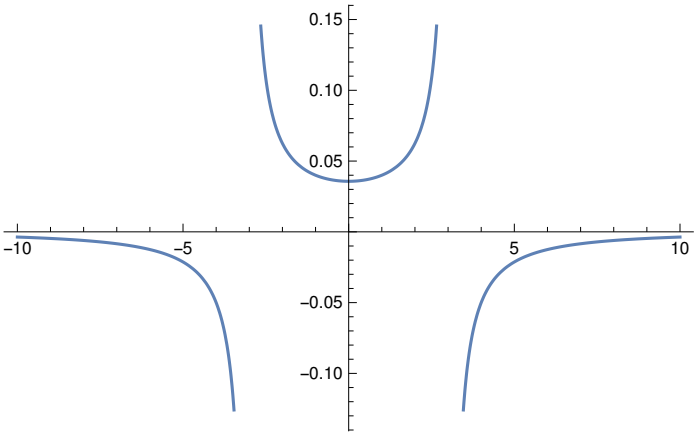
```
Global`
```

$$\frac{dy}{dx} = 6y^2x \qquad y(1) = \frac{1}{25}$$

```
With[{
   de = {y'[x] == 6 y[x]^2 x},
   initialConditions = {y[1] == 1/25}
  },
 sol = DSolveValue[Join[de, initialConditions], y[x], x]
]
```

$$\frac{1}{28 - 3x^2}$$

```
Plot[sol, {x, -10, 10}]
```




```
With[{context = "separableequationsexample1`"},
 If[Context[] == context, End[], "Not in context"]]
```

```
separableequationsexample1`
```

```
makeTemplate["Separable Equations Example 2"]
```

Separable Equations Example 2

```
With[{context = "separableequationsexample2`"},
 If[Context[] ≠ context, Begin[context]]];
Dynamic[Refresh[Context[], UpdateInterval → 1]]
```

```
Global`
```

$$y' = \frac{3x^2 + 4x - 4}{2y - 4} \qquad y(1) = 3$$

```
With[{
   de = {y'[x] == (3 x^2 + 4 x - 4)/(2 y[x] - 4)},
   initialConditions = {y[1] == 3}
  },
  sol = DSolveValue[Join[de, initialConditions], y[x], x]
]
```

DSolveValue: For some branches of the general solution, the given boundary conditions lead to an empty solution.

$$2 + \sqrt{2 - 4x + 2x^2 + x^3}$$

```
sol /. x → -5
```

$$2 + i\sqrt{53}$$

```
Reduce[2 + Sqrt[2 - 4 x + 2 x^2 + x^3] >= 0, x]
```

$$x \geq \text{Root}\left[2 - 4\#1 + 2\#1^2 + \#1^3 \&,\ 1\right]$$

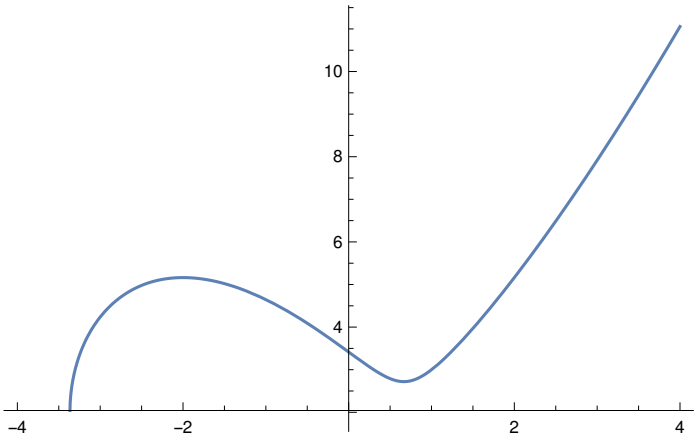
```
sol
```

$$2 + \sqrt{2 - 4x + 2x^2 + x^3}$$

```
Solve[2 + Sqrt[2 - 4 x + 2 x^2 + x^3] == 0, x]
```

$$\{\}$$

```
Plot[sol, {x, -4, 4}]
```




```
With[{context = "separableequationsexample2`"},
 If[Context[] == context, End[], "Not in context"]]
```

```
separableequationsexample2`
```

Separable Equations Example 3

```
With[{context = "separableequationsexample3`"},
 If[Context[] ≠ context, Begin[context]]];
Dynamic[Refresh[Context[], UpdateInterval → 1]]
```

```
Global`
```

$$y' = \frac{xy^3}{\sqrt{1+x^2}} \qquad y(0) = -1$$

```
With[{
  de = {y'[x] == x y[x]^3 / Sqrt[1 + x^2]},
  initialConditions = {y[0] == -1}
 },
 sol = DSolveValue[Join[de, initialConditions], y[x], x]
]
```

DSolveValue: For some branches of the general solution, the given boundary conditions lead to an empty solution.

$$-\frac{1}{\sqrt{3-2\sqrt{1+x^2}}}$$

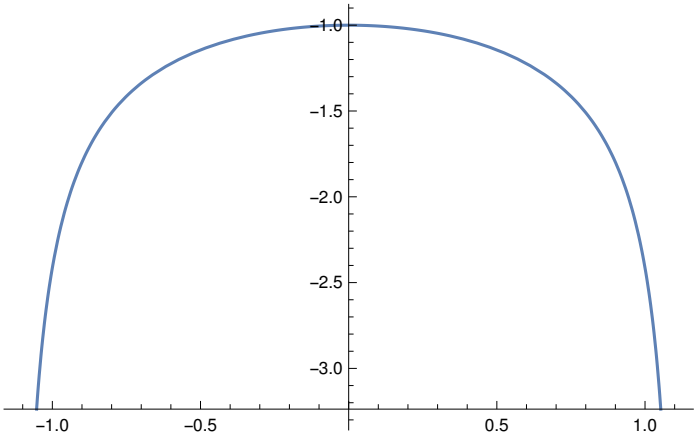
```
validity = Solve[Denominator[sol] == 0, x]
```

$$\left\{\left\{x \to -\frac{\sqrt{5}}{2}\right\}, \left\{x \to \frac{\sqrt{5}}{2}\right\}\right\}$$

```
x /. validity
```

$$\left\{-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right\}$$

```
Plot[sol, {x, -Sqrt[5]/2, Sqrt[5]/2}]
```



```
With[{context = "separableequationsexample3`"},
 If[Context[] == context, End[], "Not in context"]]
```

```
separableequationsexample3`
```

Separable Equations Example 4

```
With[{context = "separableequationsexample4`"},
 If[Context[] ≠ context, Begin[context]]];
Dynamic[Refresh[Context[], UpdateInterval → 1]]
```

```
Global`
```

$$y' = e^{-y}(2x-4) \qquad y(5) = 0$$

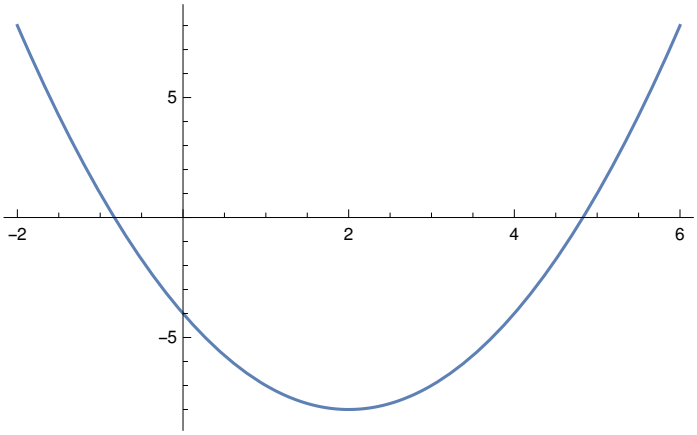
In[89]:=
```
With[{
  de = {y'[x] == e^-y[x] (2 x - 4)},
  initialConditions = {y[5] == 0}
 },
 sol = DSolveValue[Join[de, initialConditions], y[x], x]
]
```

... Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[89]=
$$\text{Log}\left[-4+2\left(-2x+\frac{x^2}{2}\right)\right]$$

In[99]:=
```
Plot[-4 + 2 (-2 x + x^2/2), {x, -2, 6}]
```

Out[99]=

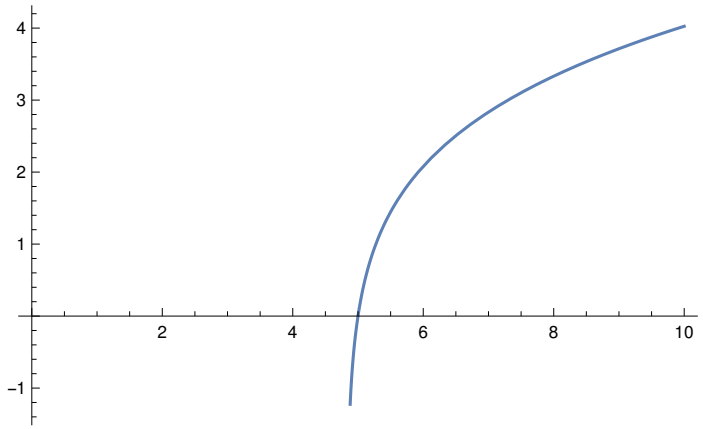


In[100]:=
```
validity = Solve[-4 + 2 (-2 x + x^2/2) == 0, x];
x /. validity
```

Out[101]= $\left\{2\left(1-\sqrt{2}\right), 2\left(1+\sqrt{2}\right)\right\}$

In[97]:=
```
Plot[sol, {x, 0, 10}]
```

Out[97]=



In[102]:=
```
With[{context = "separableequationsexample4`"},
 If[Context[] == context, End[], "Not in context"]]
```

Out[102]= separableequationsexample4`

Separable Equations Example 5

In[104]:=
```
With[{context = "separableequationsexample5`"},
 If[Context[] ≠ context, Begin[context]]];
Dynamic[Refresh[Context[], UpdateInterval → 1]]
```

Out[104]= Global`

$$\frac{dr}{d\theta} = \frac{r^2}{\theta} \qquad r(1) = 2$$

In[108]:=
```
With[{
  de = {r'[θ] == r[θ]^2/θ},
  initialConditions = {r[1] == 2}
 },
 sol = DSolveValue[Join[de, initialConditions], r[θ], θ]
]
```

Out[108]=
$$-\frac{2}{-1+2\,\text{Log}[\theta]}$$

In[114]:=
```
Denominator[-2/(-1 + 2 Log[θ])]
```

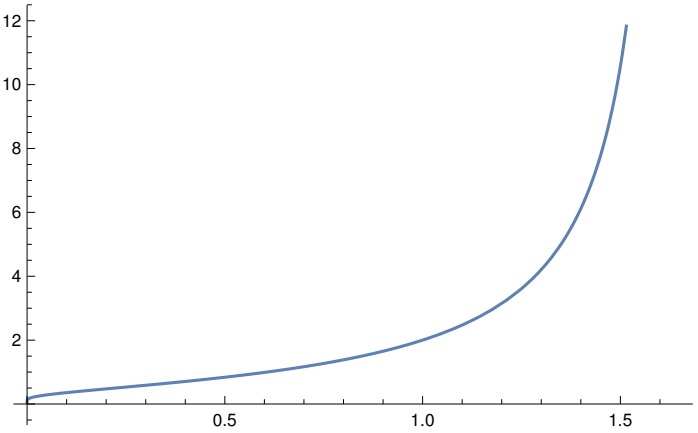
Out[114]=
$$-1+2\,\text{Log}[\theta]$$

In[115]:=
```
Solve[-1 + 2 Log[θ] == 0, θ]
```

Out[115]=
$$\left\{\left\{\theta \to \sqrt{e}\right\}\right\}$$

In[120]:=
```
Plot[sol, {θ, 0, Sqrt[e]}]
```

Out[120]=



In[122]:=
```
With[{context = "separableequationsexample5`"},
 If[Context[] == context, End[], "Not in context"]]
```

Out[122]=
```
separableequationsexample5`
```

Modeling Example 1

In[4]:=
```
With[{context = "modelingexample1`"}, If[Context[] ≠ context, Begin[context]]];
Dynamic[Refresh[Context[], UpdateInterval → 1]]
```

Out[4]= Global`

$$Q'(t) = (9)\left(\frac{1}{5}(1+\cos(t))\right) - (6)\left(\frac{Q(t)}{600+3t}\right) \qquad Q(0) = 5$$

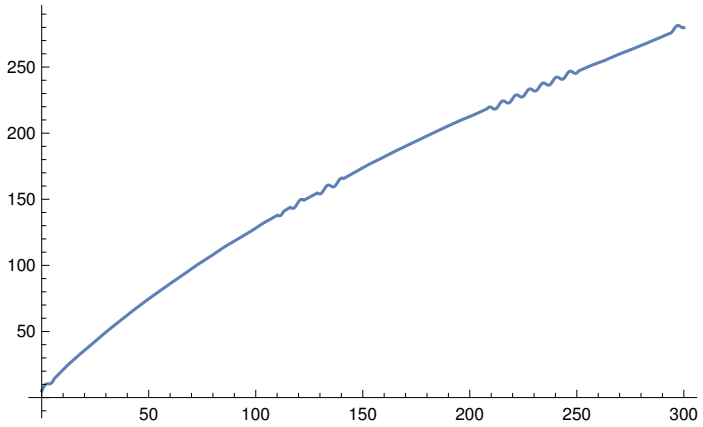
$$Q'(t) = \frac{9}{5}(1+\cos(t)) - \frac{2Q(t)}{200+t} \qquad Q(0) = 5$$

In[5]:=
```
With[{
  de = {q'[t] == 9/5 (1 + Cos[t]) - 2 q[t]/(200 + t)},
  initialConditions = {q[0] == 5}
 },
 sol = DSolveValue[Join[de, initialConditions], q[t], t]
]
```

Out[5]=
$$\frac{1}{5\,(200+t)^2}\left(996\,400 + 360\,000\,t + 1800\,t^2 + 3\,t^3 + 3600\,\text{Cos}[t] + 18\,t\,\text{Cos}[t] + 359\,982\,\text{Sin}[t] + 3600\,t\,\text{Sin}[t] + 9\,t^2\,\text{Sin}[t]\right)$$

In[9]:=
```
Plot[sol, {t, 0, 300}]
```

Out[9]=



In[10]:=
```
With[{context = "modelingexample1`"}, If[Context[] == context, End[], "Not in context"]]
```

Out[10]= modelingexample1`

In[11]:=
```
exportNotebookPDF[]
```