1

$$\frac{d\,y(t)}{dt} + 2y(t) = x(t)$$

$$\mathcal{L}\left\{\frac{d}{dt}f(t)\right\} = sF(s) - f(0^-)$$

$$\mathcal{L}\left\{\frac{d^2}{dt^2}f(t)\right\} = s^2F(s) - sf(0^-) - f'(0^-)$$

$$sY(s) - y(0^-) + 2Y(s) = X(s)$$

$$y(0^-) = 1, \quad x(t) = u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{else} \end{cases}$$

$$sY(s) - 1 + 2Y(s) = \frac{1}{s}$$

$$(s+2)Y(s) = \frac{s+1}{s}$$

$$Y(s) = \frac{s+1}{s(s+2)}$$

$$Y(s) = \frac{1}{2s} + \frac{1}{2(2+s)}$$

$$y(t) = \frac{1}{2}u(t) + \frac{1}{2}e^{-2t} \quad \text{for } t \geq 0$$

2 a

$$m \frac{d^2}{dt} y(t) + c \frac{d}{dt} y(t) + k\,y(t) = c \frac{d}{dt} x(t) + k\,x(t)$$

$x(t) = u(t)$

$y(0^-) = 0, \; x(0^-) = 0, \; y'(0^-) = 0$

$$m s^2 Y(s) + c Y(s) + k Y(s) = c X(s) + k X(s)$$

$$Y(s)(ms^2 + cs + k) = (cs + k) X(s),$$

$$Y(s) = \frac{cs+k}{ms^2+cs+k} \quad \frac{1}{s}$$

(b) $$Y(s) = \frac{cs + k}{s(ms^2 + cs + k)}$$

$$Y(s) = \frac{k}{a_1 a_2 s} + \frac{a_1 c + k}{a(a_1 - a_2)(s - a_1)} + \frac{a_2 c + k}{a_2(a_2 - a_1)(s - a_2)}$$

a_1 a_2 are roots of

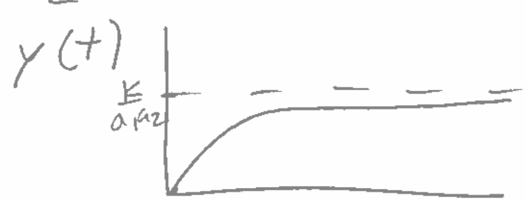
$$ms^2 + cs + k$$

$$a_1 = \frac{-c + \sqrt{c^2 - 4mk}}{2m}$$

$$a_2 = \frac{-c - \sqrt{c^2 - 4mk}}{2m}$$

(c) $$y(t) = \frac{k}{a_1 a_2} u(t) + \frac{a_1 c + k}{a_1(a_1 - a_2)} e^{a_1 t} + \frac{a_2 c + k}{a_2(a_2 - a_1)} e^{a_2 t}, \quad t \geq 0$$

$a_1, a_2 \in -\mathbb{R}$



$a_1, a_2 \in \mathbb{C}$

$e^{(\sigma + j\omega)t}$