QEA
BSET 1
Nathan Yee

1

a

10,000 Hz $\Rightarrow$ 10 $\frac{\text{samples}}{\text{cycle}}$

5,000 Hz $\rightarrow$ 5 $\frac{\text{samples}}{\text{cycle}}$

b Increase sampling period:

- Less samples
- Lower resolution representation

c ~~We only s~~

i We sample at a much slower rate

ii 2000 Hz $\rightarrow$ 2 samples per cycle
1000 Hz $\rightarrow$ 1 sample per cycle

iii All samples have magnitude 0.

d 2000 Hz $\rightarrow$ samples at peaks and valleys
1000 Hz $\rightarrow$ samples at valleys only

(2)

(a) Both setups produce a CT sound of 440 Hz. This is because our sampling rate for a 2560 Hz waveform ~~a~~ is the same as a 440 Hz signal sampled at 3000 Hz.

(b) I would expect to hear 440 Hz and 2560 Hz. The ~~high saw~~ faster sampling rate should be able to capture both frequencies.

(3)

~~wb has a very~~ ~~heights~~
wb has equalish ~~amplitudes~~ for a very wide band of frequencies

lf has high amplitudes for only low frequencies

~~wb is~~
wb sounds higher pitch than lf

(4) $Ae^{j\theta} \cdot Be^{j\phi} = ABe^{j(\theta+\phi)} = AB\cos(\theta+\phi) + jAB\sin(\theta+\phi)$

$|e^{j\theta}| = 1$

$e^{j\pi} = -1 + j0$

$e^{j2\pi} = 1 + j0$

5

$\begin{pmatrix}2\\3\end{pmatrix}$ $\quad c_1 = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}$ $\quad c_2 = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix}$

$$q = (c_1^T q)\, c_1 + (c_2^T q)\, c_2$$

$$\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}2\\3\end{pmatrix} c_1 + \begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}2\\3\end{pmatrix} c_2$$

$$\begin{pmatrix}2\\3\end{pmatrix} = \left(\frac{3}{\sqrt{2}} + \sqrt{2}\right) c_1 + \left(-\frac{3}{\sqrt{2}} + \sqrt{2}\right) c_2$$

$$\begin{pmatrix}2\\3\end{pmatrix} = \left(\frac{5}{\sqrt{2}}\right) c_1 + \left(-\frac{1}{\sqrt{2}}\right) c_2$$

$$\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$$

⑥ $N = 8, \ i = 0, \ k = 1$

$V_i^H \cdot V_k = 0$

Mathematica

⑦ Geometric summation formula

$\cancel{S_N = a}$

$$\sum_{k=0}^{N-1} (ar^k) = a\left(\frac{1-r^n}{1-r}\right)$$

$$\frac{1}{N} \sum_{l=0}^{N-1} \left(e^{j2\pi \frac{k-i}{N}}\right)^{l}$$

$$\frac{1}{N}\left(\frac{1-\left(e^{j2\pi \frac{k-i}{N}}\right)^{N}}{1-\left(e^{j2\pi \frac{k-i}{N}}\right)}\right)$$

$$\frac{1}{N}\left(\frac{1-e^{j2\pi(k-i)}}{1-e^{\left(j2\pi \frac{k-i}{N}\right)}}\right)$$

$$\frac{1}{N}\left(\frac{1-1}{1-\text{not } 1}\right)$$

$$\frac{1}{N}(0) = 0$$

8

$$V_i = V_k$$

$$|V_i| = 1$$

$$|V_k| = 1$$

$$V_i \cdot V_k = |V_i||V_k|\cos(0)$$

$$V_i \cdot V_k = 1$$

9

$$x = a_0 v_0 + a_1 v_1 + \dots + a_n v_n$$

$$v_i^H x = v_i^H (a_0 v_0 + a_1 v_1 + \dots + a_n v_n)$$

$$v_i^H x = a_i v_i^H v_i$$

$$v_i^H x = a_i$$

$$x = (v_0^H x) v_0 + (v_1^H x) v_1 + (v_2^H x) v_2 + \dots + (v_{N-1}^H x) v_N$$

(10)

$$X = \begin{pmatrix} \cos(\frac{\pi}{4}\cdot 0) \\ \cos(\frac{\pi}{4}\cdot 1) \\ \vdots \\ \cos(\frac{\pi}{4}\cdot 63) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}e^{j\frac{\pi}{4}\cdot 0} + \frac{1}{2}e^{-j\frac{\pi}{4}\cdot 0} \\ \frac{1}{2}e^{j\frac{\pi}{4}\cdot 1} + \frac{1}{2}e^{j\frac{\pi}{4}\cdot 1} \\ \vdots \\ \frac{1}{2}e^{j\frac{\pi}{4}\cdot 63} + \frac{1}{2}e^{-j\frac{\pi}{4}\cdot 63} \end{pmatrix}$$

$$X = \underbrace{\frac{1}{2}\begin{pmatrix} e^{j\frac{\pi}{4}\cdot 0} \\ e^{j\frac{\pi}{4}\cdot 1} \\ \vdots \\ e^{j\frac{\pi}{4}\cdot 63} \end{pmatrix}} + \frac{1}{2}\begin{pmatrix} e^{-j\frac{\pi}{4}\cdot 0} \\ e^{-j\frac{\pi}{4}\cdot 1} \\ \vdots \\ e^{-j\frac{\pi}{4}\cdot 63} \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix} e^{j\frac{2\pi}{64}\cdot 8\cdot 1} \\ e^{j\frac{2\pi}{64}\cdot 8\cdot 2} \\ \vdots \\ e^{j\frac{2\pi}{64}\cdot 8\cdot 63} \end{pmatrix} \quad \text{if } i=8$$

$i=56$

$$4V_{56} = \begin{pmatrix} e^{j\frac{2\pi}{4}\cdot 1} \\ \vdots \\ e^{j\frac{2\pi}{4}\cdot 63\cdot 56} \end{pmatrix}$$

$4V_8$ $\qquad$ $4V_{56}$

$$Wx = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 4 \\ \vdots \\ 4 \\ \vdots \\ 0 \end{pmatrix}$$

$8+1 = 9$th entry

$56+1 = 57$th entry

(11) 64 point DFT of vector x

$$x_n = \left(\frac{\pi}{4}(n-1)\right)$$

$$x = \cos\left(pi/4 \cdot [0:1:63]\right)$$

Mathematica

(12) $X_{i+N} = X_i$

(a)

$$X_i = \frac{1}{\sqrt{N}} \sum_{k=0}^{N} x[k]\, e^{-j\frac{2\pi}{N}ik} \qquad (21)$$

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N} x[k]\, e^{j\frac{2\pi}{N}nk} \qquad (2c)$$

$$X_i = \frac{1}{\sqrt{N}} \sum_{k=0}^{N} x[k]\, e^{-j\frac{2\pi}{N}(i+N)k}$$

$$= \frac{1}{\sqrt{N}} \sum_{k=0}^{N} x[k]\, e^{-j\frac{\pi}{N}ki}\; e^{-j\frac{\pi}{N}kN}$$

$$= \frac{1}{\sqrt{N}} \sum_{k=0}^{N} x[k]\, e^{-j\frac{2\pi}{N}ki} \cdot 1 = X_i$$

(b)

(b) N is even

$X_k = X_{-1}$ period $= N$

$X_k = X_{-1+N}$

$k = N-1$

(c) $X_k = X_{-\frac{N}{2}}$

$X_k = X_{-\frac{N}{2}+N}$

$X_k = X_{\frac{N}{2}}$

$k = \frac{N}{2}$