In class 3

6a

$$- r_x r_x \omega_x - r_x r_y \omega_y - \cancel{r_x} r_z \omega_z$$

a

$$L_x = m\left((r_x^2 + r_y^2 + r_z^2)\omega_x - r_x(r_x\omega_x + r_y\omega_y + r_z\omega_z)\right)$$

$$L_y = m\left((r_x^2 + r_y^2 + r_z^2)\omega_y - r_y(r_x\omega_x + r_y\omega_y + r_z\omega_z)\right)$$

$$L_z = m\left((r_x^2 + r_y^2 + r_z^2)\omega_z - r_z(r_x\omega_x + r_y\omega_y + r_z\omega_z)\right)$$

b

$$\begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

$$L_x = I_{xx}\omega_x + I_{xy}\omega_y + I_{xz}\omega_z$$

$$I_{xx} = m\left((r_x^2 + r_y^2 + r_z^2) - r_x r_x\right)$$

$$I_{xy} = m(-r_x r_y)$$

$$I_{xz} = m(-r_x r_z)$$

$$I_{yx} = m(-r_y r_x)$$

$$I_{yy} = m\left((r_x^2 + r_y^2 + r_z^2) - r_y r_y\right)$$

$$I_{zz} = m(-r_y r_z)$$

$$I_{zx} = m(-r_z r_x)$$

$$I_{zy} = m(-r_z r_y)$$

$$I_{zz} = m\left((r_x^2 + r_y^2 + r_z^2) - r_z r_z\right)$$

$$\begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix} = m\begin{bmatrix} r^2 - r_x r_x & -r_x r_y & -r_x r_z \\ -r_y r_x & r^2 - r_y r_y & -r_y r_z \\ -r_z r_x & -r_z r_y & r^2 - r_z r_z \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

$$r^2 = (r_x^2 + r_y^2 + r_z^2)$$

---

c $r^2 - r_x r_x = r_y^2 + r_z^2$

For rotation in ω_x

Distance in yz plane

6 unique terms

6 In class 3

d

$$\left[\begin{array}{l} m\sum_{i=1}^{N} r_i^2 - r_{ix} r_{ix} = \sum_{i=1}^{N} r_{iy}^2 + r_{iz}^2 \\ \sum_{i=1}^{N} - r_{iy} r_{ix} \\ \\ \end{array}\right.$$

e

$$\left[\begin{array}{l} \int r_y^2 + r_z^2 \, dm \\ \int r_y r_x \, dm \\ \\ \end{array}\right.$$

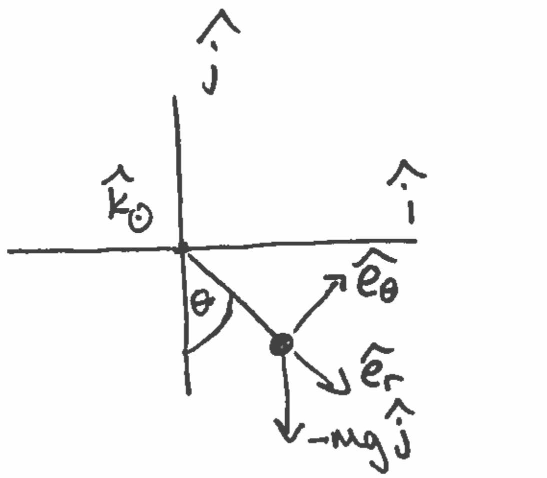
②

BSET 3

$$\sum \vec{M}_o = \frac{d}{dt}\Big|_{o1} \vec{H}_o$$

M = moment/torque

H = angular momentum



Want

$\cancel{\ddot{\theta} = -\frac{g}{l}\sin\theta}$

$$\ddot{\theta} = -\frac{g}{l}\sin\theta$$

$M_o = \vec{r}_{M/o} \times \vec{F}$, $\vec{r}_{M/o} = l\hat{e}_r$

$F = -mg\hat{j}$

$M_o = l\hat{e}_r \times -mg\hat{j}$

$M_o = -mgl\hat{k}\sin(180-\theta)$

$\vec{H}_o = \vec{r}_{M/o} \times M \frac{d}{dt}\Big|_{o1} \vec{r}_{MO}$

$\frac{d}{dt}\Big|_{o1} \vec{r}_{MO} = l\dot{\hat{e}}_r = l\dot{\theta}\hat{e}_\theta$

$\cancel{\vec{H}_o = \vec{r}_{MO} \times l\dot{\theta}\hat{e}_\theta}$

$\vec{H}_o = l\hat{e}_r \times ml\dot{\theta}\hat{e}_\theta$

$\vec{H}_o = m\dot{\theta}l^2\hat{k}$

$\frac{d}{dt}\Big|_{o1} \vec{H}_o = m\ddot{\theta}l^2\hat{k}$

$m\ddot{\theta}l^2\hat{k} = -mgl\sin(180-\theta)\hat{k}$

$\ddot{\theta} = -\frac{g}{l}\sin(180-\theta)$

$\ddot{\theta} = -\frac{g}{l}\sin(\theta)$

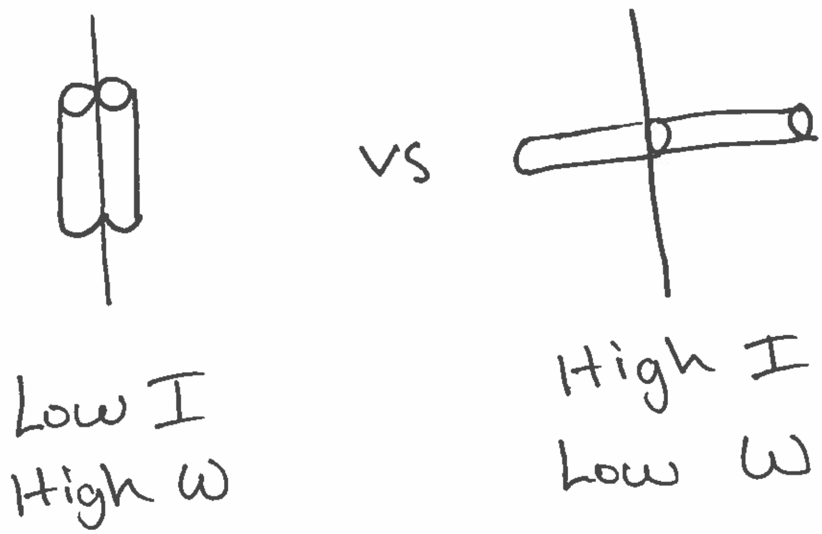
③

ⓐ Angular momentum is conserved because no outside forces.

$$\vec{H} = \sum \vec{H}_o = \Sigma(\vec{r} \times m\vec{v})$$

If some of my r's increase (arms out) then the v must decrease.

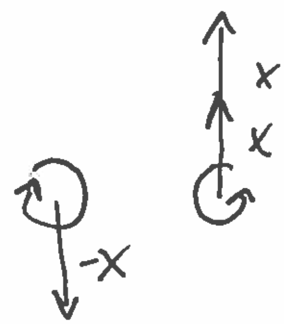
ⓑ



④ When wheel is handed to me, the angular

ⓐ momentum of the system is: wheel $= x$, me $= 0$

after I turn the wheel over; wheel $= -x$, me $= 2x$



ⓑ $\vec{L}_{total} = 3\int_0^{2\pi} \vec{r} \times \vec{v}\, d\theta$

$\vec{r} = 30$ cm

$M = 3$ kg

$\omega = 1.5 \frac{\text{cycles}}{\text{sec}}$

$\omega = 3\pi \frac{\text{rad}}{\text{sec}}$

$= 3\int_0^{2\pi} .3 \times (.3\ 3\pi)\, d\theta$

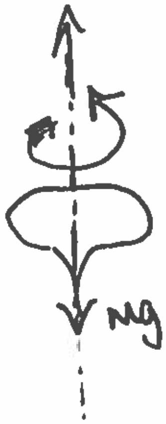
$= 3\int_0^{2\pi}$

$= 16 \frac{kg\, m^2}{s}$

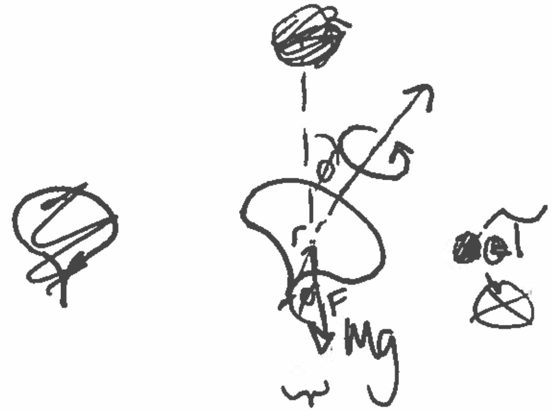
I would be spinning with $2 \cdot 16 \frac{kg\, m^2}{s}$ worth of angular momentum

5

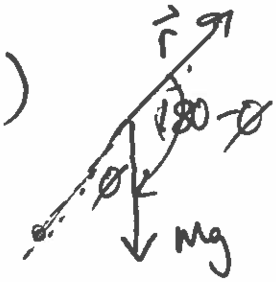
a



b



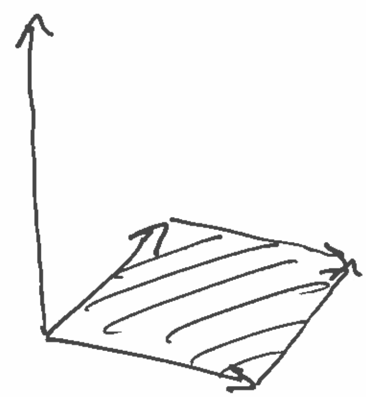
c $\tau = \vec{r} \times \vec{F} = h\,mg\sin(\phi)$



$\tau \otimes$ at com

d $\tau = \frac{d}{dt} L$

$\otimes$ τ is into page



e ~~perpendicular to~~

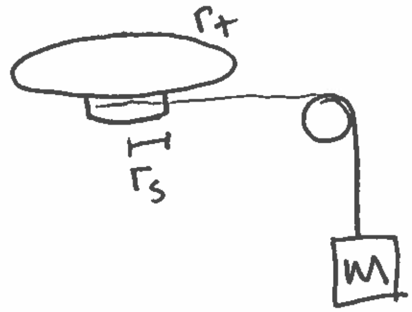
~~Direction of torque~~

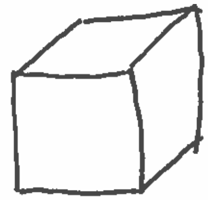
f

(e)  after Δt 

(f) We circle L around Z axis

(6)

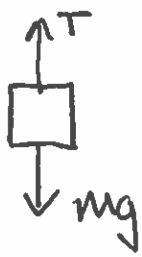




$$I_{Total} = I_{Rotating\ table} + I_{cube}$$

$$I_T = I_R + I_c$$

FBD for mass

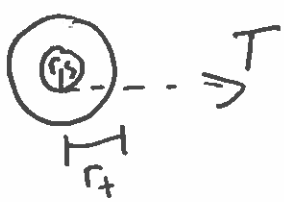


$$\Sigma F = ma = T - mg$$ where a will be negative

$$T = ma - mg$$

Torque on rotating table

Top view



$$\tau = \vec{r} \times \vec{F}$$

$$\tau = r_s T$$

$$\tau = I\alpha$$

$$\alpha = \frac{\tau}{I}$$

$$a = r_s \alpha$$

$$a = r_s \left(\frac{r_s T}{I} \right)$$

$$a = \frac{r_s^2 T}{I}$$

$$T = \frac{aI}{r_s^2}$$

6 continued

$$T = ma - mg \qquad T = \frac{a I_R}{r_s^2}$$

$$ma - mg = \frac{a I}{8 r_s^2}$$

$$I_R = \frac{r_s^2(ma - mg)}{a}$$

7 Based on the previous exercise (6) I have two expressions.

$$I_T = I_r + I_c$$

$$I_T = \frac{r_s^2 (ma - mg)}{a}$$

- We can find I_r by testing when no cube is added
- Next we can test the cube on its 3 axis
- Since the foam is massless relative to ~~the~~ the point mass weight, each side should give us the inertia along that axis.
- From the 3 inertias, I_{xx}, I_{yy}, I_{zz}, we can calculate the position of the mass