QEA - Out of Class Assignment 4: Driven Systems

In[3]:=
```
SetDirectory@NotebookDirectory[];
<< "../General.m"
```

In[29]:=
$$\frac{s+1}{s\,(s+2)}$$

Out[29]=
$$\frac{1+s}{s\,(2+s)}$$

In[32]:=
$$\text{Apart}\left[\frac{1+s}{s\,(2+s)},\ s\right]$$

Out[32]=
$$\frac{1}{2\,s}+\frac{1}{2\,(2+s)}$$

---

2)

In[10]:=
$$\text{de} = m\,y''[t]+c\,y[t]+k\,y[t] == c\,D[\text{HeavisideTheta}[t],\ t]+k\,\text{HeavisideTheta}[t]$$

Out[10]=
$$c\,y[t]+k\,y[t]+m\,y''[t] == c\,\text{DiracDelta}[t]+k\,\text{HeavisideTheta}[t]$$

In[12]:=
$$\text{LaplaceTransform}[\text{de},\ t,\ s]$$

Out[12]=
$$c\,\text{LaplaceTransform}[y[t],\ t,\ s]+k\,\text{LaplaceTransform}[y[t],\ t,\ s]+ m\left(s^2\,\text{LaplaceTransform}[y[t],\ t,\ s]-s\,y[0]-y'[0]\right) == c+\frac{k}{s}$$

In[62]:=
$$\text{params} = \langle|\,a1\to\frac{-c+\text{Sqrt}\left[c^2-4\,m\,k\right]}{2\,m},\ a2\to\frac{-c-\text{Sqrt}\left[c^2-4\,m\,k\right]}{2\,m}\,|\rangle$$

$$y[t] = \frac{k}{a1\,a2}\,\text{HeavisideTheta}[t]+\frac{a1\,c+k}{a1\,(a1-a2)}\,e^{a1\,t}+\frac{a2\,c+k}{a2\,(a2-a1)}\,e^{a2\,t}$$

Out[62]=
$$\left\langle\left|\,a1\to\frac{-c+\sqrt{c^2-4\,k\,m}}{2\,m},\ a2\to\frac{-c-\sqrt{c^2-4\,k\,m}}{2\,m}\,\right|\right\rangle$$

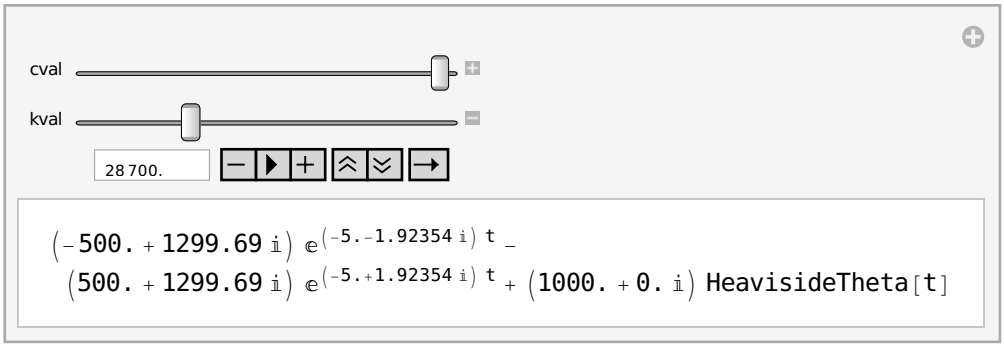
Out[63]=
$$\frac{e^{a1\,t}\,(a1\,c+k)}{a1\,(a1-a2)}+\frac{e^{a2\,t}\,(a2\,c+k)}{a2\,(-a1+a2)}+\frac{k\,\text{HeavisideTheta}[t]}{a1\,a2}$$

Large values of c seem to give the lowest oscillations after the bump in the

road.

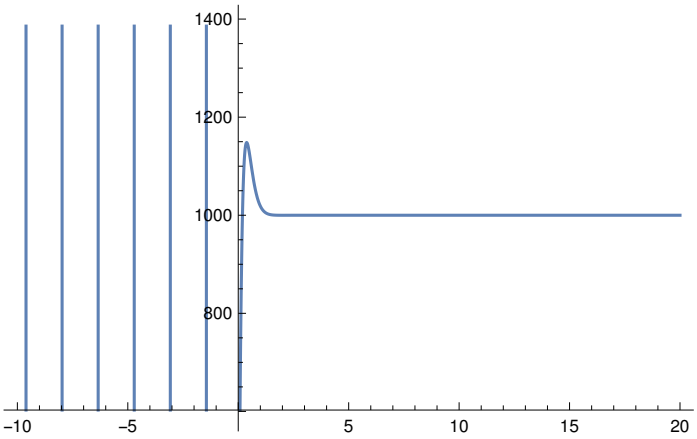
In[74]:=
```
Manipulate[
  constants = <|m → 10^3, c → cval, k → kval|>;
  equation = y[t] /. params /. constants,
  {cval, 1, 10^4}, {kval, 10^3, 10^5}]
```

Out[74]=



In[75]:=
```
Dynamic[Plot[equation, {t, -10, 20}]]
```

Out[75]=



Now let's make a1 and a2 the same. This happens when $c^2 - 4mk == 0$

In[78]:=
```
c^2 - 4 m k == 0 /. m → 10^3
```

Out[78]=
$$c^2 - 4000\,k == 0$$

In[82]:=
```
Solve[c^2 - 4000 k == 0, {k}]
```

Out[82]=
$$\left\{\left\{k \to \frac{c^2}{4000}\right\}\right\}$$

In[98]:=
```
value = 10;
constants = <|m → 10^3, c → value, k → value^2/4000|>;
equation2 = y[t] /. params /. constants
```

Power: Infinite expression $\frac{1}{0}$ encountered.

Power: Infinite expression $\frac{1}{0}$ encountered.

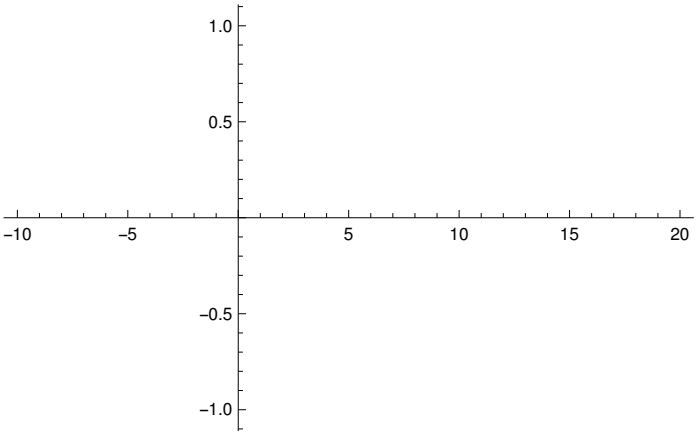
Infinity: Indeterminate expression ComplexInfinity + ComplexInfinity + 1000 HeavisideTheta[t] encountered.

Out[100]= Indeterminate

It appears that a1 and a2 can't be the same value as we get divide by zero errors

In[83]:=
```
Dynamic[Plot[equation2, {t, -10, 20}]]
```

Out[83]=



In[101]:=
```
exportNotebookPDF[]
```