

Signal processing

Time domain

Frequency domain

Sampling

Bytes

Sampling

Radio Communications system

~~Frequency~~

Antennae

AM/FM

One way/Two way

~~Phase~~

Frequency, Amplitude, phase, pulse width, resonance

①

Flow in

2.1m

$$\begin{aligned} 45\% \text{ precipitation} &= .945 \\ 55\% \text{ aquifer} &= 1.155 \end{aligned}$$

Flow out

2.1

$$\begin{aligned} 26\% \text{ evaporation} &= .5460 \\ 74\% \text{ seepage} &= 1.554 \end{aligned}$$

How long to dry out if precipitation 25% below normal
 $\text{prec} = .945 \cdot .75 = .7087$

$$\text{Flow in} - \text{Flow out} = \frac{-.2362 \text{ m}}{\text{year}}$$

$$12.9 \text{ m} - .2362 \text{ m} \times = 0$$

$$-.2362 \times = -12.9$$

$$x = \frac{12.9}{.2362}$$

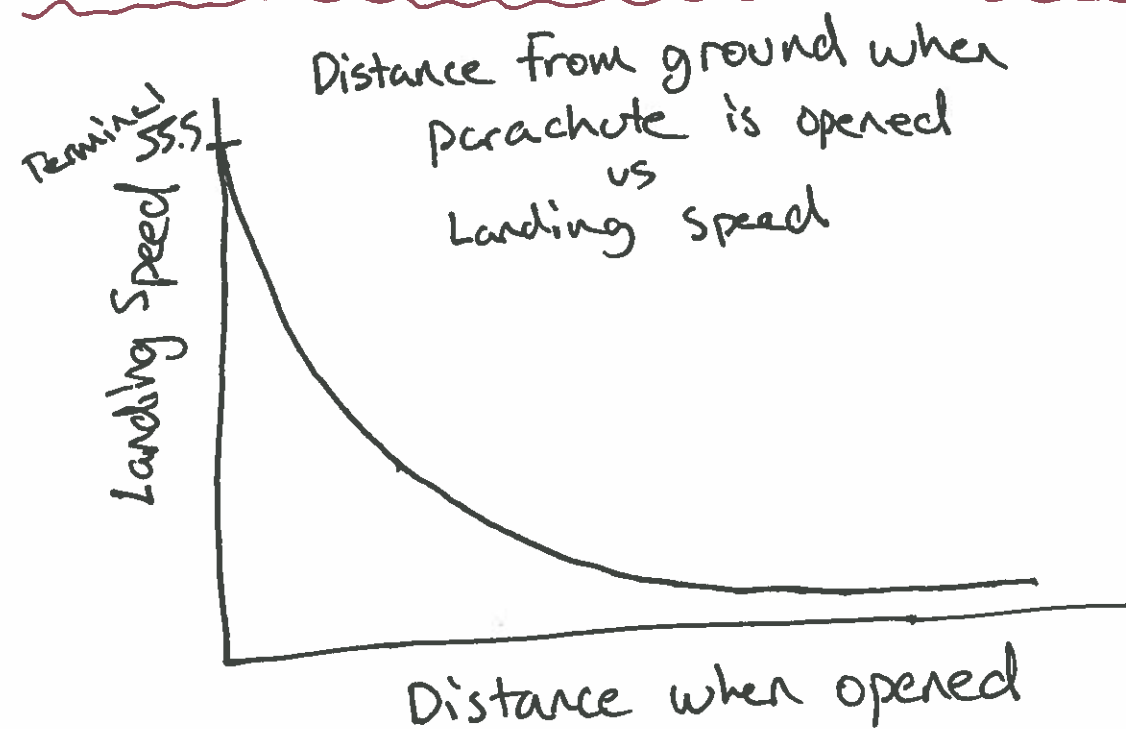
$$x = 54.6147$$

(2)

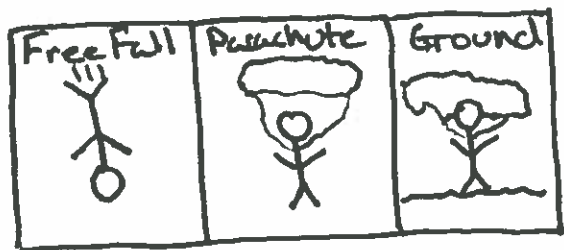
Top speed = terminal velocity

So, starting (HIGH) altitude
is not important

Terminal velocity for human = 55.5 m/s



Assume parachute is immediately ~~in full~~ opened



Position \Rightarrow

Velocity $\Rightarrow \dot{x} = v$

Acceleration $\Rightarrow \dot{v} = \text{gravity}$

Air Res_{para} + Air Res_{person} - grav

$$\dot{x} = v$$

$$\dot{v} = \alpha v^2 + \beta v^2 - g$$

③

BSET 10

MODSIM
QUESTIONS

$$\dot{X} = V$$

$$\dot{V} = -\alpha X - \beta V^2$$

a

$$\dot{V} = \frac{M}{s^2}$$

$$\alpha X = \frac{1}{s^2} M$$

$$\alpha = \frac{1}{s^2}$$

$$\beta V^2 = \frac{1}{M} \frac{M^2}{s^2}$$

$$\beta = \frac{1}{M}$$

b

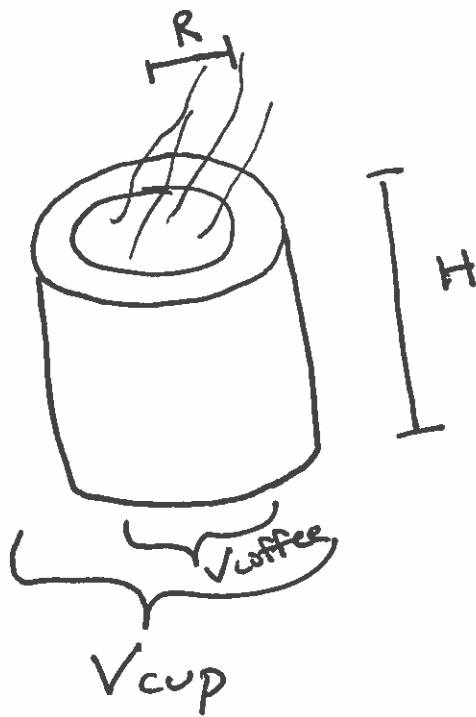
~~Code~~
~~Image~~

6

~~CODE: modsim3.m~~

~~PLOT: modsim3.pdf~~

4



QEA
MODSIM
QUESTIONS

Energy coffee = Temp coffee · Specific Heat coffee

$\frac{dE}{dt}$ coffee = Loss conduction

$$\frac{dE}{dt} = - \frac{\text{mug Cond} \cdot \text{mug Area}}{\text{mug thickness}} \cdot (\text{Temp Coffee} - \text{Temp Room})$$

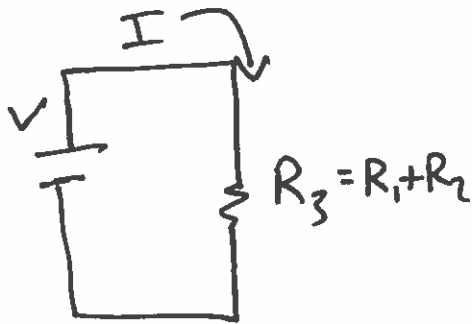
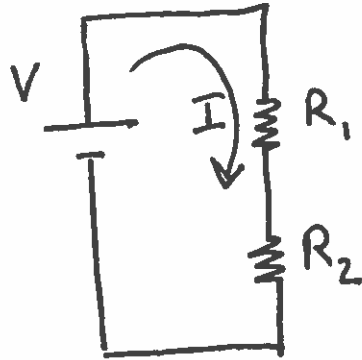
①

$$V = 12V$$

$$R_1 = 10k\Omega$$

$$R_2 = 20k\Omega$$

QEA
ISIM
QUESTIONS

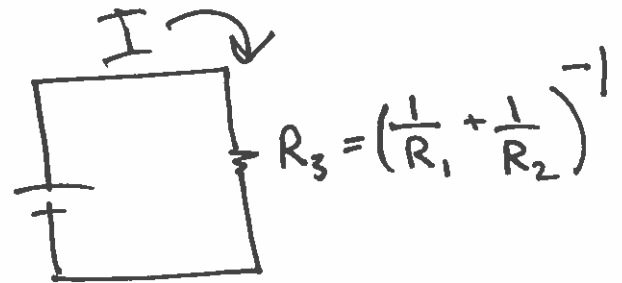
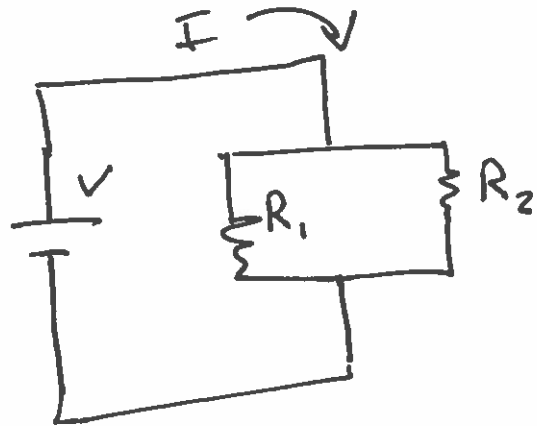


$$R_3 = 30k\Omega$$

$$V = IR$$

$$12V = I \cdot 30k\Omega$$

$$I = .4 \text{ mA}$$



$$R_3 = \left(\frac{2}{20k} + \frac{1}{20k} \right)^{-1}$$

$$= \left(\frac{3}{20k} \right)^{-1}$$

$$= \frac{20k}{3}$$

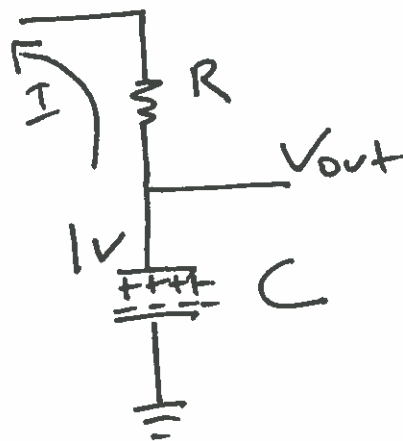
$$R_3 = 6.67k\Omega$$

$$12V = I \cdot 6.67k\Omega$$

$$I = 1.8 \text{ mA}$$

②

V_{in}



$$I = I_R = I_C$$

QEA-TSIM
BSET-0

$$V_{in} = 0$$

$$V_{out_0} = 1$$

$$R = 10 \text{ k}\Omega$$

$$C = 1 \mu\text{F}$$

Resistor

$$V_R = IR$$

Capacitor

$$I = C \frac{dV}{dt}$$

$$Q = CV$$

$$V_C = \frac{Q}{C}$$

$$I = -\frac{dQ}{dt}$$

Kirchoff

$$V_C - V_R = 0$$

$$\frac{Q}{C} - IR = 0$$

$$e^{\frac{t}{RC}} = e^{\ln\left(\frac{Q}{Q_0}\right)}$$

$$\frac{Q}{C} - \left(-\frac{dQ}{dt}\right)R = 0$$

$$\int_0^t \frac{-dt}{RC} = \int_{Q_0}^Q \frac{dQ}{Q}$$

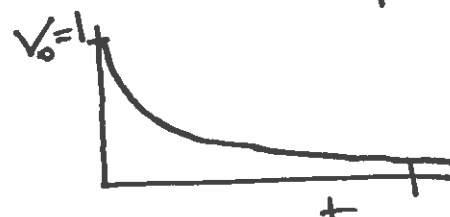
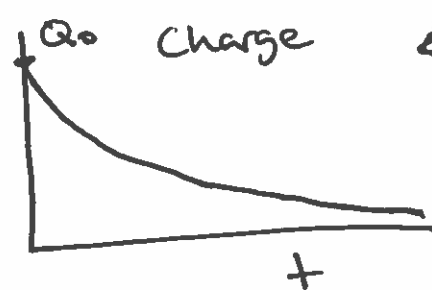
$$\frac{-t}{RC} = \ln(Q) - \ln(Q_0)$$

$$\frac{-t}{RC} = \frac{\ln(Q)}{\ln(Q_0)}$$

$$e^{-\frac{t}{RC}} = \frac{Q}{Q_0}$$

$$Q = Q_0 e^{-\frac{t}{RC}}, \quad Q_0 = CV_0$$

$$V = V_0 e^{-\frac{t}{RC}}$$



3

QEA

ISIM

QUESTIONS

$$\frac{(1+j)(3+j)(-2-j)}{(j)(3+4j)(5+j)}$$

Cartesian

$$\frac{(1+j)(3+j)(-2-j)}{(j)(3+4j)(5+j)}$$

$$\frac{(3+1j+3j-1)(-2-j)}{(3j-4)(5+j)}$$

$$\frac{(2+4j)(-2-j)}{(-4+3j)(5+j)}$$

$$\frac{(-4-2j-8j+4)}{(-20-4j+15j-3)}$$

$$\frac{-10j}{-23+11j} \left(\frac{11-23j}{11-23j} \right)$$

$$\frac{-230-110j}{650}$$

$$\begin{aligned} &= \frac{-11}{65} + \frac{23}{65}j \\ &= -0.169 + 0.354j \end{aligned}$$

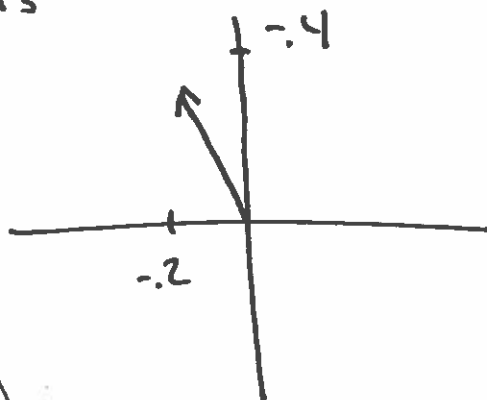
Polar

$$(\sqrt{2} \angle \frac{\pi}{24}) (\sqrt{10} \angle \text{Arctan}(\frac{1}{3}))$$

Arithmetic not worth my time

Mathematics!

$$\sqrt{\frac{2}{13}} e^{j(\pi - \text{Arctan}(\frac{23}{11}))}$$

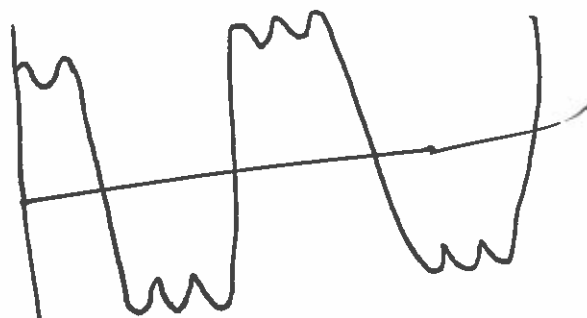


④ $z(t) = e^{jt} - .3e^{j3t} + .2e^{j5t}$

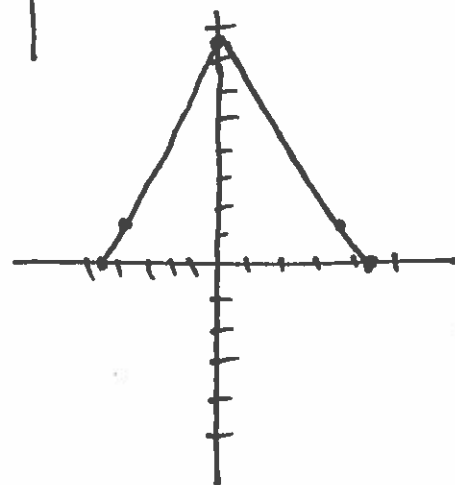
QEA
BSET 1
ISIM

~~| t | $z(t)$ | $z(t)$ |
|-----------------|--|--------------------|
| 0 | $e^0 - .3e^0 + .2e^0$ | $.9e^{j0}$ |
| $\frac{\pi}{4}$ | $e^{j\frac{\pi}{4}} - .3e^{j\frac{3\pi}{4}} + .2e^{j\frac{5\pi}{4}}$ | $.8544e^{j1.7427}$ |
| $\frac{\pi}{2}$ | | $.8544e^{j2.715}$ |~~

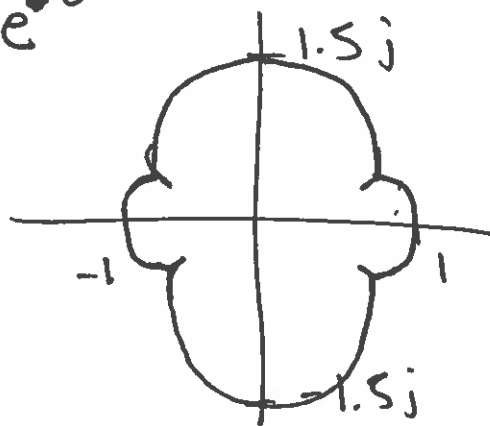
$.77 + .353j$



t	$z(t)$	$z(t)$
0	$.9 + 0j$	$.9e^{j0}$
$\frac{\pi}{4}$	$.77 + .35j$	$.855e^{j.427}$
$\frac{\pi}{2}$	$0 + 1.5j$	$1.5e^{j1.57}$
$\frac{3\pi}{4}$	$-.77 + .35j$	$.8544e^{j2.715}$
π	$-.9 + 0j$	$-.9e^{j\pi}$



Mathematica



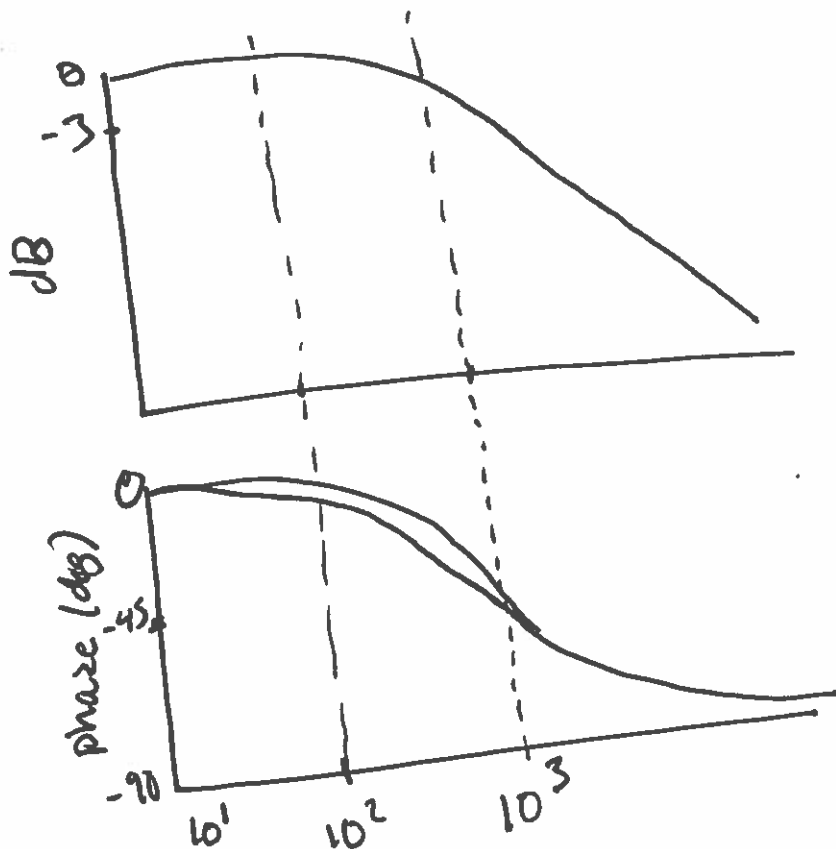
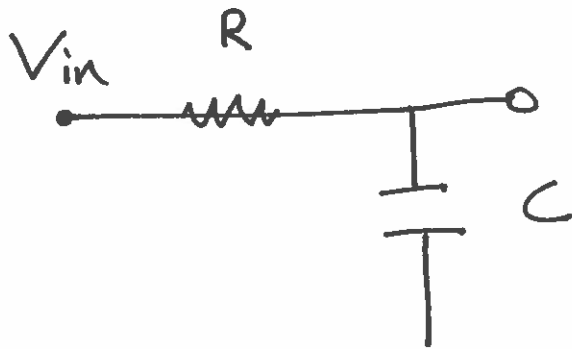
QEA
ISIM
QUESTIONS

⑤ Low pass

$$\omega_c = \frac{1}{RC} = 1 \text{ kHz}$$

$$R = 1000 \Omega = 1 \text{ k}\Omega$$

$$C = ~~1000~~ 1 \mu\text{F}$$



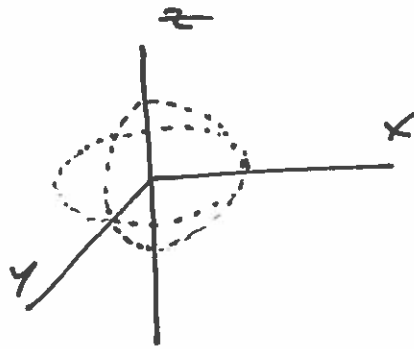
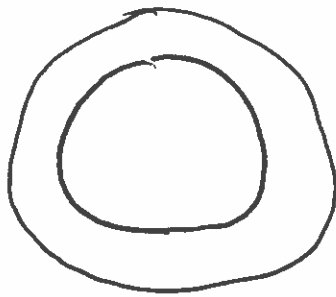
Integrate:

sphere, Hemisphere ~~$\frac{1}{4}$ sphere, $\frac{1}{8}$ sphere~~

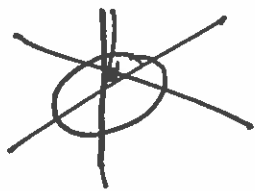
QEA

BSET 40

QEA REVIEW

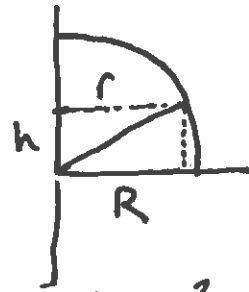


Add up layers of circles



At $z=0$ largest circle

$$r^2 = x^2 + y^2$$



$$R^2 = h^2 + r^2$$

$$\sqrt{R^2 - h^2} = r$$

A circle at z height h will have ~~area~~ radius $\sqrt{R^2 - h^2}$

smallest $h = -R$

largest $h = R$

Area at height $h \Rightarrow A = \pi r^2 = \pi \sqrt{R^2 - h^2}^2 = \pi(R^2 - h^2)$

$$\int_{-R}^R (\pi R^2 - \pi h^2) dh$$

$$\int_{-R}^R \pi R^2 dh - \int_{-R}^R \pi h^2 dh$$

$$\pi R^2 \int_{-R}^R dh - \pi \int_{-R}^R h^2 dh$$

$$\left[\pi R^2 (R + R) - \pi \frac{h^3}{3} \right]_{-R}^R$$

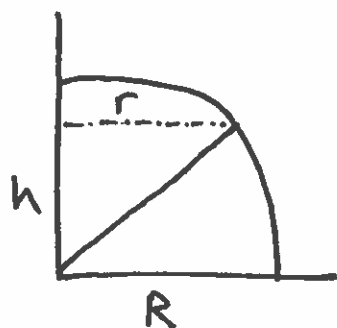
$$2\pi R^3 - \pi \left(\frac{R^3}{3} - \frac{(-R)^3}{3} \right)$$

$$\frac{6}{3} \pi R^3 - \frac{2\pi R^3}{3}$$

$$\frac{4}{3} \pi R^3$$

integrate half sphere

QEA BSET 0
QEA REVIEW



~~Area~~

Area of circle at height h

~~Area~~

$$r = \sqrt{R^2 + h^2}$$

$$A = \pi r^2 = \pi \sqrt{R^2 + h^2}^2 = \pi (R^2 + h^2)$$

$$\int_0^R \pi (R^2 + h^2) dh$$

$$\int_0^R \pi R^2 + h^2 dh$$

$$\int_0^R \pi R^2 dh + \int_0^R \pi h^2 dh$$

$$\pi R^2 \int_0^R dh + \pi \int_0^R h^2 dh$$

$$\pi R^2 (R) + \pi \left(\frac{R^3}{3} \right)$$

$$\frac{3}{3} \pi R^3 + \frac{\pi R^3}{3}$$

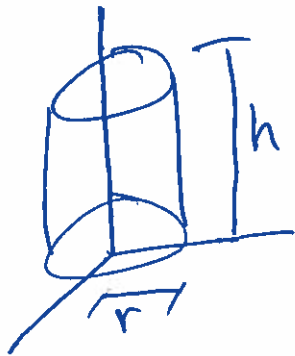
$$\frac{4\pi R^3}{3}$$

Strange. Got volume of regular sphere

MORE INTEGRALS

QEA BSET 0

$\theta R z$

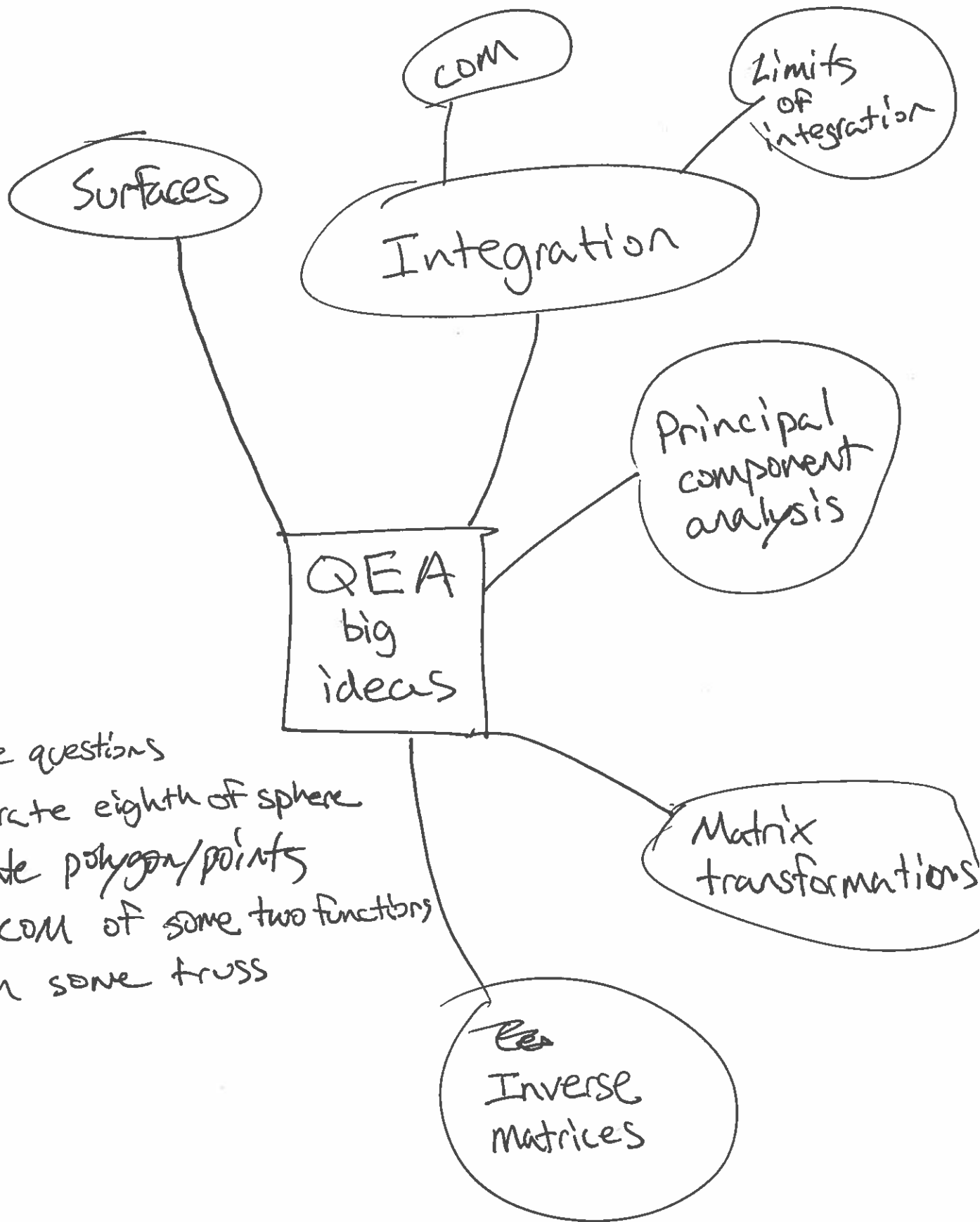


$$0 \leq \theta \leq 2\pi$$

$$0 \leq R \leq r$$

$$0 \leq z \leq h$$

$$\int_0^{2\pi} \int_0^r \int_0^h r \, dz \, dr \, d\theta$$



Possible questions

Integrate eighth of sphere

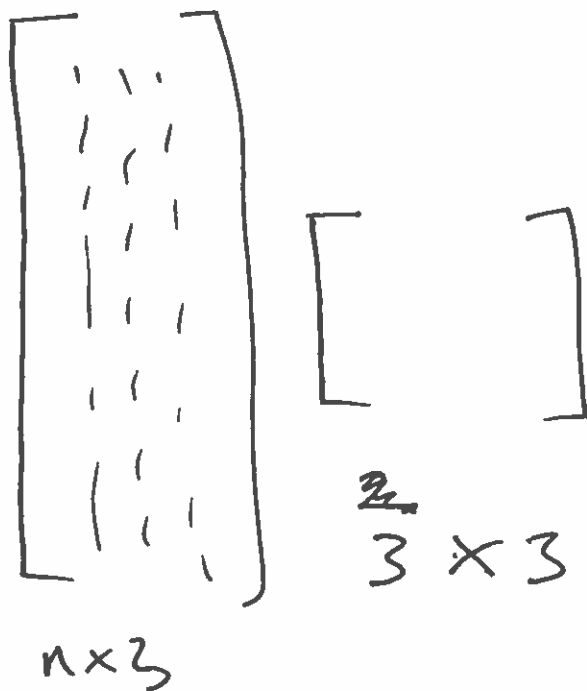
Animate polygon/points

Find COM of some two functions

Design some truss

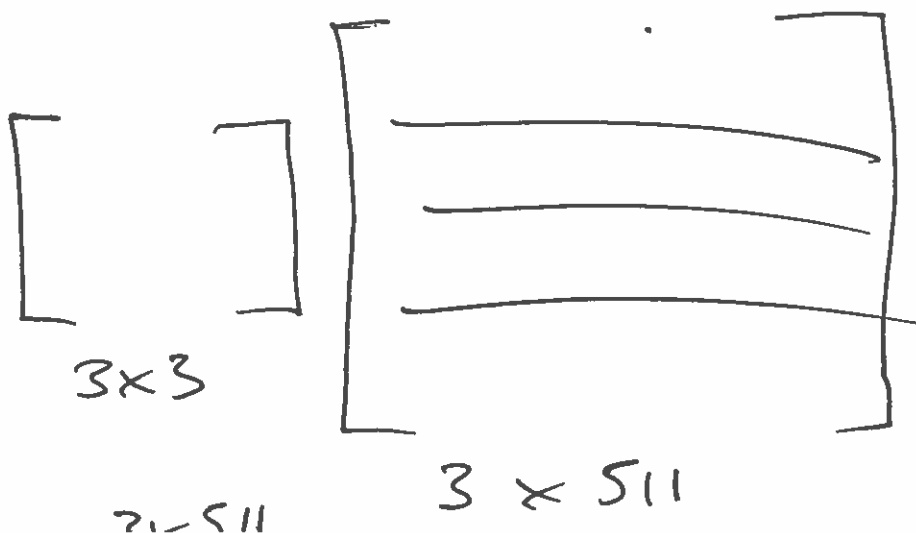
Animate points/polygon in
Mathematica to remember
matrix transformations

QEA-BSET
QEA-REVIEW



$$n \times 3 \cdot 3 \times 3$$

\downarrow
 $n \times 3$



①

QEA BSET 0
DIFFERENCE
EQUATIONS

$$X_{n+1} = 2X_n + 3$$

$$X_0 = X_0$$

$$X_1 = 2X_0 + 3$$

$$X_2 = 2(2X_0 + 3) + 3 = 4X_0 + 6 + 3 \\ = 2^{2 \cdot 1} X_0 + 2^{2-1} \cdot 3$$

$$X_n = 2^n X_0 + 2^{n-1} \cdot 3$$

②

$$W_3 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} X_3 \\ X_2 \end{pmatrix} = \begin{pmatrix} X_3 + X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} X_4 \\ X_3 \end{pmatrix}$$

$\begin{matrix} 2 \times 2 & 2 \times 1 \\ & 2 \times 1 \end{matrix}$

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \quad \det(A - \lambda I) \neq 0$$

$$A - \lambda I = \begin{pmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = 0 = (1-\lambda)(-\lambda) - (1)(1)$$

~~$$\lambda^2 + \lambda - 1 = 0$$~~

$$\lambda^2 - \lambda - 1 = 0$$

~~$$\lambda = \frac{1 \pm \sqrt{1+4}}{2}$$~~

$$\lambda_1 = \frac{1+\sqrt{5}}{2} \quad \lambda_2 = \frac{1-\sqrt{5}}{2}$$

For $\lambda = \frac{1+\sqrt{5}}{2}$,

$$\begin{bmatrix} 1 - \frac{1+\sqrt{5}}{2} & 1 \\ 1 & -\frac{1+\sqrt{5}}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1+\sqrt{5}}{2} & 1 \\ 1 & -\frac{1+\sqrt{5}}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$V_1 = \begin{pmatrix} \frac{1+\sqrt{5}}{2} \\ 1 \end{pmatrix}$$

$$V_2 = \begin{pmatrix} \frac{1-\sqrt{5}}{2} \\ 1 \end{pmatrix}$$

$$\textcircled{3} \quad w_n = c_1 \lambda_1^n v_1 + c_2 \lambda_2^n v_2$$

DIFFERENCE
EQUATIONS

④

DIFFERENCE
EQUATIONS

$$w_n = c_1 \lambda_1^n v_1 + c_2 \lambda_2^n v_2$$

$$w_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} \begin{pmatrix} \frac{1+\sqrt{5}}{2} \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} \begin{pmatrix} \frac{1-\sqrt{5}}{2} \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} \begin{pmatrix} \frac{1+\sqrt{5}}{2} \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} \begin{pmatrix} \frac{1-\sqrt{5}}{2} \\ 1 \end{pmatrix}$$

$$1 = c_1 \frac{1+\sqrt{5}}{2} + c_2 \frac{1-\sqrt{5}}{2}$$

$$1 = c_1 + c_2$$

$$c_1 = \frac{1}{10} (5+\sqrt{5}), \quad c_2 = \frac{1}{10} (5-\sqrt{5})$$

$$w_n = \frac{1}{10} (5+\sqrt{5}) \begin{pmatrix} 2 \\ 1 \end{pmatrix} \left(\frac{1+\sqrt{5}}{2} \right)^n + \frac{1}{10} (5-\sqrt{5}) \begin{pmatrix} 2 \\ 1 \end{pmatrix} \left(\frac{1-\sqrt{5}}{2} \right)^n$$

$$x_n = \frac{1}{10} (5+\sqrt{5}) \left(\frac{1+\sqrt{5}}{2} \right)^n$$

⑤

$$x_n = \frac{1}{10} (5+\sqrt{5}) \left(\frac{1+\sqrt{5}}{2} \right)^n (1) + \frac{1}{10} (5-\sqrt{5}) \left(\frac{1-\sqrt{5}}{2} \right)^n (1)$$

$$x_{100} = 5.73 \times 10^{20}$$