

IR class 3

$$- r_x r_x \omega_x - r_x r_y \omega_y - r_x r_z \omega_z$$

$$(a) L_x = m \left((r_x^2 + r_y^2 + r_z^2) \omega_x - r_x (r_x \omega_x + r_y \omega_y + r_z \omega_z) \right)$$

$$L_y = m \left((r_x^2 + r_y^2 + r_z^2) \omega_y - r_y (r_x \omega_x + r_y \omega_y + r_z \omega_z) \right)$$

$$L_z = m \left((r_x^2 + r_y^2 + r_z^2) \omega_z - r_z (r_x \omega_x + r_y \omega_y + r_z \omega_z) \right)$$

$$(b) \begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

$$L_x = I_{xx} \omega_x + I_{xy} \omega_y + I_{xz} \omega_z$$

$$I_{xx} = m(r_y^2 + r_z^2)$$

$$I_{xx} = m(r_x^2 + r_y^2 + r_z^2) - r_x r_x$$

$$I_{xy} = m(-r_x r_y)$$

$$I_{xz} = m(-r_x r_z)$$

$$I_{yx} = m(-r_y r_x)$$

$$I_{yy} = m(r_x^2 + r_z^2)$$

$$I_{zz} = m(r_x^2 + r_y^2)$$

$$I_{zx} = m(-r_z r_x)$$

$$I_{zy} = m(-r_z r_y)$$

$$I_{zz} = m(r_x^2 + r_y^2 + r_z^2) - r_z r_z$$

$$\begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix} = \begin{bmatrix} r^2 - r_x r_x & -r_x r_y & -r_x r_z \\ -r_y r_x & r^2 - r_y r_y & -r_y r_z \\ -r_z r_x & -r_z r_y & r^2 - r_z r_z \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

$$r^2 = (r_x^2 + r_y^2 + r_z^2)$$

$$(c) r^2 - r_x r_x = r_y^2 + r_z^2$$

For rotation in ω_x
Distance in yz plane

6 unique terms

⑥ In class 3

①

$$\left[\begin{aligned} \sum_{i=1}^N r_i^2 - r_{ix} r_{ix} &= \sum_{i=1}^N r_{iy}^2 + r_{iz}^2 \\ \sum_{i=1}^N -r_{iy} r_{ix} & \end{aligned} \right]$$

or

②

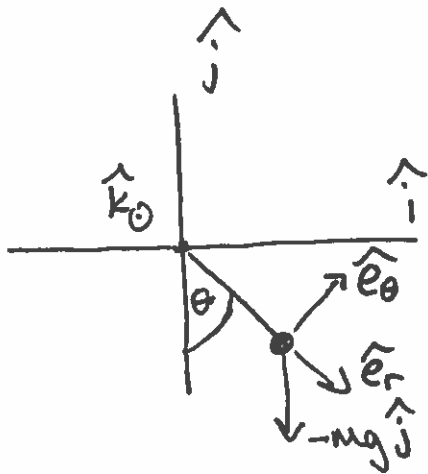
$$\left[\begin{aligned} \int r_y^2 + r_z^2 \, dm \\ \int r_y r_x \, dm \end{aligned} \right]$$

②

$$\Sigma \vec{M}_O = \frac{d}{dt} \bigg|_O \vec{H}_O$$

M = moment/torque

H = angular momentum



Want

~~$$\ddot{\theta} = -\frac{g}{l} \sin(\theta)$$~~

$$\ddot{\theta} = -\frac{g}{l} \sin \theta$$

$$M_O = \vec{r}_{m/O} \times \vec{F}, \quad \vec{r}_{m/O} = l \hat{e}_r$$

$$F = -mg \hat{j}$$

$$M_O = l \hat{e}_r \times -mg \hat{j}$$

$$M_O = -mgl \hat{k} \sin(180 - \theta)$$

$$\vec{H}_O = \vec{r}_{m/O} \times M \frac{d}{dt} \bigg|_O \vec{r}_{m/O}$$

~~$$\frac{d}{dt} \bigg|_O \vec{r}_{m/O} = l \hat{e}_r = l \dot{\theta} \hat{e}_\theta$$~~

~~$$\vec{H}_O = \vec{r}_{m/O} \times l \dot{\theta} \hat{e}_\theta$$~~

$$\vec{H}_O = l \hat{e}_r \times ml \dot{\theta} \hat{e}_\theta$$

$$\vec{H}_O = m \dot{\theta} l^2 \hat{k}$$

$$\frac{d}{dt} \bigg|_O \vec{H}_O = m \ddot{\theta} l^2 \hat{k}$$

$$m \ddot{\theta} l^2 \hat{k} = -mgl \sin(180 - \theta) \hat{k}$$

$$\ddot{\theta} = -\frac{g}{l} \sin(180 - \theta)$$

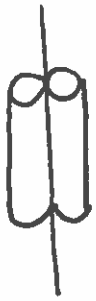
$$\ddot{\theta} = -\frac{g}{l} \sin(\theta)$$

- ③ (a) Angular momentum is conserved because no outside forces.

$$\vec{H} = \sum \vec{H}_0 = \sum (\vec{r} \times m\vec{v})$$

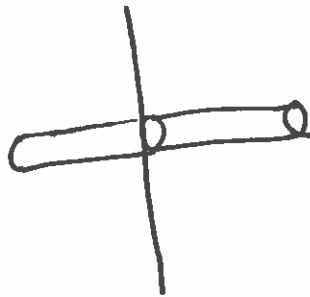
If some of my r 's increase (arms out)
then the v must decrease.

(b)



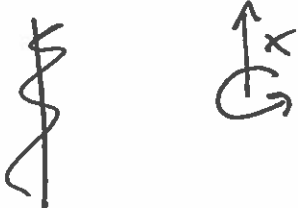
Low I
High ω

vs

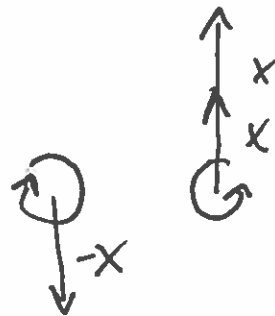


High I
Low ω

- ④ when wheel is handed to me, the angular
 (a) momentum of the system is: wheel = x , me = 0

~~the~~  after I turn the wheel

over; wheel = $-x$, me = $2x$



⑥

$$\begin{aligned}\vec{L}_{\text{total}} &= 3 \int_0^{2\pi} \vec{r} \times \vec{v} \, d\theta \\ &= 3 \int_0^{2\pi} 3 \times (3 \cdot 3\pi) \, d\theta \\ &= 3 \int_0^{2\pi} \dots \, d\theta \\ &= \cancel{16} \, 16 \, \frac{\text{kg m}^2}{\text{s}}\end{aligned}$$

$\vec{r} = 30 \text{ cm}$
 $M = 3 \text{ kg}$
 $\omega = 1.5 \frac{\text{cycles}}{\text{sec}}$
 $\omega = 3\pi \frac{\text{rad}}{\text{sec}}$

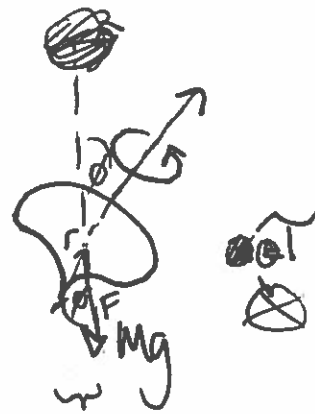
I would be spinning with ~~2.16~~ $2.16 \frac{\text{kg m}^2}{\text{s}}$
 worth of angular momentum

5

a



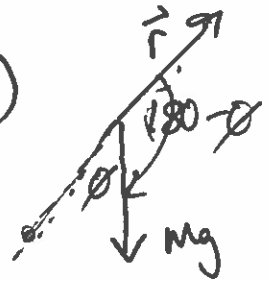
b



c

$$\tau = \vec{r} \times \vec{F} = h \cancel{mg} \sin(\cancel{\theta}) \cancel{\theta}$$

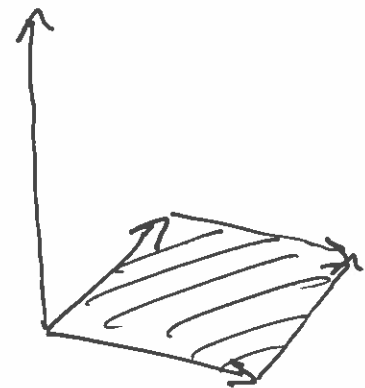
τ at com



d

$$\tau = \frac{d}{dt} L$$

τ is into page



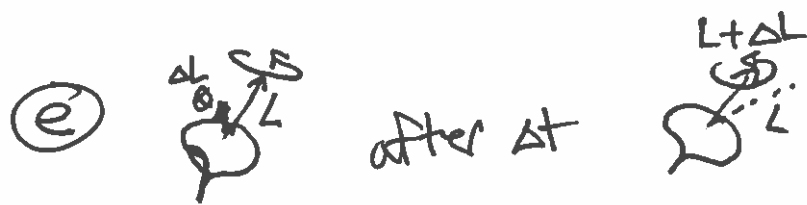
e

perpendicular to ~~the~~



~~Direction of torque~~

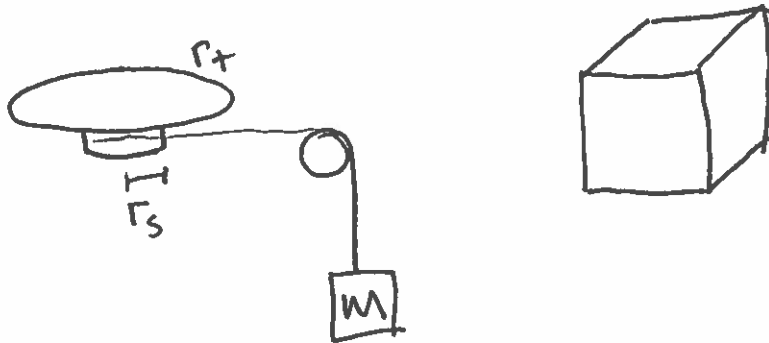
f



f We circle L around Z axis



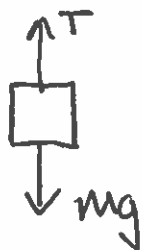
6



$$I_{\text{Total}} = I_{\text{Rotating table}} + I_{\text{cube}}$$

$$I_T = I_R + I_c$$

FBD for mass

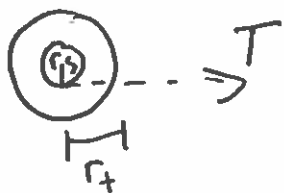


$$\Sigma F = ma = T - mg, \text{ where } a \text{ will be negative}$$

$$T = ma - mg$$

Torque on rotating table

Top view



$$\tau = \vec{r} \times \vec{F}$$

$$\tau = r_s T$$

$$a = r_s \left(\frac{r_s T}{I} \right)$$

$$a = \frac{r_s^2 T}{I}$$

$$T = \frac{aI}{r_s^2}$$

$$\tau = I\alpha \quad \alpha = \frac{\tau}{I}$$

$$\tau = I\alpha$$

$$\alpha = \frac{\tau}{I}$$

$$a = r_s \alpha$$

⑥ continued

$$T = ma - mg$$

$$T = \frac{a I_R}{r_s^2}$$

$$ma - mg = \frac{a I}{r_s^2}$$

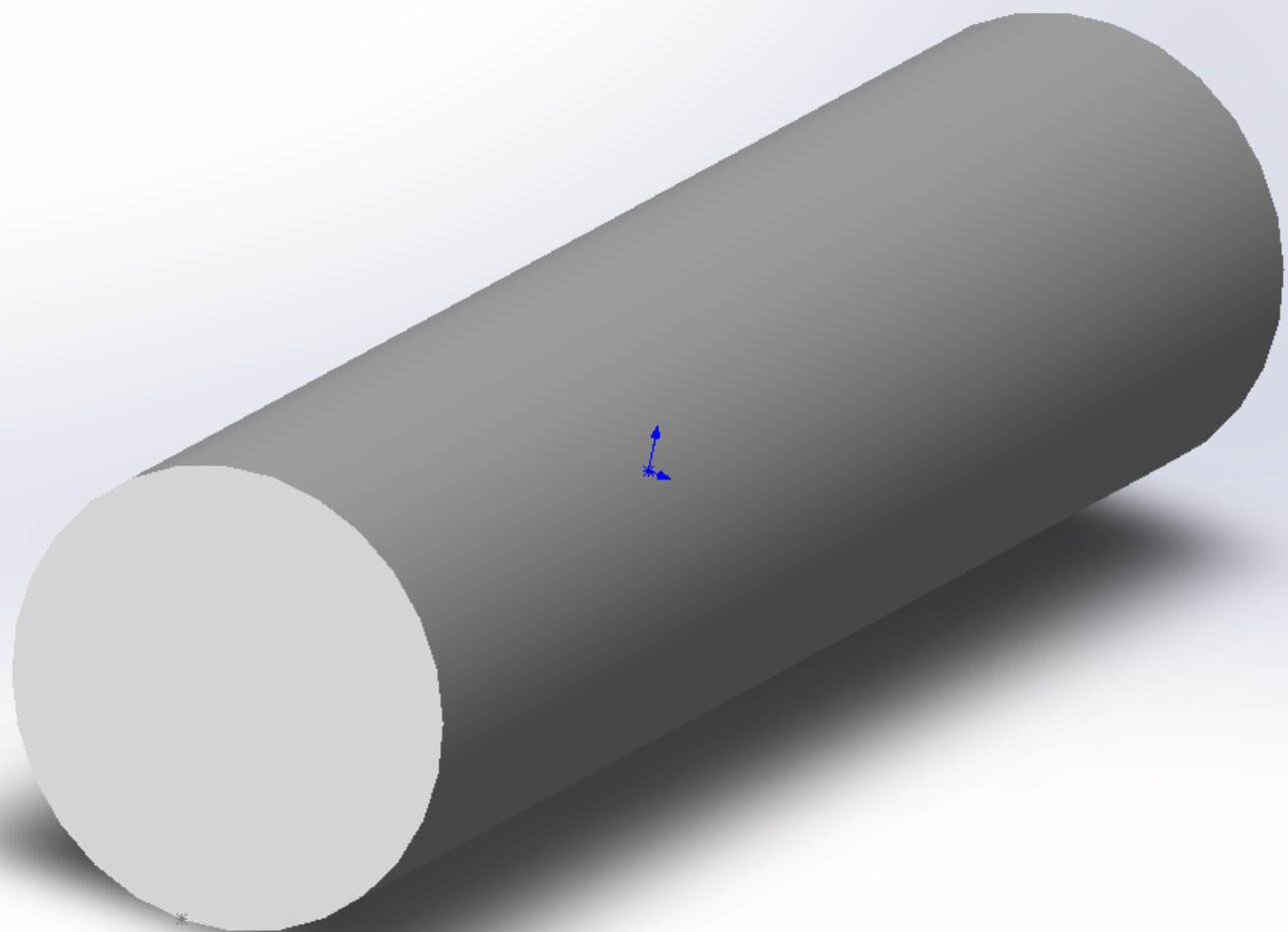
$$I_R = \frac{r_s^2 (ma - mg)}{a}$$

⑦ Based on the previous exercise (6) I have two expressions.

$$I_T = I_r + I_c$$

$$I_T = \frac{r_s^2 (ma - mg)}{a}$$

- We can find I_r by testing when no cube is added
- Next we can test the cube on its 3 axis
- Since the foam is massless relative to ~~the~~ the point mass weight, each side should give us the inertia along that axis.
- From the 3 inertias, I_{xx} , I_{yy} , I_{zz} , we can calculate the position of the mass



Mass properties of Part1
Configuration: Default
Coordinate system: -- default --

Density = 0.04 pounds per cubic inch

Mass = 0.93 pounds

Volume = 25.13 cubic inches

Surface area = 56.55 square inches

Center of mass: (inches)

X = 0.00

Y = 0.00

Z = 0.00

Principal axes of inertia and principal moments of inertia: (pounds * square inches)

Taken at the center of mass.

Ix = (0.00, 0.00, 1.00) Px = 0.46

Iy = (0.00, -1.00, 0.00) Py = 5.17

Iz = (1.00, 0.00, 0.00) Pz = 5.17

Moments of inertia: (pounds * square inches)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 5.17 Lxy = 0.00 Lxz = 0.00

Lyx = 0.00 Lyy = 5.17 Lyz = 0.00

Lzx = 0.00 Lzy = 0.00 Lzz = 0.46

Moments of inertia: (pounds * square inches)

Taken at the output coordinate system.

Ixx = 5.17 Ixy = 0.00 Ixz = 0.00

Iyx = 0.00 Iyy = 5.17 Iyz = 0.00

Izx = 0.00 Izy = 0.00 Izz = 0.46

Mass properties of Part1
Configuration: Default
Coordinate system: Coordinate System1

Density = 0.04 pounds per cubic inch

Mass = 0.93 pounds

Volume = 25.13 cubic inches

Surface area = 56.55 square inches

Center of mass: (inches)

X = 0.00

Y = 1.00

Z = -4.00

Principal axes of inertia and principal moments of inertia: (pounds * square inches)

Taken at the center of mass.

Ix = (0.00, 0.00, 1.00) Px = 0.46

Iy = (0.00, -1.00, 0.00) Py = 5.17

Iz = (1.00, 0.00, 0.00) Pz = 5.17

Moments of inertia: (pounds * square inches)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 5.17 Lxy = 0.00 Lxz = 0.00

Lyx = 0.00 Lyy = 5.17 Lyz = 0.00

Lzx = 0.00 Lzy = 0.00 Lzz = 0.46

Moments of inertia: (pounds * square inches)

Taken at the output coordinate system.

Ixx = 20.92 Ixy = 0.00 Ixz = 0.00

Iyx = 0.00 Iyy = 19.99 Iyz = -3.70

Izx = 0.00 Izy = -3.70 Izz = 1.39