Module 2, B-set 2: All Pendulums. All the Time.

Jane! Get me off this crazy thing!

November 3, 2016

Learning Goals

By the end of this building block, you should be able to...

- Derive scalar equations of motion for particle masses.
- Numerically solve for EOM's for position.
- Find velocity and acceleration expressions using the general vector formulation with rotating reference frames.

Overview and Orientation

We want to be able to control the position and attitude of our cubesat. We will apply forces (*e.g.*, thrusters) to change position and apply moments to change orientation. Forces are related to position through acceleration ($\sum \vec{F} = m\vec{a}$). As $\vec{F}$ and $\vec{a}$ are vector quantities, they can be written with respect to any (space-fixed or body-fixed) reference frame. Some reference frames are more useful or convenient that others. So it it likely that when we'll use different frames to express $\vec{F}$ and $\vec{a}$.

We will continue to practice and build skill formulating equations of motion based and interpreting simulation results through visualization or animation.

WARNING: By the time you're 4 or 5 hours into this, you should feel confident that you can complete the B-set in another 4-5 hours. If not, this is when you should **ask for help.** This means **talk to a colleague**, or **talk to a ninja**, or **track down an instructor**, or **send an email to an instructor**.

Complete the Projectile and Helical Slide Simulations (1 hour)

Make them work. Plot stuff.

The Simple Planar Pendulum (3 hours)

Consider a simple planar pendulum which consists of a mass, m, suspended by a rod (≈massless) of length, l. Gravity acts in the downward vertical direction. The position of the mass can be described by the angle θ or by the Cartesian coordinates (x,y).

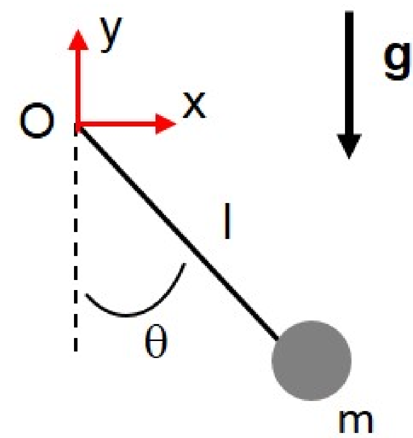


Figure 1: Planar pendulum.

1. Using the standard approach (FBD, coordinate system, constraint equations, $\vec{F} = m\vec{a}$), formulate equations of motion for the mass in terms of Cartesian coordinates. Note that here's we're looking for three equations: one for $\ddot{x}$, one for $\ddot{y}$, and a constraint equation that describes the constraint that the mass has to stay a distance l from the origin.

2. Formulate equations of motion of the mass using the "convenient" coordinate system (hint: it's cylindrical coordinates). Solution to the equation of motion will yield the angular position and velocity of the mass and the tension in the rod. Why is a separate constraint equation not needed here?

3. Show that the equations of motion found in Questions 1 and 2 are equivalent.

4. Create a computational simulation of the pendulum for a selected set of parameters and initial conditions.

5. Calculate the tension in the rod; reaction force components, R_x and R_y; and the kinetic, potential, and total energy of the system as functions of time. Plot these quantities for a selected case.

6. Using the equation of motion from Question 2, separately incorporate two types of energy dissipation into your simulation. The first is a viscous damping force that is linearly proportional to the angular velocity of the mass. The second is aerodynamic drag that is linearly proportional to the square of the angular velocity (remember to account for the direction of the drag force). Create a set of physically reasonable parameters and simulate the motion of the pendulum. Discuss differences in how the two types of damping effect the motion.

7. With a partner, build a simple pendulum. Using video capture and an appropriate tool (*e.g.*, tracker) make measurements that can be compared to your simulations. Take a picture of your set-up and describe your measurement procedure.

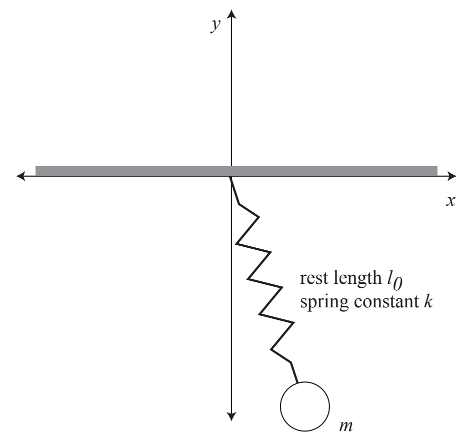
8. Is viscous damping or aerodynamic drag a better representation for energy dissipation for the pendulum you built? How could you find drag coefficients from experimental data?

9. (Optional) Create an animation of the motion of the pendulum.

Springy Simple Pendulum (1 hour)

Now that you've worked through the simple pendulum, we'll consider two additional cases. For each we'd like you to do the drill: FBD, constraints, coordinates, equations of motion, etc.

10. Here's the first case – it should look familiar! Consider the axial stiffness of the rod to be well represented as a radial linear spring. The spring coefficient is k. The unstretched length of the spring is l. For m = 1.0 kg, k = 2.0 N/m (2.0 is a little soft, try k=100 N/m), l = 1.0m, plot the trajectory of the pendulum for some initial conditions.



Spinny Simple Pendulum (3 hours)

11. And here's one that will make you want to kill the person who wrote this problem.[1]

    Now take the pivot point, O, to rotate about the y-axis with constant angular velocity, Ω. The pendulum is still constrained to swing in a plane which is rotating about the y-axis. Create a simulation for the motion of this pendulum. Plot the trajectory for a representative case.

[1] whose initials are CL, in case you were wondering.