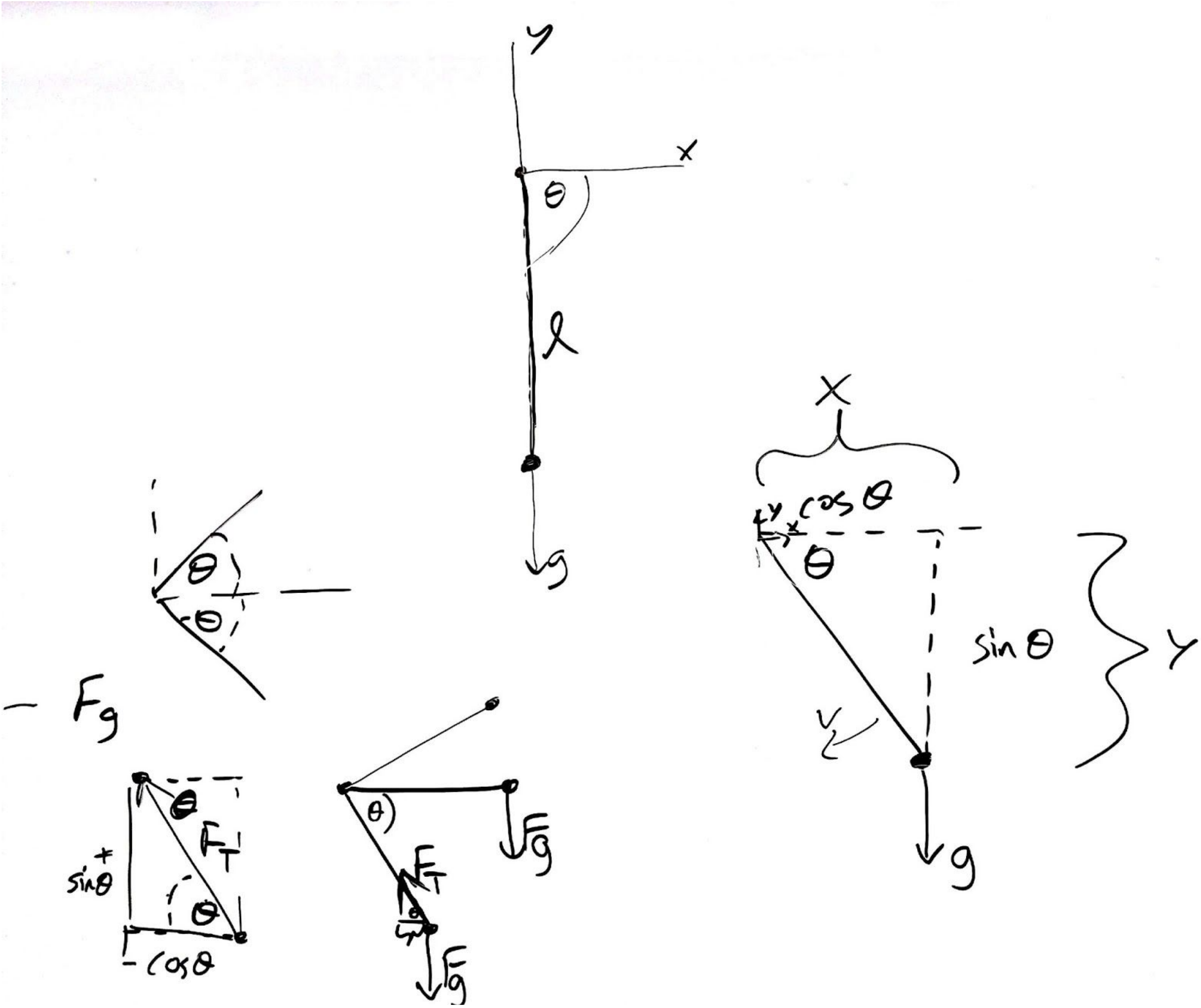
Nathan Yee
QEA - Bset2
Attitude Control

1. Free body diagrams and problem setup with differential equations at the end.



$$x^2+y^2-l^2=0$$

$$l=C=\sqrt{x^2+y^2}$$

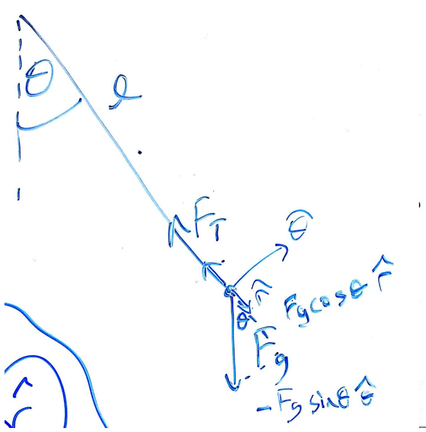
$$m\ddot{x}=-F_T\cos\theta$$

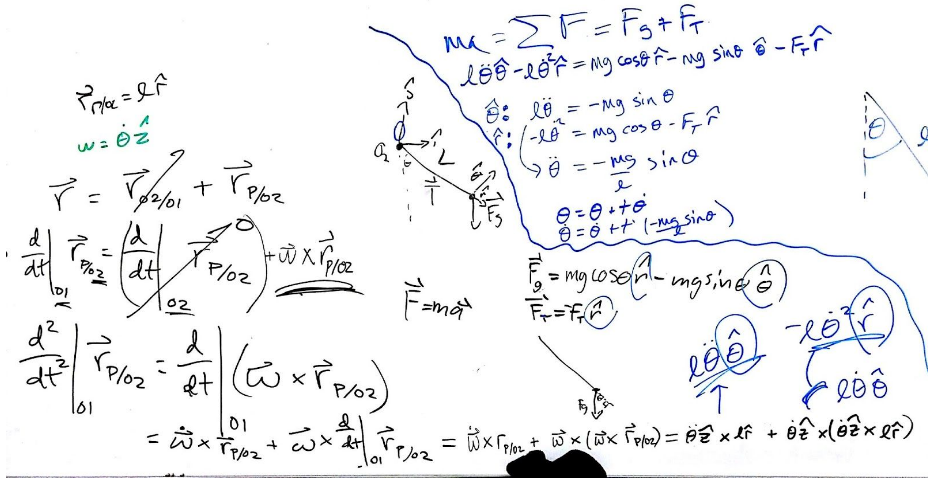
$$m\ddot{y}=F_T\sin\theta-F_g$$

$$\ddot{x}=\frac{-gy\cos\theta-\cos\theta\,\dot{x}-\cos\theta\,\dot{y}}{-x\cos\theta+y\sin\theta}$$

$$\ddot{y}=\frac{-gx\cos\theta+\dot{x}\sin\theta+\dot{y}\sin\theta}{-x\cos\theta+y\sin\theta}$$

2. Setup with differential equation shown at the end.





$$ma=\sum F=F_g+F_T$$

$$l\ddot{\theta}\hat{\theta}-l\dot{\theta}^2\hat{r}=mg\cos\theta\,\hat{r}-mg\sin\theta\,\hat{\theta}-F_T\hat{r}$$

$\hat{\theta}$: $l\ddot{\theta}=-mg\sin\theta$

$\hat{r}$: $-l\dot{\theta}^2=mg\cos\theta-F_T\hat{r}$

$\rightarrow \ddot{\theta}=-\frac{mg}{l}\sin\theta$

$$\theta=\theta+t\dot{\theta}$$

$$\dot{\theta}=\dot{\theta}+t\left(-\frac{mg}{l}\sin\theta\right)$$

3. I set my systems up with a different theta and I didn't feel like redoing either one. I'm pretty sure both will work though.

$$\ddot{\theta} = \frac{-mg}{l} \sin\theta \quad \text{Not same } \theta$$

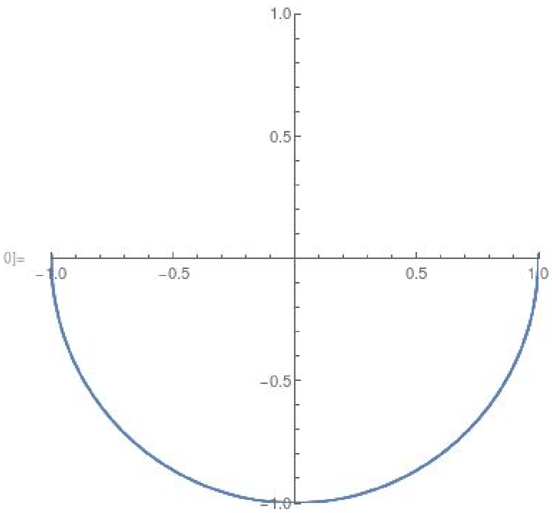
$$\ddot{x} = \frac{-g y \cos\theta - \cos\theta \dot{x} - \cos\theta \dot{y}}{x \cos\theta - y \sin\theta}$$

$$\ddot{y} = \frac{-g y \cos\theta + \sin\theta x + \sin\theta \dot{y}}{-x \cos\theta + y \sin\theta}$$

4. Simulation was done using polar coordinates

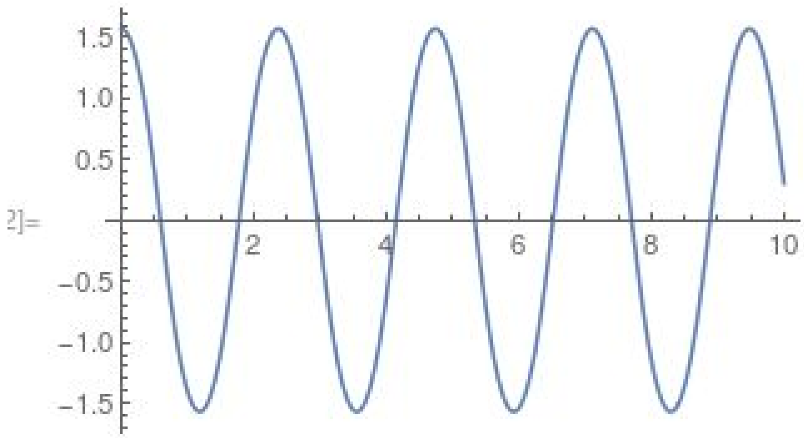
Plot of the path the pendulum covers starting from $\theta[0] = \frac{\pi}{2}$ and $\theta'[0]$=0

```
ParametricPlot[{Sin[sol], -Cos[sol]}, {t, 0, 10},
  PlotRange → {{-1, 1}, {-1, 1}}]
```



Plot of θ over time with initial conditions $\theta[0] = \frac{\pi}{2}$ and $\theta'[0]$=0

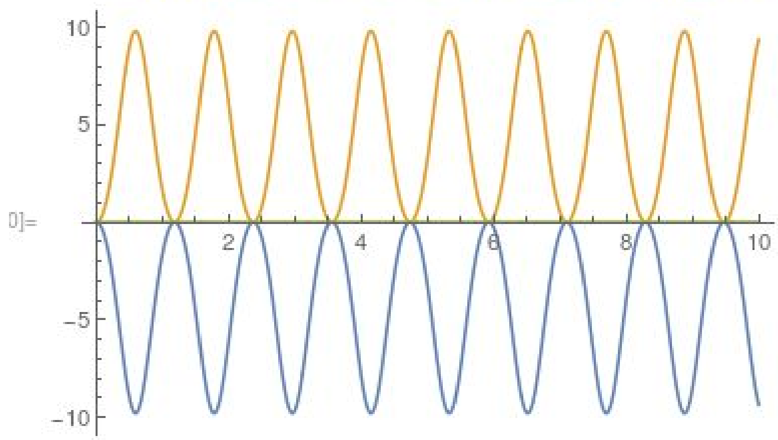
```
Plot[sol, {t, 0, 10}]
```



5. Given our initial conditions, both potential energy and kinetic energy begin at 0.

Plot of potential energy (blue), kinetic energy (yellow), and total energy (green)

```
Plot[{potential, rKinetic, potential + rKinetic}, {t, 0, 10}]
```

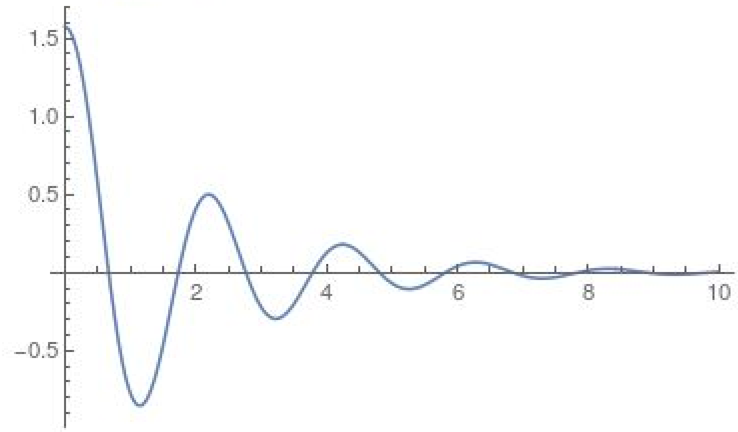


6. I'm not sure if I'm doing the damping correctly. To do this, I subtract a theta'[t] from my differential equation. If I use theta'[t]^2, I can no longer solve for the differential equation.

```
de = {θ''[t] == -m g Sin[θ[t]]/l - θ'[t]};
```

This appears to be falling off at an exponential rate.

```
Plot[sol, {t, 0, 10}, PlotRange → Full]
```



7. .
8. .
9. Made an animation. Just needed to adjust a few things and throw an animate function on the front!

10. End result of math shown below. Here we have the case where both l and theta vary with time.

(10)

$$\hat{i}: \quad l\ddot{\theta} + 2\dot{l}\dot{\theta} = -mg\sin\theta$$

$$\hat{j}: \quad -\ddot{l} - l\dot{\theta}^2 = k(l - l_0) - mg\cos\theta$$

$$\ddot{\theta} = \frac{-mg\sin\theta - 2\dot{l}\dot{\theta}}{l}$$

$$\ddot{l} = -k(l - l_0) + mg\cos\theta - l\dot{\theta}^2$$

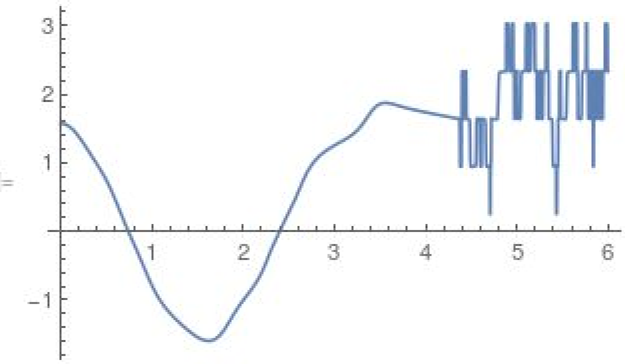
Solve differential equations with specified initial conditions:

```
With[{m = 1, g = 9.8, l0 = 1, k = 2}, initialConditions = {θ[0] == π/2, θ'[0] == 0, l[0] == 1, l'[0] == 0};
  de = {θ''[t] == -m g (Sin[θ[t]] - 2 l'[t] θ'[t])/l[t], l''[t] == -k (l[t] - l0) + m g Cos[θ[t]] - l[t] θ'[t]^2};
  {θSol, lSol} = NDSolveValue[Join[initialConditions, de], {θ[t], l[t]}, {t, 0, 6}]
 ]
```

It seems that theta'^2 has broken my differential equation solver once again.

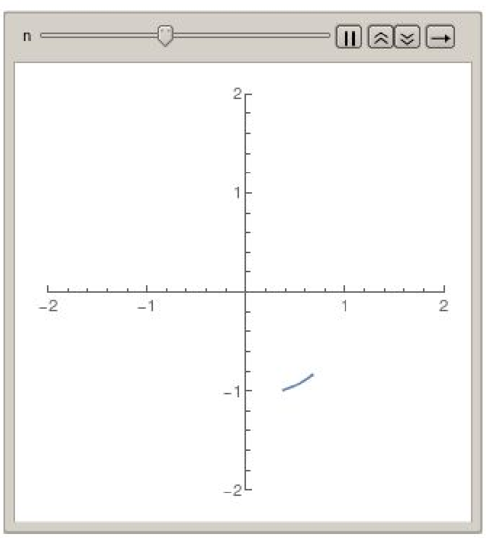
Plot of the angle over time. Those bars at the end look really strange. $\theta'[t]^2$ must have broken it again.

```
Plot[θSol, {t, 0, 6}]
```



I have the same problem of whenever I have a $\theta'[t]^2$, my differential equation solver just breaks.

```
Animate[ParametricPlot[
   {lSol Sin[θSol], -lSol Cos[θSol]},
   {t, n, n + .1},
   PlotRange → {{-2, 2}, {-2, 2}}],
  {n, 0, 5.9}]
```



11. .

Plot of the length over time

```
Plot[lSol, {t, 0, 6}]
```