Eigenstructure Assignment based PSO for LQR tuning applied to Active Suspension System

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***Abstract*—This paper presents a novel eigenstucture assignment (EA) based technique to optimize the weighing matrices of LQR using Particle Swarm Optimization (PSO) applied to vibration suppression of active suspension system. The motivation for the proposed study is to formulate a single objective function which can optimize the gains of state feedback controller for a MIMO system irrespective of the random search algorithms. Unlike the standard fitness functions like ISE and ITAE, the fitness function formulated using EA technique improves the convergence of PSO by increasing the orthogonality between the left and right eigen vectors. One of the key benefits of EA technique is that by increasing the stability horizon of closed loop system it enhances the robustness against the exogeneous disturbances. Widening the angle between the left and right eigen vectors of a closed loop system, the EA technique pushes the closed loop poles away from the imaginary axis towards left and enhances the speed of response of the system as well. The statistical analysis of the LQR optimization using PSO is also reported to assess the precision and repeatability of the algorithm. The performance of the proposed technique is assessed on a quarter car model with active suspension system through hardware in loop testing. As compared with the ISE and ITAE, experimental results accentuate that the EA technique can offer improved ride comfort and road handling by suppressing the vibrations of the vehicle body.**

***Index Terms*—Active suspension system, Ride Comfort, LQR, PSO, Eigenstructure assignment.**

I. Introduction

Suspension System is a mechanical system that connects the vehicle body to the tire and provides ride comfort by minimizing the vibrations due to the uneven road profile. Thus, a suspension system is an important part of an automobile which is responsible for providing the passengers with comfort and at the same time, also responsible for providing good road handling which are two contradictory objectives. The prime reason for suspension systems being widely used are to give better road handling performance while at the same time providing ride comfort.

Suspension systems are of 3 types, namely, passive, active and semi-active suspension systems. The passive and semi-active suspension system consist of a damper system between the vehicle body and the tire but differ in the aspect that the spring constant and damping ratio in semi-active suspension system are variables. Unlike the passive suspension system, an active suspension system consists of an actuator which exerts the force necessary to control the vertical motion between the tire and the vehicle.

Since, an actuator cannot exert an infinite amount of force and needs to exert force only at the necessary time, a feedback controller is needed to regulate the amount of force exerted by the actuator. This feedback controller is designed using LQR strategy and needs to be optimized so that a minimum amount of force results in obtaining best performance from the system. The LQR control gain is an optimal pole placement gain which is achieved based on the minimization of the actuation energy, and guarantees the stability of the system.

Hence, the problem of optimizing LQR controller is still is a topic of research [10]. LQR minimizes a quadratic cost function that contains two penalty matrices: a state weighing matrix (Q) and a control weighing matrix (R). The selection of the penalty matrices decides the performance of the controller. One of the methods to select the Q and R matrices is a trial-and-error approach. However, it is time consuming and is infeasible to obtain a satisfactory performance. Another iterative method is Brysons technique [1] in which the weighing matrices are initialized by normalizing the state feedback variables with respect to their largest permissible value. Then, just as in the trial-and-error method, the values of the weighing matrices are changed to approach the minimum index value. Another popular technique is pole placement method [2] in which the weighing matrices are assigned based on the poles, which satisfy the transient and steady state performance criteria. Nevertheless, the pole placement technique fails to consider the state and input constraints of the system.

Hence, the weight selection of LQR can also be formulated as an optimization problem and the meta heuristic algorithms like genetic algorithms, ant colony optimization, artificial bee colony, PSO etc. can be used to determine the Q and R matrices. Kundu and Nigam [3] have implemented Genetic Algorithm using ISE, ITAE and IAE as the fitness functions to be optimized while Wang et al. [4] have used Pareto-based Multi-objective Binary Probability Optimization Algorithm (MBPOA) to find the Q and R matrices by optimizing two objective functions Maximum Deviation and IAE simultaneously. Several authors have optimized the controller developing fitness functions specific to the application. Abdulla et al. [5] have used an objective function dependent on the weighted sum of rise time, peak time and a steady state error of the closed loop transfer function. The function is then optimized using GA, PSO and ABC algorithms. Nagarkar and Vikhe [6] have used Multiobjecive GA to optimize LQR controller for active suspension system in which they have taken all the objectives of system as fitness functions and have optimized them simultaneously. Yong et al. [7] have used evolutionary algorithms to optimize multiple objective functions based on maximizing the real part of eigenvalues in the left side of the s plane for the closed loop system while at the same time minimizing the controller gain and amplitude of deviation from stable state. Ant colony optimization was used to maximize the norm of the difference between the desired and actual eigenvalues and also at the same time maximize the Euclidian distance between the desired and actual eigen vectors [8]. Neto et al. [9] have used GA to minimize the fitness function in which the eigenvalue sensitivities of closed loop system are normalized and summed while simultaneously executing constraints on the eigenstructure so as to obtain desirable performance and have extended their work in [10] and used GA to select Q and R matrices and Artificial neural networks to find solution of Algebraic Ricatti Equation.

However, the objective function used to find the weighting matrices by adopting the aforementioned algorithms varies. The task of tuning an evolutionary algorithm is an obstacle in itself because using incorrect tuning parameters will lead the meta-heuristic technique to become similar to a random search and will heavily impact the convergence of the algorithm. Therefore, objective functions with user designed parameters will provide another challenge of tuning the design parameters while optimization of multi-objective function poses the problem of selecting one optimal solution out of the many solutions. Thus, to reduce the complexity and the difficulty, we have designed a single objective function which does not require tuning and is independent of the application for which LQR is being used ,thereby, allowing us to focus more on tuning of the evolutionary algorithm. To validate the performance of EA based PSO for LQR tuning, we also present the experimental validation on a quarter car model with active suspension system to achieve trade off among the following performance measures: ride comfort, vehicle handling, and passenger safety.

The remainder of the paper is organized as follows. Section 2 describes the mathematical modeling of the active suspension system. Section 3 presents the LQR controller. Section 4 explains the PSO technique for weight optimization. Section 5 details the EA technique for improving the convergence of PSO. Section 6 investigates the performance of the aforementioned objective function and compares it with the performance of ITAE and ISE objective functions. Finally, the paper ends with the concluding remarks in Section 7.

II. Mathematical Modeling

Fig. 1 shows the quarter car model with active suspension system [11]. The unsprung mass, (M_{us}) represents the mass of the tire and its flexibility is represented by the spring (K_{us}) and internal damping is represented by the damper (B_{us}). The sprung mass M_s represents one-fourth of the mass of the vehicle body. The spring (K_s), the damper or shock absorber (B_s) and the actuator (F_c) together represent the active suspension system.

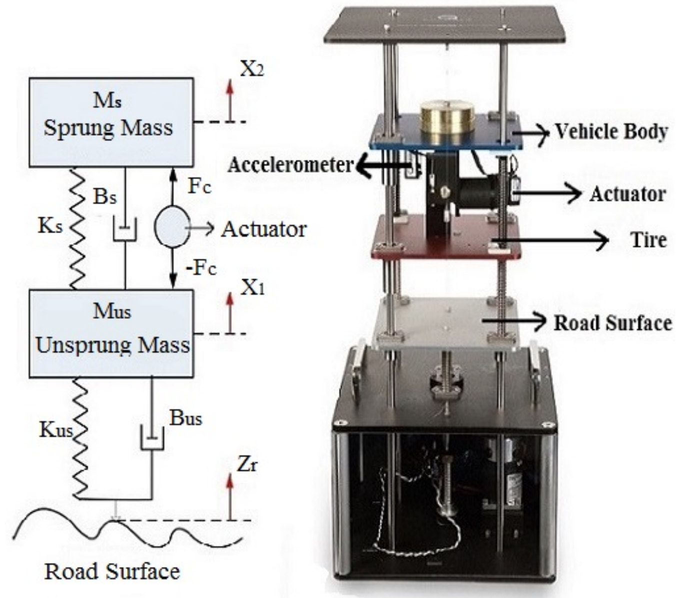


Fig. 1: Double Mass-Damper-Spring model of Active Suspension System

Table I gives the parameters of the quarter car model with the active suspension system. The equations of motion for the system derived from the free body diagram after eliminating the force of gravity and linearizing around equilibrium point is given by

$$\begin{aligned} M_{us}\ddot{z}_{us} = & -B_s\dot{z_{us}} - B_{us}\dot{z_{us}} - F_c + B_s\dot{z}_s \\ & + B_{us}\dot{z}_r - (z_{us}-z_s)K_s - (z_{us}-z_r)K_{us} \end{aligned} \tag{1}$$

$$M_s\ddot{z}_s = B_s\dot{z_{us}} + F_c - B_s\dot{z}_s - (z_s-z_{us})K_s \tag{2}$$

The system consists of two inputs: road disturbance and the controller force exerted by actuator which is the control signal. Using the following state and input vectors, we obtain the state space model of the active suspension system. The state variables, the inputs to the system and the outputs are defined as:

$$x = \begin{bmatrix} Z_s - Z_{us} & \dot{Z}_s & Z_{us}-Z_r & \dot{Z_{us}} \end{bmatrix}^T, u = \begin{bmatrix} \dot{Z}_r & F_c \end{bmatrix}^T$$

The first state represents the suspension deflection while the second state represents the vertical velocity of the vehicle body. Tire deflection and the tire vertical velocity are represented by the third and the fourth states, respectively. The measured outputs are the suspension travel and the body acceleration.

$$y = \begin{bmatrix} \dot{Z}_s - Z_{us} & \ddot{Z}_s \end{bmatrix}^T$$

Hence, the state space representation of the active suspension system is given as:

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}\right\} \tag{3}$$

The system matrix A, the control matrix B, the output matrix C and the feedforward matrix D are:

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{-K_s}{M_s} & \frac{-B_s}{M_s} & 0 & \frac{B_s}{M_s} \\ 0 & 0 & 0 & 1 \\ \frac{K_s}{M_{us}} & \frac{B_s}{M_{us}} & \frac{-K_{us}}{M_{us}} & \frac{-(B_s+B_{us})}{M_{us}} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{M_s} \\ -1 & 0 \\ \frac{B_{us}}{M_{us}} & \frac{-1}{M_{us}} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{K_s}{M_s} & -\frac{B_s}{M_s} & 0 & \frac{B_s}{M_s} \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{M_s} \end{bmatrix}$$

TABLE I: Active suspension system plant parameters

Symbol	*Description*	*Value*
M_s	Sprung mass	2.45 kg
M_{us}	Unsprung mass	1 kg
K_s	Suspension stiffness	900 N/m
K_{us}	Tire stiffness	1250 N/m
B_s	Suspension damping coefficient	7.5 Nsec/m
B_{us}	Tire damping coefficient	5 Nsec/m

III. LQR Controller

The LQR controller is used to the find the feedback gains for the system if the matrices A and B are controllable. The controller minimizes the following quadratic performance index to realize an optimal controller force F_c.

$$J = \int_0^\infty (x(t)^T Q x(t) + R F_c^2(t)) dt \tag{4}$$

Here, the weighing matrix Q penalizes the states of the system while the matrix R reflects the constraints put on the controller force by penalizing it. The weighing matrices Q and R are essentially tuning variables which affect the performance of the controller. It should be noted that weighing matrix Q should be symmetric positive semi-definite matrix(eigen values of the matrix should be greater than or equal to 0), while R should be a symmetric positive-definite matrix (eigen values of the matrix should be positive only). The solution which minimizes the above mentioned function is given by:

$$F_c = -Kx \tag{5}$$

where K is the feedback gain of the system calculated by:

$$K = -R^{-1} B^T P \tag{6}$$

where P is the solution of the algebraic riccati equation given by:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{7}$$

The closed loop equation after calculating the gains is then given by:

$$\dot{x} = (A - BK)x \tag{8}$$

IV. Particle Swarm Optimization

Particle Swarm optimization is a meta-heuristic algorithm which minimizes a given objective function by initializing a population or swarm of particles which are essentially the candidate solutions with the same velocity in different directions. Over a number of iterations, the particles are made to move into the solution space and the movement of each particle is guided by its own best position or local best and the globally best or swarms best position. The updates are made into particles position and velocity using simple mathematical operations such that the entire swarm moves towards the globally best solution [12]. Fig. 2 illustrates the working of PSO.

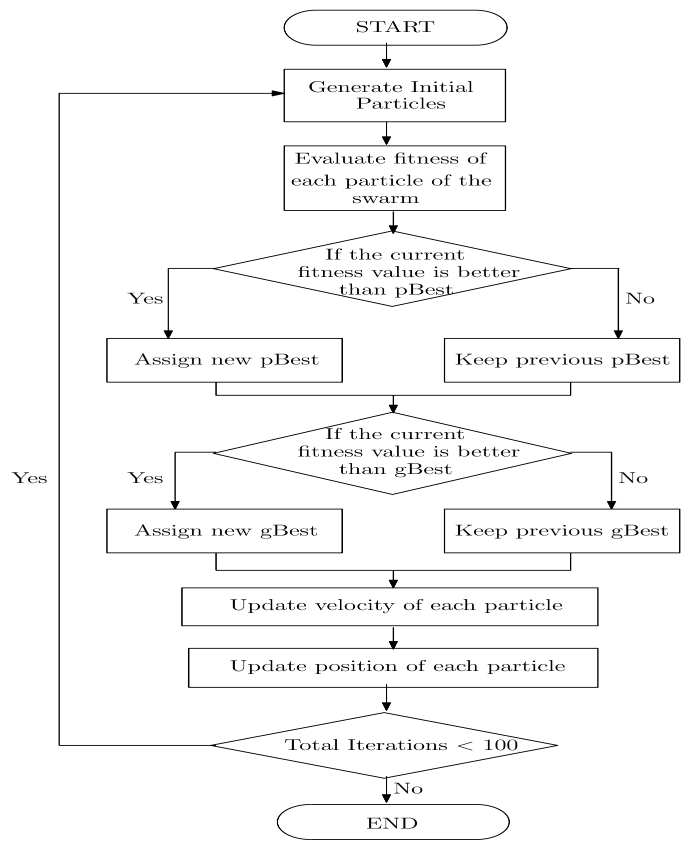


Fig. 2: Flowchart of PSO optimized LQR

The velocity and the position of the particles of the swarm are updated as follows:

$$v_{in}^{(t+1)} = w v_{in}^t + c_1 \psi_1 (p_{in}^t - x_{in}^t) + c_2 \psi_2 (p_{gn}^t - x_{in}^t) \tag{9}$$

$$x_{in}^{(t+1)} = x_{in}^t + v_{in}^{(t+1)} \tag{10}$$

where, $v_{in}^{(t)}$: Component in dimension n of the i^{th} particle velocity in iteration t
x_{in}^t : Component in dimension n of the i^{th} particle position in iteration t
c_1, c_2 : Cognitive and social factors (acceleration coefficients)
p_{in} : Best position achieved so far by the i^{th} particle in n^{th}

dimension
p_{gn} : Global best of all the particles in n^{th} dimension
ψ_1, ψ_2 : Random number in the [0, 1] interval
w : Inertia weight factor

V. Eigenstructure Assignment based Function

Eigenvalue problems are common occurrence in a lot of areas of science and engineering. An eigenvector of a matrix A is a vector with no change in its original direction on application of linear transformation on it. The factor by which the vector is scaled is called the eigen value. The eigenstructure is represented by the eigenvalues and eigenvectors. The linear algebra associated with the system in equation (3) is represented as follows:

$$Av_i = \lambda_i v_i \tag{11a}$$

$$w_i^T A = \lambda_i w_i^T \tag{11b}$$

where v_i are the right eigen vectors and w_i are the left eigen vectors and λ_i are the eigen values of the system.
The significance of the eigen values and eigen vectors for a closed loop feedback system is evident from the fact that the speed of dynamic response is influenced by the systems eigen values while the shape of the response is governed by the closed eigenvectors of the system [13]. The time response of the system in equation (3) is given as:

$$\begin{aligned} y(t) = C\sum_{i=1}^{n} v_i e^{\lambda_i t} w_i^T x(0) \\ + \sum_{i=1}^{n} v_i w_i^T \int_0^t C e^{\lambda_i (t-\tau)} B u(\tau) d\tau \end{aligned} \tag{12}$$

where x(0) represents the initial conditions of the system.
Equation (12) shows the dependence of the time response and transient response on the systems eigen values and eigen vectors. The first term represents the transient response of the system. The system nodes are denoted by:

$$n(t) = w_i^T x(0) e^{\lambda_i t} \tag{13}$$

The proposed method for eigenvector assignment is implemented by finding the right and left eigen vectors of the closed loop feedback system. The right eigenstructure assignment is widely used to suppress vibrations of flexible structures and for disturbance decoupling problems while the left eigenstructure assignment is used to define controllability measure [14]. It has been well established that the sensitivity of eigenvalues of a square matrix to the perturbation in its parameters is directly related to the ill conditioning of the matrix of eigenvectors to inversion [15]. The differential changes in equation (11a) can be written as:

$$dAv_i + Adv_i = d\lambda_i v_i + \lambda_i dv_i \tag{14}$$

By premultipying the equation (14) with w_i^T and using equation (11b), the eigenvalue perturbation can be expressed as:

$$d\lambda_i = \frac{w_i^T dA v_i}{w_i^T v_i} \tag{15}$$

The change in corresponding vector is given as:

$$dv_i = \sum_{j=1}^{n} d_{ij} v_i \tag{16}$$

where $d_{ij} = 0$ for $i = j$

$d_{ij} = \dfrac{w_i^T dA v_i}{(\lambda_i - \lambda_j)(w_i^T v_i)}$ for $i \neq j$

From equations (15) and (16), the following bounds for eigenvalue and eigenvector results:

$$|d\lambda_i| \leq \frac{||dA||_2 ||w_i||_2 ||v_i||_2}{|w_i^T v_i|} = k_i ||dA|| \tag{17}$$

$||\ ||_2$ is the L2-norm of the vectors and k_i (from bounds $1 < k_i < \infty$) is the condition number of the λ_i given by:

$$k_i = \frac{||w_i||_2 ||v_i||_2}{|w_i^T v_i|} \tag{18}$$

The condition number is the measure of sensitivity of eigenvalues to perturbations and uncertainty in the system.

Robustness in eigenstructure assignment is the degree of insensitivity of the closed loop eigenvalues to the perturbations in the system. The interpretation of insensitivity and condition number is that a perturbation O(e) in A can cause an O(e/$|Cos\theta|$) perturbation in the eigenvalue λ_i. Thus, if left and right eigenvectors are orthonormal, then a large change will occur in the eigenvalues.

A. EA Fitness Function

The Q and R matrices are taken as diagonal matrices and the particle is modeled as: All the elements of the particle are

Q_{11}	Q_{22}	Q_{33}	Q_{44}	R

variables which are initialized by random values, by the metaheuristic algorithm, within a user specified bounds. These values are then used to calculate the gains for LQR controller as illustrated in Fig. 3. The gains for each particle are then used to derive left and right eigenvector for closed loop system formed by each particle. We know that closed loop gains change the position of the poles. The cosine of the angle between the left and the right eigenvectors for an eigenvalue is given by:

$$N = \frac{W^T V}{||W||_2 ||V||_2} \tag{19}$$

where W and V are the set of the left and the right eigen vectors respectively.
$V = [v_i] = [v_1, v_2, ..., v_n]$
$W = [w_i] = [w_1, w_2, ..., w_n]$

Since, a MIMO system has multiple eigen values and hence, multiple eigen vectors, the fitness function is given as:

$$min\ S = \left| \sum N_i \right| \tag{20}$$

where S is the sum of the cosine of the angles between the left and the right eigen vectors. Minimizing S orthogonalizes the left and the right eigen vectors which, in turn, causes perturbations in the closed loop eigenvalues, thereby, causing the poles of the closed loop system to be very well separated and move towards the left half of the s plane. With further iterations of PSO, the algorithm converges towards a solution space where the eigenvalues get very well separated and dispersed in the left half of the s plane within the upper bounds of the eigenvalue sensitivity yet far away from open loop poles, thereby becoming, well conditioned and more insensitive to perturbations compared to set of eigenvalues in previous iteration, and hence, improving the stability of the system and increasing robustness.

As soon as the iteration based terminating condition is met the algorithm stops and the particle with the best position gives the lowest value of the cost function and thus, resulting in optimal value of Q and R matrices.

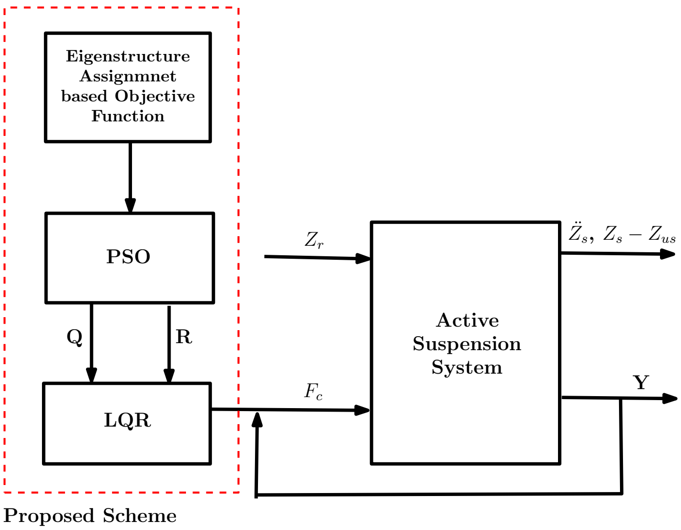


Fig. 3: PSO optimized LQR controller model

B. ITAE and ISE Fitness Functions

We also consider the following two standard fitness functions to assess the convergence of EA based technique. ITAE is defined as the integration of absolute error multiplied by time over time and is mathematically expressed as:

$$ITAE = \int |\epsilon| t dt \tag{21}$$

ISE is defined as the integration of square of error over time and is mathematically expressed as:

$$ISE = \int \epsilon^2 dt \tag{22}$$

VI. Results & Discussion

The PSO tuned LQR is implemented in MATLAB R2017a. The parameters used to tune the PSO algorithm are given in Table II.

TABLE II: Tuning Parameters specified for PSO

Parameter	*Value*
Number of variables	5
Number of iterations	100
Size of the swarm	100
Inertia weight	1
Acceleration constant, c_1	1.98
Acceleration constant, c_2	1.95

Fig. 4. shows the convergence plot of PSO over 100 iterations for three objective functions mentioned in Section 3. The statistical parameters related to convergence of objective functions is reported in Table III. From the plot and the statistical parameters, it is noticed that PSO almost always converges for ISE but it is seen in the later sections that the performance obtained by optimizing LQR using ISE is unsatisfactory. ISE eliminates large errors from the system but the oscillations of small amplitude still remain while ITAE second to ISE in convergence weighs the errors more heavily later than in the starting of the response. The minimum cost and the mean of the fitness function is the least in case of EA based PSO tuning which implies that EA has a better convergence compared to ITAE and ISE.

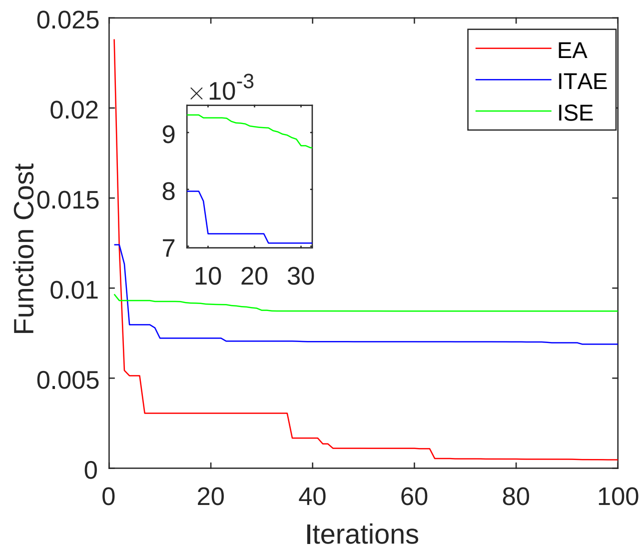


Fig. 4: Convergence of PSO for various objective functions

TABLE III: Statistical parameters of PSO optimization

Parameter	*EA*	*ITAE*	*ISE*
Minimum Cost	0.000460603	0.006952334	0.008722766
Maximum Cost	0.002998509	0.007901573	0.008724415
Range	0.002537905	0.000949239	0.000001649
Mean	0.001436203	0.00731934	0.008723259
Standard Deviation	0.000771318	0.000316853	0.000000658

The Q and R matrices obtained by optimization of LQR controller for active suspension system, using the EA based objective function are:

$$Q = \begin{bmatrix} 248.9755 & 0 & 0 & 0 \\ 0 & 72.7961 & 0 & 0 \\ 0 & 0 & 25.1644 & 0 \\ 0 & 0 & 0 & 35.4063 \end{bmatrix}, R = [0.0065]$$

Similarly, the Q and R matrices obtained by optimization of LQR controller using ITAE and ISE as objective function are:

$$Q = \begin{bmatrix} 25.4384 & 0 & 0 & 0 \\ 0 & 32.1139 & 0 & 0 \\ 0 & 0 & 99.3586 & 0 \\ 0 & 0 & 0 & 82.4724 \end{bmatrix}, R = [0.0081]$$

$$Q = \begin{bmatrix} 500 & 0 & 0 & 0 \\ 0 & 5.1502 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 17.2294 \end{bmatrix}, R = [0.004]$$

A. *Simulation Results*

From Fig. 5 and 6, it is evident that active suspension system is better than passive suspension system in road handling as well as in providing ride comfort to the passengers. We can note that the tire oscillations are eliminated in active suspension system which provides better road handling while reduced vertical body acceleration provides better ride comfort.

Fig. 7 illustrates the vehicle body and tire deflection responses for the three objective functions. The performance measures to asses the response are given in Table IV.

TABLE IV: Performance measures

For Sprung Mass

Fitness Function	*Rise time*	*Settling time*	*Overshoot %*
EA	0.245	0.459	18.65 %
ITAE	0.262	0.842	20.6 %
ISE	0.265	1.39	19.75 %

For Unsprung Mass

FitnessFunction	*Rise time*	*Settling time*	*Overshoot%*
EA	0.119	0.4	0 %
ITAE	0.326	0.933	42 %
ISE	0.326	1.48	50.05 %

We can note that the EA based technique offers the quickest response and minimum deviation from the desired trajectory for both the sprung and unsprung mass.

B. *Experimental Validation*

Fig. 8 shows the experimental test-bed which consists of Quanser active suspension system, current amplifier and a DAQ device. The system consists of three plates which represent the vehicle body (blue in colour), the tire (red in colour) and the ground (white in colour). A DC motor is connected between the top and the middle plates to emulate the suspension mechanism. The road excitation or input is provided by another DC motor connected to the lowermost plate through a lead screw and gearing mechanism. The motion of the plates is detected by high resolution optical encoders (1024 counts/rev for blue plate and 1000 counts/rev for the red plate) while the acceleration of the top plate is measured by an accelerometer (sensitivity 9.81 m/s^2/Volt). The current amplifier, AMPAQ-L2 drives the suspension motor and the road simulation motors. The output of the active suspension system is transmitted to the computer system through the DAQ board.

The results of hardware implementation for the three fitness functions are shown below:

From Fig. 9, it can be seen that the hardware results are

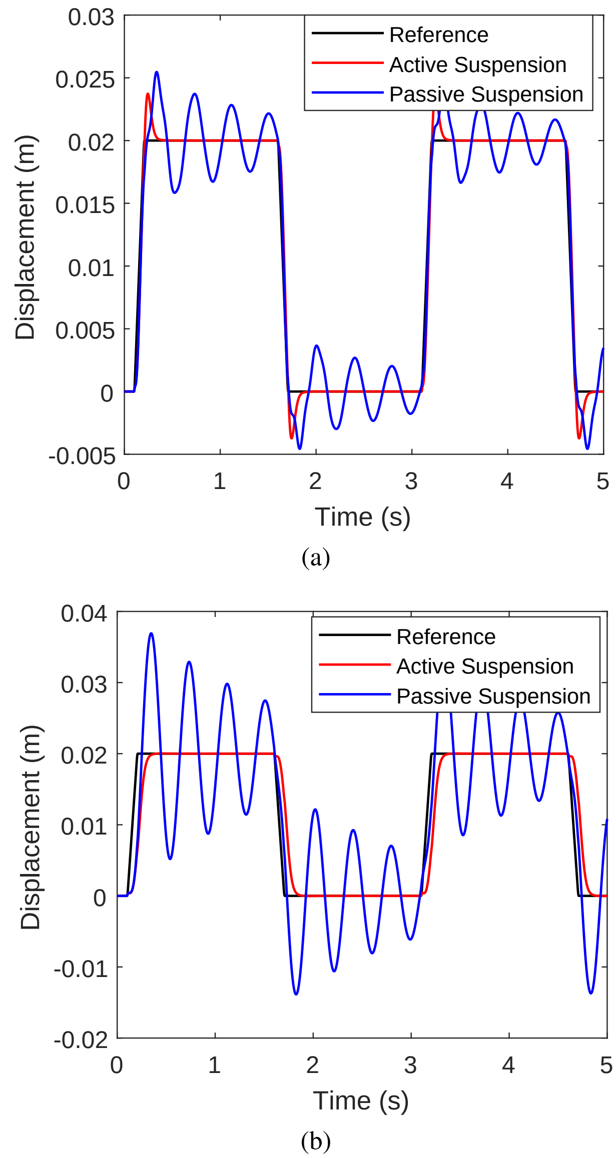


Fig. 5: Passive vs. Active (a) Response of Sprung Mass (Vehicle Body) (b) Response of Unsprung Mass (Tire)

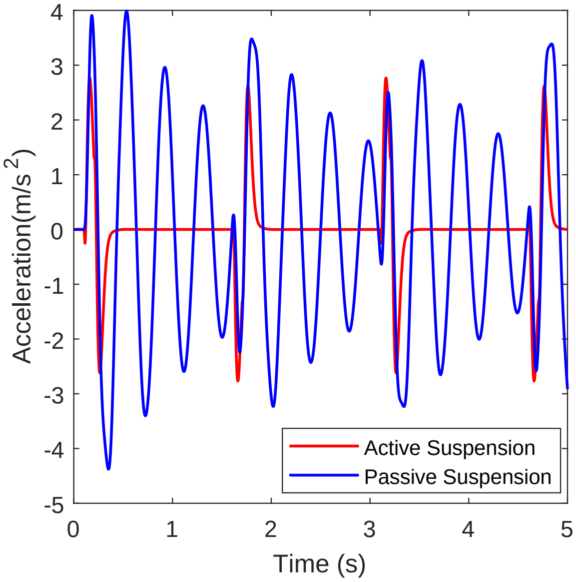
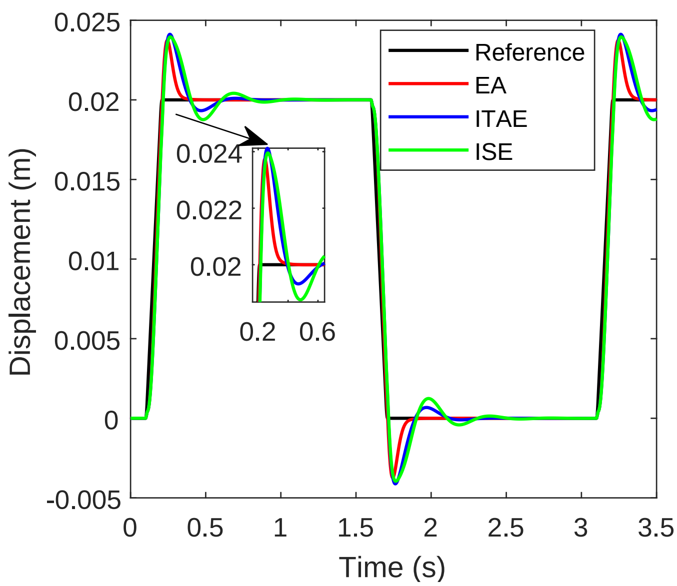


Fig. 6: Passive vs. Active: Vertical Body Acceleration

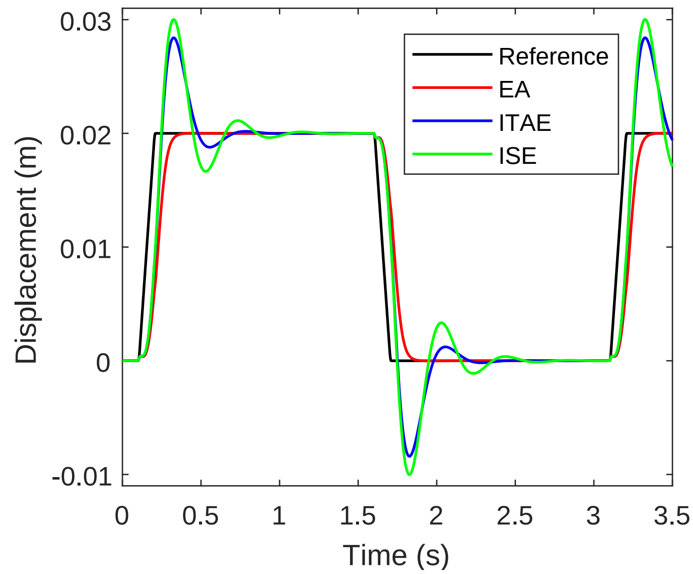


(a)

in good agreement with software simulations and helps in verifying our claim that the performance obtained by optimizing the LQR controller using the proposed Eigenstructure assignment function is better than the optimizations performed using the ITAE and ISE functions. However, we can note that the offset in suspension deflection for the three fitness functions is mainly due to the weight of the wires of the encoders and accelerometer.

VII. CONCLUSION

This paper has presented a novel eigenstructure assignment based fitness function to optimize the controller gains of LQR



(b)

Fig. 7: EA vs. ITAE vs. ISE (a) Response of Sprung Mass (Vehicle Body) (b) Response of Unsprung Mass (Tire)

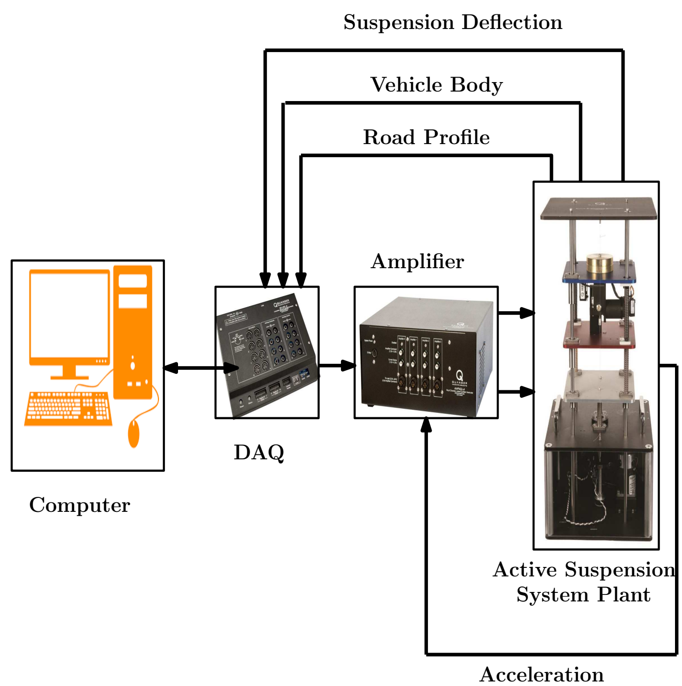
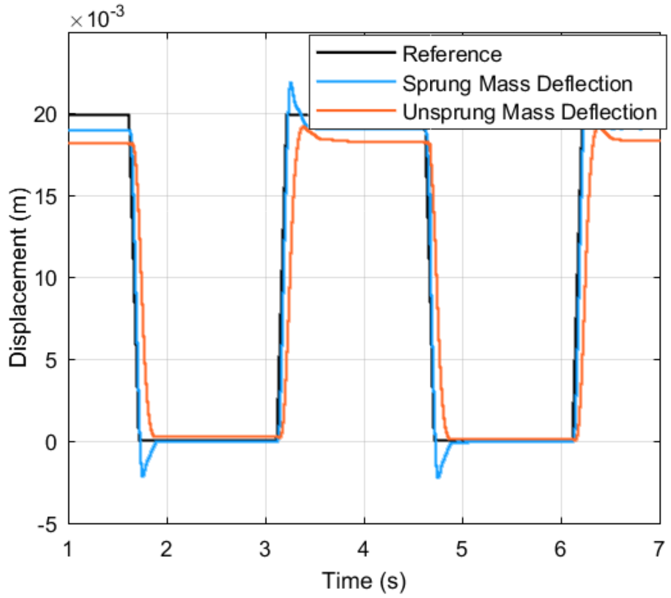
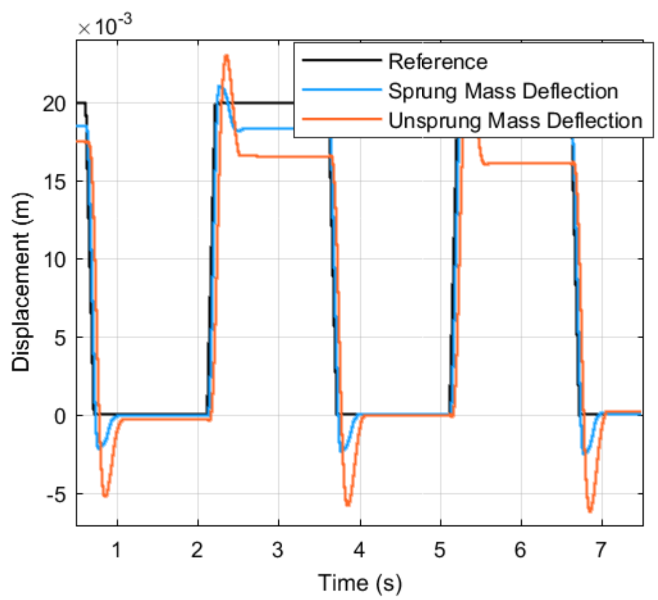


Fig. 8: Hardware Connection Diagram

using PSO implemented on an active suspension system. The dynamics of the quarter car model with active suspension is obtained using the Newton's force balance equations. As minimizing the vehicle body acceleration and the suspension deflection are two conflicting objectives, we have formulated the state feedback control design as an LQR optimization problem and used PSO to minimize the trade off between the

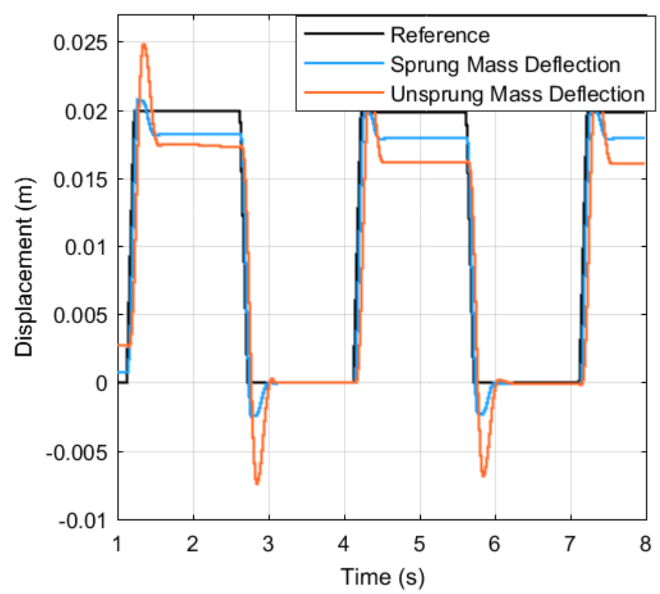


(a)



(b)

ride comfort and road handling. By increasing the orthogonality between the left and right eigen vectors for a MIMO system, we have presented a single objective function which can offer better convergence than the standard fitness functions like ISE and ITAE. Moreover, the proposed technique removes the obstacle of tuning the objective function and lay more focus on tuning the algorithm. Simulation and experimental results substantiate that the proposed fitness function can not only improve the rate of convergence but also the transient performance.



(c)

Fig. 9: Hardware Response (a) EA (b) ITAE (c) ISE

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