

$n = 50$

最坏情况:

$$T(n) = T(n-1) + T(1) + \theta(n) = T(n-1) + \theta(n)$$

$$a_n = (n-1) + (n-2) + \dots + 1 = \frac{n(n-1)}{2} = 1225$$

最好情况:

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + \theta(n)$$

$$a_n = a_{\lfloor \frac{n-1}{2} \rfloor} + a_{\lceil \frac{n-1}{2} \rceil} + n - 1$$

最好情况下的比较次数:

$$\begin{aligned} a_{50} &= a_{24} + a_{25} + 49 \\ &= a_{11} + a_{12} + 23 + 2a_{12} + 24 + 49 \\ &= a_{11} + 3a_{12} + 72 + 24 \\ &= 2a_5 + 10 + 3(a_5 + a_6 + 11) + 96 \\ &= 5a_5 + 3a_6 + 139 \\ &= 5(2a_2 + 4) + 3(a_2 + a_3 + 5) + 139 \\ &= 13a_2 + 3a_3 + 174 \\ &= 13a_2 + 3(2a_1 + 2) + 174 \\ &= 6a_1 + 13a_2 + 180 \\ &\text{易得, } a_1 = 0, a_2 = 1 \\ &\text{原式} = 193 \end{aligned}$$

综上,  $a_{50}$  最大为1225,  $a_{50}$  最小为193