Technical University of Denmark
Department of Management Engineering

42112
Mathematical Programming Modelling

Examination Assignment

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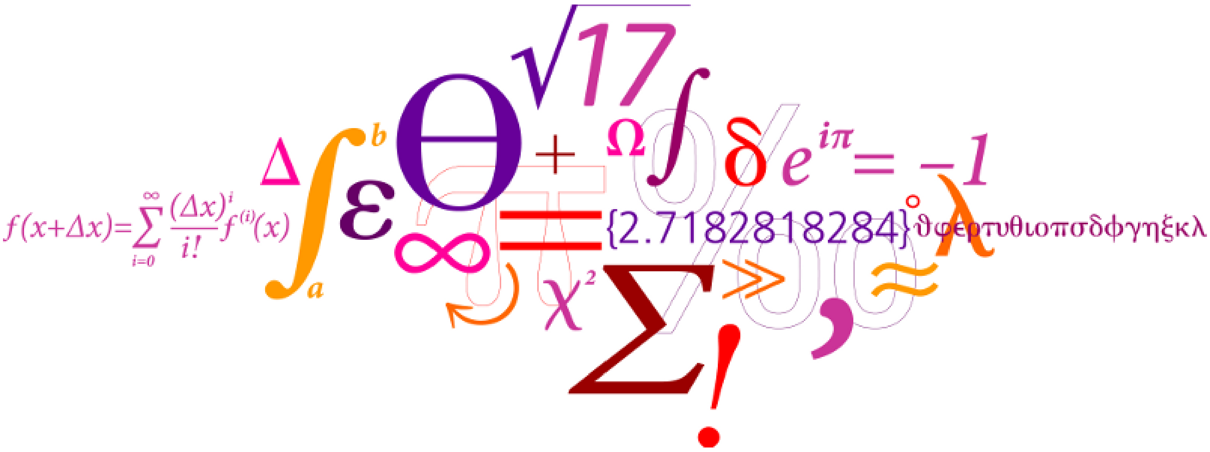


Management Science
Department of Management Engineering

Contents

1 Science Week

1.1 Assignment S1

1.1.1 Problem formulation

In this assignment a local primary school timetabler asked for help in order to plan the upcoming Science Week, where pupils from the 6^{th} to the 9^{th} grade are participating. In order to plan the schedule for every single pupil and teacher, different targets are given.
Given sets are:

- Days: {"MONDAY", "TUESDAY", "WEDNESDAY", "FRIDAY"}
- Modules: {"8.00-10.00", "10.00-12.00", "13.00-15.00"}
- Courses: {1,...,66}
- Pupils: {1,...,278}

The given parameters are:

- CourseLB_c : The minimum number of pupils needed for course c to take place.
- CourseUB_c : The maximum number of pupils for course c.
- $\text{pupil\_courses}_{p,c}$: 1, if pupil p wishes to participate in course c.

Decision Variables:

- $\text{x}_{d,h,c,p}$: Binary variable which becomes 1, if pupil p will participate in course c, in module h, on day d.
- $\text{y}_{d,h,c}$: Integer variable which indicates how much times course c is taught in module h, on day d.

1.1.2 Mathematical Model

The target in S1 is to grant as many pupil wishes as possible. So the objective function will be:

$$\max\{\textstyle\sum_{d,h,c,p} x_{d,h,c,p} * pupil\_courses_{p,c}\} \,\forall\, d,h,c,p$$

The objective function maximizes the number of granted wishes by summing all pupils p that will participate in course c, in module h, on day d, if this was on his/her wishlist.
The constraints for this model are:

- Each pupil should participate at most in 12 courses, since there are 3 different modules for 4 days:

$$\textstyle\sum_{d,h,c} x_{d,h,c,p} \leq 12, \,\forall\, p$$

- The number of participating pupils for each course c, in module h, on day d should be higher than the given minimum number of participants for each course, times the times course c is taught in module h, on day d, and lower than the given maximum number of participants for each course, times the times course c is taught in module h, on day d:

$$CourseLB_c * y_{d,h,c} \leq \sum_p x_{d,h,c,p} \leq CourseUB_c * y_{d,h,c}, \; \forall \, d, h, c$$

- Each pupil can take a course only once:

$$\sum_{d,h} x_{d,h,c,p} \leq 1, \; \forall \, c, p$$

- Each pupil can take at most one course per timeslot:

$$\sum_c x_{d,h,c,p} \leq 1, \; \forall \, d, h, p$$

1.1.3 Results

The result of the aforementioned model is 3336, which means that 3336 pupil wishes can be granted. If we multiply the number of students by the maximum of 12 courses we get the same number, meaning that every wish is granted in Assignment S1.

1.1.4 Comments

```
Pupil: 12
        1       2       3       4
1       16      7       2       63
2       5       11      34      61
3       39      40      30      51
```

Figure 1: Sample weekplan of pupil 12 S1

As it can be seen, Pupil 12 has all his wishes granted(hypothetically), as he or she has a full weekplan. In order to check if these courses were on his wishlist, these are the ones he wished to participate in:
1,2,5,7,11,16,30,34,39,40,43,51,53,61,63,66.
(NOTE: the codes for printing the above plan, wishlist etc. are in document help_assignment1.jl)

1.2 Assignment S2

1.2.1 Problem formulation

The model of Assignment S2 is an extension of the model used in Assignment S1. The maximization of pupils wishes still remains the target, but restrictions have been added. These restrictions concern the availability, ability and maximal working hours of every teacher.
The set of teachers has been added:

- Teachers: {1,...,27}

Added parameters are:

- teacher_courses$_{t,c}$: 1, if teacher t can teach course c.
- teacher_availability$_{t,d}$: 1, if teacher t is available on day d.
- teacher_capacity$_{t}$: maximum number of hours a teacher can teach.

Finally a decision variable has to be added:

- z$_{d,h,c,t}$: Binary variable which becomes 1, if teacher t will teach course c, in module h, on day d.

1.2.2 Mathematical Model

The objective function stays the same:

$$\max\{\sum_{d,h,c,p} x_{d,h,c,p} * pupil\_courses_{p,c}\}, \forall\, d, h, c, p$$

The model for S2 shares also the same constraints as S1, but these need to be extended. The added constraints are:

- Each teacher can teach only one course per module:

$$y_{d,h,c} = \sum_t z_{d,h,c,t}, \forall\, d, h, c$$

- Each teacher can teach up to three modules per day, on the days he or she are available:

$$\sum_{h,c} z_{d,h,c,t} \leq teacher\_days_{t,d} * 3, \forall\, d, t$$

- Each teacher has a maximum number of hours he can teach:

$$\sum_{d,h,c} z_{d,h,c,t} \leq teacher\_capacity_t, \forall\, t$$

- Each teacher that is assigned to teach a course, must be capable to do so:

$$z_{d,h,c,t} \leq teacher\_courses_{t,c}, \forall\, d, h, c, t$$

1.2.3 Results

After extending the model for the restrictions of Assignment S2,
the objective value is 2570 with a gap of 7.9377%. So, the number of wishes granted decreased a lot.

1.2.4 Comments

```
Pupil: 12
        1       2       3       4
1       51      66      0       1
2       2       43      11      7
3       5       34      0       63
julia>
```

Figure 2: Sample weekplan of pupil 12 S2

As it can be seen, adding teachers in the equation changed the weekplan of pupil 12, as he is now participating in 10 of the max 12 courses he could participate.

```
Teacher: 18
        1       2       3       4
1       0       25      0       0
2       0       0       0       0
3       0       24      0       0
```

Figure 3: Sample weekplan of teacher 18 S2

Teacher 18 can only teach on Tuesday, has the capacity of working 7 modules(which makes working only one day a binding constraint i.e. 3 modules max) and can teach beside others, courses 24 and 25.

1.3 Assignment S3

1.3.1 Problem formulation

In this extension of the model the target is that teachers do not have an idle hour in between two modules. In order to ensure that no teacher gets an idle middle slot, a new variable and two new constraints need to be added.

1.3.2 Mathematical Model

The model for Assignment S3 shares the same objective function as S1 and S2 i.e. maximizing the number of pupils wishes. There are also no sets or parameters to be added. What is needed is a new variable:

- $s_{d,h,t}$: Binary variable which becomes 1, if teacher t starts teaching in module h, on day d.

There are also two new constraints to be added:

- Each teacher should start only once per day:

$$\sum_h s_{d,h,t} \leq 1, \; \forall \, d, t$$

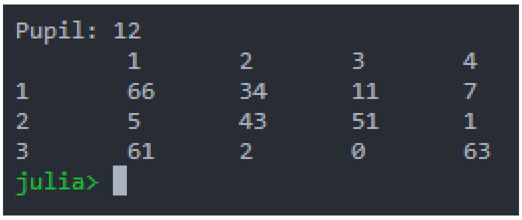
- A teacher can teach a course if he started teaching the previous module, or if he starts teaching in this module.

$$\sum_c z_{d,h-1,c,t} + s_{d,h,t} \geq \sum_c z_{d,h,c,t}, \; \forall \, d, h, t$$

1.3.3 Results

After ensuring that no teacher will have an idle slot,
the objective value is 2563 with a gap of 8.2325%, while the best bound 2774, remained unchanged.

1.3.4 Comments

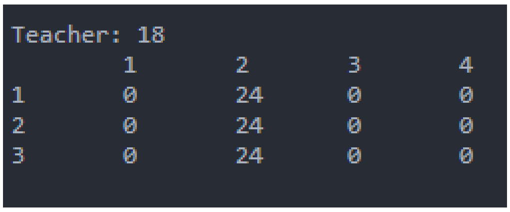

Pupil: 12

	1	2	3	4
1	66	34	11	7
2	5	43	51	1
3	61	2	0	63

julia>

Figure 4: Sample weekplan of pupil 12 S3

As it can be observed, the number of wishes granted for pupil 12 increased from 10 to 11 comparing Assignment S2 and S3. This is due to setting a gap from optimum in order for the problem to be solved in reasonable time(this can also be observed from the best bounds the solver is showing).


Teacher: 18

	1	2	3	4
1	0	24	0	0
2	0	24	0	0
3	0	24	0	0

Figure 5: Sample weekplan of teacher 18 S3

In Assignment S2 Teacher 18 had an idle hour in between two modules(Figure 3). After extending the model in order not to have idle hours in between, Teacher 18 works on all 3 modules of Tuesday.

1.4 Assignment S4

1.4.1 Problem formulation

In Assignment S4, everything stays the same as before. A new thing to consider is the grade each pupil belongs.

1.4.2 Mathematical Model

As mentioned above, everything stays the same as before except the new parameter:

- grade_p : Shows the grade of each student p.

Besides that three new constraints are added:

- The number of pupils from grades 6.1, 6.2, 6.3 needs to be more than 2.

$$\sum_{pp} x_{d,h,c,pp} \geq 2 * x_{d,h,c,p} \text{ , } if\ grade_p = 61, \forall\, d, h, c, p$$

$$\sum_{pp} x_{d,h,c,pp} \geq 2 * x_{d,h,c,p} \text{ , } if\ grade_p = 62, \forall\, d, h, c, p$$

$$\sum_{pp} x_{d,h,c,pp} \geq 2 * x_{d,h,c,p} \text{ , } if\ grade_p = 63, \forall\, d, h, c, p$$

1.4.3 Results

After adding the constraints for Assignment S4 the best bound of 2774 remains the same. The objective value is 2639 with a gap of 5.1156%

1.4.4 Comments

```
Pupil: 12
         1        2        3        4
1        66       43       51       63
2        0        2        34       1
3        5        0        11       7
julia>
```

Figure 6: Sample weekplan of pupil 12 S4

After ensuring that every pupil from the 6^{th} grade (6.1, 6.2, 6.3) will have a classmate for every course he is participating in, pupil 12 has again 10 out of his 18 wishes granted(Pupil 12 belongs to grade 61).

```
Teacher: 18
         1        2        3        4
1        0        24       0        0
2        0        24       0        0
3        0        24       0        0
```

Figure 7: Sample weekplan of teacher 18 S4

The weekplan of Teacher 18 remains unaffected by the new constraints.

2 Handball Referee Scheduling

2.1 H1

2.1.1 Problem formulation

In this assignment, we are asked to model and solve the scheduling problem of handball referees. Our aim is to create a schedule such that the overall distance that the referees have to cover is minimum.
We have the following sets at our disposal:

- referfree_arena_distances $_{r,a}$:
  Contains the distances from a referee(r)'s home to the arena(a) and back.
- match_areana $_{m,a}$:
  It is a boolean matrix Is true if the match(m) takes place in arena(a) false otherwise.
- match_time $_{m,d}$:
  It is a boolean matrix Is true if the match(m) takes place in day(d) false otherwise.
- ref_not_available $_{r,d}$:
  Indicates if the referee(r) is unavailable to work at day(d) or not.
- arena_occupied $_{a,d}$:
  Contains how many matches take place in arena(a) on day(d).
- R : number of referees.
- M : number of matches.
- D : number of match days.
- A : number of arenas.

Decision Variable :

- $x_{r,d,a}$: If referee(r) has to officiate at arena(a) on day(d)

Mathematical Model

- Objective :
  The model minimizes the objective function (total traveled distance) which is a product of two terms: the decision variable $x_{r,d,a}$ and the referfree_arena_distances $_{r,a}$:

$$Min: \sum_{d=1}^{D}\sum_{r=1}^{R}\sum_{a=1}^{A}(x_{r,d,a} * referfree\_arena\_distances_{r,a})$$

- Constraints :

  1. Two referees must be assigned to each match.

$$\sum_{r=1}^{R}(x_{r,d,a}) = 2 * arena\_occupied_{a,d} \quad \forall a, d$$

  2. A referee can officiate one match a day

$$\sum_{a=1}^{A}(x_{r,d,a}) <= 1 \quad \forall r, d$$

  3. Referee can officiate only the days that he is available.

$$\sum_{a=1}^{A}(x_{r,d,a}) <= 1 - ref\_not\_available_{r,d} \quad \forall r, d$$

2.1.2 Results

Given the above constraints we get an objective value of 8866 units of distance.

2.1.3 Comments

At this question we created a new helping matrix (arena_occupied) by calculating the product of match_areana, match_time.

2.2 H2

2.2.1 Problem formulation

On this section we will also take under consideration the referees' pair preference. In order to combine the two objective we will build two models, one for maximizing the pairs and another one for minimizing the traveled distance. Next, we will execute those models sequentially in all combinations. Finally, we will get two results, one that minimizes the travel distance first and then maximize the pairs under the given distance and one that maximizes the pairs first and then minimize the traveled distance without worsening the pair score.
On top of the sets we have on H1 we will also use:

- ref_pair $_{r,rr}$:
  Indicates if the referee (r) prefers to be paired with referee (rr).

Decision Variable :

- $x_{r,m}$: If referee(r) has to officiate the match (m).

Mathematical Model 1

- Objective of Model 1 :
  The first model minimizes the objective function (total traveled distance) which is a product of three terms: the decision variable $x_{r,m}$, the referfee_arena_distance $_{r,a}$ and the match_areana$_{m,a}$

$$Min: \sum_{m=1}^{M}\sum_{r=1}^{R}\sum_{a=1}^{A}(x_{r,m} * referfree\_arena\_distances_{r,a} * match\_areana_{m,r})$$

- Constraints of Model 1

  1. Two referees must be assigned to each match.

$$\sum_{r=1}^{R}(x_{x,m}) = 2 \quad \forall m$$

  2. A referee can officiate one match a day

$$\sum_{m=1}^{M}(x_{r,m} * match\_time_{m,d}) <= 1 \quad \forall r, d$$

  3. Referee can officiate only the days t)hat he is available.

$$x_{r,m} * match\_time_{m,d} <= 1 - ref\_not\_available_{r,d} \quad \forall r, m, d$$

Mathematical Model 2

- Objective of Model 2 This model maximizes the pairs with respect to the constraints of model 1. The objective function is a three-term product: $x_{r,m}$, ref_pair$_{r,rr}$ and $x_{rr,m}$

$$Max: \sum_{r=1}^{R}\sum_{m=1}^{M}\sum_{rr=r}^{R}(x_{r,m} * ref\_pair_{r,rr} * x_{rr,m})$$

- Constraints of Model 1

  - Same as model 1.

2.2.2 Combination 1

On the first combination we will focus on minimizing the traveled distance. Then we will find the best pair combination for that optimal distance. To do that we solve the Model 1 and we store the objective value in a variable called target_distance. Next we add an extra constraint on Model 2 to guarantee that the max pair solution will not exceed the target_distance. The constraint is:

$$\sum_{m=1}^{M}\sum_{r=1}^{R}\sum_{a=1}^{A}(x_{r,m} * referfree\_arena\_distances_{r,a} * match\_areana_{m,r}) <= target\_distance$$

We can notice that the extra constrain we put on Model 2 is based on the objective function of model 1.

2.2.3 Combination 2

On the second combination we will focus on maximizing the preferable pairs. Then we will find the best total distance covered for that optimal number of pairs. To do that we solve the Model 2 and we store the objective value in a variable called target_pairs. Next we add an extra constraint on Model 1 to guarantee that the min distance solution will not violate the target_pairs. The constraint is:

$$\sum_{r=1}^{R}\sum_{m=1}^{M}\sum_{rr=r}^{R}(x_{r,m} * ref\_pair_{r,rr} * x_{rr,m})$$

We can notice that the extra constrain we put on Model 1 is based on the objective function of model 2.

2.2.4 Results

- Combination 1
  Target_distance = 8866
  Pairs given from simple solution of Model 1 = 10
  Max pairs from Modified Model 2 = 12 20% improvement.

- Combination 2
  Target_pairs = 151
  Distance given from simple solution of Model 2 = 31096
  Min Distance from Modified Model 1 = 15990
  48% Reduction.

2.2.5 Comments

At this section we can notice that combining models we can get an improved solution given several objective functions.

2.3 H3

2.3.1 Problem formulation

In this section, there are some extra requests to take in mind in order to improve the quality schedule. So we will minimize the times a referee is officiating a team. If these are more than 3 then a cost of "1" will be added for every match. The same goes for if the referee is officiating a team sequentially. The objective is to minimize these two costs. We also need to add constraints such that the distance travelled is less or equal than 1.25 times the min distance found in Q1 and the referee pairs at least 0.75 times the pairs found in Q2.

Mathematical Model

Ìn order to write this model we need to preprocess data. We construct an Integer Array team_match_ind$_{t,23}$. From the given data we found out that each team plays no more than 23 games. So this array shows which game team t plays in which order. Next we construct an array match_per_team$_t$ to count how many games a team plays.

The variable used are:

- x$_{r,m}$: Binary, becomes 1 if referee r officiates match m
- y$_{r,t}$: Integer, summation of referee r officiating team t
- z$_{r,t}$: Integer, counts how many games referee r officiates team t sequentially
- c$_{r,t}$: Integer, cost of team t sequentially officiated by referee r

The objective function will look as follows:

$$min \sum_{r=1}^{R}\sum_{t=1}^{T} z_{r,t} + c_{r,t} \text{ , } \forall\, r, t$$

Despite the constraints that were added before, about having two referees per match, each referee playing at most one game per matchday and the referees availability, now 5 new constraints need to be added. These are:

- Variable y$_{r,t}$ equals the summation referee r officiates team t.

$$\sum_{m} x_{r,m} * team\_match_{t,m} = y_{r,t} \text{ , } \forall\, r, t$$

- Non-negative variable z$_{r,t}$ gets a value larger than zero if referee r officiated team t more than three times.

$$y_{r,t} - z_{r,t} \leq 3 :, \forall\, r, t$$

- The solution found must be at most 25% worse than the one from Q1.

$$\sum_{r}\sum_{m}\sum_{a} x_{r,m} * referfee\_arena\_distances_{r,a} * match\_areana_{ma} \leq target\_distance * 1.25$$

- We need to have at least 75% of the referee pairs that from Q2

$$\sum_{r}\sum_{m}\sum_{rr} x_{r,m} * ref\_pair_{r,rr} * x_{rr,m} \geq target\_pairs * 0.75$$

- for each consecutive match of team t that referee r is officiating a cost is considered.

$$c_{r,t} = \sum_{n=1:match\_per\_team_{t-1}} x_{r,a_{t,n}} * x_{r,a_{t,n+1}} \text{ , } \forall\, r, t$$

2.3.2 Results

- The minimum cost for the above model is 1.

2.3.3 Comments

The combination of the three models shows that we only have a cost of 1, while being very close to the minimum distance and maximal pairs. This shows that any model can be improved if it is seen by a different perspective.