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Learning and Reasoning by Analogy

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We use analogy when we say something is a Cinderella story and when we learn about resistors by thinking about water pipes. We also use analogy when we learn subjects like economics, medicine, and law. This paper presents a theory of analogy and describes an implemented system that embodies the theory. The specific competence to be understood is that of using analogies to do certain kinds of learning and reasoning. Learning takes place when analogy is used to generate a constraint description in one domain, given a constraint description in another, as when we learn Ohm's law by way of knowledge about water pipes. Reasoning takes place when analogy is used to answer questions about one situation, given another situation that is supposed to be a precedent, as when we answer questions about *Hamlet* by way of knowledge about *Macbeth*.

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1. Analogy

Much thinking is done by analogy. We face a situation, we recall a similar situation, we match them up, we reason, and we learn. We use analogy when we say some situation is likely to be a Cinderella story and when we learn about resistors by thinking about water pipes. Experts in economics, medicine, and law use analogy to relate new situations to case studies.

This paper presents a theory of analogy and describes an implemented system that embodies the theory. The paper begins with a presentation of some examples that further illustrate the sort of reasoning and learning to be understood, followed by a discussion of how one can tell that there has been some success. Next, there is a specification of a representation and an exploration of principles that seem to enable reasoning and learning to be done with the aid of the representation. Finally, an implemented system is presented that actually does reason and learn.

The implemented system has a number of key ingredients, the following in particular:

—*Extensible-relations representation.* Situations are represented using relations between pairs of parts. Supplementary descriptions can be attached to the relations when elaboration is needed.

—*Importance-dominated matching.* The similarity between two situations is measured by finding the best possible match according to what is important in the situations as exhibited by the situations themselves. Various kinds of constraint relations help determine importance. Cause is a common importance-determining constraint.

—*Analogy-driven constraint learning.* A constraint such as Ohm's law is learned as a byproduct of mapping the parts of a situation in a well-understood domain into the parts of another situation in an ill-understood domain.

—*Analogy-driven reasoning.* Some questions about a situation ask if a particular relation holds. Causes found in a remembered situation can supply suggestive precedents.

—*Classification-exploiting hypothesizing.* Memory is searched for situations that are likely to be similar to a new, given situation. The search assumes that the useful remembered situations will involve the same sorts of things as the new one at some level of classification.

The system is presumed to work with situations that are subject to certain restrictions:

—*Symbolic sufficiency.* A situation can be described by using a repertoire of classes, properties, acts, and other relations. The repertoire need not be so large as to make matching complicated.

—*Description-determined similarity.* A situation is similar to another if the important relations in their descriptions can be placed in correspondence.

—*Constraint-determined importance.* The important relations of a situation are the ones explicitly said to be important by some teacher or implicitly known to be important by being involved in constraint

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relations. Often the constraint relations have to do with various forms of cause.

—*Historical continuity.* A situation that is similar to a past situation generally leads to similar results or conclusions.

2. The Criteria for Success and the Competence to be Understood

In any scientific work, it is necessary to have some way of determining success. For this work, claims for success are with respect to the following criteria: (1) there must be an implemented program that performs a specified task; (2) the implemented program must perform by virtue of identifiable principles.

In Artificial Intelligence, having such criteria for success in mind helps avoid tendencies either to be romantically speculative about the power of vague ideas or to be overcome by the performance of working, but ad hoc programs. For this particular work, part of the specified task is to do reasoning and learning by analogy as required by the following representative scenario:

> A teacher tells a student that the voltage across a resistor can be calculated by thinking about the water pressure across a length of pipe. The student correctly finds the voltage without knowing Ohm's law.
>
> The teacher instructs the student in two different voltage-resistance-current situations and tells the student to formulate a law. The student invents Ohm's law.
>
> The teacher suggests generalizing from the water-pipe law and Ohm's law. The student formulates a linear constraint that involves forces and flows.

Thus, practice with some specific situations in one domain enables the invention of a specific law. In the other direction, once several forms of the same sort of law are known, comparison enables generalization.

Another part of the specified task is to reason by analogy, as required in the following representative examples:

> A plot outline is given in terms of 40 or 50 facts. The plot appears reminiscent of several of Shakespeare's tragedies. Analysis suggests that it is most like *Macbeth.* Someone asks if the person that corresponds to Macbeth will end up dead. Reasoning by analogy suggests asking if the person that corresponds to Lady Macbeth persuaded the Macbeth equivalent to murder the Duncan equivalent.
>
> The case of Smith versus Wesson establishes a precedent for assault cases. Smith pointed a rifle at Wesson to frighten him. The rifle was not loaded, and it was therefore harmless. Nevertheless, an assault has taken place because Wesson did not know that the rifle was not loaded. Subsequently, Smith versus Wesson is retrieved when the case of Villain versus Victim is considered. Villain pointed a pistol at Victim in order to frighten him. The pistol was a harmless toy. Reasoning by analogy suggests asking if Victim knew that the pistol was harmless.

All of these examples, having to do with both learning and reasoning, have been handled successfully in a series of experiments using an implemented system. Given that the examples suggest an interesting level of competence, it remains to be more precise by showing the input to and the output from the implemented programs and to demonstrate that the implemented programs work by virtue of identifiable principles.

The term *competence*, incidentally, is used in the sense intended by Chomsky, namely to refer to the *knowledge* necessary for any particular set of algorithms to exhibit stated behavior. *Performance* has to do with the *use* of knowledge to exhibit stated behavior. Implementation of a particular set of algorithms, from this point of view, is done so that performance can help evaluate progress toward understanding competence.

3. Representing Situations Using Extensible Relations

Broadly speaking, a representation is a vocabulary of symbols together with some conventions for arranging them. A good representation is one that has the following coupled characteristics:

It makes the important facts explicit;
It suppresses irrelevant detail;
It is perspicuous;
It exposes constraint;
It can be computed from a natural input.

In this section, a particular representation will be explained. It is based on two key assumptions: first, that what needs to be represented can be expressed in simple English and second, that much of what can be expressed in simple English can be thought of as consisting of an act-specifying verb and some noun-centered word groups that the act ties together.

For the simplest sentences, the nouns involved are just the agent and the object of the act. For others, the agent and object are supplemented by other things that participate in act description, such as an instrument, a time or location, or perhaps a source or destination if the act involves motion. In the following, for example, an instrument is specified:

Prince Charming found Cinderella with her glass shoe.

Knowing the kinds of things that are involved in describing an act and understanding how to recognize those things in sentences is the objective of *case-grammar theories* of sentence meaning. Agents, objects, instruments, and similar terms are used as names for *case slots.* Sentence analysis is viewed as the job of filling case slots using sentences as raw material.

3.1. Extensible-Relation Representation

From the perspective of case grammar, the most obvious way to strip a situation description of its syntactic nuances and to represent its approximate meaning is to use the material supplied by the sentences in the description to fill in an act-oriented schema, as illustrated by the following rendering.

Action: Find
Agent: Charming
Object: Cinderella
Instrument: [name for Cinderella's shoe]

The disadvantage of such a representation is that it favors situation matching by finding correspondences between act-oriented schemata, rather than between situation parts. Introspectively, I feel that I form analogies by pairing off situation parts, using acts and other relations as evidence, not the other way around. Consequently, I use an alternative, object-oriented representation. Parts of situations are represented as nodes that are tied together with relations forming a kind of semantic net. When more than an agent and an object is involved in an act, a supplementary description is tied to the act-specifying relation, making the representation just as capable of bearing case-like information. The supplementary description itself consists of a node related to other nodes as illustrated by Figure 1. As a consequence of the ability of the supplementary description node to say a lot about a relation, this is called an *extensible-relation* representation.

Using this extensible-relation representation, the agent and the object involved in an act have an explicit prominence. Since agents and objects are generally among the parts of a situation, the representation favors situation matching by finding correspondences between situation parts, which I think seems natural.

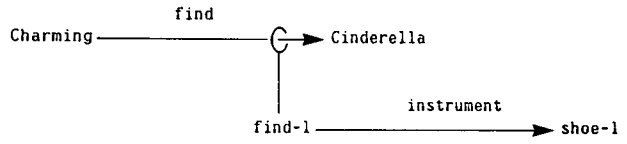
Recall that this discussion began with the assumption that much of simple English consists of sentences whose information content can be captured by a representation that views sentences from the perspective of case grammar. There are other kinds of sentences, however. Some indicate that an object belongs to some class; some indicate that an object has some property; and some indicate that a particular relationship holds between two objects:

Charming is a prince.
Cinderella is beautiful.
Cinderella loves Charming.

Information about classes and properties is easily conveyed by using A-KIND-OF and HAS-PROPERTY relations. Information about relationships between two objects is handled by state relations such as LOVES just as if the relationship were an act. Note that A-KIND-OF, HAS-PROPERTY, and state relations can be tied to supplementary description nodes although no such nodes are demanded by the examples just given.

Note that the relations from and to a supplementary node all come from a limited vocabulary because the purpose of a supplementary node is to tie together all the participants in a constraint. This limited vocabulary consists of the following: first, case names, such as *instrument*, used to describe constraints couched in the form of simple act-centered sentences; second, case-like names, such as *multiplier*, used to describe algebraic constraints; and third, names like *cause*, used to show how constraints themselves are constrained.

Fig. 1. A FIND relation further described by a supplementary description node.



The actual implementation was done using a version of FRL, an acronym for a LISP-based Frame Representation Language, developed by Roberts and Goldstein [14, 15]. FRL was used because FRL has handy mechanisms for asserting relations and attaching supplementary descriptions to them. In FRL terms, an agent—act–object combination is expressed as a *frame*, a *slot* in the frame, and a *value* in the slot. A supplementary description node for an agent–act–object combination is expressed in the form of a so-called *comment frame* attached to the frame–slot–value combination.

As a consequence of the FRL implementation, my habit is to use the terms *frame* and *value* when referring to nodes, to use the term *slot* when referring to relations of all kinds, and to use the term *comment frame* when referring to the nodes which bear supplementary descriptions. Note that comment frames are part of the knowledge representation—they are not notes to human programmers.

As it stands, many standard questions have been put off, particularly those involved in the recording of information about quantification, negation, disjunction, and perspective. These questions were put off since dealing with them was not forced by the situations considered in this work so far.

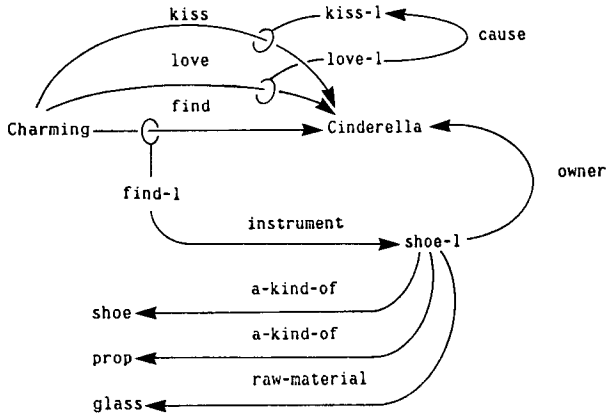
3.2. English-Like Input

It was pointed out that one characteristic of a good representation is that it can be computed from a natural input. This means that the extensible-relation representation of a situation should be computable from a simple English description of the situation or from something that is at least close to simple English.

In fact, a translator that translates English-like situation descriptions into descriptions in extensible-relation representation has been designed and implemented. It was designed with a view toward compromise between maximum input transparency and minimum translator complexity. It requires only minimal syntactic, semantic, and reference-finding machinery since it accepts a kind of parenthesized English notation in which the case names are explicitly given. For example, part of the sentence "Prince Charming found Cinderella with her glass shoe," is expressed as follows:

Charming find Cinderella [instrument shoe-1].

Fig. 2. The input translator produces a constellation of nodes and relations from a few English-like sentences.



This sentence illustrates the basic form for conveying an agent–act–object combination with case information. Material enclosed in square brackets gives case-like information to be hung on the comment frame that further describes the agent–act–object combination just asserted. Thus the expression [instrument shoe-1] generates a comment frame for the Charming-find-Cinderella combination and hangs a case-filler combination on it. More about the shoe is conveyed as follows:

Shoe-1 a-kind-of shoe prop—raw-material glass—owner Cinderella.

This sentence illustrates that the input language permits a kind of elision. When the same agent–act combination is used with several objects, the objects are just enumerated one after the other. When the same agent is involved with several acts, hyphens keep things separated.

A minimal reference feature makes it possible to use the following, simpler forms, in which specific names are avoided:

Charming find Cinderella [instrument a shoe].

The shoe is a prop—raw-material glass—owner cinderella.

Expressions like "a shoe" generate instances attached to the named class, SHOE in this case. Expressions like "the shoe" simply refer to the last instance created of the named elass. Expressions like "is a prop" are taken to mean that an A-KIND-OF relation is implied.

Finally, it is often useful to have a direct way to refer to an agent–act–object comment frame. This is done by embedding the agent, act, and object involved in curly brackets. Thus either of the following indicates that Charming's love for Cinderella causes him to kiss her:

Charming love Cinderella [cause {Charming kiss Cinderella}].

{Charming love Cinderella} cause {Charming kiss Cinderella}.

Figure 2 shows what all of this information looks like when translated into the graphical form of the extensible-relation representation.

3.3. Demons

In addition to English-like input, it is good to have some deductions made automatically, reducing the need for tedious attention to details. For example, given that Cinderella is married to Prince Charming, Prince Charming is clearly married to Cinderella. Similarly when one person kills another, it is clear that the person killed is dead.

One way to arrange for such obvious deductions is to use procedures that are invoked when relations are inserted in the data base. Such procedures are commonly called *if-added demons*. In the version of FRL used, if-added demons that trigger on the use of a particular relation are placed in a frame describing that relation.

In the end, demons prove so important that there has to be some concern about whether they can be learned. In fact, they can be because using a demon is like doing an analogy in miniature. The ideas that make it possible to accumulate big chunks of experience are the same as the ideas that explain how it is possible to remember cause–effect relations at the level of demons.

An immediate question is whether demon use should be limited, and if so, by what control scheme. At the moment, there are no answers.

3.4. Capturing Story Plots Requires Attention to Cause

Using extensible relations, ten story plots were set down. The following is what one version of one of these, namely *Macbeth*, looks like in the English-like notation:

MA is a story.

{Macbeth is a noble} before {Macbeth is a king}. Macbeth marry Lady-macbeth. Lady-macbeth is a woman—has-property greedy ambitious. Duncan is a king. Macduff is a noble—has-property loyal angry. Weird-sisters is a hag group—has-property old ugly weird—number 3.

Weird-sisters predict {Macbeth murder Duncan}. Macbeth desire {Macbeth a-kind-of king} [cause {Macbeth murder Duncan}]. Lady-macbeth persuade {Macbeth murder Duncan}. Macbeth murder Duncan [coagent Lady-macbeth—instrument knife]. Lady-macbeth kill Lady-macbeth. Macbeth murder Duncan [cause {Macduff kill Macbeth}].

In addition to *Macbeth*, the plots set down include three other Shakespearean tragedies, one of his comedies, two plays by Ibsen, and some random things, all selected by thumbing through an encyclopedia of plots. The purpose was to discover if the representation and accompanying vocabulary are adequate for reasoning with plots by analogy. The following observations were made:

A few basic English words adequately supplied by far the bulk of those needed for slot names and classes. Most of the ones I use are in Ogden's thousand word Basic English vocabulary [12] and in the first thousand or two most frequent English words [2].

Cause is important because cause constrains relations pairs.

Many people have attended to the role of cause, particularly Schank [17] and Wilks [19]. To handle cause here, the following conventions were honored, based on the work of Givon [5]:

Acts, relations, and people can cause or prevent acts and relations. The inverses of CAUSE and PREVENT are CAUSED-BY and PREVENTED-BY. See Figure 3(a) for what happens given this fragment:

Lady-Macbeth cause {Macbeth murder Duncan}.

People can persuade and dissuade. Persuade indicates cause and a CAUSE relation is therefore generated by a demon whenever PERSUADE is used. Dissuade similarly indicates prevent. Since persuaders and dissuaders normally intend for something to happen, INTEND relations are generated too, again by demons. Since the person persuaded or dissuaded retains control of what is happening, a demon-placed CONTROL relation so indicates. See Figure 3(b) for an example showing what happens given this:

Lady-Macbeth persuade {Macbeth murder Duncan}.

People can also order and forbid. For the moment ORDER and FORBID simply carry demons that place PERSUADE and DISSUADE relations and trigger their demons in turn.

People can force. Force is like order and persuade, except that the person forcing has control of what is happening. See Figure 3(c) for an example showing what happens given this use of a FORCE relation:

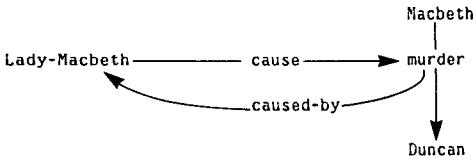
Lady-Macbeth force {Macbeth murder Duncan}.

4. Determining Analogous Parts Using Importance-Dominated Matching

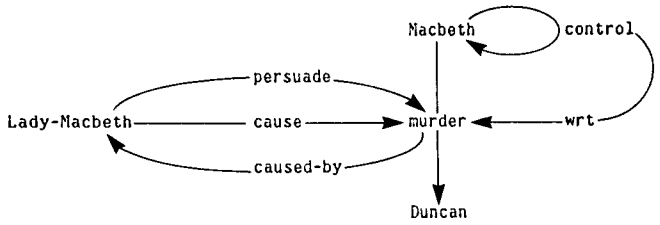
Analogy is based on the assumption that if two situations are similar in some respects, then they must be similar in other respects as well. To determine if two situations are similar, the parts of the situations must be placed in correspondence. The purpose of matching is to establish the best way to do this. In general, the best way will leave some of the parts' classes, properties, acts, and other relations unpaired, because if two situations were exactly alike, nothing could be inferred about one by using the other. A matcher for use in analogy must be flexible, not rigid.

It is easy to be seduced into worrying about matching for its own sake, without attention to the sorts of things to be matched. This typically leads to the invention of all sorts of mechanisms of doubtful value in practice. Consequently, the matcher described here was developed by implementing only those mechanisms needed to attend to the factors that demonstrably influence similarity.

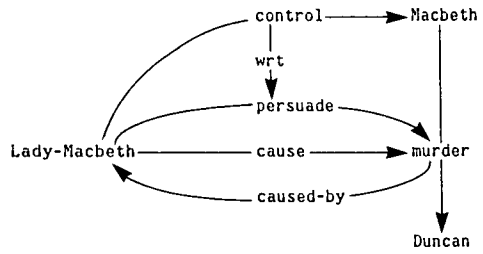
Fig. 3. The relations placed by CAUSE, PERSUADE, and FORCE.



3a: Lady-Macbeth cause {Macbeth murder Duncan}.



3b: Lady-Macbeth persuade {Macbeth murder Duncan}.



3c: Lady-Macbeth force {Macbeth murder Duncan}.

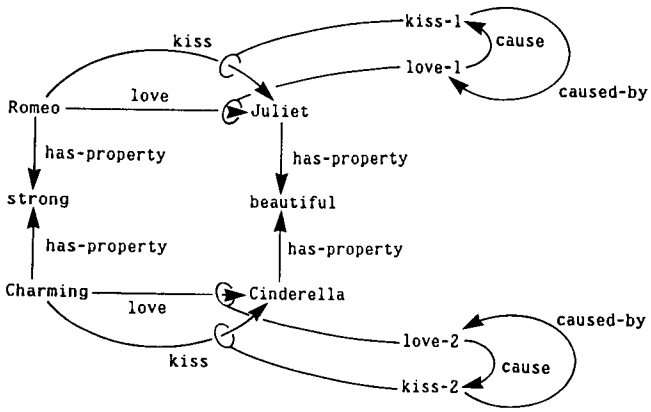
Recall that the parts of a situation are the new instances generated by the input interface whenever the articles *a* and *an* are used. Given that matching is to pair the parts of two situations, three general issues require thought: first, how is the space of all possible matches of the parts of two situations to be searched; second, what is to constitute a quantum of evidence for a particular match, and third, how are the quanta of evidence for a particular match to be combined to produce an overall measure of similarity.

The issue of how to search the space of all possible matches seemed the best issue to neglect in the early stages of this research. The reason is that efficiency is best addressed after it is established that there is something worthwhile to be efficient about. Consequently, the implemented matcher does a brute force search of the space of possible matches, calculates how good each is, and announces the best.

The price paid is that the situations matched may not involve more than a handful of parts. In general, if there are $N1$ parts in one situation and $N2$ in another, then the number of ways the situations can be paired up is $N1!/(N1\text{-}N2)!$, given that $N1$ is equal to or greater than $N2$.

At first thought, trying all possible pairings seems hopeless since the number of possible pairings gets big fast. For small $N1$ and $N2$, the number is manageable. The implemented matcher, when compiled, handles 100 or so pairing possibilities without excessive strain. For

Fig. 4. Matching *Cinderella* with *Romeo and Juliet* produces a match score of six. Having the same properties accounts for two points; having the same relations between the people accounts for two more; and having corresponding causal relations between the comment frames accounts for the final two.



larger numbers, something must be done to constrain the number of pairings considered.

Given x way of searching the space of all possible matches, the next issues have to do with finding and combining evidence so that the best match can be identified. Thought about these issues led to the following conclusions:

—To exploit an analogy between two situations starts with establishing the best correspondence between the parts of the situations using the parts' classes, properties, acts, or other relations as evidence, depending on what is important.

—To establish what is important in a situation may require attention to constraint. This in turn usually means looking at the causal relations exhibited in the situation.

—To establish enough of what is important in a situation may require expansion of some facts in the direction of more detail or abstraction of some facts in the direction of more generality.

—To establish enough of what is important in a situation may require asking some questions.

—To combine evidence, simply counting the individual items of evidence is sufficient to handle the particular tasks involved in determining success.

Plainly, the issue of what constitutes an item of evidence requires further discussion. Examples will deal with both story plots and natural laws.

4.1. Finding Correspondence Can Require Attention to Properties and Relations

Suppose, for example, that Prince Charming and Cinderella are the characters in one plot and Romeo and Juliet are the characters in another. (These names were picked so that the combinations are mnemonic—the plots are not developed in this illustration.)

Charming job entertaining—has-property brave strong—love Cinderella—kiss Cinderella.

Cinderella job cleaning—has-property beautiful.

Romeo job fighting cleaning—has-property strong—love Juliet—kiss Juliet.

Juliet has-property beautiful.

Knowing just these facts intuitively indicates that CHARMING corresponds to ROMEO and CINDERELLA to JULIET. The implemented matcher agrees because it tries all possible matches and because its simple scoring module awards one point to each shared relation. In doing its job, the matcher produces a set of paired frames for each possible match. Each set of paired frames is referred to as a list of linked pairs. Each list of linked pairs is evaluated by calculating a similarity score for each linked pair in the list and adding the results together. The score for each linked pair is calculated by scoring one point if the two linked frames contain the same value in some particular slot or if two linked frames contain the two parts of another linked pair in some particular slot.

The matcher therefore produces a score of four for the intuitively correct match of CHARMING and CINDERELLA with ROMEO and JULIET, but only one for the incorrect one.

The details of the matcher's scoring module are not important. The important thing is that it seems reasonable to use both relations and properties as evidence in lieu of specific information about what is important. We will see that relations and properties are not sufficient, however.

4.2. Finding Correspondence Can Require Attention to Corresponding Comments

For the sake of illustration, suppose Prince Charming's love for Cinderella causes him to kiss her and Romeo's love for Juliet has the same result, indicated by the following:

{Charming love Cinderella} cause {Charming kiss Cinderella}.

{Romeo love Juliet} cause {Romeo kiss Juliet}.

Certainly there is evidence for pairing CHARMING with ROMEO and CINDERELLA with JULIET since such a pairing places the LOVE and the KISS slots in correspondence. However it is evident that pairing the characters in this way also puts LOVE-1 and KISS-1 in correspondence with LOVE-2 and KISS-2, which puts a CAUSE and a CAUSED-BY relation in correspondence, producing additional evidence of similarity.

To account for the additional similarity available in comment frames, the implemented matcher adds corresponding comment frames to each possible list of linked pairs after it is formed but before it is scored. Thus the corresponding comment frames are considered linked when scoring other frames, and they themselves are scored. Thus the total score should be six for the two

situations under consideration. Figure 4 illustrates the combinations that lead to this score in graphic form.

4.3. Finding Correspondence Can Require Attention to Classification Information

So far it would not help to add the following information:

Charming is a prince. Romeo is a boy.

Cinderella is a princess. Juliet is a girl.

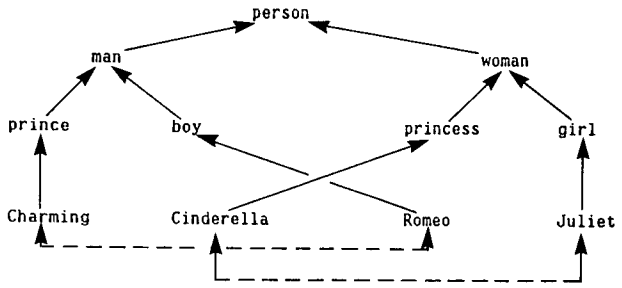
Knowing that CHARMING is A-KIND-OF PRINCE and ROMEO is A-KIND-OF BOY lends no direct strength to their similarity, nor does it help to know that CINDERELLA is A-KIND-OF PRINCESS and JULIET is A-KIND-OF GIRL. These facts do lend indirect strength to the two pairs because the given classifications indicate common classification at higher levels. After all, a prince and a boy are the same sex as are a princess and a girl.

To account for this indirect evidence of similarity, the implemented matcher treats all A-KIND-OF slots as if they contained everything that is found by tracing through the A-KIND-OF hierarchy that leads from them. The A-KIND-OF slot of CHARMING contains only PRINCE, but it is treated as if it contained PRINCE, MAN, and PERSON. Similarly ROMEO's A-KIND-OF slot is treated as if it contained not only BOY, but also MAN and PERSON. Consequently the A-KIND-OF SLOT contributes a score of two when CHARMING and PRINCE are paired. Similarly CINDERELLA is a PRINCESS, a WOMAN, and a PERSON while JULIET is a GIRL, a WOMAN and a PERSON. Their A-KIND-OF slots contribute a score of two as well. Thus the A-KIND-OF slots of all the parts lead to a match score of ten, four more than before. Figure 5 shows the A-KIND-OF hierarchy that gives these extra points.

An objection to this type of scoring is that the A-KIND-OF hierarchy might involve long chains that would tend to cause classification information to dominate matching. Happily, long A-KIND-OF chains do not occur in the situations considered in this work so far, so the objection has been noted, but not thoroughly studied.

If long A-KIND-OF chains should prove to be a problem, the work of Rosch et al. may be relevant [16]. They argue that the world of human experience is such that there is a so-called basic level of class abstraction in the A-KIND-OF hierarchy. At this basic level, two things are true: at the next level up, the members of the classes share substantially fewer properties than at the basic level; and at the next level down in the hierarchy, the members of the classes share about the same number of properties as at the basic level. Concepts like guitar, apple, hammer, shirt, table, and car are at the basic level. Musical instrument, fruit, tool, clothing, furniture, and vehicle are higher. Grand piano, MacIntosh apple, ball-peen hammer, dress shirt, kitchen table, and sports car are lower.

Fig. 5. The A-KIND-OF hierarchy is exploited in matching. The A-KIND-OF slots contribute four points to the scoring of the match with ROMEO paired with CHARMING and CINDERELLA with JULIET even though each has something different in its A-KIND-OF slot.



Above the basic level, common class membership means little. Below the basic level, common class membership adds little. Consequently, it might be reasonable to score matching points only for common classification at the basic level, ignoring common classification above and below, finessing the problem of long A-KIND-OF chains.

4.4. Constraint Makes Some Relations More Important than Others

There is some debate about whether a matcher should distinguish among the kinds of information available for matching. One view is that classification information is the most important while another view favors properties. Still another, milder view is that all information is important, but to a varying degree that has to be accounted for by a weighting scheme, possibly context dependent. This can quickly give the matcher an ad hoc feel. It is disturbing when a program must be tuned up by fooling with a system of parameters.

Nevertheless, some relations are more important than others because they lead to the conclusions that are to be exploited in the analogy process. The important relations are sometimes A-KIND-OFs, sometimes HAS-PROPERTYs, and sometimes other relations, some of which are normally incidental.

Thankfully, importance tends to be taught by teachers, either explicitly or implicitly. Explicit teaching is done when a teacher says, perhaps without justification, that some fact is important. Implicit teaching is done when a teacher includes some fact in a constraint, typically in the form of a causal chain. Thus I take the following position:

> Any relation can be important in matching. The importance of a particular relation can be determined by remembering what teachers have said about it or by noting whether it is involved in constraining something. Causing something is an important way of constraining.

For the most part, the examples in this paper assume a beneficient teacher who gives only the relevant facts and who does not deliberately try to confuse the system by shoveling detritus at it. It is important, however, to

understand that mechanisms have been implemented that pay attention to importance on demand.

In particular, the matcher can be told to use only relations that have comment frames with IMPORTANT in the HAS-PROPERTY slot. The HAS-PROPERTY slot of a comment frame can have IMPORTANT placed in it directly as in the following example:

Macbeth kill Duncan [has-property important].

Alternatively, the HAS-PROPERTY slot can have IMPORTANT put in by a demon placed in the CAUSE frame. Using this demon, all frames at either end of a cause relation are noted to be important, as well as the cause relation itself.

Actually, the implemented strategy represents one end of a spectrum of possibilities. As it stands, relations never become globally important. A looser strategy would make a relation important everywhere in a situation if it is determined to be important somewhere in the situation. A still looser strategy would make a relation important everywhere in a situation if it is important somewhere in some other situation of the same general class.

4.5. Matching Large Groups May Require Some Preliminary Classification

As the size of two groups to be matched becomes large, trying all possibilities becomes intractable. There are two choices: Throw away the exhaustive matcher and do something else, or somehow prune the collection of matching alternatives that the matcher generates. The implemented matcher prunes:

> One way to limit the matching alternatives is to restrict the pairings to those that link together only frames of the same class, as specified by instructions to the matcher.

For example, if there are two groups of people to be matched, and each contains, say, three men and four women, then the total number of match alternatives is:

$N1!/(N1-N2)! = 7! = 5040.$

But if the matcher is instructed to link men only with men and women only with women, then the number is:

$M1!/(M1-M2)! \times W1!/(W1-W2) = 3! \times 4! = 6 \times 24 = 144.$

The smaller number is only 3 percent of the larger. Of course it is no longer possible to discover a male Cinderella, a defect that may suggest a similar difficulty when people must deal with analogies involving many parts. To prevent too many blunders of this sort requires some way of selecting a good set of classes for the matcher. There may be some way of doing this by inspecting the A-KIND-OF hierarchy in the vicinity of the frames involved in the match.

4.6. Matching Finds Corresponding Parts in Plots

Here are some results showing the match scores between the four Shakespearean tragedies and one comedy:

	MA	HA	JU	OT	TA
MAcbeth	78	49	45	21	9
HAmlet	49	108	35	22	9
JUlius Caesar	45	35	91	28	8
OThello	21	22	28	71	10
TAming of the Shrew	9	9	8	10	50

The choice of Shakespearean tragedies was somewhat ill-advised since they lean toward the macabre. Nevertheless, it is interesting that the tendency to have evil, murder, and death everywhere in sight does make them more similar to each other than to the comedy.

The average score on the diagonal is 80. Evidently the average number of facts known about each plot is therefore 80. Some of the facts are derived by demons and others are implied by the A-KIND-OF connections. A demon on MURDER creates a KILL and links the MURDER and the KILL together with a CAUSE relation. A demon on KILL leads to instances of HAS-PROPERTY and CAUSE because killing someone causes that person to have the property of being dead. Also, murderers are noted to have the EVIL property.

It is instructive to look at the best and worst off-diagonal matches to see if they make sense. Evidently *Macbeth* and *Hamlet* show the most similarity. The matcher announces its view as follows:

49. Values match with 78. and 108. possible—Best match is 13. better than next best.

(49. (10. MACBETH CLAUDIUS)
(5. DUNCAN GHOST)
(4. LADY-MACBETH GERTRUDE)
(4. MURDER-1 MURDER-2)
(4. KILL-1 KILL-4)
(4. KILL-3 KILL-5)
(3. MACDUFF HAMLET)
(3. KILL-2 KILL-8)
(2. HQ-3 HQ-5)
(2. HQ-2 HQ-9)
(2. HQ-1 HQ-4)
(1. WEIRD-SISTERS LAERTES)
(1. CAUSE-6 CAUSE-11)
(1. CAUSE-5 CAUSE-20)
(1. CAUSE-1 CAUSE-8)
(1. CAUSE-2 CAUSE-9)
(1. CAUSE-7 CAUSE-12)
(0. AKO-2 AKO-3))

This makes some sense. Macbeth and Claudius both kill a king so as to become king and both are killed in turn. Their victims are Duncan and the Ghost. Macduff and Hamlet kill them. Their wives are Lady Macbeth and Gertrude.

On the other hand, *The Taming of the Shrew* and *Julius Caesar* show little similarity. In fact, there are only

two points of similarity beyond the fact that there are four people to pair up and two of the four have the same sex. The score of eight is at the level of background noise.

Evidently, the plots as given to the matcher do not exhibit much diverting detail because the general shape of the table is the same when the matcher counts only the relations that are demonstrably important. The following revised table shows this:

	MA	HA	JU	OT	TA
MAcbeth	16	12	11	5	0
HAmlet	12	28	9	5	0
JUlius Caesar	11	9	21	7	0
OThello	5	5	7	21	0
TAming of the Shrew	0	0	0	0	11

The scores are much reduced, but still *Macbeth* and *Hamlet* are similar while *The Taming of the Shrew* and *Julius Caesar* are not.

In producing the scores in this table, the matcher counted only relations marked as important. Demons on CAUSE and PREVENT do the marking.

4.7. Abstraction May Be Necessary Before Matching

Matching may require some preliminary act abstraction, exploiting the fact that acts like KILL imply more abstract acts like HURT. Suppose, for example, that two situations are proposed, one involving a tragic event, and the other, a person in conflict with himself:

> TRAGIC-EVENT is a situation.
> Evil-person is a person. Good-person is a person. Evil-person hurt Good-person. Evil-person has-property evil. Good-person has-property good.
> SELF-CONFLICT is a situation.
> Grubbla is a person. Grubbla has-conflict-with Grubbla.

As they stand, these situations are abstractions of some of the things that go on in the ten experimental plots, but neither has anything explicitly in common with any of them, other than that people are involved and that some explicit hurting goes on in Ibsen's *A Doll's House* and Shaw's *Pygmalion*. Good and evil are nowhere to be found.

A kind of abstraction can make good matches happen anyway. It is easy to put demons on HAS-PROPERTY and MURDER so that loyal people are noted to be GOOD and murderers are noted to be EVIL. Other demons can generate HURT and HAS-CONFLICT-WITH relations, given KILL. For the tragic event situation, these demons enable the matcher to produce the following results, where the numbers give the actual matching scores and the percentages of the maximum possible scores:

The matches, in order of quality, are:

5. 83.% Macbeth
5. 83.% Hamlet
5. 83.% Othello
5. 83.% Julius Caesar
4. 66.% Dolls House
4. 66.% Pygmalion
4. 66.% Adam and Eve
3. 50.% Cinderella
2. 33.% Taming of the Shrew
2. 33.% Hedda Gabbler

The act of murder made Macbeth an evil person and established that he hurt Duncan. The plot lacks perfect match with the description of a tragic event because it was not stated that Duncan was a good person, nor was it deduced. Hamlet and Julius Caesar similarly fail to match perfectly. The Ghost and Caesar are not known to be good.

Othello fails because Othello kills Desdaemona, but killing, unlike murdering, does not imply a person is evil. Desdaemona, however, is good because she is loyal.

Now consider the other situation, the one that involves self-conflict:

The matches, in order of quality, are:

2. 100.% Hamlet
2. 100.% Othello
2. 100.% Julius Caesar
2. 100.% Hedda Gabbler
1. 50.% Macbeth
1. 50.% Taming of the Shrew
1. 50.% Dolls House
1. 50.% Pygmalion
1. 50.% Cinderella
1. 50.% Adam and Eve

To kill means to hurt which means to have conflict with. Evidently the good matches are the ones with suicides.

A form of irony, incidentally, could be found the same way. But there were no such incidents, at least as described:

> IRONIC-EVENT is a situation.
> Unfortunate-person is a person. Unfortunate-person want a desired-act—attempt the desired-act [cause an actual-act]. The desired-act opposite the actual-act.

All these examples argue for the following conclusion:

> Simple deduction in the direction of abstraction facilitates some matches.

4.8. Many Similarity Measures Are Possible

The similarity measure used in the implemented matcher is just the total number of points of evidence exhibited when the parts in two situations are optimally paired. Thus the similarity is a measure of overlap.

Many other authors have considered the question of similarity measurement, although not in the context of the representation used in this paper. In particular, Tversky considers situations in which two objects defined by feature sets are to be compared [18]. He argues persuasively that similarity should be determined not

Fig. 6. Water-situation-3, a situation involving a water pipe, can be matched to the general water pipe description easily.

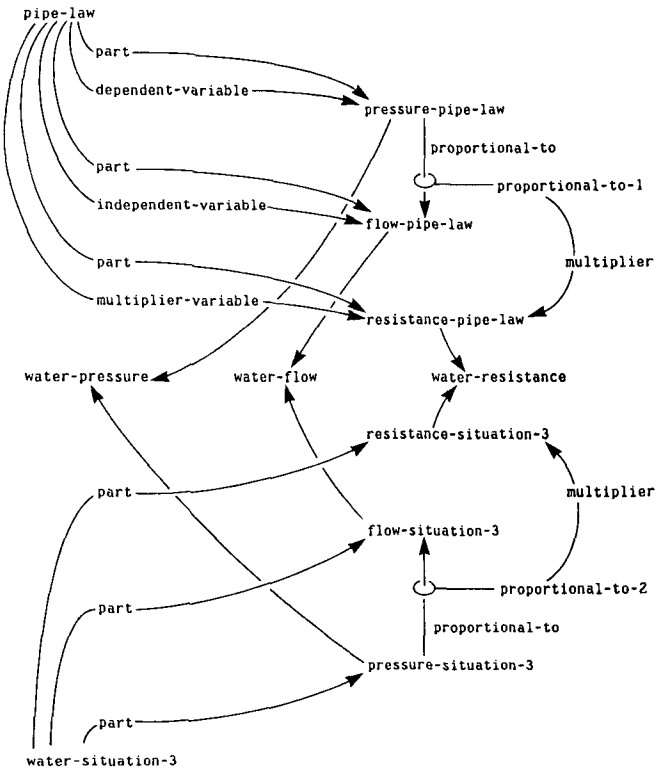
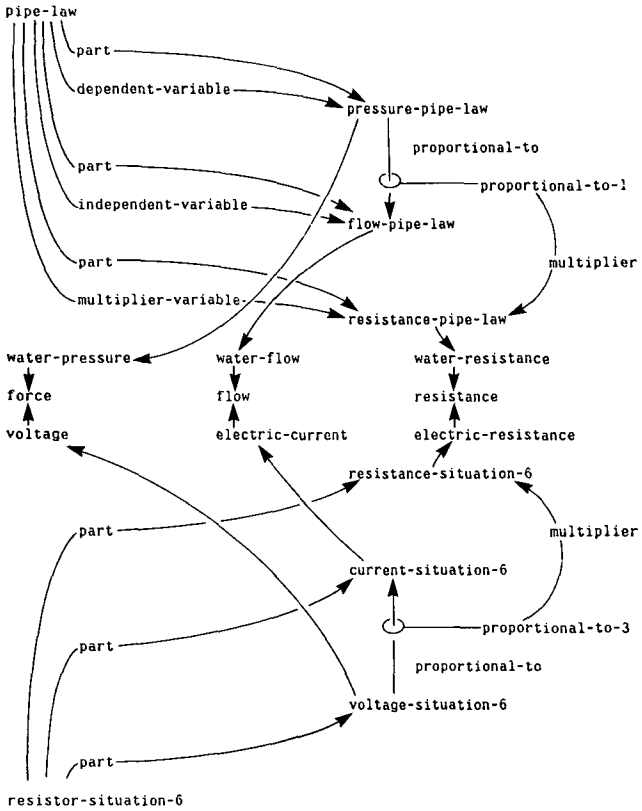


Fig. 7. Resistor-situation-6, a situation involving a resistor, can be matched to the general water pipe description.



only by the features that correspond, but also by those that do not. For determining the similarity of feature set A to feature set B, he recommends this formula:

$$\text{SIMILARITY}(A, B) = \theta f(A \cup B) - \alpha f(A - B) - \beta f(B - A)$$

For some θ, α, and β where f is typically a function that satisfies additivity:

$$f(X \cup Y) = f(X) + f(Y).$$

In this paper similarity is measured between groups of frames rather than feature sets. $A \cup B$ is analogous to the slot-value combinations that are in one or both of the groups of frames; $A - B$ and $B - A$ would be analogous to the slot-value combinations that are in one group of frames but not in the other; f is just a function which counts; θ is 1; and both α and β are 1. If the analogs to $A - B$ and $B - A$ were used with unequal α and β, the measure would be unsymmetric—one situation would be more similar to a second than the second would be to it.

4.9. Matching Work on Physical Laws as well as on Plots

Consider the relation between the water pressure and water flow in a pipe. It is possible to describe the kind of thing each is, as well as about how each is related to the other and to the resistance of the pipe, using the same symbol-arrangement conventions that we have been working with. A new vocabulary is needed, however:

PIPE-LAW is a constraint—dependent-variable pressure-pipe-law—independent-variable flow-pipe-law—multiplier resistance-pipe-law.

Pressure-pipe-law is a water-pressure. Flow-pipe-law is a water-flow. Resistance-pipe-law is a water-resistance.

Pressure-pipe-law is proportional-to flow-pipe-law [multiplier resistance-pipe-law].

Matching a specific situation against this constraint is like matching one play plot against another. Moreover, a specific situation and a general law need not be so closely related, as when an electrical situation involving a resistor is matched against the pipe law.

Note that abstraction may be necessary with physical laws just as it is with plots. Suppose, for example, that A is proportional to B in one situation while X is determined by Y in the other. Or suppose that A is known to be a kind of force in one situation and X is known to cause something in the other. Matching should be possible because being proportional to something is a way of being determined by something and forces are things that cause. As with plots, when matching seems too difficult at first, some demons are required to make simple abstraction-oriented deductions. No changes are made to the matcher itself.

Fig. 8. The descriptive part of Ohm's law, as abstracted from two situations analyzed through the water-pipe analogy.

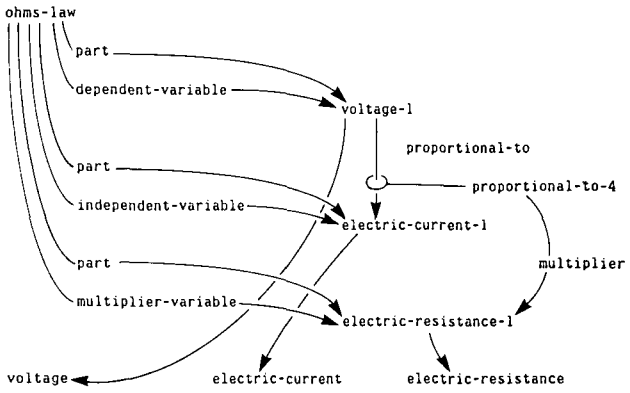
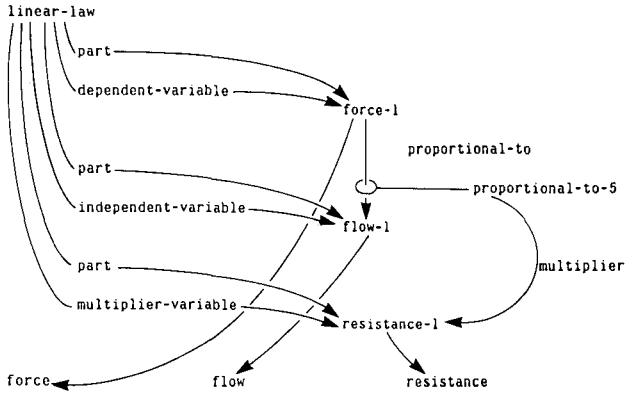


Fig. 9. The descriptive part of the general linear law, as abstracted from the water pipe law and Ohm's law.



5. Learning New Constraints by Identifying Analogous Parts

Understanding matching has prepared us for understanding how knowledge from one domain can be transferred to another as when we learn Ohm's law by being told that resistors are like water pipes.

The reason matching helps is that applying a constraint can be viewed as consisting of two steps: First, identification of the role each part plays in the constraint and second, computation of a result. Thus matching helps because it identifies the role each part plays in a constraint, making it possible to pair off the parameters of a computation against the available arguments. This point of view is reminiscent of a suggestion in Moore and Newell [11].

Figure 6 illustrates the role of matching in applying general knowledge about water pipes to a specific water pipe situation. The general description of water pipe situations identifies the parts through both the PART relation and through the role-establishing relations DEPENDENT-VARIABLE, INDEPENDENT-VARIABLE, and MULTIPLIER. The description of the specific water pipe situation only identifies the parts through the PART relation, providing no explicit means of identifying the role the parts are to play in computing, say, the water pressure, given the pipe resistance and water flow. A procedure looking for the DEPENDENT-VARIABLE, INDEPENDENT-VARIABLE, and MULTIPLIER relations cannot work on the specific situation until the parts of the specific situation are identified with those of the general situation.

In the example given, establishing the match is no problem because the parts of the specific situation have enough classification, property, and other information to insure match.

5.1. Matching Enables Computation in Analogous Domains

So far we have looked at an example in which it was necessary to match the parts of a specific situation against those of a general description within one domain. Now we look at an example in which the specific situation and the general situation are in different domains.

Figure 7 illustrates an example in which it is desired to calculate the voltage across a resistor. As shown, the parts of the specific resistor situation are described by classification, property, and other information, but it is assumed that there is no general description of resistor situations and no procedure for computing the voltage across a resistor.

If a teacher announces that resistors are like water pipes, however, the voltage can be computed. It is only necessary to match the parts of the specific resistor situation with those of the general water pipe situation and to use the procedure for calculating water pressure on the resistor-situation parts identified as the INDEPENDENT-VARIABLE and the MULTIPLIER. As it stands, there is enough evidence to insure the correct match of the parts, although the strength of the match is naturally weaker than it was when the specific situation and the general situation both involved water.

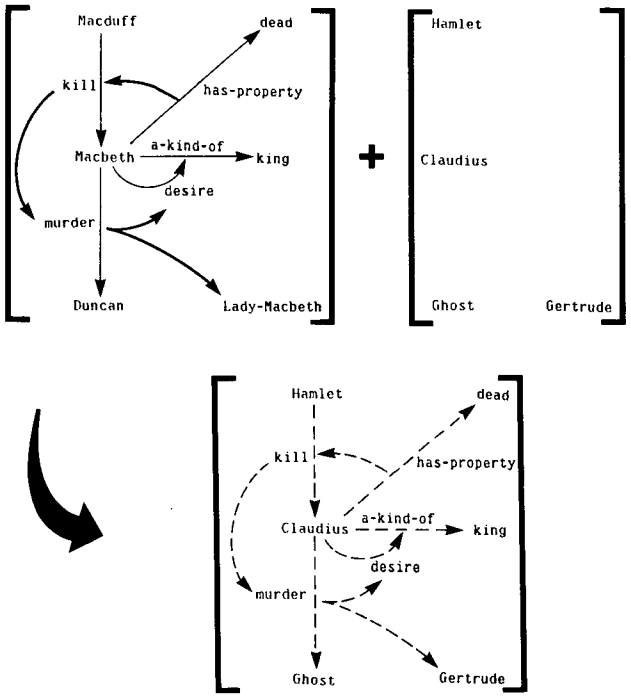
5.2. Specific Laws Can Be Learned

It is possible to generate a general description of resistor situations once two specific situations have been analyzed through the water-pipe analogy. The idea is simple: Copy the classification, property, and other information that is common to both specific situations. Figure 8 illustrates the result, given two analyzed resistor situations.

With the new description of resistor situations, the procedure that computes water pressure can be applied to resistor situations with more confidence, since resistor situations will match the general resistor-situation description better than they will match the general water pipe situation. The newly generated, general description of resistor situations, together with the same procedure that worked with water pipes, constitute Ohm's law. Generating the description constitutes a kind of learning. Let us recapitulate the steps:

Two specific situations are analyzed using a general situation description in another domain.

Fig. 10. The causal connections in *Macbeth*. If *Macbeth* is matched with a situation having no connections, then asking if the Macbeth person dies leads to asking questions determined by the causes. This is done by overlaying the CAUSE connections in *Macbeth* on those in the new situation, a stripped down version of *Hamlet* here. The first question is about Gertrude and the cause of a murder. If the answer is unknown, the second question is about the Claudius person and whether he desires to be king.



—The common parts of the specific situation descriptions determine a general description in the same domain.

—The procedural part of the law in the other domain is brought over without change.

It is possible to push the learning further back, acquiring the procedural part of the law by examples, rather than by copying, but work on this is not yet solid enough to report.

5.3. General Laws Can Be Learned

Specific laws are learned by jointly analyzing two worked-out situations in one domain. General laws are learned by jointly analyzing two specific laws in different domains. The learning procedure is the same.

Figure 9 illustrates the result, given the water-pipe law and Ohm's law. Again the key step is to form a new description out of what is common to both given descriptions. The procedural part, noted to be the same in both specific laws, is adopted without change.

6. Reasoning Using Analogy-Driven Situation Analysis

How is it possible to know if some relation holds in one situation, given that the situation is analogous to another, well-understood situation? The answer is that the constraint relations in the well-understood situation suggest the right relations to check and the right questions to ask. Here we consider only special cases of constraint involving cause. The key assumption is that the cause structure of a well-understood situation is likely to say something about the possible cause structure in a situation to be analyzed.

6.1. The Cause Relations in Situations Make Common-Sense Reasoning Possible

To be more precise, the following steps are taken in the current implementation when a user asks about some relation:

—First, the relation in question may actually be in the situation being analyzed. If so, no further action is needed.

—Second, in the well-understood situation, the relation in question may be caused by a person. If so, ask if the corresponding person in the situation being analyzed causes the relation.

—Third, in the well-understood situation, the relation in question may be caused by another relation. If so, try to justify that other relation in the situation being analyzed by recursion.

If there are several causes for a relation in the well-understood situation, all must be verified. If any PREVENTED-BY relations are encountered along the way, they are investigated as follows:

> If, in the well-understood situation, a person prevents a relation, ask if the corresponding person in the situation being analyzed prevents the relation.
>
> If, in the well-understood situation, a relation is prevented by another relation, try to justify that other relation in the situation being analyzed by recursion.

Success in pursuing any of a relation's PREVENTED-BY values means the corresponding relation in the given situation cannot be established.

To see how all this works out, consider the following stripped-down version of *Hamlet*:

HA is a story.

Ghost is a king. Claudius is a man—marry Gertrude. Gertrude is a woman. Hamlet is a man—has-property loyal.

As it stands, there is barely enough said to do an unambiguous match against *Macbeth*, but given that a user asks questions, it may still make sense to use the analogy. In particular, the following traces what can be done given the cause connections in *Macbeth*, shown in Figure 10, together with a question about whether Claudius dies:

(CHECK 'CLAUDIUS 'HAS-PROPERTY 'DEAD IN HA USING MA)
Does GERTRUDE cause the MURDER slot of CLAUDIUS to have GHOST in it?
>NO (meaning the user does not know)
Does CLAUDIUS DESIRE [CLAUDIUS A-KIND-OF KING]?
>YES (assumed for illustration)
Evidently CLAUDIUS DESIRE [CLAUDIUS A-KIND-OF KING].
Evidently there is sufficient CAUSE for the MURDER slot of CLAUDIUS to have GHOST in it.

Evidently there is sufficient CAUSE for the KILL slot of HAMLET to have CLAUDIUS in it.
Evidently there is sufficient CAUSE for the HAS-PROPERTY slot of CLAUDIUS to have DEAD in it.
[CLAUDIUS HAS-PROPERTY DEAD] is verified by the precedent in MA

It makes sense. By using analogy, a kind of commonsense reasoning is exhibited. Initial experiments indicate that the same ideas work in law where dealing with legal precedent seems like a combination of situation identification and analogy-driven situation analysis.

7. Finding Analogies by Classification-Exploiting Hypothesizing

Before two situations can be matched, it is necessary to find the remembered situation that is most relevant to the situation under analysis. Two mechanisms for such hypothesizing have been implemented: one mechanism uses a situation to guide a search through a network of possibilities related by SIMILAR-TO; and another mechanism uses a situation to probe into an A-KIND-OF tree augmented with indexing information.

The mechanism using SIMILAR-TO relations was devised by Winston [21] and developed by Minsky [10]. A straightforward implementation did not seem particularly illuminating. To be sure, a similarity net was constructed from the ten experimental situations, but ten is not enough to demonstrate anything. With a data base of the size used, only weak illustrations were possible.

7.1. Using a Situation to Probe into an A-KIND-OF Tree

The mechanism that uses an A-KIND-OF tree is more important, for it is suggestive of how information retrieval might be done. To begin, a list is made of everything a situation's parts are a kind of. Each element of this list is checked to see if it has an APPEARS-IN slot. Values in such slots are used to hypothesize situations that are likely to match well.

APPEARS-IN slots are filled as situations are remembered. Everything above a situation's parts in the A-KIND-OF tree has the situation placed in its APPEARS-IN slot.

Suppose, for example, that we start with a *tabula rasa* and supply the following:

Prince is a man. Man is a person. Princess is a woman. Woman is a person. Boy is a man. Girl is a woman.

CI is a story. Charming is a prince. Cinderella is a princess. ⟨Plus other relations involving Charming and Cinderella⟩

SN is a story. Snow-White is a princess. ⟨Plus other relations involving Snow White⟩

The effect is to place CI in the APPEARS-IN slot of PRINCE, MAN, PERSON, PRINCESS, and WOMAN, and to place SN in the APPEARS-IN slot of PRINCESS, WOMAN, and PERSON.

When looking for situations to match a given situation, the A-KIND-OF tree standing above each part is searched for APPEARS-IN slots. These slots then vote for the remembered situations they contain. In the actual implementation, the voting is weighted in two ways:

—Each encountered instance of APPEARS-IN casts votes in inverse proportion to the number of values present. This reduces the weight of APPEARS-IN votes coming from frequently occurring classes.

—Each encountered instance of APPEARS-IN casts votes in proportion to the number of slots in the part of the situation associated with it. This increases the weight of APPEARS-IN votes coming from the more important parts of the situation.

Suppose, for example, that the following situation is given:

RJ is a situation. Romeo is a boy. Juliet is a girl. ⟨Plus other facts about Romeo and Juliet⟩

Suppose Romeo has six slots and Juliet has four. Romeo is connected to MAN, which has only CI in its APPEARS-IN slot, and to PERSON, which has both CI and SN. Thus Romeo's contribution is nine votes for CI and three for SN. Juliet's is connected to WOMAN and PERSON, each of which has both CI and SN in the APPEARS-IN slot. Thus Juliet's contribution is four votes for each. CI beats SN, 13 to 7.

Using relations for indexing and retrieving, incidentally, requires no further machinery, since relation-describing frames can be plot parts just like other things. The following would do this for an instance of KISS used in the CINDERELLA frame, insuring that CI would end up in the APPEARS-IN slot of KISS.

{Cinderella kiss Charming} is a kiss—part of Ci.

It also may be reasonable to index and retrieve on pairs of situation parts. As it stands, the implementation allows a search for plausible situations that have parts that are a kind of PRINCE and a kind of MARRY. Using pairs for indexing and retrieval would enable a system to look for, say, situations in which a particular prince is married to someone. This has not received much thought yet.

The current method was used on each of the experimental situations. The situation used as a probe was the unambiguous first hypothesis except when the probe was one of the two Ibsen plays. This is not strange. The Ibsen people are just people, men and women, not princes, generals, or other distinguishing things.

7.2. It Is Not Clear if Classification-Exploiting Hypothesizing Scales Well

In the end, it must be determined if the A-KIND-OF-based identification method is robust enough to be useful when the number of situations is increased to a practical size. The previously mentioned work of Rosch et al. may be relevant [16]. They argue that people tend to recognize and specify concepts at the so-called basic

level first. This might mean that the most useful situation-hypothesizing information would be at that level. Plainly the higher level classifications would not be useful because the situations involving a class like PERSON would be too numerous to provide useful constraint or even to record.

If there are only, say, a thousand basic classes, and if each situation uses, say, four objects and relations in a prominent way, then there could be on the order of 10^{10} situations with different combinations. There would be plenty of room for an expert to know a lot. Of course the space of combinations certainly is filled unevenly by the situations that are useful. A better analysis or some experimentation is needed, therefore, to see if some form of A-KIND-OF-based identification will work in practical situations. Obvious things to think about are context-constrained use of the APPEARS-IN slots and analysis-time discovery of useful classification information.

8. Other Questions

It is not possible to discuss all that has been done in a palatable length paper. Among the details left out are explanations of the following:

- —The form of the representation when reduced to list structure.
- —The form of the English-like interface when reduced to a program.
- —The method by which it is possible to improve match in difficult circumstances by automatically generating questions to be answered by users or by some deduction system.
- —The experimental results exhibited by the matcher when dealing with variously described resistor and water-pipe situations.
- —The method by which it is possible to learn the procedural part of laws.
- —The method by which it is possible to learn the procedural part of laws.
- —The method by which it is possible to learn demons, together with the arguments suggesting that the use of a demon is like the exploitation of a miniature analogy.
- —The way the parts of one group of situations can be matched to the parts of another group of situations. (This is required for identifying the analogy between the basic constraints of the electrical world and those in the mechanical world. The resistor, capacitor, and inductance laws constitute the first group, and the damper, spring, and momentum laws, the second.)

All of these points are discussed elsewhere [22].

9. Conclusion: Simple Mechanisms Have Promise

This paper is about a set of ideas that enable certain types of learning and reasoning to take place by analogy in domains that satisfy certain restrictions, namely:

Symbolic sufficiency;
Description-determined similarity;
Constraint-determined importance;
Historical continuity.

The learning and reasoning ideas developed for such domains are these:

Extensible-relations representation;
Importance-dominated matching;
Analogy-driven constraint learning;
Analogy-driven reasoning;
Classification-exploiting hypothesizing.

It is now clear that simple situations can be analyzed by attacking them with the analogy process using stored situations ranging from small, demon-sized incidents to *Macbeth*-sized plots. Once analyzed, a situation becomes eligible to be remembered for use in analyzing newer, perhaps bigger situations. Experience makes an analogy-making system smarter, or at least more experienced.

10. Related Work

In some ways, this work has roots in my previous learning systems. The first of these is known as the ARCH system [20], and the second, FOX [21], both being identified by the typical things involved in the learning. In addition, many of the ideas in this paper were influenced by other precedents. In particular, Schank [17], Wilks [19], and others stimulated work on the problem of how thinking is determined by stored experience and emphasized the importance of cause relations. Filmore [4] popularized case grammar. Evans [3] broke ground with his work on the geometric analogy problem. Moore and Newell [11] suggested something quite reminiscent of the way laws are learned. Minksy [10] and Goldstein and Roberts [14, 15] worked out key representation ideas. Martin [8], Meldman [9], and Rieger [13] demonstrated the utility of hard work on details of vocabulary. Lenat's success with his mathematical discovery system provoked renewed interest in the entire area of computer learning [7].

Since the experiments reported in this paper were done, a better, more natural interface has been designed and implemented by Katz [6]. Similarly, a more efficient matcher has been designed and implemented by Brotsky [1]. Conversion to Katz' interface and Brotsky's matcher is underway.

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Programming Techniques and Data Structures

R. Rivest Editor

Deletion in Two-Dimensional Quad Trees

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An algorithm for deletion in two-dimensional quad trees that handles the problem in a manner analogous to deletion in binary search trees is presented. The algorithm is compared with a proposed method for deletion which reinserts all of the nodes in the subtrees of the deleted node. The objective of the new algorithm is to reduce the number of nodes that need to be reinserted. Analysis for complete quad trees shows that the number of nodes requiring reinsertion is reduced to as low as $\frac{2}{3}$ of that required by the old algorithm. Simulation tests verify this result. Reduction of the number of insertions has a similar effect on the number of comparison operations. In addition, the total path length (and balance) of the resulting tree is observed to remain constant or increase slightly when the new algorithm for deletion is used, whereas use of the old algorithm results in a significant increase in the total path length for large trees.

Key Words and Phrases: binary tree deletion, quad trees, associative searching, information retrieval, binary search trees, sorting, searching, geographic databases

CR Categories: 3.70, 4.34, 4.79

Introduction

A number of data structures have been proposed for retrieval on composite keys [1, 3, 8]. We are interested in the quad tree of [3]. It is useful whenever there is a need to perform operations such as searching a two-dimensional structure. For example, we may wish to find all

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