

$$24. (1 + \cot^2(x))(1 - \cos^2(x)) = 1 \quad \text{para todo } x \text{ real, } x \neq k\pi$$

$$= \left(1 + \left(\frac{\cos(x)}{\sin(x)}\right)^2\right) \cdot \sin^2(x) = 1$$

$$= \left(1 + \frac{\cos^2(x)}{\sin^2(x)}\right) \cdot \sin^2(x) = 1$$

$$= \frac{\sin^2(x) + \cos^2(x)}{\sin^2(x)} \cdot \sin^2(x) = 1$$

$$= \frac{\sin^2(x) + \cos^2(x)}{\sin^2(x)} \cdot \sin^2(x) = 1$$

$$= \frac{1}{\sin^2(x)} \cdot \sin^2(x) = 1$$

$$1 = 1$$

$$25. 2 \sec(x) \cdot \tan(x) = \frac{1}{\cos \sec(x) - 1} + \frac{1}{\cos \sec(x) + 1} \quad \text{para todo } x, x \neq \frac{\pi}{2} + k\pi$$

$$2 \cdot \frac{1}{\cos(x)} \cdot \frac{\sin(x)}{\cos(x)} = \frac{1}{\sin(x)} + \frac{1}{1 - \sin(x)}$$

$$\frac{2}{\cos(x)} \cdot \frac{\sin(x)}{\cos(x)} = \frac{2 \sin(x)}{\cos^2(x)} \cdot \frac{1 - \sin(x)}{\sin(x)}$$

$$26. (1 - \frac{1}{\sec(x)})^2 + (1 - \frac{1}{\csc(x)})^2 = (\sec(x) - \csc(x))^2$$

para todo  $x$  real,  $x \neq k\pi$

$$\left(1 - \frac{\sin(x)}{\cos(x)}\right)^2 + \left(1 - \frac{\cos(x)}{\sin(x)}\right)^2 = \left(\frac{1}{\cos(x)} - \frac{1}{\sin(x)}\right)^2$$

$$\frac{2\sin(x)}{1 - \sin(x)^2} + \frac{2\cos(x)}{1 - \cos(x)^2} = \frac{1 - \sin(x)}{\sin(x)} + \frac{1 + \sin(x)}{\sin(x)}$$

$$2\sin(x) = \frac{1}{\sin(x)} + \frac{1}{\sin(x)} \quad 2\sin(x) = \frac{1}{\sin(x)} + \frac{1}{\sin(x)} = 0$$

$$0 = 0$$

$$27. \frac{1 - \cos(x)}{\sin(x) \cdot \cos(x)} + \sin(x) = \frac{1 - \cos(x)}{\tan(x)}$$

$$\frac{1 - \cos(x)}{\sin(x) \cdot \cos(x)} + \frac{\sin(x)}{1} = \frac{1 - \cos(x)}{\sin(x)} + \frac{\sin(x)}{\cos(x)}$$

$$\frac{1 - \cos(x) + \sin(x)^2 \cdot \cos(x)}{\sin(x) \cdot \cos(x)} = \frac{1 - \cos(x) + \cos(x)}{1} + \frac{\sin(x)}{\cos(x)}$$

$$\frac{1 - \cos(x) + \sin(x)^2 \cdot \cos(x)}{\sin(x) \cdot \cos(x)} = \frac{\cos(x) - \cos(x)^2}{\sin(x)} + \frac{\sin(x)}{\cos(x)}$$

$$\frac{1 - \cos(x) \cdot \cos(x)^2}{\sin(x) \cdot \cos(x)} = \frac{\cos(x)^2 - \cos(x)^2 + \sin(x)^2}{\sin(x) \cdot \cos(x)}$$

$$\frac{1 - \cos(x)^2}{\sin(x) \cdot \cos(x)} = \frac{1 - \cos(x)^2}{\sin(x) \cdot \cos(x)}$$

$$28. \cos^4(x) + \sin^4(x) + 2(\sin(x) \cdot \cos(x))^2 = 1$$

$$\cos(x)^4 + \sin(x)^4 + 2(\sin(x)^2 \cos(x)^2)$$

$$\cos(x)^4 + 2\sin(x)^2 \cos(x)^2 + \sin(x)^4$$

$$(\cos(x)^2 + \sin(x)^2)^2$$

$$1^2 = 1$$

$$1 = 1$$

$$29. \frac{\sin(x)}{\cos \sin(x)} + \frac{\cos(x)}{\sin(x)} = 1$$

$$\frac{\sin(x)}{1} + \frac{\cos(x)}{1} = 1$$

30. to



$$33) \frac{\cot^2(x)}{1 + \cot^2(x)} = \frac{\cos^2(x)}{\sin^2(x)}$$

$$\frac{\left(\frac{\cos(x)}{\sin(x)}\right)^2}{1 + \frac{\cos^2(x)}{\sin^2(x)}} = \frac{\frac{\cos^2(x)}{\sin^2(x)} \cdot \frac{\sin^2(x)}{\sin^2(x)}}{\frac{\sin^2(x) + \cos^2(x)}{\sin^2(x)}} = \frac{\cos^2(x)}{1}$$

$$34) \frac{\sin^3(x) - \cos^3(x)}{\sin(x) - \cos(x)} = \frac{1 + \sin(x) \cdot \cos(x)}{1}$$

$$\frac{(\sin(x) - \cos(x)) \cdot (\sin^2(x) + \sin(x)\cos(x) + \cos^2(x))}{\sin(x) - \cos(x)}$$

$$\frac{(\sin(x) + \cos(x)) \cdot (1 + \sin(x)\cos(x))}{\sin(x) - \cos(x)}$$

$$1 + \sin(x)\cos(x)$$

$$= 1 + \sin(x)\cos(x)$$

$$36) \frac{2(\sin(x) + \cos(x))}{(\cos(x) + \cot^2(x))} = \frac{2(1 + \sin(x) + \cos(x))}{1}$$

$$\left( \frac{\sin(x) + \sin(x)}{\cos(x)} \right) \left( \frac{\cos(x) + \cos(x)}{\sin(x)} \right)$$

$$\left( \frac{\sin(x) \cdot \cos(x) + \sin(x)}{\cos(x)} \right) \left( \frac{\cos(x) \cdot \sin(x) + \cos(x)}{\sin(x)} \right)$$

$$2 \cdot (\cos(x) + 1)(\sin(x) + 1)$$

$$2 \cos(x) + 2 \sin(x) + 2 \cos(x) + 2 \sin(x) + 2$$

$$2 \sin(x) \cos(x) + 2 \cos(x) + 2 \sin(x) + 2$$

$$\sin(2x) + 2 \cos(x) + 2 \sin(x) + 2 = \sin(2x) + 2 \sin(x) +$$

$$2 \cos(x) + 2$$

$$\cdot (1 + \sin(x) + \cos(x))(1 + \sin(x) + \cos(x))$$

$$1 + \sin(x) + \cos(x) + \sin(x) + \sin(x)^2 + \sin(x) \cos(x) +$$

$$\cos(x) + \sin(x) \cos(x) + \cos(x)^2$$

$$= 1 + 2 \sin(x) + 2 \cos(x) + 2 \sin(x) \cos(x) + \sin(x)^2 + \cos(x)^2$$

$$2 + 2 \sin(x) + 2 \cos(x) + \sin(2x) = \sin(2x) + 2 \sin(x) + 2 \cos(x) +$$