An Abaqus-Matlab Tutorial for Jointed Systems

International Committee on Jointed Structures Seminar Series

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Table of Contents

Detailed Instructions are hosted through Github in a repository named Nidish96/Abaqus4Joints

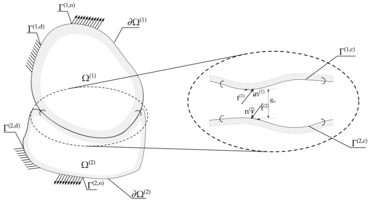
Acknowledgements

- Prof. Matthew Brake
- Dr. Justin Porter, Maeve Karpov
- Prof. Matt Allen

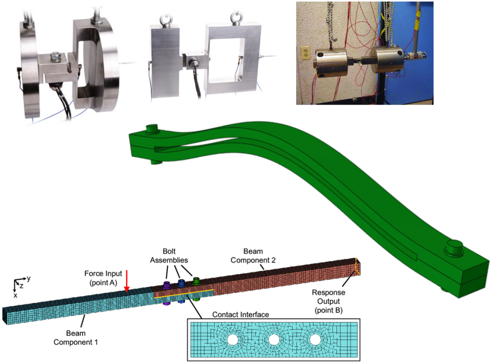
The research needs of the joints community are quite unique and specific

1. Introduction

The Generic Contact Problem



Joints Benchmarks (see jointmechanics.org)



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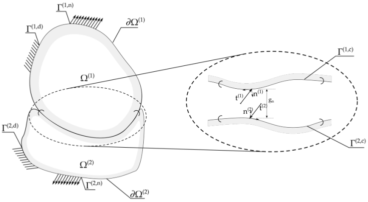

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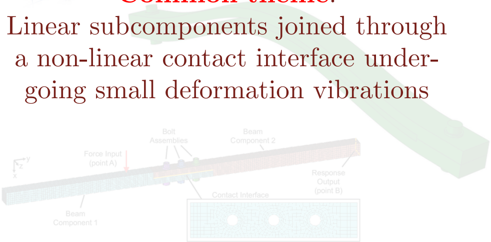
Linear subcomponents joined through a non-linear contact interface undergoing small deformation vibrations



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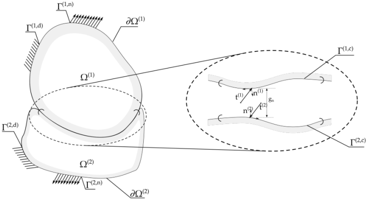

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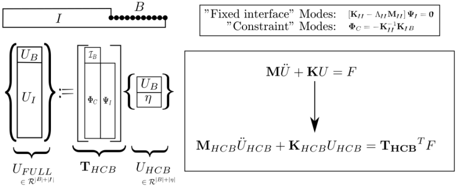
The HCB/CMS Methodology



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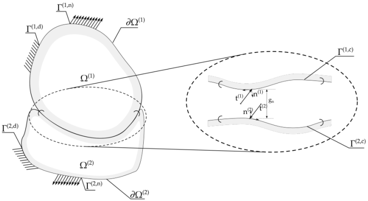

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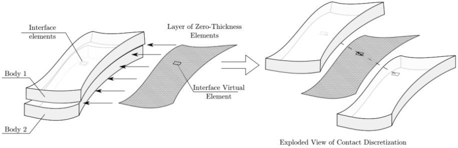
Interface Virtual Element Representations



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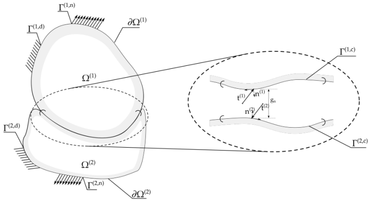

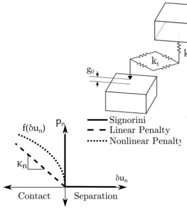

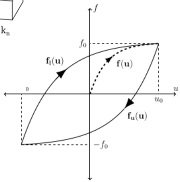

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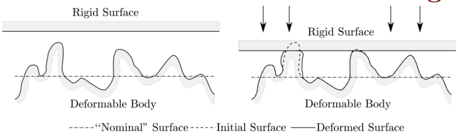
Interface Constitutive Modeling



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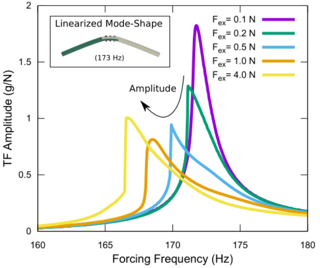
Nonlinear Responses



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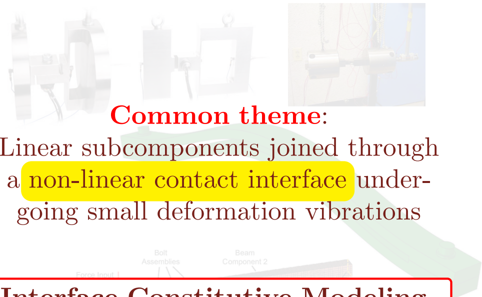

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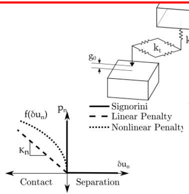

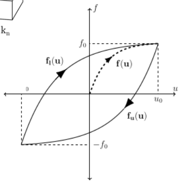

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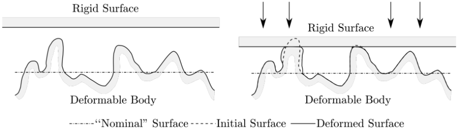

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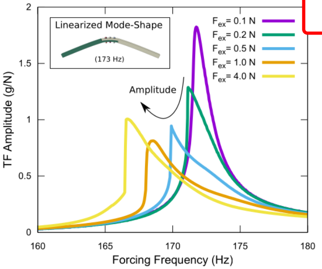

Goal: Off-Load as much as possible to **focus on joints**.

Common theme:
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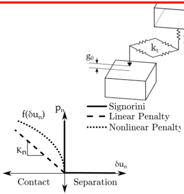

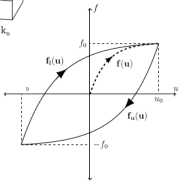

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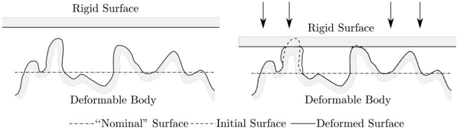
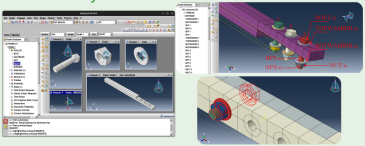


An Overview of the Modeling Pipeline

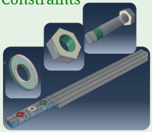
2.Outline of the Steps

Pre-Processing

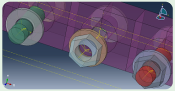
Assembly



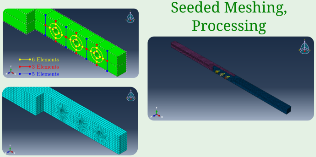
Constraints



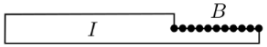
Bolt Prestress as Static Load



Seeded Meshing, Processing



Component Mode Synthesis



"Fixed interface" Modes: $[\mathbf{K}_{II} - \Lambda_{II}\mathbf{M}_{II}]\,\mathbf{\Psi}_I = \mathbf{0}$
"Constraint" Modes: $\mathbf{\Phi}_C = -\mathbf{K}_{II}^{-1}\mathbf{K}_{IB}$

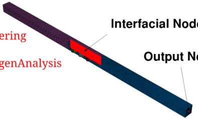
$$\underbrace{\begin{Bmatrix} U_B \\ U_I \end{Bmatrix}}_{U_{FULL} \in \mathcal{R}^{|B|+|I|}} := \underbrace{\begin{bmatrix} \mathcal{I}_B & \\ \Phi_C & \Psi_I \end{bmatrix}}_{\mathbf{T}_{HCB}} \underbrace{\begin{Bmatrix} U_B \\ \eta \end{Bmatrix}}_{U_{HCB} \in \mathcal{R}^{|B|+|\eta|}}$$

$$\mathbf{M}\ddot{U} + \mathbf{K}U = F$$

$$\downarrow$$

$$\mathbf{M}_{HCB}\ddot{U}_{HCB} + \mathbf{K}_{HCB}U_{HCB} = \mathbf{T_{HCB}}^T F$$

+ Reorganize NodeSet numbering
+ Choose recovery nodes
+ Conduct Fixed Interface EigenAnalysis



Matrix Extraction

+ Use the provided python script to convert the mtx files into MATLAB mat files

```
** ----------------------------------------------------------------
**
** STEP: HCBCMS
**
*Step, name=HCBCMS, nlgeom=NO
*Substructure Generate, overwrite, type=Z1, recovery matrix=YES,
      nset=OutNodes, mass matrix=YES
*Select Eigenmodes, generate
1, 20, 1
*Damping Controls, structural=COMBINED, viscous=COMBINED
*Retained Nodal Dofs, sorted=NO
BOTS_NDS, 1, 3
TOPS_NDS, 1, 3
**
** LOAD CASES
**
*Substructure Load Case, name=LCASE
** Name: BoltLoad-1   Type: Concentrated force   Scale factor: 1
*Cload
BPT-1_Set-1, 3, 1.
** Name: BoltLoad-2   Type: Concentrated force   Scale factor: 1
*Cload
BPT-2_Set-1, 3, 1.
** Name: BoltLoad-3   Type: Concentrated force   Scale factor: 1
*Cload
BPT-3_Set-1, 3, 1.
** Name: NutLoad-1   Type: Concentrated force   Scale factor: 1
*Cload
NPT-1_Set-1, 3, -1.
** Name: NutLoad-2   Type: Concentrated force   Scale factor: 1
*Cload
NPT-2_Set-1, 3, -1.
** Name: NutLoad-3   Type: Concentrated force   Scale factor: 1
*Cload
NPT-3_Set-1, 3, -1.
*Substructure Matrix Output, FILE NAME=Modelmats, MASS=YES,
      STIFFNESS=YES, SLOAD=YES, RECOVERY MATRIX=YES
*End Step
```

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2.Outline of the Steps

Pre-Processing

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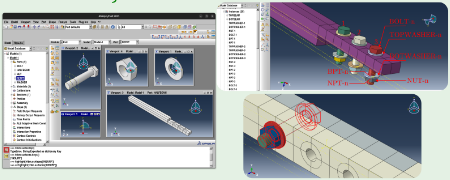

Load

Seeded Meshing, Processing

Component Mode Synthesis

"Fixed interface" Modes: $[K_{II} - \lambda_i M_{II}]\Phi_I = 0$
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$$\underbrace{\begin{Bmatrix} U_B \\ U_I \end{Bmatrix}}_{U_{FULL}} := \underbrace{\begin{bmatrix} I_B & \\ \Phi_C & \Phi_I \end{bmatrix}}_{T_{HCB}} \underbrace{\begin{Bmatrix} U_B \\ \eta \end{Bmatrix}}_{U_{HCB}}$$

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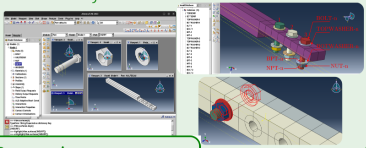
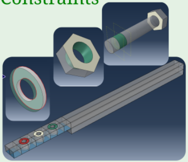


Constraints



Bolt Prestress as Static Load

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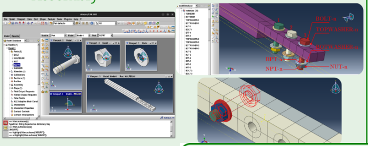

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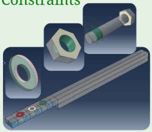

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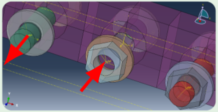
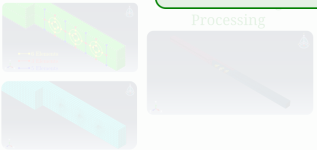


Processing



Component Mode Synthesis

"Fixed interface" Modes:
"Constraint" Modes:

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"Pull the Bolts, Push the Washers"

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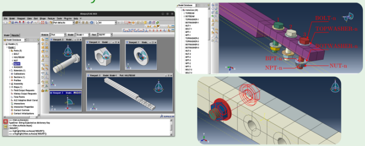

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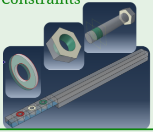

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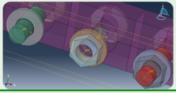

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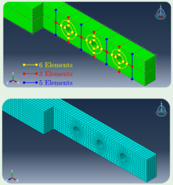

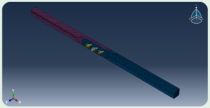

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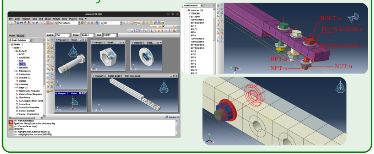

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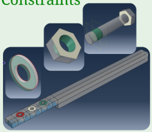

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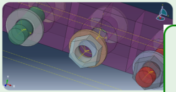

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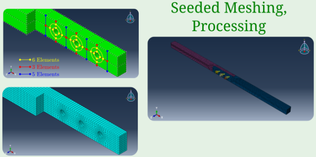

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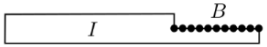

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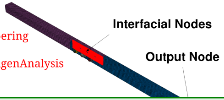

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1, 20, 1
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*Substructure Matrix Output, FILE NAME=Modelmats, MASS=YES,
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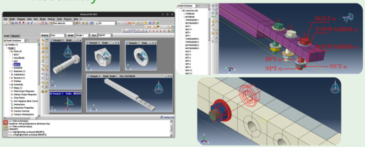
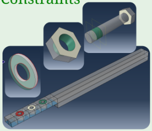


Constraints



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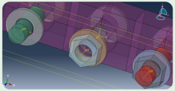
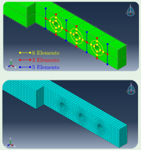
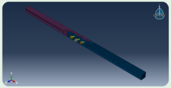
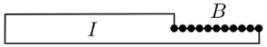


Seeded Meshing, Processing





Component Mode Synthesis



"Fixed interface" Modes: $[\mathbf{K}_{II} - \Lambda_{II}\mathbf{M}_{II}]\,\mathbf{\Psi}_I = \mathbf{0}$

"Constraint" Modes: $\mathbf{\Phi}_C = -\mathbf{K}_{II}^{-1}\mathbf{K}_{IB}$

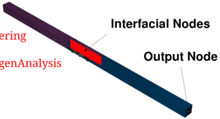
$$\underbrace{\begin{Bmatrix} U_B \\ U_I \end{Bmatrix}}_{U_{FULL} \in \mathcal{R}^{|B|+|I|}} := \underbrace{\begin{bmatrix} \mathcal{I}_B & \\ \Phi_C & \Psi_I \end{bmatrix}}_{\mathbf{T}_{HCB}} \underbrace{\begin{Bmatrix} U_B \\ \eta \end{Bmatrix}}_{U_{HCB} \in \mathcal{R}^{|B|+|\eta|}}$$

$$\mathbf{M}\ddot{U} + \mathbf{K}U = F$$

$$\downarrow$$

$$\mathbf{M}_{HCB}\ddot{U}_{HCB} + \mathbf{K}_{HCB}U_{HCB} = \mathbf{T_{HCB}}^T F$$

+ Reorganize NodeSet numbering
+ Choose recovery nodes
+ Conduct Fixed Interface EigenAnalysis



Matrix Extraction

+ Use the provided python script to convert the mtx files into MATLAB mat files

```
** ----------------------------------------------------------------
**
** STEP: HCBCMS
**
*Step, name=HCBCMS, nlgeom=NO
*Substructure Generate, overwrite, type=Z1, recovery matrix=YES,
      nset=OutNodes, mass matrix=YES
*Select Eigenmodes, generate
1, 20, 1
*Damping Controls, structural=COMBINED, viscous=COMBINED
*Retained Nodal Dofs, sorted=NO
BOTS_NDS, 1, 3
TOPS_NDS, 1, 3
**
** LOAD CASES
**
*Substructure Load Case, name=LCASE
** Name: BoltLoad-1   Type: Concentrated force   Scale factor: 1
*Cload
BPT-1_Set-1, 3, 1.
** Name: BoltLoad-2   Type: Concentrated force   Scale factor: 1
*Cload
BPT-2_Set-1, 3, 1.
** Name: BoltLoad-3   Type: Concentrated force   Scale factor: 1
*Cload
BPT-3_Set-1, 3, 1.
** Name: NutLoad-1   Type: Concentrated force   Scale factor: 1
*Cload
NPT-1_Set-1, 3, -1.
** Name: NutLoad-2   Type: Concentrated force   Scale factor: 1
*Cload
NPT-2_Set-1, 3, -1.
** Name: NutLoad-3   Type: Concentrated force   Scale factor: 1
*Cload
NPT-3_Set-1, 3, -1.
*Substructure Matrix Output, FILE NAME=Modelmats, MASS=YES,
      STIFFNESS=YES, SLOAD=YES, RECOVERY MATRIX=YES
*End Step
```

2.1. Relative Coordinates Pipeline

Outline of the Steps

- Suppose that $\underline{u}_T, \underline{u}_B, \underline{u}_I$ are the vectors of top, bottom, and internal nodal DoFs.
- Then the governing equations look like:

$$\begin{bmatrix} \underline{\underline{M}}_{TT} & \underline{\underline{M}}_{TB} & \underline{\underline{M}}_{TI} \\ & \underline{\underline{M}}_{BB} & \underline{\underline{M}}_{BI} \\ \text{sym} & & \underline{\underline{M}}_{II} \end{bmatrix} \begin{bmatrix} \ddot{\underline{u}}_T \\ \ddot{\underline{u}}_B \\ \ddot{\underline{u}}_I \end{bmatrix} + \begin{bmatrix} \underline{\underline{K}}_{TT} & \underline{\underline{K}}_{TB} & \underline{\underline{K}}_{TI} \\ & \underline{\underline{K}}_{BB} & \underline{\underline{K}}_{BI} \\ \text{sym} & & \underline{\underline{K}}_{II} \end{bmatrix} \begin{bmatrix} \underline{u}_T \\ \underline{u}_B \\ \underline{u}_I \end{bmatrix} + \begin{bmatrix} \underline{F}_T^{(c)} \\ \underline{F}_B^{(c)} \\ \underline{0} \end{bmatrix} = \underline{F}_e(t)$$

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Outline of the Steps

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- Under the relative coordinate transformation,

$$\begin{bmatrix} \underline{u}_T \\ \underline{u}_B \\ \underline{u}_I \end{bmatrix} = \begin{bmatrix} \underline{\underline{I}}_T & \underline{\underline{I}}_T & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{I}}_T & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{I}}_I \end{bmatrix} \begin{bmatrix} \Delta\underline{u} \\ \underline{u}_B \\ \underline{u}_I \end{bmatrix}, \qquad \Delta\underline{u} = \underline{u}_T - \underline{u}_B.$$

2.1. Relative Coordinates Pipeline

Outline of the Steps

- Suppose that $\underline{u}_T, \underline{u}_B, \underline{u}_I$ are the vectors of top, bottom, and internal nodal DoFs.
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- Under the relative coordinate transformation,

Transformed Equations of Motion

$$\begin{bmatrix} \underline{\underline{M}}_{TT} & \underline{\underline{M}}_{TT} + \underline{\underline{M}}_{TB} & \underline{\underline{M}}_{TI} \\ & \begin{matrix} \underline{\underline{M}}_{TT} + \underline{\underline{M}}_{BB} + \\ \underline{\underline{M}}_{TB} + \underline{\underline{M}}_{TB}^T \end{matrix} & \underline{\underline{M}}_{TI} + \underline{\underline{M}}_{BI} \\ \text{sym} & & \underline{\underline{M}}_{II} \end{bmatrix} \begin{bmatrix} \underline{\ddot{u}}_T \\ \underline{\ddot{u}}_B \\ \underline{\ddot{u}}_I \end{bmatrix} + \begin{bmatrix} & & \\ & \overline{K} & \\ & & \end{bmatrix} \begin{bmatrix} \underline{u}_T \\ \underline{u}_B \\ \underline{u}_I \end{bmatrix} + \begin{bmatrix} \underline{F}_T^{(c)} \\ \underline{0} \\ \underline{0} \end{bmatrix} = \underline{F}_e(t).$$

2.1. Relative Coordinates Pipeline

Outline of the Steps

- Suppose that $\underline{u}_T, \underline{u}_B, \underline{u}_I$ are the vectors of top, bottom, and internal nodal DoFs.
- Then the governing equations look like:

$$\begin{bmatrix} \underline{\underline{M}}_{TT} & \underline{\underline{M}}_{TB} & \underline{\underline{M}}_{TI} \\ & \underline{\underline{M}}_{BB} & \underline{\underline{M}}_{BI} \\ \text{sym} & & \underline{\underline{M}}_{II} \end{bmatrix} \begin{bmatrix} \underline{\ddot{u}}_T \\ \underline{\ddot{u}}_B \\ \underline{\ddot{u}}_I \end{bmatrix} + \begin{bmatrix} \underline{\underline{K}}_{TT} & \underline{\underline{K}}_{TB} & \underline{\underline{K}}_{TI} \\ & \underline{\underline{K}}_{BB} & \underline{\underline{K}}_{BI} \\ \text{sym} & & \underline{\underline{K}}_{II} \end{bmatrix} \begin{bmatrix} \underline{u}_T \\ \underline{u}_B \\ \underline{u}_I \end{bmatrix} + \begin{bmatrix} \underline{F}_T^{(c)} \\ \underline{F}_B^{(c)} \\ \underline{0} \end{bmatrix} = \underline{F}_e(t)$$

- Under the relative coordinate transformation,

Transformed Equations of Motion

$$\begin{bmatrix} \underline{\underline{M}}_{TT} & \underline{\underline{M}}_{TT} + \underline{\underline{M}}_{TB} & \underline{\underline{M}}_{TI} \\ & \underline{\underline{M}}_{TT} + \underline{\underline{M}}_{BB} + \underline{\underline{M}}_{TB} + \underline{\underline{M}}_{TB}^T & \underline{\underline{M}}_{TI} + \underline{\underline{M}}_{BI} \\ \text{sym} & & \underline{\underline{M}}_{II} \end{bmatrix} \begin{bmatrix} \underline{\ddot{u}}_T \\ \underline{\ddot{u}}_B \\ \underline{\ddot{u}}_I \end{bmatrix} + \begin{bmatrix} \overline{K} \end{bmatrix} \begin{bmatrix} \underline{u}_T \\ \underline{u}_B \\ \underline{u}_I \end{bmatrix} + \begin{bmatrix} \underline{F}_T^{(c)} \\ \underline{0} \\ \underline{0} \end{bmatrix} = \underline{F}_e(t).$$

2.1. Relative Coordinates Pipeline

Outline of the Steps

- Suppose that $\underline{u}_T, \underline{u}_B, \underline{u}_I$ are the vectors of top, bottom, and internal nodal DoFs.
- Then the governing equations look like:

$$\begin{bmatrix} \underline{\underline{M}}_{T} \cdots \\ \\ \text{sym} \cdots \end{bmatrix} \cdots = \underline{F}_e(t)$$

It is clearly advantageous (when possible) to use the relative coordinate transformation as early as possible in the CMS process.
Thankfully, this is possible fully in Abaqus itself!

- Under the relative coordinate transformation,

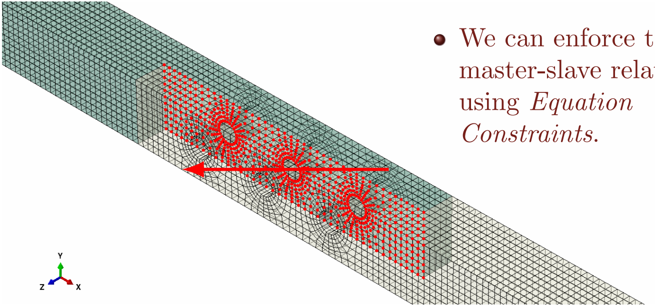
Transformed Equations of Motion

$$\begin{bmatrix} \underline{\underline{M}}_{TT} & \underline{\underline{M}}_{TT} + \underline{\underline{M}}_{TB} & \underline{\underline{M}}_{TI} \\ & \begin{matrix} \underline{\underline{M}}_{TT} + \underline{\underline{M}}_{BB} + \\ \underline{\underline{M}}_{TB} + \underline{\underline{M}}_{TB}^{T} \end{matrix} & \underline{\underline{M}}_{TI} + \underline{\underline{M}}_{BI} \\ \text{sym} & & \underline{\underline{M}}_{II} \end{bmatrix} \begin{bmatrix} \underline{\ddot{u}}_T \\ \underline{\ddot{u}}_B \\ \underline{\ddot{u}}_I \end{bmatrix} + \begin{bmatrix} & & \\ & \overline{K} & \\ & & \end{bmatrix} \begin{bmatrix} \underline{u}_T \\ \underline{u}_B \\ \underline{u}_I \end{bmatrix} + \begin{bmatrix} \underline{F}_T^{(c)} \\ \underline{0} \\ \underline{0} \end{bmatrix} = \underline{F}_e(t).$$

2.1. Relative Coordinates Pipeline

Outline of the Steps

- By adding Reference Points to the model at the location of the interfacial nodes, we can create **slave nodes whos DoF will be the relative DoF of the corresponding nodes on the interface**.
- This can be done quite easily with Python scripting (the website has a script for the BRB which can be adapted to arbitrary contexts).



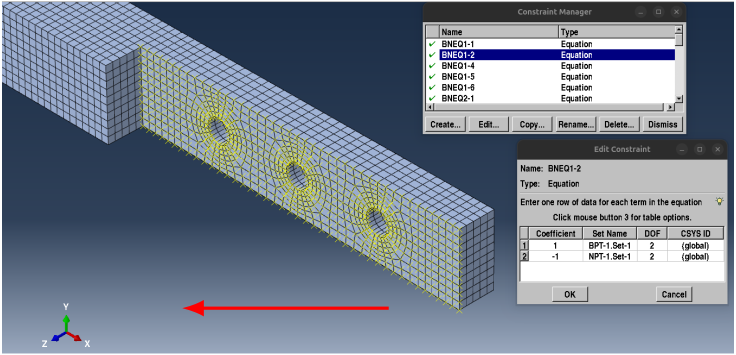
- We can enforce the master-slave relationship using *Equation Constraints.*

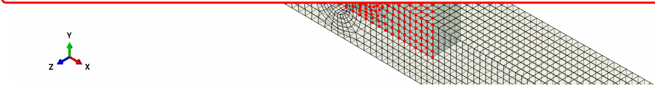
2.1. Relative Coordinates Pipeline

Outline of the Steps

- By adding Reference Points to the model at the location of the interfacial no ... ve D...
- Th ... as a scr...

...And this is what it looks like on ABAQUS.





2.1. Relative Coordinates Pipeline

Outline of the Steps

- By adding Reference Points to the model at the location of the interfacial no… …ve D…
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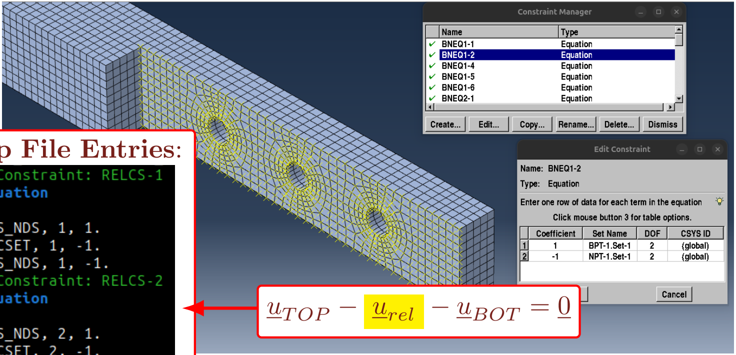
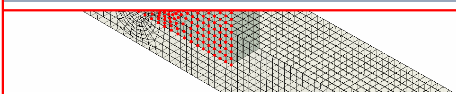


$$\underline{u}_{TOP} - \boxed{\underline{u}_{rel}} - \underline{u}_{BOT} = \underline{0}$$

.inp File Entries:

```
** Constraint: RELCS-1
*Equation
3
TOPS_NDS, 1, 1.
RELCSET, 1, -1.
BOTS_NDS, 1, -1.
** Constraint: RELCS-2
*Equation
3
TOPS_NDS, 2, 1.
RELCSET, 2, -1.
BOTS_NDS, 2, -1.
** Constraint: RELCS-3
*Equation
3
TOPS_NDS, 3, 1.
RELCSET, 3, -1.
BOTS_NDS, 3, -1.
```



Nonlinear Analysis in Under 100 Lines of MATLAB Code!

3. Nonlinear Analysis in MATLAB

4. Outro and Recommendations

- When we're interested in working on friction modeling at the interfacial level, it makes sense to off-load everything else to a software.
- Try to think of ways to utilize the ABAQUS solvers as much as possible (for CMS modal analysis, etc.).

Some Drawbacks of this Pipeline

- Applicability is **strictly restricted to small displacement contact**.
- Geometrical Nonlinearities can't be present. If so, try an ICE fitting coupled with the relative coordinates approach.

Thank You!

All the detailed instructions are hosted through Github in a repository named Nidish96/Abaqus4Joints

Comments, suggestions and contributions welcome!
Please email me at nidish@iitm.ac.in.

References I