An evolutionary game theory model for devaluing rhinos

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Abstract

Rhino poaching has escalated in recent years due the demand for rhino horn in Asian countries. Rhino horn is used in Traditional Chinese Medicine and as a status symbol of success and wealth. Wild life managers attempt to minimise the rhino casualties with approaches such as devaluation of the rhino horn. The most common strategy of devaluing horns includes dehorning. In [7] game theory modelling was used to examined the interaction of poachers and wild life managers. A manager can either 'dehorn' their rhinos or leave the horn attached. Poachers may chose to to behave 'selectively' or 'indiscriminately'. The approach described in this paper builds upon [7] and investigates the interactions between the poachers using evolutionary game theory. Evolutionary game theory, allows us to explore the evolutionary stabilities of the strategies available to a poacher. The purpose of this work is to discover whether conditions which encourages the poachers to behave selectively, they only kill those rhinos with full horns, exist. Notwithstanding, the analytical results prove that poachers will never adopt a selective behaviour as long as there is the slightest gain from a partial horn. Additionally, poachers behaving indiscriminate can be proven to be stable and robust. Thus, we conclude that dehorning is not the dominant strategy for a wild life manager, and we suggest that money are spend on security.

1 Introduction

The illegal trade in rhino horn supports aggressive poaching syndicates and a black market [11]. This lucrative market entices people to invest their time and energy to gain a 'winfall' in the form of a rhino horn, through the poaching of rhinos. In recent years poaching has escalated to an unpresidented level resulting in concerns over their future existence [13]. In response, rhino conservation has seen increased ilitarisation with 'boots on the ground' and 'eyes in the sky' [2]. An alternative method is to devalue the horn itself, one of the main methods being the removal so that only a stub is left. The first attempt at large-scale rhino dehorning as an anti-poaching measure was in Damordond, Namibia, in 1989 [10]. Other methods of devaluing the horn that have been suggested include the insertion of poisons, dyes or GPS trackers [4, 13]. However, like dehorning, they cannot remove all the potential gain from an intact horn (poison and dyes fade or GPS trackers can be removed). This paper builds on the work of [7] and considers the general strategy of devaluing horns, which includes dehorning.

Rhino populations now persist largely in protected areas or on private land, and require intensive protection [3]. For wildlife managers law enforcement is often one of the main methods of deterring poaching, however rhino managers can remove the poaching incentive by devaluing their rhinos [10]. In [10] they found the optimum proportion to dehorn using mean horn length as a measure of the proportion of rhinos dehorned. They showed, with realistic parameter values, that the optimal strategy is to dehorn as many rhinos as possible. A manager does not need to choose between law enforcement or devaluing, but perhaps adopt a combination of the two; especially given that devaluing rhinos comes at a cost to the manager, and the process comes with a risk to the rhinos.

A recent paper modelled the interaction between a rhino manager and poachers using game theory [7]. The authors consider a working year of a single rhino manager. A manager is assumed to have standard yearly resources which can be allocated on devaluing a proportion of their rhinos or spent on security. It is assumed that all rhinos initially have intact horns. Poachers may either only kill rhinos with full horns, 'selective poachers', or kill all rhinos they encounter, 'indiscriminate poachers'. If all rhinos are left by the rhino manager with their intact horns, it does not pay poachers to be selective so they will chose to be indiscriminate. Conversely, if all poachers are selective, it pays rhino managers to invest in devaluing their rhinos. This dynamic is represented in Fig. 1. Assuming poachers and managers will always behave so as to maximise their payoff, there are two equilibriums: either all devalued and all poachers selective; or all horns intact and all poachers indiscriminate. Essentially, either the managers win, the top left quadrant of Fig. 1, or the poachers win, the bottom right quadrant of Fig. 1. The paper concludes that poachers will always choose to behave indiscriminately, and thus the game settles to the top left quadrant, i.e., the poachers win.

The work of [7] did not take in to account the population dynamic effect of these strategies. In a population full of selective poachers would their be a benefit to a single poacher becoming indiscriminate or vice versa? This notion is explored here using evolutionary game theory [12]. The game is not that of two players anymore (manager and poacher)

Figure 1: The game between rhino manager and rhino poachers. The system settles to one of two equilibriums, either devaluing is effective or not.

but an infinite population of poachers is considered. This allows for the interaction between poachers over multiple plays of the game to be explored with the rhino manager being the one that creates the conditions of the population.

In evolutionary game theory, we assume infinite populations and in our model this will be represented by $\chi = (x_1, x_2)$ with x_1 proportion of the population using a strategy of the first type and x_2 of the second, denoted by s_1, s_2 respectively. We assume there is a utility function u_1 and u_2 that maps the population to a fitness for each type,

$$u_1(\chi) \qquad u_2(\chi).$$

In evolutionary game theory these utilities are used to dictate the evolution of the population over time, according to the following differential equations,

$$\begin{cases} \dfrac{dx_1}{dt} = x_1(u_1(\chi) - \phi), \\[2em] \dfrac{dx_2}{dt} = x_2(u_2(\chi) - \phi). \end{cases} \tag{1}$$

The overall population is assumed to remain stable thus, $x_1 + x_2 = 1$ and

$$\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0 \Rightarrow x_1(u_1(\chi) - \phi) + x_2(u_2(\chi) - \phi) = 0. \tag{2}$$

Thus the average fitness can be written as,

$$\phi = x_1 u_1(\chi) + x_2 u_2(\chi). \tag{3}$$

By substituting (3) and $x_2 = 1 - x_1$ in (1),

$$\frac{dx_1}{dt} = x_1(1 - x_1)(u_1(\chi) - u_2(\chi)). \tag{4}$$

The equilibria of the differential equation (4) are given by, $x_1 = 0$, $x_1 = 1$, and $0 < x_1 < 1$ for $u_1(\chi) = u_2(\chi)$.

These equilibria correspond to stability of the population: the differential equation (4) no longer changes. Further to this, evolutionary game theory aims to qualify these equilibria subject to evolutionary changes.

The notion of evolutionary stability can be checked only for the stable strategies. For a stable strategy to be an Evolutionary Stable Strategy (ESS) it must remain the best response even in a mutated population χ_ϵ. A mutated population is called the post entry population, where a small proportion ϵ starts deviating and adopts a different strategy.

(a) A diagram of a stable strategy which is not ESS.   (b) A diagram of a stable strategy which is ESS.
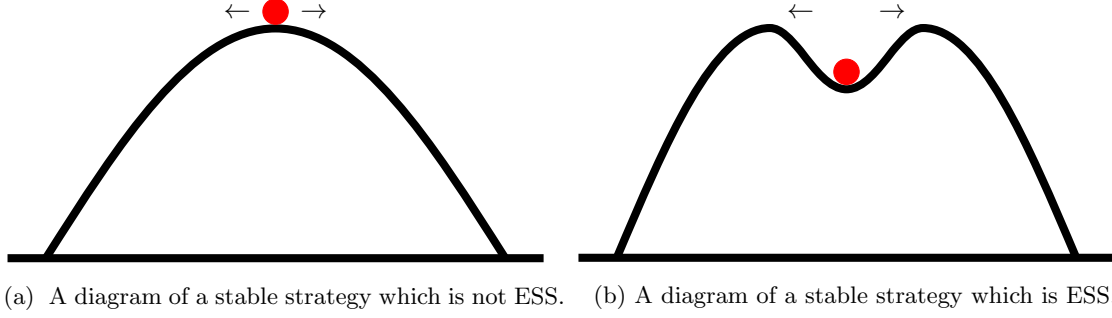
Figure 2: Diagrams of stable strategies.

Fig. 2 illustrates two potential strategies: without outside stimulation neither marble would move. In Fig. 2b however the stability can be described as "stronger": with a small nudge the marble would return to its stable position. In Fig. 2a any non zero nudge would move the marble out of its equilibria.

In Section 2, we determine expressions for u_1, u_2 that correspond to a population of wild rhino poachers and we explore the stability of the equilibria identified in [7]. The results contained in this paper are proven analytically, and more specifically it is shown that:

- all poachers behaving selectively is trivially unstable,

- a mixed population where selective and indiscriminate poachers learn to co exist can not hold in sensible scenarios,

- all poachers adopting an indiscriminate strategy is an evolutionary stable strategy.

This implies that under almost all conditions, no matter what current proportion of of poachers are acting selectively, the population will eventually turn in to a population of only indiscriminate poachers.

2   The Utility Model

As discussed briefly in Section 1, a wild rhino poacher can adopt two strategies, to either behave selectively or indiscriminately. Calculating the utility for each strategy, the gain and cost that poachers are exposed to must be taken into account. The poacher incurs a loss from seeking a rhino, and the risk involved. The gain depends upon the value of horn, the proportion of horn remaining after the manager has devalued the rhino horn and the number of rhinos (devalued and not).

Let us first consider the gain to the poacher, where θ is the amount of horn taken. We assume rhino horn value is determined by weight only, a reasonable assumption as rhino horn is sold in a grounded form [1]. Referring to Fig. 1, clearly if the horn is intact, the amount of horn gained is $\theta = 1$ for both the selective and the indiscriminate poacher. If the rhino horn has been devalued, and the poacher is selective, the amount of horn gained is $\theta = 0$ as the poacher does not kill. However, if the poacher is behaving indiscriminately, the amount of horn gained is $\theta = \theta_r$ (for some $0 < \theta_r < 1$). Therefore, the amount of horn gained in the general case is

$$\theta(r,s) = s(1-r) + (1-s)((\theta_r - 1)r + 1), \tag{5}$$

where r is the proportion of rhinos that have been devalued, and s is the proportion of selective poachers. Note that since $\theta_r, r, s \in [0,1]$, then $\theta(r,s) > 0$. Standard supply and demand arguments imply that the value of rhino horn decreases as the quantity of horn increases [8]. Thus at any given time the expected gain for a poacher is

$$H\theta(r,s)^{-\alpha}, \tag{6}$$

where H is a scaling factor associated with the value of a full horn, and $\alpha \geq 0$ is a constant that determines the precise relationship between the quantity and value of the horn. Fig. 3, verifies that the gain curve corresponds to a demand curve.

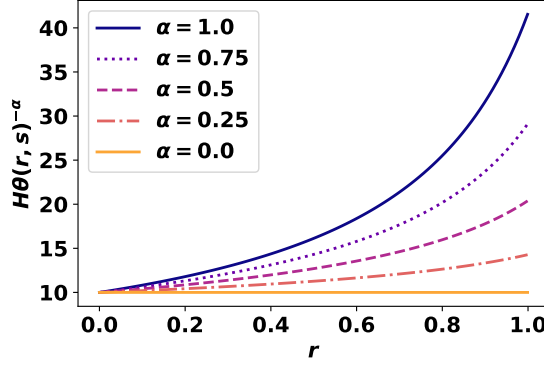An individual interacts with the population, denoted as $\chi = (x, 1-x)$. Thus, the gain is either

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Figure 3: $H\theta(r,s)^{-\alpha}$ for values $H = 10, \theta_r = 0.3$ and $s = 0.2..$

$$\begin{cases} \theta(r,1)H\theta(r,x)^{-\alpha} & \text{selective poacher} \\ \theta(r,0)H\theta(r,x)^{-\alpha} & \text{indiscriminate poacher} \end{cases} \tag{7}$$

depending on the chosen strategy of the individual, see Fig. 1.

Secondly we consider the costs incurred by the poacher. Let us denote the number of rhinos that will be considered at risk given r and s as $\phi(r,s)$. The rhinos **not** at risk are the devalued ones that cross the paths of selective poachers. Thus:

$$\psi(r,s) = 1 - rs. \tag{8}$$

Additionally, the poachers are also exposed to a risk. The risk to the poacher is the opposite of the proportion of rhinos devalued r, since the rhino manager can spend more on security if the cost of devaluing is low.

$$(1-r)^{\beta}, \tag{9}$$

where $\beta \geq 0$ is a constant that determines the precise relationship between the proportion of rhinos not devalued and the security on the grounds. Therefore, at any given time the expected cost for a poacher is,

$$F\psi(r,s)^{\gamma}(1-r)^{\beta} = F(1-rs)^{\gamma}(1-r)^{\beta} \tag{10}$$

where F and $\gamma \geq 0$ are constants that determine the precise relationship between the proportion of vulnerable rhinos and the probability of finding a rhino, such that γ close to zero indicates very sparse rhinos. Fig. 4, verifies the decreasing relationship between r and the cost.

Note that for a indiscriminate poachers $s = 0$ the seeking cost (9) will always be 1, thus the cost of finding a rhino is greater than the same cost for a selective poacher. However, a selective poacher needs more time to secure an 'available' rhino, if they exist at all. Hence, an additional cost that tends to infinity as $r \to 1$ must be applied,

$$\begin{cases} \frac{1}{\psi(r,1)} = \frac{1}{1-r} & \text{selective poacher} \\ \frac{1}{\psi(r,0)} = 1 & \text{indiscriminate poacher} \end{cases} \tag{11}$$

To summarise, the cost incurred by a given individual when interacting with the population is given by

$$\begin{cases} \frac{1}{1-r}F(1-rx)^{\gamma}(1-r)^{\beta} & \text{selective poacher} \\ F(1-rx)^{\gamma}(1-r)^{\beta} & \text{indiscriminate poacher} \end{cases} \tag{12}$$

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(a) For values, $s = 0.7, F = 5$ and $\beta = 0.95$.   (b) For values, $s = 1, F = 5$ and $\gamma = 0.95$.
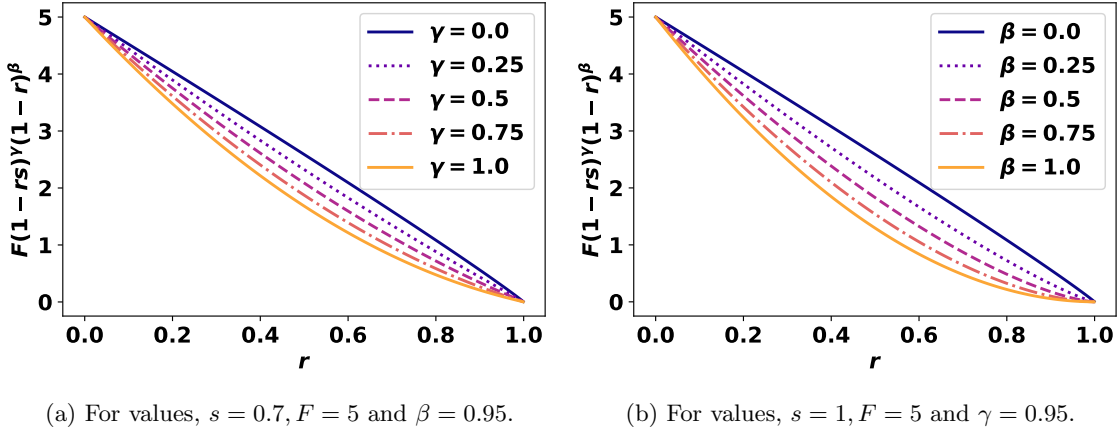
Figure 4: $F(1 - rs)^\gamma (1 - r)^\beta$.

As a result, the utility of the poachers can now be defined. Let $\sigma = (s, 1 - s)$ denote the strategy of an individual. Thus $\sigma = (1, 0)$ represent an individual poacher who is selective and $\sigma = (0, 1)$ represent an individual poacher who is indiscriminate. Combining (7) and (12) gives the utility function for the individual poacher σ in the population χ,

$$u(\sigma, \chi) = s u_1(\chi) + (1 - s) u_2(\chi), \tag{13}$$

where

$$u_1(\chi) = \theta(r, 1) H \theta(r, x)^{-\alpha} - \frac{1}{1 - r} F \psi(r, x)^\gamma (1 - r)^\beta, \tag{14}$$

$$u_2(\chi) = \theta(r, 0) H \theta(r, x)^{-\alpha} - F \psi(r, x)^\gamma (1 - r)^\beta. \tag{15}$$

Substituting (14) and (15) into (13) gives

$$u(\sigma, \chi) = H(\theta_r r(1 - s) - r + 1) \theta(r, x)^{-\alpha} - F\left(1 - s + \frac{s}{1 - r}\right) (1 - rx)^\gamma (1 - r)^\beta. \tag{16}$$

In Section 3, the notions of stability and evolutionary stability of these two strategies as well as a potential mixed strategy will be investigated.

3 Evolutionary Stability

By definition, for a strategy to be an ESS it must first be a best response to an environment where the entire population is playing the same strategy. In our model there are three possible stable distributions based on the equilibria of equation (4),

- All poachers are selective $s = 1$;

- All poachers are indiscriminate $s = 0$;

- Mixed population of selective and indiscriminate poachers.

Each of the equilibria will be examined in the following subsections.

3.1 All poachers are selective $s = 1$

Theorem 1. *Using the utility model descried in Section 2, a population of selective poachers ($\sigma = (1,0)$) is unstable.*

Proof. For $\sigma = (1,0)$ to be a best response to itself the utility of behaving selectively in a population of selective poachers must be greater than the utility of a poacher behaving indiscriminately in a population of selective poachers,

$$u((1,0),(1,0)) > u((0,1),(1,0)) \tag{17}$$

where,

$$u((1,0),(1,0)) = H(1-r)^{1-\alpha} - F(1-r)^{\beta+\gamma-1}, \tag{18}$$

and

$$u((0,1),(1,0)) = H(\theta_r r + 1 - r)(1-r)^{-\alpha} - F(1-r)^{\beta+\gamma}. \tag{19}$$

Setting (18) to be greater than (19) gives the condition,

$$H\theta_r r < F[1 - (1-r)^{-1}](1-r)^{\gamma+\beta+\alpha} \tag{20}$$

The left-hand size will always be negative since $1 - (1-r)^{-1} \leq 0$ for any r, on the other hand the right-hand side is always positive. Thus, (20) can never hold. $\qquad\square$

This implies that whilst [7] identified the individual poachers acting selectively as an equilibrium it is an extremely week equilibrium from the point of view of population stability: a slight change and the population will change.

3.2 All poachers are indiscriminate $s = 0$

Theorem 2. *Using the utility model descried in Section 2, a population of indiscriminate poachers ($\sigma = (0,1)$) is evolutionarily stable.*

Proof. In order for all indiscriminate $\sigma = (0,1)$ to be an ESS it must remain the best response in a mutated population, thus,

$$u((0,1),\chi_\epsilon) > u(\chi_\epsilon,\chi_\epsilon), \tag{21}$$

must hold. Where $\chi_\epsilon = (\epsilon, 1 - \epsilon)$ giving:

$$u((0,1),\chi_\epsilon) = H(\theta_r r - r + 1)\theta(r,x_\epsilon)^{-\alpha} - F(1-\chi_\epsilon)^\gamma (1-r)^\beta, \tag{22}$$

$$u(\chi_\epsilon,\chi_\epsilon) = H(\theta_r r - r(1-x_\epsilon)) + 1)\theta(r,x_\epsilon)^{-\alpha} - F(1-x_\epsilon)^\gamma (1-r)^\beta \left(1 - x_\epsilon + \frac{x_\epsilon}{1-r}\right). \tag{23}$$

Let the difference of (22) and (23) be denoted as,

$$\delta = u((0,1),\chi_\epsilon) - u(\chi_\epsilon,\chi_\epsilon), \tag{24}$$

$$\delta = H\theta(r,\chi_\epsilon)^{-\alpha}\theta_r r x_\epsilon - F(1-x_\epsilon)^\gamma (1-r)^\beta x_\epsilon \left(\frac{-r}{1-r}\right) \tag{25}$$

all indiscriminate will be an ESS only if $\delta > 0$ for any small value of ϵ. Thus only if,

$$H\theta(r,\chi_\epsilon)^{-\alpha}\theta_r r x_\epsilon > F(1-x_\epsilon)^\gamma (1-r)^\beta x_\epsilon \left(\frac{-r}{1-r}\right) \tag{26}$$

The right-hand of inequality (26) is always negative since $(\frac{-r}{1-r}) < 0$ for all r. On the contrary, the left-hand side is always positive for all r, thus the inequality always holds. It is proven that all indiscriminate is an evolutionary stable strategy. $\qquad\square$

This shows that a population of indiscriminate poachers not only corresponds to the equilibrium of the individual based game identified in [7] but also is a very robust and attractive equilibrium at the population level.

3.3 Mixed population of selective and indiscriminate poachers

Theorem 3. *Using the utility model descried in section 2, a mixed stable strategy ($\sigma = (s^*, 1 - s^*)$) exists only if $F > H$.*

Proof. A mixed population $\sigma = (s^*, 1 - s^*)$ is said to be stable for a given s^* only if,

$$u((1,0), (s^*, 1 - s^*)) = u((0,1), (s^*, 1 - s^*)). \tag{27}$$

The left-hand side is,

$$u((1,0), (s^*, 1 - s^*)) = H(1 - r)\theta(r, s^*)^{-\alpha} - F(1 - r)(1 - rs^*)^\gamma (1 - r)^\beta.$$

The right-hand side is

$$u((0,1), (s^*, 1 - s^*)) = H(\theta_r + 1 - r)\theta(r, s^*)^{-\alpha} - F(1 - rs^*)^\gamma (1 - r)^\beta.$$

Substituting these into (27) gives an expression to solve for s^*,

$$-H\theta_r r\theta(r, s^*)^{-\alpha} + Fr(1 - rs^*)^\gamma (1 - r)^\beta = 0. \tag{28}$$

$$\frac{F}{H} = \frac{\theta_r}{(1 - rs)^\gamma} + \frac{1}{(1 - rs)^\gamma (1 - r)^\beta r(1 - s)} \tag{29}$$

Note that the right-hand side of equation (29) is always greater to 1, since $\frac{1}{(1-rs)^\gamma(1-r)^\beta r(1-s)} \geq 1$. Thus for a mixed stable strategy to exists F must be greater or equal to H. $\qquad\square$

For poaching to be a rational option the cost associated with securing a horn will never surpass it's value: if it did then there would be no poaching. Thus, for a given scenario of our model to be reasonable F will always be smaller than H. Thus, a mixed strategy can not stable.

In this section we have studied analytically the stability of all the possible equilibria. We have proven the instability of any population with selective poachers and the evolutionary stability of the indiscriminate behaviour. All of these theoretic results have been verified empirically, the data for this has been archived at [].

In Section 4, we discuss some of the insights we gain from the results of this section.

4 Discussion and Conclusions

In this section we discuss the insights of our findings. Our results question the effectiveness of how wild life managers have been dealing with poachers for more than 20 years.

Dehorning wild rhinos was introduced in 1989 as a mean of securing the safety of the animal. However, dehorning can be effective if and only if poachers chose to behave in a selective manner. Exploring the dynamics of a selective population it was proven than being selective is not a best response even in a population where everyone is behaving selectively. More specifically, even a mixed population with a small percentage of the population behaving selectively will not persist. Thus, a poacher would never adopt a selective strategy.

For a realistic and generic utility model it was found that the only strategy that was proven to be stable is the indiscriminate one. Moreover, it was proven that to be evolutionary stable as well. Meaning, that for any given starting population, the poachers would evolve to adopt an indiscriminate behaviour. This is illustrated in Fig. 5 ((4) is solved numerically using [6]).

Poachers behaving indiscriminately is an undesirable case and these results indicate that the life of wild rhinos cannot be secured by dehorning. Our results disagree with the claim of [10] that managers should dehorn as many rhinos as possible. We believe that the resources should be assigned to security.
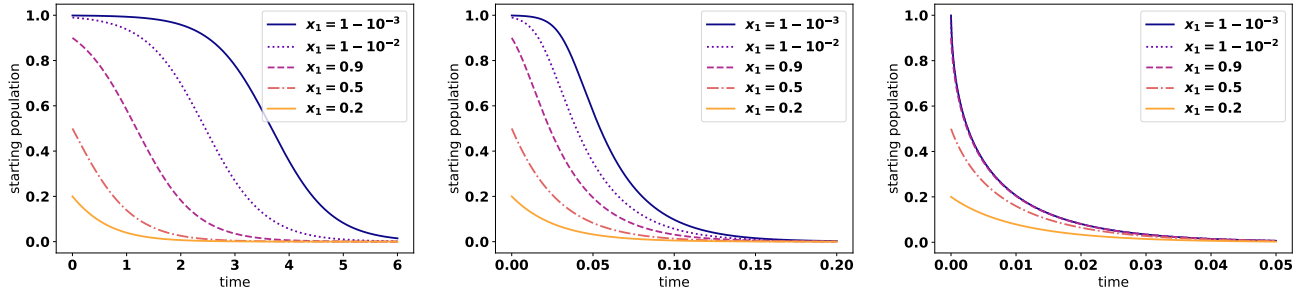
(a) Low proportion of dehorned rhinos: $r = 0.05$

(b) Majority of dehorned rhinos: $r = 0.65$

(c) High proportion of dehorned rhinos: $r = 0.99$

Figure 5: The change of the population over time with different starting populations. For $F = 5, H = 50, \alpha = 2, \beta = 2, \gamma = 1, \theta_r = 0.6$.

Acknowledgements

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