An Evolutionary Game Theoretic Model of Rhino Horn Devaluation

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Declaration of interest: None.

Abstract

Rhino populations are at a critical level due to the demand for rhino horn and the subsequent poaching. Wildlife managers attempt to secure rhinos with approaches to devalue the horn, the most common of which is dehorning. Game theory has been used to examine the interaction of poachers and wildlife managers where a manager can either 'dehorn' their rhinos or leave the horn attached and poachers may behave 'selectively' or 'indiscriminately'. The approach described in this paper builds on this previous work and investigates the interactions between the poachers. We build an evolutionary game theoretic model and determine which strategy is preferred by a poacher in various different populations of poachers. The purpose of this work is to discover whether conditions which encourage the poachers to behave selectively exist, that is, they only kill those rhinos with full horns.

The analytical results show that full devaluation of all Rhinos will likely lead to indiscriminate poaching. In turn it shows the devaluing of Rhinos can only be effective when implemented along with a strong incentive framework. This paper aims to contribute to the necessary research required for informed discussion about the lively debate on legalising rhino horn trade.

1   Introduction

Rhino populations now persist largely in protected areas or on private land, and require intensive protection [7] because the demand for rhino horn continues to pose a serious threat [2]. The illegal trade in rhino horn supports aggressive poaching syndicates and a black market [17, 22]. This lucrative market entices people to invest their time and energy to gain a 'windfall' in the form of a rhino horn, through the poaching of rhinos. In recent years poaching has escalated to an unprecedented level resulting in concerns over their future existence [4, 19]. Several methods include addressing the problem by reducing the demand in the market. In [4] the authors suggest meeting the demand for rhino horn through a legal market by farming the rhino horn from live rhinos. This controversial proposal is an active conversation, with the actual quantity of horn that could be farmed being estimated recently by [20]. Moreover, the debate is not limited to rhinos - [10] considered ivory and stated that by enforcing a domestic ivory trade ban we can reduce the market's demand.

Nonetheless, for wildlife managers law enforcement is often one of the main methods to deter poachers. In response, rhino conservation has seen increased militarisation with 'boots on the ground' and 'eyes in the sky' [5]. An alternative method is to devalue the horn itself, one of the main methods being the removal so that only a stub is left. The first attempt at large-scale rhino dehorning as an anti-poaching measure was in

Damaraland, Namibia, in 1989 [16]. Other methods of devaluing the horn that have been suggested include the insertion of poisons, dyes or GPS trackers [8, 19]. However, like dehorning, they cannot remove all the potential gain from an intact horn (poison and dyes fade or GPS trackers can be removed and have been found to affect only a small proportion of the horn). In [16] they found the optimum proportion to dehorn using mean horn length as a measure of the proportion of rhinos dehorned. They showed, with realistic parameter values, that the optimal strategy is to dehorn as many rhinos as possible. A manager does not need to choose between law enforcement or devaluing, but perhaps adopt a combination of the two; especially given that devaluing rhinos comes at a cost to the manager, and the process comes with a risk to the rhinos.

A recent paper modelled the interaction between a rhino manager and poachers using game theory [12]. The authors consider a working year of a single rhino manager. A manager is assumed to have standard yearly resources which can be allocated on devaluing a proportion of their rhinos or spent on security. It is assumed that all rhinos initially have intact horns. Poachers may either only kill rhinos with full horns, 'selective poachers', or kill all rhinos they encounter, 'indiscriminate poachers'. This strategy may be preferred to avoid tracking a devalued rhino again, and/or to gain the value from the partial horn. If all rhinos are left by the rhino manager with their intact horns, it does not pay poachers to be selective so they will chose to be indiscriminate since being selective incurs an additional cost to discern the status of the rhino. Conversely, if all poachers are selective, it pays rhino managers to invest in devaluing their rhinos. Assuming poachers and managers will always behave so as to maximise their payoff, there are two equilibriums: either all rhinos are devalued and all poachers are selective; or all horns are intact and all poachers are indiscriminate. Essentially, either the managers win and rhinos survive or the poachers win and rhinos are killed. The paper [12] concludes that poachers will always choose to behave indiscriminately, and thus the game settles to the bottom right quadrant, i.e., the poachers win.

In this manuscript, we explore the population dynamic effects associated to the interactions described by [12]. More specifically, the interaction between poachers.In a population full of indiscriminate poachers is there a benefit to a single poacher becoming selective or vice versa? This notion is explored here using evolutionary game theory [18]. The game is not that of two players anymore (manager and poacher) but now the players are an infinite population of poachers. This allows for the interaction between poachers over multiple plays of the game to be explored with the rhino manager being the one that creates the conditions of the population.

Note that poachers are, in practice finite, and each has individual factors that will affect a poacher's behaviour. An infinite population model corresponds to either an asymptotic generalisation or overall de-

scriptive behaviour.

In evolutionary game theory, we assume infinite populations and in our model this is represented by $\chi = (x_1, x_2)$ with x_1 being the proportion of the population using a strategy of the first type and x_2 of the second. We assume there are utility functions u_1 and u_2 that map the population to a fitness for each strategy, given by,

$$u_1(\chi) \text{ and } u_2(\chi).$$

In evolutionary game theory these utilities are used to dictate the evolution of the population over time, according to the following differential equations,

$$
\begin{cases}
\dfrac{dx_1}{dt} = x_1(u_1(\chi) - \phi), \\[4em]
\dfrac{dx_2}{dt} = x_2(u_2(\chi) - \phi),
\end{cases}
\tag{1}
$$

In some settings these utilities are referred to as fitness and/or are mapped to a further measure of fitness. This is not the case considered here (it is assumed all evolutionary dynamics are considered by the utility measures).

where ϕ is the average fitness of the whole population. Here, the overall population is assumed to remain stable thus, $x_1 + x_2 = 1$ and

$$\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0 \Rightarrow x_1(u_1(\chi) - \phi) + x_2(u_2(\chi) - \phi) = 0. \tag{2}$$

Recalling that $x_1 + x_2 = 1$ the average fitness can be written as,

$$\phi = x_1 u_1(\chi) + x_2 u_2(\chi). \tag{3}$$

By substituting (3) and $x_2 = 1 - x_1$ in (1),

$$\frac{dx_1}{dt} = x_1(1 - x_1)(u_1(\chi) - u_2(\chi)). \tag{4}$$

The equilibria of the differential equation (4) are given by, $x_1 = 0$, $x_1 = 1$, and $0 < x_1 < 1$ for $u_1(\chi) = u_2(\chi)$. These equilibria correspond to stability of the population: the differential equation (4) no longer changes.

The notion of evolutionary stability can be checked only for these stable strategies. For a stable strategy to be an Evolutionary Stable Strategy (ESS) it must remain the best response even in a mutated population χ_ϵ. A mutated population is the post entry population where a small proportion $\epsilon > 0$ starts deviating and adopts a different strategy. Fig. 1 illustrates two potential strategies: without outside stimulation neither marble would move. In Fig. 1b however the stability can be described as "stronger": with a small nudge the marble would return to its stable position. In Fig. 1a any non-zero nudge would move the marble out of its equilibria.

(a) A diagram of a stable strategy which is not ESS.
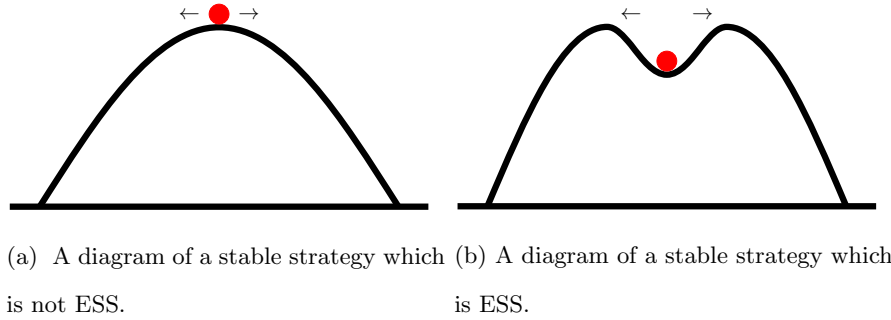
(b) A diagram of a stable strategy which is ESS.

Figure 1: Diagrams of stable strategies.

In Section 2, we determine expressions for u_1, u_2 that correspond to a population of wild rhino poachers and we explore the stability of the equilibria identified in [12]. The results contained in this paper are proven analytically, and more specifically it is shown that:

• In the presence of sufficient risk: a population of selective poachers is stable.

• Full devaluation of all rhinos will lead to indiscriminate poachers.

This implies that under almost all conditions, no matter what current proportion of of poachers are acting selectively, the population will eventually turn into a population of only indiscriminate poachers.

5

2 The Utility Model

As discussed briefly in Section 1, a rhino poacher can adopt two strategies, to either behave selectively or indiscriminately. To calculate the utility for each strategy, the gain and cost that poachers are exposed to must be taken into account. The poacher incurs a loss from seeking a rhino, and the risk involved. The gain depends upon the value of horn, the proportion of horn remaining after the manager has devalued the rhino horn and the number of rhinos (devalued and not).

Let us first consider the gain to the poacher, where θ is the amount of horn taken. We assume rhino horn value is determined by weight only, a reasonable assumption as rhino horn is sold in a grounded form [1]. Clearly if the horn is intact, the amount of horn gained is $\theta = 1$ for both the selective and the indiscriminate poacher. If the rhino horn has been devalued, and the poacher is selective, the amount of horn gained is $\theta = 0$ as the poacher does not kill. However, if the poacher is behaving indiscriminately, the proportion of horn gained is $\theta = \theta_r$ (for some $0 < \theta_r < 1$). Therefore, the amount of horn gained in the general case is

$$\theta(r, x) = x(1 - r) + (1 - x)((\theta_r - 1)r + 1), \tag{5}$$

where r is the proportion of rhinos that have been devalued, and x is the proportion of selective poachers and $1 - x$ is the proportion of indiscriminate poachers. Note that since $\theta_r, r, x \in [0, 1]$, then $\theta(r, x) > 0$, that is, some horn will be taken. Standard supply and demand arguments imply that the value of rhino horn decreases as the quantity of horn available increases [14]. Thus at any given time the expected gain is

$$H\theta(r, x)^{-\alpha}, \tag{6}$$

where H is a scaling factor associated with the value of a full horn, and $\alpha \geq 0$ is a constant that determines the precise relationship between the quantity and value of the horn. Fig. 2, verifies that the gain curve corresponds to a demand curve.

An individual interacts with the population, denoted as $\chi = (x, 1 - x)$. For simplicity, from herein the population χ is referred to in terms of the proportion of selective poachers only, x. Therefore, the gain for a
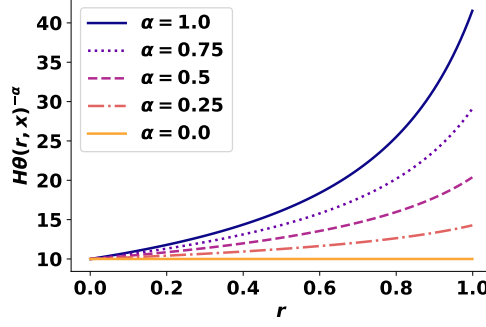
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Figure 2: $H\theta(r,x)^{-\alpha}$ for values $H = 10, \theta_r = 0.3$ and $x = 0.2$.

poacher in the population x is either

$$
\begin{cases}
\theta(r,1)H\theta(r,x)^{-\alpha} & \text{selective poacher} \\
\\
\theta(r,0)H\theta(r,x)^{-\alpha} & \text{indiscriminate poacher}
\end{cases}
\tag{7}
$$

depending on the chosen strategy of the individual.

Secondly we consider the costs incurred by the poacher.

It is assumed that a given poacher will spend sufficient time in the park to obtain the equivalent of at least a single Rhinoceros's horn. For selective poachers this implies searching the park for a fully valued horn and for indiscriminate poachers this implies either finding a fully valued horn or finding N_r total Rhinoceroses where $N_r = \lceil 1/\theta_r \rceil$.

Figure 3 shows a random walk that any given poacher will follow in the park. Both types of poacher will exit the park as soon as they encounter a fully valued Rhino, which at every encounter is assumed to happen with probability $1 - r$. However, the indiscriminate poachers may also exit the park if they encounter N_r devalued Rhinos in a row. Each step on the random walk is assumed to last 1 time unit: during which a Rhino is found. To capture the fact that indiscriminate poachers will spend a different amount of time to selective poachers with each Rhino the parameter t is introduced which corresponds to the amount of time it takes to find and kill a Rhino (thus $t \geq 1$).

Using this, the expected time spent in the park T_1, T_2 by poachers of both types can be obtained:
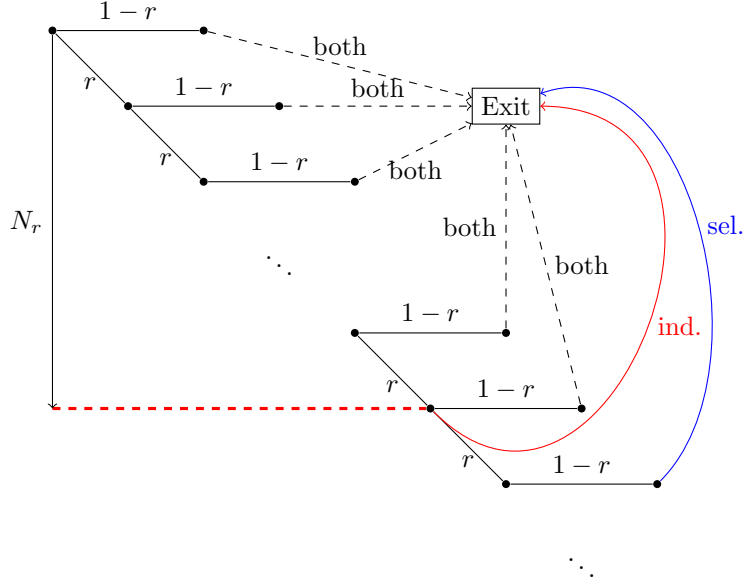
For selective poachers:

7

Figure 3: Illustrative random walk showing the points at which an indiscriminate or a selective poacher will leave the park

$$T_1 = (1-r)t + r(1-r)(1+t) + r^2(1-r)(2+t) + \ldots \tag{8}$$

$$= (1-r)\sum_{i=0}^{\infty} r^i(i+t) \tag{9}$$

$$= (1-r)\left(1/r\sum_{i=0}^{\infty} ir^{(i+1)} + t\sum_{i=0}^{\infty} r^i\right) \tag{10}$$

$$= (1-r)\left(\frac{r}{(1-r)^2} + \frac{t}{1-r}\right) \qquad \text{using standard formulae for geometric series} \tag{11}$$

$$= \frac{r+t(1-r)}{1-r} \tag{12}$$

For indiscriminate poachers:

$$T_2 = (1-r)t + r(1-r)2t + r^2(1-r)3t + \cdots + r^{N_r-2}(1-r)(N_r-1)t + r^{N_r-1}N_r t \tag{13}$$

$$= (1-r)t \sum_{i=1}^{N_r-1} ir^{i-1} + r^{N_r-1}N_r t \tag{14}$$

$$= (1-r)t \left(\frac{1}{r(r-1)^2} \left(N_r r r^{N_r} - N_r r^{N_r} - rr^{N_r} + r \right) \right) + r^{N_r-1}N_r t \tag{15}$$

$$= \frac{t(1-r^{N_r})}{(1-r)} \tag{16}$$

Additionally, the poachers are also exposed to a risk. The risk to the poacher is directly related to the proportion of rhinos not devalued, $1-r$, since the rhino manager can spend more on security if the cost of devaluing is low. In real life this is not always the case. The cost of security can be extremely high thus it cannot be guaranteed that much security will be added from the saved money. However, our model assumes that there is a proportional and negative relationship between the measures.

$$(1-r)^\beta, \tag{17}$$

where $\beta \geq 0$ is a constant that determines the precise relationship between the proportion of rhinos not devalued and the security on the grounds. Therefore, at any given time the expected cost for a poacher is,

$$FT_i(1-r)^\beta \text{ for } i \in \{1, 2\} \tag{18}$$

where F is a constants that determines the precise relationship. Fig. 4 verifies the decreasing relationship between r and the cost.

One final consideration given to the utility model is the incorporation of a disincentive to indiscriminate poachers. Numerous interpretations can be incorporated with this:

- more severe punishment for indiscriminate killing of rhinos;

- educational interventions that highlight the negative aspects of indiscriminate killing;

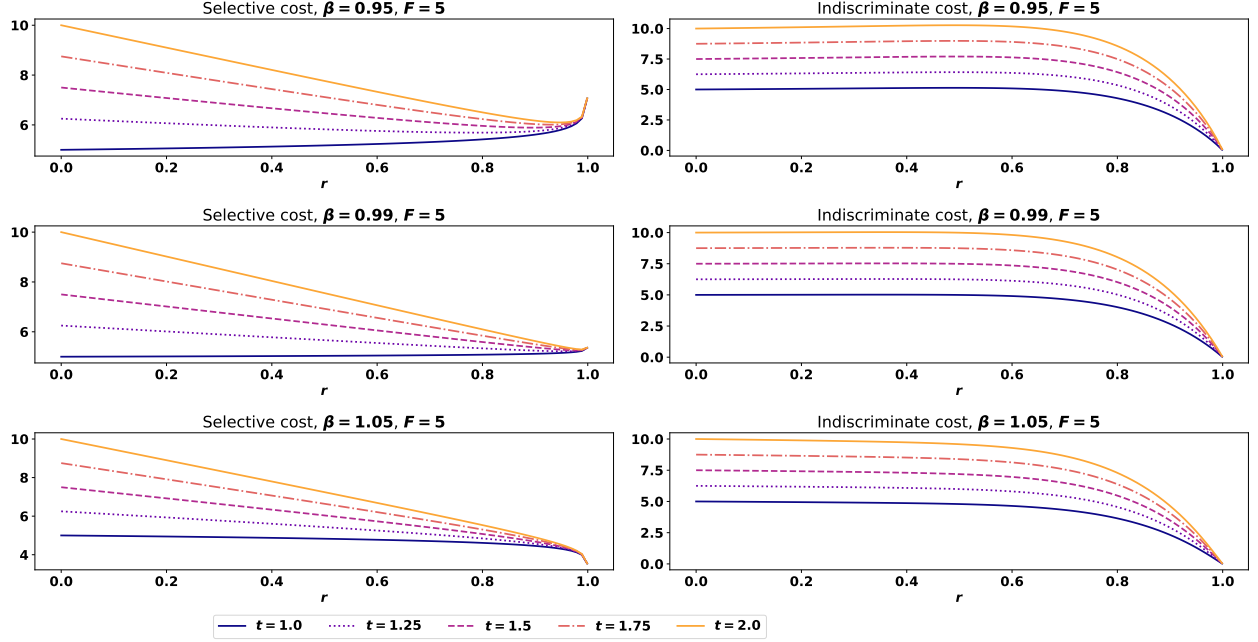- the possibility of a better alternative being offered to selective poachers.

Figure 4: Costs associated to both poachers for varying values of r and t.

This will be captured by a constant Γ.

Combining (7) and (18) gives the utility functions for selective poachers, $u_1(x)$, and indiscriminate poachers, $u_2(x)$,

$$u_1(x) = \theta(r,1)H\theta(r,x)^{-\alpha} - (r + t(1-r))F(1-r)^{\beta-1}, \tag{19}$$

$$u_2(x) = \theta(r,0)H\theta(r,x)^{-\alpha} - t(1 - r^{N_r})F(1-r)^{\beta-1} - \Gamma \tag{20}$$

Let $\sigma = (s, 1-s)$ denote an individual where $\sigma = (1,0)$ represents a selective poacher, and $\sigma = (0,1)$ represents an indiscriminate poacher. For simplicity, from herein the individual σ is referred to in terms of the probability of behaving selectively, $\sigma = s$. Thus the general utility function for an individual poacher in the population with a proportion of $0 \leq x \leq 1$ selective poachers is

$$u(s,x) = su_1(x) + (1-s)u_2(x). \tag{21}$$

10

Substituting (19) and (20) into (21) and using (5) gives,

$$u(s,x) \quad = \quad H(\theta_r r(1-s) - r + 1)\theta(r,x)^{-\alpha} - F\left(sr + st(1-r) + (1-s)t(1-r^{N_r})\right)(1-r)^{\beta-1} - (1-s)t \quad (22)$$

Figure 5 shows the evolution of the system over time for a variety of initial populations and parameters. This is done using numerical integration implemented in [6].
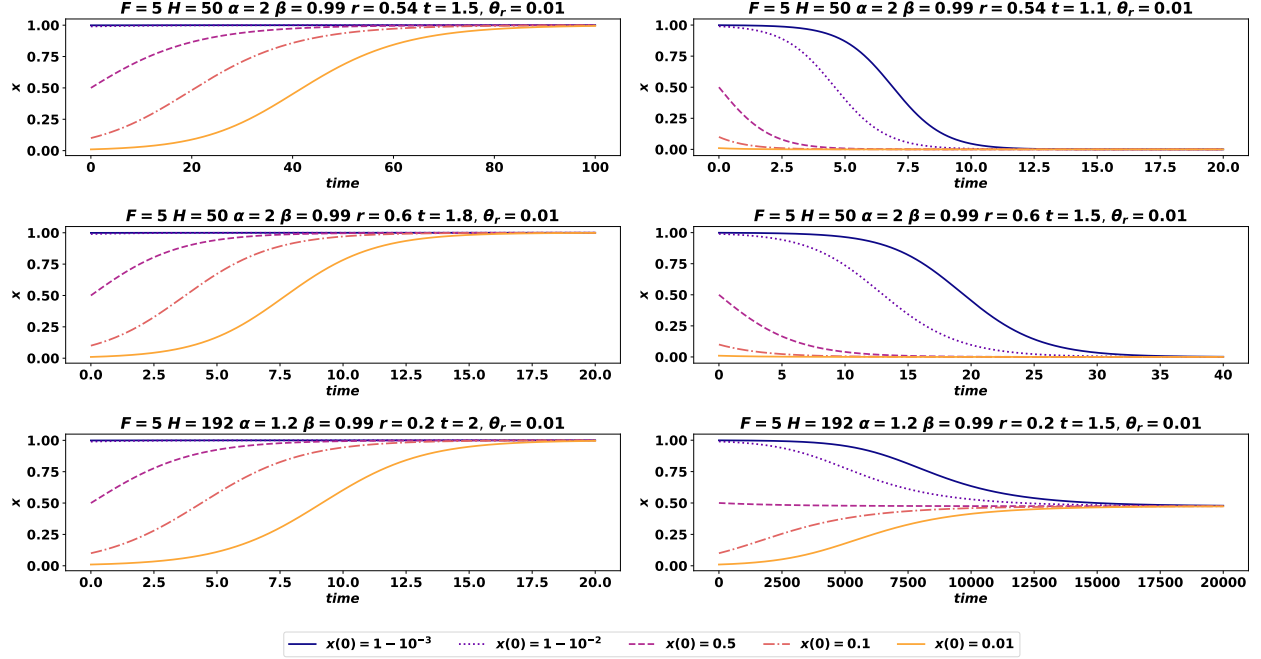

Figure 5: The change of the population over time with different starting populations. For $F = 5, H = 50, \alpha = 2, \beta = .99, t = 1.5, \theta_r = 0.01, \Gamma = 0$.

Two different outcomes seem to be evolutionarily stable, for the higher values of r the security is low and the only way to obtain utility from poaching is to act indiscriminately. When r is lower, then there is less utility and sufficient full valued Rhinos to ensure the risk of acting selectively is sufficiently low.

In Section 3, these observations will be confirmed theoretically.

3 Evolutionary Stability

By definition, for a strategy to be an ESS it must first be a best response to an environment where the entire population is playing the same strategy. In our model there are three possible stable distributions based on

the equilibria of equation (4):

- all poachers are selective;

- all poachers are indiscriminate;

- mixed population of selective and indiscriminate poachers.

An ESS corresponds to asymptotic behaviour near the equilibria of (4), this correspond to the concept of Lyapunov stability [13].

For simplicity, denote the right hand side of (4) as f. In this setting, when x is near to some equilibria x^* (so that $f(x^*) = 0$ then the evolutionary game can be linearized (using standard Taylor Series expansion) as:

$$\frac{d(x^* + \epsilon)}{dt} = J(x^*)\epsilon \tag{23}$$

where:

$$J(a) = \left.\frac{df}{dx}\right|_{x=a} \tag{24}$$

This gives a standard approach for evaluating equilibria of the underlying game. For a given equilibria x^*, $J(x^*) < 0$ if and only if x^* is an ESS.

Using equations (19) and (20):

$$J(a) = \frac{1}{(r-1)\left(-ar\theta_r + r\theta_r - r + 1\right)^{\alpha+1}} \left(J_1 - J_2\right) + \Gamma(1 - 2a) \tag{25}$$

where:

$$J_1 = F\left(-r+1\right)^{\beta}\left(-ar\theta_r + r\theta_r - r + 1\right)^{\alpha+1}\left(2art - 2ar - 2ar^{\lceil\frac{1}{\theta_r}\rceil}t - rt + r + r^{\lceil\frac{1}{\theta_r}\rceil}t\right) \tag{26}$$

$$J_2 = Ha\alpha r^2\theta_r^2\left(-a+1\right)\left(r-1\right) + Hr\theta_r\left(2a-1\right)\left(r-1\right)\left(ar\theta_r - r\theta_r + r - 1\right) \tag{27}$$

Theorem 1. *Using the utility model described in Section 2, a population of selective poachers is stable if and only if:*

$$t > \frac{1}{1 - r^{\lceil \frac{1}{\theta_r} \rceil - 1}} \frac{F + H\theta_r (1 - r)^{1 - \alpha - \beta} - \Gamma/r(1 - r)^{1 - \beta}}{F} \tag{28}$$

178 *Proof.* Direct substitution gives:

$$J(1) = \frac{1}{(-r + 1)^{\alpha + 1} (r - 1)} \left(F(-r + 1)^\beta (-r + 1)^{\alpha + 1} \left(rt - r - r^{\lceil \frac{1}{\theta_r} \rceil} t \right) - Hr\theta_r (r - 1)^2 \right) - \Gamma \tag{29}$$

$$= \left(F(1 - r)^{\beta - 1} \left(r - t(r - r^{\lceil \frac{1}{\theta_r} \rceil}) \right) + Hr\theta_r (1 - r)^{-\alpha} \right) - \Gamma \tag{30}$$

179 The required condition is $J(1) < 0$:

$$F(1 - r)^{\beta - 1} r + Hr\theta_r (1 - r)^{-\alpha} - \Gamma < F(1 - r)^{\beta - 1} t(r - r^{\lceil \frac{1}{\theta_r} \rceil}) \tag{31}$$

$$\frac{r}{r - r^{\lceil \frac{1}{\theta_r} \rceil}} \frac{F + H\theta_r (1 - r)^{1 - \beta - \alpha} - \Gamma/r(1 - r)^{1 - \beta}}{F} < t \tag{32}$$

180 which gives the required result.

181 $\square$

182 • Note that limit of rhs as $r \to 1^-$ is ∞ so simply devaluing all rhinos is not a valid approach.

183 • The equilibrium predicted in [12] can be obtained in specific setting.

184 In this section we have analytically studied the stability of all the possible equilibria. We have proven

185 that all potential equilibria are possible. All of these theoretic results have been verified empirically, and the

186 data for this has been archived at [9].

187 Figure 6 shows a number of scenarios:

188 • Scenario 1: $F = 5$ $H = 50$ $r = 0.45$ $\alpha = 2$ $\beta = 0.99$, $t = 2$ $\theta_r = 0.05$, $\Gamma = 0$

189 • Scenario 2: $F = 5$ $H = 50$ $r = 0.4$ $\alpha = 2.5$ $\beta = 0.99$, $t = 1.8$ $\theta_r = 0.05$, $\Gamma = 0$

190 • Scenario 3: $F = 5$ $H = 25$ $r = 0.45$ $\alpha = 2$ $\beta = 0.99$, $t = 2$ $\theta_r = 0.05$, $\Gamma = 0$

191 • Scenario 4: $F = 5$ $H = 25$ $r = 0.4$ $\alpha = 2.5$ $\beta = 0.99$, $t = 1.8$ $\theta_r = 0.05$, $\Gamma = 0$

192 • Scenario 5: $F = 5$ $H = 25$ $r = 0.99$ $\alpha = 2$ $\beta = 0.99$, $t = 2$ $\theta_r = 0.05$, $\Gamma = 4$
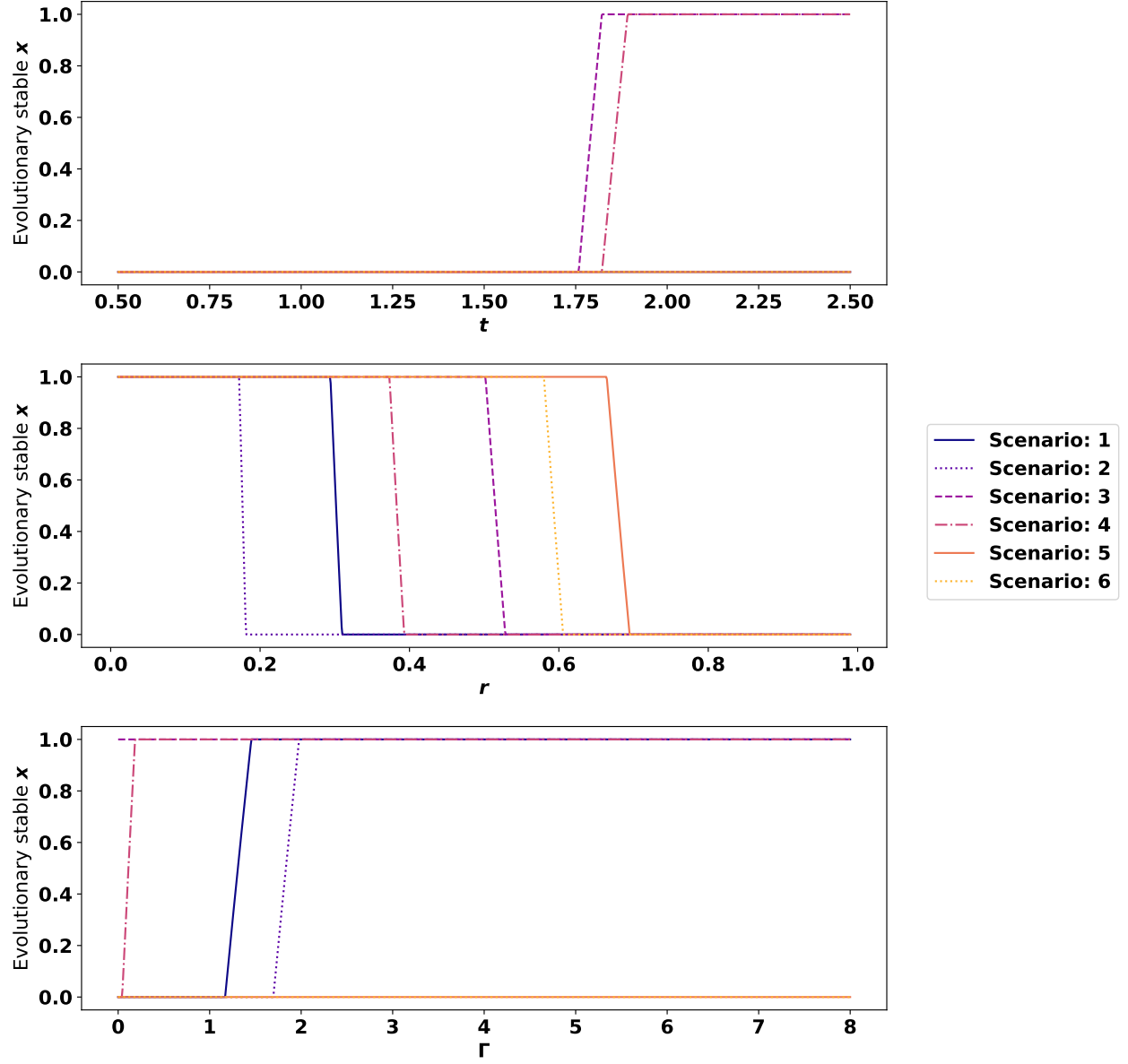
Figure 6: Evolutionary stable populations for varying values of t, r, Γ for 6 difference scenarios.

- Scenario 6: $F = 5$ $H = 25$ $r = 0.99$ $\alpha = 2.5$ $\beta = 0.99$, $t = 1.8$ $\theta_r = 0.05$, $\Gamma = 4$

These simulations confirm Theorem 1:

- Too high value of r force the population to become indiscriminate even with a high disincentive.

- For all scenarios a value of r does exist for which a selective population will subsist.

This confirms that devaluing alone is not a solution and in fact can potentially have averse consequences: combinations of dehorning and education (creating a disincentive) is needed.

4   Discussion and Conclusions

In this work the dynamics of a selective population were explored. It was shown that given sufficient risk associated with killing a Rhino it would be possible for a selective population of poachers to subsist..

Numerical results indicate that even when there is a large disincentive, devaluing of all Rhinos would lead to a population of indiscriminate poachers. Therefore, the proportion of rhinos which could be saved is limited, so approaches which aim to reduce demand (as suggested in [4]) would have more potential.

The disincentive factor can have several interpretations. According to [3], engaging the rural communities that neighbour or live with wildlife is the key to fighting the illegal trade of wildlife. Strengthening disincentives for illegal behaviour can be interpreted as the disincentive factor. Likewise, increasing incentives for wildlife stewardship, decreasing the cost of living with wildlife, and supporting livelihood that is not related to poaching can serve as incentives for selective behaviour.

Note that the proportion of devalued rhinos r is continuous over $[0, 1]$ in the model. However, standard practice of a given park manager in almost all cases is to either devalue all the animals in a defined enclosed area, or none at all. This is thought to be because partial devaluing tends to disturb rhino social structures. The theoretic model presented here, whilst allowing for consideration at the park level with park managers playing a mixed strategy, also can be considered at the macroeconomic level. Where r represents the quantity of devalued rhinos available across multiple parks.

The insights gained, notably that a population of selective poachers is sustainable only if a large disincentive is in place (even if all rhinos have been devalued) have implication at the long term national policy level.

Following discussions with environmental specialists it is clear that devaluing is empirically thought to be one of the best responses to poaching. This indicates that whilst of theoretic and macroeconomic interest, the

modelling approach investigated in this work has potential for further work. For example, a detailed study of two neighbouring parks with differing policies could be studied using a game theoretic model. Another interesting study would be to introduce a third strategy available to poachers: this would represent the possibility of not poaching (perhaps finding another source of income) and/or leaving the current environment to poach elsewhere.

Authors' contributions

All authors conceived the ideas and designed the methodology. NG and VK developed the source code needed for the numerical experiments and generating the data. All authors contributed critically to the drafts and gave final approval for publication.

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Data Accessibility

The data generated for this work have been archived and are available online [9].

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