An Evolutionary Game Theoretic Model of Rhino Horn Devaluation

Nikoleta E. Glynatsi, Vincent Knight, Tamsin E. Lee

Keywords: Evolutionary dynamics; Evolutionary stability; Game theory; Poachers' interactions; Rhinoceros; Wildlife

Declaration of interest: None.

Abstract

Rhino populations are at a critical level due to the demand for rhino horn and the subsequent poaching. Wildlife managers attempt to secure rhinos with approaches to devalue the horn, the most common of which is dehorning. Game theory has been used to examine the interaction of poachers and wildlife managers where a manager can either 'dehorn' their rhinos or leave the horn attached and poachers may behave 'selectively' or 'indiscriminately'. The approach described in this paper builds on this previous work and investigates the interactions between the poachers. We build an evolutionary game theoretic model and determine which strategy is preferred by a poacher in various different populations of poachers. The purpose of this work is to discover whether conditions which encourage the poachers to behave selectively exist, that is, they only kill those rhinos with full horns. Notwithstanding, the analytical results prove that poachers will never adopt a selective strategy as long as there is gain from a partial horn. Additionally, poachers behaving indiscriminately is stable and robust. However, the model is adapted further to include a disincentive factor, which may represent factors such as harsher punishment, or lower demand for horn. With a disincentive, poachers can be encouraged to behave selectively, but only when there are few devalued rhinos

The analytical results show that full devaluation of all rhinos will likely lead to indiscriminate poaching. In turn it shows that devaluing of rhinos can only be effective when implemented along with a strong disincentive framework. This paper aims to contribute to the necessary research required for informed discussion about the lively debate on legalising rhino horn trade.

1 Introduction

Rhino populations now persist largely in protected areas or on private land, and require intensive protection [13] because the demand for rhino horn continues to pose a serious threat [2]. The illegal trade in rhino horn supports aggressive poaching syndicates and a black market [28, 34]. This lucrative market entices people to invest their time and energy to gain a 'windfall' in the form of a rhino horn, through the poaching of rhinos. In recent years poaching has escalated to an unprecedented level resulting in concerns over their future existence

Standard economic theory predicts extinction through poaching alone is unlikely due to escalating costs as the number of remaining species approaches zero [7]. However, the rarity of rhino horn makes it a luxury good, or financial investment for the wealthy [14], and thus the increased cost and risk to poach does not increase as rapidly as the increased gain - the anthropogenic Allee effect [5, 7]. However, the anthropogenic Allee effect was recently revisited [18] to highlight that the relationship is even more complex and pessimistic. The value of rhino horn can inflate, even with a large population size, due to an increase in the cost (i.e. risk) to poach. Therefore measures to protect rhino horn may actually be increasing the gain to poachers. It is not clear whether this relationship has contributed to the escalation in rhino poaching over recent years. Nonetheless, it is clear that the future existence of rhinos is endangered because of poaching [4, 30]. Several methods include addressing the problem by reducing the demand in the marketThis rationale leads to debate about legalising rhino horn trade, which in turn may reduce demand. In [4] the authors suggest meeting the demand for rhino horn through a legal market by farming the rhino horn from live rhinos. This controversial proposal is an active conversation, with In fact recently the actual quantity of horn that could be farmed being estimated recently was estimated by [31]. Moreover, the debate However [8] argues that because the demand for horn is so high, legalising trade may lead to practices that maximise profit, but are not suitable for sustainable rhino populations, and thus rhinos may be 'traded on extinction'. The potential impact of various policies are nicely summarised in [9], where de-horning is noted to be promising for 'in-country intervention'. Nonetheless, preventing poaching covers in-country and global issues, and thus legalising rhino horn trade is a controversial and active conversation, which is not limited to rhinos - [17] considered ivory and stated that by enforcing a domestic ivory trade ban we can reduce the market's demand.

NonethelessAs it stands, for wildlife managers law enforcement is often one of the main methods to deter poachers. In response, rhino Rhino conservation has seen increased militarisation with 'boots on the ground' and 'eyes in the sky' [11]. An alternative method is to devalue the horn itself, one of the main methods being the removal so that only a stub is left. The first attempt at large-scale rhino dehorning as an anti-poaching measure was in Damaraland, Namibia, in 1989 [25]. Other methods of devaluing the horn that have been suggested include the insertion of poisons, dyes or GPS trackers [15, 30][15, 30]. However, like dehorning, they cannot remove all the potential gain from an intact horn (poison and dyes fade or GPS trackers can be removed and have been found to affect only a small proportion of the horn). In [25] [25, 26] they found the optimum proportion to dehorn using mean horn length as a measure of the proportion of rhinos dehorned. They showed, with realistic parameter values, that the optimal strategy is to dehorn as many rhinos as possible. A manager does not need to choose between law enforcement or devaluing, but perhaps adopt a combination of the two; especially given that devaluing rhinos comes at a cost to the manager, and the process comes with a risk to the rhinos.

A recent paper modelled the interaction between a rhino manager and poachers using game theory [21]. The authors consider a working year of a single rhino manager. A manager is assumed to have standard yearly resources which can be allocated on devaluing a proportion of their rhinos or spent on security. It is assumed that all rhinos initially have intact horns. Poachers may either only kill rhinos with full horns, 'selective poachers', or kill all rhinos they encounter, 'indiscriminate poachers'. This strategy may be preferred to avoid tracking a devalued rhino again, and/or to gain the value from the partial horn. If all rhinos are left by the rhino manager with their intact horns, it does not pay poachers to be selective so they will chose to be indiscriminate since being selective incurs an additional cost to discern the status of the rhino. Conversely, if all poachers are selective, it pays rhino managers to invest in devaluing their rhinos. This dynamic is represented in Fig. 1. Assuming poachers and managers will always behave so as to maximise their payoff, there are two equilibriums: either all rhinos are devalued and all poachers are selective; or all horns are intact and all poachers are indiscriminate. Essentially, either the managers winand rhinos survive , the top left quadrant of Fig. 1, or the poachers winand rhinos are killed, the bottom right quadrant of Fig. 1. The paper [21] concludes that poachers will always choose to behave indiscriminately, and thus the game settles to the bottom right top left quadrant, i.e., the poachers win.

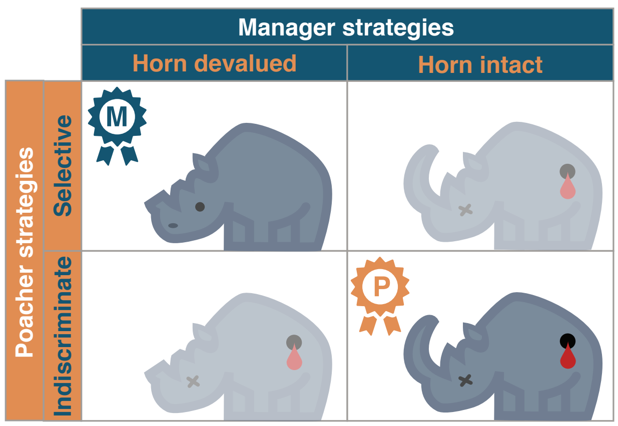

Figure 1: The game between rhino manager and rhino poachers. The system settles to one of two equilibriums, either devaluing is effective or not.

At the extremes, we could consider the game as one of opportunistic exploitation [5]. That is, consider intact rhinos and devalued rhinos as two species, where one is more valuable than the other. Opportunistic exploitation advances upon the theory of anthropogenic Allee effect to consider two species which are exploited together. Specifically, when a highly valued species becomes rarer, a secondary, less valuable species is then targeted. As with opportunistic exploitation on a larger scale, rhino managers need to account for the multispecies system.

In this manuscript, we explore the population dynamic effects associated to the interactions described by [21]. More specifically, the interaction between poachers. In a population full of indiscriminate poachers is there a benefit to a single poacher becoming selective or vice versa? This notion is explored here using evolutionary game theory [29]. The game is not that of two players anymore (manager and poacher) but now the players are an infinite population of poachers. This allows for the interaction between poachers over multiple plays of the game to be explored with the rhino manager being the one that creates the conditions of the population.

Note that poachers are, in practice finite, and each has individual factors that will affect a poacher's behaviour. An infinite population model corresponds to either an asymptotic generalisation or overall descriptive behaviour.

In evolutionary game theory, we assume infinite populations and in our model this is represented by $\chi = (x_1, x_2)$ (x_1, x_2) with x_1 being the proportion of the population using a strategy of the first type and x_2 of the second. We assume there are utility functions u_1 and u_2 that map the population to a fitness for each strategy, given by,

$$u_1(\chi) \text{ and } u_2(\chi).$$

In evolutionary game theory these utilities are used to dictate the evolution of the population over time, according to the following differential replicator equations,

$$\begin{cases} \dfrac{dx_1}{dt} = x_1(u_1(\chi) - \phi), \\ \\ \dfrac{dx_2}{dt} = x_2(u_2(\chi) - \phi), \end{cases} \tag{1}$$

where ϕ is the average fitness of the whole population [27]. In some settings these utilities are referred to as fitness and/or are mapped to a further measure of fitness. This is not the case considered here (it is assumed all evolutionary dynamics are considered by the utility measures).

where ϕ is the average fitness of the whole population. Here, the overall population is assumed to remain stable thus, $x_1 + x_2 = 1$ and

$$\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0 \Rightarrow x_1(u_1(\chi) - \phi) + x_2(u_2(\chi) - \phi) = 0. \tag{2}$$

Recalling that $x_1 + x_2 = 1$ the average fitness can be written as,

$$\phi = x_1 u_1(\chi) + x_2 u_2(\chi). \tag{3}$$

By substituting (3) and $x_2 = 1 - x_1$ in (1),

$$\frac{dx_1}{dt} = x_1(1 - x_1)(u_1(\chi) - u_2(\chi)). \tag{4}$$

The equilibria of the differential equation (4) are given by, $x_1 = 0$, $x_1 = 1$, and $0 < x_1 < 1$ for $u_1(\chi) = u_2(\chi)$. These equilibria correspond to stability of the population: the differential equation (4) no longer changes.

The notion of evolutionary stability can be checked only for these stable strategies. For a stable strategy to be an Evolutionary Stable Strategy (ESS) it must remain the best response even in a mutated population χ_ϵ. A mutated population is the post entry population where a small proportion $\epsilon > 0$ starts deviating and adopts a different strategy. Fig. **??** illustrates two potential strategies: without outside stimulation neither marble would move. In Fig. **??** however the stability can be described as "stronger": with a small nudge the marble would return to its stable position. In Fig. **??** any non-zero nudge would move the marble out of its equilibria.

A diagram of a stable strategy which is not ESS. A diagram of a stable strategy which is ESS. Diagrams of stable strategies.

In Section 2, we determine expressions for u_1, u_2 that correspond to a population of wild rhino poachers and we explore the stability of the equilibria identified in [21]. The results contained in this paper are proven analytically, and more specifically it is shown that:

- all poachers behaving selectively is trivially unstable,
- a mixed population where selective and indiscriminate poachers learn to coexist can not hold, In the presence of sufficient risk: a population of selective poachers is stable.
- all poachers adopting an indiscriminate strategy is an evolutionary stable strategyFull devaluation of all rhinos will lead to indiscriminate poachers.

This implies that under almost all conditions, no matter what current proportion of of poachers are acting selectively, the population will eventually turn into a population of only indiscriminate poachers.

2 The Utility Model

As discussed briefly in Section 1, a rhino poacher can adopt two strategies, to either behave selectively or indiscriminately. To calculate the utility for each strategy, the gain and cost that poachers are exposed to must be taken into account. The poacher incurs a loss from seeking a rhino, and the risk involved. The gain depends upon the value of horn, the proportion of horn remaining after the manager has devalued the rhino horn and the number of rhinos (devalued and not).

Let us first consider the gain to the poacher, where θ is the amount of horn taken. We assume rhino horn value is determined by weight only, a reasonable assumption as rhino horn is sold in a grounded form [1]. Clearly if the horn is intact, the amount of horn gained is $\theta = 1$ for both the selective and the indiscriminate poacher. If the rhino horn has been devalued, and the poacher is selective, the amount of horn gained is $\theta = 0$ as the poacher does not kill. However, if the poacher is behaving indiscriminately, the proportion of horn gained value gained from the horn is $\theta = \theta_r$ (for some $0 < \theta_r < 1$). Therefore, the amount of horn gained in the general case is

$$\theta(r,x) = x(1-r) + (1-x)((1 - \mathrm{r} + \mathrm{r}\theta_r - 1)r + 1), \tag{5}$$

where r is the proportion of rhinos that have been devalued, and x is the proportion of selective poachers and $1-x$ is the proportion of indiscriminate poachers. Note that since $\theta_r, r, x \in [0,1]$, then $\theta(r,x) > 0$, that is, some horn will be taken. Standard supply and demand arguments imply that the value of rhino horn decreases as the quantity of horn available increases [23]. Thus at any given time the expected gain is

$$H\theta(r,x)^{-\alpha}, \tag{6}$$

where H is a scaling factor associated with the value of a full horn, and $\alpha \geq 0$ is a constant that determines the precise relationship between the quantity and value of the horn. Fig. 2, verifies that the gain curve corresponds to a demand curve: we see that as r increases so that the supply of rhinos decreases the value is higher and vice versa. We have chosen a simple function to model the demand (and thus gain) of rhino horn value, relative to the proportion of rhinos devalued. However, demand for illegal wildlife generally involves more factors than simply supply. Additional factors include, but are not limited to, social stigma, tourism revenues, government corruption, and rich countries being willing to pay to ensure species existence [6, 32].

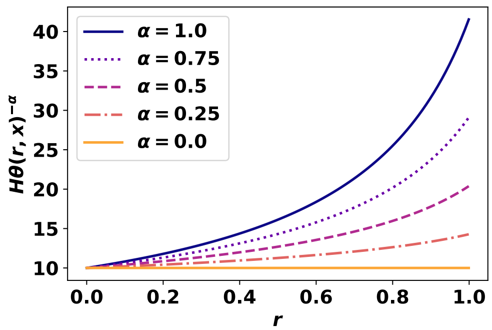

Figure 2: $H\theta(r,x)^{-\alpha}$ for values $H = 10, \theta_r = 0.3$ and $x = 0.2$.

An individual interacts with the population , denoted as $\chi = (x, 1-x)$. For simplicity, from herein the population χ is referred to in terms of which is uniquely determined by x, the proportion of selective poachersonly, x. Therefore, the gain for a poacher in the population x is either

$$\begin{cases} \theta(r,1)H\theta(r,x)^{-\alpha} & \text{selective poacher} \\ \theta(r,0)H\theta(r,x)^{-\alpha} & \text{indiscriminate poacher} \end{cases} \tag{7}$$

depending on the chosen strategy of the individual.

Secondly we consider the costs incurred by the poacher. Let us denote the number of rhinos that will be considered at risk given r and x as $\psi(r,x)$. The rhinos **not** at risk are the devalued ones that cross the paths of selective poachers. Thus the cost to seek a rhino depends on r and x by, It is assumed that a given poacher will spend sufficient time in the park to obtain the equivalent of at least a single rhinoceros's horn. For selective poachers this implies searching the park for a fully valued horn and for indiscriminate poachers this implies either finding a fully valued horn or finding N_r total rhinoceroses where $N_r = \lceil \frac{1}{\theta_r} \rceil$.

$$\psi(r,x) = 1 - rx.$$

Figure 3 shows a random walk that any given poacher will follow in the park. Both types of poacher will exit the park as soon as they encounter a fully valued rhino, which at every encounter is assumed to happen with probability $1-r$. However, the indiscriminate poachers may also exit the park if they encounter N_r devalued rhinos in a row. Each step on the random walk is assumed to last 1 time unit: during which a rhino is found. To capture the fact that indiscriminate poachers will spend a different amount of time to selective poachers with each rhino the parameter τ is introduced which corresponds to the amount of time it takes to find and kill a rhino (thus $\tau \geq 1$).

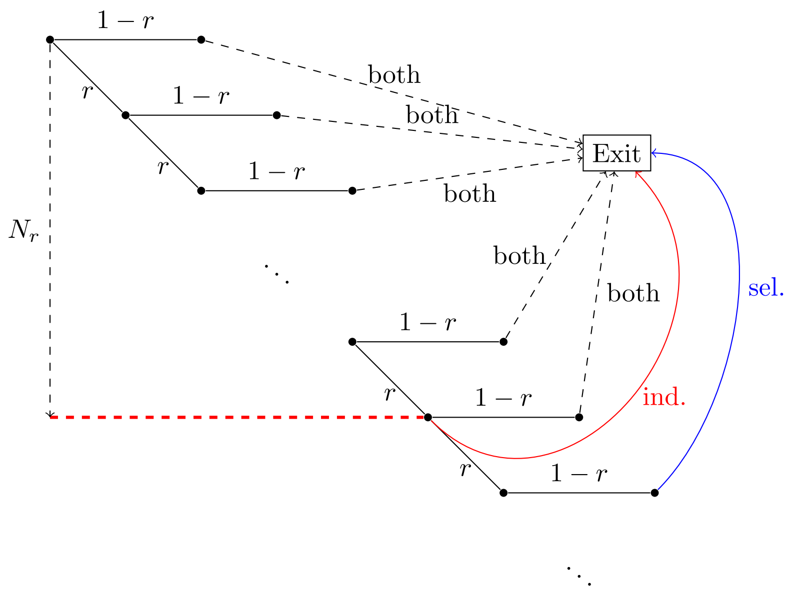

Figure 3: Illustrative random walk showing the points at which an indiscriminate or a selective poacher will leave the park.

Using this, the expected time spent in the park T_1, T_2 by poachers of both types can be obtained:

For selective poachers:

$$\begin{aligned}
T_1 &= (1-r)\tau + r(1-r)(1+\tau) + r^2(1-r)(2+\tau) + \dots \\
&= (1-r)\sum_{i=0}^{\infty} r^i(i+\tau) \\
&= (1-r)\left(\frac{1}{r}\sum_{i=0}^{\infty} i r^{(i+1)} + \tau \sum_{i=0}^{\infty} r^i\right) \\
&= (1-r)\left(\frac{r}{(1-r)^2} + \frac{\tau}{1-r}\right) \qquad \text{using standard formula for geometric series} \\
&= \frac{r + \tau(1-r)}{1-r}
\end{aligned} \tag{8}$$

For indiscriminate poachers:

$$
\begin{aligned}
T_2 &= (1-r)\tau + r(1-r)2\tau + r^2(1-r)3\tau + \cdots + r^{N_r-2}(1-r)(N_r-1)\tau + r^{N_r-1}N_r\tau \\
&= (1-r)\tau \sum_{i=1}^{N_r-1} i r^{i-1} + r^{N_r-1}N_r\tau \\
&= (1-r)\tau \left(\frac{1}{r\,(r-1)^2}\left(N_r r r^{N_r} - N_r r^{N_r} - r r^{N_r} + r\right) \right) + r^{N_r-1}N_r\tau \\
&= \frac{\tau(1-r^{N_r})}{(1-r)}
\end{aligned} \tag{9}
$$

Additionally, the poachers are also exposed to a risk. The risk to the poacher is directly related to the proportion of rhinos not devalued, $1-r$, since the rhino manager can spend more on security if the cost of devaluing is low. In real life this is not always the case. The cost of security can be extremely high thus it cannot be guaranteed that much security will be added from the saved money. However, our model assumes that there is a proportional and negative relationship between the measures.

$$
(1-r)^{\beta}, \tag{10}
$$

where $\beta \geq 0$ is a constant that determines the precise relationship between the proportion of rhinos not devalued and the security on the grounds. Therefore, at any given time the expected cost for a poacher is,

$$
F\psi(r,x)^{\gamma}(1-r)^{\beta} = F(1-rx)^{\gamma}(1-r)^{\beta}
$$

$$
FT_i(1-r)^{\beta} \text{ for } i \in \{1,2\} \tag{11}
$$

where F and $\gamma \geq 0$ are constants that determine is a constants that determines the precise relationshipbetween the proportion of vulnerable rhinos and the probability of finding a rhino, such that γ close to zero indicates very sparse rhinos. Fig. 4 verifies the decreasing relationship between r and the cost.

Note that a selective poacher needs more time to secure an 'available' rhino, if they exist at all. Hence, there is an additional cost that tends to infinity as $r \to 1$, For values, $x = 0.7$, $F = 5$ and $\beta = 0.95$.

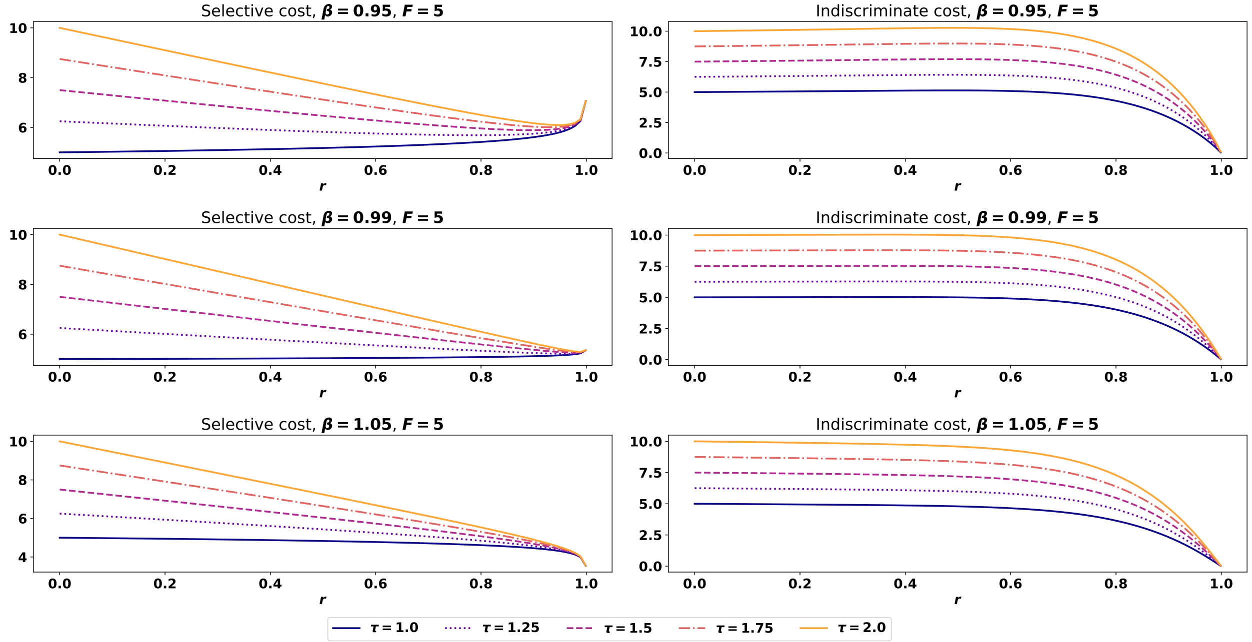

Figure 4: For Costs associated to both poachers for varying values , $x = 1, F = 5$ of r and $\gamma = 0.95\tau$. $F(1-rx)^{\gamma}(1-r)^{\beta}$.

$$\begin{cases} \frac{1}{\psi(r,1)} = \frac{1}{1-r} & \text{selective poacher} \\ \frac{1}{\psi(r,0)} = 1 & \text{indiscriminate poacher.} \end{cases}$$

One final consideration given to the utility model is the incorporation of a disincentive to indiscriminate poachers. Numerous interpretations can be incorporated with this:

To summarise, the cost incurred by a given individual poacher when interacting with the population x is given by

- more severe punishment for indiscriminate killing of rhinos;
- educational interventions that highlight the negative aspects of indiscriminate killing;
- the possibility of a better alternative being offered to selective poachers.

$$\begin{cases} \frac{1}{1-r} F\psi(r,x)^{\gamma}(1-r)^{\beta} & \text{selective poacher} \\ F\psi(r,x)^{\gamma}(1-r)^{\beta} & \text{indiscriminate poacher.} \end{cases}$$

This will be captured by a constant Γ.

Combining (7) and (11) gives the utility functions for selective poachers, $u_1(x)$, and indiscriminate poachers, $u_2(x)$,

$$u_1(x) = \theta(r,1)H\theta(r,x)^{-\alpha} - \frac{1}{1-r}F\psi(r,x)^{\gamma}(1-r)^{\beta},$$
$$u_2(x) = \theta(r,0)H\theta(r,x)^{-\alpha} - F\psi(r,x)^{\gamma}(1-r)^{\beta}.$$

$$u_1(x) = \theta(r,1)H\theta(r,x)^{-\alpha} - (r+\tau(1-r))F(1-r)^{\beta-1}, \tag{12}$$

Let $\sigma = (s, 1-s)$ denote an individual where $\sigma = (1,0)$ represents a selective poacher, and $\sigma = (0,1)$ represents an indiscriminate poacher. For simplicity, from herein the individualσ is referred to in terms of

$$u_2(x) = \theta(r,0)H\theta(r,x)^{-\alpha} - \tau(1-r^{N_r})F(1-r)^{\beta-1} - \Gamma \tag{13}$$

Given a specific individual, let s denote the probability of behaving selectively, $\sigma = s$them behaving selectively. Thus the general utility function for an individual poacher in the population with a proportion of $0 \le x \le 1$ selective poachers is

$$u(s,x) = su_1(x) + (1-s)u_2(x).$$

$$\mathsf{u}(\mathsf{s},\mathsf{x}) = \mathsf{s}\mathsf{u}_1(\mathsf{x}) + (1-\mathsf{s})\mathsf{u}_2(\mathsf{x}). \tag{14}$$

Substituting (12) and (13) into (14) and using (5) and (??) gives,

$$u(s,x) = H(\theta_r r(1-s) - r + 1)\theta(r,x)^{-\alpha} - F\left(1 - s + \frac{s}{1-r}\right)(1-rx)^{\gamma}(1-r)^{\beta}.$$

$$u(s,x) = H(\theta_r r(1-s) - r + 1)\theta(r,x)^{-\alpha} - F\left(sr + s\tau(1-r) + (1-s)\tau(1-r^{N_r})\right)(1-r)^{\beta-1} - (1-s)\Gamma \tag{15}$$

In Section 3, the notions of stability and evolutionary stability of these two strategies, as well as a potential mixed strategy will be investigatedFigure 5 shows the evolution of the system over time for a variety of initial populations and parameters.

This is done using numerical integration implemented in [12].

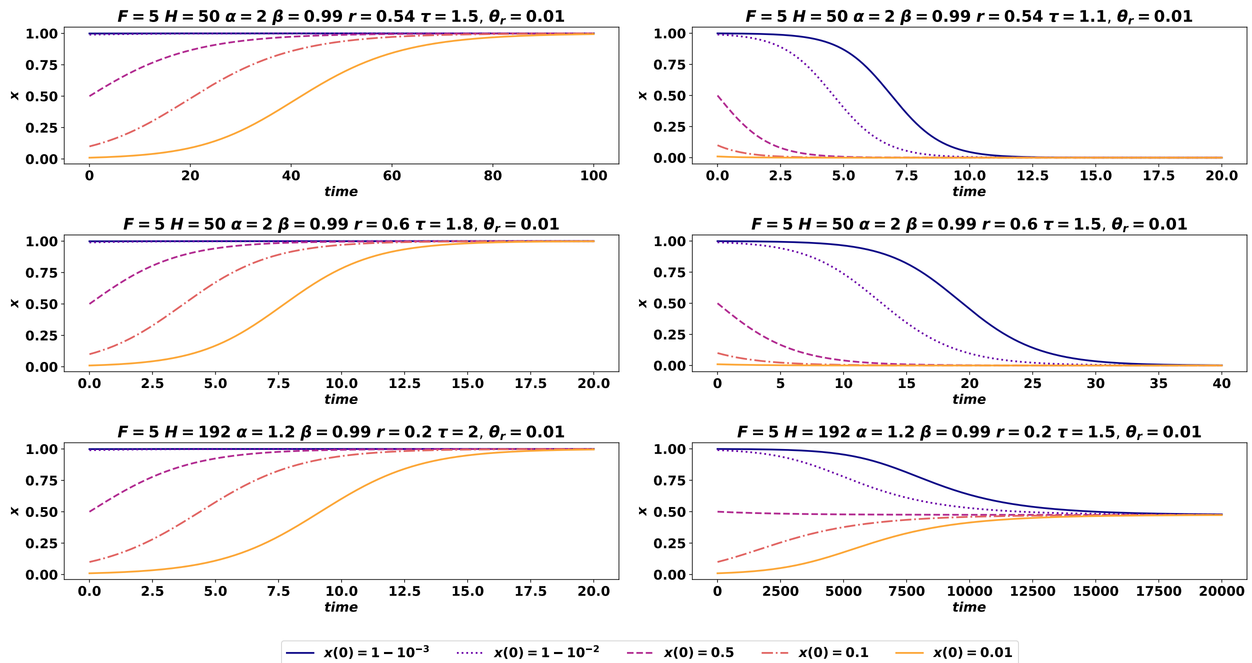

Figure 5: The change of the population over time with different starting populations. For $F = 5, H = 50, \alpha = 2, \beta = .99, \tau = 1.5, \theta_r = 0.01, \Gamma = 0$.

Two different outcomes seem to be evolutionary stable, for the higher values of r the security is low and the only way to obtain utility from poaching is to act indiscriminately. When r is lower, then there is less utility and sufficient full valued rhinos to ensure the risk of acting selectively is sufficiently low.

In Section 3, these observations will be confirmed theoretically.

3 Evolutionary Stability

By definition, for a strategy to be an ESS it must first be a best response to an environment where the entire population is playing the same strategy. In our model there are three possible stable distributions based on the equilibria of equation (4):

- all poachers are selective;
- all poachers are indiscriminate;
- mixed population of selective and indiscriminate poachers.

Each of the equilibria will be examined in the following subsections.

3.1 All poachers are selective

An ESS corresponds to asymptotic behaviour near the equilibria of (4), this correspond to the concept of Lyapunov stability [22].

Using the utility model described in Section 2, a population of selective poachers is unstable. For simplicity, denote the right hand side of (4) as f. In this setting, when x is near to some equilibria x^* so that $f(x^*) = 0$ then the evolutionary game can be linearized (using standard Taylor Series expansion) as:

For $s = 1$ to be a best response to itself the utility of behaving selectively in a population of selective poachers must be greater than the utility of a poacher behaving indiscriminately in a population of selective poachers,

$$\frac{d(x^* + \epsilon)}{dt} = J(x^*)\epsilon \tag{16}$$

$$u(1,1) > u(0,1),$$

where where:

$$\begin{aligned} u(1,1) =& H(1-r)\theta(r,1) - \alpha - F(\frac{1}{1-r})(1-r)^{\gamma}(1-r)^{\beta} \\ =& H(1-r)^{1-\alpha} - F(1-r)^{\beta+\gamma-1}, \end{aligned}$$

and

$$J(a) = \left.\frac{df}{dx}\right|_{x=a} \tag{17}$$

$$\begin{aligned} u(0,1) =& H(\theta_r r - r + 1)\theta(r,1) - \alpha - F(1-r)^{\gamma}(1-r)^{\beta} \\ =& H(\theta_r r + 1 - r)(1-r)^{-\alpha} - F(1-r)^{\beta+\gamma}. \end{aligned}$$

This gives a standard approach for evaluating equilibria of the underlying game. For a given equilibria x^*, $J(x^*) < 0$ if and only if x^* is an ESS.

Setting (??) to be greater than (??) gives the condition,

$$H\theta_r < -F(1-r)^{\gamma+\beta+\alpha-1}.$$

Using equations (12) and (13):

The right-hand side will always be negative for any r, on the other hand the left-hand side is always positive. Thus, (??) can never hold.

$$J(a) = \frac{1}{(r-1)\left(-ar\theta_r + r\theta_r - r + 1\right)^{\alpha+1}} \left(J_1 - J_2\right) + \Gamma(1-2a) \tag{18}$$

This implies that whilst [21] identified the individual poachers acting selectively as an equilibrium it is an extremely weak equilibrium from the point of view of population stability: a slight change and the population will change. where:

3.1 All poachers are indiscriminate

$$J_1 = F\left(-r+1\right)^{\beta}\left(-ar\theta_r + r\theta_r - r + 1\right)^{\alpha+1}\left(2art - 2ar - 2ar^{\lceil \frac{1}{\theta_r} \rceil}t - r\tau + r + r^{\lceil \frac{1}{\theta_r} \rceil}\tau\right)$$

$$J_2 = Ha\alpha r^2\theta_r^2\left(-a+1\right)\left(r-1\right) + Hr\theta_r\left(2a-1\right)\left(r-1\right)\left(ar\theta_r - r\theta_r + r - 1\right)$$

Theorem 1. *Using the utility model described in Section 2, a population of indiscriminate poachers is evolutionarily stable .* selective poachers is stable if and only if:

$$\tau > \frac{1}{1 - r^{\lceil \frac{1}{\theta_r} \rceil - 1}} \frac{F + H\theta_r(1-r)^{1-\alpha-\beta} - \frac{\Gamma}{r(1-r)^{1-\beta}}}{F} \tag{19}$$

Proof. In order for $s = 0$ to be an ESS it must remain the best response in a mutated population χ_ϵ, where $\chi_\epsilon = (x_\epsilon, 1-x_\epsilon)$. Following the same notation, the mutated population will be denoted as x_ϵ from herein. Thus,

$$u(0, x_\epsilon) > u(x_\epsilon, x_\epsilon),$$

must hold. From (15),

$$\begin{aligned} u(0, x_\epsilon) =& \quad H(\theta_r r - r + 1)\theta(r, x_\epsilon)^{-\alpha} - F(1 - r x_\epsilon)\gamma(1-r)\beta, \\ u(x_\epsilon, x_\epsilon) =& \quad H(\theta_r r(1-x_\epsilon) - r + 1)\theta(r, x_\epsilon)^{-\alpha} - F(1 - r x_\epsilon)\gamma(1-r)\beta\left(1 - x_\epsilon + \frac{x_\epsilon}{1-r}\right). \end{aligned}$$

Direct substitution gives:

Let the difference of (??) and (??) be denoted as,

$$J(1) = \frac{1}{(-r+1)^{\alpha+1}(r-1)}\left(F(-r+1)^{\beta}(-r+1)^{\alpha+1}\left(r\tau - r - r^{\lceil \frac{1}{\theta_r} \rceil}\tau\right) - Hr\theta_r (r-1)^2\right) - \Gamma$$
$$= \left(F(1-r)^{\beta-1}\left(r - \tau(r - r^{\lceil \frac{1}{\theta_r} \rceil})\right) + Hr\theta_r (1-r)^{-\alpha}\right) - \Gamma$$

$$\delta = u(0, x_\epsilon) - u(x_\epsilon, x_\epsilon),$$
$$= H\theta(r, x_\epsilon)^{-\alpha}\theta_r r x_\epsilon - F(1 - r x_\epsilon)\gamma(1-r)\beta x_\epsilon \left(\frac{-r}{1-r}\right).$$

All players being indiscriminate will be an ESS only if $\delta > 0$ for any small value of ϵ. Thus only if, The required condition is $J(1) < 0$:

$$H\theta(r, x_\epsilon)^{-\alpha}\theta_r r x_\epsilon > F(1 - r x_\epsilon)\gamma(1-r)\beta x_\epsilon \left(\frac{-r}{1-r}\right).$$

$$F(1-r)^{\beta-1} r + Hr\theta_r (1-r)^{-\alpha} - \Gamma < F(1-r)^{\beta-1}\tau(r - r^{\lceil \frac{1}{\theta_r} \rceil})$$
$$\frac{r}{r - r^{\lceil \frac{1}{\theta_r} \rceil}} \frac{F + H\theta_r (1-r)^{1-\beta-\alpha} - \frac{\Gamma}{r(1-r)^{1-\beta}}}{F} < \tau$$

The right-hand side of inequality (3.1) is always negative since $(\frac{-r}{1-r}) < 0$ for all r. On the contrary, the left-hand side is always positive for all r, thus the inequality always holds. Therefore, it is proven that all indiscriminate poachers is an evolutionary stable strategy. which gives the required result. □

This shows that a population of indiscriminate poachers not only corresponds to the equilibrium of the individual based game identified Note that the limit of the right hand side of equation (19) tends to infinity as $r \to 1^-$. This means that devaluing all rhinos is not a valid approach.

Furthermore, we see that the equilibria with poachers acting selectively, predicted in [21] , but is also a very robust and attractive equilibrium at the population level.

3.1 Mixed population of selective and indiscriminate poachers

Using the utility model described in section 2, a mixed stable population ($s = s^*$) never exists for $0 < r < 1$.

A mixed strategy $s = s^*$ is said to be stable for a given s^* only if,

$$u(1, s^*) = u(0, s^*).$$

236 From equation (15) the left-hand side is,

$$u(1, s^*) = H(1-r)\theta(r, s^*)^{-\alpha} - F\frac{1}{(1-r)}(1-rs^*)^\gamma(1-r)^\beta$$

237 and the right-hand side is

$$u(0, s^*) = H(\theta_r r + 1 - r)\theta(r, s^*)^{-\alpha} - F(1-rs^*)^\gamma(1-r)^\beta.$$

238 Substituting (**??**) and (**??**) into (**??**) gives can in fact be obtained in specific settings.

$$\begin{aligned} u(1, s^*) - u(0, s^*) &= H\theta(r, s^*)^{-\alpha}(-\theta_r r) + F(1-rs^*)^\gamma(1-r)^\beta(\frac{r}{1-r}) \\ &= -r(H\theta_r\theta(r, s^*)^{-\alpha} + F(1-rs^*)^\gamma(1-r)^{\beta-1}) = 0. \end{aligned}$$

239 Condition (**??**) can never hold for $0 < r < 1$ because both terms of the sum are positive. Thus a mixed stable equilibrium is never stable.
240 Note that similar theoretic results have been obtained about the evolutionary stability of indiscriminate poachers
241 but these have been omitted for the sake of clarity.

242 In this section we have analytically studied the stability of all the possible equilibria. We have proven the
243 instability of any population with selective poachers, and the evolutionary stability of the indiscriminate behaviourthat all potential
244 equilibria are possible. All of these theoretic results have been verified empirically, and the data for this has
245 been archived at [16].

246 Theorems 1, **??**, **??** are illustrated in Fig. **??** which shows the numerical solutions to (4) using (12) and (13) with $x_1 = x$. This is
247 done using numerical integration implemented in [12]. As evidenced from the scenarios considered, all populations converge to being all
248 indiscriminate.

249 The change of the population over time with different starting populations. For $F = 5, H = 50, \alpha = 2, \beta = 2, \gamma = 1, \theta_r = 0.6$.

250 Note that these results essentially follow by comparing equations (12) and (13), which show that $u_1(x) \leq u_2(x)$ for all x. Consider

the reverse scenario,

$$u_1(x) > u_2(x)$$
$$F\psi(r,x)\gamma(1-r)\beta - 1 < -H\theta(r,x) - \alpha\theta_r$$

which gives the required contradiction. This shows that the utility model used implies that given the choice of acting selectively or indiscriminately, in any given environment, it will always be a rational deviation to act indiscriminately.

In the following section a disincentive to acting indiscriminately will be introduced. This can be interpreted in many waysFigure 6 shows a number of scenarios:

- more severe punishment for indiscriminate killing of rhinos; Scenario 1: $F = 5$ $H = 50$ $r = 0.45$ $\alpha = 2$ $\beta = 0.99$, $\tau = 2$ $\theta_r = 0.05$, $\Gamma = 0$
- educational interventions that highlight the negative aspects of indiscriminate killing; Scenario 2: $F = 5$ $H = 50$ $r = 0.4$ $\alpha = 2.5$ $\beta = 0.99$, $\tau = 1.8$ $\theta_r = 0.05$, $\Gamma = 0$
- the possibility of a better alternative being offered to selective poachers. Scenario 3: $F = 5$ $H = 25$ $r = 0.45$ $\alpha = 2$ $\beta = 0.99$, $\tau = 2$ $\theta_r = 0.05$, $\Gamma = 0$
- Scenario 4: $F = 5$ $H = 25$ $r = 0.4$ $\alpha = 2.5$ $\beta = 0.99$, $\tau = 1.8$ $\theta_r = 0.05$, $\Gamma = 0$
- Scenario 5: $F = 5$ $H = 25$ $r = 0.99$ $\alpha = 2$ $\beta = 0.99$, $\tau = 2$ $\theta_r = 0.05$, $\Gamma = 4$
- Scenario 6: $F = 5$ $H = 25$ $r = 0.99$ $\alpha = 2.5$ $\beta = 0.99$, $\tau = 1.8$ $\theta_r = 0.05$, $\Gamma = 4$

4 Disincentive for indiscriminate behaviour

Let us consider the following modification of the utility to an indiscriminate poacher:

$$\tilde{u}_2(x) = u_2(x) - \Gamma$$

where $\Gamma > 0$ is some constant representing a disincentive only applied to indiscriminate poachers. This leads to the following modified form of (14):

$$\tilde{u}(s,x) = u(s,x) - (1-s)\Gamma.$$

This leads to the following theorem:

Using the modified utility model described a population of selective poachers is stable if and only if:

$$\theta_r H - F(1-r)^{\gamma+\beta+\alpha-1} < \frac{\Gamma(1-r)^{\alpha}}{r}.$$

Following a similar structure to that of Theorem 1,

$$\tilde{u}(1,1) > \tilde{u}(0,1),$$

where

$$\tilde{u}(1,1) = H(1-r)^{1-\alpha} - F(1-r)^{\beta+\gamma-1},$$

and

$$\tilde{u}(0,1) = H(\theta_r r + 1 - r)(1-r)^{-\alpha} - F(1-r)^{\beta+\gamma} - \Gamma.$$

Setting (**??**) to be greater than (**??**) gives the condition,

$$\theta_r H - F(1-r)^{\gamma+\beta+\alpha-1} < \frac{\Gamma(1-r)^{\alpha}}{r}.$$

This immediately leads to the following important remark:

Using the modified utility model described, if all rhinos have been dehorned a population of selective poachers is not stable.

All rhinos being dehorned implies that $r = 1$. Substituting this into (4) gives,

$$\theta_r H < 0,$$

which is not possible.

Theorem **??** is due to the fact that given two options, if all rhinos have been dehorned then all poachers will need to act indiscriminately to have any source of utility. Theorem **??** states that a mixed population can be stable with a disincentive, as opposed to Theorem **??** which states that a mixed stable strategy does not exist for $0 < r < 1$. This is illustrated in Fig. **??** where the evolutionary dynamics are represented for a number of scenarios .

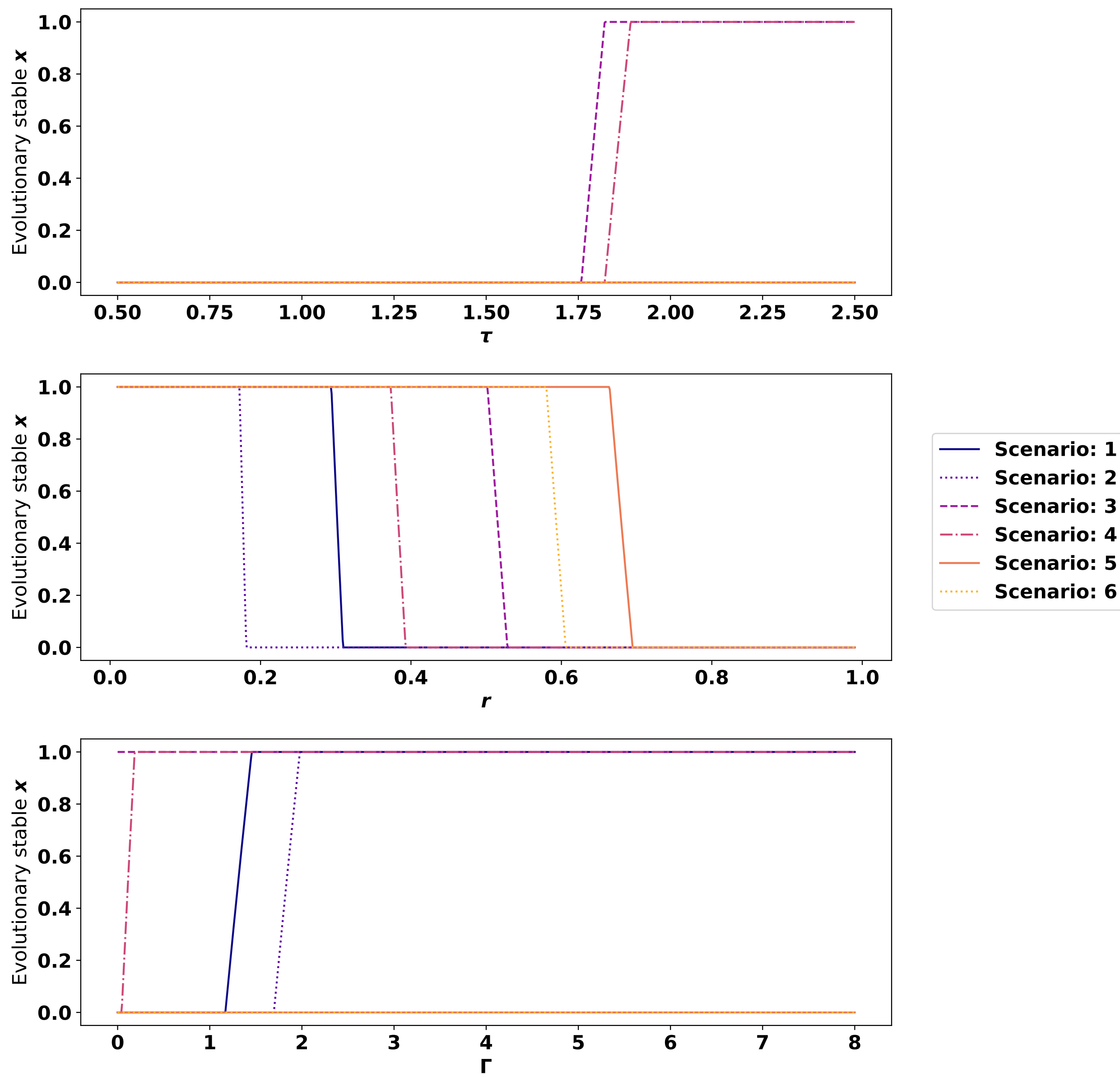

Figure 6: Evolutionary stable populations for varying values of τ, r, Γ for 6 difference scenarios.

Note that now the manager has control over the point of convergence by controlling r, as shown in Fig. 6 where a difference in the value from $r = 0.7$ to $r = 0.5$ changes the point of convergence from a mixed to a selective one. Moreover, the manager can also manipulate how fast the population convergences, as shown in the difference from $r = 0.5$ to $r = 0.2$. Fig **??** shows the equilibrium for x for a number of different values of r, and indicates that having a large These simulations confirm Theorem 1. A high value of r pushes poachers to behave indiscriminately. However all selective poachers can save at most, the r proportion of dehorned rhinos, so a low r is not ideal. As demonstrated in Fig **??**, a higher value of H also has a non-ideal effect on the poachers, as one would expect. Thus, high values of H and forces the population to become indiscriminate even with a high disincentive. Moreover, for all scenarios a value of r lead to more indiscriminate poachers. does exist for which a selective population will subsist.

The insights gained from the results are discussed in Section 4This confirms that devaluing alone is not a solution and in fact can potentially have averse consequences: combinations of devaluing and education (creating a disincentive) is needed.

The change of the population over time with different starting populations with a disincentive. For $F = 50, \alpha = 2, \beta = 2, \gamma = 1, \theta_r = 0.5, \Gamma = 300$.

The change of the convergence point with different values of r with different starting populations with a disincentive. For $F = 50, H = 150, \alpha = 2, \beta = 2, \gamma = 1, \theta_r = 0.5, \Gamma = 300$.

Effect of r for $F = 50, \beta = 2, \gamma = 1, \theta_r = 0.6$. Effect of H for $F = 50, r = 0.6, \beta = 2, \gamma = 1, \theta_r = 0.6$. Equilibrium behaviour with a disincentive.

4 Discussion and Conclusions

In this work the dynamics of a selective population were explored. It was proven that being selective is not a best response even in a population where everyone is behaving selectively. More specifically, even a mixed population with a small percentage of the population behaving selectively will not persist. Thus, a poacher would never adopt a selective strategy over an indiscriminate oneshown that given sufficient risk associated with killing a rhino it would be possible for a selective population of poachers to subsist.

Using a realistic and generic utility model it was found that the only strategy that was proven to be stable is the indiscriminate one. Moreover, it was proven to be evolutionary stable as well. Meaning, that for any given starting population, the poachers would evolve to adopt an indiscriminate behaviour.

Our results indicate however that it is possible for We have developed a game theoretic model which examines the specific question for rhino managers: how to deter poachers by devaluing horns? One of the main conclusions of the work presented here is that if there is sufficient risk associated with indiscriminate behaviour then a population of selective poachers to exist, but for this to occur a disincentive must be applied to the utility of indiscriminate poachers. Numerical results indicate that even in this case, the more rhinos which are dehorned, the less probability that a poacher will be selective. Assuming basic supply and demand arguments, the demand of a partial horn will increase by removing horns thus the probability of being selective decreases. Therefore, even in the scenario with a disincentive, the proportion of rhinos which could be saved is limited, so approaches which aim to reduce demand (as suggested in [4])would have more potential.

The disincentive factor can have several interpretations. According to [3], can be stable. The model also incorporates wider factors in a general manner such as a disincentive factor. The disincentive factor may be an increase in the monetary fine for poachers. In fact [10], who identify the most important contributors to the number of rhinos illegally killed in South Africa (between 1900 and 2013), found that increasing the monetary fine has a more significant effect than increasing the years in prison. However, the disincentive factor may also include wider influences, such as engaging the rural communities that neighbour or live with wildlife is the key to fighting the illegal trade of wildlife. Strengthening disincentives for illegal behaviour can be interpreted as the disincentive factor. Likewise, increasing incentives for wildlifestewardship, wildlife [4], or decreasing the cost of living with wildlife, and supporting a livelihood that is not related to poachingcan serve as incentives for selective behaviour. Zooming out further, it could include global issues such as an increase in ecotourism, which would provide a sustainable income for the community.

Another opportunity for wider factors, such as global issues, to be included in the model is via the supply and demand function. For example, [10] show that one of the three most important contributors to the number of rhinos illegally killed was the GDP in Far East Asia, where the demand for rhino horn is at its greatest. This finding supports [20] call for improved law enforcement and demand reduction in the Far East.

Note that the proportion of dehorned devalued rhinos r is continuous over $[0, 1]$ in the model. However, standard practice of a given park manager in almost all cases is to either dehorn devalue all the animals in a defined enclosed area, or none at all. This is thought to be because partial dehorning devaluing tends to disturb rhino social structures. The theoretic model presented here, whilst allowing forconsideration at the park level with park managers playing a mixed strategy, also can be considered at the macroeconomic level. Where r represents the quantity ofdehorned rhinos available across multiple parks.

The insights gained, notably that a population of selective poachers is sustainable only if a large disincentive is in place (even if all rhinos have been dehorned) have implication at the long term national policy levelOur results indicate that devaluing all rhinos will only decrease rhino poaching if potential poachers have a viable alternative (even in the case of a large disincentive).

The debate about the effectiveness of devaluation for preventing poachers and, is extensive and ongoing. This model answers one aspect of the topic, but larger questions remain. There are many drivers to account for, many of which are included in a systems dynamics model presented in [8] which captures the five most important factors: rhino abundance, rhino demand, a price model, an income model and a supply model. Using the optimal dehorning model of [25], the model [8] finds that poachers behaving indiscriminately will always prevail, which indicates that the risk associated with indiscriminate behaviour might not have been captured fully.

Following discussions with environmental specialists it is clear that dehorning devaluing is empirically thought to be one of the best responses to poaching. This indicates that whilst of theoretic and macroeconomic interest, the modelling approach investigated in this work has potential for further work. For example, a detailed study of two neighbouring parks with differing policies could be studied using a game theoretic model. Another interesting study would be to introduce a third strategy available to poachers: this would represent the possibility of not poaching (perhaps finding another source of income) and/or leaving the current environment to poach elsewhere. Finally, the specific rhino population could also be modelled using similar techniques and incorporated in the supply and demand model.

Authors' contributions

All authors conceived the ideas and designed the methodology. NG and VK developed the source code needed for the numerical experiments and generating the data. All authors contributed critically to the drafts and gave final approval for publication.

Acknowledgements

This work was performed using the computational facilities of the Advanced Research Computing @ Cardiff (ARCCA) Division, Cardiff University.

A variety of software libraries have been used in this work:

- The Scipy library for various algorithms [12].
- The Matplotlib library for visualisation [19].
- The SymPy library for symbolic mathematics [24].
- The Numpy library for data manipulation [33].

Finally, we would like to express our appreciation to David Roberts and Michael 't Sas-Rolfes for their valuable and constructive suggestions for this research. We would also like to thank Dr. Jonathan Gillard for his careful reading and double checking of the algebra in the paper. , and Anna Huzar for Figure 1.

Data Accessibility

The data generated for this work have been archived and are available online [16].

References

[1] Rhino Conservation - Save the Rhino @ONLINE, 2017.

[2] R. Amin, K. Thomas, R. H. Emslie, T. J. Foose, and N. Strien. An overview of the conservation status of and threats to rhinoceros species in the wild. International Zoo Yearbook, 40(1):96–117, 2006.

[3] D. Biggs, R. Cooney, D. Roe, H. T. Dublin, J. R. Allan, D. W. S. Challender, and D. Skinner. Developing a theory of change for a community-based response to illegal wildlife trade. Conservation Biology, 31(1):5–12, 2017.

[4] D. Biggs, F. Courchamp, R. Martin, and H. P. Possingham. Legal Trade of Africa's Rhino Horns. Science, 339(6123):1038–1039, 2013.

[5] Trevor A Branch, Aaron S Lobo, and Steven W Purcell. Opportunistic exploitation: an overlooked pathway to extinction. Trends in ecology & evolution, 28(7):409–413, 2013.

[6] Erwin H Bulte and G Cornelis Van Kooten. Economics of antipoaching enforcement and the ivory trade ban. American Journal of Agricultural Economics, 81(2):453–466, 1999.

[7] Franck Courchamp, Elena Angulo, Philippe Rivalan, Richard J Hall, Laetitia Signoret, Leigh Bull, and Yves Meinard. Rarity value and species extinction: the anthropogenic allee effect. PLoS biology, 4(12):e415, 2006.

[8] Douglas J Crookes. Trading on extinction: An open-access deterrence model for the south african abalone fishery. South African Journal of Science, 112(3-4):1–9, 2016.

[9] Douglas J Crookes and James N Blignaut. A categorisation and evaluation of rhino management policies. Development Southern Africa, 33(4):459–469, 2016.

[10] Enrico Di Minin, Jussi Laitila, Federico Montesino-Pouzols, Nigel Leader-Williams, Rob Slotow, Peter S Goodman, Anthony J Conway, and Atte Moilanen. Identification of policies for a sustainable legal trade in rhinoceros horn based on population projection and socioeconomic models. Conservation Biology, 29(2):545–555, 2015.

[11] R. Duffy, St John F. A., B. Buscher, and D. Brockington. The militarization of anti-poaching: undermining long term goals? Environmental Conservation, 42(4):345348, 2015.

[12] J. Eric, O. Travis, P. Pearu, et al. SciPy: Open source scientific tools for Python: v0.19.0, 2001–.

[13] S. M. Ferreira, M. Pfab, and M. Knight. Management strategies to curb rhino poaching: Alternative options using a cost-benefit approach. South African Journal of Science, 110(5-6):01–08, 2014.

[14] Yufang Gao, Kelly J Stoner, Andy TL Lee, and Susan G Clark. Rhino horn trade in china: An analysis of the art and antiques market. Biological Conservation, 201:343–347, 2016.

[15] V. Gill. Rhino horn GPS used to deter poachers. `http://news.bbc.co.uk/earth/hi/earth_news/newsid_9116000/9116106.stm`, 2010.

[16] Nikoleta E. Glynasti, Vincent Knight, and Tamsin E. Lee. Data for Evolutionary Game Theoretic Model of Rhino Poaching Manuscript. Available at: `https://doi.org/10.5281/zenodo.1408678`, September 2018.

[17] R. Harvey, C. Alden, and Y. S. Wu. Speculating a Fire Sale: Options for Chinese Authorities in Implementing a Domestic Ivory Trade Ban. Ecological Economics, 141:22 – 31, 2017.

[18] Matthew H Holden and Eve McDonald-Madden. High prices for rare species can drive large populations extinct: the anthropogenic allee effect revisited. Journal of theoretical biology, 429:170–180, 2017.

[19] J. D. Hunter. Matplotlib: A 2D graphics environment. Computing In Science & Engineering, 9(3):90–95, 2007.

[20] Katherine Lawson and Alex Vines. Global impacts of the illegal wildlife trade: The costs of crime, insecurity and institutional erosion. Chatham house, 2014.

[21] T. E. Lee and D. L. Roberts. Devaluing rhino horns as a theoretical game. Ecological Modelling, 337:73 – 78, 2016.

[22] Aleksandr Mikhailovich Lyapunov. The general problem of the stability of motion. International journal of control, 55(3):531–534, 1992.

[23] N. G. Mankiw. Macroeconomics. Worth Publishers, 2010.

[24] A. Meurer, C. P. Smith, M. Paprocki, O. Čertík, S. B. Kirpichev, M. Rocklin, A. Kumar, S. Ivanov, J. K. Moore, S. Singh, T. Rathnayake, S. Vig, B. E. Granger, R. P. Muller, F. Bonazzi, H. Gupta, S. Vats, F. Johansson, F. Pedregosa, M. J. Curry, A. R. Terrel, Š. Roučka, A. Saboo, I. Fernando, S. Kulal, R. Cimrman, and A. Scopatz. Sympy: symbolic computing in python. PeerJ Computer Science, 3, 2017.

[25] E. J. Milner-Gulland and N. Leader-Williams. A Model of Incentives for the Illegal Exploitation of Black Rhinos and Elephants: Poaching Pays in Luangwa Valley, Zambia. Journal of Applied Ecology, 29(2):388–401, 1992.

[26] EJ Milner-Gulland. How many to dehorn? a model for decision-making by rhino managers. In Animal Conservation forum, volume 2, pages 137–147. Cambridge University Press, 1999.

[27] M. A. Nowak. Evolutionary Dynamics: Exploring the Equations of Life, 2006.

[28] K. Nowell, W. L. Chyi, and C. J. Pei. The horns of a Dilemma: The market for rhino horn in Taiwan: Final Report for WWF Project II3637. 03, 1992.

[29] J. M. Smith and G. R. Price. The Logic of Animal Conflict. Nature, 246:15 – 18, 1973.

[30] R. J. Smith, D. L. Roberts, R. Duffy, and F. A. V. St John. New rhino conservation project in South Africa to understand landowner decision-making. Oryx, 47(3):323323, 2013.

[31] A. Taylor, D. Balfour, D. K. Brebner, R. Coetzee, H. Daview-Moster, P. A. Lindsey, J. Shaw, and M. Sas-Rolfes. Sustainable rhino horn production at the pointy end of the rhino horn trade debate. Biological Conservation, 216:60 – 68, 2017.

[32] G Cornelis van Kooten. Protecting the african elephant: a dynamic bioeconomic model of ivory trade. Biological Conservation, 141(8):2012–2022, 2008.

[33] S. Walt, S. C. Colbert, and G. Varoquaux. The NumPy array: a structure for efficient numerical computation. Computing in Science & Engineering, 13(2):22–30, 2011.

[34] G. L. Warchol, L. L. Zupan, and W. Clack. Transnational criminality: An analysis of the illegal wildlife market in Southern Africa. International Criminal Justice Review, 13(1):1–27, 2003.