Stability of defection, optimisation of strategies and the limits of memory in the Prisoner's Dilemma RESPONSE TO REVIEWS

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March 17, 2020

We would like to open this response by thanking the reviewers for their thoughtful comments and suggestions. We have fully taken their comments on board and made significant modifications to both the mathematical arguments and the narrative. We feel this has greatly improved the work.

Both reviewers commented on different aspects of the paper and so our modifications naturally fit in to two categories (which we will discuss in detail comment by comment):

- The first is an improvement of the theoretic results both in terms of their proofs but also the result themselves. For example, we omitted to include a detail about the derivative of quadratic form in one of our proofs and also had only given that quadratic form for the specific case of $(R, P, T, S) = (3, 0, 5, 1)$. The reviewer was very right to point out issues like this and we have addressed them all.
- The second reviewer questioned the general narrative of the paper as well as the evolutionary work. This was very helpful and we took time to reflect on the narrative of the paper. As a result of this we have changed the title to "A theory of mind: Best responses to memory-one strategies. The limitations of extortion and restricted memory" and reorganised some of the content. We have also clarified the nature of the evolutionary process we're discussing.

We will now take each comment of the reviewers in turn and highlight our efforts made to improve the work.

1 Response

Regarding the comments of Reviewer 1:

> ``Theorem 2: First, it is not clear how equation 17 is obtained.''

We have added further details on how to obtain the equation in the proof of Theorem 2 (in the current version Theorem 1).

> ``Theorem 2: Moreover, even if that's correct, it is only for a specific PD payoff matrix. One cannot state the general results in (4) from just a specific case. To obtain (4), authors need to establish it for the general PD payoff matrix.''

We agree with the comment of the reviewer. The utility was specific only for a given payoff matrix. We have generalised our result by considering the generalised payoffs (R, S, T, P) and feel that not only is this a more useful result and whilst we were worried that this closed form could be very messy it is in fact relatively elegant.

> ‘‘Theorem 3: The proof in the Appendix did not show how to obtain equations (22) - (24), the key results of the Theorem’’

We have added a sentence clarifying that this is obtained by taking the average over the opponents of the previously obtained expression for the utility.

> ‘‘Theorem 3: and what was stated in (5)’’

We have addressed this comment by specifying that it is an algebraic manipulation following equation (4).

Regarding the comments of Reviewer 2:

> ‘‘what exactly is the evolutionary process used, what are the parameters? This is not mentioned anywhere in the paper or appendix, and makes it then hard to evaluate the impact of the findings, or give any hope for reproducing the results. I realize that there is a repository containing raw data, but the lack of explanation of the setup really harms any expectation of the paper being self-contained. The issue of the evolutionary process not being explained also prevented me from fully following the argument the authors make for why best responses in the evolutionary process would be less extortionate. ’’

We agree with the comment of the reviewer. We are not running a evolutionary tournament and now understand that our wording was misleading. We study best response memory-one strategies that include self interactions which in a sense are approximating evolutionary stable best response strategies. We have reworded the analysis of best response memory-one strategies that include self interactions and it’s results.

> ‘‘Another related problem is that the authors don’t go into more detail about their SSE method. A more expansive explanation (also of Table 1) would be of critical importance, seeing that this is one of the central points of the results section.’’

We have included more details of the SSE method and a better discussion of the results based on Table 1. We have also included plots of the SSE distributions that accompany Table 1.

> ‘‘Another point I want to raise is how this work relates to the findings in Hilbe et al.: Partners and rivals in direct reciprocity (Nature Human Behaviour, 2018). That previous review article shows, among other results, the conditions under which "partner" (aiming for mutual cooperation, fighting back against defection) or "rival" (defecting/extortionate) strategies evolve in the IPD. It seems that the authors of the current work investigate a similar distinction (however, they should explain what exactly they mean with "adaptable", as it features as a contrast to "extortionate"), and thus would profit from discussing the Hilbe et al. article as well.
> ’’

We thank the reviewer for their article recommendation. The findings of Hilbe et al and how our results relate to them, are discussed in the "Results" section.

> ‘‘As a last point, I think that the flow of the paper could be improved. As it is now, the paper reads a bit like a list of results that are more or less loosely tied together (especially Section 1.2 suffers from this problem in comparison to the others). It would be good to motivate single sections in the broader context of the message the paper wants to tell. ’’

Thank you for this comment which encouraged us to reflect on the narrative of the paper, as a result we have modified the title of the manuscript and the titles of the sections. Moreover, the results of Section 1.2 have been to the Appendix, and the Theorem 1 (old version Theorem 2) has been moved in the main body.

> ‘‘The figures should be larger, such that the labels are easier to read’’

We have made this suggested change.

> ‘‘Please make a reference to the Appendix when you mention the Theorems, and maybe summarize their core statement in the main text, which would make the paper flow better.’’

We have made this suggested change.