

Institute of Cognitive Science



## **Bachelor thesis**

### **Adaptive K-Means Clustering of Data Sequences**

Clustering of video sequences with dynamic cluster count and recognition of their serial identity

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# Declaration of authorship

I hereby certify that the work presented here is, to the best of my knowledge and belief, original and the result of my own investigations, except as acknowledged, and has not been submitted, either in part or whole, for a degree at this or any other university.

Osnabrück, 23. Oktober 2016

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# 1<sup>1</sup>

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## Introduction

K-means clustering is one of the most often used clustering algorithms for image segmentation. Image segmentation is the basis for many image analysis techniques and thus highly values reliable results and computing time. While k-means clustering offers a great deal for the latter part, reliable results are sensitive towards the starting positions. The starting position is defined as the chosen initial cluster number and their respective position.

So far there exists no generally acknowledged optimal solution to compute the optimal number of clusters for any given data set a priori. One common bypass is to run the algorithm several times with different starting points. Especially for image segmentation however, many proposals have been made to create good starting points for clustering images[4].

The goal of this thesis is to adapt one of these algorithms to video sequences and improve it through the gained knowledge from continuous frames. The created clusters for each frame should then be compared to its predecessor, to determine whether clusters of previous frames persisted. Creating an algorithm to identify good starting positions for k-means clustering in video sequences can already be used for video compression on its own, but it could also be useful for many fields of video analysis as for example feature extraction and object recognition. If a cluster contains most visible features of an object, the identification of the serial identity of this particular clusters can especially be applicable in fields as object tracking.

The organization of this thesis is as follows. Chapter 2 provides some information on clustering and image segmentation fundamentals. It introduces k-means clustering and an algorithm to find good starting positions for k-means clustering in image segmentation.

Chapter 3 specifies the underlying task of this bachelor thesis and presents the developed procedures and algorithms.

In chapter 4 the results of the performance evaluation are presented successively on synthetic and real data. Thereby some necessary tweaks of the proposed algorithm are introduced and implemented.

Chapter 5 reviews the evaluation and concludes the thesis.

# 2<sup>2</sup> Theoretical basis

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## 2.1 Clustering

Clustering can broadly be described as classification of data into several groups. Clustering algorithms are unsupervised learning algorithms, which means they make no prior assumptions about the output components[5].

The vague objective is to find “*interesting* groupings of training samples”[6]. This separation into a number of groups, called clusters, happens according to a measure of similarity. The goal of the algorithm is to find clusters in a way, that samples within are more similar to each other than samples from different clusters and thus minimizing the prediction error. This is often summarized as the weighting of “intra-cluster” and “inter-cluster” distance. The first one describes the distinction from samples within a cluster, whereas the latter is the distance of one cluster to another one[7].

Hereby the similarity measurement reflects a priori knowledge, for example using standard Euclidean distance on a RGB color space equals the assumption that all values have equal importance. Nevertheless is the clustering process usually „ad hoc“; the underlying clustering process defines the relevant information itself. This makes clustering very generalizable[6].

Often clustering algorithms create prototypes. A prototype is a data object that is representative of all other objects in the cluster. In most cases this is simply the average of each data point within the cluster. Prototypes can be used in many ways, they ease the comparison of clusters among each other and can be used for data compression. Therefore they are often used as input for further data processing or data analysis techniques[1].

One of the most common usage of clustering algorithms is dimensionality reduction. The goal is to find a mapping from a  $d$ –dimensional input space  $R^d$  to some  $m$  dimensional output space  $R^m$ , where  $m < d$ [6].

$$G(x) : R^d \rightarrow R^m \quad (2.1)$$

A good mapping should also possess the inverse function.

$$F(z) = R^m \rightarrow R^d \quad (2.2)$$

Hence the reduced output can be decoded to the original input without losing too much information. Thus the main features can be kept, while the amount of data is reduced.

Clusters can either be described hierarchically (i.e. described in tree structures) or they can be purely partitional. The latter sort can be further divided into 2 groups. The first one is called „hard clustering“, each sample is definitely placed in only one cluster. The second one is called „soft clustering“ and in alternative each sample can belong to several clusters according to a related probability distribution.[8]

## 2.2 K-means clustering

K-means clustering is one of the oldest and most widely used clustering algorithms. It requires an explicit distance measure, an input of training samples and the number of classes  $k$ . It belongs to the class of hard clustering algorithms.[8]

The main concept is to represent each cluster by the vector of mean attribute values for numeric attributes or by a vector of the most frequent values for nominal attributes that are assigned to that cluster[9]. In contrast to numeric values, nominal attributes can not simply be ordered and have only a limited set of values. This kind of representation is also called cluster center and acts as a prototype.[1]

The operation of k-means clustering can be seen in algorithm 2.1. It starts with initializing random points as centroid of a cluster. This initialization process will be repeated until the desired number  $K$  of clusters is generated. The next step is to assign every training sample to the closest cluster center and update it by shifting it to the mean of all cluster members for each attribute. This step has to be repeated until it converges, due to bad initialization points this might take many iterations. Hence the usual implementation either uses a limit for the number of iterations or stops if the number of changes fall below a certain  $\epsilon$ .

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### Algorithm 2.1 Basic K-means algorithm

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- 1: select  $K$  points as initial centroids
  - 2: **repeat**
  - 3:     assign each data sample to the closest center
  - 4:     update the cluster centers by shifting them to the mean of each attribute
  - 5: **until** centroids moved no more than  $\epsilon$
-



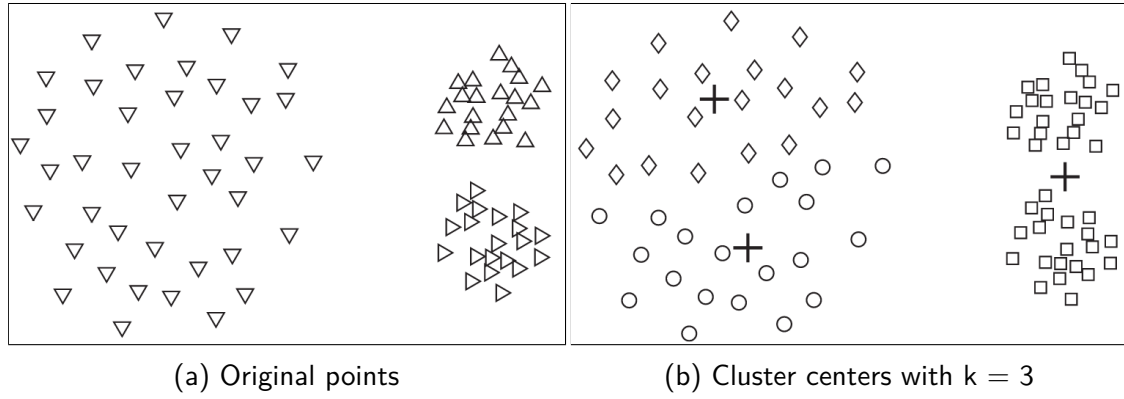


Figure 2.1: Example of k-means clustering on data with distinct groups[1]

The k-means algorithm aims to minimize an objective squared error function:

$$J = \sum_{j=1}^k \sum_{\forall i} |x_i^j - c_j|^2 \quad (2.3)$$

hereby  $c_j$  is the coordinate vector of the  $j$ th cluster,  $\{x_i^j\}$  are the points assigned to the  $j$ th cluster and  $|x_i^j - c_j|$  describes the distance[4]. It can be shown that the k-means algorithm converges to a local minimum[10].

The relevant distance measure can be chosen dependent on the task, but most of the times the normal Euclidean distance is used.

$$\text{Euclidean distance: } \frac{1}{I} \cdot \sum_i \left( \sqrt{x_i - c_i} \right)^2 \quad (2.4)$$

An advantage of k-means clustering is its low complexity but it has some weaknesses. K-means struggle with outliers, which highly influence the results. It can also only be used on data for which there is a centroid and it cannot handle non-globular clusters as seen in Figure 2.2. Finally it has the requirement of getting an accurate number of clusters as input a priori[11, 1].

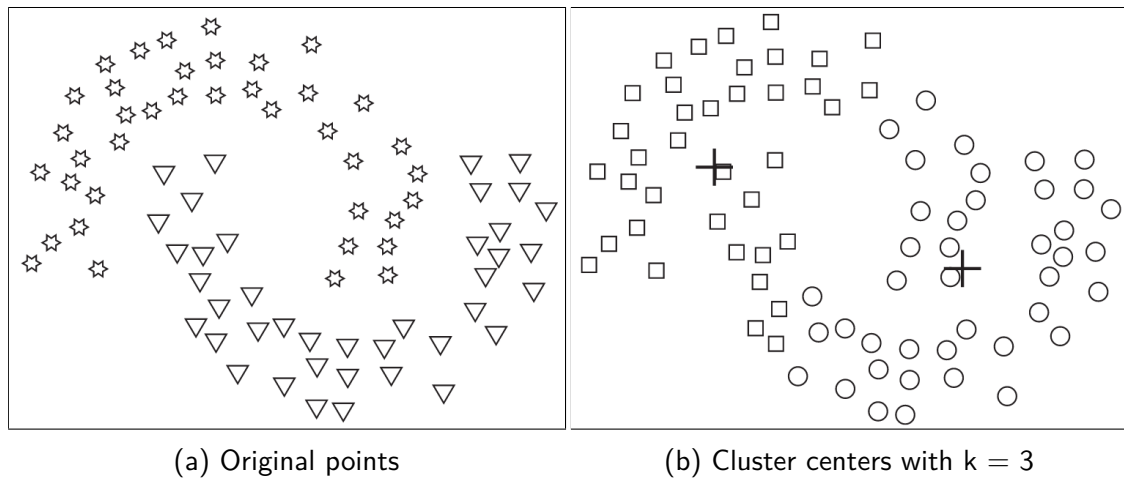


Figure 2.2: Example of k-means clustering on data with non-globular clusters[1]

## 2.3 Image color segmentation

One of the many applications for the k-means clustering algorithm is color segmentation. Image segmentation in general is the process of splitting/classifying an image into several parts, so that each region builds a homogenous segment. Combining any of the homogenous segments would result in a heterogenous segment[11] .

Segmentation algorithms are often the very first step in image processing. Subsequent steps as for example feature extraction and classification are highly reliant on the successful generation of homogenous parts. If the respective object can not be identified, it can certainly not be classified. The segmentation outcome will always be dependent on the underlying image and the object, or feature which should be identified. Therefore there is no single perfect clustering for an image, this complicates the evaluation process[2].

Segmentation algorithm can be divided into two main groups. The first one is the edge/boundary group; the associated algorithms aim to detect edges to find boundaries in between groups of pixels. The second group is region based and tries to order pixels according to their mutual similarity[2] .

Color image segmentation is usually residing in the second group. Combined with texture segmentation, it is mostly used in tasks concerned about content based retrieval[5]. Hereby images are organised/searched according to their pictured content.

Clustering algorithms share the same problem as region based segmentation and can thus be used for color segmentation. The problem of using k-means clustering for image segmentation is once again the requirement to assing the number of clusters beforehand. If a good number of clusters is known however it is optimal in minimizing the average distortion[12].

## 2.4 Finding the optimal numbers of clusters for k-means clustering

Although the k-means clustering is relatively easy to implement on a relative wide field, it has some drawbacks. The final results of the clustering algorithm vary depending on the number of clusters and especially on their initialization. So if the initial clusters are chosen randomly, it will lead to different results[13].

In order to find the optimal number of clusters and initialize them automatically Siddheswar Ray and Rose H. Turi proposed an algorithm which uses “a simple validity measure based on the intra-cluster and inter-cluster distance measure”[7].

The basic k-means algorithm minimizes the sum of squared distanced from all points to their cluster centers, hence the distances from each point to their cluster centers provides the information whether the clusters are compact. The averages of all these distances constitute the intra-cluster measurement.

$$\text{intra} = \frac{1}{N} \sum_{i=1}^K \|x - z_i\|^2 \quad (2.5)$$

where  $N$  is the number of all pixels,  $K$  is the cluster count and  $z_i$  is the actual cluster center. To be as compact as possible, we obviously want to minimize this measurement. The inter-cluster measurement in contrast describes the distances between the cluster centers and should be maximized.

$$\text{inter} = \min \left( \|z_i - z_j\|^2 \right), i = 1, 2, \dots, K - 1, j = i + 1, \dots, K \quad (2.6)$$

Only the minimum of each distance is considered, because we always want to maximize the smallest distances. The maximized values of larger values will be automatically larger as well.

In an optimal clustering we obviously want to have compact clusters, which are as distant as possible. Thus the validity measurement is defined as the ratio of the intra and inter measure.

$$\text{validity} = \frac{\text{intra}}{\text{inter}} \quad (2.7)$$

The optimal cluster count should posses the smallest validity.

The proposed algorithm of Siddheswar Ray and Rose H. Turi can be seen in algorithm 2.2. It starts with initialising every single data point into one cluster and using the average of each data point as cluster center (line 1). Repeatedly the variance of each data point will be calculated and added up, the sum of all points will be used as divisor

(line 3). The cluster with the largest value will be split (line 4). Due to the occurring minimization of intra-cluster distance in k-means clustering, this cluster would most likely be split when increasing the cluster count. Before the process will be repeated, the normal k-means clustering is executed and the validity calculated according to the new calculated cluster centers (line 5). This processes is repeated until a self-assigned number of clusters  $k_{\max}$  is reached. From this follows that the basic k-means algorithm will be run  $k_{\max}$  times.

The clusters will be split, according to the old cluster center and with regard to the minimum and maximum of each attribute.

$$z'_i = (z_{i1} - a_1, z_{i2} - a_2, z_{i3} - a_3) \quad (2.8)$$

$$z''_i = (z_{i1} + a_1, z_{i2} + a_2, z_{i3} + a_3) \quad (2.9)$$

With  $z'_i$  and  $z''_i$  being the two new cluster centers,  $z_i$  being the the old cluster centers with index 1, 2, 3 for each attribute.

$$a_j = \frac{z_{ij} - \max_j}{2} \quad (2.10)$$

Where  $\max_j$  is the maximum value for the  $j$ -component.

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**Algorithm 2.2** Find the optimal cluster count algorithm

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- 1: Initialise a single cluster, with the average of each attribute as cluster center
  - 2: **repeat**
  - 3:     calculate the variance of each attribute  $\sigma_i$  and add them up  $\sum_{i=1}^I \frac{1}{I} \sigma_i$
  - 4:     split the cluster centers with the biggest variance
  - 5:     use the standard k-means algorithm and calculate the validity
  - 6: **until** a self-assinged limit  $k_{\max}$  is reached
- 

Due to the larger inter-cluster distance, small cluster numbers tend to be selected more often. In synthetic images, with an actual perfect cluster count the calculated minimal validity will be the optimal cluster count. In natural images the best number of clusters is usually preferred. To overcome the effect of a large inter-cluster distance with a small  $k$ , the first local maximum in the validity measure needs to be found.

$$\text{val}(k-1) < \text{val}(k) > \text{val}(k+1) \quad (2.11)$$

The adjusted best validity measure will be the smallest validity in between the range of  $\text{val}(k)$  and  $\text{val}(k_{\max})$ .

It is important to notice that this algorithm is supposed for image segmentation in RGB-color space, although it can also be implemented on a different feature space as long as the attributes can be scaled properly.

## 2.5 Color spaces

A color space is a specific combination of color channels which forms a reproducible representation of colors in both digital and analog representations. An image is only a spatially organized set of numbers, where pixels describe the particular combination of color channels at a specific position. A gray image for example is a 2d-array limited to a single color channel. The most used color space is the RGB-color space. It is based on the tristimulus theory of color in the human visual system. Its name describes the 3 used color channels; red, green and blue. This is similar to the three types of cones in the human eye which absorb the respective electromagnetic wavelengths. In normal 24-bit images, each channel is usually scaled to 0–255 for the common 1 byte per colour channel. Therefore it is important to notice is that the color channels overlap. A perceived blue item will have the highest peak in the blue channel, but it will also have milder components of red and green[2, 5].

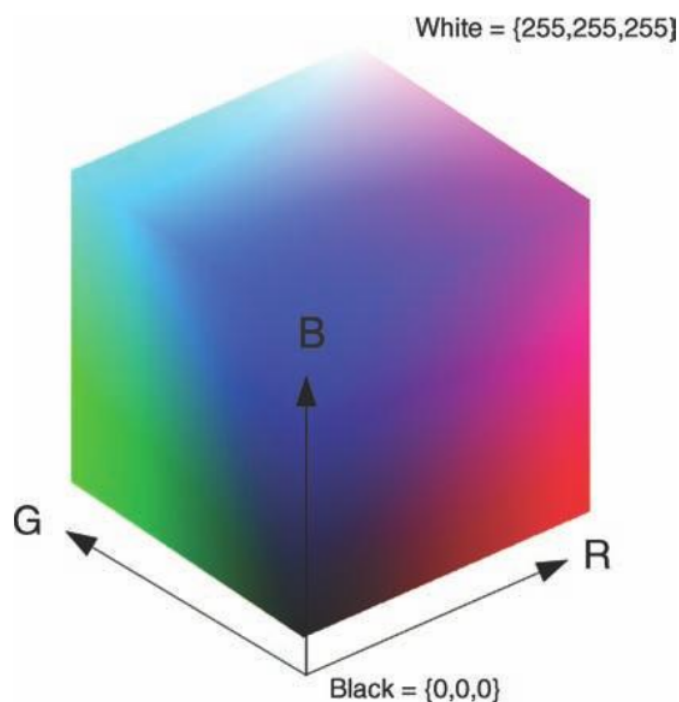


Figure 2.3: A representation of the 3d RGB cube[2]

The main problem with the RGB-colorspace in image processing is its non-linearity. Moving in a certain way in the cube might not produce a color that is perceptually consistent with the change in each of the channels.

The HSV-color space is closer to the human idea of color, because it is easier to think of a color in terms of H(ue), S(aturation) and V(alue). The value determines the brightness. The color space can be represented in a 3d-hexagone, where the central vertical axis represents the Intensity. Hue is defined as an angle in the range  $[0, 2\pi]$  and Saturation is the depth or purity of the color, it is measured as a radial distance from the central axis[14].

The HSV-color space is advantageous in comparison to the RGB-color space in color based image segmentation, because image objects are more consistently contained in the resulting hue field than in the channels of the RGB representation. This applies in particular to changing light conditions[2].

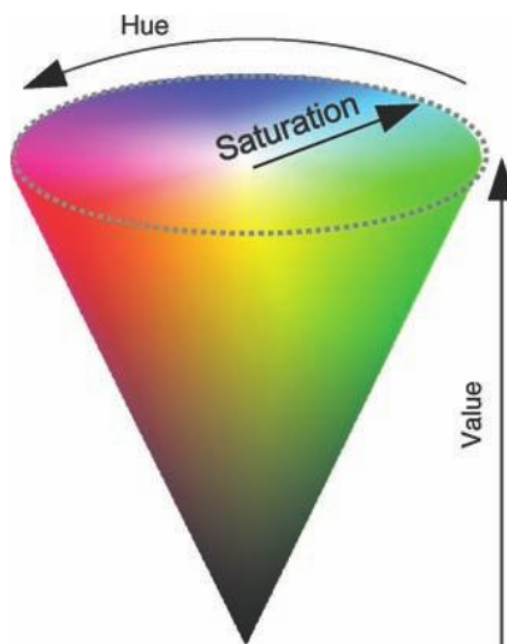


Figure 2.4: A representation of the HSV colorspace cone[2]

Both RGB and HSV spaces are however, not perceptually uniform; the distances among colors perceived by the human are not mirrored by similar distances between the points representing those colors in such spaces[5].

# 3<sup>3</sup>

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## Adaptive K-means clustering

One of the logical extensions of the k-means algorithm used for color segmentation seems to implement the algorithm not only on images, but on whole video sequences. The existing algorithms for determining the optimal cluster count could be transferred on whole videos, by simply processing them frame by frame. Usually the resulting processing time would prove unfit for most practical use if the algorithm is applied on each frame of the video. While the regular k-means computing time is feasible, at least if an iteration limit is implemented, the computing time for dynamic cluster count identification as discussed in 2.4 is tremendous, due to the repeated use of the basic k-means algorithm.

Hence, the first task of this bachelor thesis is to find a way to adapt the identification of the optimal cluster number from images to video sequences.

Without further investigation the resulting cluster have only a limited application use and can only represent the gathered knowledge of a single frame. In order to include the information of the whole video it is necessary to compare the resulting clusters of each frame. A continuous occurring in several frames should eventually be recognized.

Thus, the second task of this bachelor thesis is to implement a similarity measurement for clusters to merge them over several frames. In order to distinguish the clusters which are calculated using the k-means algorithm in a single image and the clusters which are generated through merging clusters of a video sequence, the latter kind will be referred to as “super-clusters”.

### 3.1 Dynamic cluster count algorithm in video sequences

Due to color segmentation in video sequences and images being essentially the same problem, the proposed algorithm by Siddheswar Ray and Rose H. Turi, would also work in videos[7]. Due to the assumption that we are only looking at continuous video sequences, calculating the optimal cluster count for each frame individually, without including any knowledge gained from the previous calculations, would result in redundant calculations for most videos. Thus the aim is to reduce the calculation time, while

maintaining the same results algorithm 2.2 would produce if applied on each frame individually.

Often the optimal cluster count will be unchanged from one frame to the subsequent one. The resulting cluster will only vary slightly, either due to an occurred movement or due to a change of the lightning condition. Hence it seems reasonable to look for an effective way to include the gained knowledge of the previous frames. One option would be to check whether cluster centers of the previous frame could be kept as initial center for the k-means algorithm of the current frame.

As described in 2.3 the measure to evaluate the number of clusters is the validity, its optimal value always falls on a local minimum in comparison to a higher/lower cluster count. Which means that if the cluster centers of the previous frame still constitute a local minima in the current frame it is likely that the optimal cluster count is unchanged.

Unfortunately there are two major problems with this conclusion. Even if the old cluster centers constitute a local minimum on the current frame it is not guaranteed that it is the optimal solution. As described in 2.3 the optimal cluster count for a synthetic image seems to be the global minima and in case of a real image it would be the smallest validity after the first local maxima. Neither for synthetic images, nor for natural images can be checked whether the current minimum constitutes the optimum without calculating the validity for each  $k$ .

Consider for example the validity chart in Fig 3.1. Searching for the minimal validity would result in only 2 clusters. As discussed in 2.3 the best validity in natural images would be at  $k = 8$ , because it is the minimal value after the first local maximum. But if the previous frame would have had 13 clusters, the adapted algorithm would continue to work with those clusters, because  $k = 13$  constitutes a local minimum.

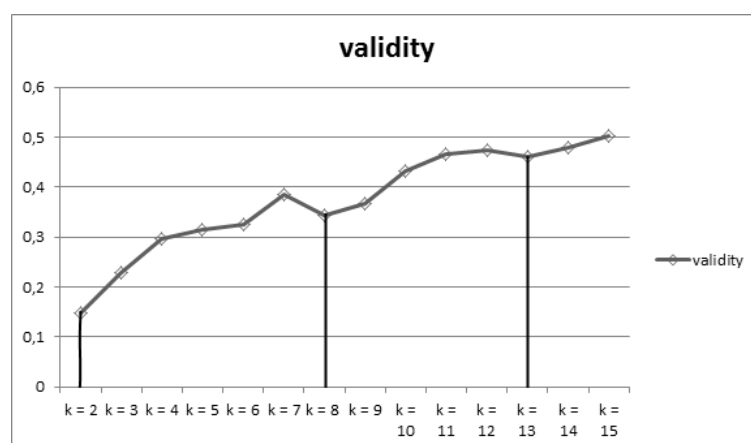


Figure 3.1: Example of a validity chart

Even if the optimal number of clusters in a frame would equal the used number of clusters in a former one, the old cluster centers might be unfit as initialization points



for k-means clustering in the current frame. To get a good initialization point, the algorithm of Siddheswar Ray and Rose H. Turi puts the whole image into a single cluster at first. The mean of each pixel builds its cluster center. Continually the cluster with the highest variance is split into two parts. Thus using the old cluster centers as initialization points for k-means clustering implies the assumption, that each time a cluster had the highest variance in the previous frame, the same cluster would have the highest variance in the current frame as well.

Despite the occurring problems, using the local minima assumption seems to be a good indicator to either keep the old cluster centers as initialization points or start all over. The stated problems are expected to occur rarely, due to mostly small occurring changes when transitioning from one frame to another. The impact of identifying the wrong cluster count in a video sequence will be further discussed in the evaluation of this algorithm.<sup>1</sup>

At the beginning of this algorithm, the normal algorithm 2.2 will be used on the first frame to get a good starting point to cluster the remaining frames. In each iteration of algorithm 2.2 the cluster centers for the current  $k$  have to be saved (line 2). This is necessary in order to check whether the old cluster centers still constitutes a local minima in the validity chart on the next frame. First the validity of the best cluster centers in the first frame has to be calculated on the current frame (line 4). As in all following steps as well, the adapted cluster centers for  $k$  have to be stored (line 5). To check whether this validity constitutes a local minimum, the validity for  $k+1$  and  $k-1$  is needed. To get this for  $k+1$  the same process as previously described can be used and the cluster with the highest variance will be split (line 6). Afterwards the basic k-means algorithm is applied and the validity is calculated (line 7).<sup>2</sup> Unfortunately it is not possible to simply remerge two clusters, because there is no similar indicator as the variance for splitting, which defines the clusters who have previously belonged together. Instead for getting the validity for  $k-1$  the corresponding cluster centers of the previous cycle has to be used as initialization point for k-means clustering (line 9), although this cluster centers have been generated using a previous frame. Therefore it is important to save all calculated cluster centers, because they might be used later. After running k-means clustering and calculating the variance it can finally be checked, if the validity for  $k$  constitutes a minimum (line 12).

If this is the case the described steps can simply be repeated on the next frame. Otherwise the old cluster center seem to be unoptimal and thus the whole algorithm 2.2 will be repeated on the current frame (line 15).

---

<sup>1</sup>see 4.4 implementation in a video sequence

<sup>2</sup>see 2.3 Optimal cluster count

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**Algorithm 3.1** The adapted algorithm to find the optimal cluster count in video sequences

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```
1: Split the video sequence into frames
2: Use algorithm 2.2 to find the optimal k for the first frame and save the cluster
   centers for each number of clusters
3: repeat for each frame
4:   Use standard k-means with existing cluster centers of k and calculate the validity
5:   Save the new cluster centers for k
6:   Split the cluster with the highest variance
7:   Use the k-means algorithm and calculate the validity for k+1
8:   Save the new cluster centers for k+1
9:   Get the cluster centers of k-1 of the previously stored iterations
10:  Use the k-means algorithm and calculate validity for k-1
11:  Save the new cluster centers for k-1
12:  if validity of k-1 > validity of k <= validity of k+1 then
13:    continue
14:  else
15:    Use algorithm 2.2 and save the cluster centers for each number of clusters
16:  end if
17: until all frames are processed
18: Group the clusters into super-clusters according to a similarity measurement
```

---

This algorithm will save computational time in cost of accuracy. In some special cases it will provide non optimal solutions, this will further be outlined in chapter 4.

## 3.2 Similarity measurement of clusters occurring over several frames

In order to find a suitable similarity measurement for comparing clusters, it is important to keep track of the goal. The fusion of several similar clusters into one super-cluster should finally provide information of the whole video. The basic k-means color segmentation algorithm as described in 2.2 is repeatedly used in 2.3, would ultimately provide only minimal information via the cluster centers.

For example, if only the Euclidean distance of each cluster center would be compared and the most similar ones would be merged, the gained information could be seen as color histogram of the whole video sequence. One improvement step is not only to include the color spaces, but also the size to compare clusters. Thus clusters which share a color with another cluster center but are ultimately unconnected, might already be separated and don't share the same super-cluster.

Without including the coordinates in any way, clusters cannot include any information about shape and may contain disjointed areas. For example in Figure 3.2 the optimal

cluster count so far would be determined as 2, because it only consists of 2 colors and a pure color segmentation algorithm is applied. One cluster contains the black background and the second one the red rectangles. To split clusters not only according to their color, but also according to their geometrical distinctness, the coordinates of each pixel need to be considered when calculating the clusters. If the geometrical distinctness is considered, the optimal clustercount in Figure 3.2 changes from 2 to 3, because the rectangles are not connected.

There are several ways to include geometrical features, but it would be optimal if reasonable results could be achieved with the k-means clustering algorithm itself. The basic idea hereby is to extend the feature space from a 3d-vector which includes the color space, to a 5d-vector which also includes the XY-coordinates.

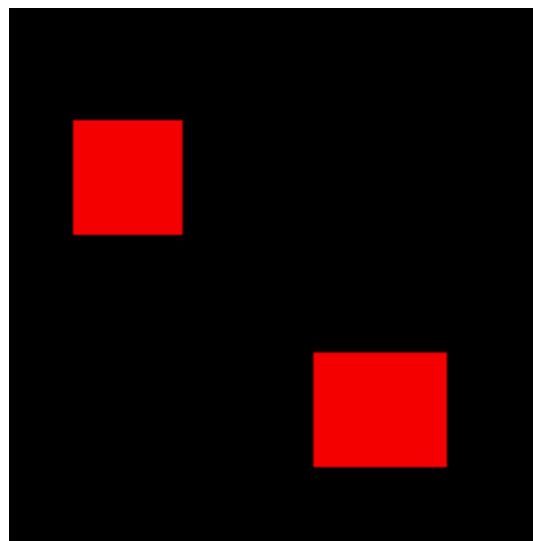


Figure 3.2: Example of an image with optimal cluster count of 2 on pure color segmentation and optimal cluster count of 3 when including geometrical features

A different approach to include geometrical knowledge is to detect shapes in the already separated clusters, created by the k-means color segmentation algorithm. The OpenCV library[15] already implements the necessary functions for doing so. First a canny algorithm will detect the edges of the respective cluster, before the openCV method “findContours(…)” will finally try to calculate the corresponding contours. Contours can be seen as boundaries, and in contrast to edges they form closed curvatures. If there are several contours convoluted in each other as for example seen in Figure 3.3, only the most outer one is relevant and will represent a cluster. In the case of the image in Figure 3.3 the only relevant contours are labeled from 0-2, because the remaining contours are all enclosed in contour 2.

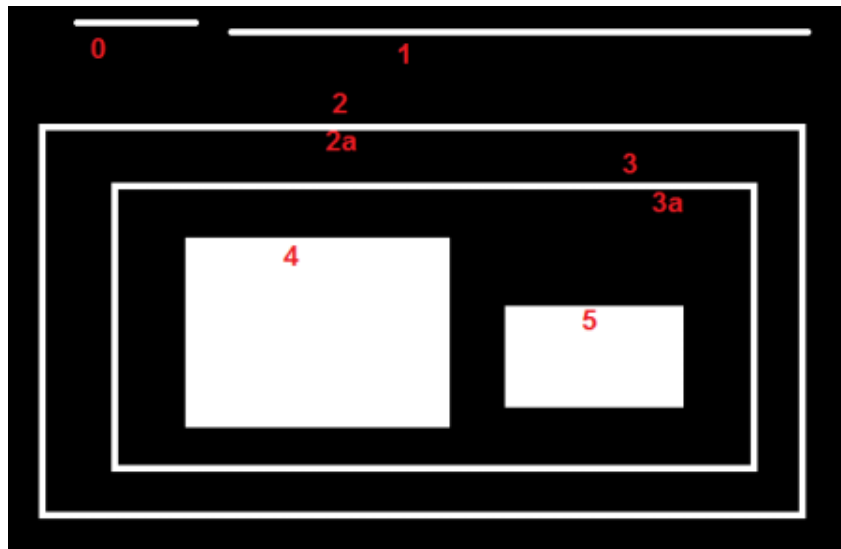


Figure 3.3: Example for hierachy in edge detection[3]

# 4<sup>4</sup> Results and Evaluation

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## 4.1 Evaluation of the optimal cluster count

### 4.1.1 On synthetic images

The results reported from Siddheswar Ray and Rose H. Turi [7] on synthetic images described in 2.3, could mostly be verified. On very simple images as seen in Fig 4.1 the automatically specified cluster count matches the actual used colors. Notice that this section is only concerned about color segmentation.

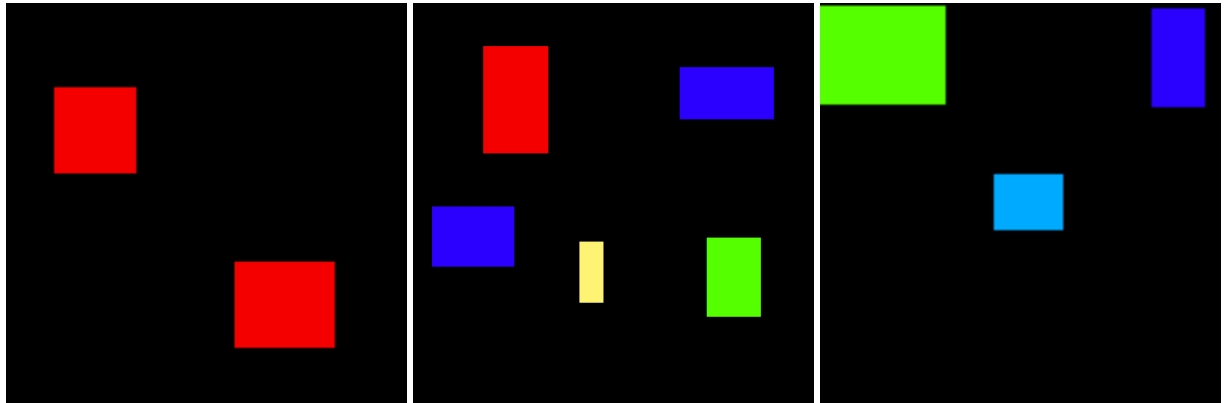


Figure 4.1: Simple generated images with known used number of colours (2, 5, 4)

Surprisingly the calculated optimal cluster count using the RGB color space and a maximal cluster count of 25 in Figure 4.2 was determined as 10 instead of the expected 5 clusters. This error occurs due to the different numbers of colors used in differing shapes. Each rectangle consists of only of one specific color, whereas the circle can be distinguished into several RGB-values. The core of the circle and everything in close distance has only one color value, but the closer the distance to the boundary the more the color changes in order to achieve an effect of anti-aliasing, this can be seen in Figure 4.3. Anti-aliasing makes the rectangle shape of pixels on a computer display less visible for the human eye. In this specific case the circle consists of 49 different RGB values. Due to the relationship of inter cluster distance and intra cluster distance the circle is split into 5 different clusters. The conversion into HSV did not result in a calculated cluster count of 5 either; instead it went up to 13 (keeping the Euclidean

distance as distance measurement). As described in 2.4 the different nature of the RGB color space and HSV color space results in different distances in between colors. Many of the 49 RGB colors will probably share the same hue value, but can be further clustered according to the saturation and brightness. Therefore this results in different clusters in comparison to the RGB color space and with it to a rise of the calculated optimal cluster count.

Given that the used colors in a natural images are expected to be a lot more than 53, the performed clustering will never be on a comparable fine level. The decrease of the intra cluster distance would lead to a lower inter cluster distance and supposedly the validity would increase.

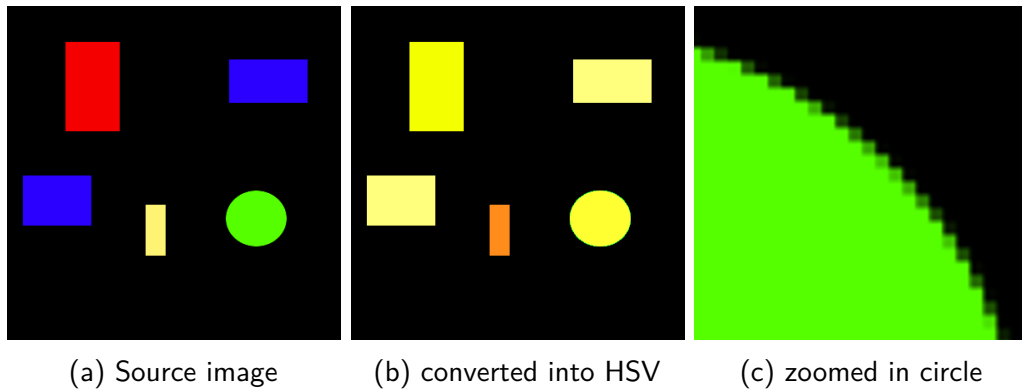


Figure 4.2: Simple generated image including a circle converted into HSV

Increasing the number of circles and adding more colors overall to testing images as for example seen in Figure 4.3, resulted in a correct computation of the cluster count in RGB color space. The particular image is built of 4564 RGB values and the right cluster count of 10 is calculated. Transferring this image into the HSV color space resulted in the computation of 10 as well.

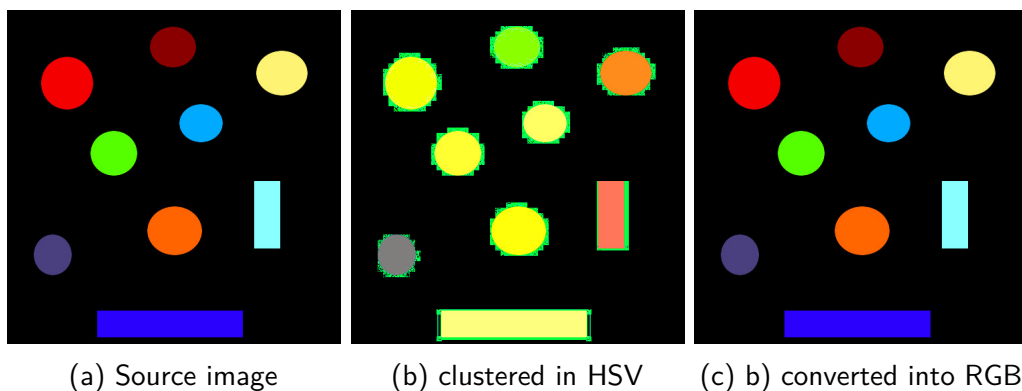


Figure 4.3: Simple generated image build of 4564 RGB values converted into HSV

### 4.1.2 On real images

Until now the cluster count was only tested on self-generated images with known optimal cluster count. On real images this is usually only roughly estimable.

The algorithm was tested clustering images of the Berkeley Segmentation Dataset[16] and provided satisfactory results. To evaluate the dynamic cluster count the clustered images, were compared to the images generated with a higher/lower number of clusters. These are also generated with algorithm 2.2. It increments the number of clusters succesively until  $k_{\max}$  is reached, hence the images in the range from  $k = 2$  to  $k = k_{\max}$  can be used for comparison purposes.

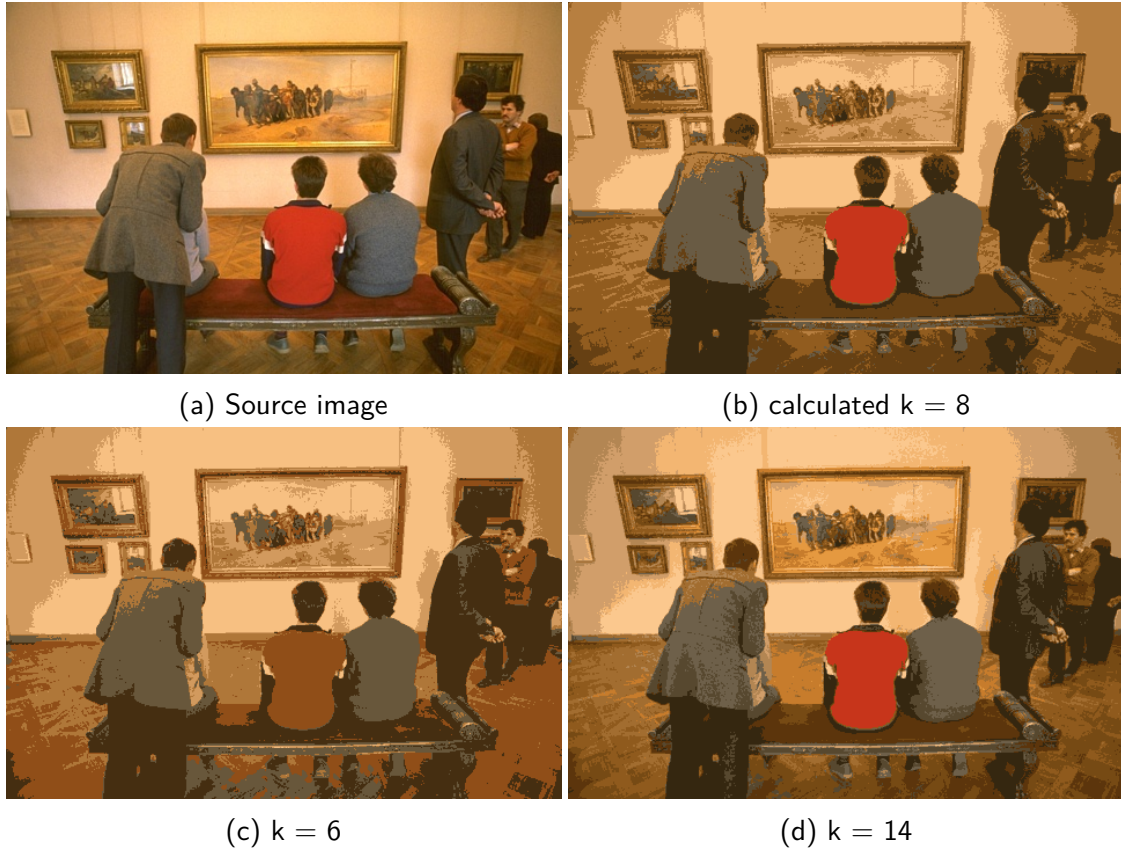


Figure 4.4: Gallery example from the Berkeley Database clustered in RGB-color space

The calculated cluster count of 8 in Figure 4.4 is the first cluster count where, the red shirt builds its own cluster. It seems as in a smaller cluster counts the inter cluster distance is bigger. In  $k > 8$  the intra cluster distance seems to be minimized but has only minimal visible impact, because of the higher inter cluster distance before. Thus  $k = 8$  seem to be the right choice.



(a) Source image



(b) calculated  $k = 4$



(c)  $k = 5$



(d)  $k = 10$

Figure 4.5: Snow shoes example from the Berkeley Database clustered in RGB-color space



The calculated cluster count of 4 in Figure 4.5 seems to retain most key features of the image, the results seen in c) for  $k=3$  seem to be similarly well. The cluster count of 10 seen in d) might be preferable due to the existence of a cluster solely for the snow shoes. The latter impression probably arises of our focus on the image center; but looking at the sky reveals that it has already been segmented into 3 sections at this point.

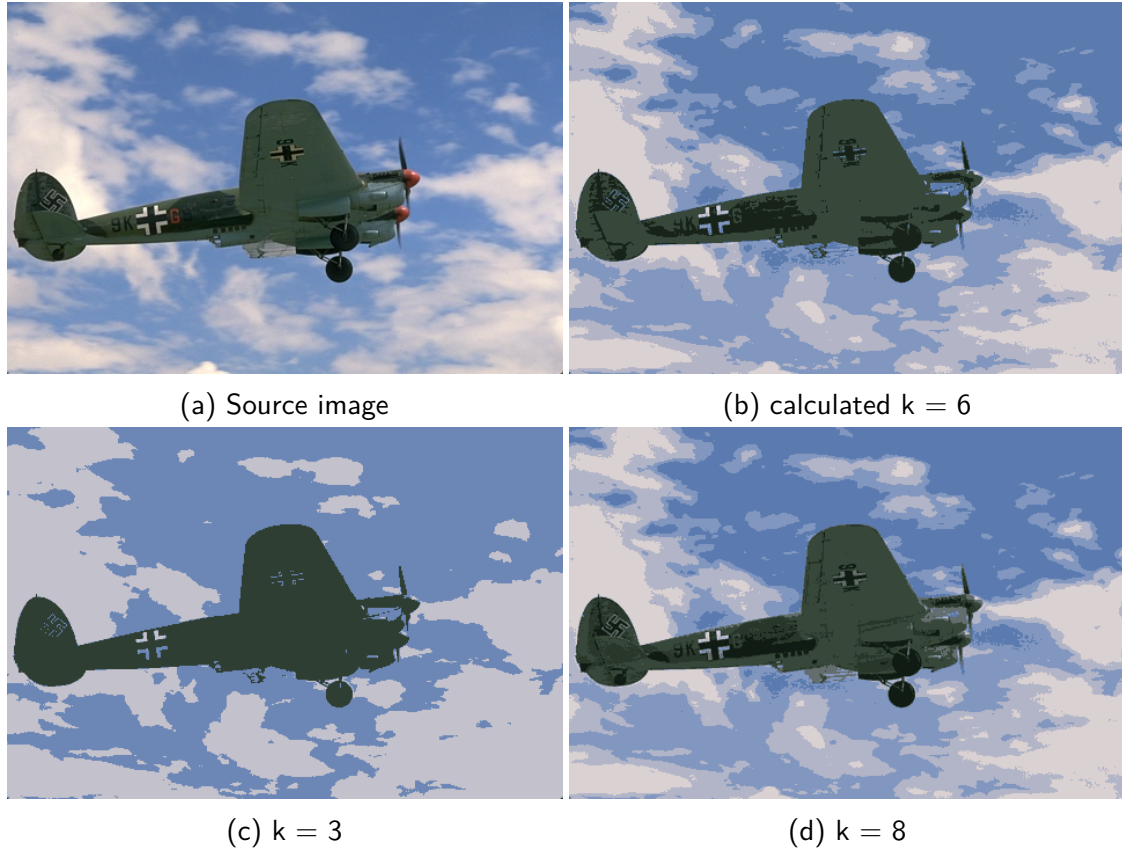


Figure 4.6: Airplane example from the Berkeley Database clustered in RGB-color space

The calculated optimal cluster count of 6 in Figure 4.6 seems to be the most adequate cluster number, although  $k=3$  seems to highlight the plane better. However, k-means clustering is an unsupervised algorithm without prior knowledge, hence the cluster count of 6 retains more information of the background as well.

Using the HSV-color space instead of the RGB-color space did not lead to considerably better results. Strikingly the calculated optimal cluster count was mostly higher in HSV as for example seen in Figure 4.7. Otherwise the same findings as in the RGB-color space could be made. Overall the comparison did not favor any color space. The results were subjectively examined on the same level. The determination of which color space to use, should depend on the final task of the clustering algorithm. Because the following work is rather on an abstract level than on implementing the algorithm on a particular task and because the underlying algorithm for determining the cluster

count was originally proposed for the RGB color space, the latter one will be used in the evaluation of the dynamic cluster count on video sequences.

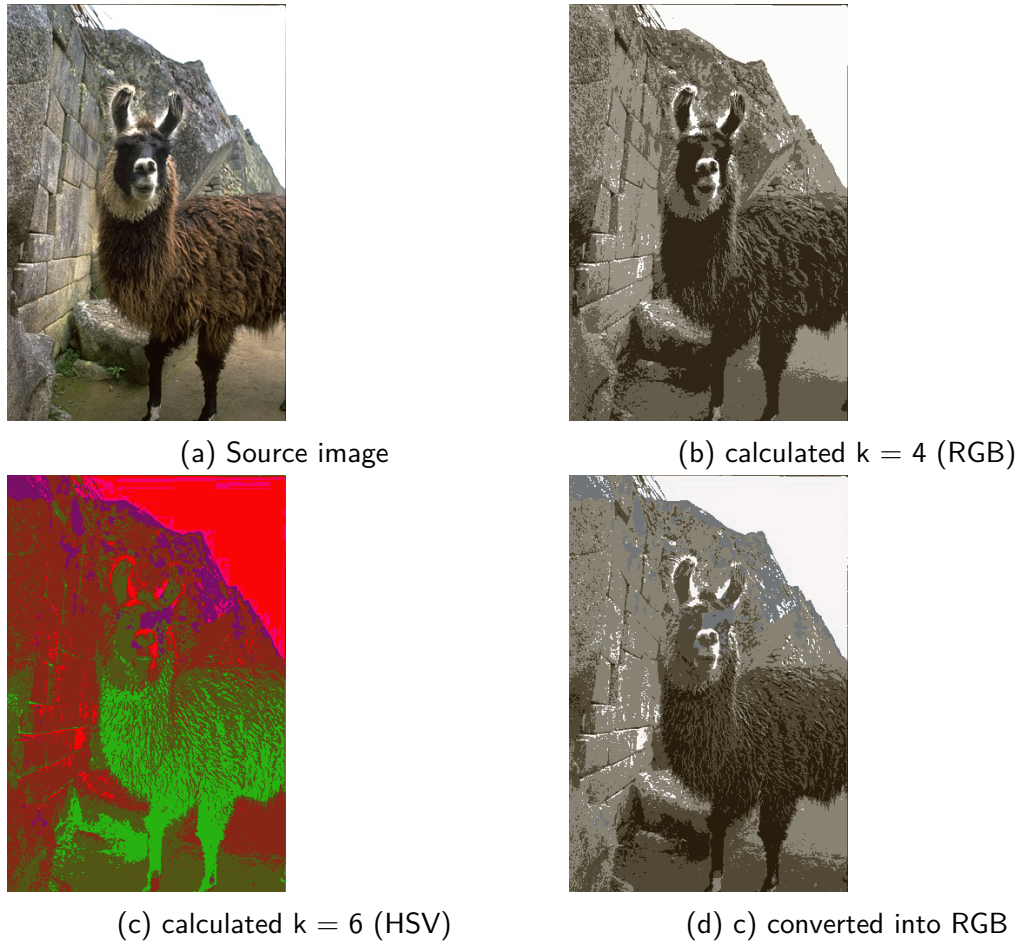


Figure 4.7: Comparison of results in RGB and HSV color spaces

## 4.2 Implementation in a video sequence

### 4.2.1 On synthetic video sequences

Implementing the dynamical cluster count as described in algorithm 3.1 into a simple synthetic video sequence revealed that the cluster count incremented correctly when necessary, but failed to detect required reductions of  $k$ .

Figure 4.8 shows an example of a video sequence. In image a) to d) the cluster count stays the same and is recognized correctly. The new appearance of a rectangle in cluster e) leads to a new cluster count calculation. In cluster g) the green rectangle is gone and a new cluster count calculation should be the result. Unfortunately the algorithm fails to identify the change and keeps the old cluster count.

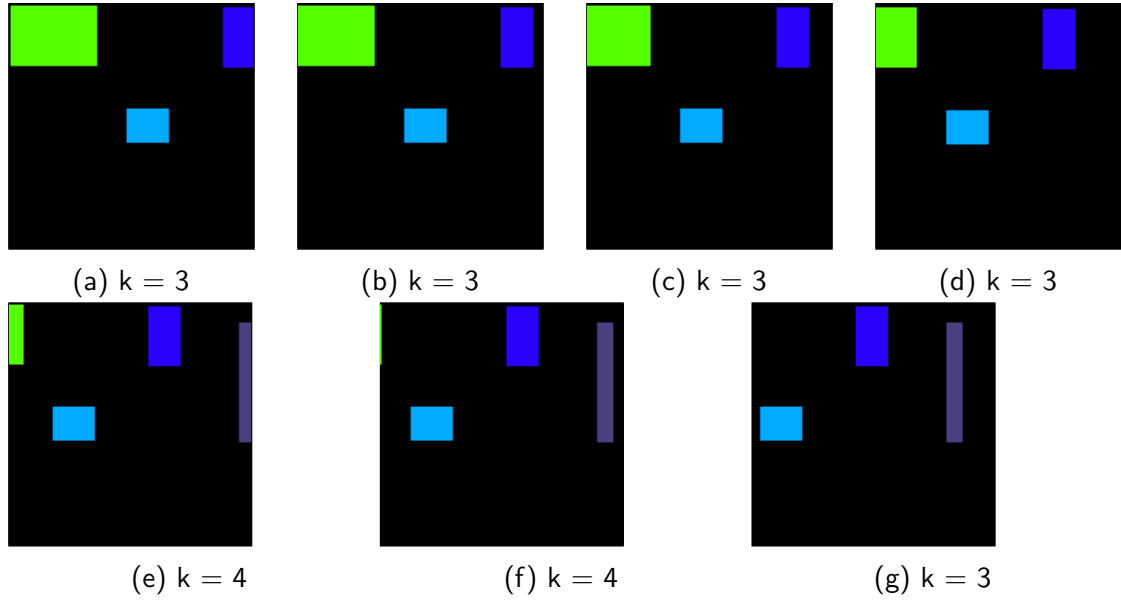


Figure 4.8: A simple video sequence in which areas are moving out of the video

This and similar tests showed that as long as something is added to the scene, the variance calculation of previous runs for determining the cluster count will still be meaningful for the current frame. If the clusters are calculated with k-means using the cluster centers of the previous frame as initialization points, the generated clusters will have a higher variance if a new area is added to the scene. Thus the validity measure will usually decrease if the number of clusters is increased, this is the same principle used in algorithm 2.2. Even if the algorithm for identifying the optimal cluster count would have split clusters in a different way if run on the current image, the disparity can still be regulated. For the transformation from  $k$  to  $k+1$ , the k-means algorithm splits the cluster with the highest variance on the current image. In contrast the initialization points for  $k-1$  are solely received via the calculation on previous frames. Therefore the calculations of the validity for  $k-1$  needs to be improved. One possibility for doing so is to use the same procedure as  $k+1$  received already. Instead of calculating the validity for  $k-1$  immediately, the cluster centers for  $k-2$  (if  $k > 3$ ) will be used as initialization points. The new clusters will be split once again as described in 2.3 and the arising cluster centers will be used to calculate the validity for  $k-1$ . Another improvement can be achieved, when the order of the validity checks is changed. The validity for  $k-1$  will be checked first, the generated clusters will be split twice and used to calculate the validity for  $k$  and  $k+1$ .

With this slight modification the algorithm achieved the right cluster count results on all self-generated simple video sequences similar to Figure 4.8 as long as the conditions remained constant. The considered cases included appearing and disappearing areas both solely and simultaneously.

However, adding blurring, noise or light effects to an image resulted most often in a

different cluster count than before, indicating unstableness in real images on changing conditions. But the impact of these effects on simple images with only few features is found to be stronger and less subtle than in most real occurring condition changes seen in natural video sequences.

To actually compare the produced cluster centers over several frames, the generated cluster centers of the first frame were used as the first centers of “super-clusters” (=merged clusters of frames). On each iteration of the algorithm, after the optimal number of clusters has been identified, the new cluster centers are compared to the centers of existing super-clusters. For comparison the usual Euclidean color space distance and the Euclidean distance of the coordinate centers of the cluster centers and the super-clusters were calculated. In order to add them, they had to be normalized.

$$d_{color} = |v_{color1} - v_{color2}| \quad (4.1)$$

$$d_{coord} = |v_{coord1} - v_{coord2}| \quad (4.2)$$

$$d_{v1v2} = d_{color} + d_{coord} \quad (4.3)$$

With  $v_1$  being the normalized cluster center and  $v_2$  being the normalized center of the super-cluster.  $v_{color}$  only includes the color information and  $v_{coord}$  only includes the center coordinates.

If the sum of both distances of the closest super-cluster was smaller than a self-determined  $\delta_{dis}$  the cluster center belonged to the super-center.

$$d_{v1v2} < d_{max\cdot\delta} = \delta_{dis} \quad (4.4)$$

In this case the center of the super-cluster had to be updated. Thus the old center of the super-cluster is simply replaced with the newest member. To get to a good  $\delta_{dis}$  the maximal possible distance has to be considered and scaled accordingly. An alternative approach is to add the center of the cluster to the center of the super-cluster and divide it by 2. This led to more stable results. As soon as one outlier appears the center is shifted in the wrong direction. Because it is likely that new clusters will not be assigned to the right super-cluster. Normally we expect clusters to move continuously in the same direction, hence the first method led mostly to better results.

Otherwise if the distance to the closest super-cluster was bigger than  $\delta_{dis}$  the cluster center is not part of an existing supercluster and its values will be the initial values of a new super-cluster.

This method generated the right results for simple synthetic video sequences using

a reasonable  $\delta$ . For example, the video sequence seen in Fig 4.8  $\sim 0.06 < \delta < 0.6$  produced right results. Overall  $\delta = 0.15$  was found to be a good threshold, a higher value for  $\delta$  often resulted in putting several clusters of one frame into a single super-cluster.

## 4.2.2 On real video sequences

To evaluate the performance of algorithm 3.1 its calculated adaptive cluster count was compared to the outcome produced by its predecessor algorithm developed by Ray and Rose H. Turi<sup>1</sup>, where each frame was processed individually. This was done on video sequences of the Freiburg-Berkeley Motion Segmentation Dataset[17], one example consisting of only 19 frames can be seen in Figure 4.9.

The adapted and unadapted algorithms produced the same cluster count on most frames, however, the adapted algorithm tended to switch the cluster count less times than the unadapted one. This outcome had to be expected, as described in 3.1 the adapted algorithm will not change the cluster count if the validity currently constitutes a local minima, whereas the unadapted algorithm will always go with the global minima.



Figure 4.9: Example of a video sequence used for evaluation

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<sup>1</sup>algorithm 2.2



Frame	k	super-clusters
1-2	6	0 1 2 3 4 5
3-11	5	0 2 1 3 4
12-16	5	0 1 2 3 4
17	6	0 3 2 4 1 5
18-19	8	0 5 2 4 1 4 3 5
8 super-clusters		

(a) results for Fig 4.9 using the unadapted algorithm

Frame	k	super-clusters
1-2	6	0 1 2 3 4 5
3-11	5	0 2 1 3 4
12-16	5	0 1 2 3 4
17	5	0 1 2 3 4
18-19	5	0 1 2 3 4
7 super-clusters		

(b) results for Fig 4.9 using the adapted algorithm

Table 4.1: The results produced on the video sequence of Figure 4.9 in RGB with  $\delta = 0.2$

Table 4.1 shows the calculated number of clusters and the respective super-clusters for the video sequence seen in Figure 4.9. The index on the list of super-clusters corresponds to the cluster of each frame which is assigned to the respective super-cluster. In this particular case the adapted algorithm produced only in 3 out of 19 frames a different outcome.

The different results produced in the adapted and unadapted algorithm don't necessarily mean that the calculated cluster count of the adapted algorithm is worse. Sometimes it might be reasonable to keep the cluster count more stable. If the cluster count is mostly unchanged, the generated clusters are more similar and thus merge into fewer super-clusters. This might be useful for tracking the movement of an object. If the object is recognized in one of the clusters of the first frame, the corresponding super-cluster might ease the recognition in the following clusters. Figure 4.10 for example shows the super-cluster labeled as 4 in Table 4.1, where most key features of the car are visible.

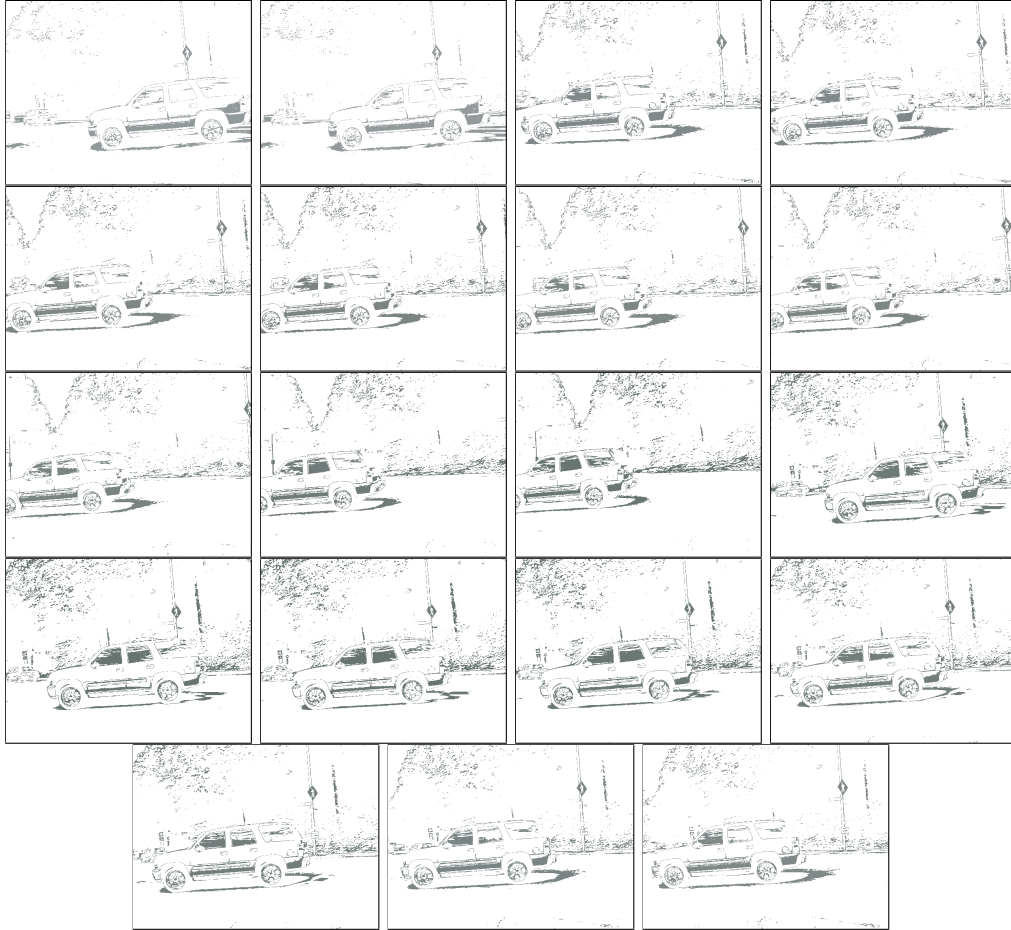


Figure 4.10: The clusters, which are put into the super-cluster 4 in Table 4.1

However, it is unlikely that the features of an object are completely put into one cluster. The super-clusters could also be used to compress the data size of the video sequence. The video sequence of Figure 4.9 could be encoded to only 7 colors when using the adapted algorithm. For other purposes it is hardly possible to rate the performance of this algorithm in reference to the super-clusters, due to the lack of similar scientific work. If the clusters would actually possess striking visual features the super-clusters could be evaluated further. Therefore it makes sense to try to include geometrical features into the segmentation process.

### 4.3 Evaluation of k-means clustering with a 5d-feature vector

Using a 5d-feature vector, consisting of the color space and the coordinates promises two main advantages. Clustered regions would be closer to semantic knowledge, because regions with similar colors are already split and thus the results are more useful for

comparing clusters over the course of a video sequence. Furthermore after the k-means algorithm has clustered each frame separately, the whole outcome can build a new dataset, which then again could be clustered via k-means to fuse the clusters of each frame into super-clusters holding information of the whole video sequence. The requirement to do so are reliable and reasonable generated results when clustering single images.

To get reasonable images the 5d-feature vector has to be normalized first.

$$\text{5d-feature Vector: } \vec{v} = \begin{pmatrix} r \\ g \\ b \\ x \\ y \end{pmatrix} \quad (4.5)$$

$$\vec{v}_{normalized} = \begin{pmatrix} \frac{r}{255} \frac{1}{3} \\ \frac{g}{255} \frac{1}{3} \\ \frac{b}{255} \frac{1}{3} \\ \frac{x}{\max X} \frac{1}{2} \alpha \\ \frac{y}{\max Y} \frac{1}{2} \alpha \end{pmatrix} \quad (4.6)$$

Where  $\alpha$  is the scaling factor. Usually the color space will be seen as more important than the geometric features, so  $\alpha$  will be close to zero.

The same normalization procedure should theoretically be applied on the variance calculation as well. Unfortunately due to the incoherent shape of the background, the cluster containing the background area had often the highest variance out of the calculated clusters. Figure 4.11 shows an example of this occurrence; the white area represents the not contained area of the specific cluster. While the algorithm gets the expected result for  $k=2$  and  $\alpha = 0.004$  (a ratio of 250:1), it fails for  $k=3-5$  and splits the background further.<sup>2</sup>

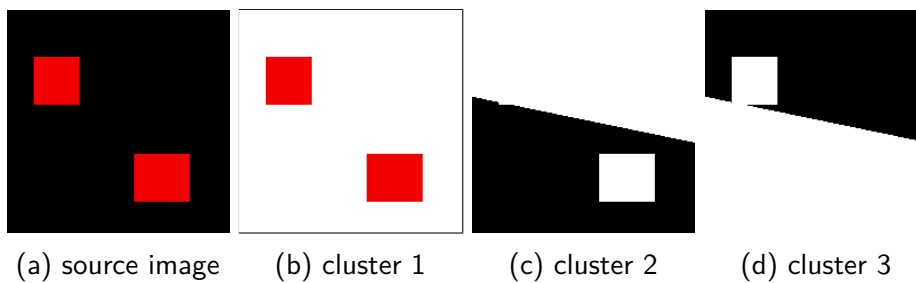


Figure 4.11: Example of single clusters when splitting the wrong cluster using a 5d-feature vector

<sup>2</sup>Using the algorithm 2.2 to determine the optimal cluster count



Ignoring the geometrical features while calculating the variance lead to better results in the case of Figure 4.11, but only by chance the rectangles were the first cluster in  $k=2$ . In order to split the right cluster a different measurement than the variance would be needed. Even if the problem of finding the right cluster to split is ignored and the right cluster is assigned manually, it didn't generate reasonable results. The major problem hereby is to find a scale of the geometric features to the color features. In some images this scale could be found, but did not result in equally good clusters on different images. The requirement to produce results on any image would be to assign the scale dynamically to each image. But for some images there might even exist no appropriate ratio of color features and geometrical features, which would lead to meaningful clusters. A potential instance of this will be further discussed in section 4.4.

## 4.4 Evaluation of the alternative approach

Due to the inefficiency using a 5d-feature vector, especially with the problem of getting a right measure for the optimal cluster count, it seems easier to separate the splitting according to color features and the splitting according to geometrical features into two processes. Instead of clustering with a 5d-feature vector, the well performing 3d-vector including only the color space can be retained and each cluster might be further divided.

We only want to divide clusters with clear geometrical boundaries further. Hence if it is possible to draw several polygons without overlapping, where the area of the polygons is made up by the pixels of the clusters, the polygons will build new clusters.

This can be achieved using the openCV library[15].<sup>3</sup> Hereby one major problem needs to be considered. The cluster(s) which hold the color information of the background might lead to the same detected boundaries as the clusters which are actually surrounded by the background.

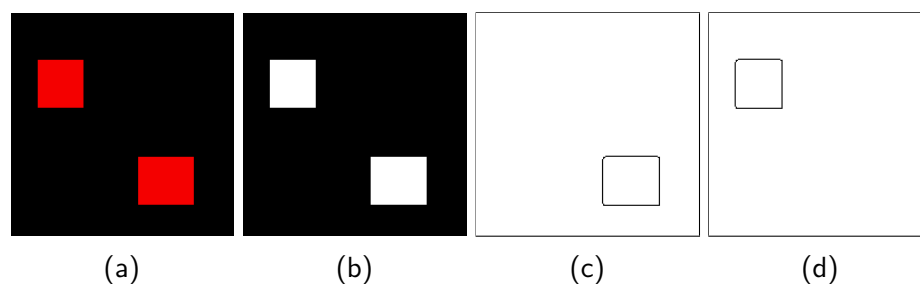


Figure 4.12: Example images for the occurring background problem when calculating boundaries

The first image of Fig 4.12 shows the image which will be clustered. This will result in two clusters, when determining the optimal cluster count for color segmentation.

<sup>3</sup>see section 3.2

The first one will only hold the two red rectangles, while the second one will hold the background. The cluster containing the background can be seen in b). The last two images c) and d) show the calculated boundaries. Unfortunately these boundaries can be calculated using any of the two clusters. Hence if adding the calculated polygons of each cluster to the number of cluster centers, the optimal cluster count would be determined as 4 instead of 3. Thus the calculated polygons should only increase the cluster count if the comprised area is filled of the pixels which are part of the cluster used for calculating it. This check needs to be implemented in the algorithm.

Furthermore cluster centers should also include coordinates in order to get coherent clusters. In each iteration of the basic k-means algorithm, the mean of the coordinates of each updated cluster will form the cluster center together with the corresponding color features.

When splitting the cluster further using the method described in 2.4, the new cluster centers will be built up of the same color feature but will be separated according to the new mean of the coordinates of each polygon.

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**Algorithm 4.1** Find geometrical features of clusters

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```

1: calculate the mean of the coordinates of each cluster
2: update the cluster center including the coordinate information
3: detect the (outer) boundaries of each cluster
4: for each cluster do
5:   if Several polygons are detected and the pixels inside the polygons are part of
     the current cluster then
6:     Delete the old cluster center of this cluster center
7:     Calculate the mean of the coordinates inside the boundary
8:     Built new cluster centers according to the calculated coordinates and the
       old color feature
9:   end if
10: end for

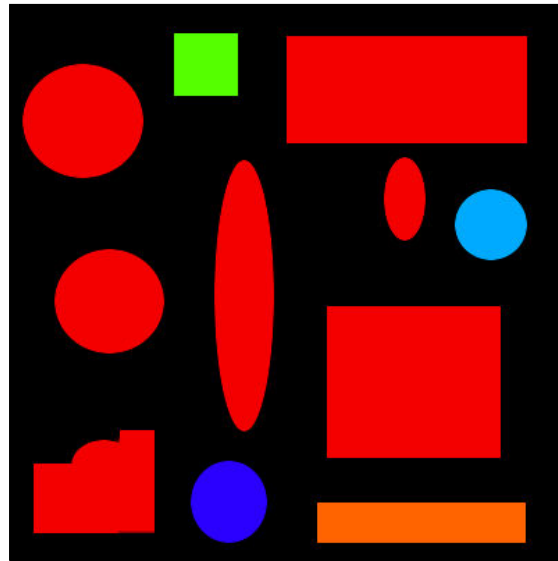
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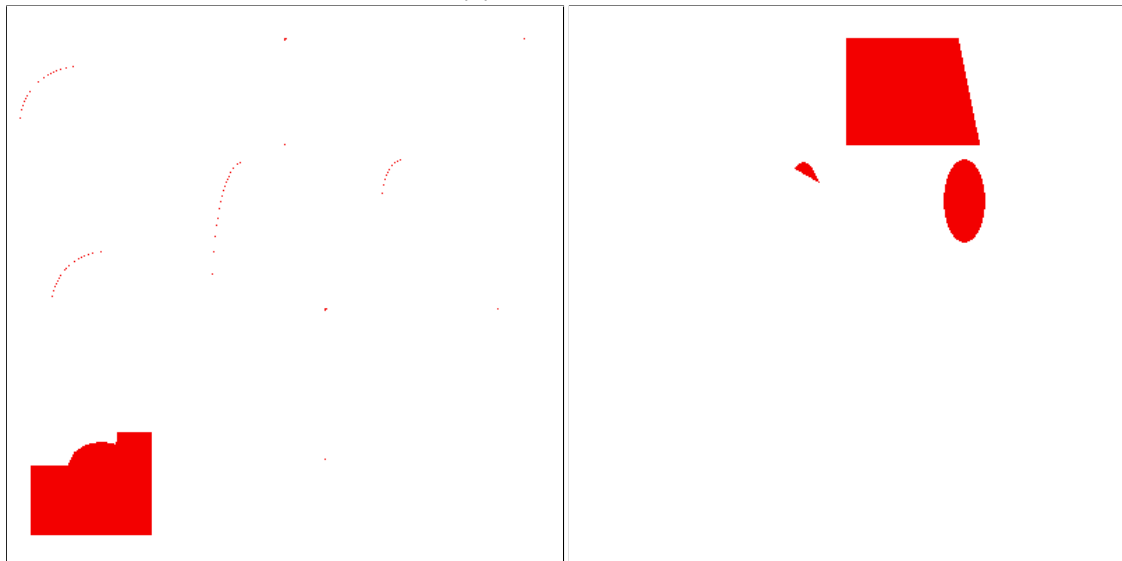
One of the advantages of the k-means algorithm is, that each pixel is assigned to the corresponding cluster center. The assignment could be done in different ways. Because the splitting of the clusters happens according to the calculated boundaries, it seems logical to check whether the pixels of the original cluster lie inside the calculated polygon and assign them accordingly. Alternatively the calculated cluster centers could be used as starting points for k-means clustering with either the 5d-vector of section 4.3 or using a 2d-vector consisting of the coordinates on the particular cluster.

Both assignment methods have some flaws, illustrated in 4.13. The created boundaries are not completely accurate, hence some pixels are not inside the polygon and will fall through the first method as seen in b). The second method doesn't consider boundaries at all, the k-means algorithm will simply check which pixel is on the shortest distance to the specific cluster. Adjusting the scale value  $\alpha$  did not lead to the desired result

and only adjusted distribution of the clusters slightly. The areas are too close together. Either the pixel distance, or the colors are required to be further divided in order to get correct results.



(a) Source image



(b) flaws via polygon check

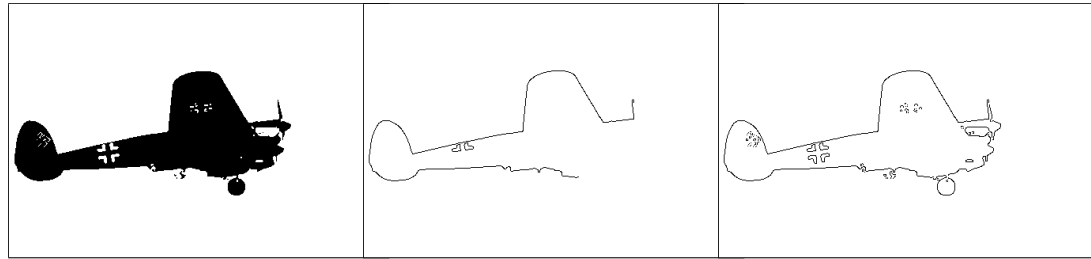
(c) flaws via k-means

Figure 4.13: Comparison of boundary cluster assignment and k-means

On simpler images as for example seen in Figure 4.1 both methods work fine and lead to the expected results. Overall, the method of checking whether a pixel lies within the calculated polygon seems to get closer to the expected results. The problem of assigning only few pixels to the wrong cluster could easily be solved. For example by implementing a noise filter.

On natural images however the described method didn't lead to an increased cluster count. In all tested images the contour detection failed to find useful polygons. Despite using a nearly optimal binary image in Figure 4.14, the resulting contours are not

completely aligned. Printing all contours in one image shows the potential and might indicate how an improved detecting algorithm will improve the results.



(a) Binary image (b) Best calculated contour (c) All contours in one image

Figure 4.14: Example of a flawed boundary detection

The source image of Figure 4.15 presents two snowshoes. The goal in segmenting this image would be to split the particular cluster so that one features the left snowshoe and one features the right one. Unfortunately no fitting polygon could be calculated and the clusters remained unchanged.



(a) Binary image of interesting cluster (b) All resulting contours in one image

Figure 4.15: Second example of a flawed boundary detection

## Chapter 5

# 5 Discussion and conclusion

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Testing the described algorithm by Ray and Rose H. Turi on images verified the formerly reported results. Although some flaws need to be considered before using it. On very simplistic images the calculated cluster count will separate the image into too many clusters due to the possible reduction of the intra cluster distance. Besides this, it should be known beforehand whether the investigated image will be a synthetic or a real one. Usually this should be no problem and is no new finding, because it is already stated in the original article.[7]

Adapting the algorithm to generate an adaptive cluster count on video sequences proved to work well on synthetic video sequences. On natural video sequences the results were satisfactory but did not always produce identical results in comparison to the independent calculated ones. Due to the high computation time, the effective use of this algorithm would require some further improvements for most of the possible tasks. Even if the current cluster count of a frame in a video sequence constitutes a local minimum in the next frame the basic k-mean algorithm has to be repeated 4 times. Otherwise the basic k-means algorithm will be repeated until  $k_{\max}$  is reached.

The improvements should be tailored to the final task. For example, one idea for object-tracking would be to recalculate the cluster count only if the object has been lost. In addition the algorithm could implement multi-threading to improve the calculation time.

Creating super-clusters which include color- and coordinate information, by comparison of cluster center over several frames, provides knowledge of the movement and continuity of existing clusters. The information yield and specification of a good threshold  $\delta_{dis}$  or even the suggested procedure for creating super-clusters in general can only be properly evaluated if the generated clusters are closer to semantic knowledge. This could be achieved by including more features in clustering than just the color information. The attempt of including geometric features via a 5d-feature vector while retaining the basic k-means algorithm did not lead to improved results. The main problem was to find a working ratio of the colors and coordinates, this ratio should furthermore be dynamic to be universal applicable. On some images the 5d-feature vectors did not deliver the desired outcome independent on the scaling ratio. This was probably caused by one of the big flaws of k-means clustering, it can only produce spherically formed clusters. This is especially problematic when the coordinates are included in the clustering process. In

this cases other clustering methods as for example mean-shift would produce better results.

The alternative approach combined the region based image segmentation of k-means clustering with a boundary detection approach. Although the splitting of clusters did ultimately not work in real images, the found edges in single clusters showed some promise. The next step would be to include actual semantic knowledge in order to find objects or features, and create clusters solely for them.

The concept of super-clusters seemed to work quite well and merged similar clusters correctly. But besides data compression, no other field of use could be tested in this thesis. However, if semantic knowledge could actually be implemented in the clustering process, the serial identity of clusters in video sequences would drastically increase and therefore also the fundamental suitability of super-clusters.

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