***** Tutorial Sheet - 1 *****

1. Evaluate following limits:

   (a) $f(x) = \begin{cases} 1+x^2 & \text{if } 0 \le x \le 1 \\ 2-x & \text{if } x > 1 \end{cases}$ at $x = 1$

   (b) $f(x) = \frac{e^{1/x}}{e^{1/x}+1}$ at x = 0

   (c) $f(x) = \begin{cases} 1 & \text{if } x < 1 \\ 2-x & \text{if } 1 < x < 2 \\ 2 & \text{if } 2 \le x \end{cases}$ at $x = 1, 2$

   (d) $f(x) = sin\left(\frac{1}{x}\right)$ at $x = 0$

2. Discuss the continuity and differentiability of the functions:

   (i) $f(x) = \begin{cases} 1+sin(x) \ ; & \text{when } 0 < x < \pi/2 \\ 2+(x-\pi/2)^2 \ ; & \text{whenf } x \ge \pi/2 \end{cases}$ at $x = \pi/2$.

   (ii) $f(x) = \begin{cases} x\left[1+\frac{sin(log(x^2))}{3}\right] \ ; & \text{when } x \ne 0 \\ 0 \ ; & \text{whenf } x = 0 \end{cases}$ at $x = 0$.

   (iii) $f(x) = \frac{x}{1+|x|}$ at $x = 0$.

   (iv) $f(x) = \begin{cases} 2+x \ ; & \text{when } x \le 0 \\ 1+x \ ; & \text{when } 0 < x < 1 \\ 3-x \ ; & \text{when } 1 \le x \le 2 \\ 3x \ ; & \text{when } x > 2 \end{cases}$ at $x = 0, 1, 2$.

3. If the following function $f(x)$ is continuous everywhere, find the values of a, b, c, where

   (a). $f(x) = \begin{cases} \frac{sin\{(a+1)x\}+sin(x)}{x} \ ; & \text{when } x < 0 \\ c \ ; & \text{when } x = 0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}} \ ; & \text{when } x > 0 \end{cases}$

   (b). $f(x) = \begin{cases} \frac{x^2-8x+7}{ax^2-5ax-14a} \ ; & \text{when } x < 7 \\ b \ ; & \text{when } x = 7 \\ \frac{\sqrt{x+2}-c}{x^2-8x+7} \ ; & \text{when } x > 7 \end{cases}$

4. Evaluate following limits:

   (a) $\lim_{x\to 0}\left[\frac{\sin^2 x - x^2}{x^2\sin^2 x}\right]$

   (b) $\lim_{x\to 1}[(1-x)\tan(\pi x/2)]$

   (c) $\lim_{x\to 2}\left[\frac{x-1}{x-2} - \frac{1}{ln(x-1)}\right]$

   (d) $\lim_{x\to\infty}\left[\frac{x^2}{e^x}\right]$

   (e) $\lim_{x\to\infty}\sqrt{\frac{x+\sin x}{x-\cos^2 x}}$

   (f) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

   (g) $\lim_{x\to 0}(x\log(\sin x))$

5. Find the n^{th} derivative of following:

   (a) $\frac{1}{5x+7}$

   (b) $x^2 sin(3x)$

6. Let $y = \tan^{-1}(x)$. Use Leibnitz's theorem to show that $(1+x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$.

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7. Let $y = a\cos(\log x) + b\sin(\log x)$. Use Leibnitz's theorem to show that $x^2y_2 + xy_1 + y = 0$ and $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

8. Let $y = e^{\tan^{-1}x}$. Show that $(1+x^2)y'' + (2x-1)y' = 0$ and hence use Liebnitz's theorem to show that $(1+x^2)y_{n+2} + \{2(n+1)x - 1\}y_{n+1} + (n^2+n)y_n = 0$.

9. Use Taylor's series to evaluate following, correct upto four decimal places:

   (a) $\cos(31)^0$

   (b) $\sqrt{17}$

   (c) $\sin(29)^0$

   (d) $\sqrt{15}$

10. Obtain the Taylor's polynomial approximation of following:

    (a) $f(x) = \sin(3x)$ about the point $a = 0$ upto degree 5

    (b) $f(x) = e^{2x}$ about the point $a = 0$ upto degree 4 in $[0, 0.5]$

    (c) $f(x) = \frac{1}{1-x}$ about the point $a = 0$ upto degree 3 in $[0, 0.25]$

    (d) $f(x) = x\sin(x)$ about the point $a = 0$ upto degree 4 in $[-1, 1]$

    (e) $f(x) = e^x$ about $x = 0$

11. Find the stationary points and identify whether these are maximum, minimum or inflection points for following functions:

    (a) $y = -x^2 - 5x - 5$

    (b) $y = 2x^2 + 4$

    (c) $y = 2x^3 - 9x^2 - 24x + 10$

    (d) $y = (3x-2)^2$

    (e) $y = \frac{3x}{x^2+1}$

    (f) $y = x^3$

    (g) $y = 4\sqrt{x} - x$

12. Find the asymptotes of the curve:

    (a) $x^3 + y^3 - 3axy = 0$

    (b) $y = \frac{x-4}{x^2+4x+3}$

    (c) $y = \frac{(x-1)^3}{x^2(x+1)}$

    (d) $y = \frac{x+1}{\sqrt{x^2-4}}$

    (e) $y = x + \frac{1}{x}$

13. Let $y = x + \frac{1}{x}$. Determine
    (i) points of relative maxima and minima,
    (ii) intervals of increasing and decreasing,
    (iii) point of inflection,
    (iv) intervals of concave upward and downward,
    (v) Asymptotes and hence draw a neat sketch of its graph labelling all information obtained above.

14. Let $y = \frac{(x-1)(x-3)}{x^2}$. Determine
    (i) points of relative maxima and minima,
    (ii) intervals of increasing and decreasing,
    (iii) point of inflection,
    (iv) intervals of concave upward and downward,
    (v) Asymptotes and hence draw a neat sketch of its graph labelling all information obtained above.

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15. Let $y^2 = \frac{x-3}{x^2-6x-7}$. Determine
(i) region,
(ii) point of intersection,
(iii) Symmetry,
(iv) Asymptotes and hence draw a neat sketch of its graph labelling all information obtained above.

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Answers

1. (a) 1
   (b) Does not exist
   (c) 1 at $x = 1$. Does not exist $x = 2$.
   (d) Does not exist.

2. (i) Continuous and differentiable.
   (ii) Continuous and not differentiable.
   (iii) Continuous and differentiable.
   (iv) Continuous at $x = 1$ and not differentiable at $x = 0, 1, 2$.

3. (a) $a = -\frac{3}{2},\ c = \frac{1}{2},\ b$ any real number.
   (b) $a = 24,\ b = \frac{1}{36},\ c = 3$.

4. (a) $-\frac{1}{3}$
   (b) $\frac{2}{\pi}$
   (c) $\frac{1}{2}$
   (d) 0
   (e) 1
   (f) e
   (g) 0

5. (a) $\dfrac{(-1)^n n! 5^n}{(5x+7)^{n+1}}$
   (b) $x^2 3^n \sin\left(3x + \dfrac{n\pi}{2}\right) + 2nx3^{n-1} \sin\left(3x + \dfrac{(n-1)\pi}{2}\right) + n(n-1)3^{n-2} \sin\left(3x + \dfrac{(n-2)\pi}{2}\right)$.

6. Proof

7. Proof

8. Proof

9. (a) 0.85717
   (b) 4.12310
   (c) 0.48481
   (d) 3.87298

10. (a) $3x - 27\dfrac{x^3}{3!} + 243\dfrac{x^5}{5!} - \ldots$
    (b) $1 + 2x + 4\dfrac{x^2}{2!} + 8\dfrac{x^3}{8!} + 16\dfrac{x^4}{4!} + \ldots$
    (c) $1 + x + x^2 + x^3 + \ldots$
    (d) $x^2 - 6x^4 + \ldots$
    (e) $1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots$

11. (a) $x = -\frac{5}{2}$ maxima
    (b) $x = 0$ minima
    (c) $x = -1$ maxima
    $x = 4$ Minima

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(d) $x = \frac{2}{3}$ minima

(e) $x = 1$ maxima
$x = -1$ minima

(f) $x = 0$ point of inflection

(g) No stationary point

12. (a) No Horizontal and Vertical asymptote
Obliques $y = -x + a$.

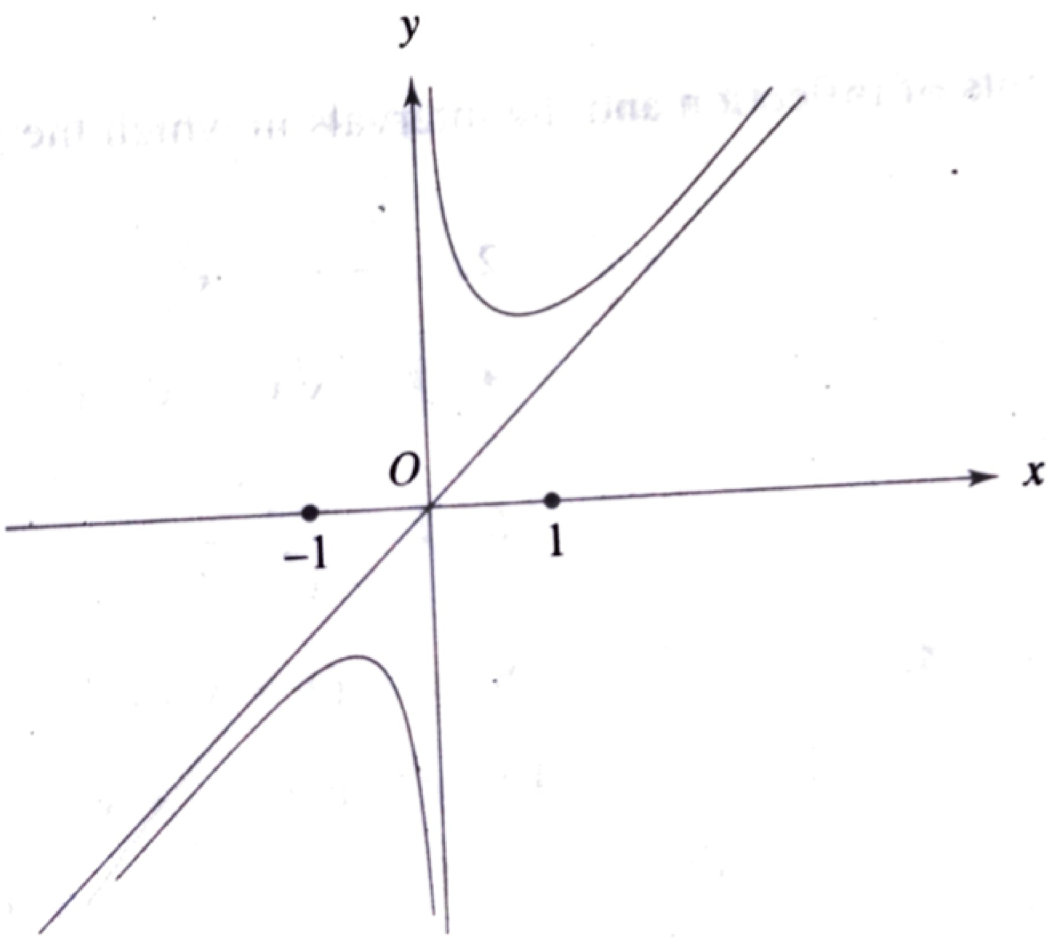
(b) Horizontal: $y = 0$
Vertical: $x = -1, x = -3$.

(c) Horizontal: $y = 1$
Vertical: $x = 0$.

(d) Horizontal: $y = 1$
Vertical: $x = \pm 2$.

(e) Horizontal: Does not exist
Vertical: $x = 0$
Obliques $y = x$.

13. Figure



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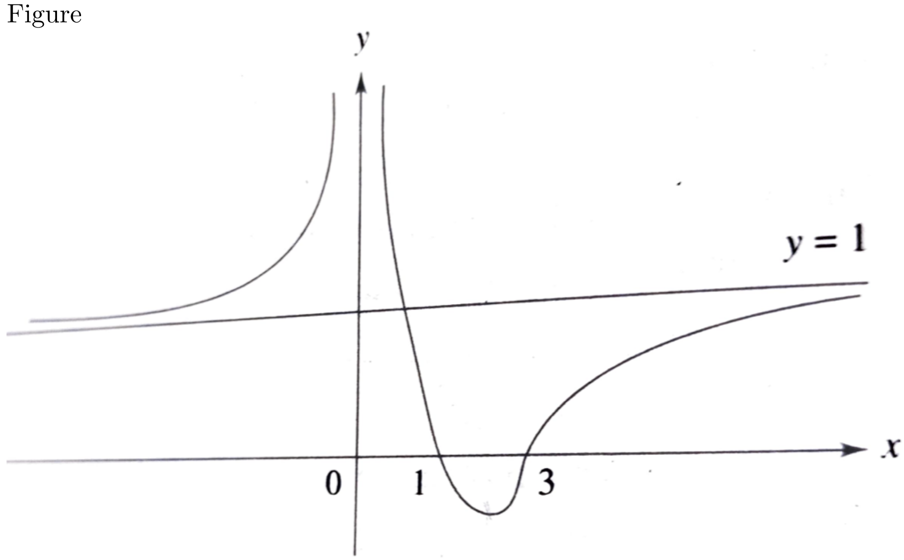
14. Figure



15. Figure