page 1 school logo – date – Supervisor + me (Dr. Cécile Malet - Dagréou)

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Projet 5

Interpolation and integration methods

p2.

Summary : This project is composed of two preliminary parts, the first one being about interpolation for what ? methods, and the second ~~one~~ about integration methods, as well as an application. The application consists in producing a pressure map around an airfoil using ~~the~~ interpolation and integration methods. explain

+ table of contents

p3

Where is your intro ?

1 Interpolation Samuel Khalifa

To ~~make analysis on the~~ analyse an airfoils' surfaces possible, a method to model ~~the airfoil's surface~~ it with a potential function was necessary. As the database only gives some points on the surface, interpolation was needed. which one ? There are multiple interpolation methods, but ~~it had~~ they have to satisfy 2 requirements: continuous and easily derivable. Cubic Splines interpolation was used as this method satisfies the requirements and was easy to implement. be & ying

what is the point of doing this ??

1.1 Cubic Spline Algorithm

The Cubic Spline approximation can be defined by equation 1 :

$$\omega_{[\![x_i;x_{i+1}]\!]}(x) = A_i(x)y_i + B_i(x)y_{i+1} + \frac{h_i^2}{6}\left(y_i^{''}\left(A_i^3(x) - A_i(x)\right) + y_{i+1}^{''}\left(B_i^3(x) - B_i(x)\right)\right) \tag{1}$$

With $h_i = x_{i+1} - x_i$, $A_i(x) = \frac{x_{i+1}-x}{h_i}$ and $B_i(x) = \frac{x-x_i}{h_i}$.

To compute the derivative of a cubic spline interpolation, ω_i has to be derived. Its derivative is

All formulas mean nothing to a non-specialist. Explain !

defined in equation 2 :

$$\omega_i^{'}(x) = \frac{y_{i+1} - y_i}{h_i} + \frac{h_i}{6}\left(y_i^{''}\left(1 - 3A_i^2(x)\right) + y_{i+1}^{''}\left(3B_i^2(x) - 1\right)\right) \tag{2}$$

To ensure equation 2 can be derived, the following condition has to be met: $\forall i \in [\![1;n]\!]\ \omega_{i-1}^{'}(x_i) = \omega_i^{'}(x_i)$. This condition can be modelled with equation 3 :

$$\frac{h_{i-1}}{6}y_{i-1}^{''} + \frac{h_{i-1} + h_i}{3}y_i^{''} + \frac{h_i}{6}y_{i+1}^{''} = \frac{y_{i+1} - y_i}{h_i} - \frac{y_i - y_{i-1}}{h_{i-1}} \tag{3}$$

It is needed to specify the value of $y_0^{''}$ and $y_{n+1}^{''}$. For the rest of the experiments, the N natural S spline method was used. It ~~The "natural spline" method~~ sets both $y_0^{''}$ and $y_{n+1}^{''}$ to 0.

To get a single function that returns the correct $\omega_i(x)$ for a given x in $[\![x_0; x_n]\!]$, a function $range(x)$ that searched for the correct range $[\![x_i; x_{i+1}]\!]$ for a given x was implemented using a binary search algorithm for efficiency's sake as the list of x_i is in increasing order. Now, the function $\omega(x)$ that gives the interpolation function for all x in $[\![x_0; x_n]\!]$ is defined in equation 4 :

$$\omega(x) = \omega_{range(x)}(x) \tag{4}$$

To check whether the C cubic S spline method ~~implementation~~ was correct implemented or not, the results ~~of our implementation~~ were compared with those ~~the ones~~ from a state of the art library for cubic spline interpolation: `scipy`. font

and ? conclusions from comparison ?

1.2 Interpolation of the goe05k A airfoil

An airflow can be separated into 2 surfaces: the extrados (upper surface) and ~~the~~ intrados (lower surface). The database only gives points across the 2 surfaces as seen in F figure 1a . → should be here !

Therefore, the cubic spline interpolation method was used to model each part of the airfoil as a function to make the analysis on the airfoil possible, as seen ~~in~~ F figure 1b.

be consistent . either you capitalise throughout or you don't .

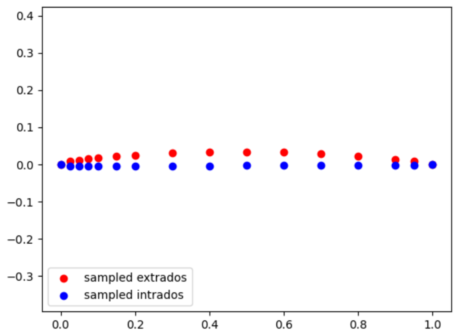


Fig. 1a

(a) The sampled points of the goe05k airfoil.

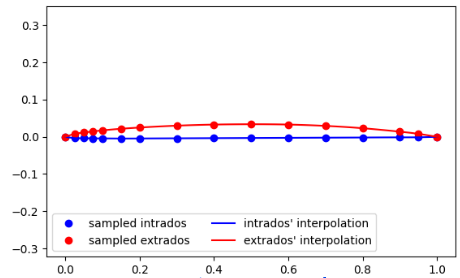


Fig. 1b

(b) The interpolated goe05k airfoil.

2 Integration

Thibaut Brintet

In order to approximate the pressure on either side of the airfoil, an integral computation algorithm needs to be implemented. Three different computation methods were ~~implemented~~ considered, the goal ~~was~~ being to find the algorithm with the highest convergence speed.

why?

2.1 Trapezoidal method

Thibaut Brintet

The trapezoidal method approximates the area under the curve of the function by dividing it into trapezoids and summing their areas.

Considering n as the number of trapezoids used, the time complexity ~~of the~~ algorithm 's is $\mathcal{O}(n)$. By increasing n, the accuracy ~~of the~~ result 's is improved, but the execution time is longer.

Despite its simplicity, the level of accuracy given by this method is very good for performing simple computations.

However, it converges more slowly than other numerical integration techniques, such as Simpson's rule or Romberg's method, and it may require a large number of sub-intervals to achieve the desired level ~~of~~ accuracy. which are? what do they do? offer?

2.2 Simpson's rule

Arthur Le Floch

Simpson's rule gives a second-order approximation of the integral of f over $[a, b]$, which means that the result is approximated using a parabola instead of a straight line, as in the trapezoidal method. Its implementation is based on an approximation similar to the one used in the previous method (see equation 5).

$$\int_{x_i}^{x_{i+1}} f(x)dx \approx \frac{x_{i+1} - x_i}{3} \left[f(x_i) + 4 \times f\left(\frac{x_i + x_{i+1}}{2}\right) + f(x_{i+1}) \right] \tag{5}$$

Given this approximation, the integral of the function f can be approximated by summing these results over all the sub-intervals, $[x_i, x_{i+1}]$ in $[a, b]$, as shown in equation 6.

$$\int_a^b f(x)dx \approx \sum_{i=0}^{n-1} \int_{x_i}^{x_{i+1}} f(x)dx \tag{6}$$

~~The~~ time complexity ~~of this~~ method is $\mathcal{O}(n)$, where n is the number of sub-intervals.

2.3 Romberg's method

Arthur Le Floch

Romberg's method gives a more accurate approximation of the integral of f over $[a, b]$ than the previous ones. This method fills a table with the results of the trapezoidal method ~~for the~~ first column, with a given number of evaluations. The results of the previous rows are then used to compute the next rows, using both equations 7 and 8.

It can be noted that the different rows represent integration methods using polynomials of increasing order. Thus, this method generalizes the ones that were implemented before. meaning ?

$$R_{n,0} = h_n \times \left(\frac{f(a) + f(b)}{2} + \sum_{k=0}^{2^{n-1}-1} f(x_k) \right) \text{ where } h_n = \frac{b-a}{2^n} \text{ and } x_k = a + k \times h_n \tag{7}$$

$$R_{n,m} = \frac{4^m R_{n,m-1} - R_{n-1,m-1}}{4^m - 1} \text{ where } n, m > 0 \tag{8}$$

In order to fill the table in the correct order, the columns must be filled from left to right, and the rows from top to bottom. Once the table is filled, the result of the integral is the bottom right element of the table.

~~The~~ time complexity ~~of~~ this method 's depends on the table's size ~~of the table, and~~ I in terms of memory, a 2-dimensional array is required, as well as other values during the computation.

and what does that tell/give you ?

2.4 Integration with a given precision

Arthur Le Floch

The following method ~~gives~~ offers the possibility ~~to~~ of computeing the integral of a function with a given specific precision. Its implementation uses the same algorithm ~~as~~ from the Romberg's method, except that it stops when the algorithm reaches the expected precision.

In fact, the algorithm stops when the equation 9 is satisfied, i.e. when the difference between the current result and the previous one is less than the given precision.

$$|R_{n,m} - R_{n-1,m-1}| \leq \epsilon \tag{9}$$

This method seemed to be the longest to apply, but is the most accurate one among all the methods that were implemented. And why is that significant ??

2.5 Comparison of convergence speeds — why compare them? point?

Thibaut Brintet

The convergence speed of each method is ~~available on~~ displayed in Figure 1. — can it be - you already have 1a + 1b!

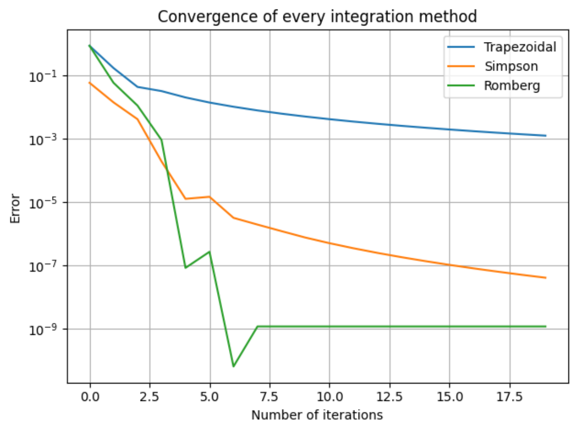
The trapezoidal method has a linear convergence rate, which means that the error decreases at a rate proportional to $1/n$, where n is the number of sub-intervals. This rate of convergence is not very fast, but it is generally sufficient for most applications.

Simpson's rule has a faster convergence rate than the trapezoidal method, which is proportional to $1/n^2$. This means that it typically requires fewer sub-intervals to achieve a given level of accuracy compared to the trapezoidal method. However, it can be sensitive to the shape of the function being integrated, and may not always be applicable.

Romberg's method has a much faster convergence rate than both the trapezoidal method and Simpson's rule. Its convergence rate is exponential, which means that the error decreases at a rate proportional to 2^{-2m}, where m is the number of iterations. This means that it can achieve a high accuracy level ~~of accuracy~~ with relatively few function evaluations, making it very efficient for many applications. like what? However, it requires more memory to store ~~the~~ intermediate results, and can be more computationally intensive in some cases. like what?

examples?

Overall, the choice of integration method depends on the specific application and the desired level of accuracy. The trapezoidal method is simple and robust, and is often a good choice for simple computations. Simpson's rule is more accurate than the trapezoidal method, but may be less robust in some cases. Romberg's method is the most accurate and efficient of the three, but requires more memory and computational resources.



2

3

Figure 1: Comparison between different integration methods on $f(x) = \frac{1}{1+x^2}$ over $[-1, 2]$.

3 Modeling the airflow & pressure map

airplane? explain! Connect to reality & real world!

Mohamed Aymane Kherraz

The goal of this part was to use the previous functions to demonstrate the flow and behavior of air around the foils. In other words, ~~to~~ show ing how air moves around the foils in order to produce that lift power and as little air turbulence as possible. Moreover, the aim was ~~to be able~~ to draw the pressure map that the foils were ~~are~~ going to be put under in order to deduce information like ~~what is~~ the maximum pressure a wing can ~~is supposed to~~ handle and which parts can need ~~to~~ handle more or less pressure.

3.1 Modeling the airflow

next page - no title at bottom of a page

Mohamed Aymane Kherraz

An ~~The~~ airfoil is supposed to operate in a laminar flow, which is a system where each air particle moves along a curve that does not ~~n't~~ cross the path of another air particle, also known in ~~the~~ physics ~~field~~ as no turbulence. The closest particle to both surfaces moves exactly along the curves of air ~~the~~ airfoil, and the further the particle is, the flatter its path is. This can be summarized by saying that there is no disturbance in the supposed air system.

The next goal was to be able to draw the curves of N particles equally distributed in space. Many approximations were made in the field of fluid mechanics to simplify the motion ~~movement~~ equation and still be as accurate as possible. Let h_{max} (i.e., h_{min}) be the maximum (i.e., minimum) height of the airfoil, f be the function of the upper surface of the foil, and g be the function of the lower surface of the air foil. The general formula of the trajectory of any air particle above the upper surface is:

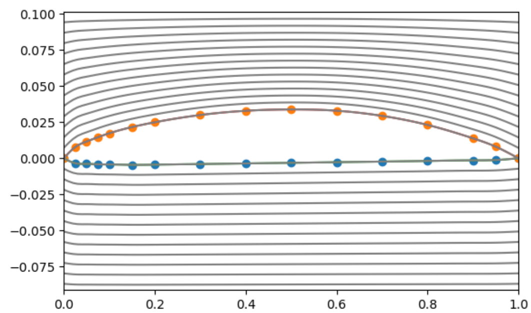
$$y = f_\lambda(x) = (1-\lambda)f(x) + \lambda \times 3h_{max} \quad \text{with} \quad \lambda \in [0;1] \tag{10}$$

For particles below the lower surface, the motion ~~movement~~ equation goes like the following:

$$y = g_\lambda(x) = (1-\lambda)g(x) - \lambda \times 3h_{max} \quad \text{with} \quad \lambda \in [0;1] \tag{11}$$

To draw the airflow for $N = 30$ particles equally distributed in space, the first step was to draw the interpolation of the airfoil, which was done in the graph 1b, using the cubic spline interpolation previously defined and described in ~~the~~ section 1.1.

The interval $[0;1]$ was separated into 15 equal sections as λ options like the following: 15 particles above the upper surface using the equation 10 and the rest under the lower surface using the equation 11. This application ~~gives the graph 2.~~ is illustrated by Figure 3



and what does that reveal ??

Figure 2 3: The path of 30 particles in the space of an airfoil.

3.2 Pressure map M

at last !!

Marwa HAJJI LAAMOURI

All of the above amounts to plotting a pressure map around the wing, from which it is possible to deduce useful information about the airfoil's aerodynamic performance ~~of the airfoil~~. Figure 3 4 shows an example ~~of~~ pressure map.

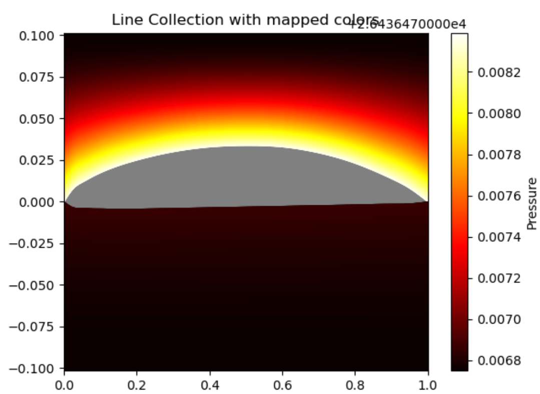


Figure 3 4: ~~Example of~~ Pressure map example.

The value of a pressure map lies in ~~its~~ visually highlighting ~~the~~ important information related to ~~the~~ pressure around the airfoil being studied, ~~rendering it more fluidly processed~~. It so appears that, in the context of the mathematical modelisation chosen and the approximations made, ~~the~~ air travels along a longer path right above the outer edge of the wing and slows down as we move higher up, while ~~the~~ air right below the inner edge of the wing seems to flow slower. ~~The~~ C color coding~~, in particular,~~ facilitates the understanding of the map: darker areas express slower rates ~~of~~ airflow, while lighter areas express faster airflow rates. The gray area corresponds to the wing itself, in which no flow naturally exists.

& processing it + fluidly.

but The function `plot_pressure` generates the map by reading data from an input file, which contains, ~~in particular,~~ the coordinates of the points tracing the airfoil's outer and inner edges ~~of the airfoil~~. Cubic spline interpolation is then used to smooth out the wing's delimitations ~~of the wing~~.

As described above, we use a parameter lambda to define the air particles flowing above and below the wing. For each of those, the airflow and line pressure are then calculated.

Finally, the graph is ready to be plotted. → & where is it ??

Conclusion

Marwa HAJJI LAAMOURI

In conclusion, this project focused on two key numerical techniques: cubic spline interpolation and integration. These methods were developped in the first two parts of the project and then put to the test in an application, in which the main objective was to produce a pressure map around an airfoil. The first step was to choose an interpolation method to model the airfoil's surface. the Cubic Splines method was selected for its properties of continuity and easy derivability. It should be noted that a binary search algorithm was used to optimize the performance of the interpolation algorithm by reducing ~~the~~ search time. In order to ensure correctness, the implementation was compared with Scipy's implementation. which is point?

In the second part of the project, three integration methods were compared: the Trapezoidal method, Simpson's rule, and Romberg's method. The Trapezoidal method was simple and gave good results for simple computations but was slow to converge compared to the other methods. Simpson's rule provided a more accurate approximation by using a parabolic function to estimate the integral, resulting in faster convergence. It was then concluded that Romberg's method proved to be the most accurate and fastest, using extrapolation to improve the results.

The application of these methods to produce a pressure map around an airfoil was successful, with the pressure distribution accurately computed. The results showed ~~the areas of~~ high and low pressure areas on the airfoil's surface, which is valuable information for aerodynamic analysis. The methods used in this project can be applied to other fields where numerical techniques are required, such as physics, engineering, and finance. ?? like what? examples?

Good work & good potential, but pretty much inaccessible & incomprehensible for a non-specialist. You need to explain, justify what you do.

→ technical terms: definitions, glossary

→ why doing this, not that? results, conclusions.

→ connect w/ the real world. Too theoretical.

- no bibliography, references?

- revise figure nbs -