Homework4 solution

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所有结论正确，过程合理的解答均可得到全部分数，结论错误，过程正确也可以得到相应的步骤分。

1 3.28

Read in a photo and convert to a matrix. Perform a singular value decomposition of the matrix. Reconstruct the photo using only 1,2,4, and 16 singular values.

1. Print the reconstructed photo. How good is the quality of the reconstructed photo?

2. What percent of the Frobenius norm is captured in each case?

Solution: (25 分)

样例 Matlab 代码: ·····(15′)

```
t=2; \%Here t is the number of sigular values that we want to use

A=imread('test.jpg');

A=double(A);

for i=1:3

[s,v,d]=svd((A(:,:,i)));

target(:,:,i) = s(:,:)*v(:,1:t)*d(:,1:t)';

sum1=norm(target(:,:,i), 'fro');

sum2=norm(A(:,:,i), 'fro');

end

ratio=sum1/sum2;

imwrite(uint8(target),'result4.jpg');
```

第一问: ·····(5′)

在保留前 1，2，4 个奇异向量时效果不怎么好，16 个的时候基本上能看出轮廓、分辨出问题。回答时只要提到“效果不怎么好”或者“基本能看出轮廓”或是其它与所提交的图片相匹配的结论均可。

只有图片没有评价不得分。

第二问: ·····(5′)

取决于具体的图片，正常范围在 80% 到 100% 之间，显然单调递增。具体数值取决于图片。

2 4.2

Does $lim_{t\to\infty}a(t)-a(t+1)=0$ *imply that* $a(t)$ *converges to some value? Hint: consider the average cumulative sum of the digits in the sequence* $10^2 1^4 0^8 1^{16}\ldots$

只要举出并论证了反例均可得到全部分数，举出反例 10 分，论证 15 分

Solution 1:

Let $a(t)=\sum_{i=1}^{t}\frac{1}{i}$, it's easy to verify that $a(t)$ goes to ∞, while $lim_{t\to\infty}a(t)-a(t+1)=lim_{t\to\infty}\frac{1}{t}=0$

Solution 2:

Consider the average cumulative sum of the digits in the sequence $10^2 1^4 0^8 1^{16}\ldots$

when $n=2^{2k}-1$, the average cumulative sum is $\frac{1}{3}$ (the number of 0 is always exactly 2 times the number of 1) ·····(5′)

when $n=2^{2k+1}-1$,the average cumulative sum goes to $\frac{2}{3}$ as $k\to\infty$ (the number of 1 is almost 2 times the number of 0) ·····(5′)

And $lim_{t\to\infty}a(t)-a(t+1)\le lim_{t\to\infty}\frac{2}{t}=0$ ·····(5′)

3 4.4

A Markov chain is said to be symmetric if for all i *and* j, $p_{ij}=p_{ji}$. *What is the stationary distribution of a connected symmetric chain? Prove your answer.*

答案 10 分，论证 15 分，没有说这个马氏链是连通的就说明 stationary distribution 是唯一的扣 5 分，没证唯一性扣 5 分（这两项不重复扣）（Theorem 4.2 和 Lemma 4.3 都必须在连通的情况下使用）

Solution:

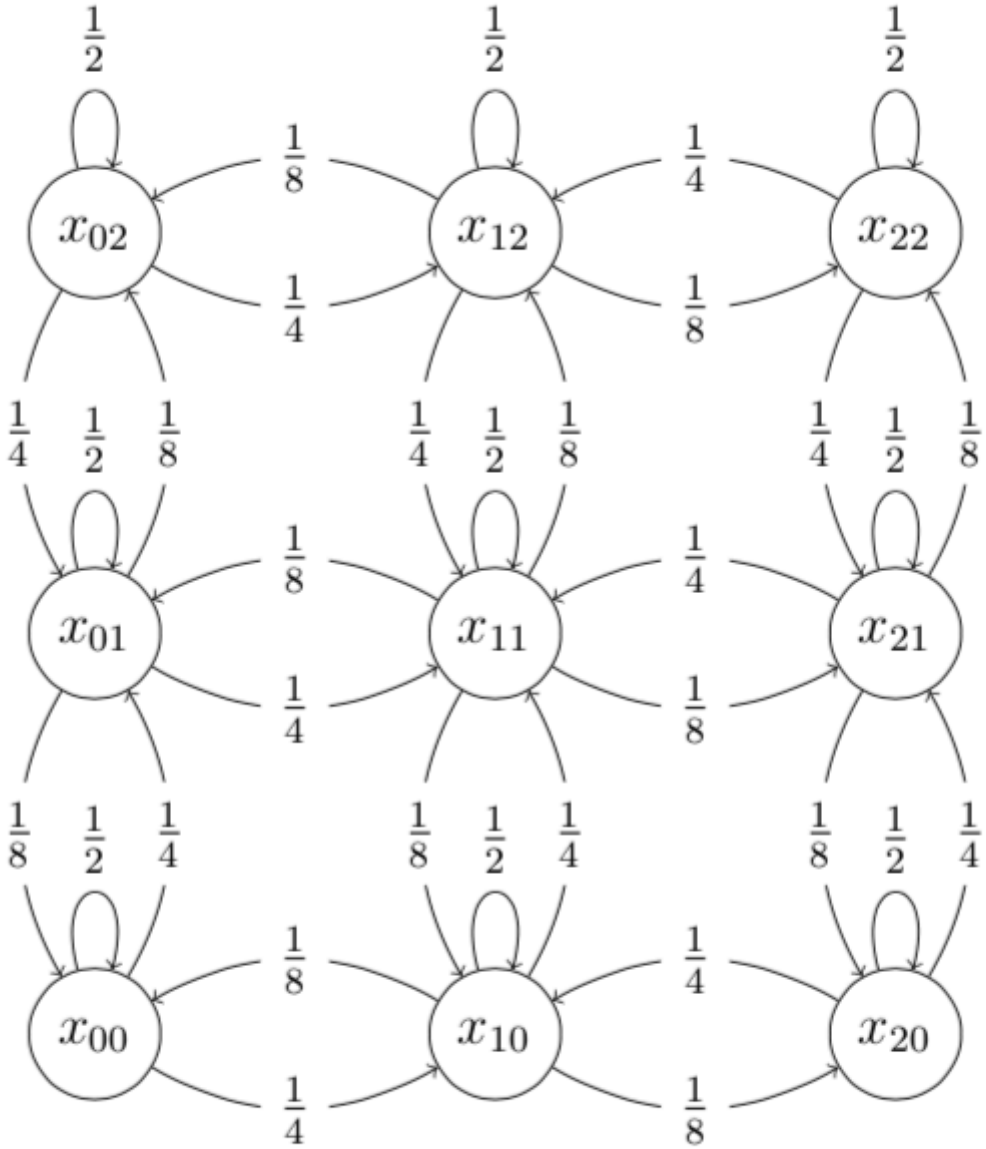
Because we are considering connected symmetric chain, $p_{ij}=p_{ji}$, and according to Lemma 4.3, a stationary distribution satisfies $p_i p_{ij}=p_j p_{ji}$ for all i and its adjacent vertex j. Since for adjacent pair i,j, we have $p_{ij}=p_{ji}\neq 0, p_i=p_j$ so $p_i=\frac{1}{n}$ for all i.

4 4.7

Using the Metropolis-Hasting Algorithm create a Markov chain whose stationary probability is that given in the following table. Use the 3×3 *lattice for the underlying graph.*

Solution:

手算或编程算均可，过程 5 分，结论 20 分，参考答案如下两图。如果采用了不同的 lattice，只要结论正确即可得到全部分数。


x_{02}
x_{12}
x_{22}
x_{01}
x_{11}
x_{21}
x_{00}
x_{10}
x_{20}

p	00	01	02	10	11	12	20	21	22
00	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{4}$	0	0	0	0	0
01	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{8}$	0	$\frac{1}{4}$	0	0	0	0
02	0	$\frac{1}{4}$	$\frac{1}{2}$	0	0	$\frac{1}{4}$	0	0	0
10	$\frac{1}{8}$	0	0	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{8}$	0	0
11	0	$\frac{1}{8}$	0	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{8}$	0	$\frac{1}{8}$	0
12	0	0	$\frac{1}{8}$	0	$\frac{1}{4}$	$\frac{1}{2}$	0	0	$\frac{1}{8}$
20	0	0	0	$\frac{1}{4}$	0	0	$\frac{1}{2}$	$\frac{1}{4}$	0
21	0	0	0	0	$\frac{1}{4}$	0	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{8}$
22	0	0	0	0	0	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$