



# 计算机视觉表征与识别

## Chapter 4: Template, Pyramid, and Filter Banks

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<http://mcg.nju.edu.cn/>



# Assignment 1



## Format requirements

- (i) Project materials including report template: `proj1.zip`
- (ii) Please use the provided `proj1_report.pptx` template to write your report, and convert the slide deck into a PDF for your submission. The report should contain your name, student ID, and e-mail address;
- (iii) You should choose python to write your code, and provide a README file to describe how to execute the code;
- (iv) Pack your `proj1_report.pdf`, `proj1_code` and `README` into a zip file, named with your student ID, like MG1933001.zip. If you have an improved version, add an extra '\_' with a number, like MG1933001\_1.zip. We will take the final submitted version as your results.
- (v) Do **not** install any additional packages inside the conda environment. The TAs will use the same environment as defined in the config files we provide you, so anything that's not in there by default will probably cause your code to break during grading. Do not use absolute paths in your code or your code will break. Use relative paths like the starter code already does. Failure to follow any of these instructions will lead to point deductions.



# Assignment 1



## Submission Way

- (i) Please submit your results to email [cvcourse.nju@gmail.com](mailto:cvcourse.nju@gmail.com) , the email subject is "Assignment 1";
- (ii) The deadline is 23:59 on April 10, 2020. No submission after this deadline is acceptable.

## About Plagiarize

DO NOT PLAGIARIZE! We have no tolerance for plagiarizing and will penalize it with giving zero score. You may refer to some others' materials, please make citations such that one can tell which part is actually yours.

## Evaluation Criterion

We mainly evaluate your submission according to your code and report. Efficient implementation, elegant code style, concise and logical report are all important factors towards a high score.



# Image filtering



- Compute a function of the local neighborhood at each pixel in the image
  - Function specified by a “filter” or mask saying how to combine values from neighbors.
- Uses of filtering:
  - Enhance an image (denoise, resize, etc)
  - Extract information (texture, edges, etc)
  - Detect patterns (template matching)



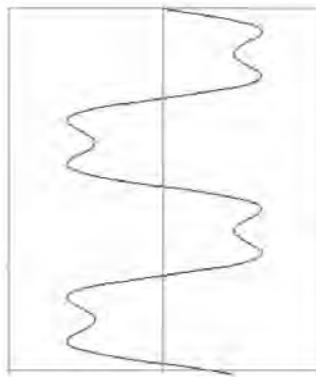
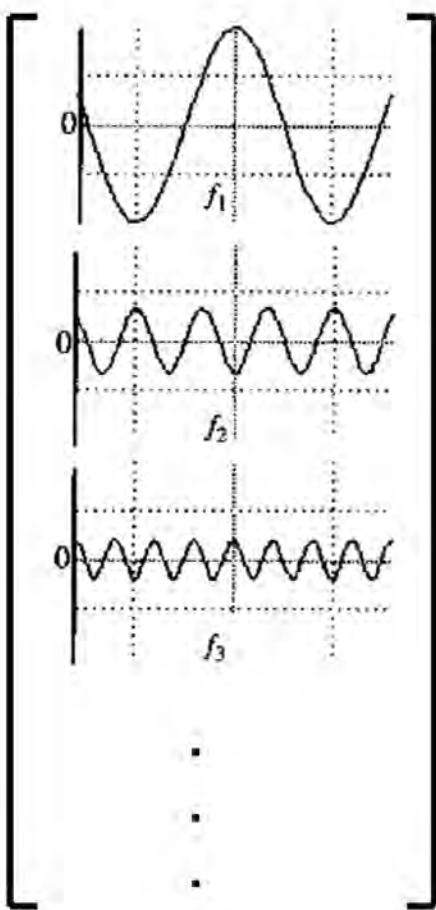
# Three views of filtering



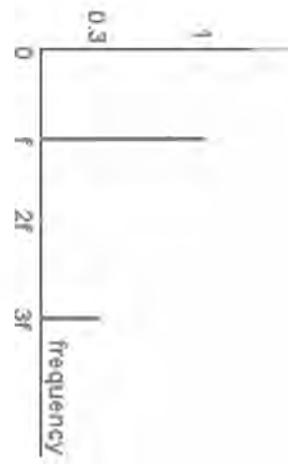
- Image filters in spatial domain
  - Filter is a mathematical operation on values of each patch
  - Smoothing, sharpening, measuring texture
- **Image filters in the frequency domain**
  - Filtering is a way to modify the frequencies of images
  - Denoising, sampling, image compression
- Templates and Image Pyramids
  - Filtering is a way to match a template to the image
  - Detection, coarse-to-fine registration

# Fourier series: just a change of basis

$$M \quad f(x) = F(\omega)$$



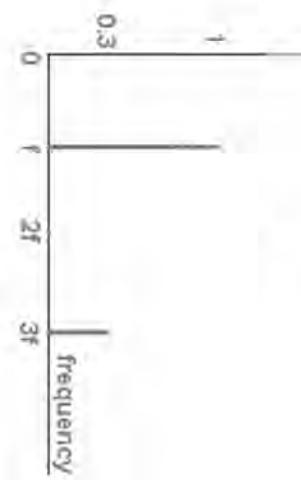
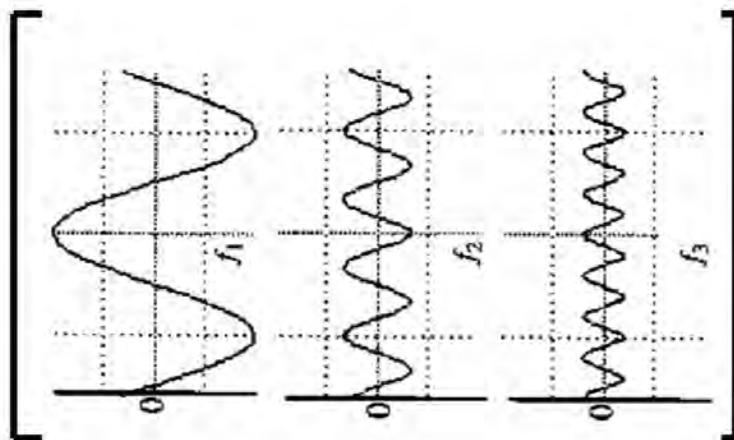
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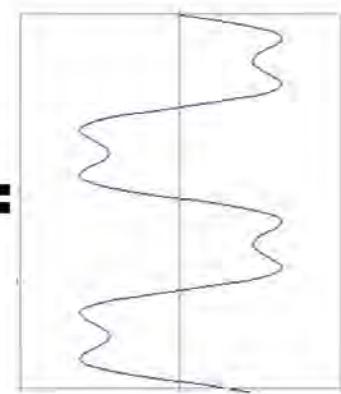
# Inverse FT: Just a change of basis

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$$\mathcal{M}^{-1} F(\omega) = f(x)$$



=



# The Discrete Fourier transform

Discrete Fourier Transform (DFT) transforms a signal  $f[n]$  into  $F[u]$  as:

$$F[u] = \sum_{n=0}^{N-1} f[n] \exp\left(-2\pi j \frac{un}{N}\right)$$

The inverse of the DFT is:

$$f[n] = \frac{1}{N} \sum_{u=0}^{N-1} F[u] \exp\left(2\pi j \frac{un}{N}\right)$$

The signal  $f[n]$  is a weighted linear combination of complex exponentials with weights  $F[u]$

# The Discrete Fourier transform

Discrete Fourier Transform (DFT) transforms a signal  $f[n]$  into  $F[u]$  as:

$$F[u] = \sum_{n=0}^{N-1} f[n] \exp\left(-2\pi j \frac{un}{N}\right)$$

Discrete Fourier Transform (DFT) is a linear operator. Therefore, we can write:

$$\mathbf{F} = \begin{matrix} & \xrightarrow{n} \\ \mathbf{u} & \downarrow \\ \text{exp}\left(-2\pi j \frac{un}{N}\right) \end{matrix}$$

**NxN array**

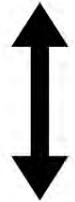
Lets visualize the transform coefficients

69

# Visualizing the Fourier transform

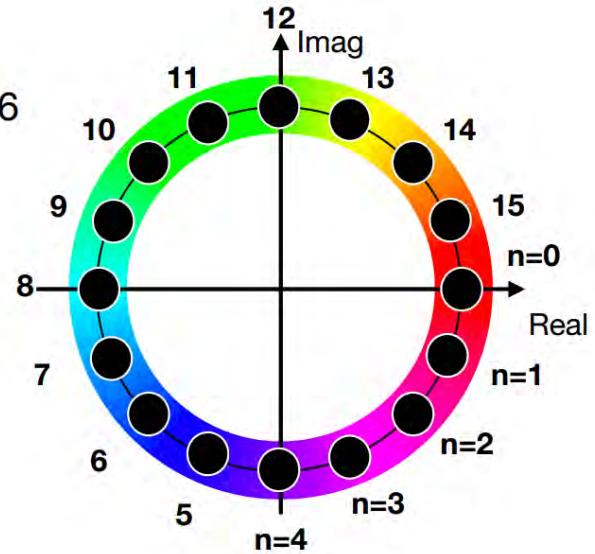
$$F[u] = \sum_{n=0}^{N-1} f[n] \exp\left(-2\pi j \frac{un}{N}\right)$$

$$\exp(\alpha j) = \cos(\alpha) + j \sin(\alpha)$$



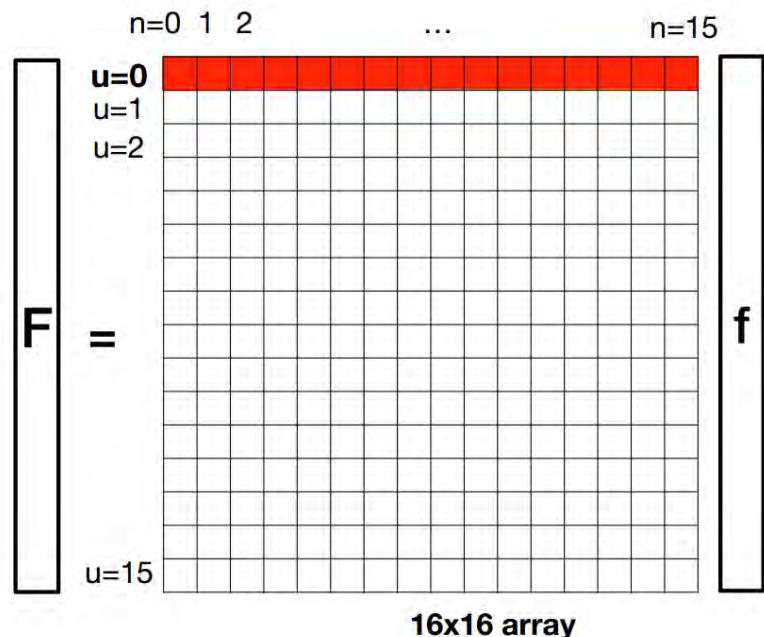
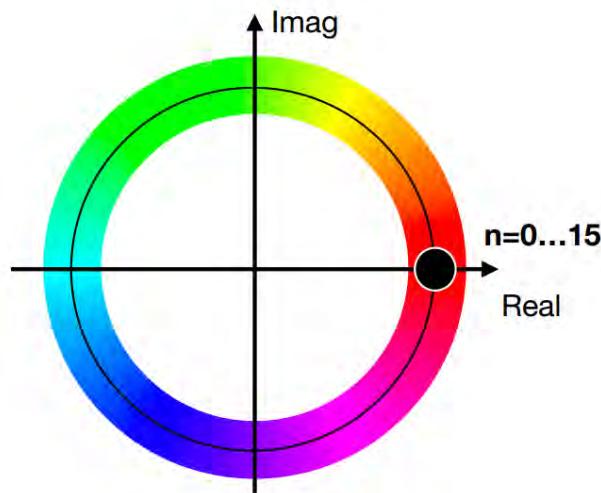
$$\cos\left(2\pi \frac{un}{N}\right) - j \sin\left(2\pi \frac{un}{N}\right)$$

For:  
u=1  
N=16



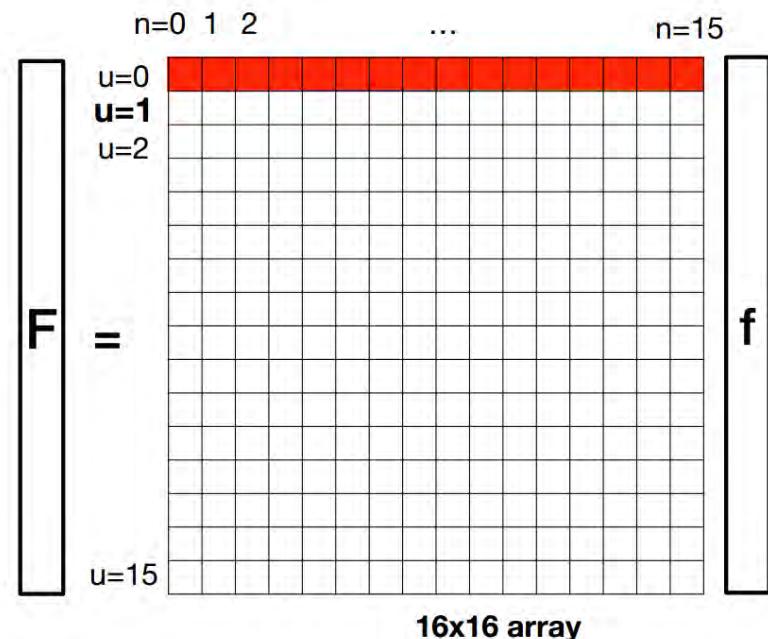
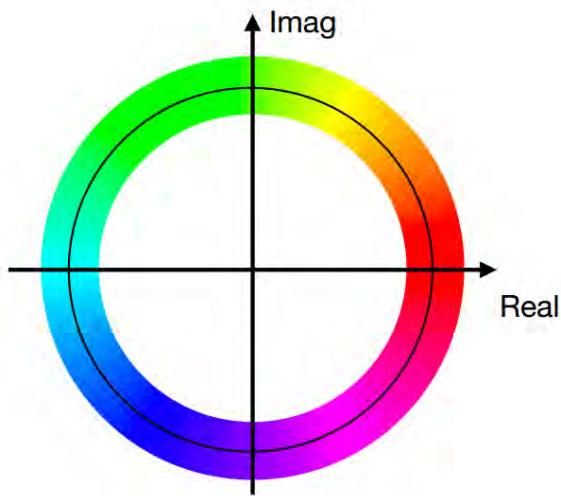
# Visualizing the transform coefficients

$$\exp\left(-2\pi j \frac{un}{N}\right) \quad \text{For } N=16$$



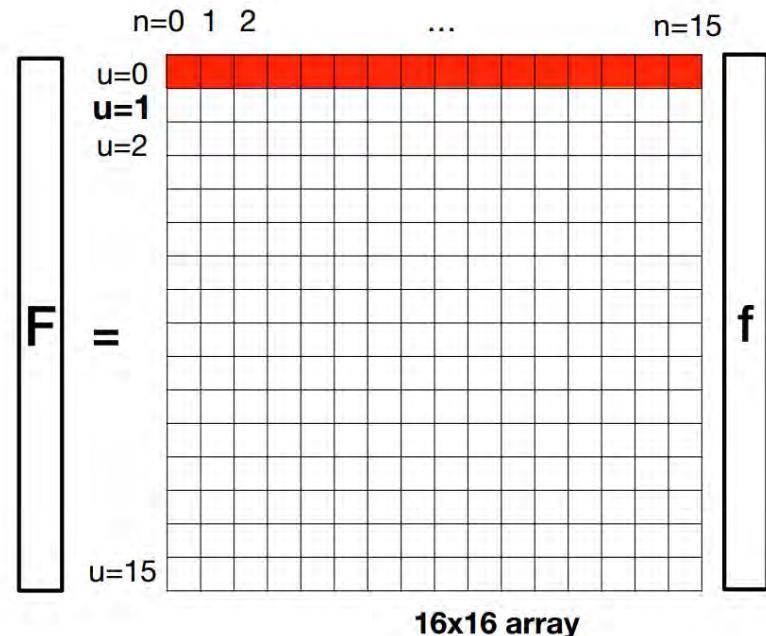
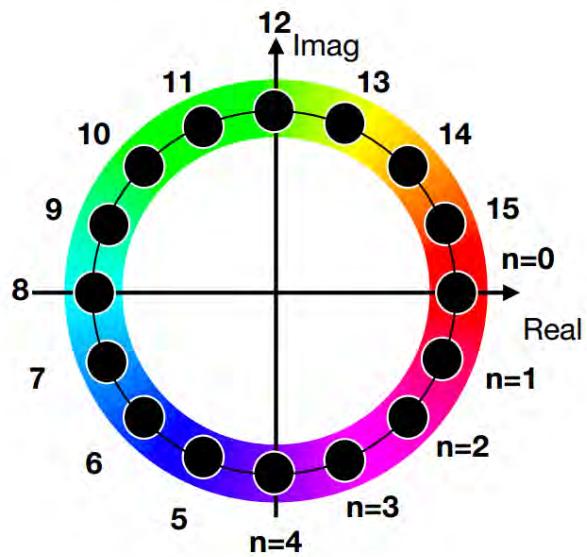
# Visualizing the transform coefficients

$$\exp\left(-2\pi j \frac{un}{N}\right) \quad \text{For } N=16$$



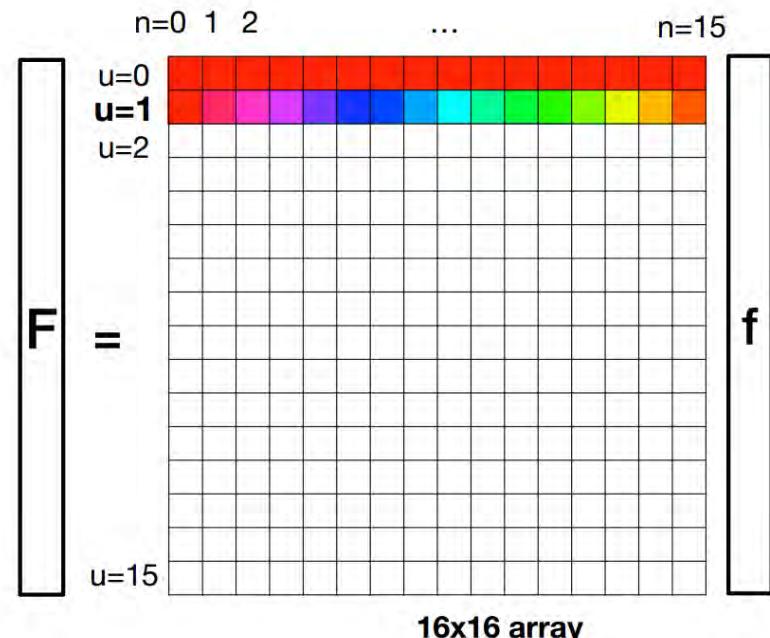
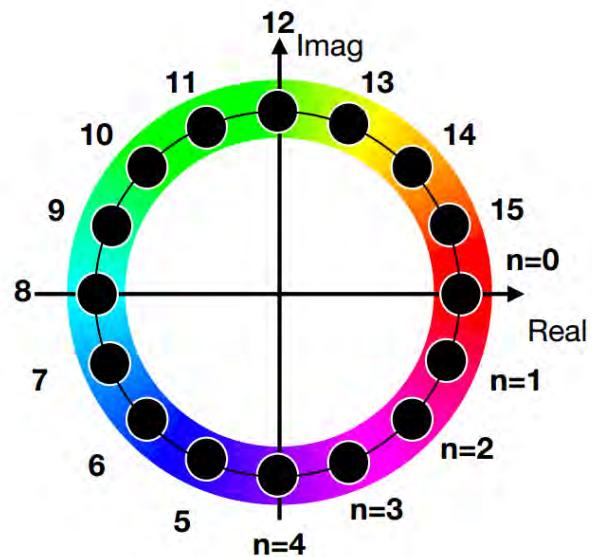
# Visualizing the transform coefficients

$$\exp\left(-2\pi j \frac{un}{N}\right) \quad \text{For } N=16$$



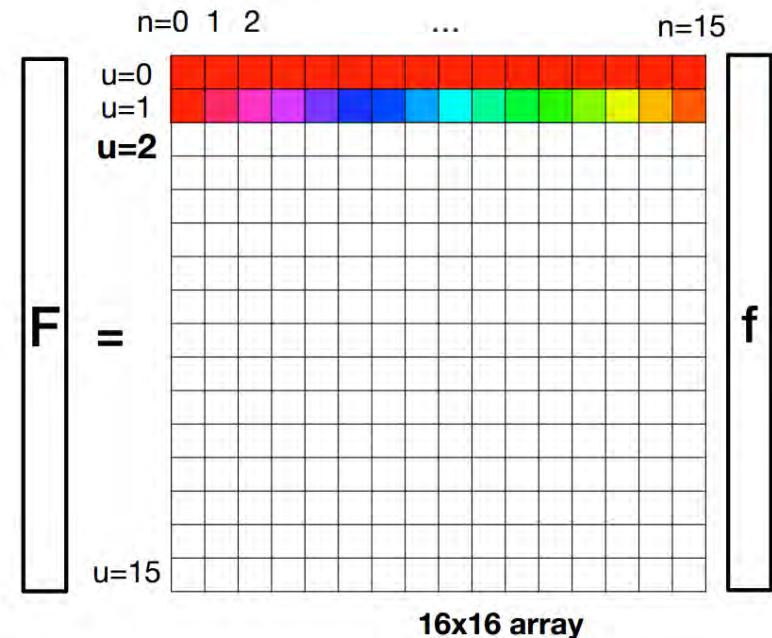
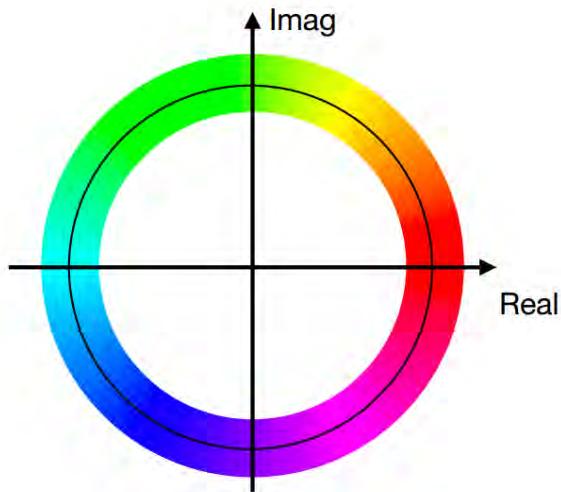
# Visualizing the transform coefficients

$$\exp\left(-2\pi j \frac{un}{N}\right) \quad \text{For } N=16$$



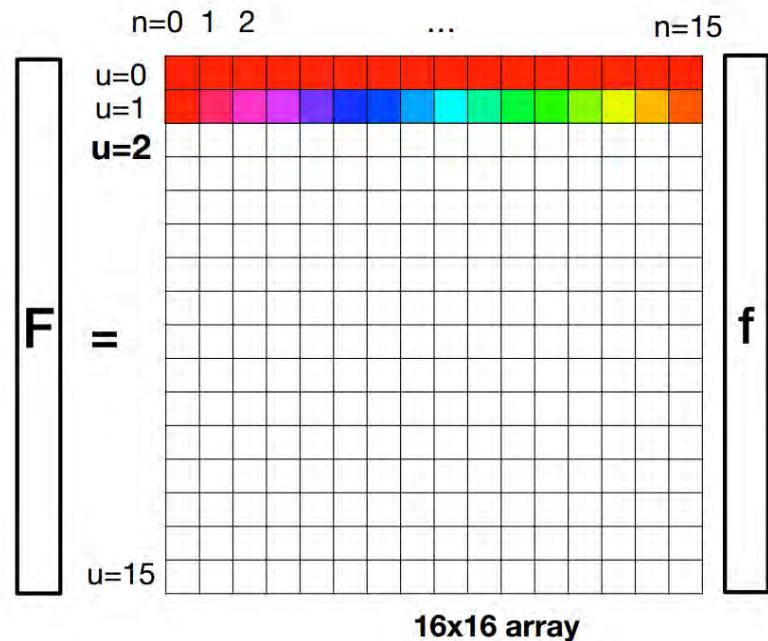
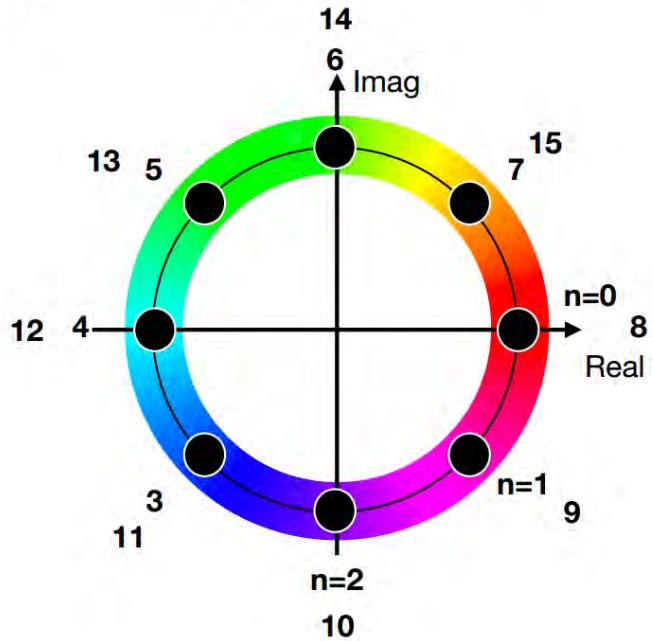
# Visualizing the transform coefficients

$$\exp\left(-2\pi j \frac{un}{N}\right) \quad \text{For } N=16$$



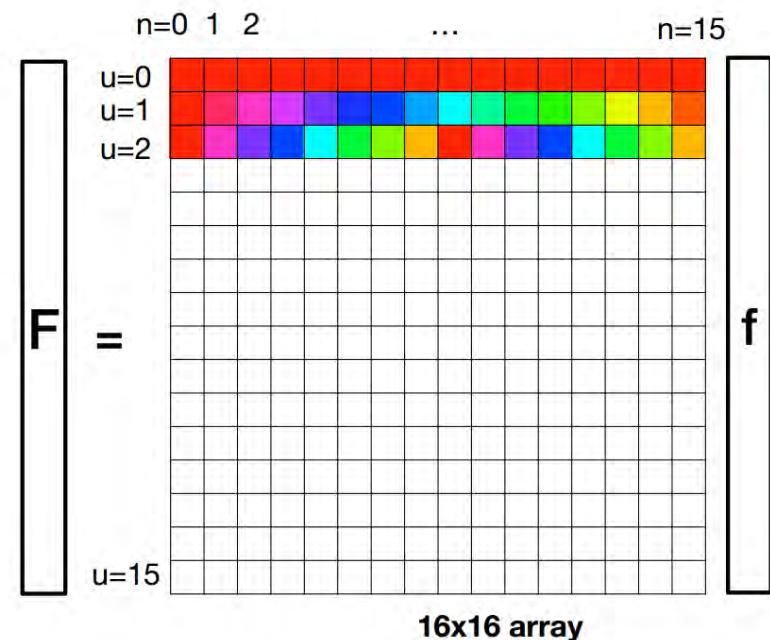
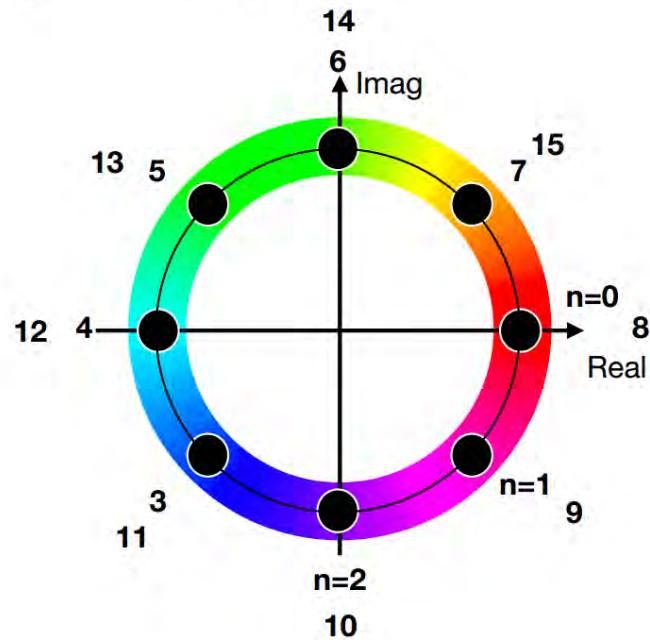
# Visualizing the transform coefficients

$$\exp\left(-2\pi j \frac{un}{N}\right) \quad \text{For } N=16$$



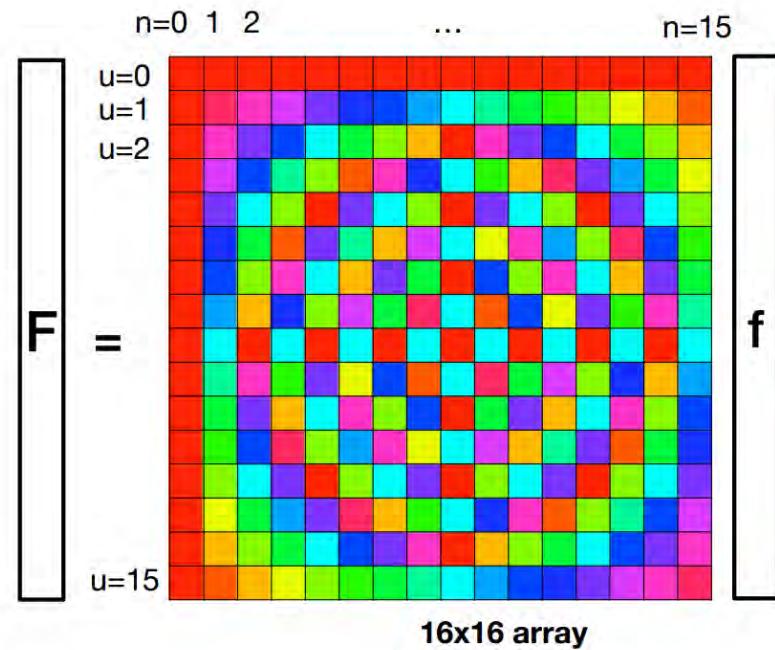
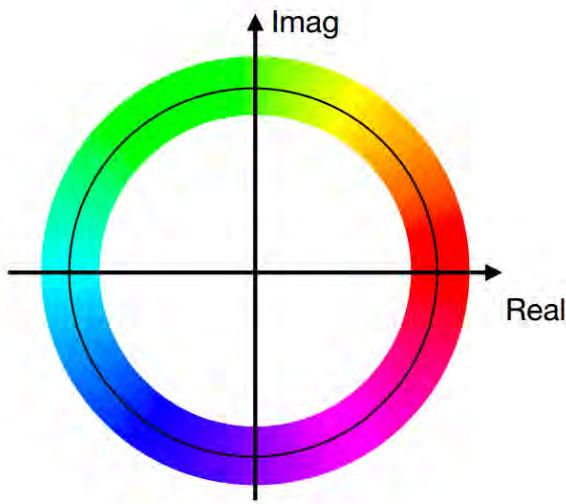
# Visualizing the transform coefficients

$$\exp\left(-2\pi j \frac{un}{N}\right) \quad \text{For } N=16$$



# Visualizing the transform coefficients

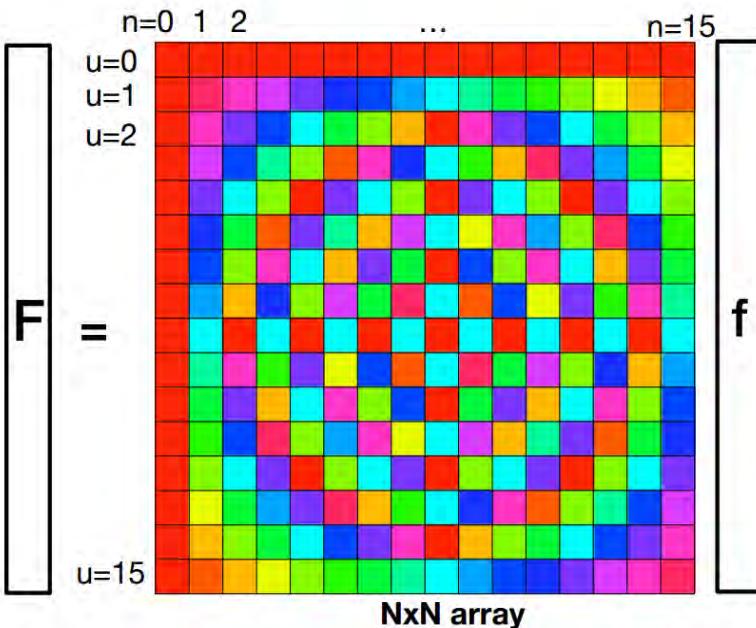
$$\exp\left(-2\pi j \frac{un}{N}\right) \quad \text{For } N=16$$



# The inverse of the Discrete Fourier transform

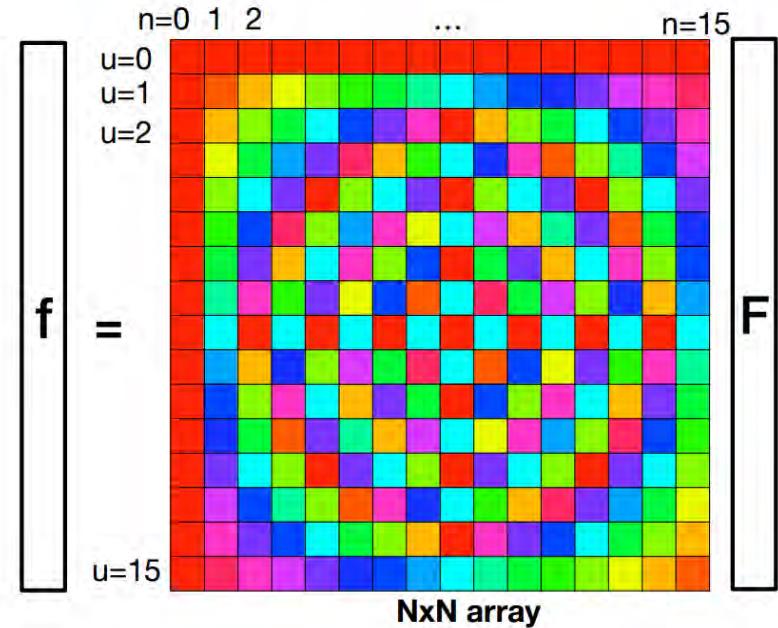
Discrete Fourier Transform (DFT):

$$F[u] = \sum_{n=0}^{N-1} f[n] \exp\left(-2\pi j \frac{un}{N}\right)$$



Its inverse:

$$f[n] = \frac{1}{N} \sum_{u=0}^{N-1} F[u] \exp\left(2\pi j \frac{un}{N}\right)$$



# Summary

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The spatial function  $f(x, y)$

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} du dv$$

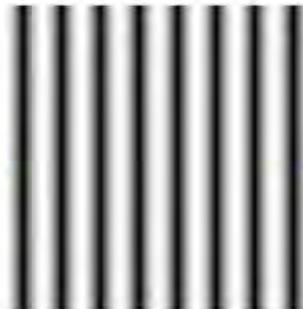
is decomposed into a weighted sum of 2D orthogonal basis functions in a similar manner to decomposing a vector onto a basis using scalar products.

$f(x, y)$

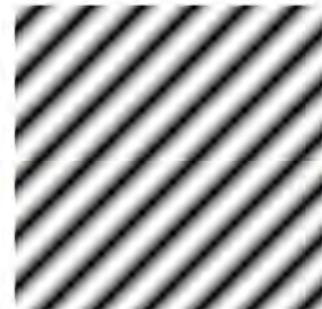


$= \alpha$

$+ \beta$



$+ \gamma$



$+ \dots$

---

## 2-D DFT

变换对公式：1D → 2D推广

$$F(u, v) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) \exp[-j2\pi(\frac{ux}{M} + \frac{vy}{N})]$$

$$f(x, y) = \frac{1}{\sqrt{MN}} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) \exp[j2\pi(\frac{ux}{M} + \frac{vy}{N})]$$

频谱 (幅度)

$$|F(u, v)| = \left[ R^2(u, v) + I^2(u, v) \right]^{1/2}$$

相位角

$$\phi(u, v) = \arctan[I(u, v)/R(u, v)]$$

功率谱

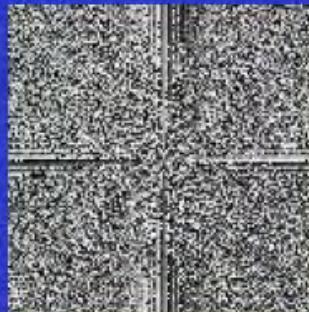
$$P(u, v) = |F(u, v)|^2 = R^2(u, v) + I^2(u, v)$$



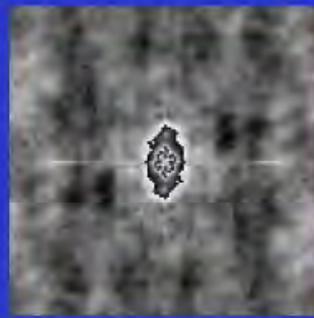
原图像



幅度谱



相位谱



由幅度谱重建

相位谱为0



由相位谱重建

幅度谱为常数

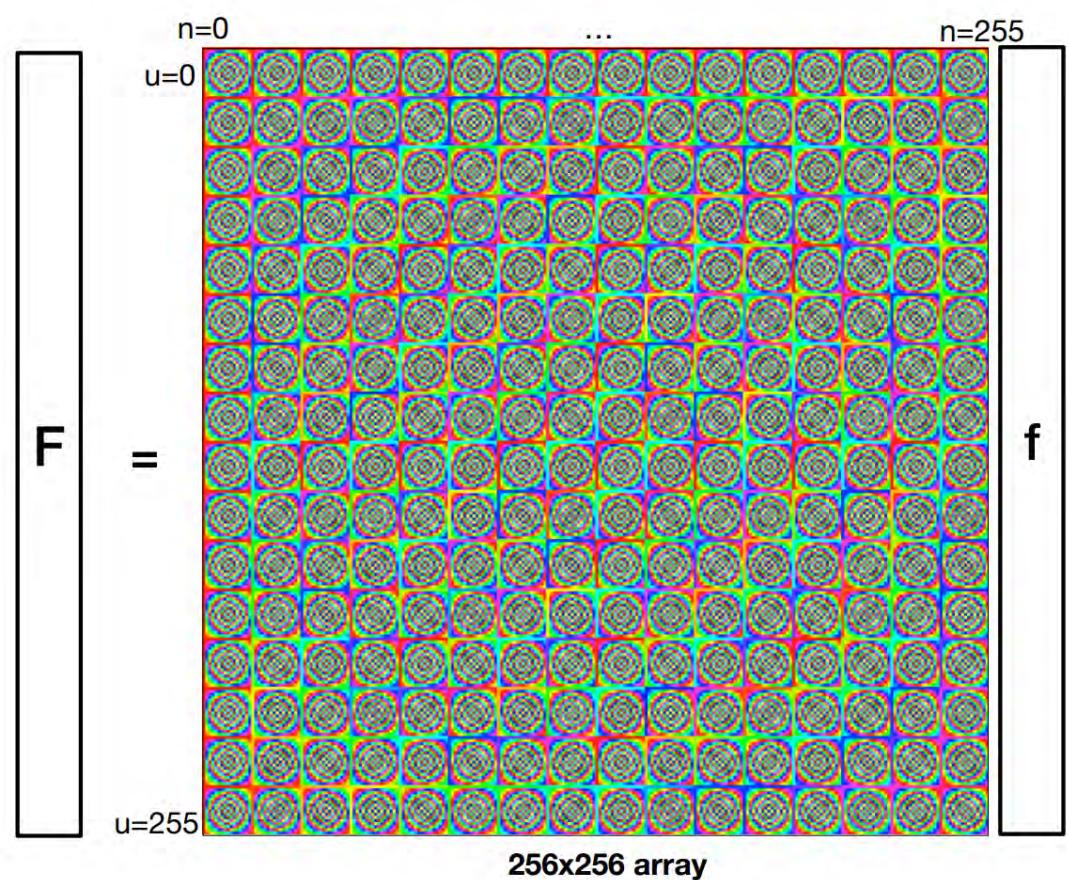


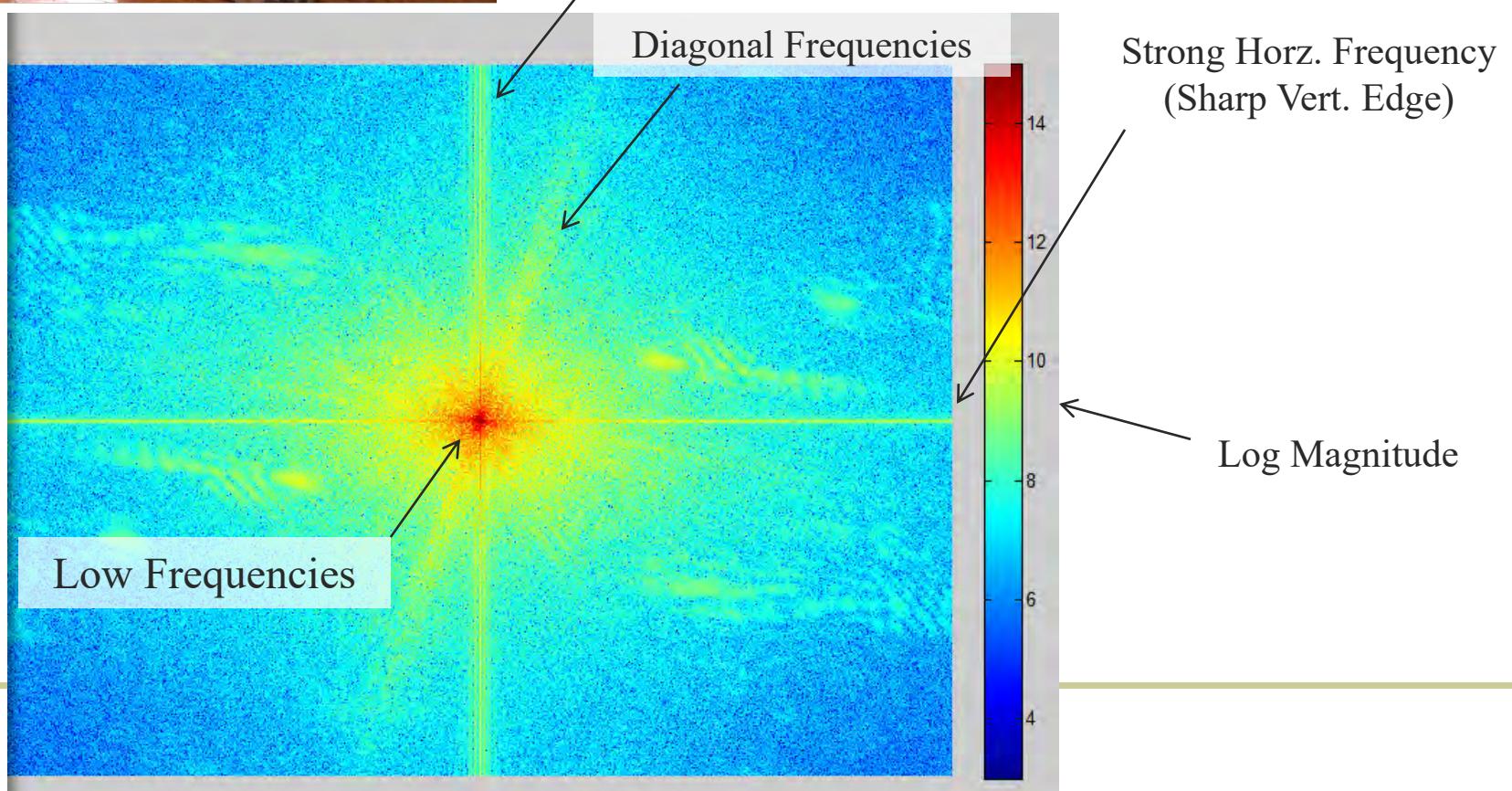
结论：相位谱可能具有更重要的应用

# Visualizing the 2D DFT coefficients

$$F[u, v] = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} f[n, m] \exp\left(-2\pi j \left(\frac{un}{N} + \frac{vm}{M}\right)\right)$$

For N=M=16







# The Convolution Theorem



- The Fourier transform of the convolution of two functions is the product of their Fourier transforms

$$\mathcal{F}[g * h] = \mathcal{F}[g]\mathcal{F}[h]$$

- The inverse Fourier transform of the product of two Fourier transforms is the convolution of the two inverse Fourier transforms

$$\mathcal{F}^{-1}[gh] = \mathcal{F}^{-1}[g] * \mathcal{F}^{-1}[h]$$

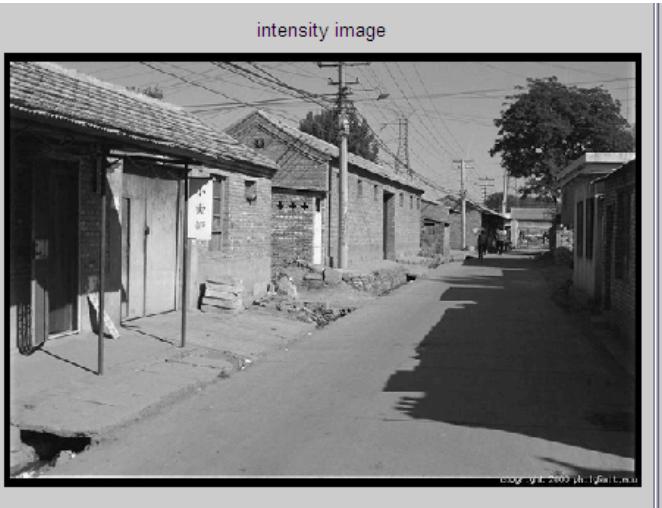
- **Convolution** in spatial domain is equivalent to **multiplication** in frequency domain!



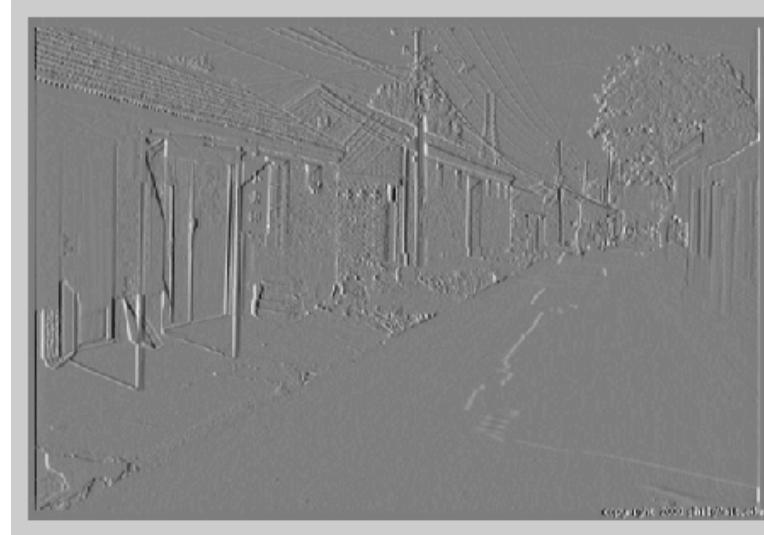
# Filtering in spatial domain



1	0	-1
2	0	-2
1	0	-1



$$\ast \quad = \quad$$
A convolution operation diagram showing a 3x3 kernel (represented by a yellow asterisk) multiplied by a 3x3 input patch (represented by a gray square divided into a 3x3 grid of colored squares), resulting in a single output value.

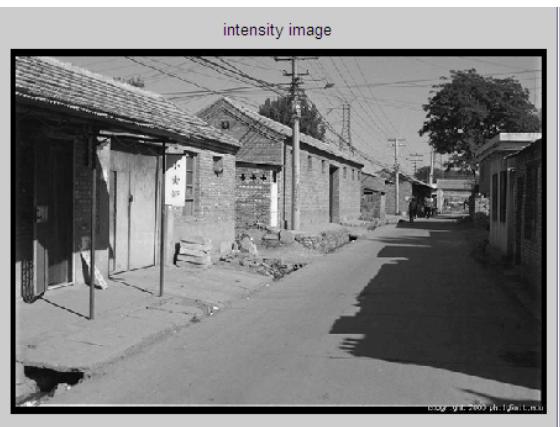




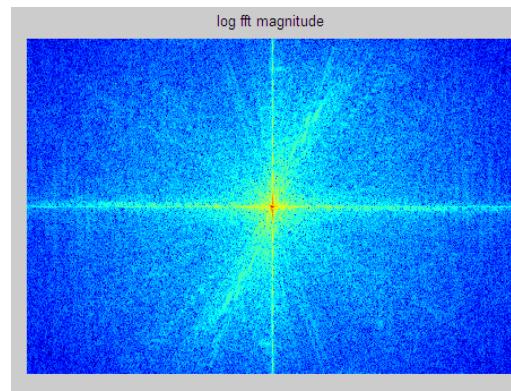
# Filtering in frequency domain



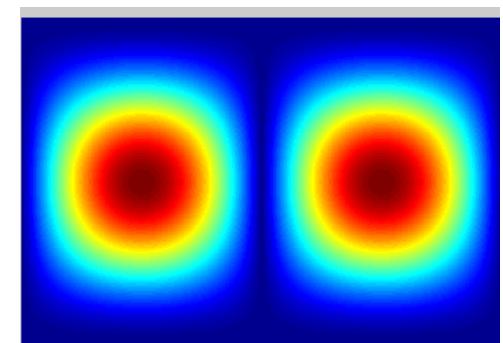
FFT



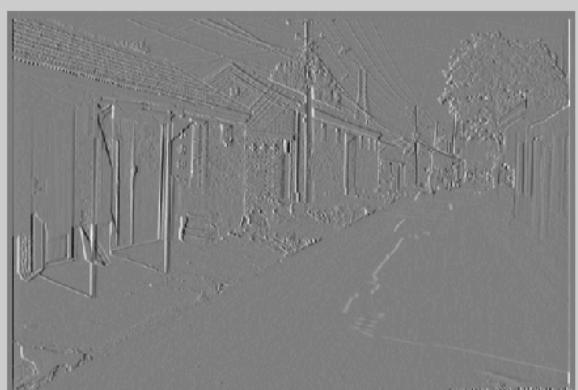
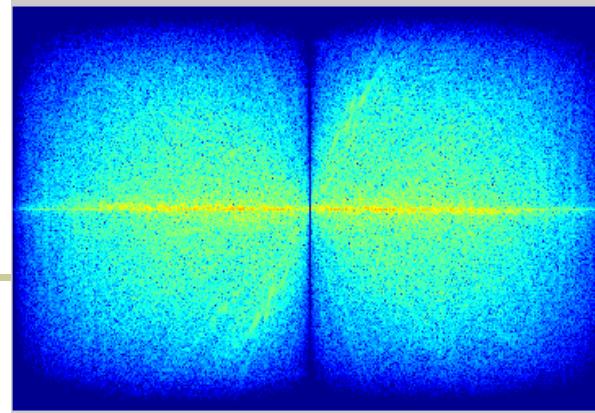
FFT



||



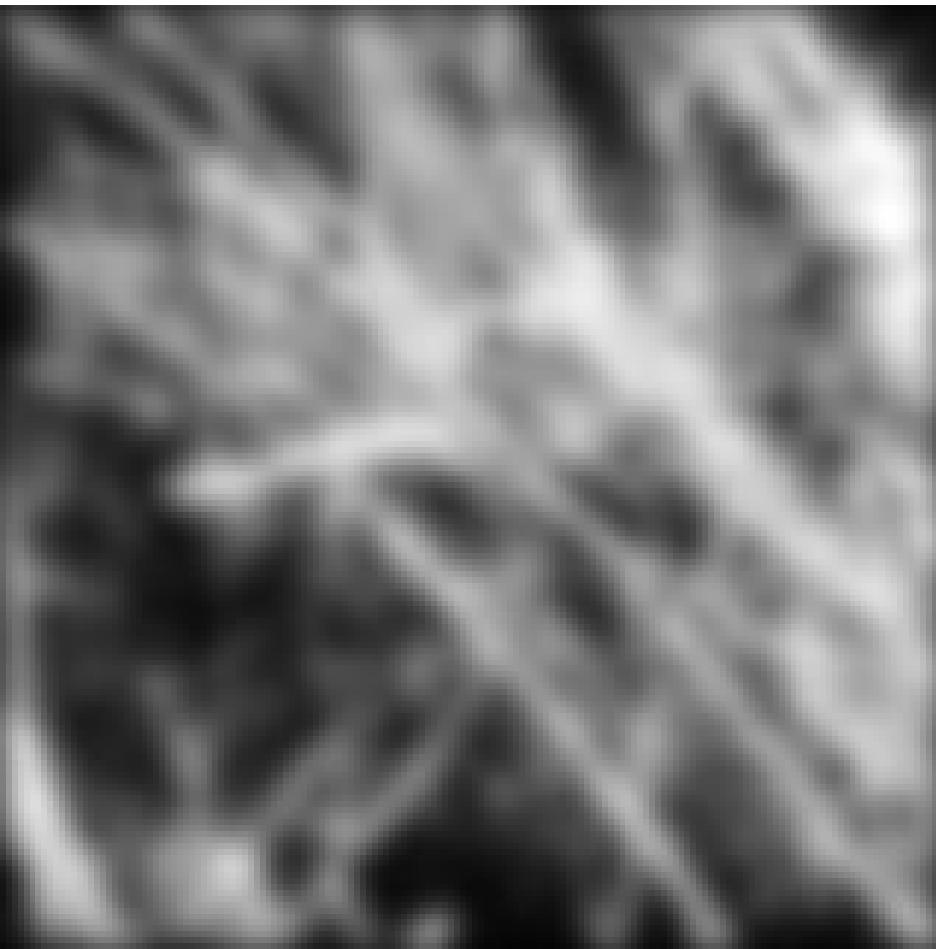
Inverse FFT



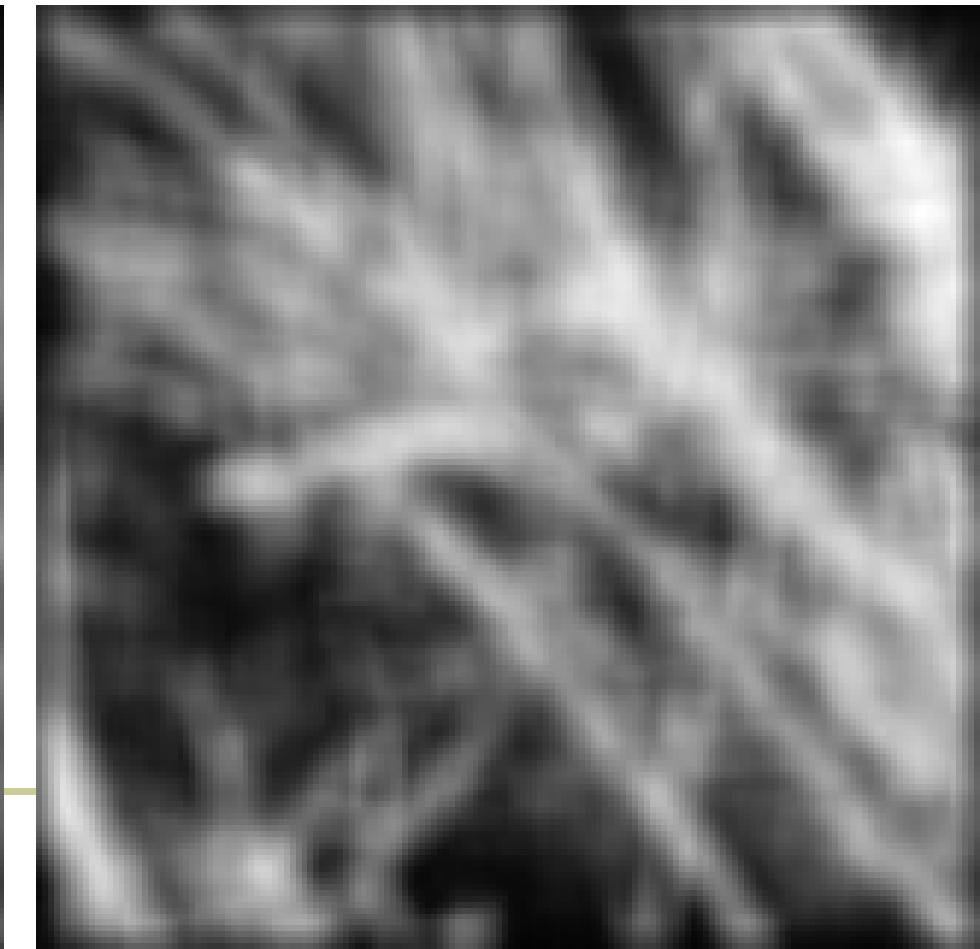


# Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

Gaussian

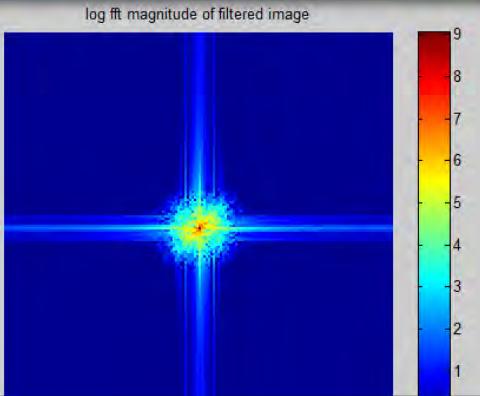
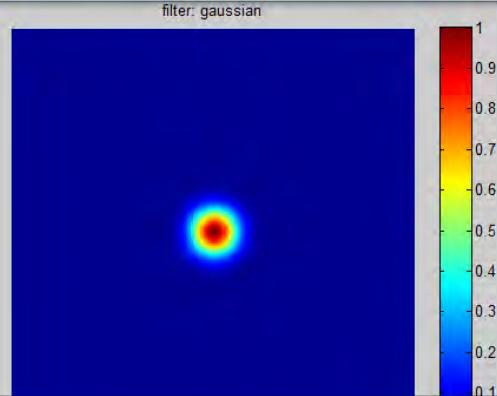
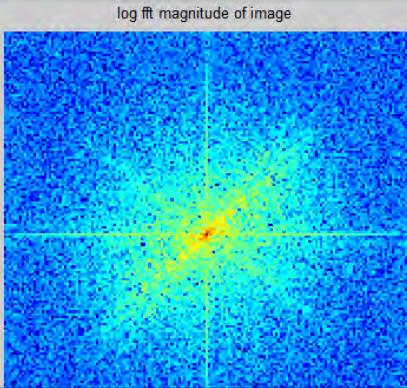
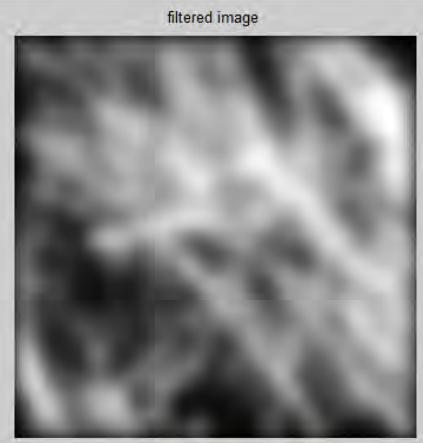


Box filter



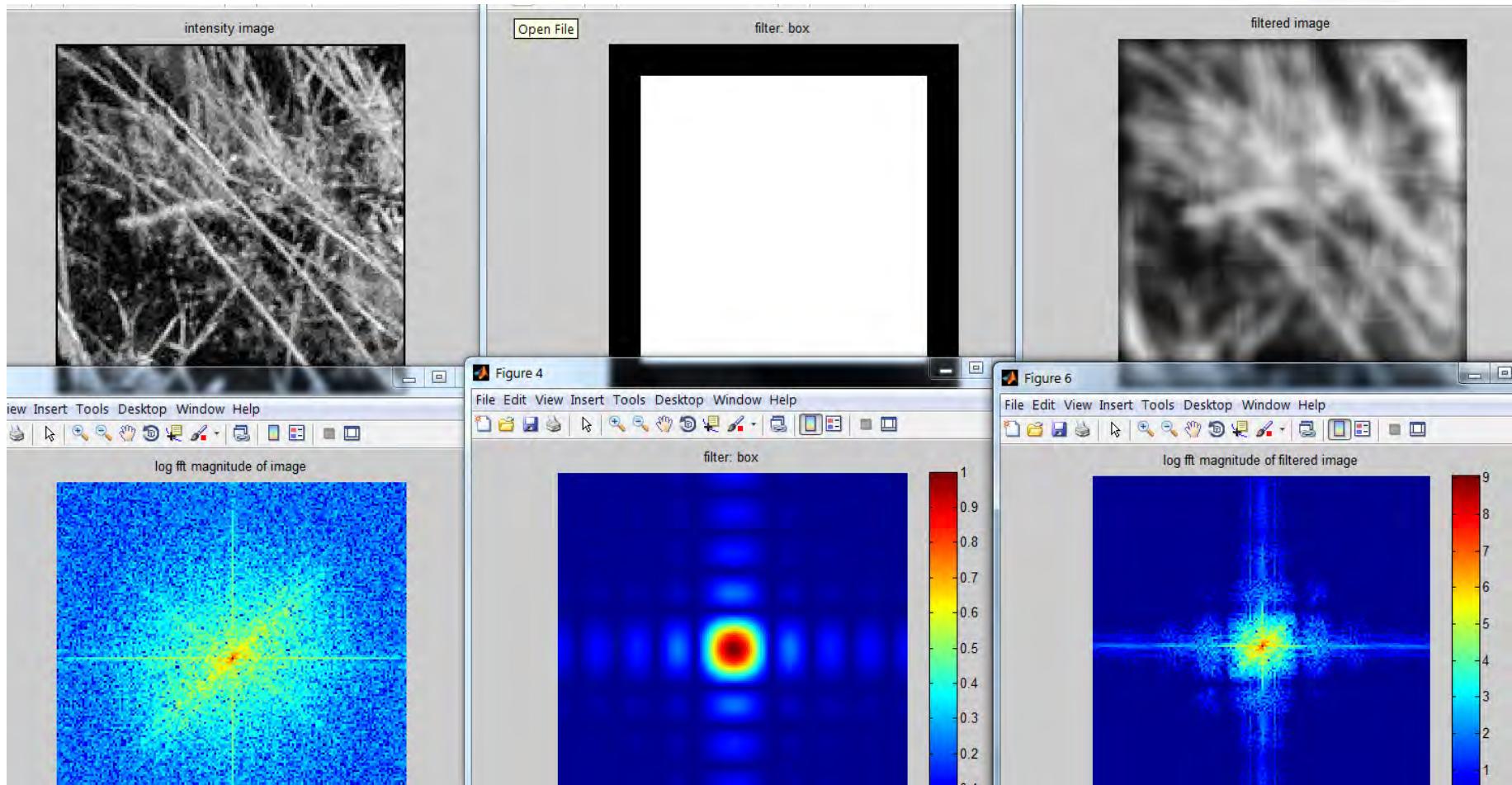


# Gaussian Filter





# Box Filter





# Sampling



**Why does a lower resolution image still make sense to us? What do we lose?**

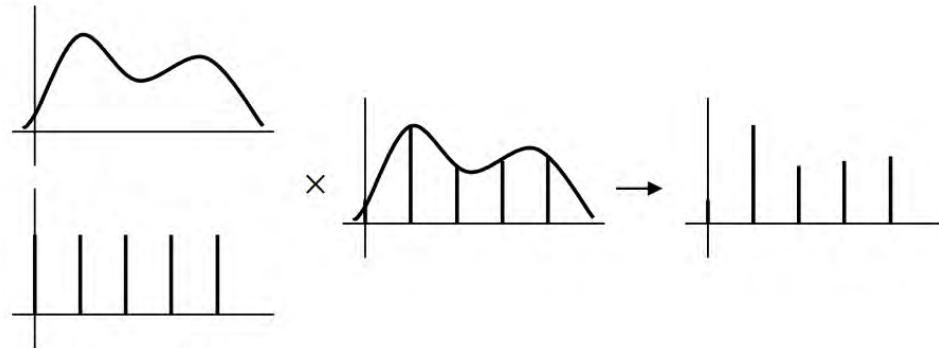




# Fourier Interpretation: Sampling



- Sampling in the spatial domain is like multiplying with a spike function.



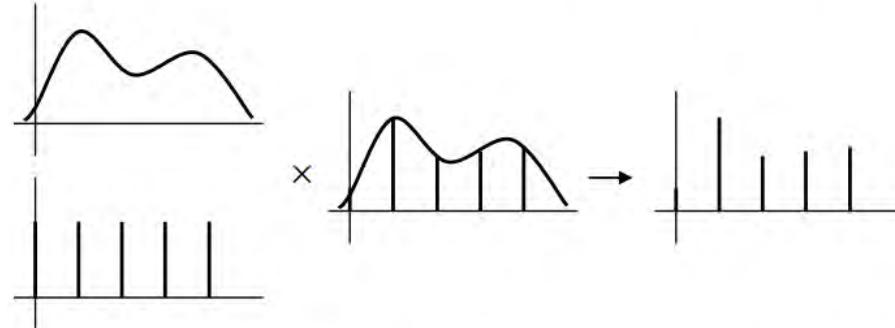
- Sampling in the frequency domain is like...

?

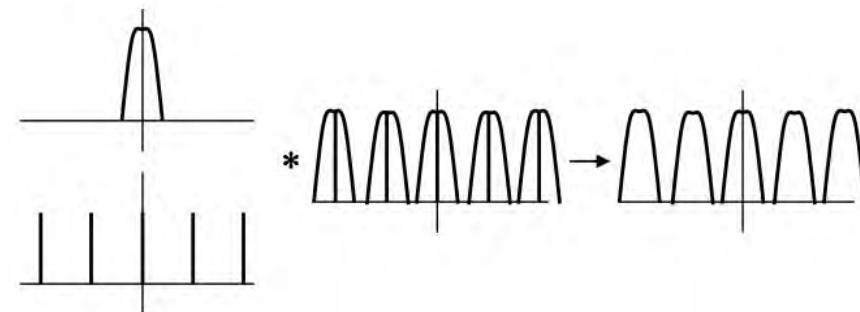


# Fourier Interpretation: Sampling

- Sampling in the spatial domain is like multiplying with a spike function.

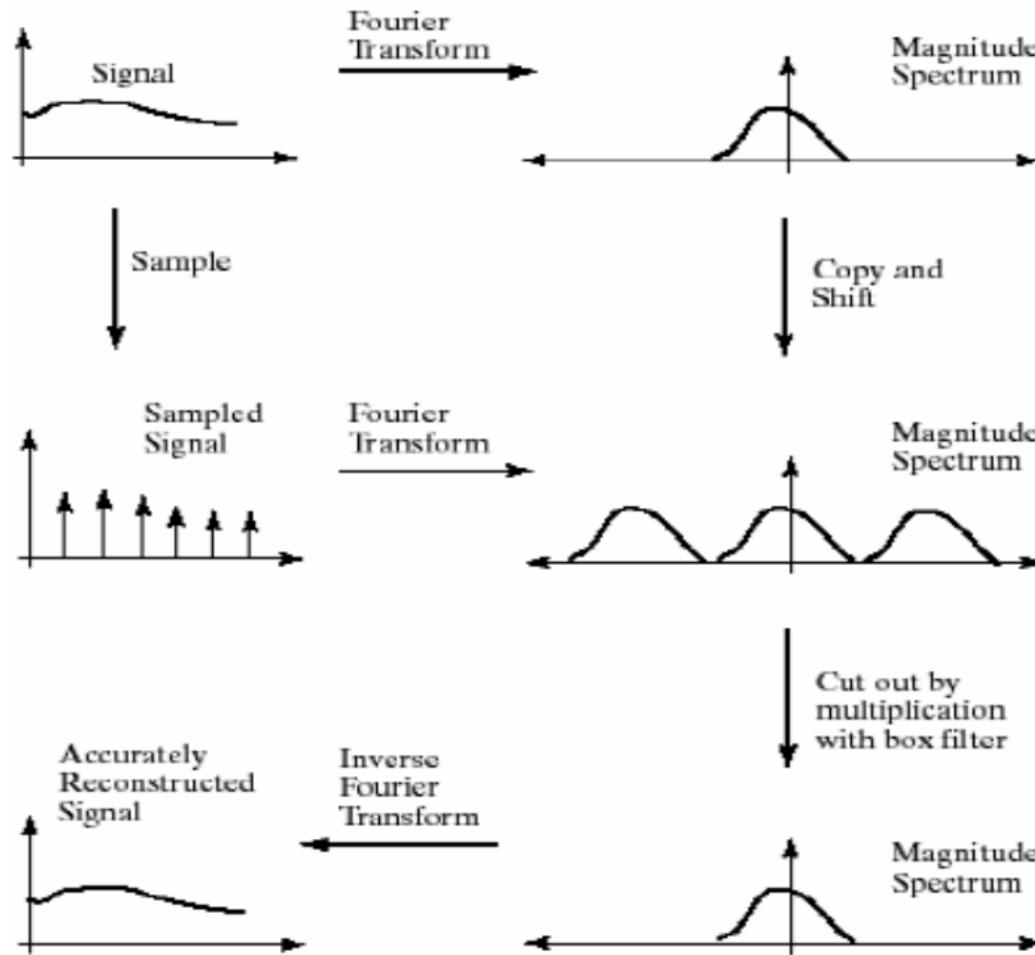


- Sampling in the frequency domain is like convolving with a spike function.



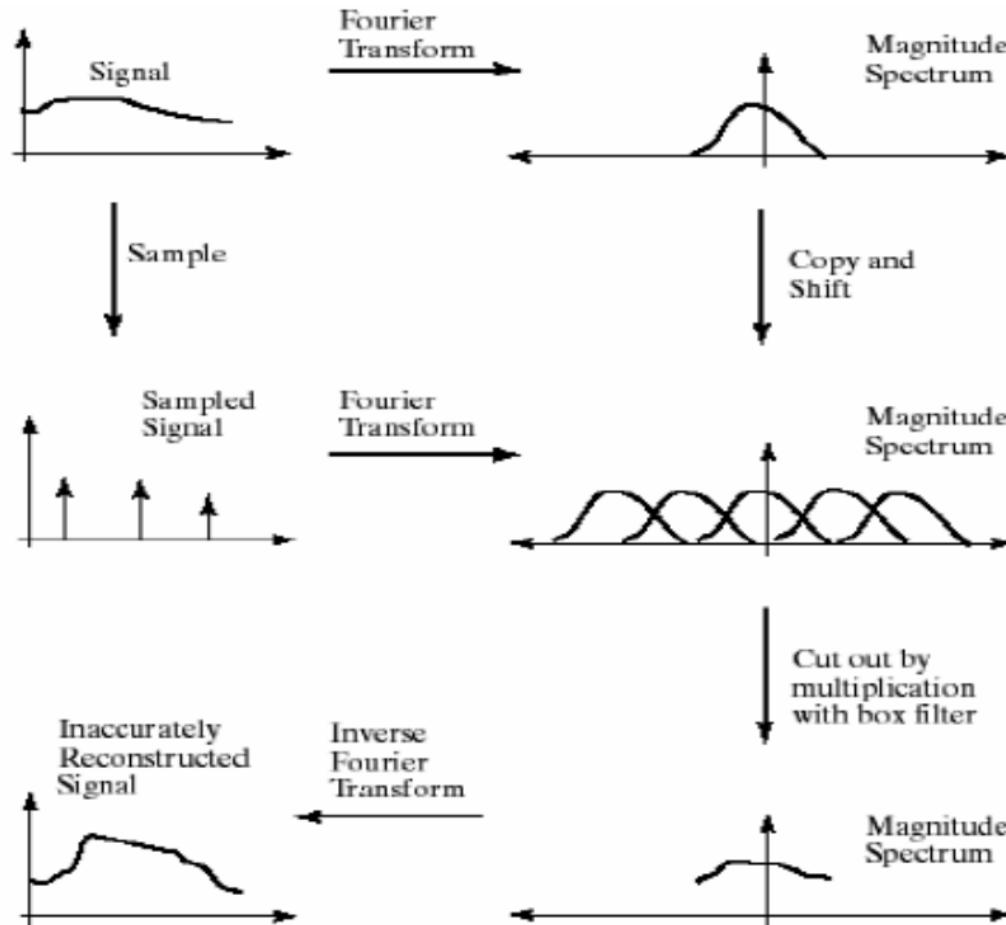


# Fourier Interpretation: Sampling





# Fourier Interpretation: Sampling





# Sampling and aliasing



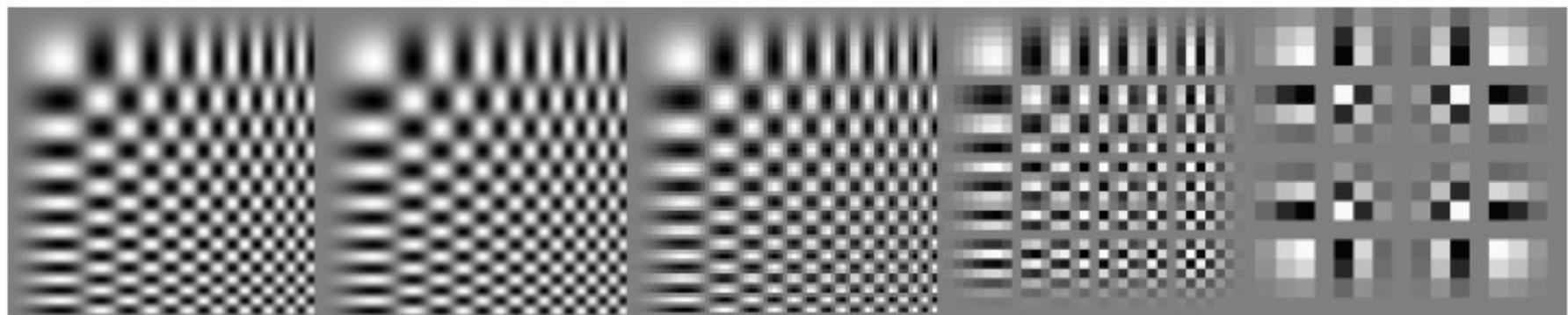
256x256

128x128

64x64

32x32

16x16





# Anti-aliasing

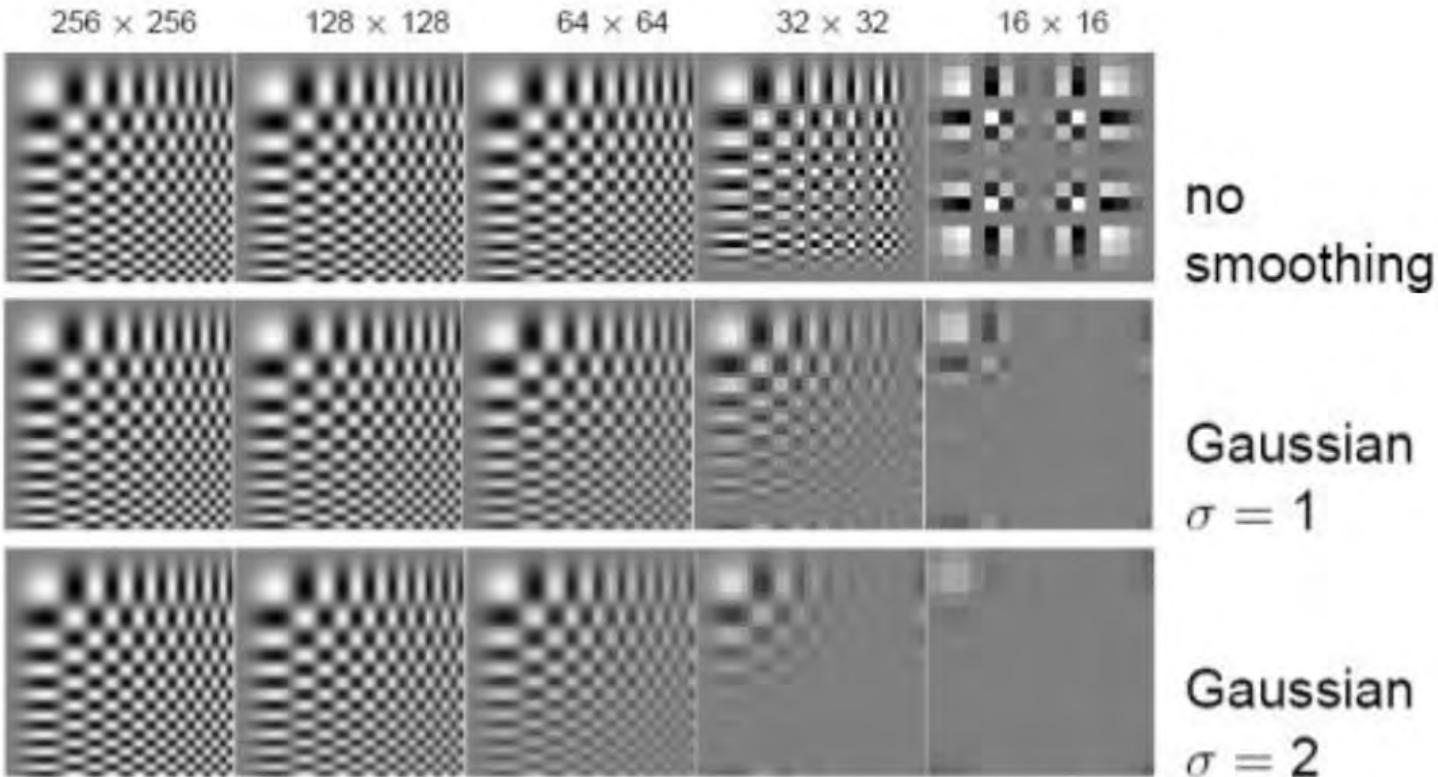


## Solutions:

- Sample more often
- Get rid of all frequencies that are greater than half the new sampling frequency
  - Will lose information
  - But it's better than aliasing
  - Apply a smoothing filter



# Anti-aliasing



Note: We cannot recover the high frequencies, but we can avoid artifacts by smoothing before resampling.



# Three views of filtering



- Image filters in spatial domain
  - Filter is a mathematical operation on values of each patch
  - Smoothing, sharpening, measuring texture
- Image filters in the frequency domain
  - Filtering is a way to modify the frequencies of images
  - Denoising, sampling, image compression
- **Templates and Image Pyramids**
  - Filtering is a way to match a template to the image
  - Detection, coarse-to-fine registration



# Today's class



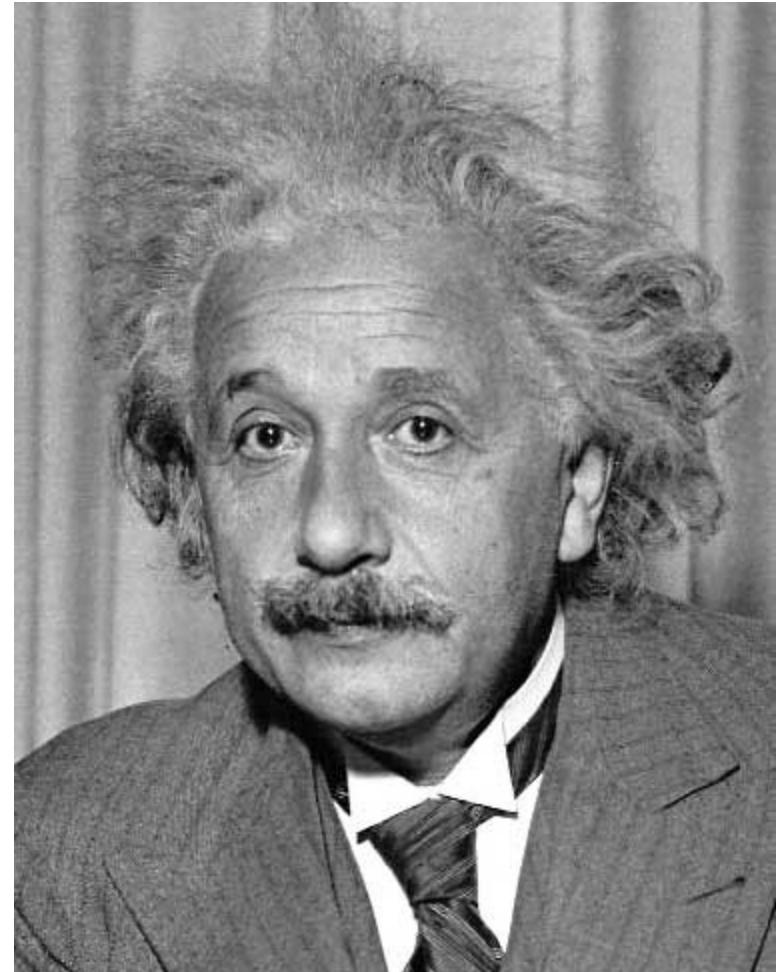
- **Template matching**
- **Gaussian Pyramids**
  - Application for recognition
  - Pyramids representation in deep learning
- **Laplacian Pyramids**
  - Application for image blending
  - Hybrid images
- **Steerable pyramids:**
  - Filter banks and texture analysis



# Template matching



- Goal: find in image
- Main challenge: What is a good similarity or distance measure between two patches?
  - Correlation
  - Zero-mean correlation
  - Sum Square Difference
  - Normalized Cross Correlation





# Matching with filters

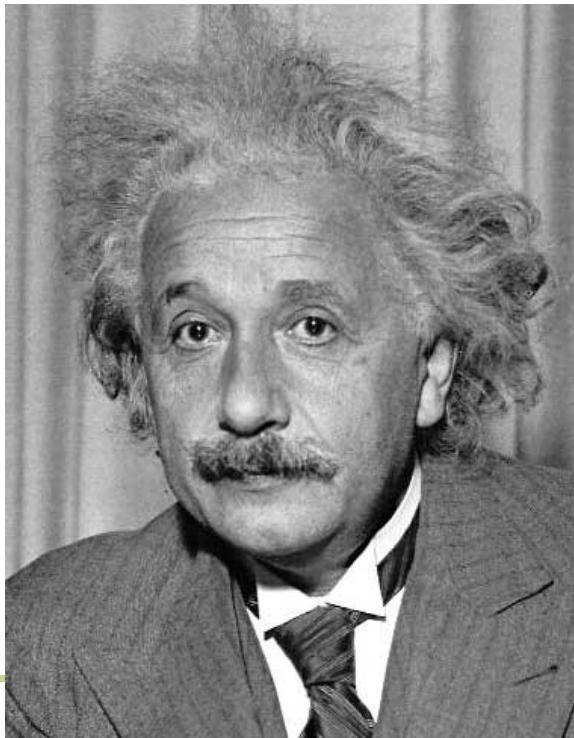


- Goal: find in image
- Method 0: filter the image with eye patch

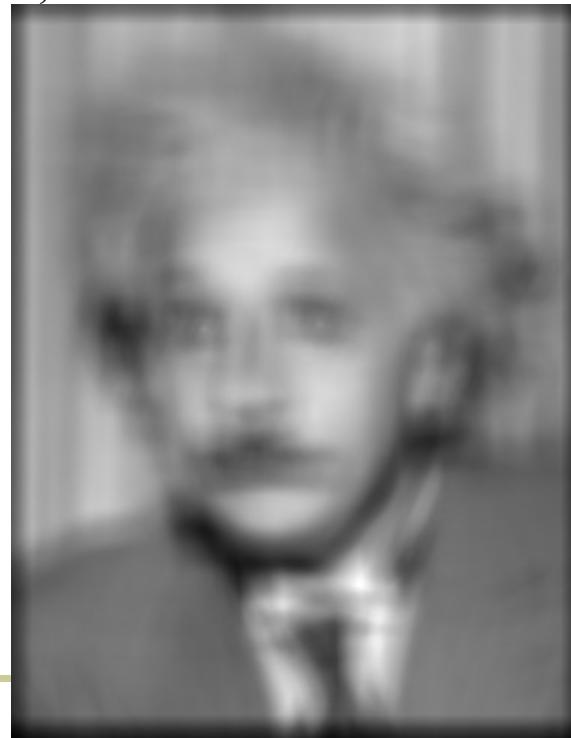
$$h[m,n] = \sum_{k,l} g[k,l] f[m+k, n+l]$$



$f$  = image  
 $g$  = filter



Input



Filtered Image

What went wrong?

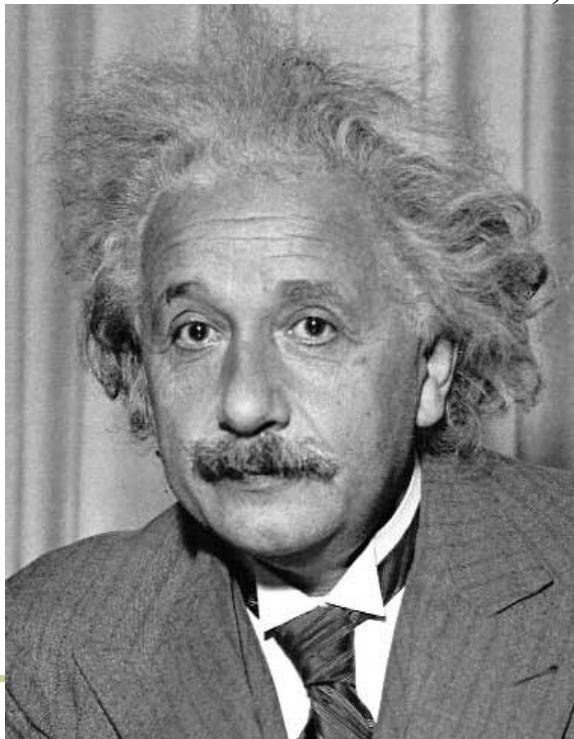


# Matching with filters

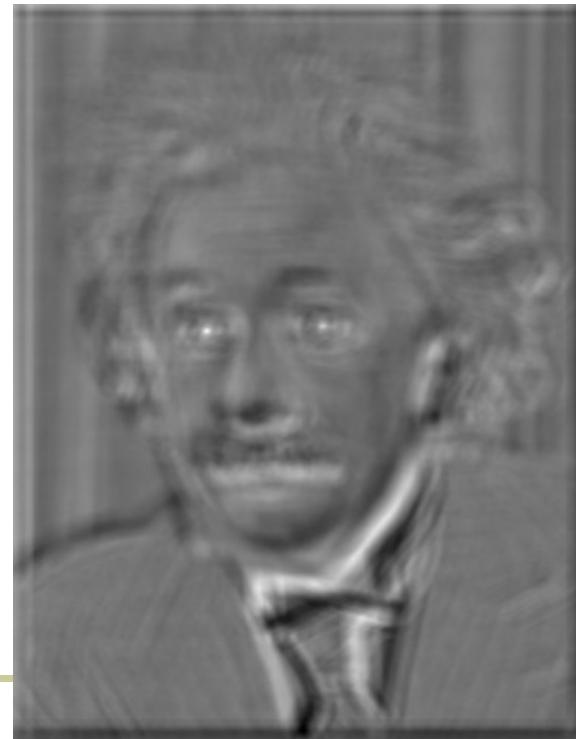


- Goal: find  in image
- Method 1: filter the image with zero-mean eye

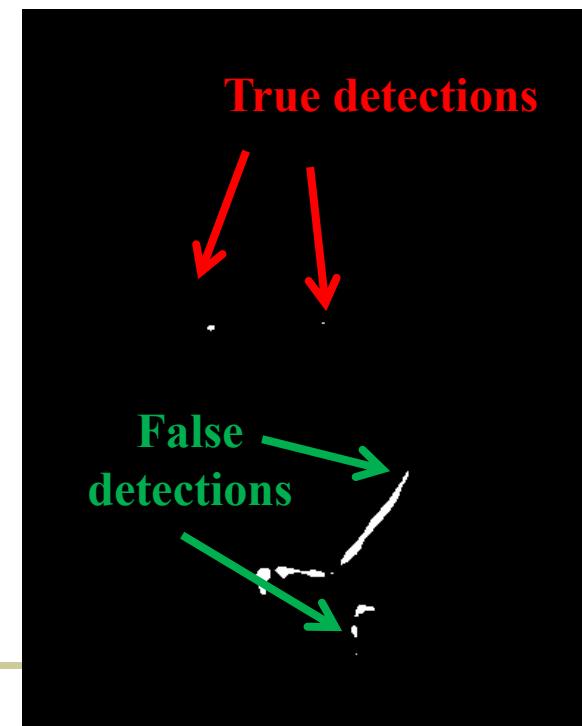
$$h[m, n] = \sum_{k, l} (g[k, l] - \bar{g}) \underbrace{(f[m + k, n + l])}_{\text{mean of template } g}$$



Input



Filtered Image (scaled)



Thresholded Image

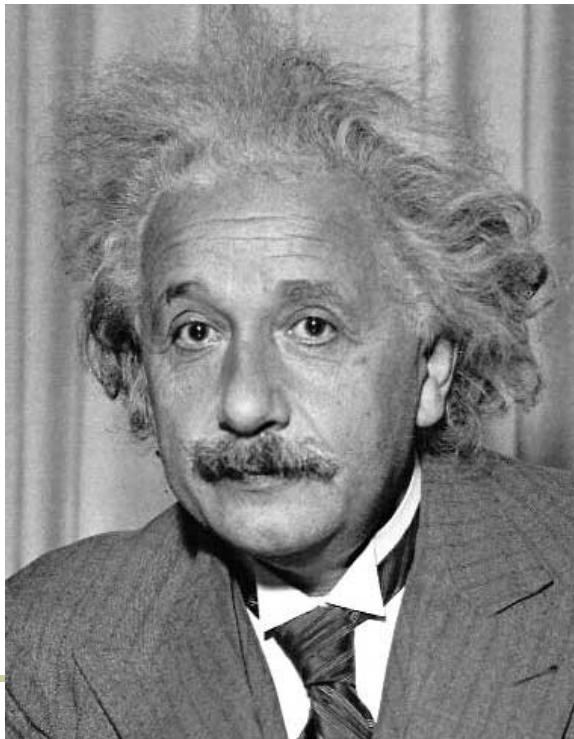


# Matching with filters



- Goal: find in image
- Method 2: SSD

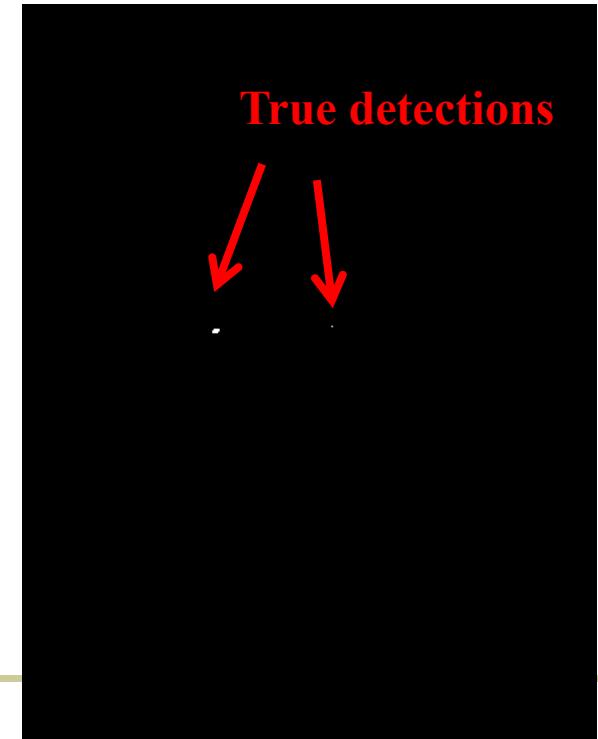
$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k, n+l])^2$$



Input



1 -  $\text{sqrt}(\text{SSD})$



True detections

Thresholded Image



# Matching with filters



Can SSD be implemented with linear filters?

$$h[m, n] = \sum_{k, l} (g[k, l] - f[m + k, n + l])^2$$



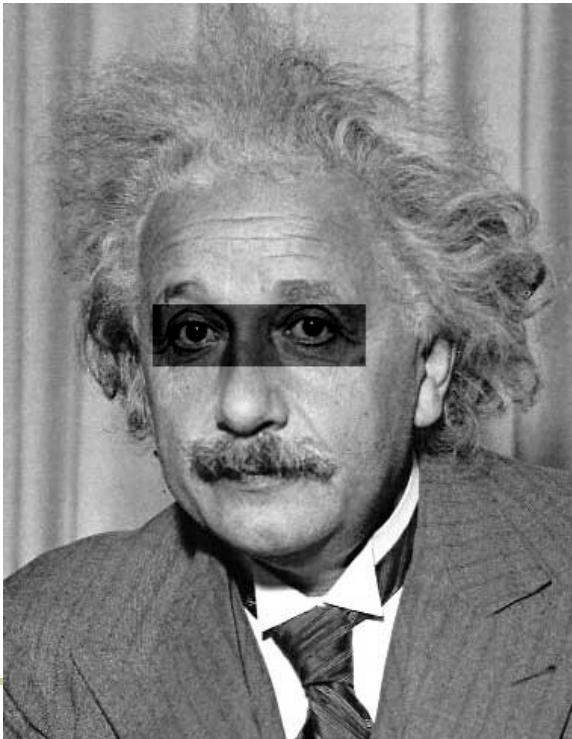
# Matching with filters



- Goal: find in image
- Method 2: SSD

What's the potential downside  
of SSD?

$$h[m, n] = \sum_{k, l} (g[k, l] - f[m + k, n + l])^2$$



Input



1 - sqrt(SSD)



# Matching with filters



- Goal: find in image
- Method 3: Normalized cross-correlation

$$h[m, n] = \frac{\sum_{k,l} (g[k, l] - \bar{g})(f[m + k, n + l] - \bar{f}_{m,n})}{\left( \sum_{k,l} (g[k, l] - \bar{g})^2 \sum_{k,l} (f[m + k, n + l] - \bar{f}_{m,n})^2 \right)^{0.5}}$$

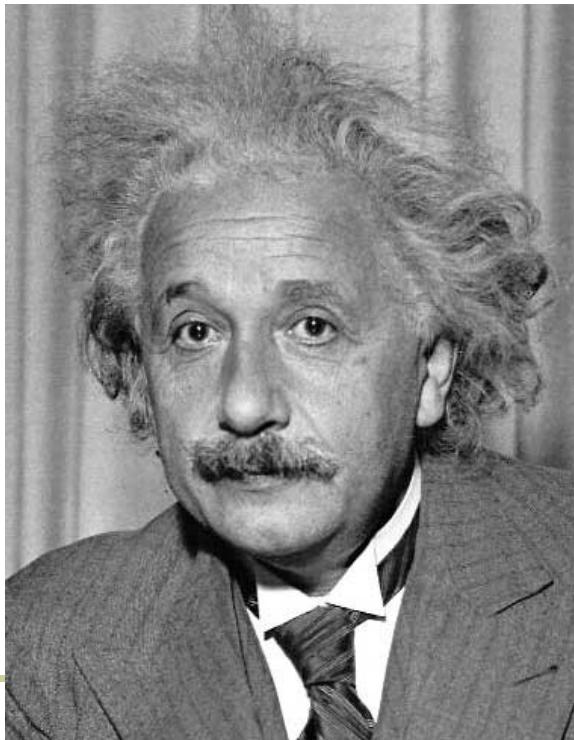
Matlab: `normxcorr2(template, im)`



# Matching with filters



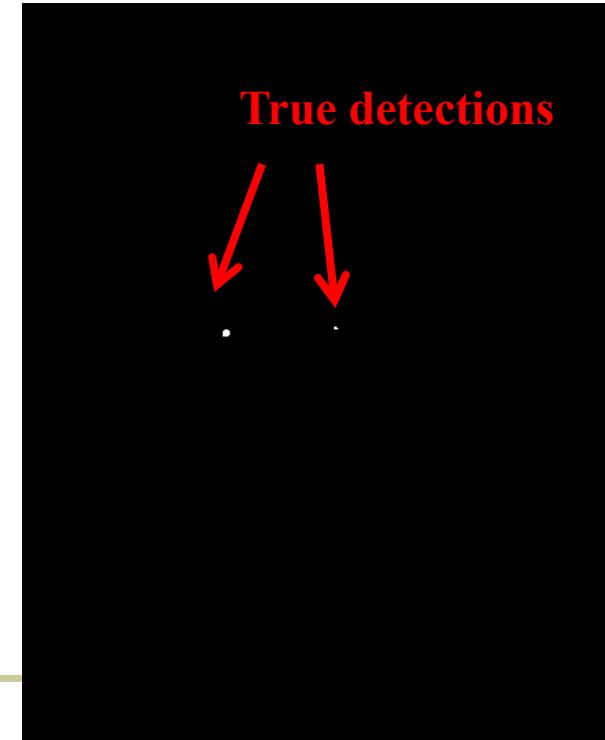
- Goal: find in image
- Method 3: Normalized cross-correlation



Input



Normalized X-Correlation



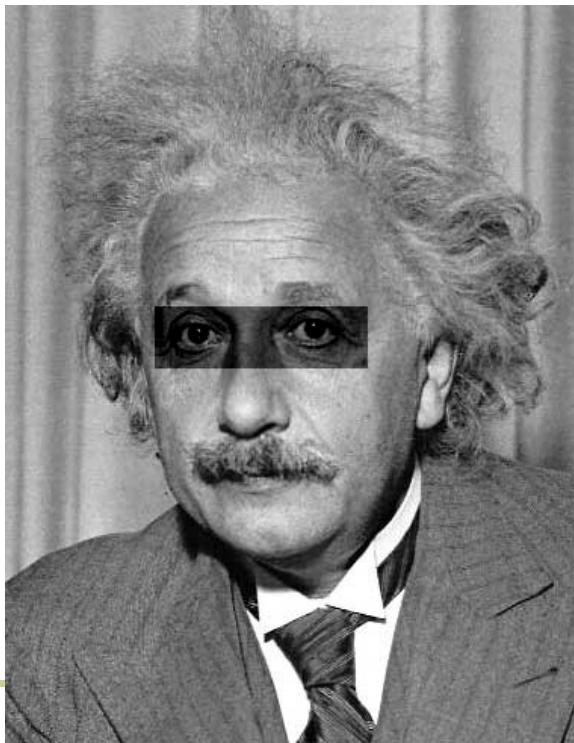
Thresholded Image



# Matching with filters



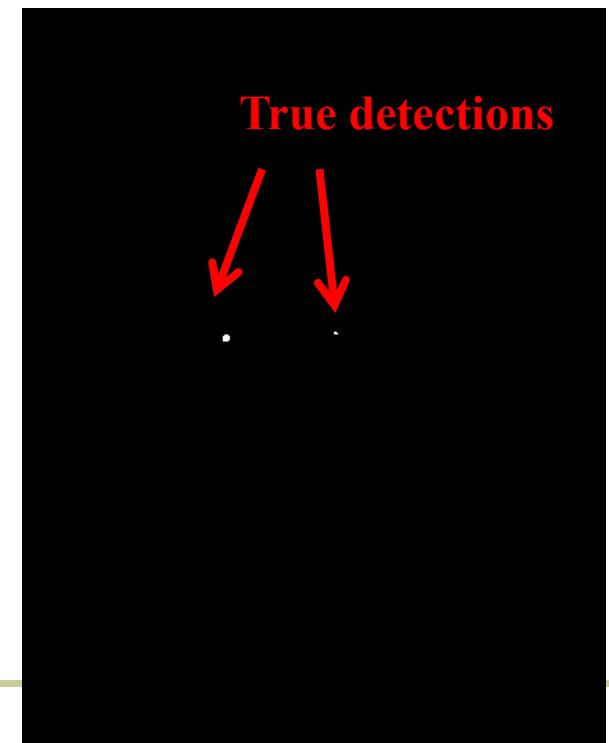
- Goal: find in image
- Method 3: Normalized cross-correlation



Input



Normalized X-Correlation



True detections



Q: What is the best method to use?



A: Depends

- Zero-mean filter: fastest but not a great matcher
- SSD: next fastest, sensitive to overall intensity
- Normalized cross-correlation: slowest, invariant to local average intensity and contrast



# Today's class



- Template matching
- **Gaussian Pyramids**
  - Application for recognition
  - Pyramids representation in deep learning
- **Laplacian Pyramids**
  - Application for image blending
  - Hybrid images
- Steerable pyramids:
  - filter banks and texture analysis



# Translation invariance



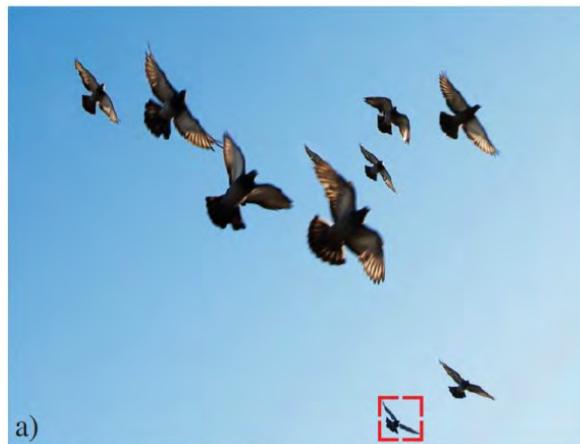
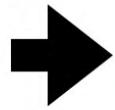


# We need translation and scale invariance

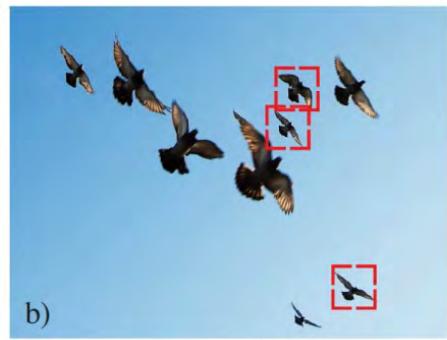




# Image pyramids



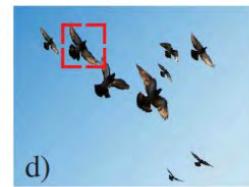
a)



b)



c)



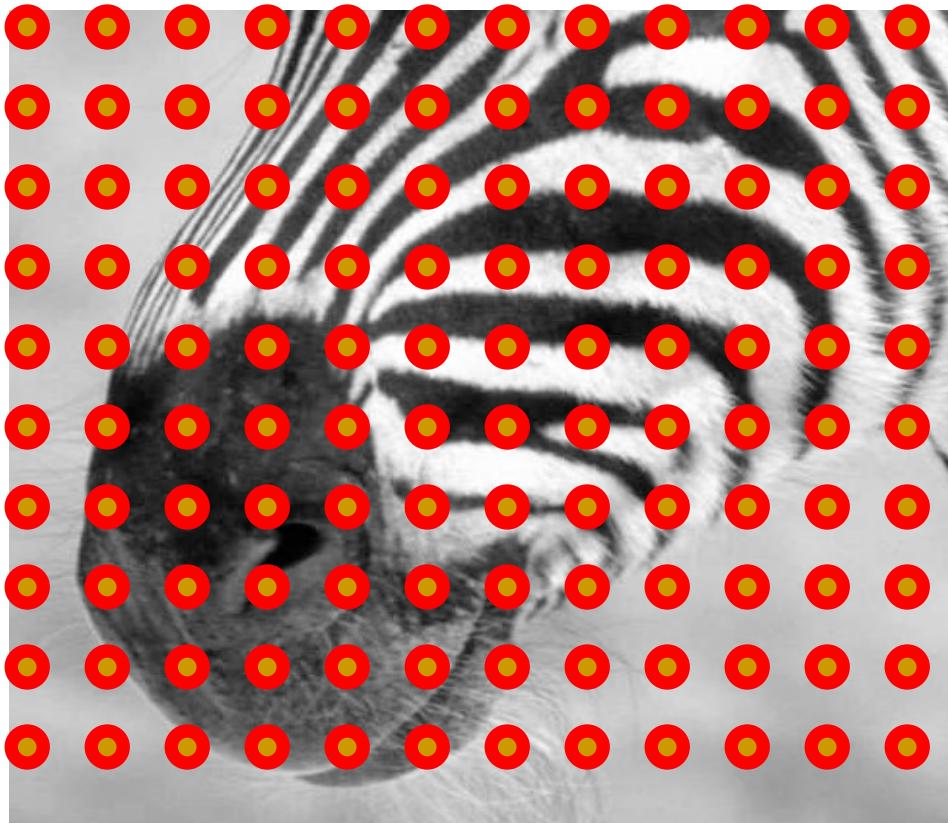
d)



e)



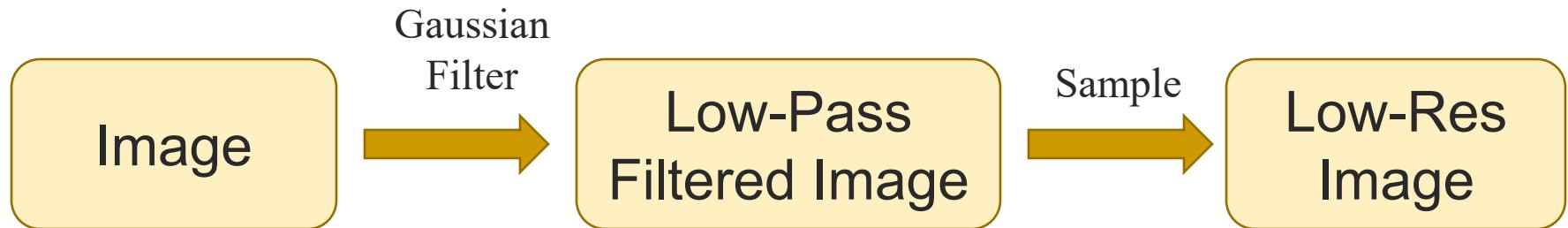
# Subsampling by a factor of 2



Throw away every other row and column  
to create a 1/2 size image



# Recall: sampling



E. H. Adelson | C. H. Anderson | J. R. Bergen | P. J. Burt | J. M. Ogden

# Pyramid methods in image processing

[http://persci.mit.edu/pub\\_pdfs/RCA84.pdf](http://persci.mit.edu/pub_pdfs/RCA84.pdf)



# Image pyramids



- **Gaussian pyramid**
  - Application for recognition
- **Laplacian pyramid**
  - Application for image blending

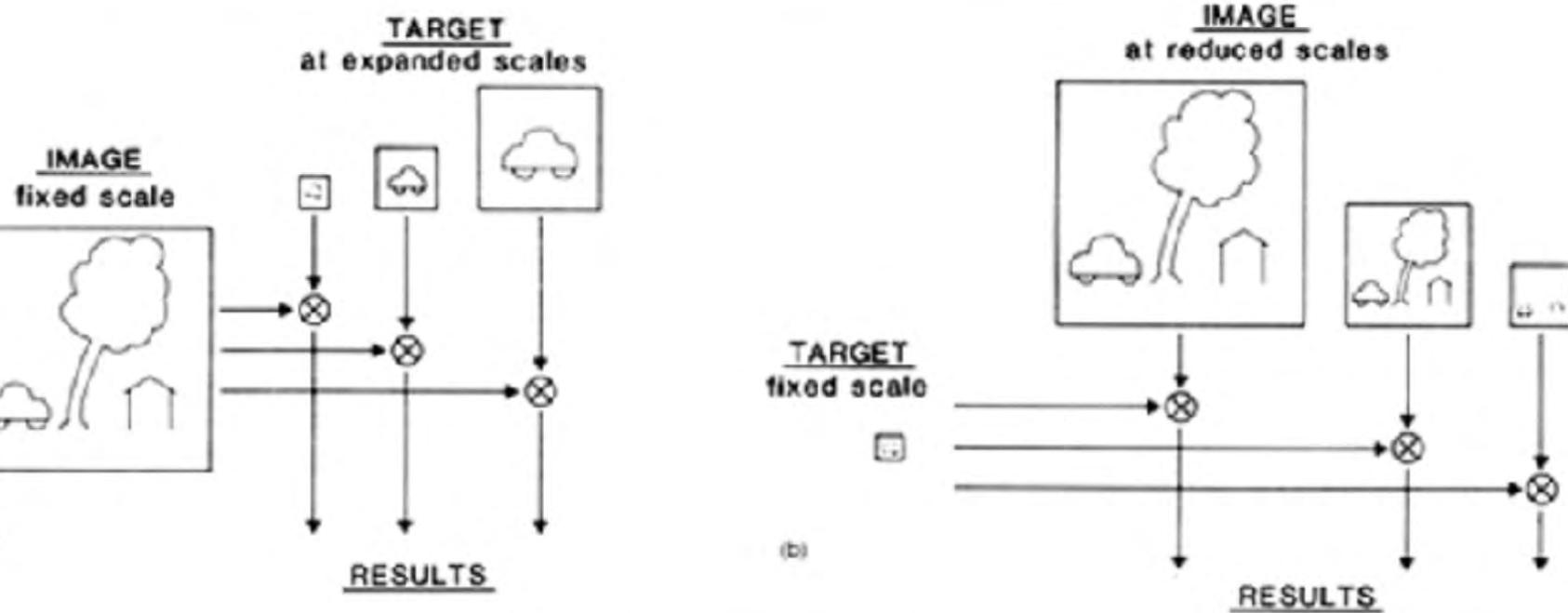


Fig. 1. Two methods of searching for a target pattern over many scales. In the first approach, (a), copies of the target pattern are constructed at several expanded scales, and each is convolved with the original image. In the second approach, (b), a single copy of the target is convolved with

copies of the image reduced in scale. The target should be just large enough to resolve critical details. The two approaches should give equivalent results, but the second is more efficient by the fourth power of the scale factor (image convolutions are represented by 'O').



**Fig. 2a.** The Gaussian pyramid. The original image,  $G_0$ , is repeatedly filtered and subsampled to generate the sequence of reduced resolution image  $G_1$ ,  $G_2$ , etc. These comprise a set of lowpass-filtered copies of the original image in which the bandwidth decreases in one-octave steps.



$I_{r,0}$



$G_{r,1}$



$G_{r,2}$

**Fig. 2b.** Levels of the Gaussian pyramid expanded to the size of the original image.  
The effects of lowpass filtering are now clearly apparent.



**Gaussian Pyramid**

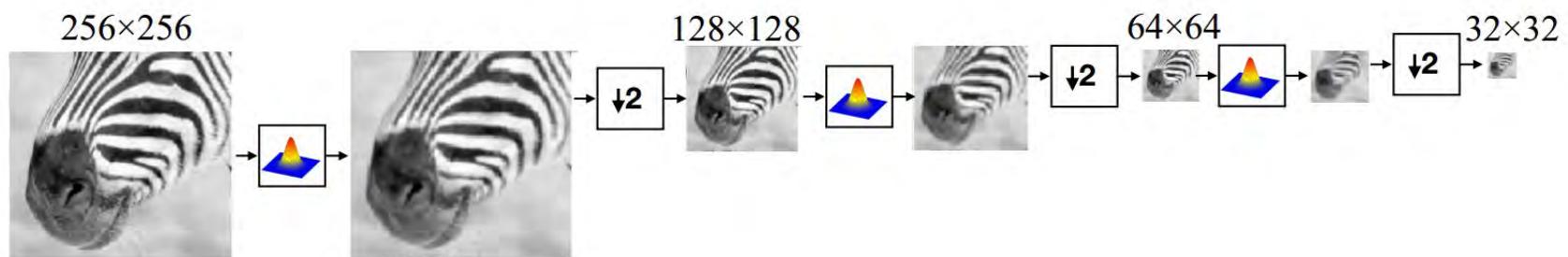
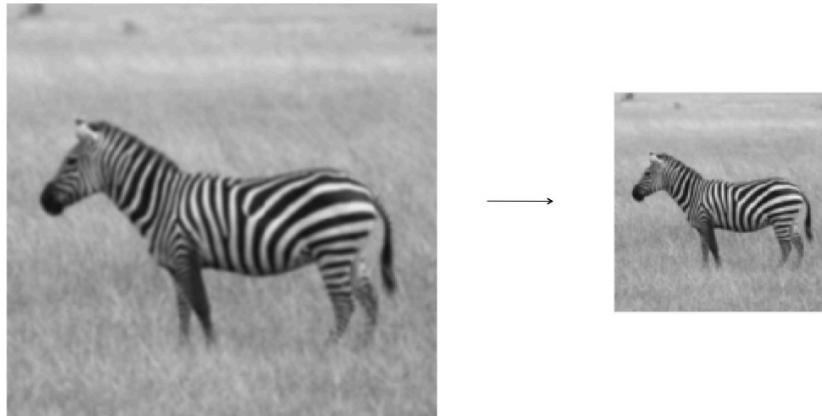


# Gaussian Pyramid



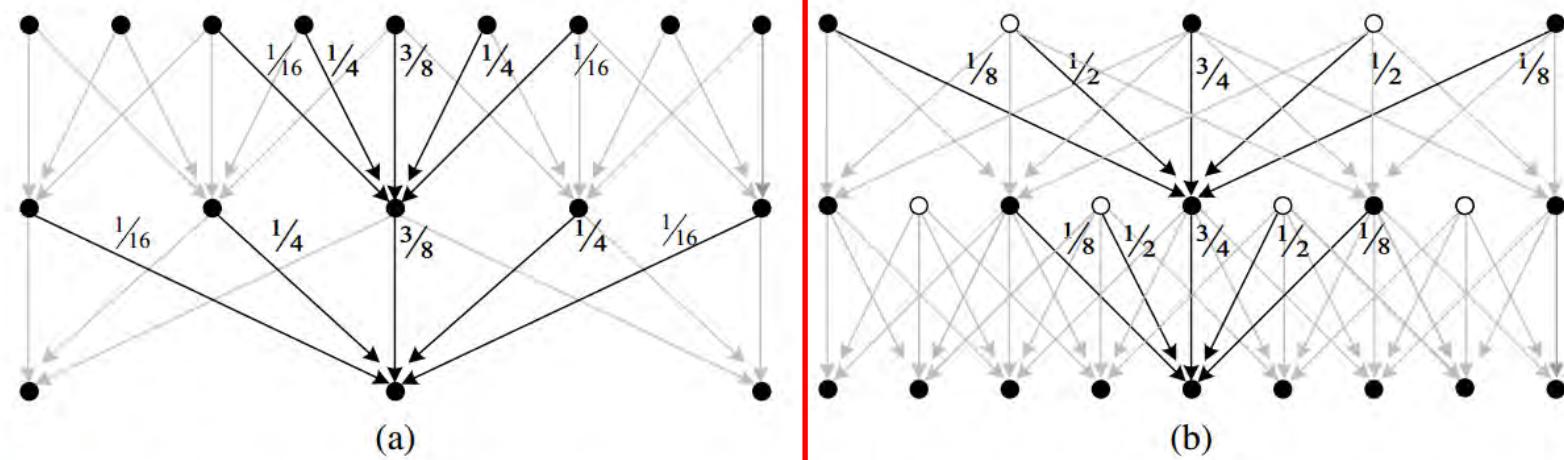
For each level

1. Blur input image with a Gaussian filter
2. Downsample image





# Downsampling & Upsampling



**Figure 3.33** The Gaussian pyramid shown as a signal processing diagram: The (a) analysis and (b) re-synthesis stages are shown as using similar computations. The white circles indicate zero values inserted by the  $\uparrow 2$  upsampling operation. Notice how the reconstruction filter coefficients are twice the analysis coefficients. The computation is shown as flowing down the page, regardless of whether we are going from coarse to fine or *vice versa*.

# The Gaussian pyramid

$$\frac{1}{16} \begin{bmatrix} 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

256×256



$$\xrightarrow{[1, 4, 6, 4, 1]}$$



$g_0$



128×128

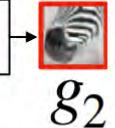
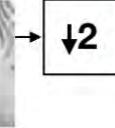


$g_1$

$$\downarrow 2$$



$$[1, 4, 6, 4, 1]$$



$$\downarrow 2$$

$$g_2$$

$$g_1 = G_0 g_0$$

$$G_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\frac{1}{16} \begin{bmatrix} 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 \end{bmatrix}$$

# The Gaussian pyramid

$$\frac{1}{16} \begin{bmatrix} 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

256×256



$[1, 4, 6, 4, 1]$



$g_0$



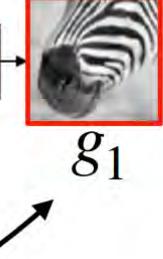
128×128

$g_1$

$$G_0 = \frac{1}{16} \begin{bmatrix} 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 \end{bmatrix}$$



128×128



$\downarrow 2$

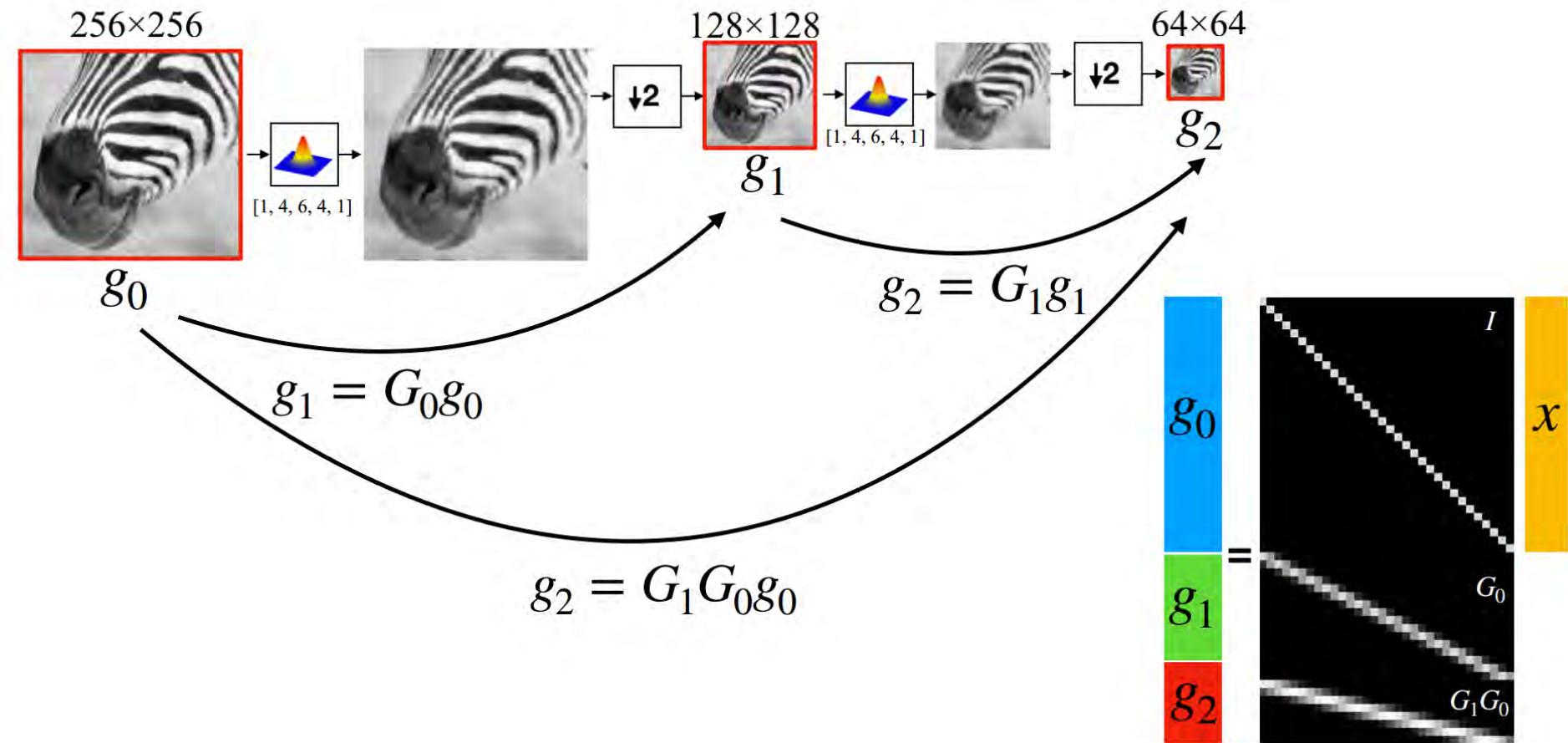
64×64



$g_2$

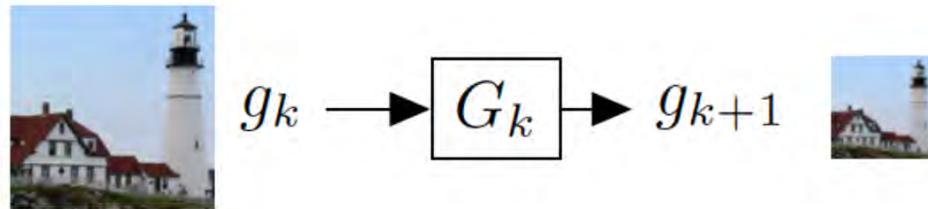
$\downarrow 2$

# The Gaussian pyramid





# Gaussian Pyramid



For each level

1. Blur input image with a Gaussian filter
2. Downsample image



# Gaussian pyramids used for



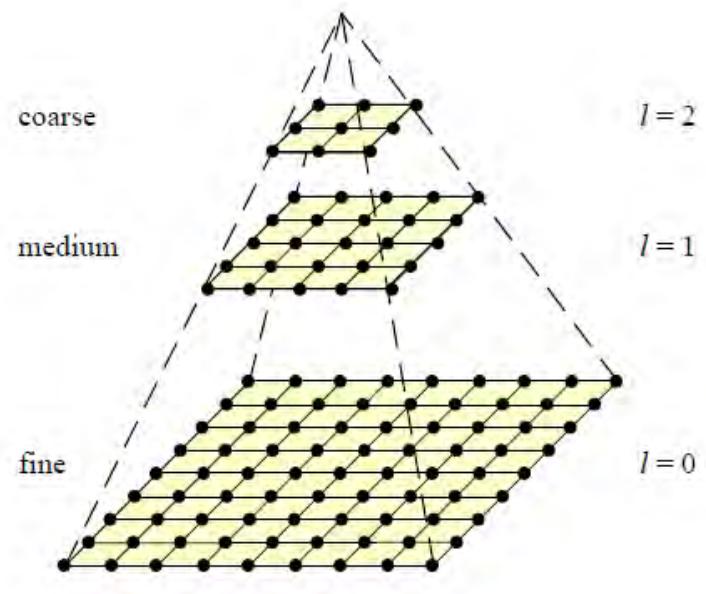
- Up or down sample images
- Multi-resolution image analysis
  - Look for an object over various spatial scales
  - Coarse-to-fine image registration: form blur estimate or the motion analysis on very low-resolution image, upsample and repeat
  - Often a successful strategy for avoiding local minima in complicated estimation tasks



# Coarse-to-fine Image Registration



1. Compute Gaussian pyramid
2. Align with coarse pyramid
3. Successively align with finer pyramids
  - Search smaller range



Why is this faster?

Are we guaranteed to get the same result?



# Template Matching with Image Pyramids

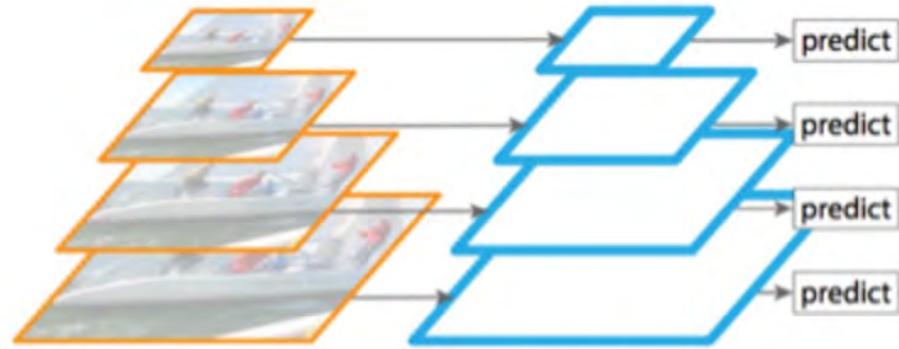
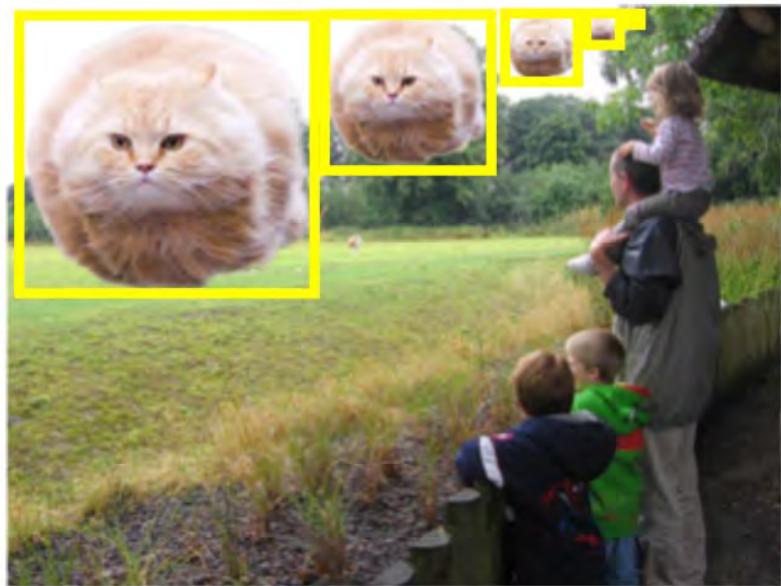


Input: Image, Template

1. Match template at current scale
2. Downsample image
  - In practice, scale step of 1.1 to 1.2
3. Repeat 1-2 until image is very small
4. Take responses above some threshold, perhaps with non-maxima suppression



# From Image Pyramid to Feature Pyramid



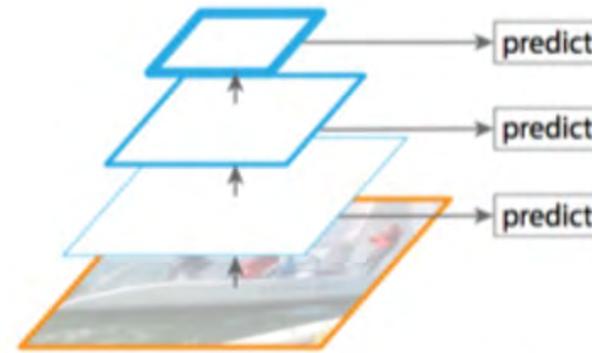
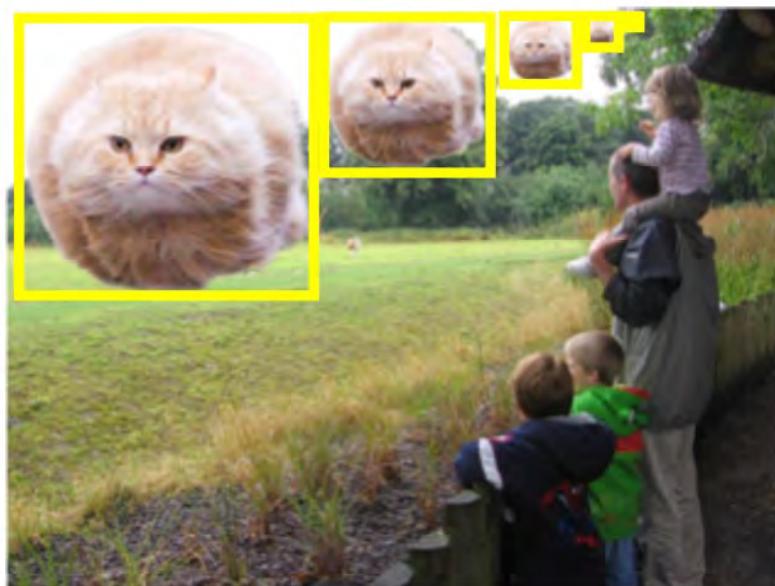
(a) Featurized image pyramid

**Standard solution – slow!**

(E.g., Viola & Jones, HOG, DPM, SPP-net, multi-scale Fast R-CNN, ...)



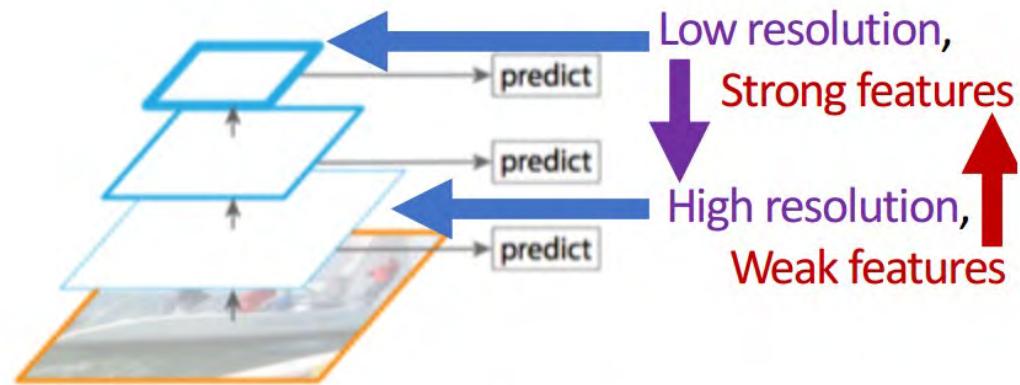
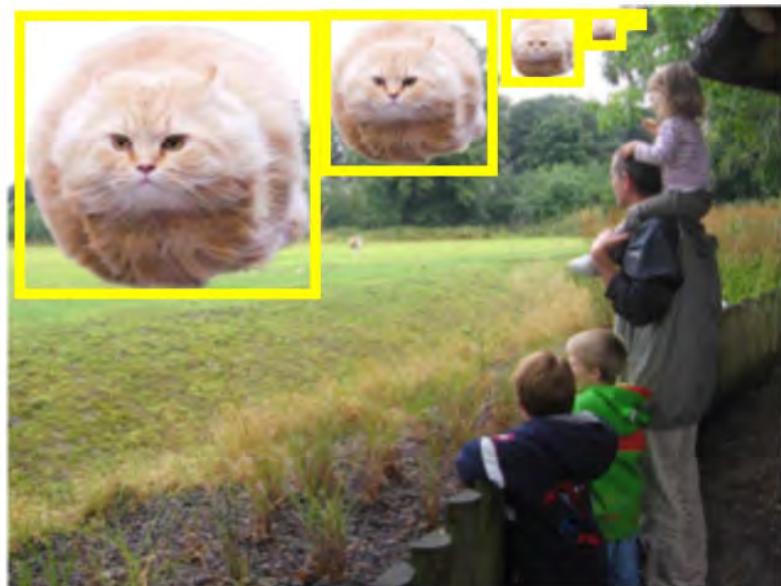
# Feature Pyramid Networks



(c) Pyramidal feature hierarchy  
Use the internal pyramid – *fast, suboptimal*  
(E.g.,  $\approx$  SSD, ...)



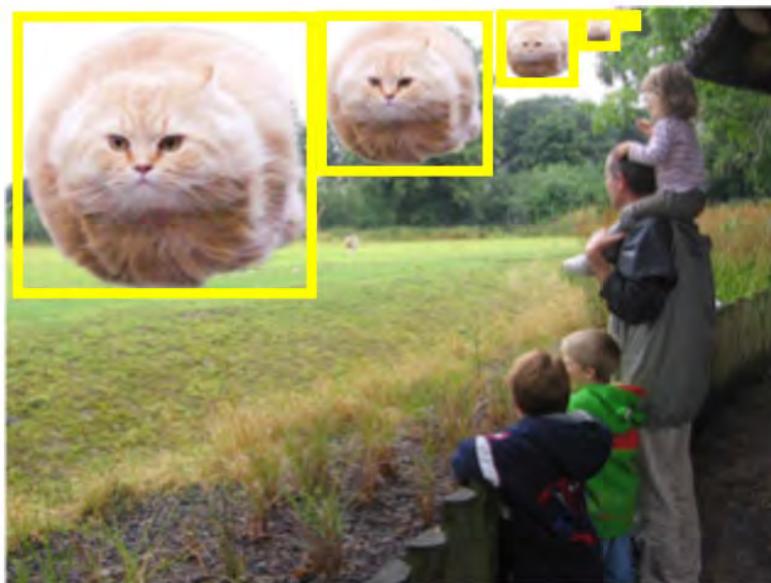
# Feature Pyramid Networks



(c) Pyramidal feature hierarchy  
Use the internal pyramid – *fast, suboptimal*  
(E.g.,  $\approx$  SSD, ...)



# Feature Pyramid Networks

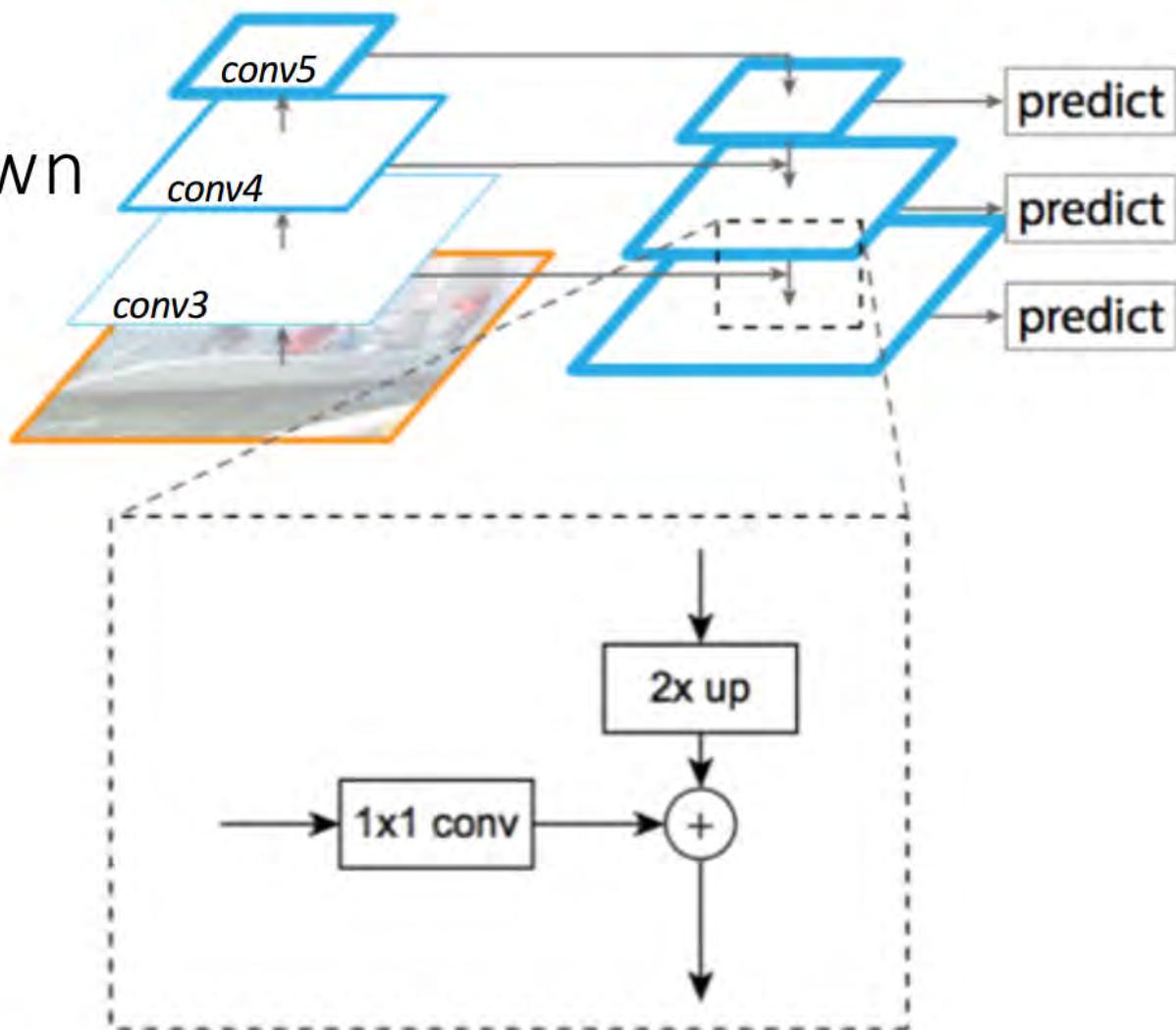


(d) Feature Pyramid Network

Top-down enrichment of high-res features –  
*fast, less suboptimal*

Lin et al. Feature Pyramid Networks for Object Detection. CVPR 2017.

# FPN Top-down Refinement Module



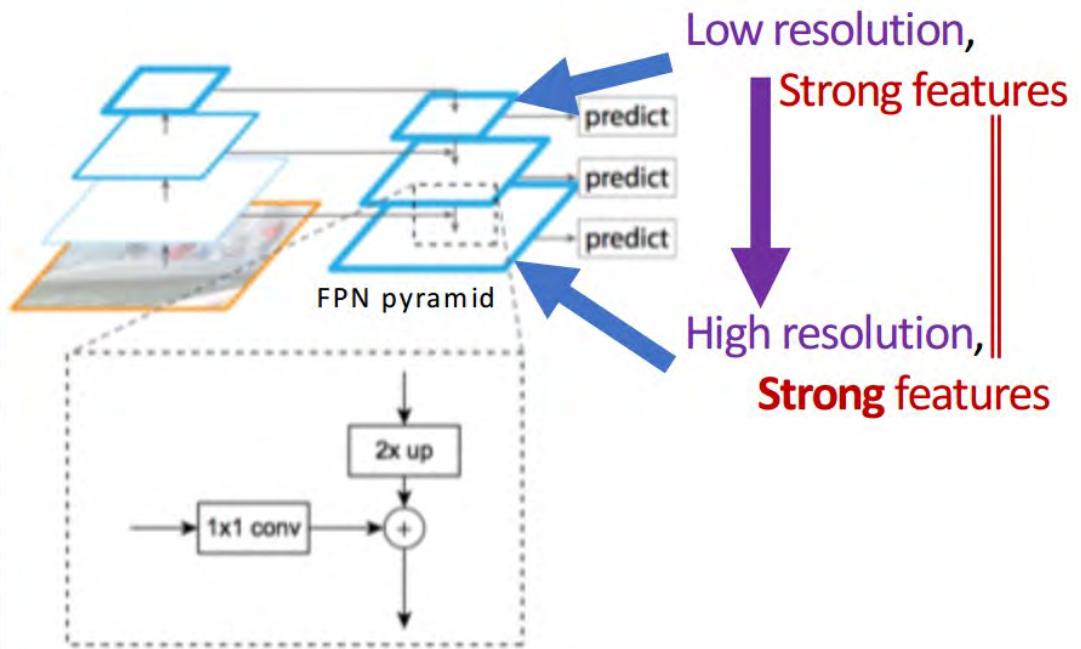
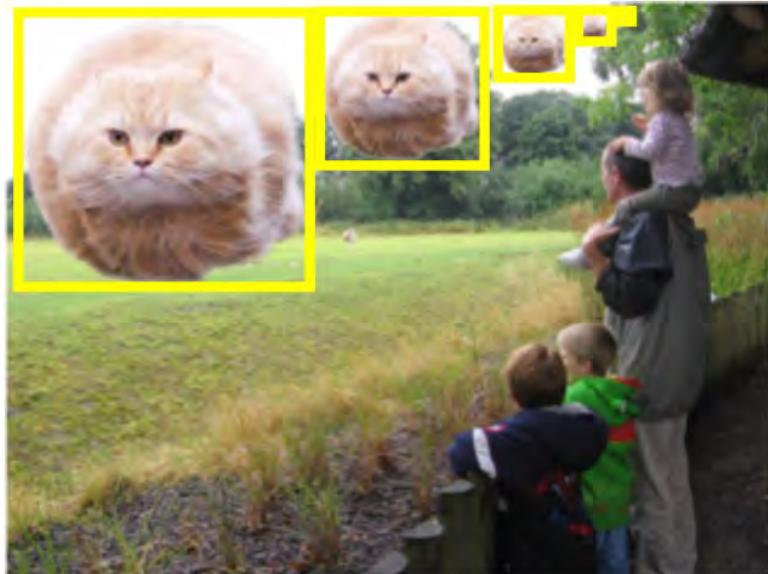
Lin et al. Feature Pyramid Networks for Object Detection. CVPR 2017.



# Feature Pyramid Networks



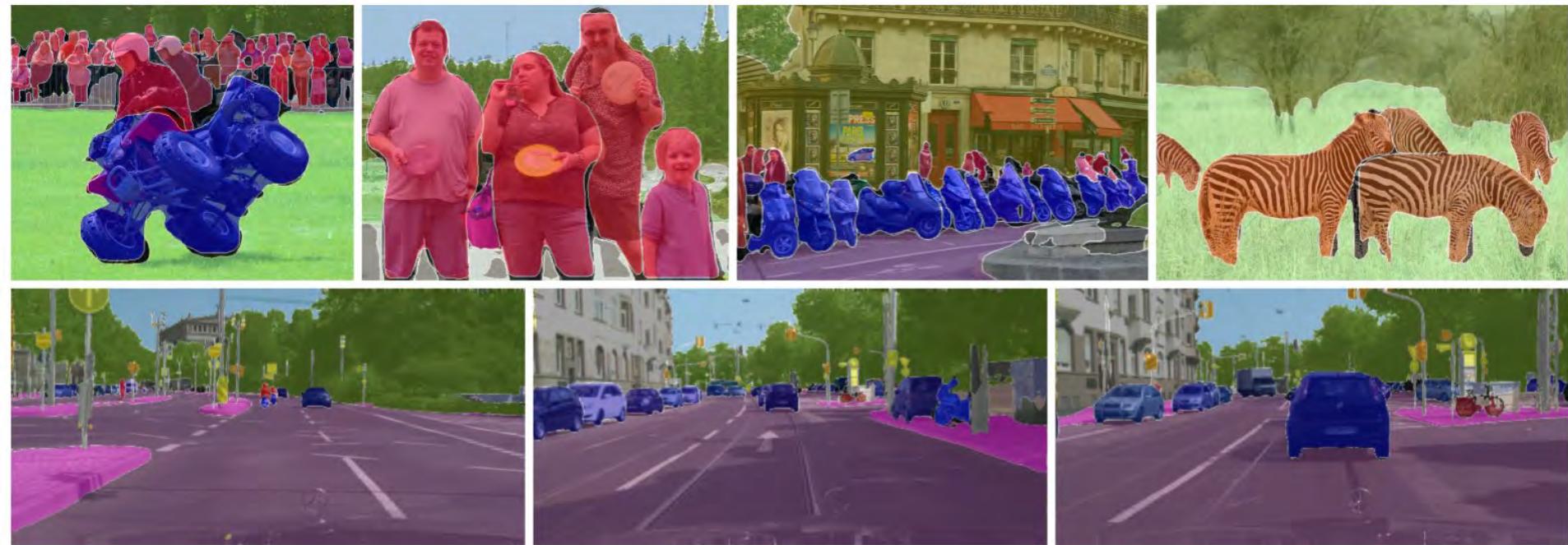
No Compromise on Feature Quality, still Fast



Lin et al. Feature Pyramid Networks for Object Detection. CVPR 2017. See also: Shrivastava's TDM.

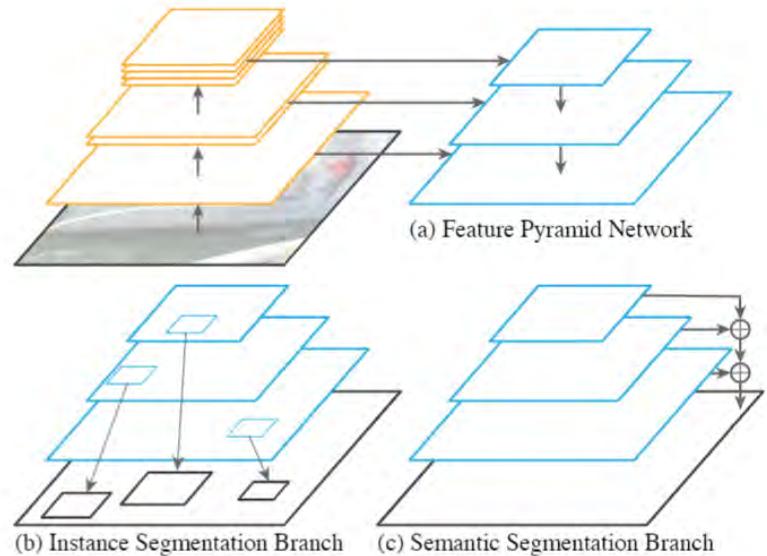
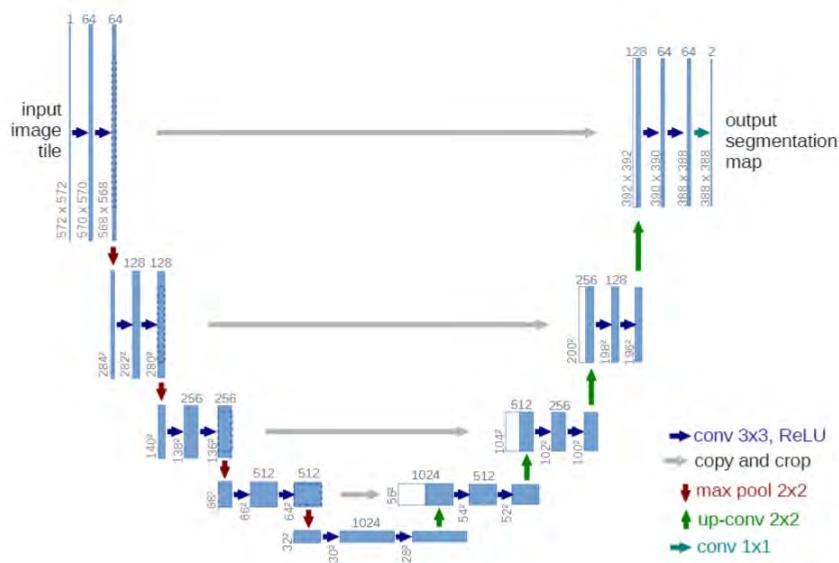


# Segmentation





# Pyramid in segmentation

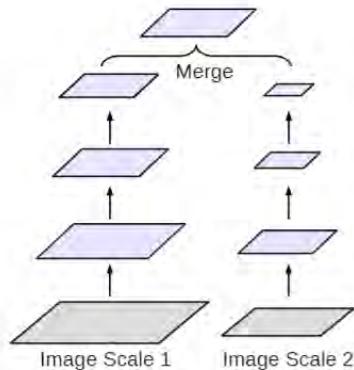


**U-Net: Convolutional Networks for Biomedical Image Segmentation**

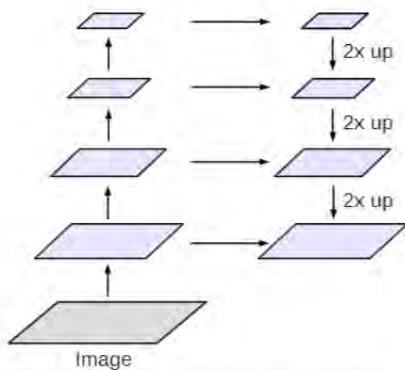
**Panoptic Feature Pyramid Networks**



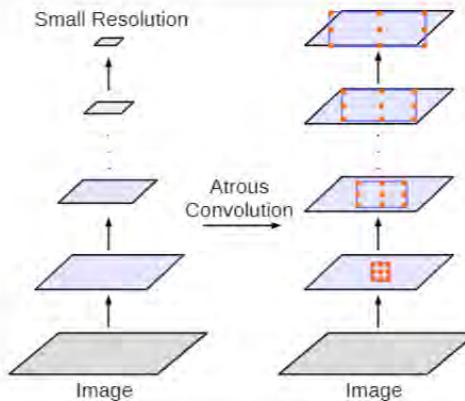
# Atrous Convolution



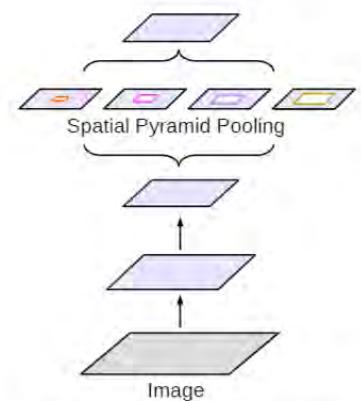
(a) Image Pyramid



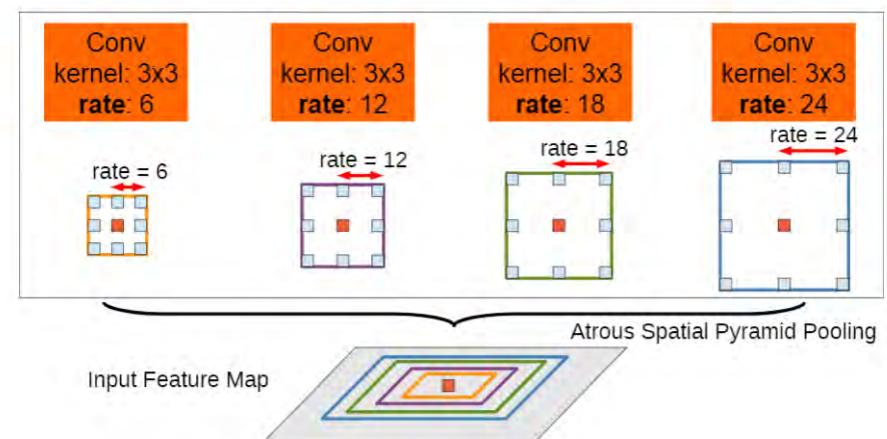
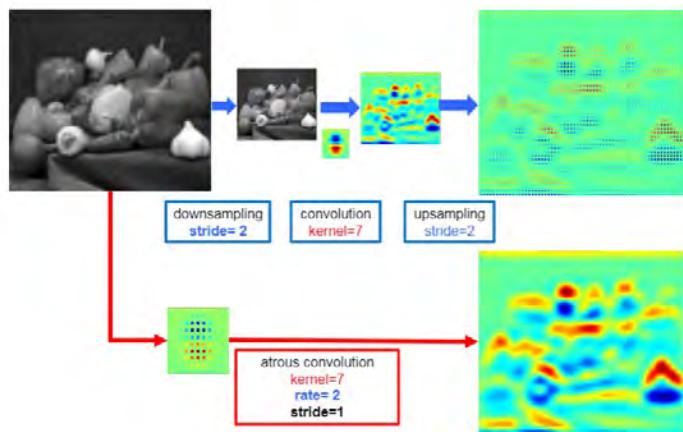
(b) Encoder-Decoder  
Figure 2. Alternative architectures to capture multi-scale context.



(c) Deeper w. Atrous Convolution

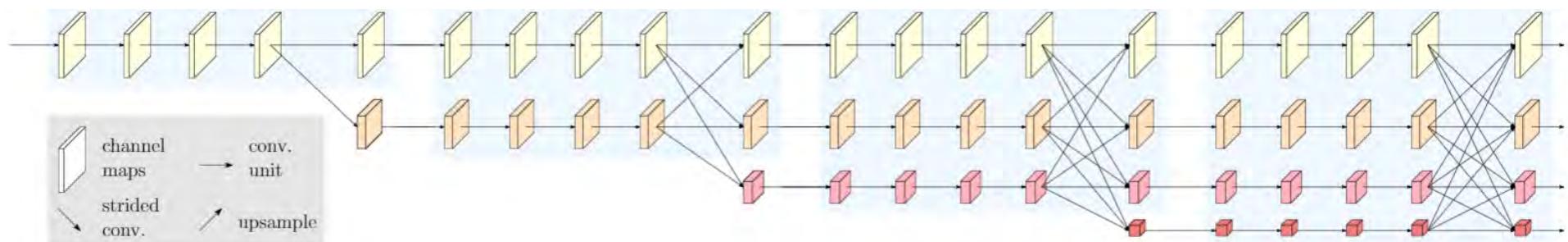
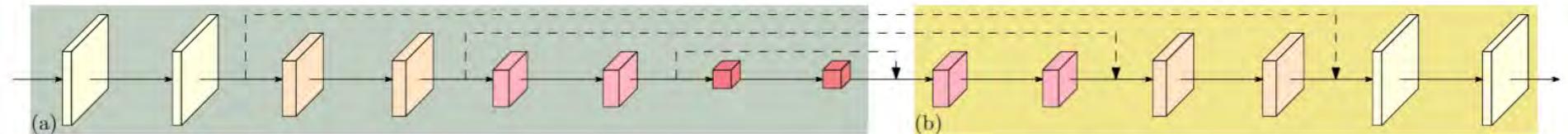


(d) Spatial Pyramid Pooling

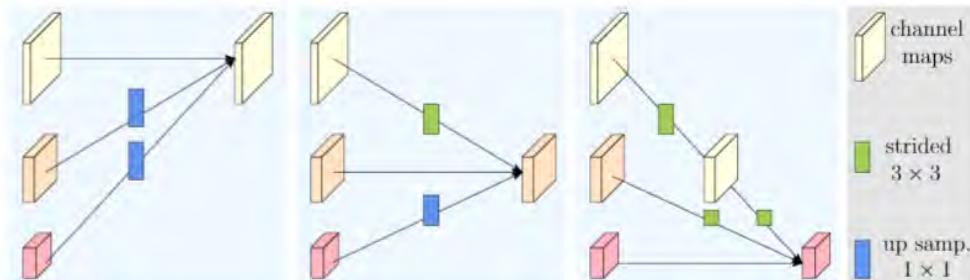




# HRNet



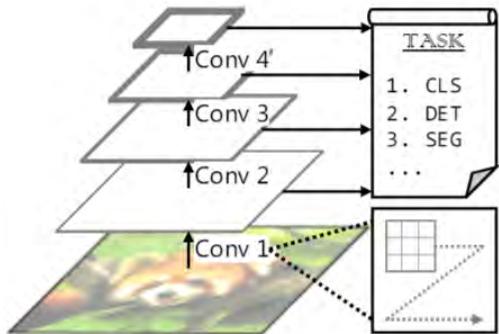
$$\begin{array}{ccccccc} \mathcal{N}_{11} & \rightarrow & \mathcal{N}_{21} & \rightarrow & \mathcal{N}_{31} & \rightarrow & \mathcal{N}_{41} \\ & \searrow & & & \searrow & & \\ & & \mathcal{N}_{22} & \rightarrow & \mathcal{N}_{32} & \rightarrow & \mathcal{N}_{42} \\ & & & & \searrow & & \\ & & & & \mathcal{N}_{33} & \rightarrow & \mathcal{N}_{43} \\ & & & & & & \searrow \\ & & & & & & \mathcal{N}_{44}, \end{array}$$



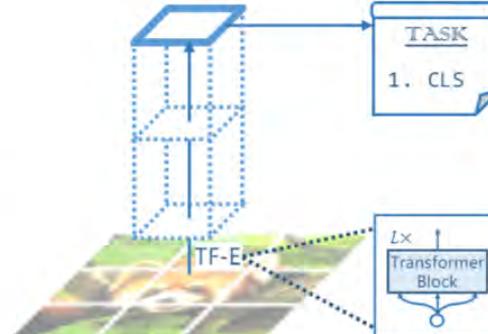
# Deep High-Resolution Representation Learning for Visual Recognition



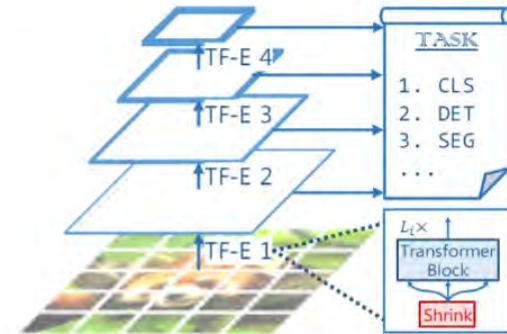
# Pyramid in Transformer



(a) CNNs: VGG [41], ResNet [15], etc.

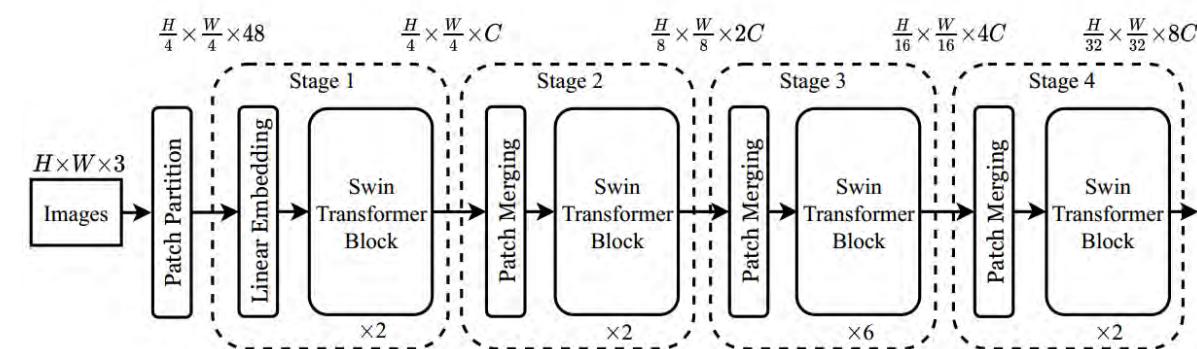
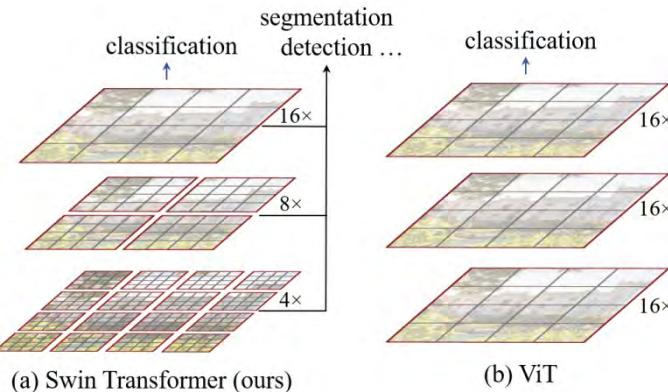


(b) Vision Transformer [10]



(c) Pyramid Vision Transformer (ours)

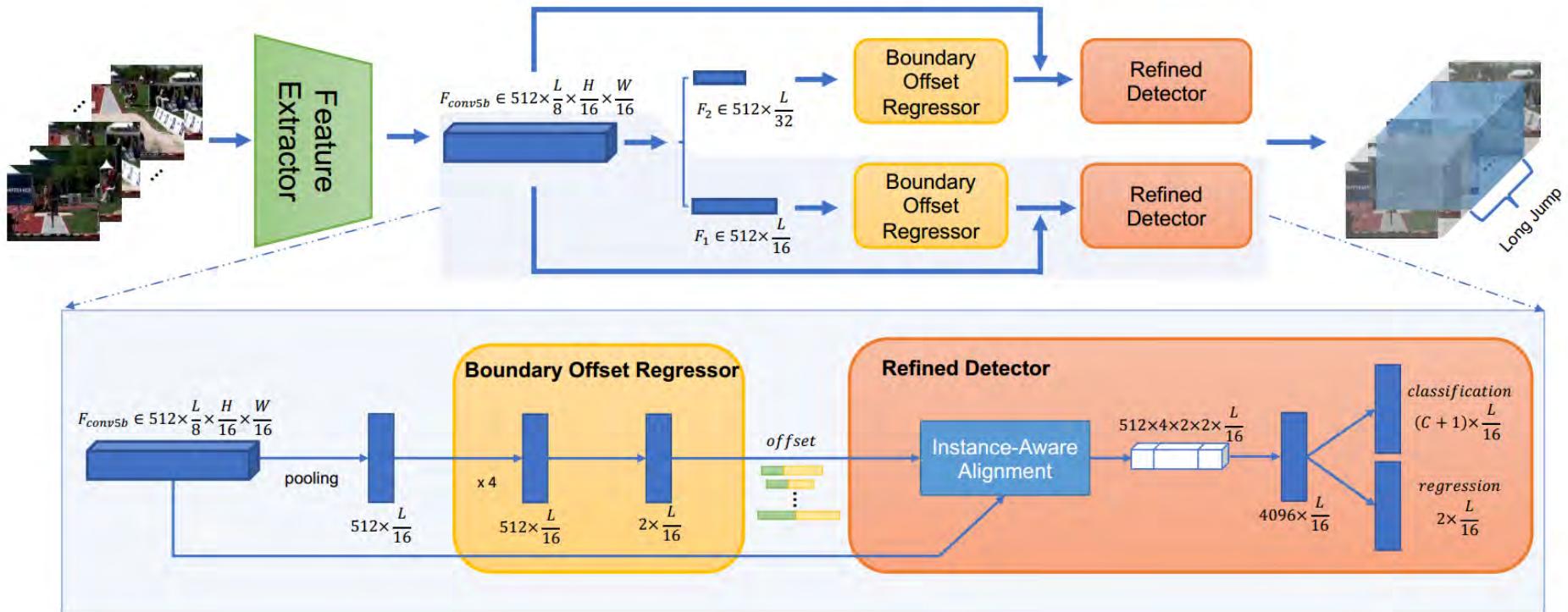
## Pyramid Vision Transformer: A Versatile Backbone for Dense Prediction without Convolutions



## Swin Transformer: Hierarchical Vision Transformer using Shifted Windows



# Temporal Pyramid: Image to Video



Instance-Aware Alignment for Anchor Free Temporal Action Detection



# Image pyramids



- Gaussian pyramid
  - Application for recognition
- Laplacian pyramid
  - Application for image blending



$L_0$



$L_1$



$L_2$



$L_3$



$L_4$

**Fig. 4a.** The Laplacian pyramid. Each level of this band-pass pyramid represents the difference between successive levels of the Gaussian pyramid.



$L_{0,0}$



$L_{1,0}$



$L_{2,2}$

**Fig. 4b.** Levels of the Laplacian pyramid expanded to the size of the original image.  
Note that edge and bar features are enhanced and segregated by size.



512

256

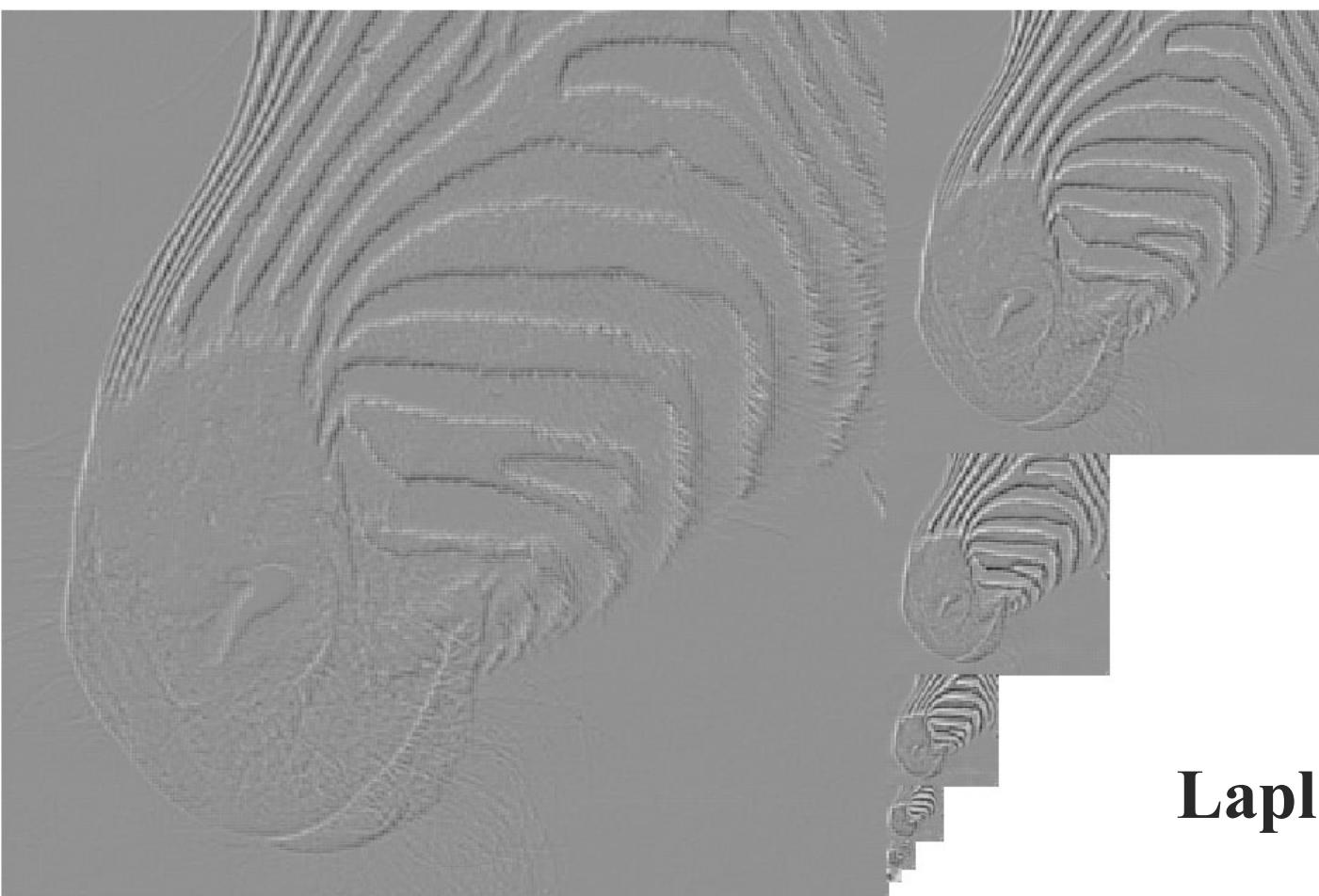
128

64

32

16

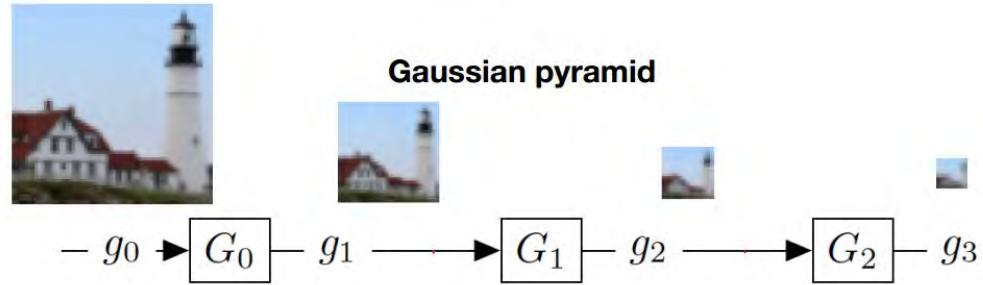
8



**Laplacian Pyramid**

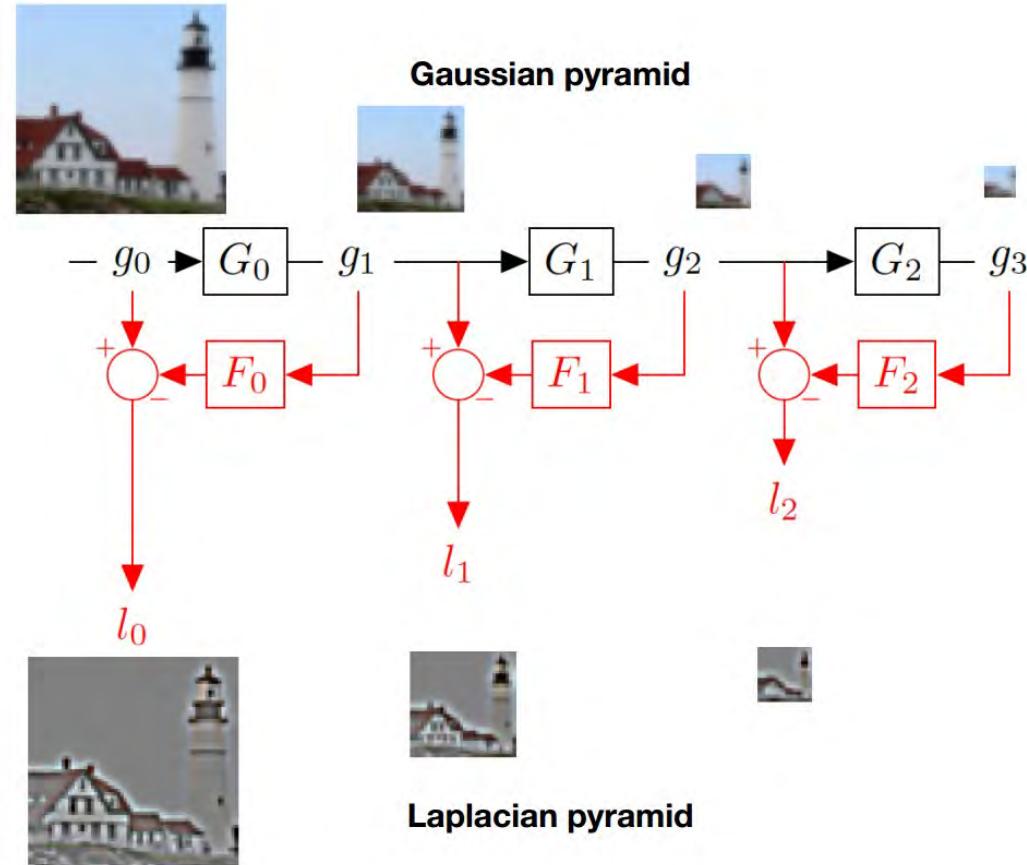


# The Laplacian Pyramid





# The Laplacian Pyramid

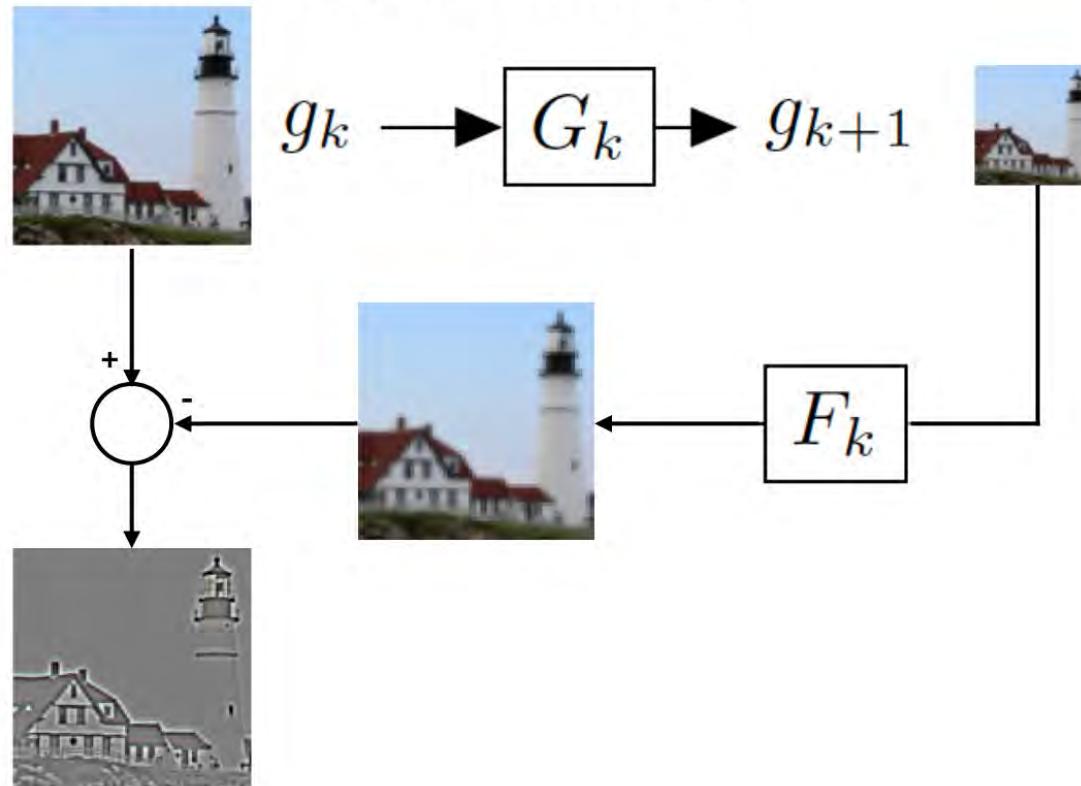




# The Laplacian Pyramid

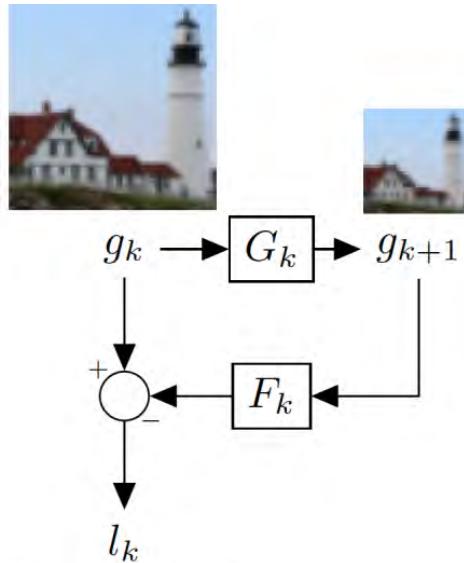


Compute the difference between upsampled Gaussian pyramid level  $k+1$  and Gaussian pyramid level  $k$ .





# The Laplacian Pyramid



**Blurring and downsampling:**

$$G_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \frac{1}{16}$$

(Downsampling by 2)

**Upsampling and blurring:**

$$\begin{bmatrix} 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 \end{bmatrix}$$

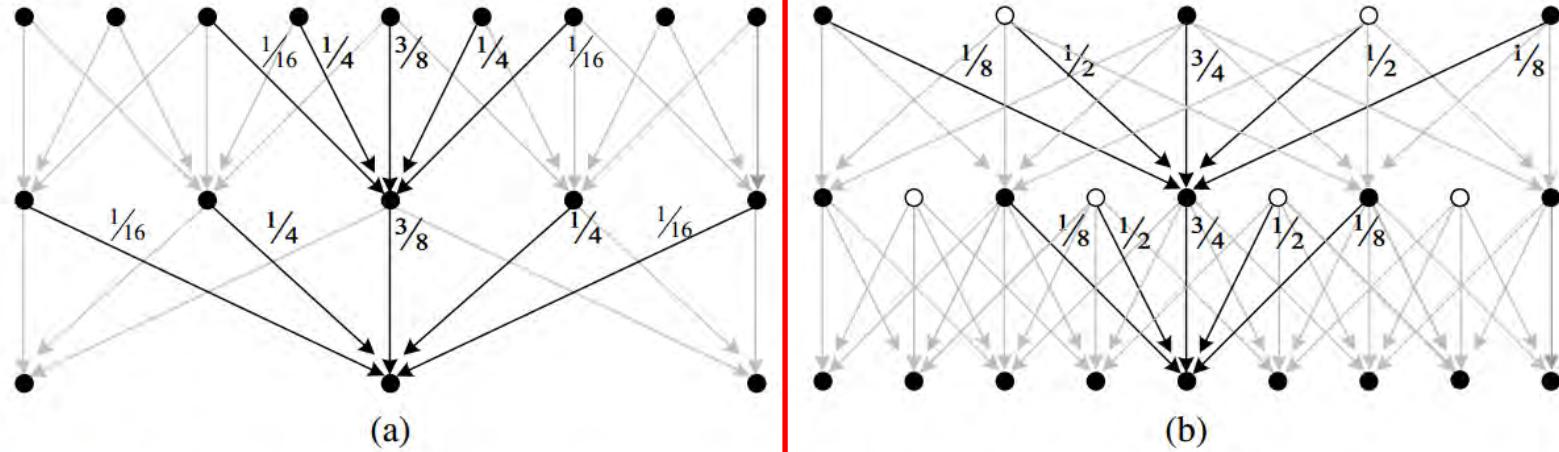
(blur)

$$F_0 =$$





# Downsampling & Upsampling



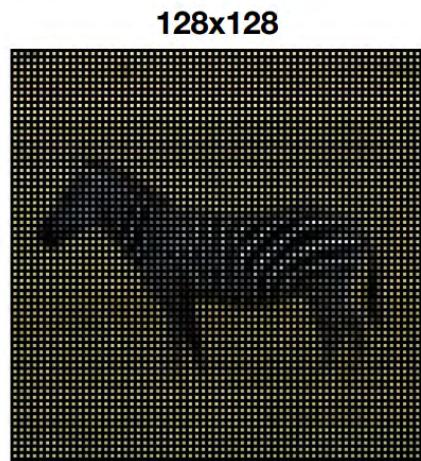
**Figure 3.33** The Gaussian pyramid shown as a signal processing diagram: The (a) analysis and (b) re-synthesis stages are shown as using similar computations. The white circles indicate zero values inserted by the  $\uparrow 2$  upsampling operation. Notice how the reconstruction filter coefficients are twice the analysis coefficients. The computation is shown as flowing down the page, regardless of whether we are going from coarse to fine or *vice versa*.



# Upsampling



Insert zeros  
→

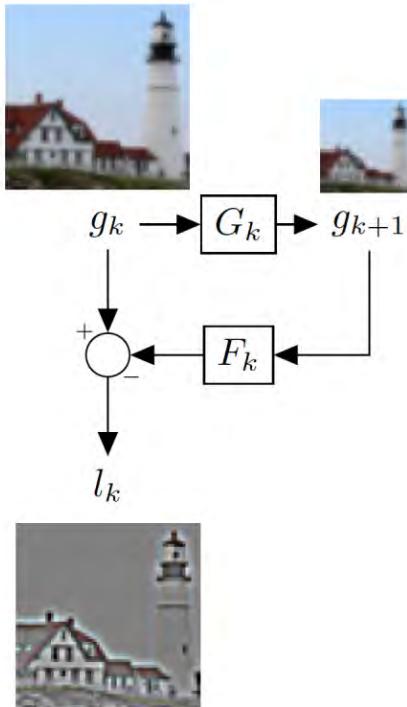


$$\odot \begin{bmatrix} 0.25 & 0.5 & 0.25 \\ 0.5 & 1 & 0.5 \\ 0.25 & 0.5 & 0.25 \end{bmatrix} =$$





# The Laplacian Pyramid



### **Blurring and downsampling:**

$$G_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \frac{1}{16}$$

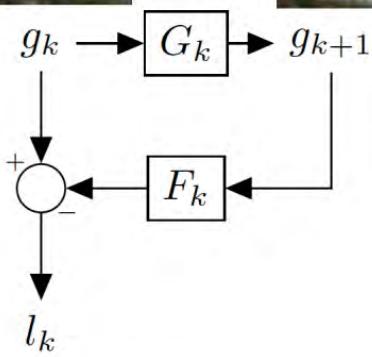
(Downsampling by 2)

## Upsampling and blurring:

$$l_0 = (I_0 - F_0 G_0) g_0$$



# The Laplacian Pyramid



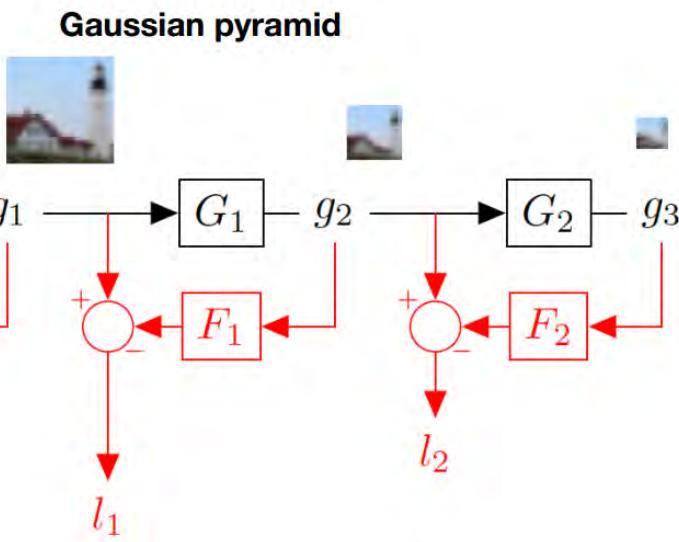
$$l_0 = (I_0 - F_0 G_0) g_0$$

$$= \frac{1}{256} \begin{bmatrix} 182 & -56 & -24 & -8 & -2 & 0 & 0 & 0 \\ -56 & 192 & -56 & -32 & -8 & 0 & 0 & 0 \\ -24 & -56 & 180 & -56 & -24 & -8 & -2 & 0 \\ -8 & -32 & -56 & 192 & -56 & -32 & -8 & 0 \\ -2 & -8 & -24 & -56 & 180 & -56 & -24 & -8 \\ 0 & 0 & -8 & -32 & -56 & 192 & -56 & -32 \\ 0 & 0 & -2 & -8 & -24 & -56 & 182 & -48 \\ 0 & 0 & 0 & 0 & -8 & -32 & -48 & 224 \end{bmatrix} x$$

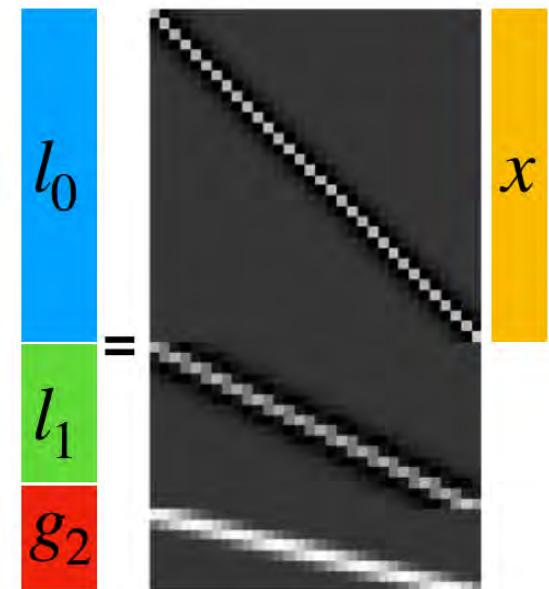




# The Laplacian Pyramid

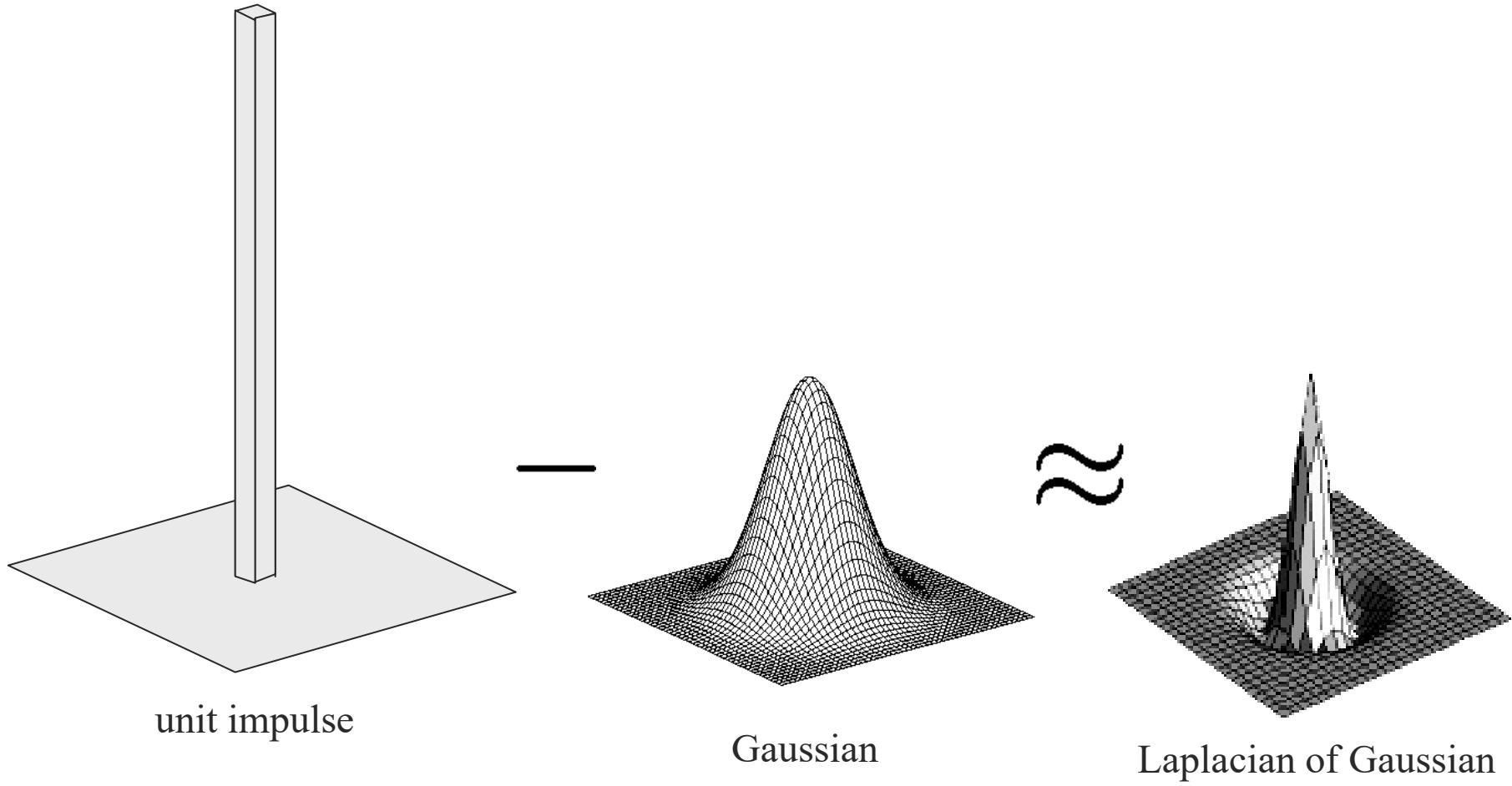


Laplacian pyramid



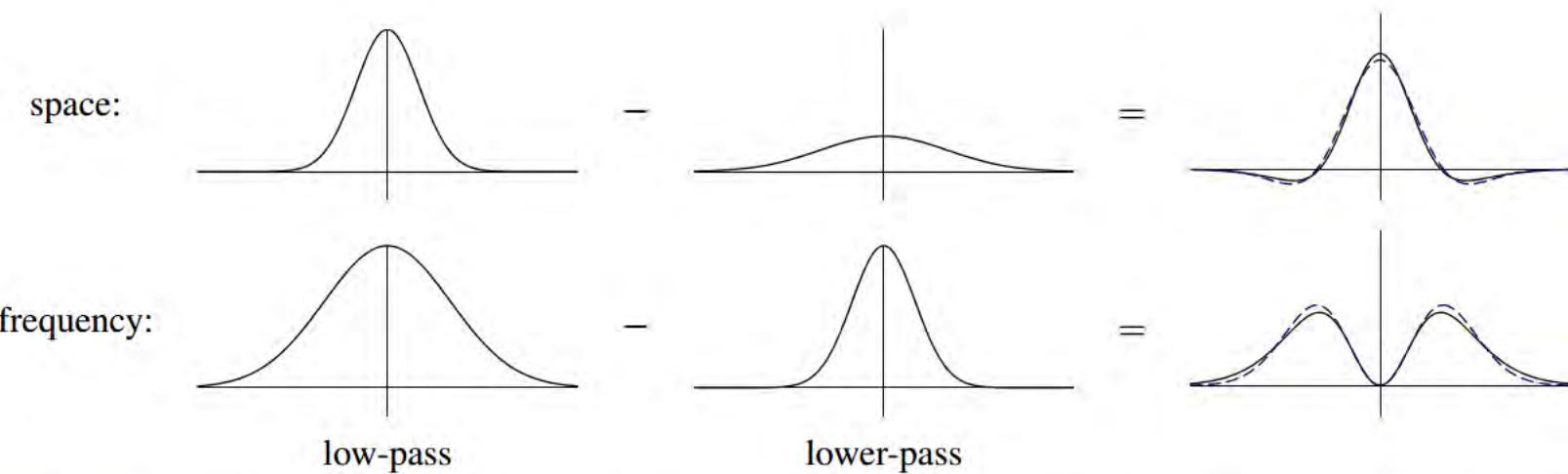


# Laplacian filter





# LoG vs. DoG



**Figure 3.35** The difference of two low-pass filters results in a band-pass filter. The dashed blue lines show the close fit to a half-octave Laplacian of Gaussian.

$$\text{DoG}\{I; \sigma_1, \sigma_2\} = G_{\sigma_1} * I - G_{\sigma_2} * I = (G_{\sigma_1} - G_{\sigma_2}) * I.$$

$$\text{LoG}\{I; \sigma\} = \nabla^2(G_\sigma * I) = (\nabla^2 G_\sigma) * I,$$

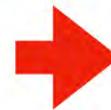
$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$



# The Laplacian Pyramid



Gaussian  
residual



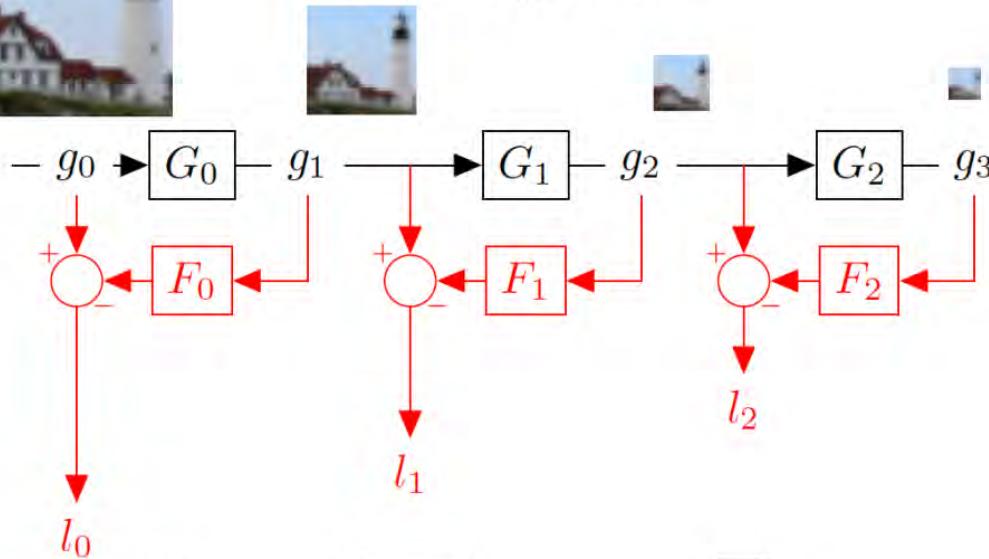
Laplacian pyramid

Can we invert the  
Laplacian Pyramid?

# The Laplacian Pyramid



Gaussian pyramid



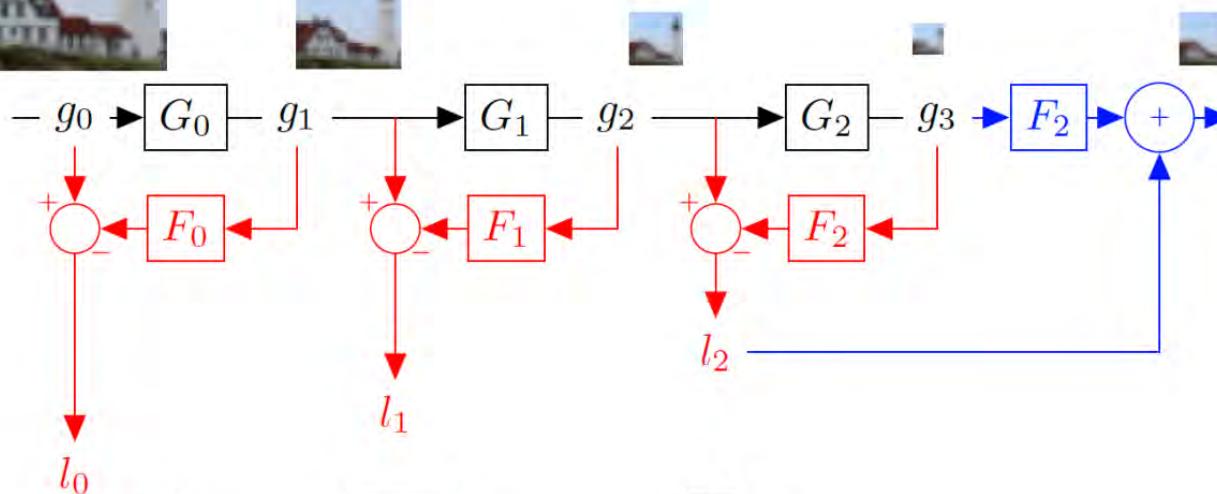
Laplacian pyramid



# The Laplacian Pyramid

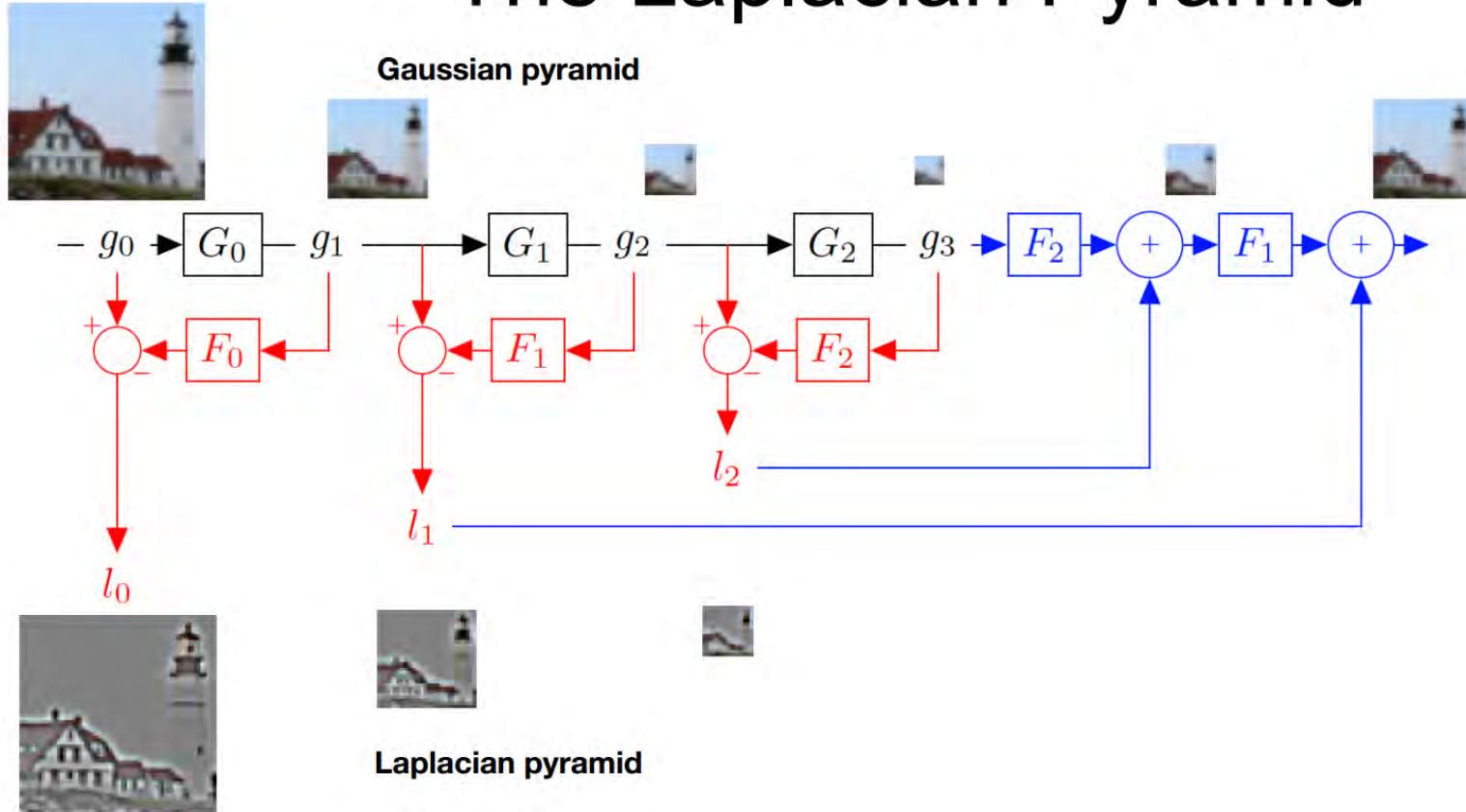


Gaussian pyramid



Laplacian pyramid

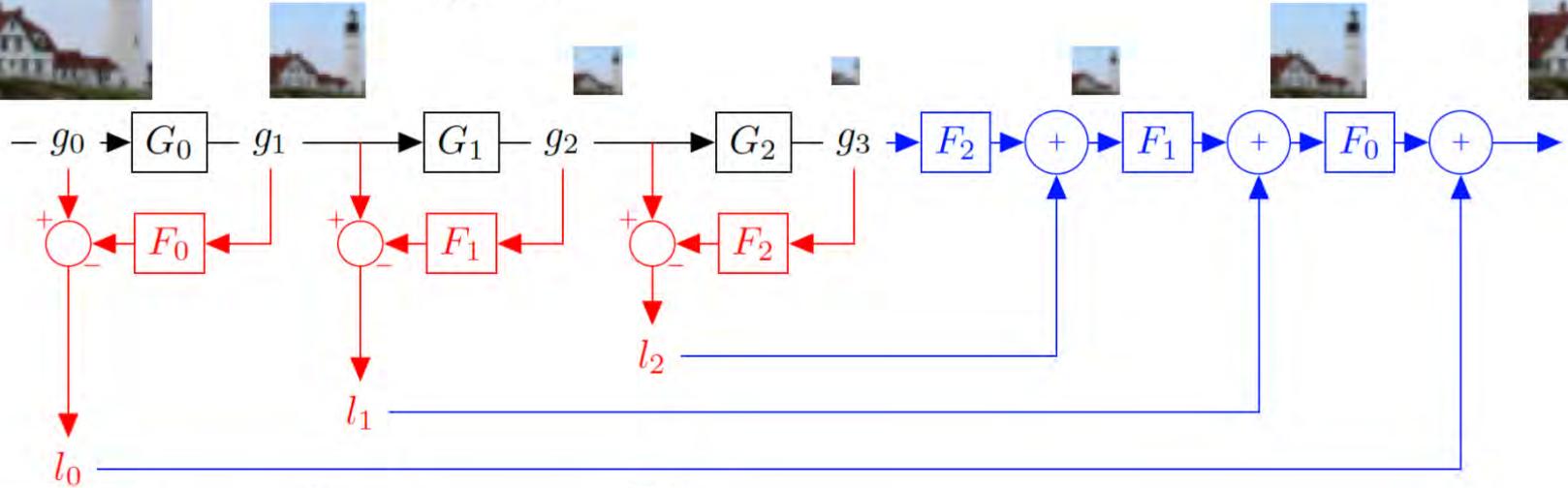
# The Laplacian Pyramid



# The Laplacian Pyramid



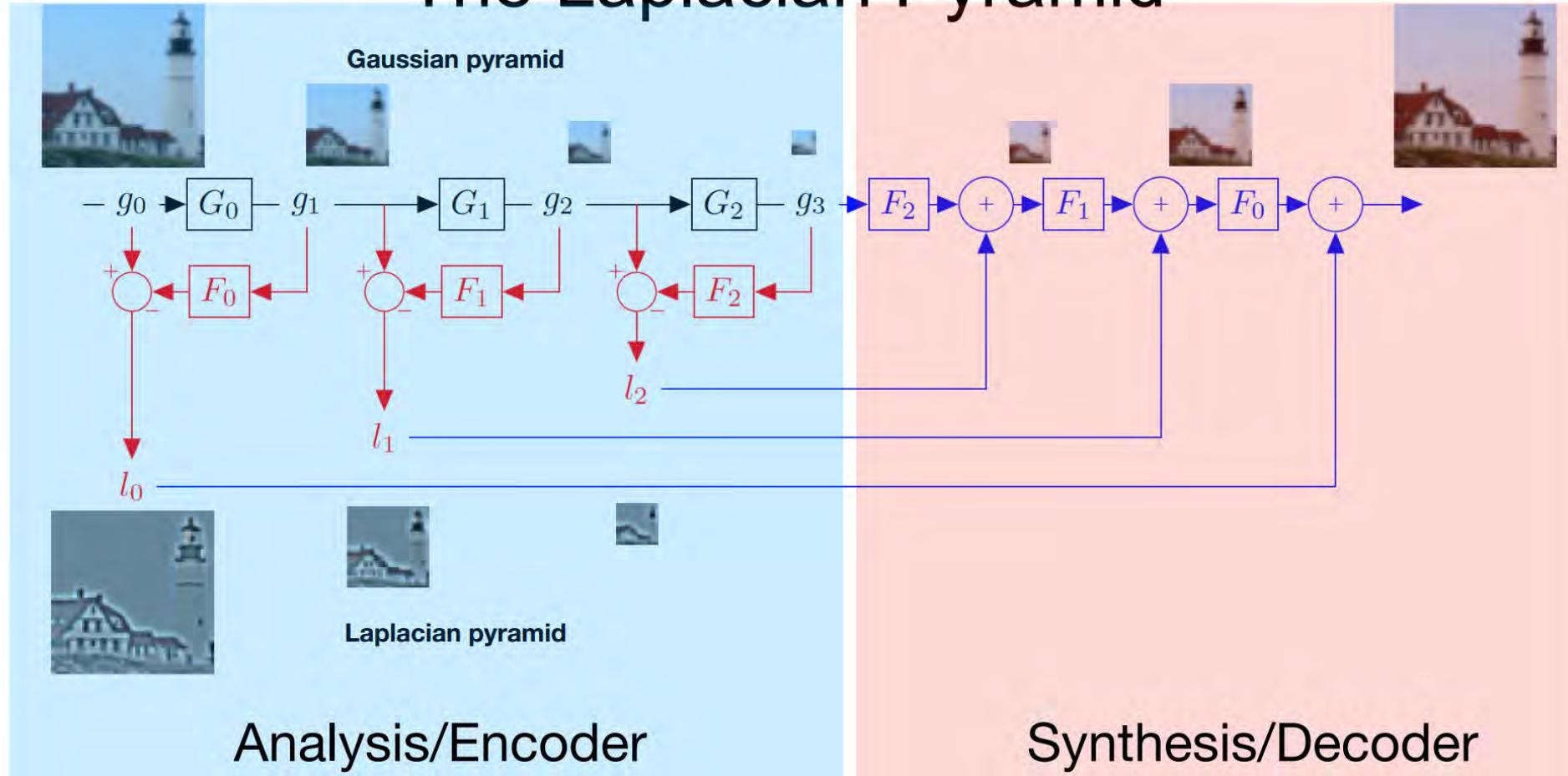
Gaussian pyramid



Laplacian pyramid



# The Laplacian Pyramid



# Laplacian pyramid reconstruction algorithm: recover $x_1$ from $L_1, L_2, L_3$ and $x_4$

$G\#$  is the blur-and-downsample operator at pyramid level  $\#$

$F\#$  is the blur-and-upsample operator at pyramid level  $\#$

First, form Gaussian pyramid:

$$x_2 = G_1 x_1$$

$$x_3 = G_2 x_2$$

$$x_4 = G_3 x_3$$

Then the Laplacian pyramid elements are:

$$L_1 = (I - F_1 G_1) x_1$$

$$L_2 = (I - F_2 G_2) x_2$$

$$L_3 = (I - F_3 G_3) x_3$$

Reconstruction of original image ( $x_1$ ) from Laplacian pyramid elements and the smallest level of the Gaussian pyramid,  $x_4$ :

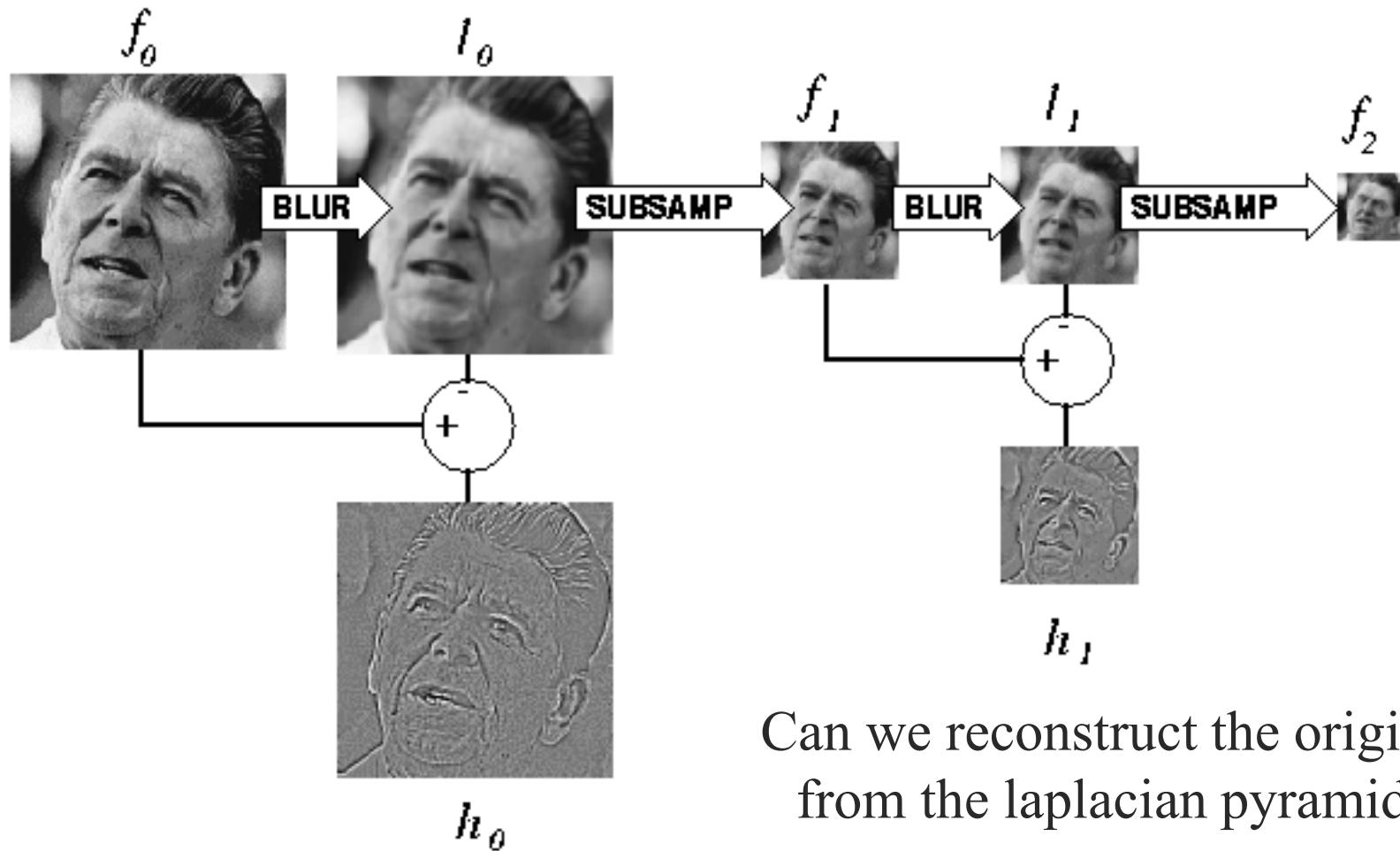
$$x_3 = L_3 + F_3 x_4$$

$$x_2 = L_2 + F_2 x_3$$

$$x_1 = L_1 + F_1 x_2$$



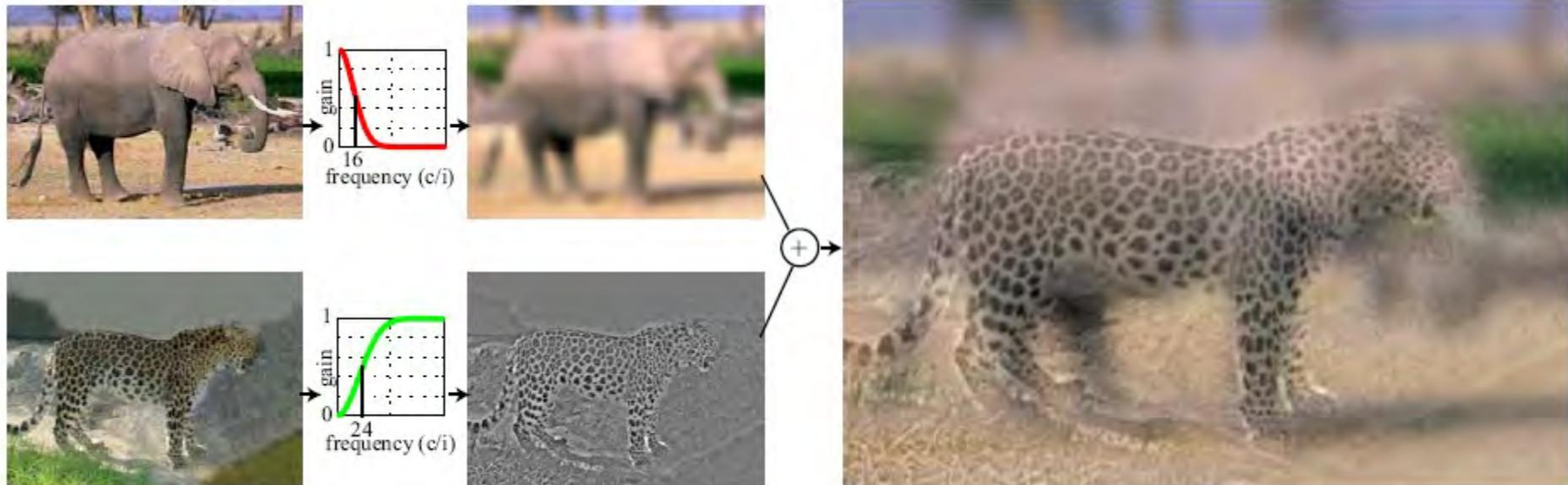
# Computing Gaussian/Laplacian Pyramid



Can we reconstruct the original  
from the laplacian pyramid?



# Hybrid Images

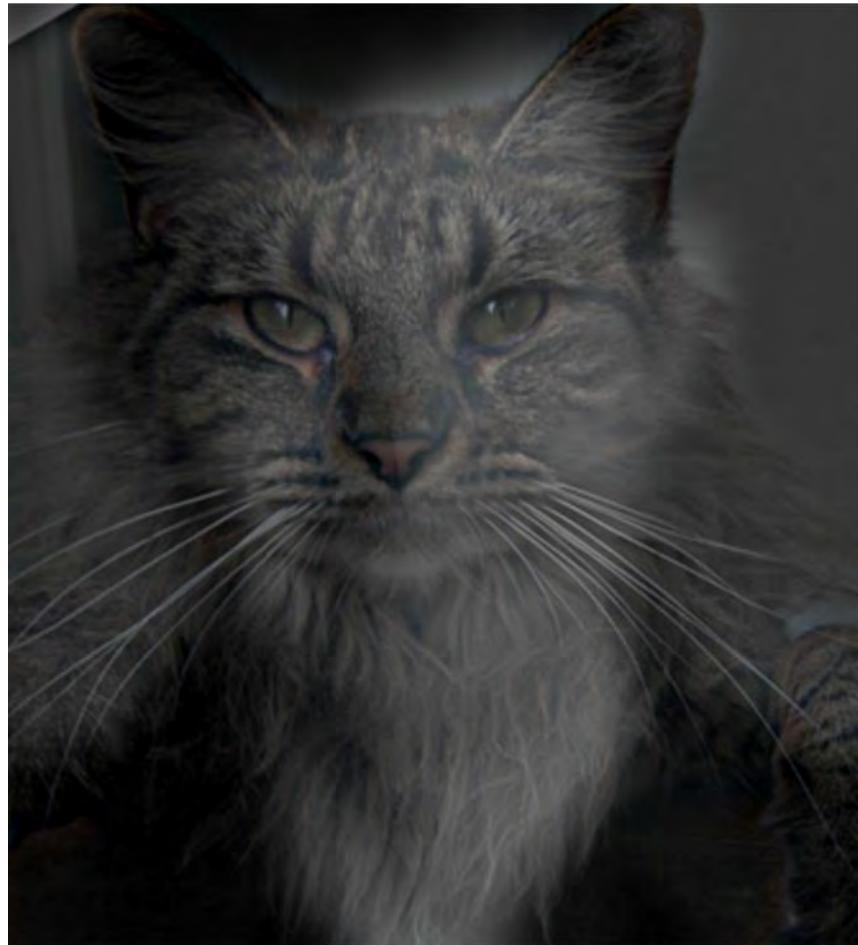


- A. Oliva, A. Torralba, P.G. Schyns,  
“Hybrid Images,” SIGGRAPH 2006





# Hybrid Image





# Creating the Gaussian/Laplacian Pyramid



Image =  $G_1$

Smooth, then downsample



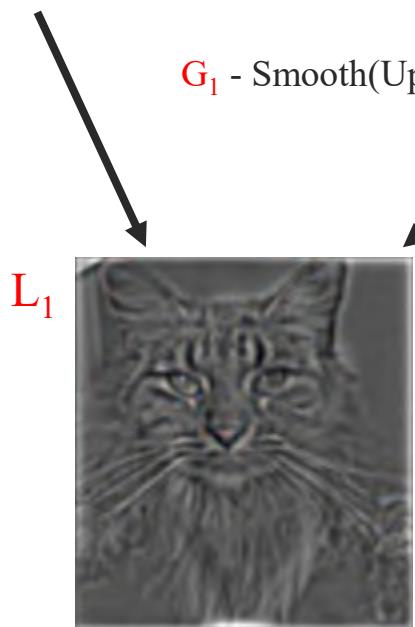
Downsample  
(Smooth( $G_1$ ))



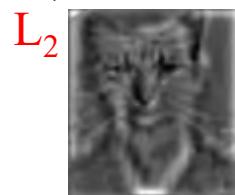
Downsample  
(Smooth( $G_2$ ))



$\dots G_N = L_N$



$G_1 - \text{Smooth}(\text{Upsample}(G_2))$



$G_2 - \text{Smooth}(\text{Upsample}(G_3))$



$G_3 - \text{Smooth}(\text{Upsample}(G_4))$

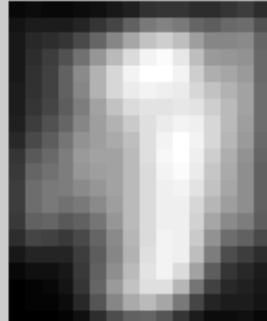
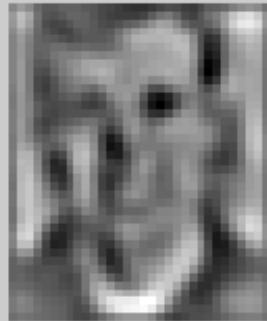
- Use same filter for smoothing in each step (e.g., Gaussian with  $\sigma = 2$ )
  - Downsample/upsample with “nearest” interpolation



# Hybrid Image in Laplacian Pyramid



High frequency → Low frequency





# Reconstructing image from Laplacian pyramid



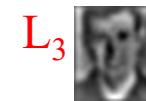
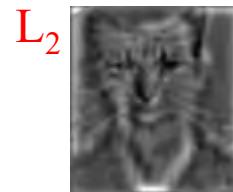
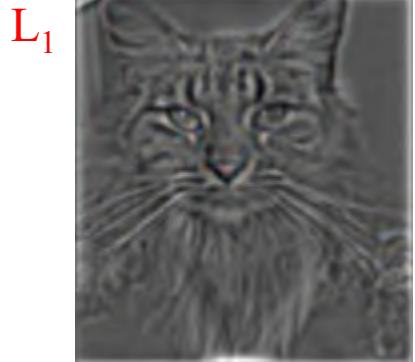
$$\text{Image} = L_1 + \text{Smooth}(\text{Upsample}(G_2))$$



$$G_2 = L_2 + \text{Smooth}(\text{Upsample}(G_3))$$



$$G_3 = L_3 + \text{Smooth}(\text{Upsample}(L_4))$$



- Use same filter for smoothing as in deconstruction
  - Upsample with “nearest” interpolation
  - Reconstruction will be nearly lossless



# Laplacian pyramid applications



- Texture synthesis
- Image compression
- Noise removal
- Computing image features (e.g., SIFT)
- Image Blending...



# Image Blending



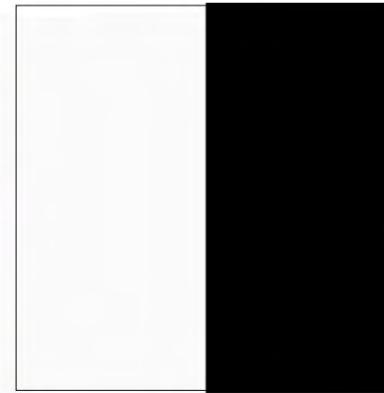


# Image Blending





# Image Blending

 $I^A$  $I^B$  $m$  $I$ 

$$I = m * I^A + (1 - m) * I^B$$



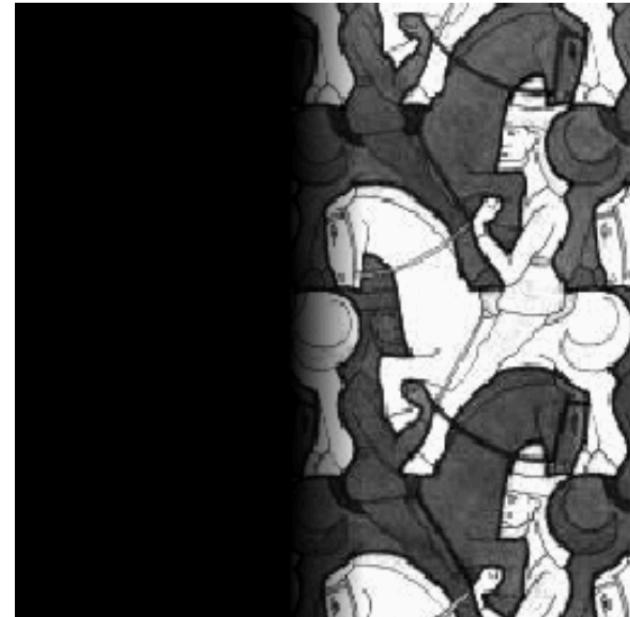
# Image Blending



## Feathering

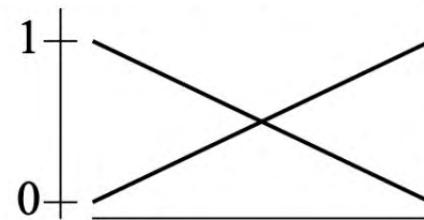
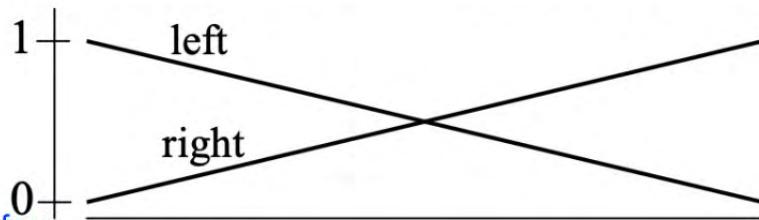


+





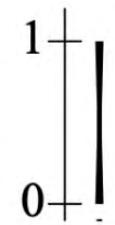
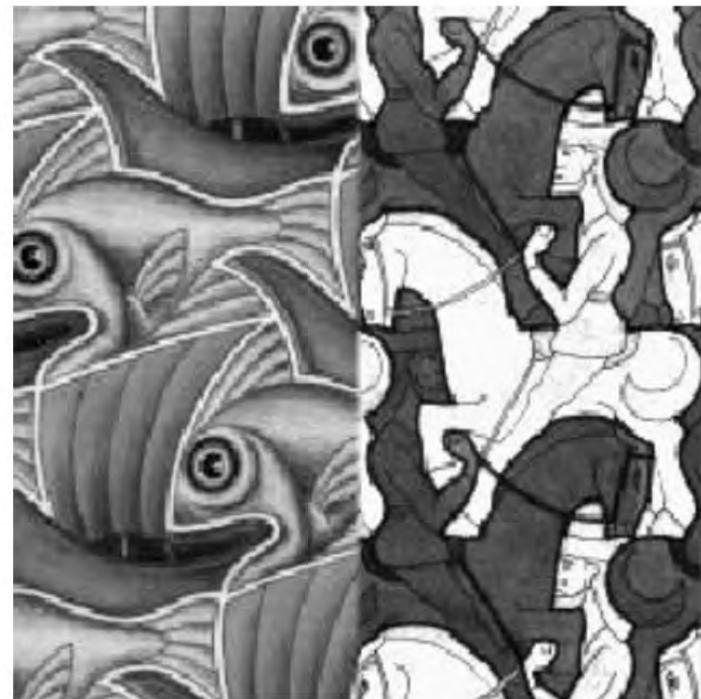
# Affect of window size



Slide by A. Efros



# Affect of window size



Slide by A. Efros



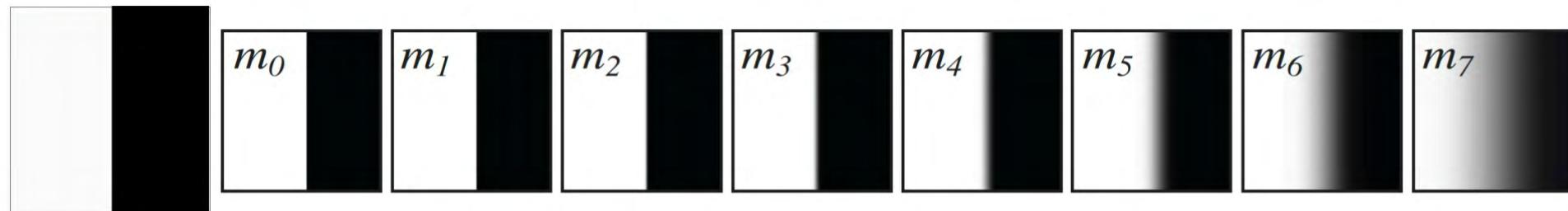
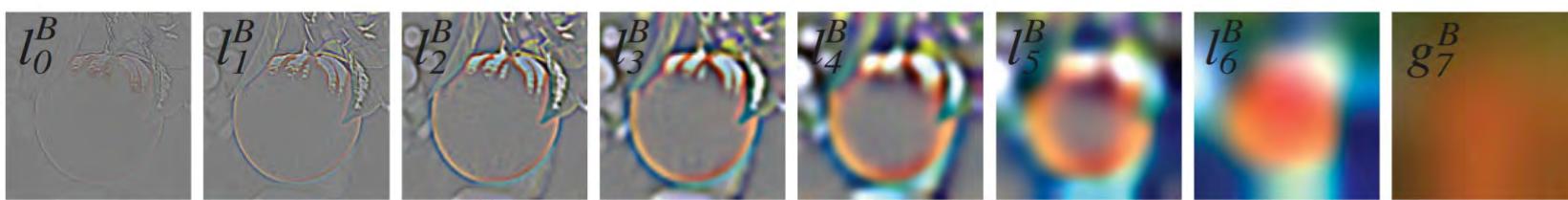
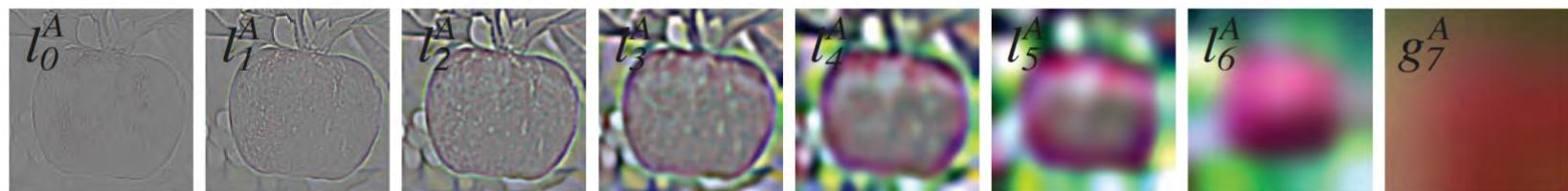
# Good Window Size



“Optimal” Window: smooth but not ghosted



# Image Blending with the Laplacian Pyramid



$$l_k = l_k^A * m_k + l_k^B * (1 - m_k)$$



# Image Blending with the Laplacian Pyramid





# Image Blending with the Laplacian Pyramid



- Build Laplacian pyramid for both images: LA, LB
- Build Gaussian pyramid for mask: G
- Build a combined Laplacian pyramid:
- Collapse L to obtain the blended image



# Image pyramids

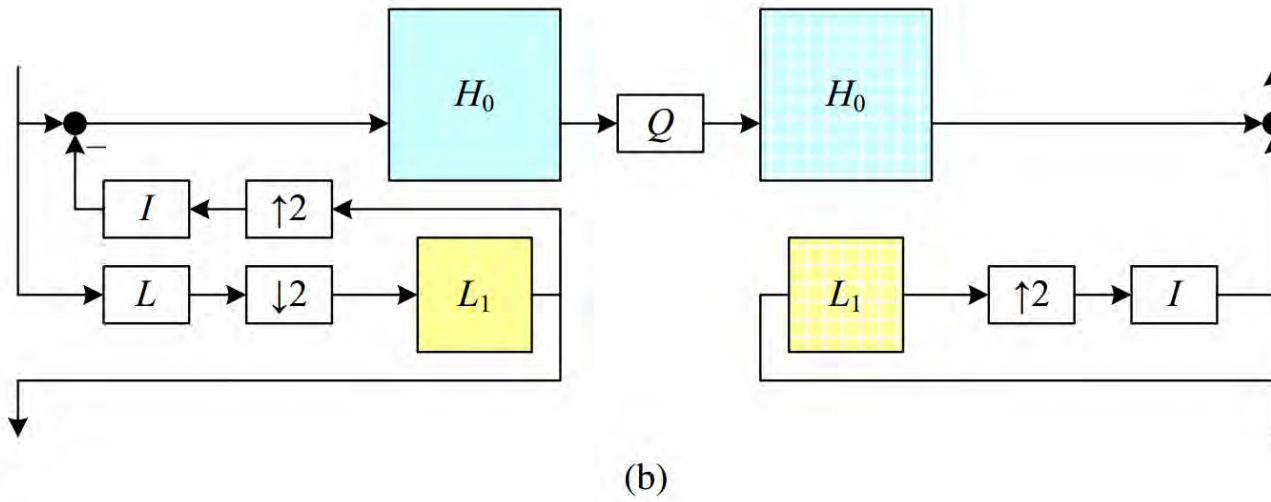
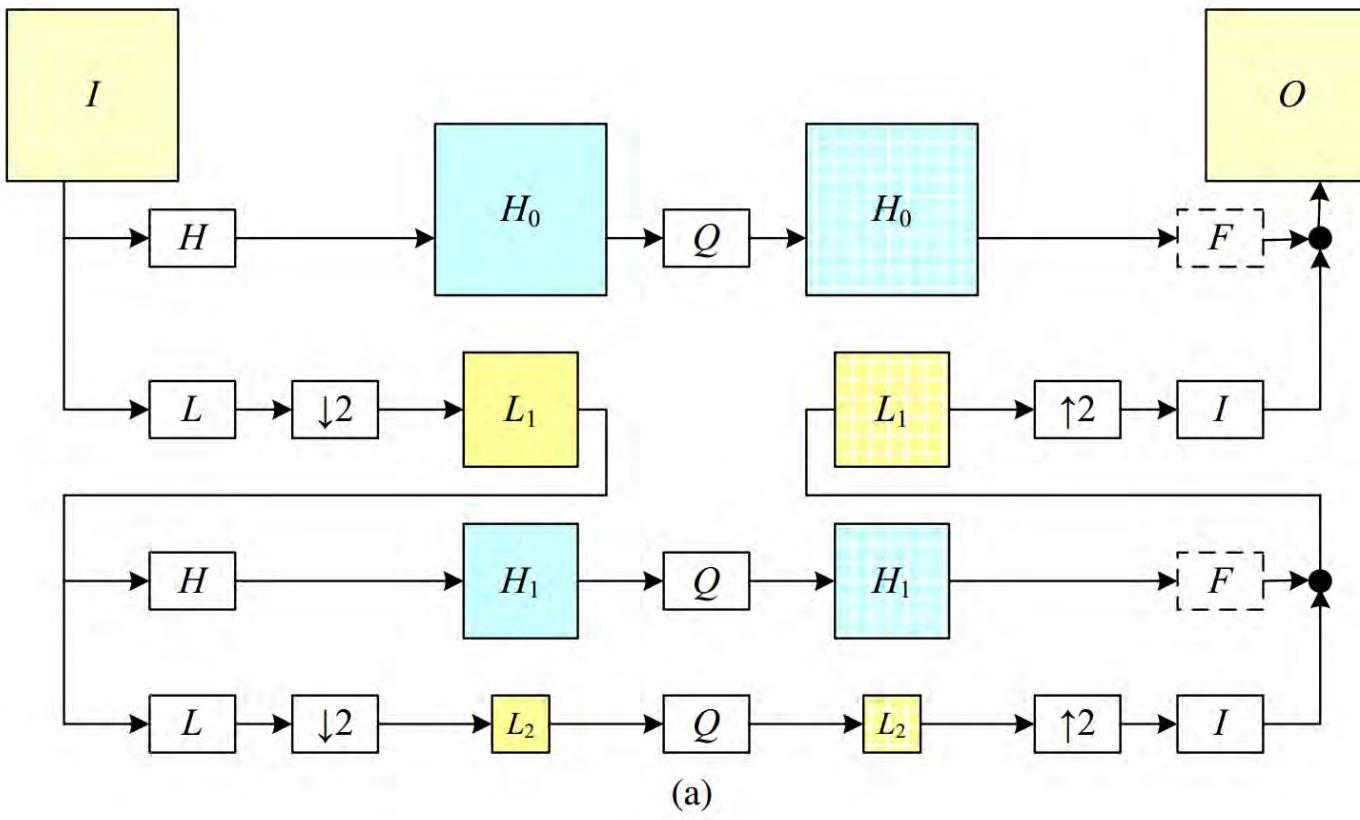
- Gaussian
- Laplacian



Progressively blurred and subsampled versions of the image. Adds scale invariance to fixed-size algorithms.



Shows the information added in Gaussian pyramid at each spatial scale. Useful for noise reduction & coding.





# Today's class



- Template matching
- Gaussian Pyramids
  - Application for recognition
  - Pyramids representation in deep learning
- Laplacian Pyramids
  - Application for image blending
  - Hybrid images
- **Steerable Pyramids:**
  - filter banks and texture analysis

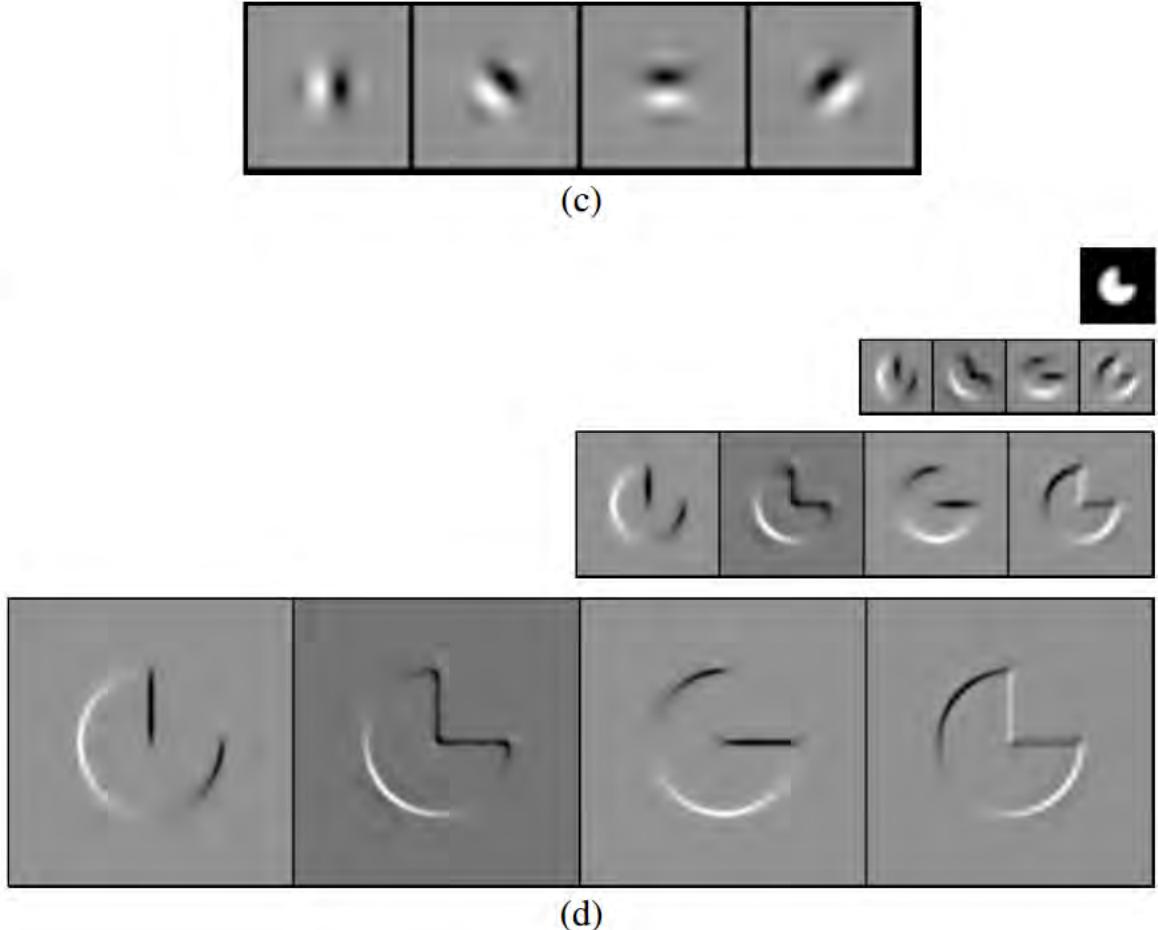
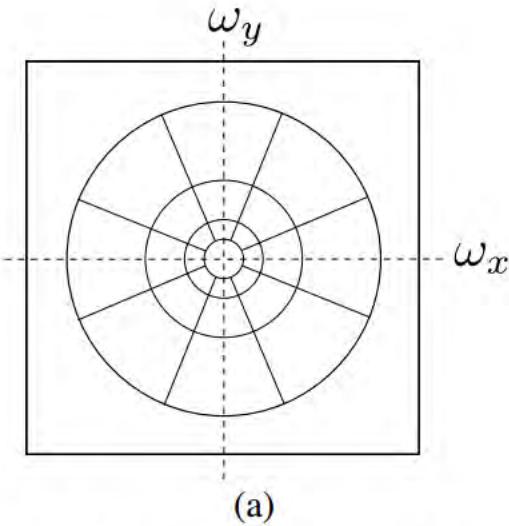


# Orientations



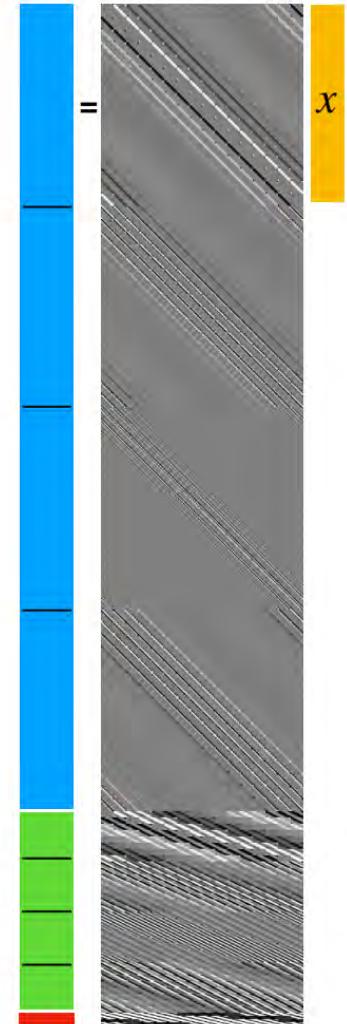
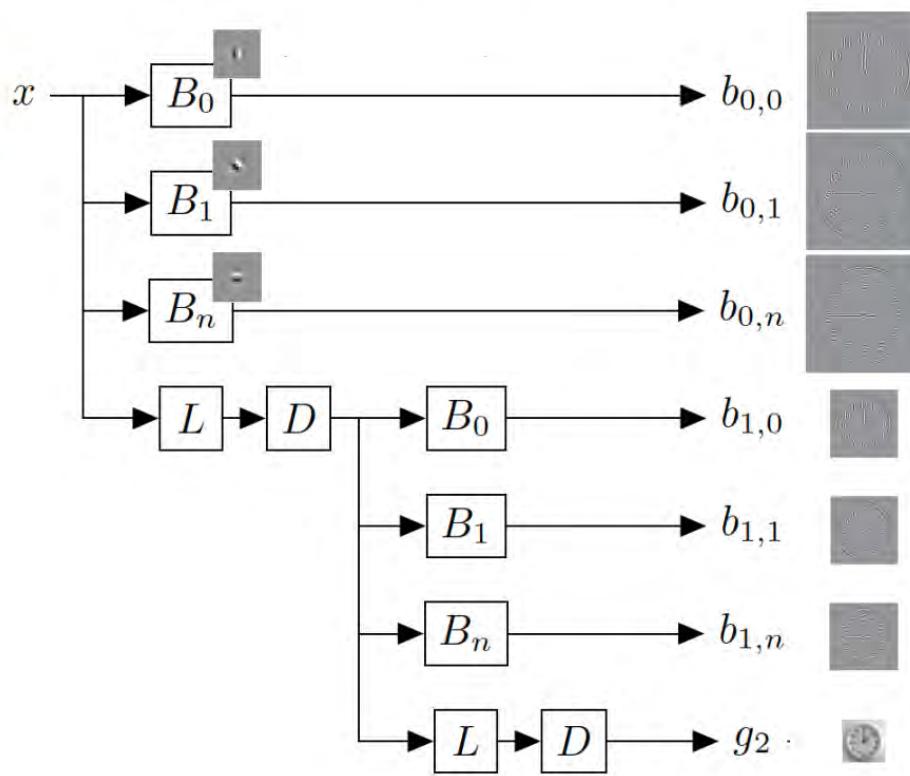


# Steerable Pyramid



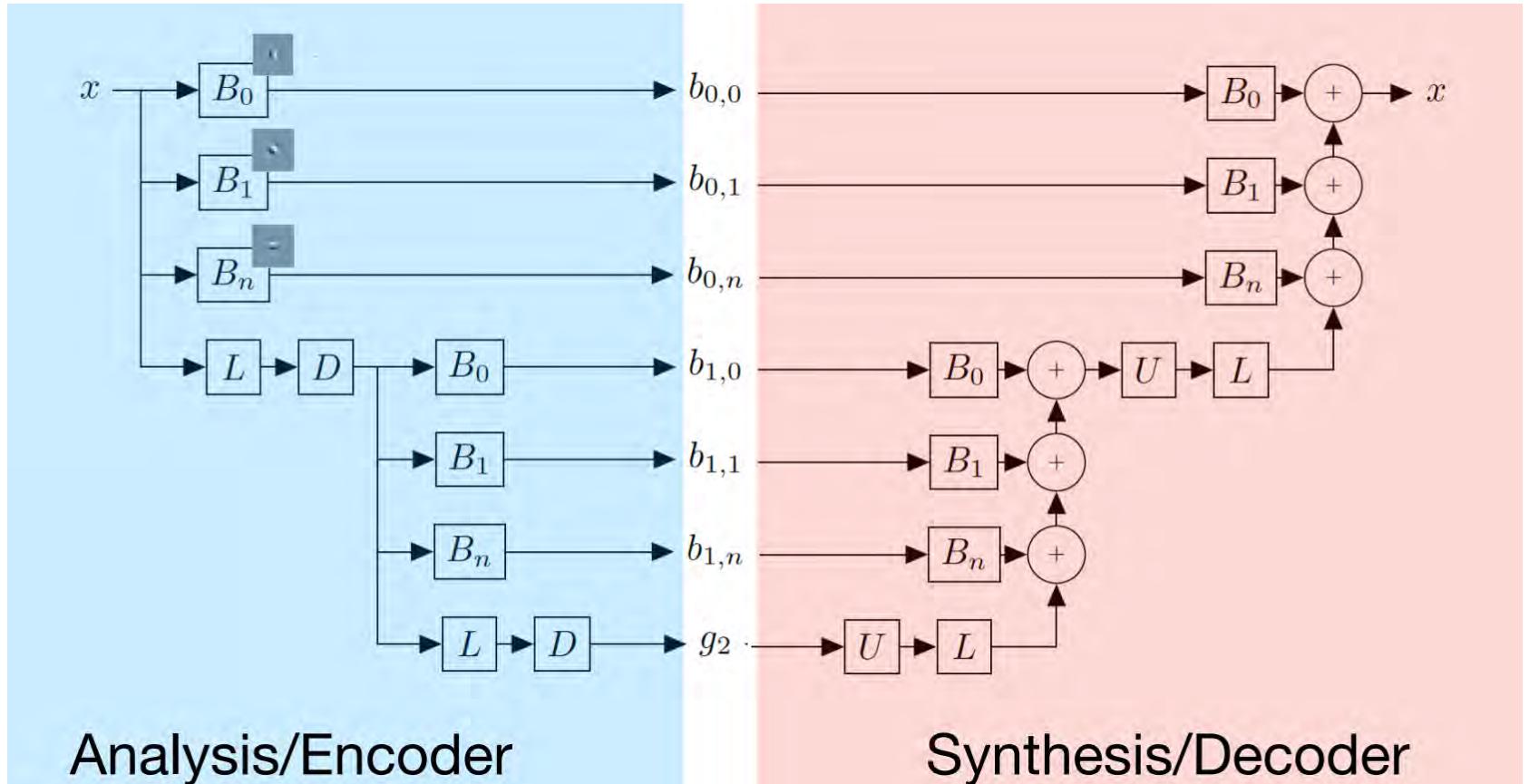
**Figure 3.40** Steerable shiftable multiscale transforms (Simoncelli, Freeman, Adelson *et al.* 1992) © 1992 IEEE: (a) radial multi-scale frequency domain decomposition; (b) original image; (c) a set of four steerable filters; (d) the radial multi-scale wavelet decomposition.

# Steerable Pyramid





# Steerable Pyramid



Analysis/Encoder

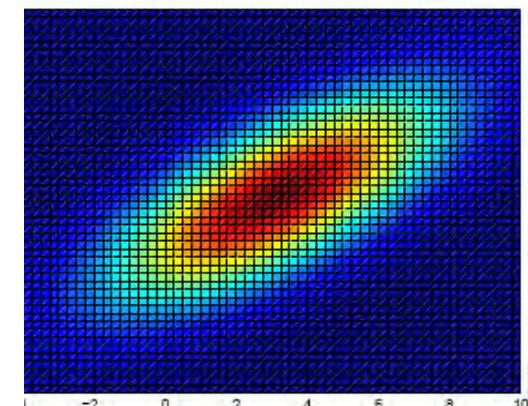
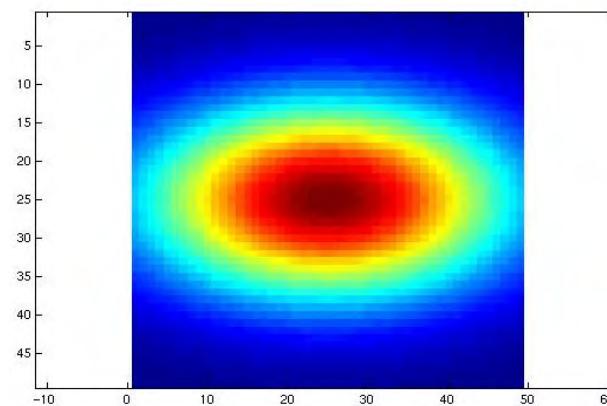
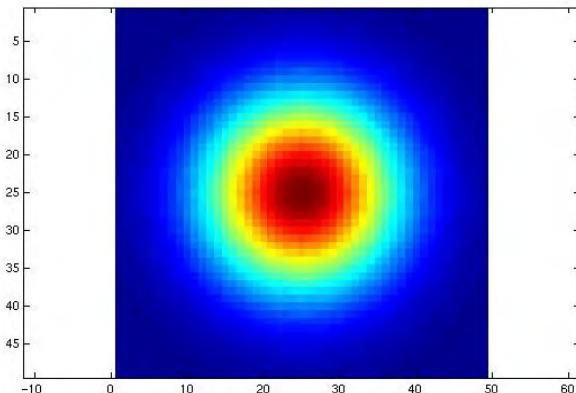
Synthesis/Decoder



# Multivariate Gaussian



$$p(x; \mu, \Sigma) = \frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp \left( -\frac{1}{2} (x - \mu)^T \Sigma^{-1} (x - \mu) \right)$$



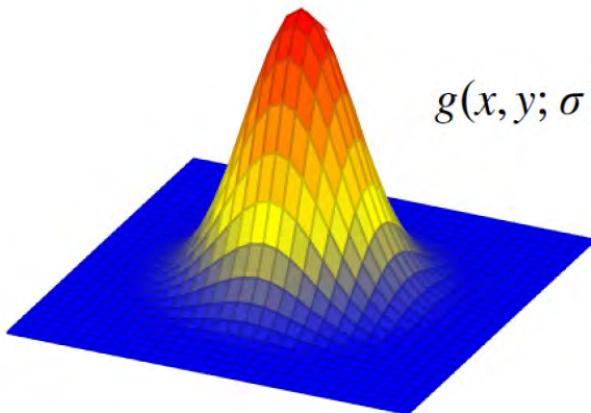
$$\Sigma = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 16 & 0 \\ 0 & 9 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 10 & 5 \\ 5 & 5 \end{bmatrix}$$



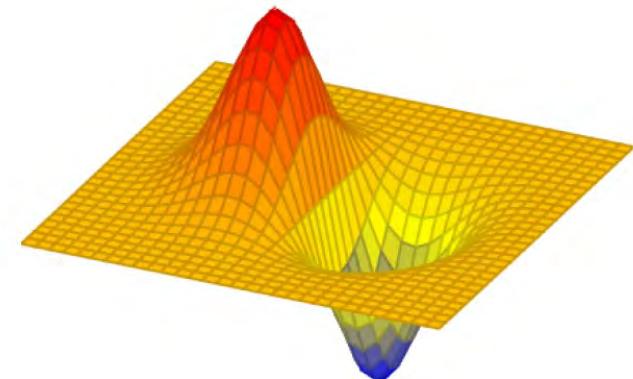
# Gaussian Derivative



$$g(x, y; \sigma) = \frac{1}{2\pi\sigma^2} \exp -\frac{x^2 + y^2}{2\sigma^2}$$

The continuous derivative is:

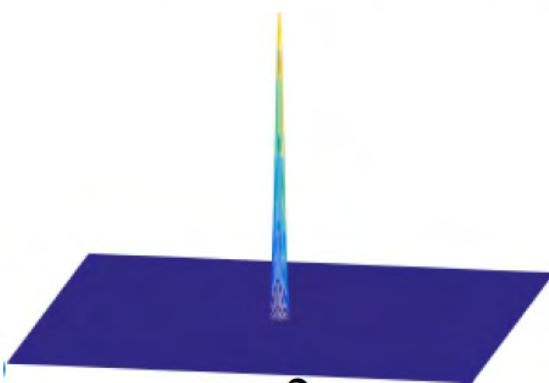
$$\begin{aligned} g_x(x, y; \sigma) &= \frac{\partial g(x, y; \sigma)}{\partial x} = \\ &= \frac{-x}{2\pi\sigma^4} \exp -\frac{x^2 + y^2}{2\sigma^2} \\ &= \frac{-x}{\sigma^2} g(x, y; \sigma) \end{aligned}$$



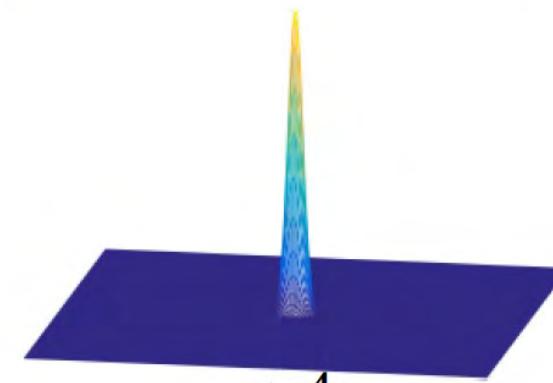
104



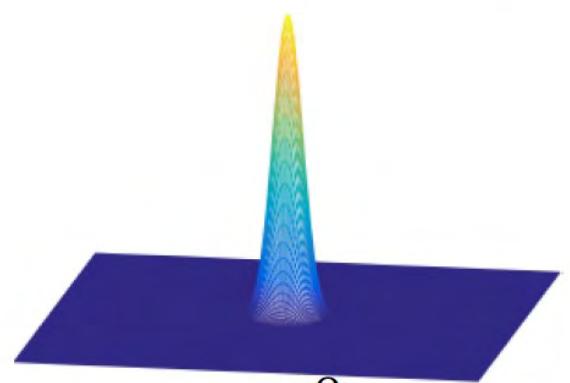
# Gaussian Scale



$\sigma=2$



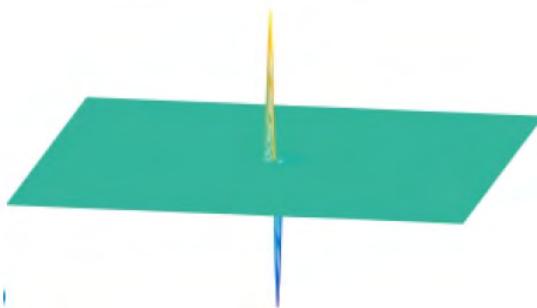
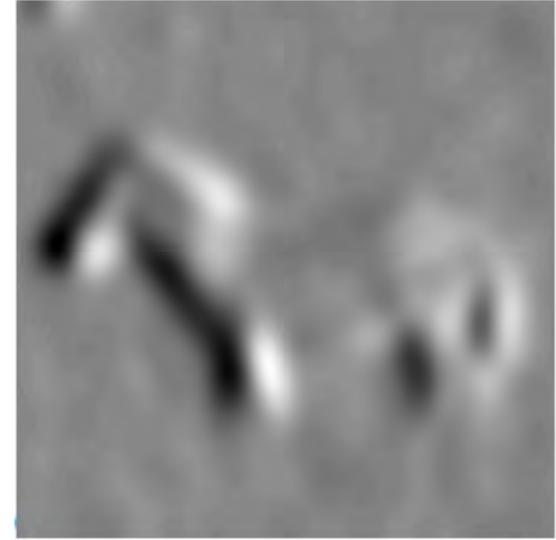
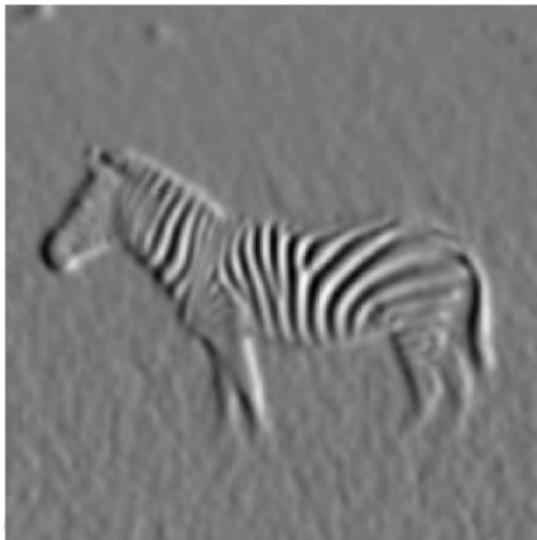
$\sigma=4$



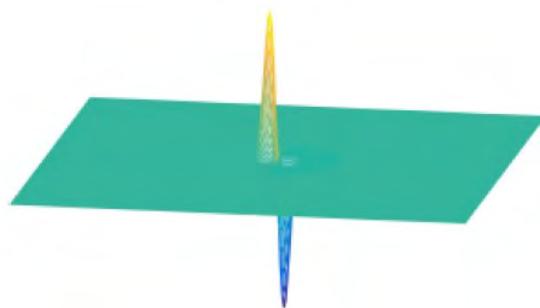
$\sigma=8$



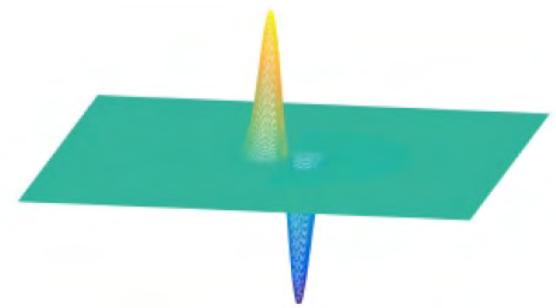
# Derivatives of Gaussian: Scale



$\sigma=2$



$\sigma=4$



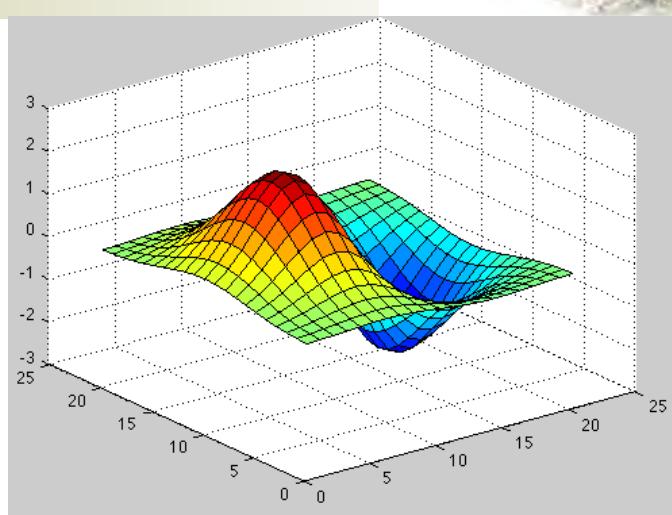
$\sigma=8$



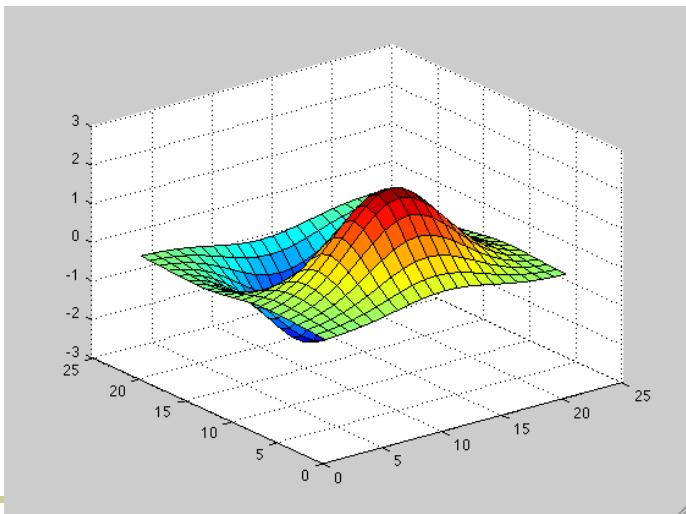
# Orientation



$$g_x(x,y) = \frac{\partial g(x,y)}{\partial x} = \frac{-x}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



$$g_y(x,y) = \frac{\partial g(x,y)}{\partial y} = \frac{-y}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

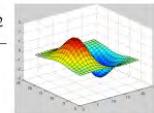




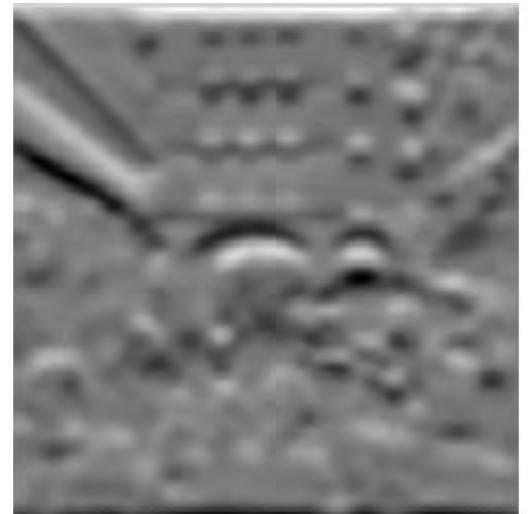
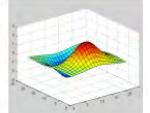
# Orientation



$$g_x(x,y) = \frac{\partial g(x,y)}{\partial x} = \frac{-x}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



$$g_y(x,y) = \frac{\partial g(x,y)}{\partial y} = \frac{-y}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



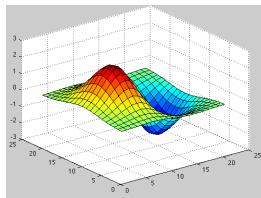
What about other orientations not axis aligned?



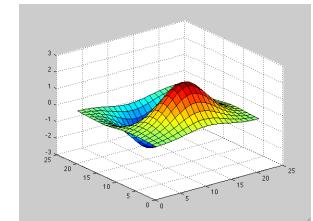
# Orientation



$$g_x(x,y) = \frac{\partial g(x,y)}{\partial x} = \frac{-x}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



$$g_y(x,y) = \frac{\partial g(x,y)}{\partial y} = \frac{-y}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



The smoothed directional gradient is a linear combination of two kernels

$$u^T \nabla g \otimes I = (\cos(\alpha)g_x(x,y) + \sin(\alpha)g_y(x,y)) \otimes I(x,y) =$$

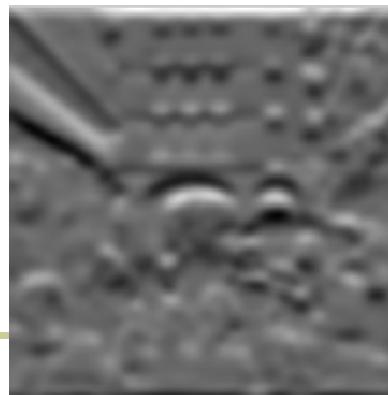
Any orientation can be computed as a linear combination of two filtered images

$$= \cos(\alpha)g_x(x,y) \otimes I(x,y) + \sin(\alpha)g_y(x,y) \otimes I(x,y) =$$

$$= \cos(\alpha)$$



$$+ \sin(\alpha)$$



$$=$$





# Simple example of steerable filter

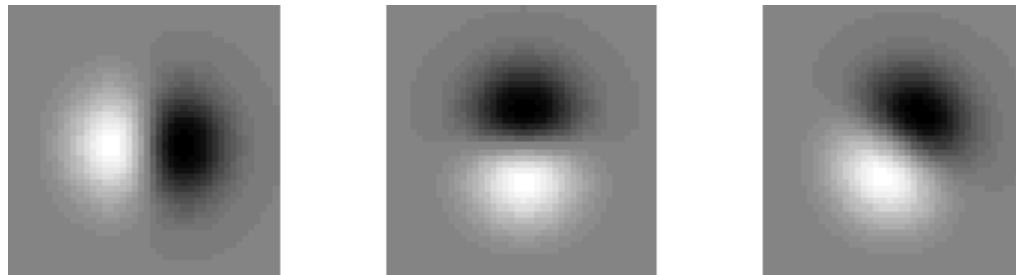


“Steerability”-- the ability to synthesize a filter of any orientation from a linear combination of filters at fixed orientations.

$$G_{\theta}^1 = \cos(\theta)G_0^1 + \sin(\theta)G_{90}^1$$

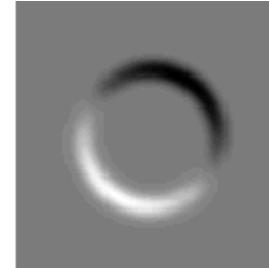
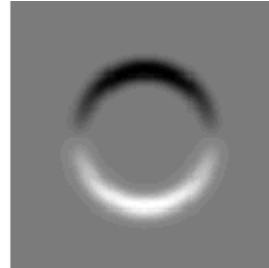
$0^\circ$                      $90^\circ$                     Synthesized  $30^\circ$

Filter Set:



Response:

Raw Image



Taken from:  
W. Freeman, T.  
Adelson, “The Design  
and Use of Steerable  
Filters”, IEEE  
Trans. Patt. Anal. and  
Machine Intell.,  
vol 13, #9, pp 891-900,  
Sept 1991



# Orientation analysis

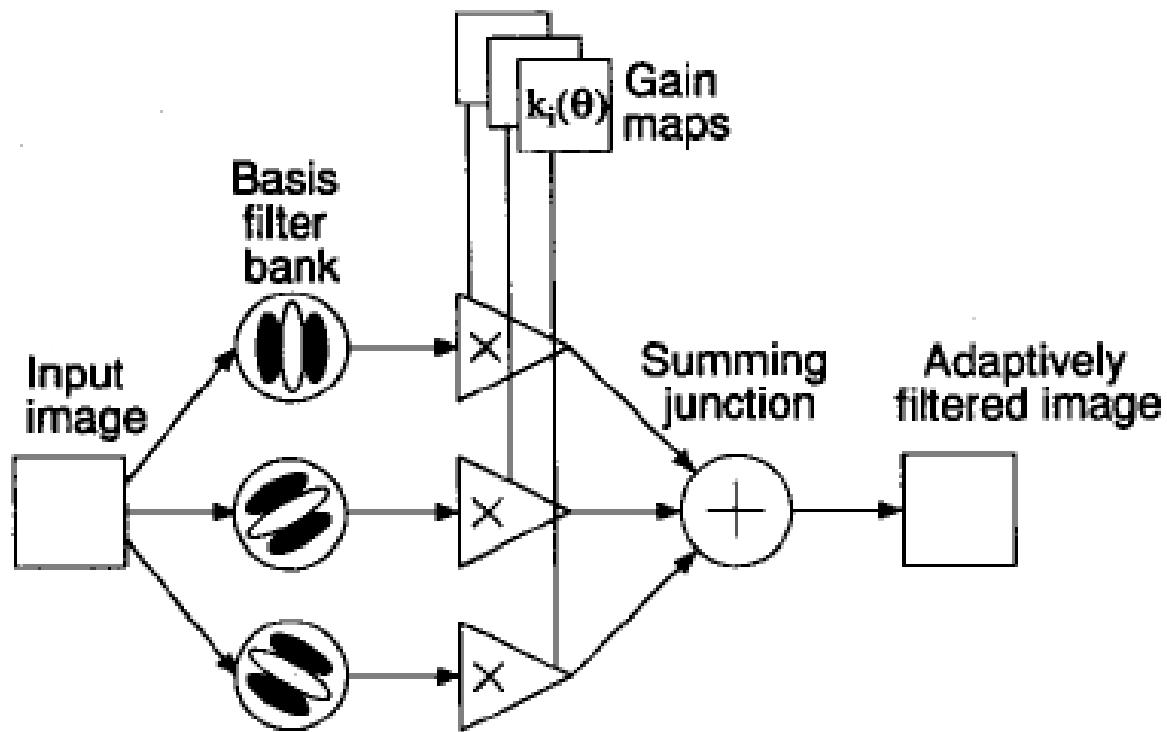
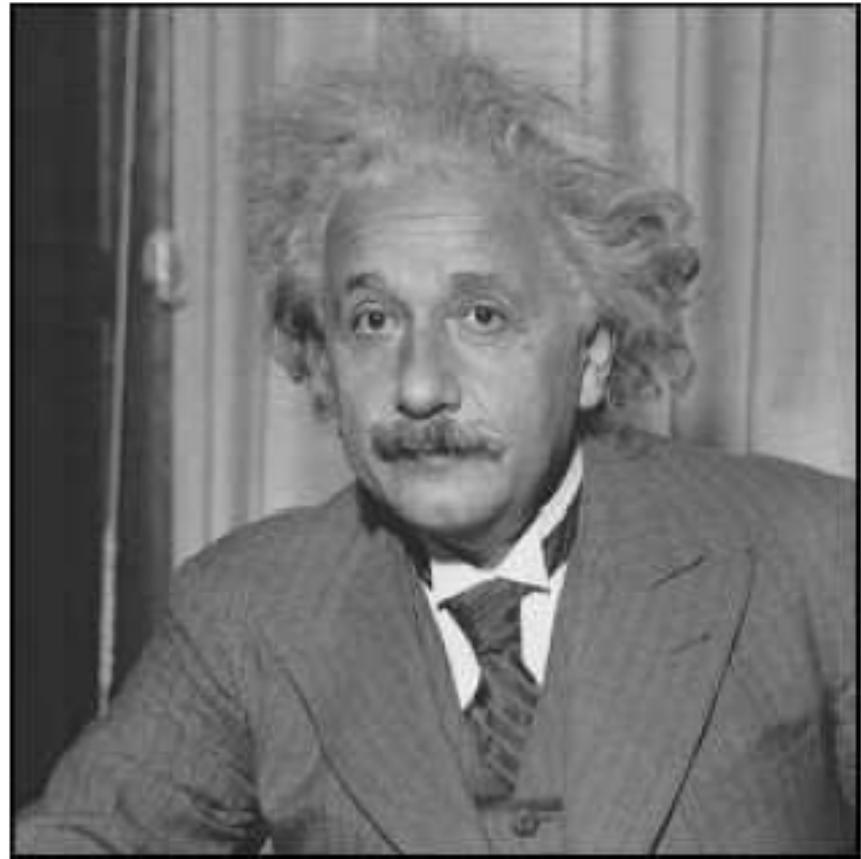


Fig. 3. Steerable filter system block diagram. A bank of dedicated filters process the image. Their outputs are multiplied by a set of gain maps that adaptively control the orientation of the synthesized filter.



# Orientation analysis



(a)



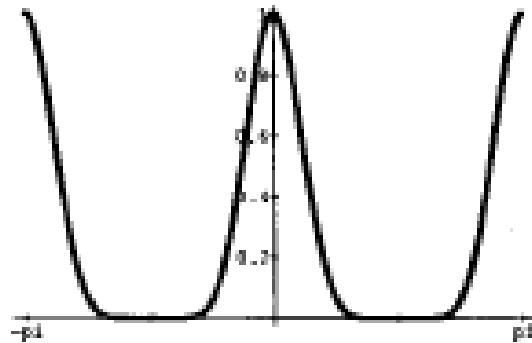
(b)



# Orientation analysis



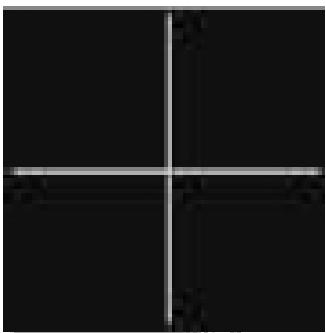
(a)



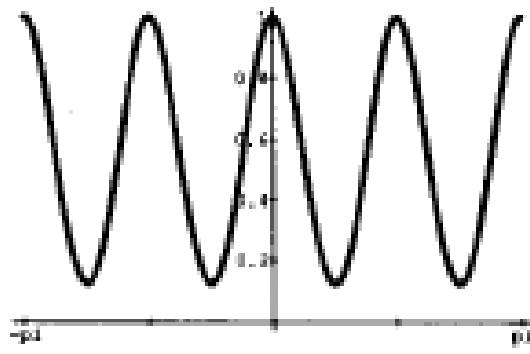
(c)



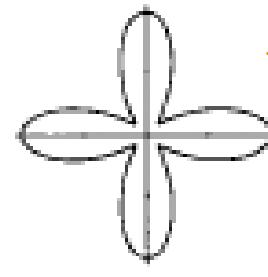
(e)



(b)



(d)



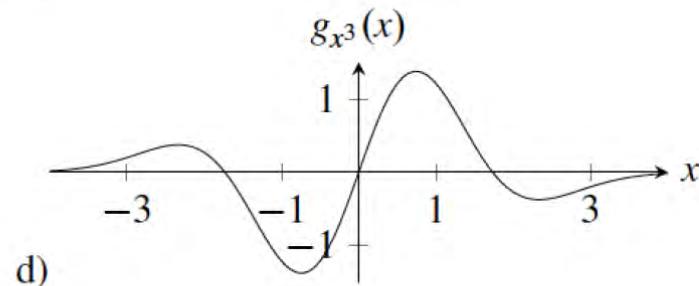
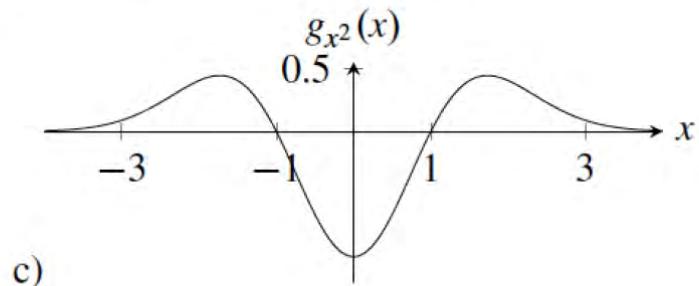
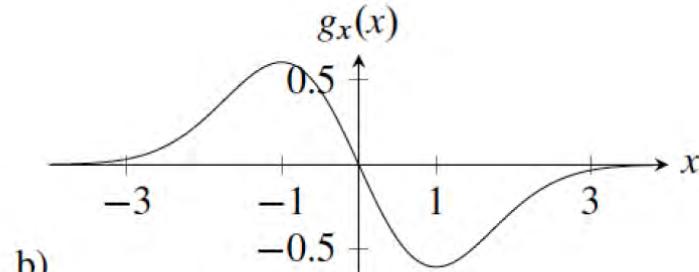
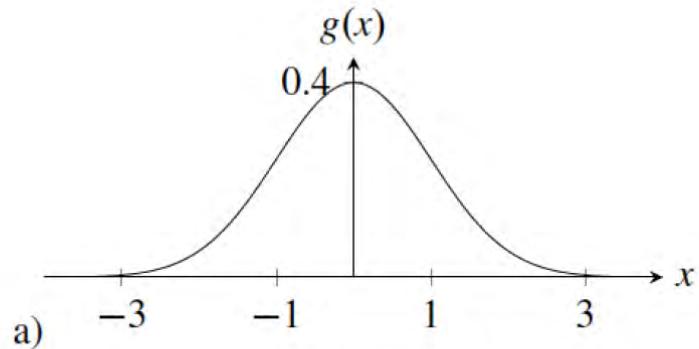
(f)



High resolution in orientation requires many oriented filters as basis (high order gaussian derivatives or fine-tuned Gabor wavelets).



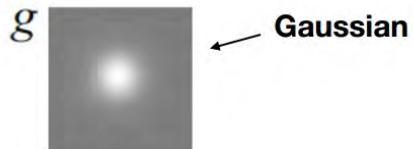
# N-th order Gaussian derivatives



$$g_{x^n, y^m}(x, y; \sigma) = \frac{\partial^{n+m} g(x, y)}{\partial x^n \partial y^m} = \left( \frac{-1}{\sigma \sqrt{2}} \right)^{n+m} H_n \left( \frac{x}{\sigma \sqrt{2}} \right) H_m \left( \frac{y}{\sigma \sqrt{2}} \right) g(x, y; \sigma)$$



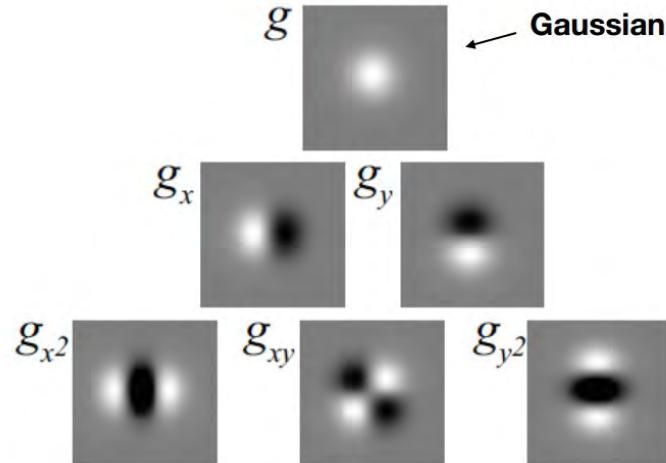
# N-th order Gaussian derivatives



$$g_{x^n, y^m}(x, y; \sigma) = \frac{\partial^{n+m} g(x, y)}{\partial x^n \partial y^m} = \left( \frac{-1}{\sigma \sqrt{2}} \right)^{n+m} H_n \left( \frac{x}{\sigma \sqrt{2}} \right) H_m \left( \frac{y}{\sigma \sqrt{2}} \right) g(x, y; \sigma)$$



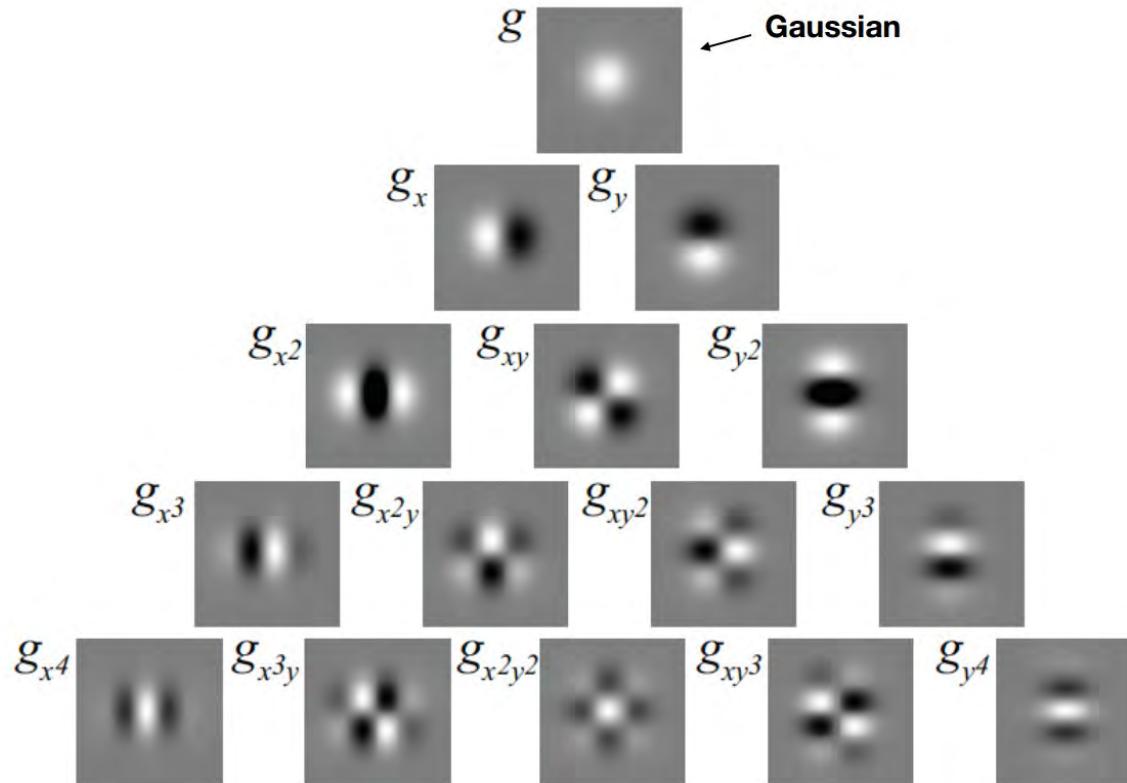
# N-th order Gaussian derivatives



$$g_{x^n, y^m}(x, y; \sigma) = \frac{\partial^{n+m} g(x, y)}{\partial x^n \partial y^m} = \left( \frac{-1}{\sigma \sqrt{2}} \right)^{n+m} H_n \left( \frac{x}{\sigma \sqrt{2}} \right) H_m \left( \frac{y}{\sigma \sqrt{2}} \right) g(x, y; \sigma)$$



# N-th order Gaussian derivatives



$$g_{x^n, y^m}(x, y; \sigma) = \frac{\partial^{n+m} g(x, y)}{\partial x^n \partial y^m} = \left( \frac{-1}{\sigma \sqrt{2}} \right)^{n+m} H_n \left( \frac{x}{\sigma \sqrt{2}} \right) H_m \left( \frac{y}{\sigma \sqrt{2}} \right) g(x, y; \sigma)$$



# Filter bank

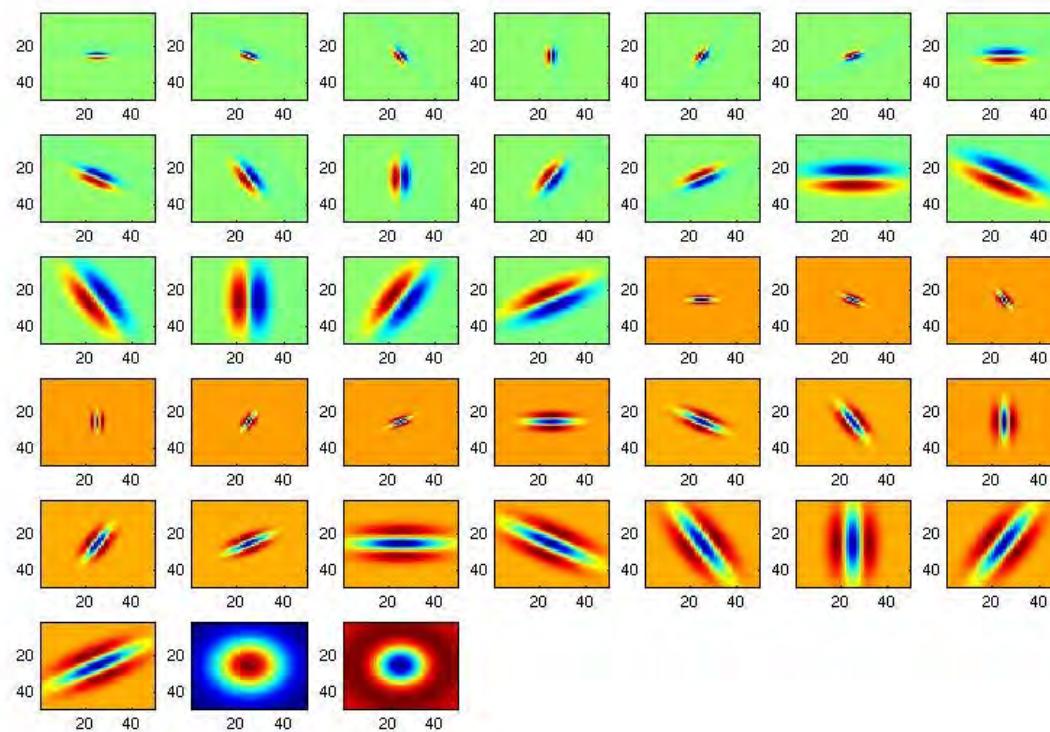
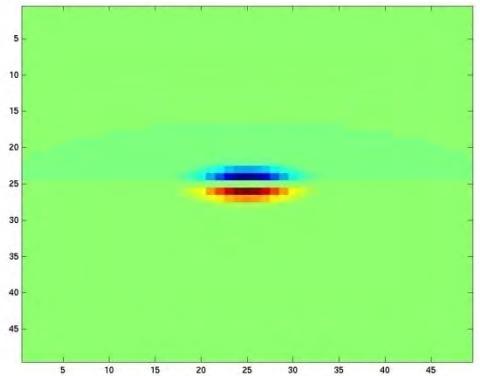
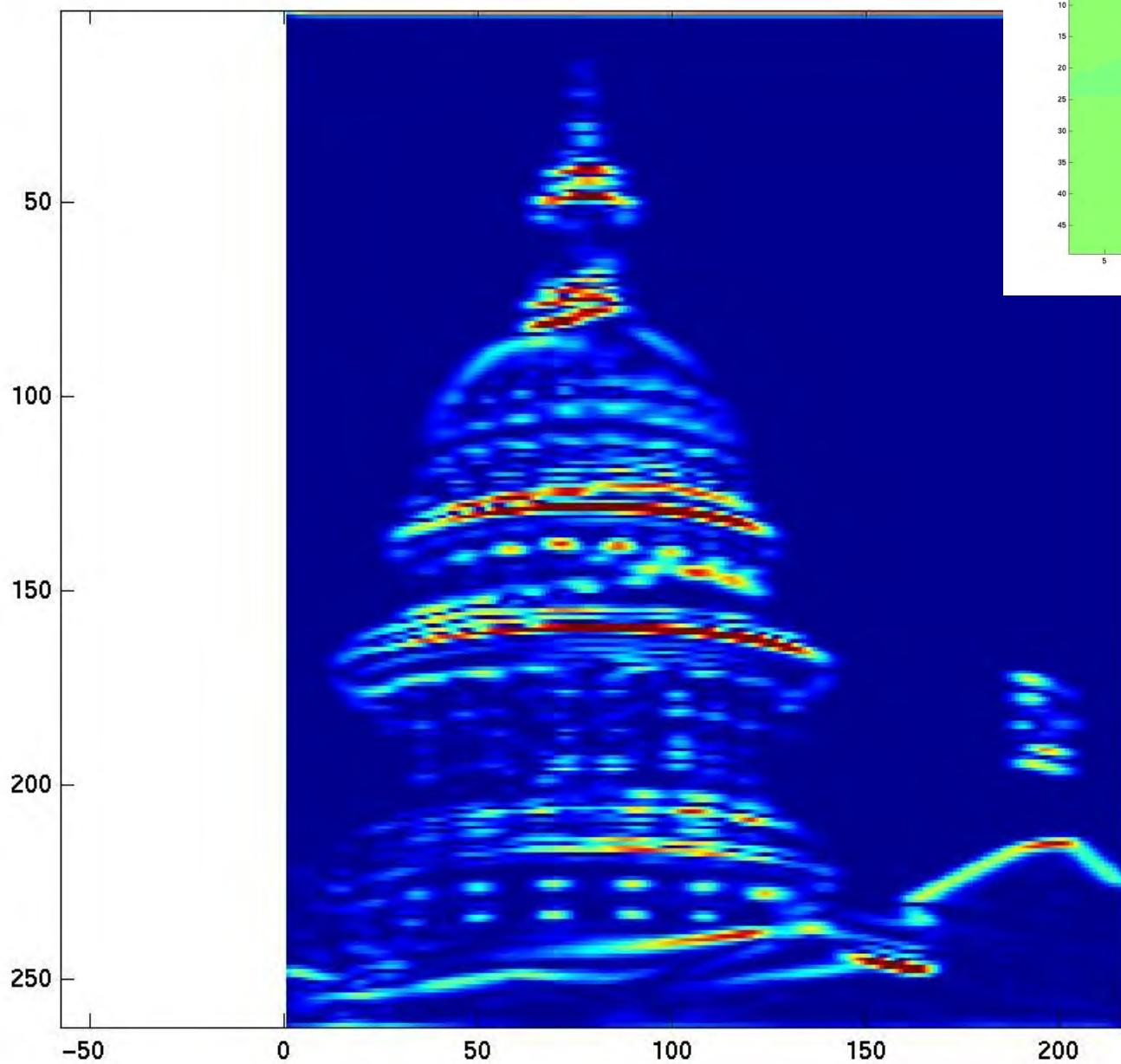


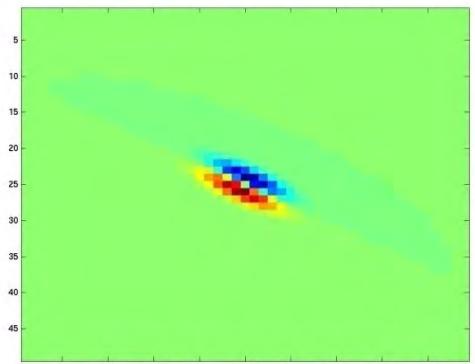
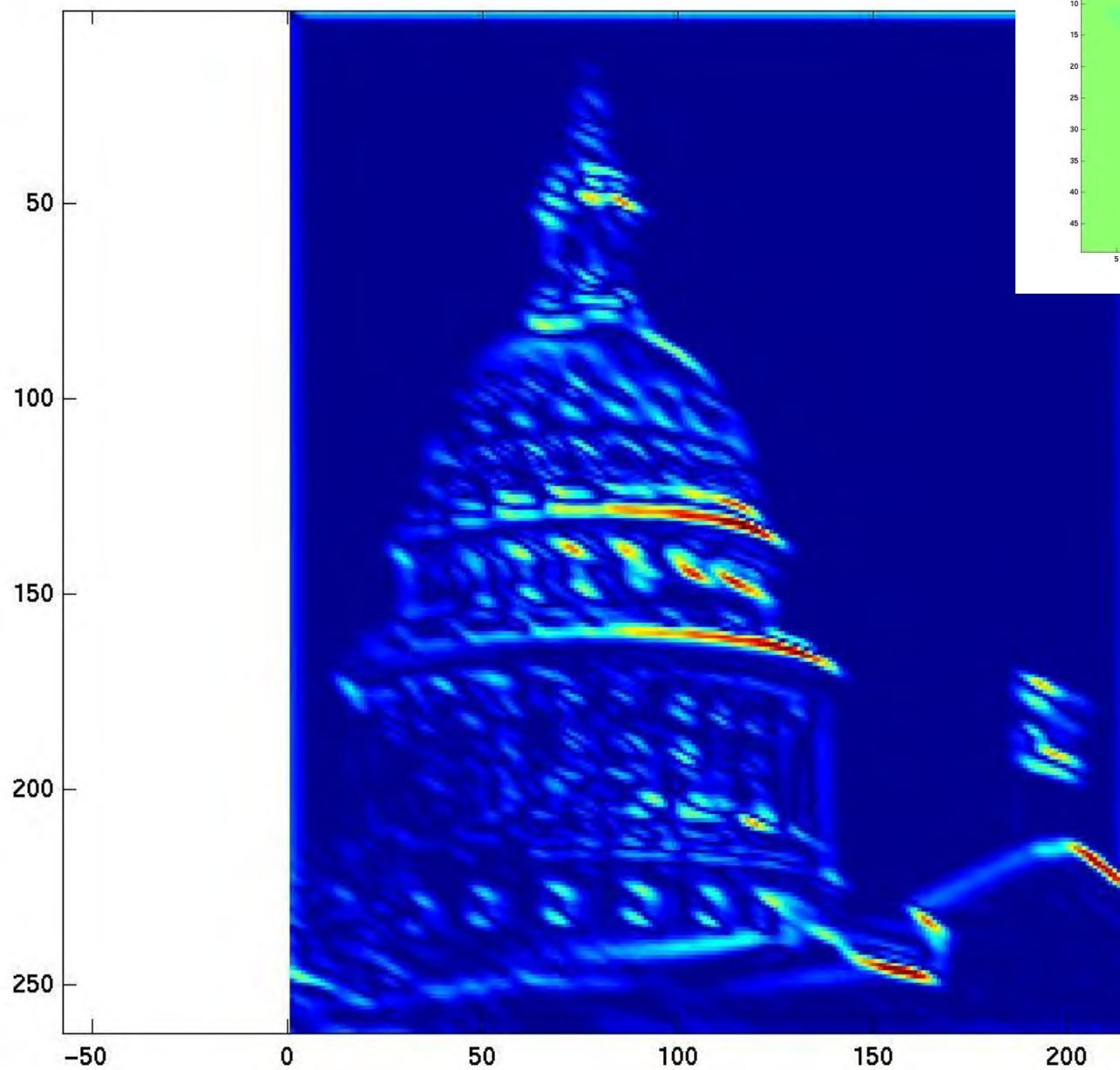
Image from <http://www.texasexplorer.com/austincap2.jpg>



Slide credit:  
Kristen Grauman



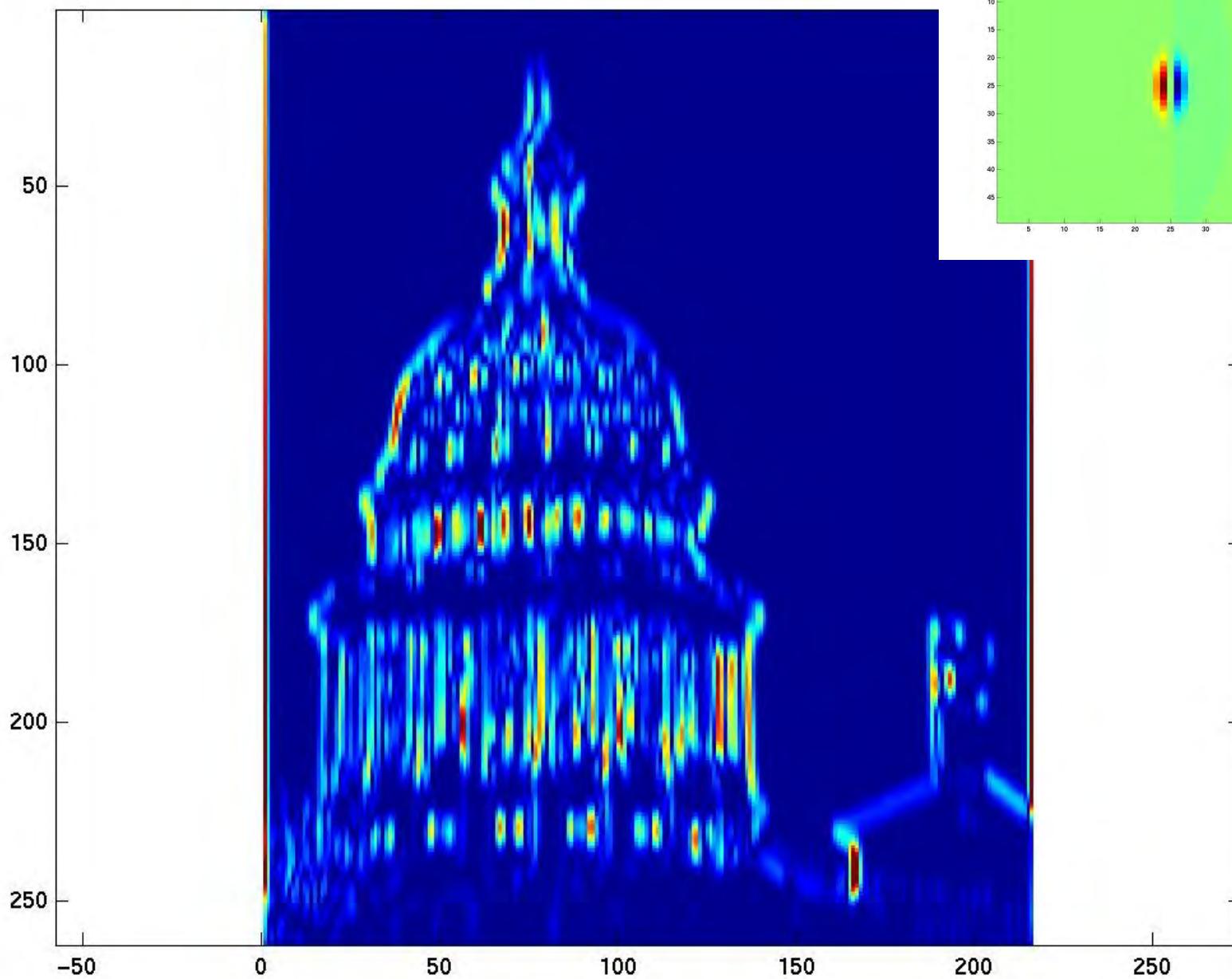
Slide credit:  
Kristen Grauman



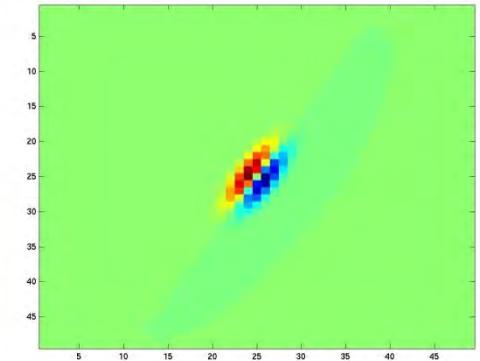
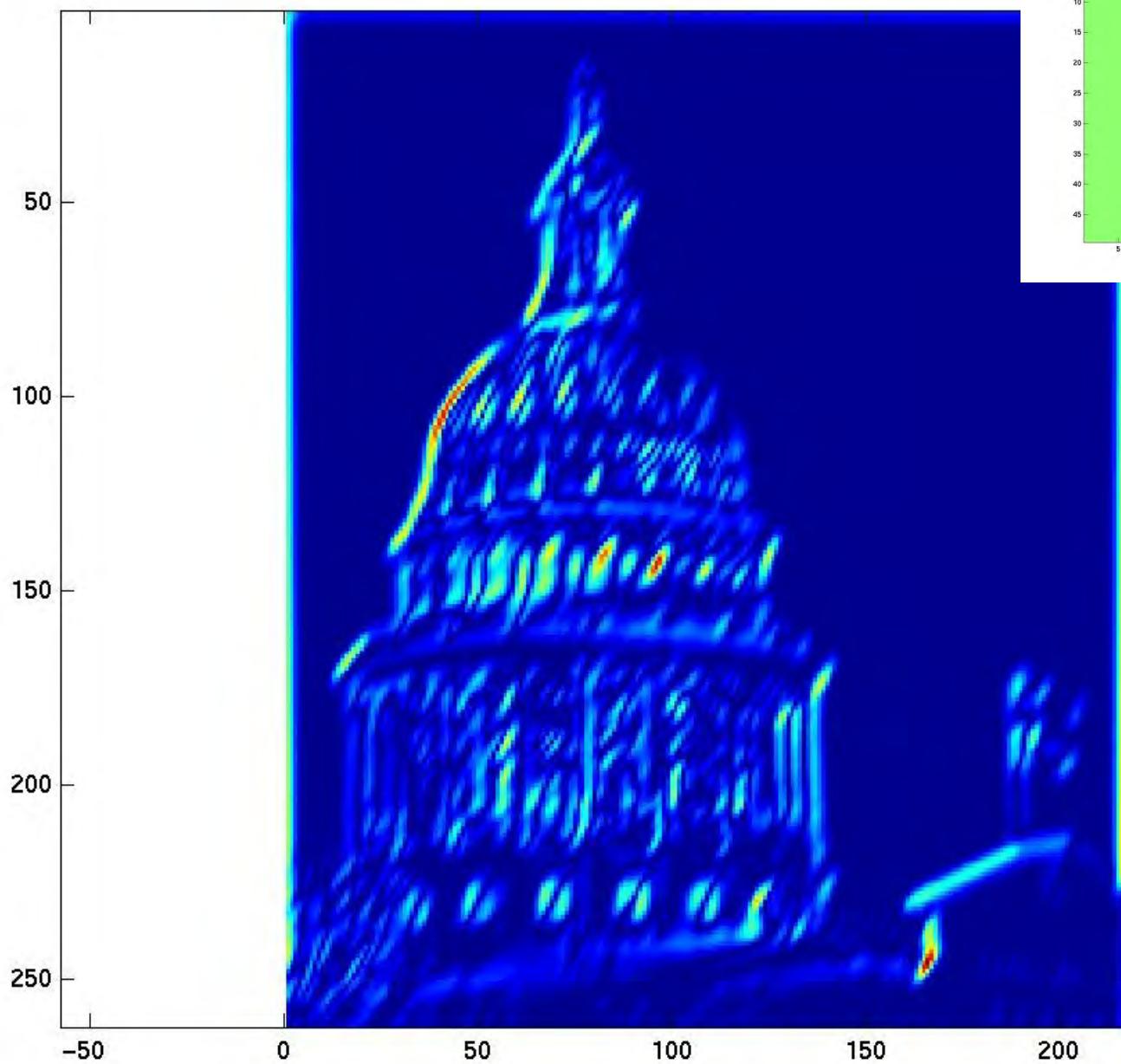
Slide credit:  
Kristen Grauman



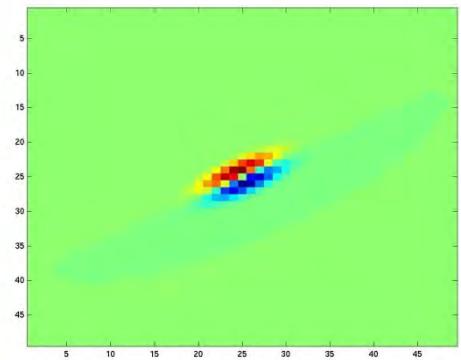
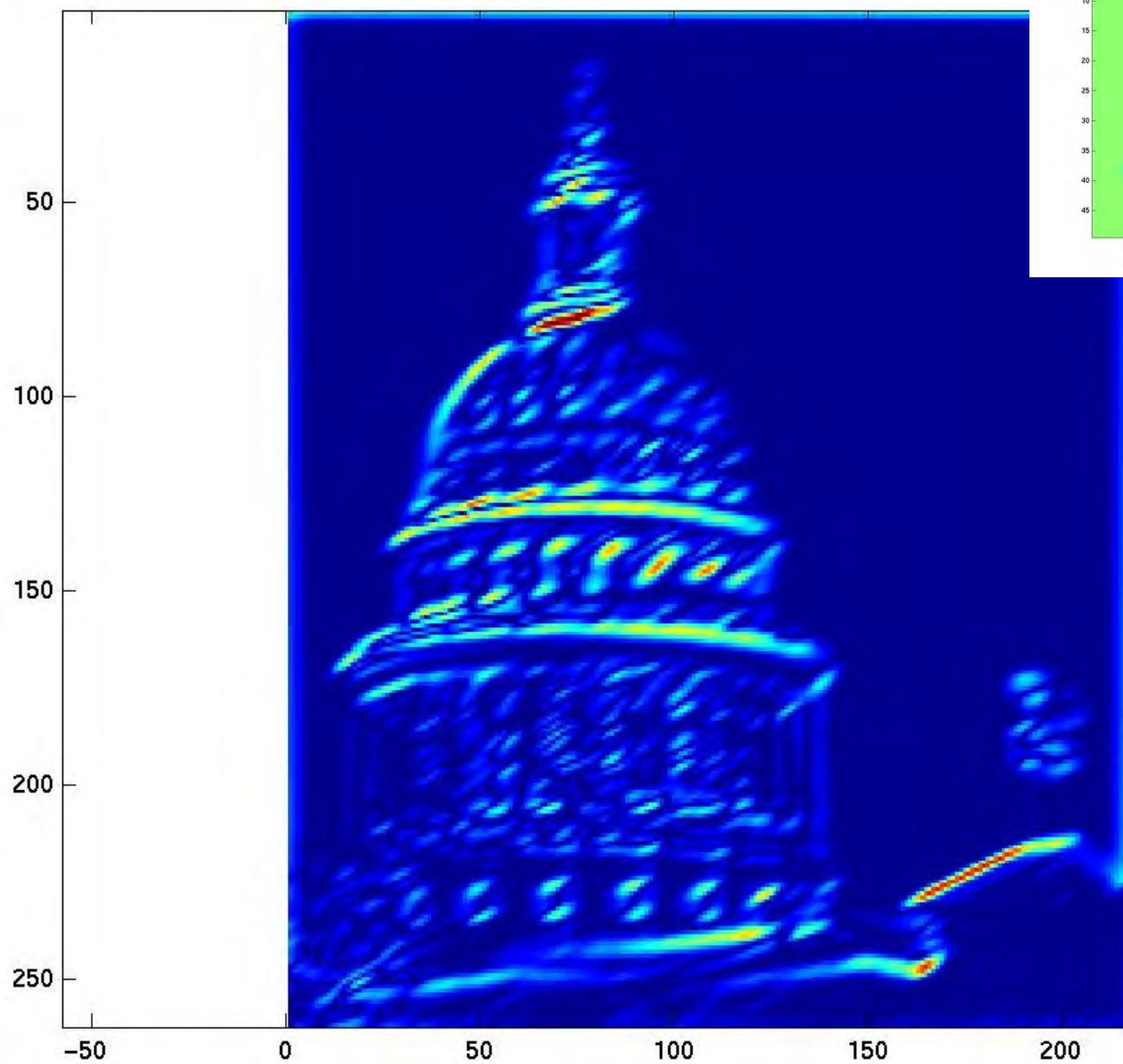
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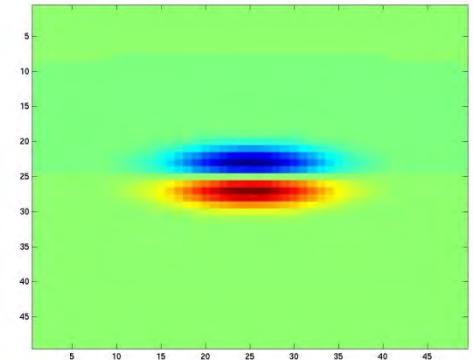
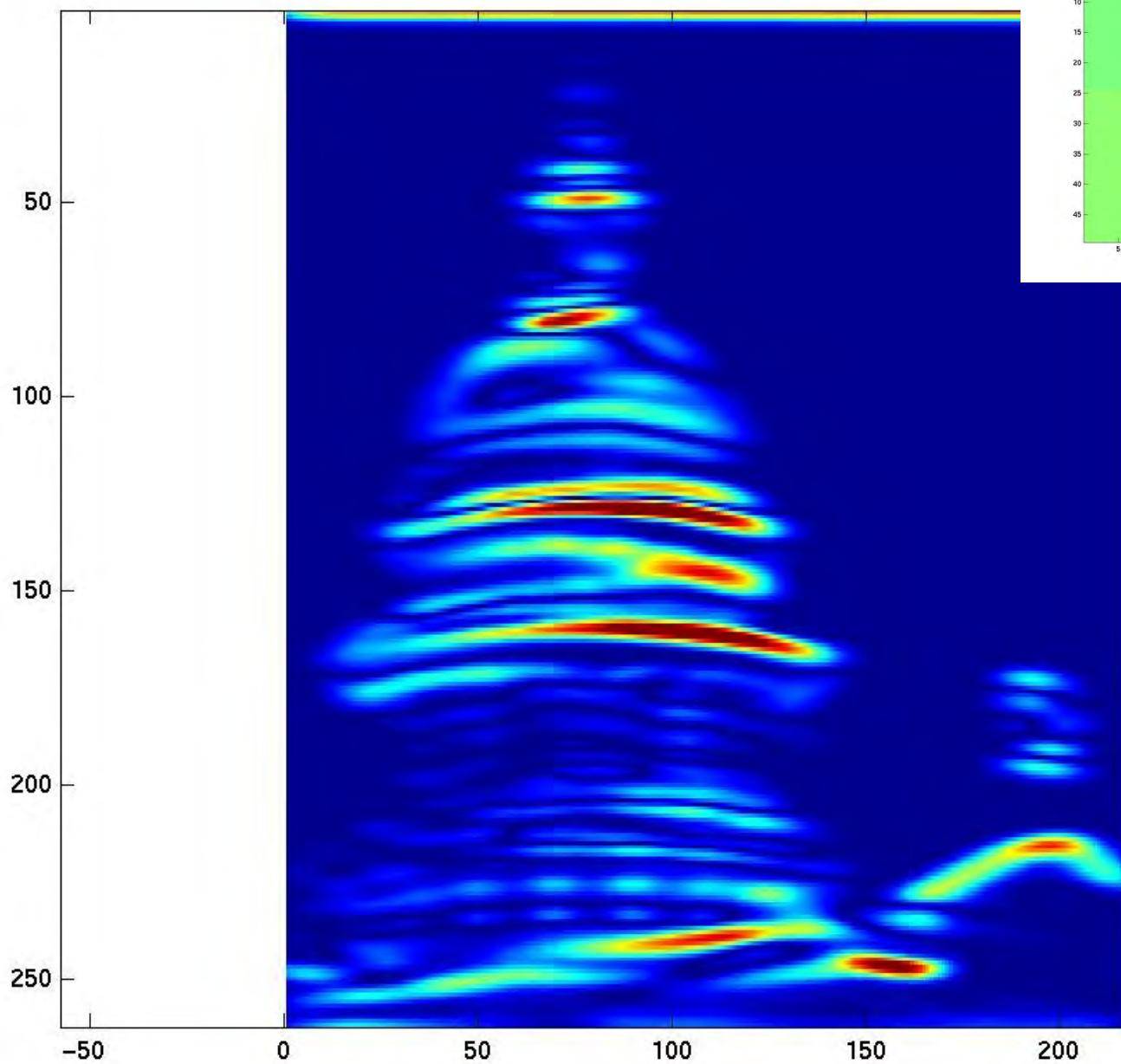
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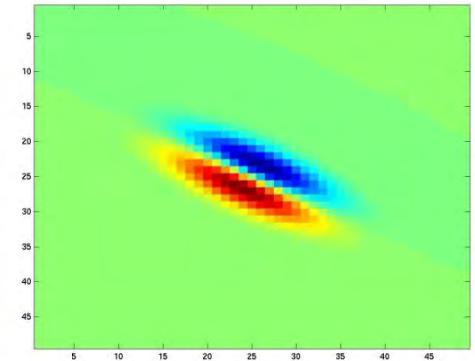
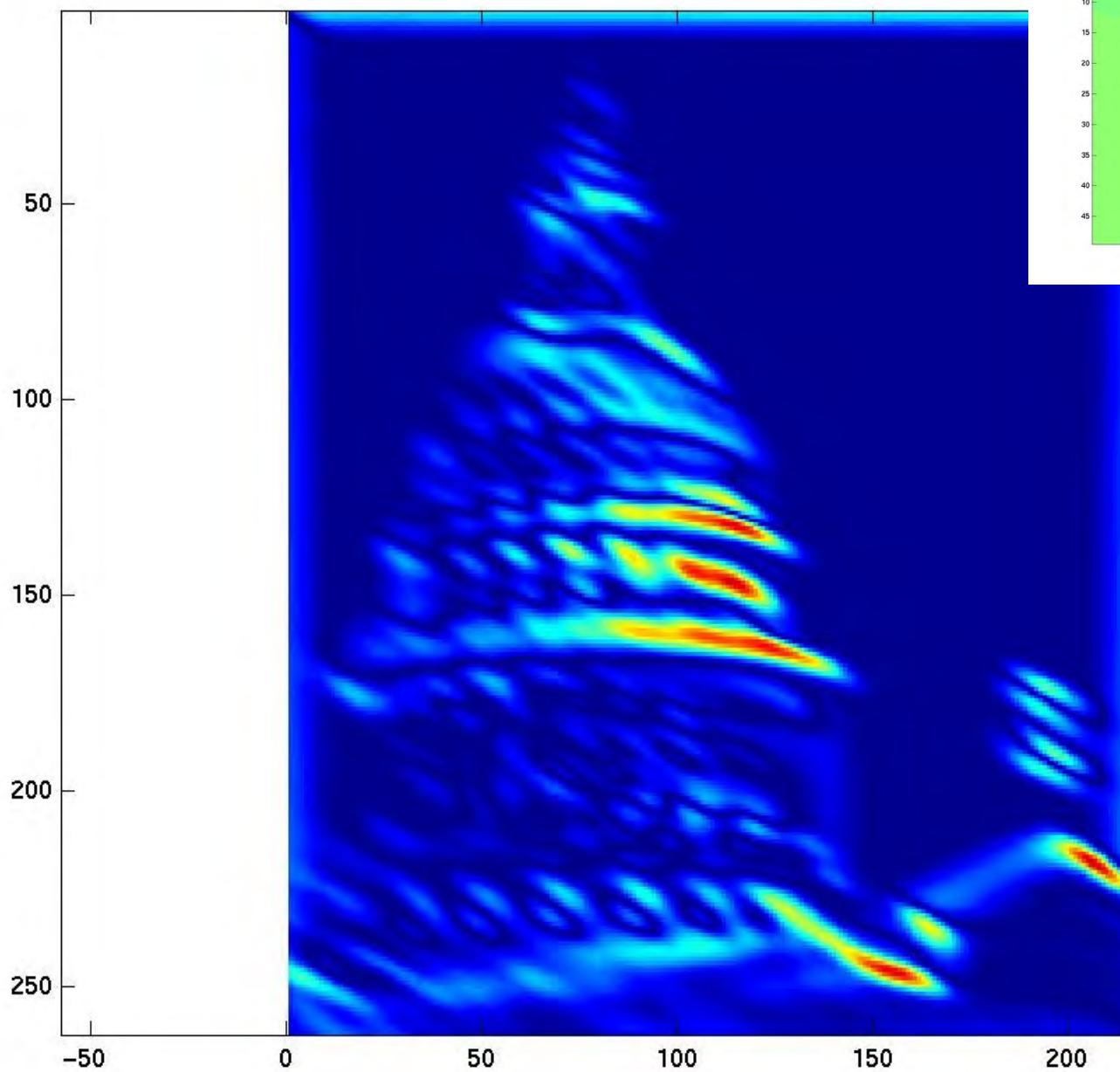
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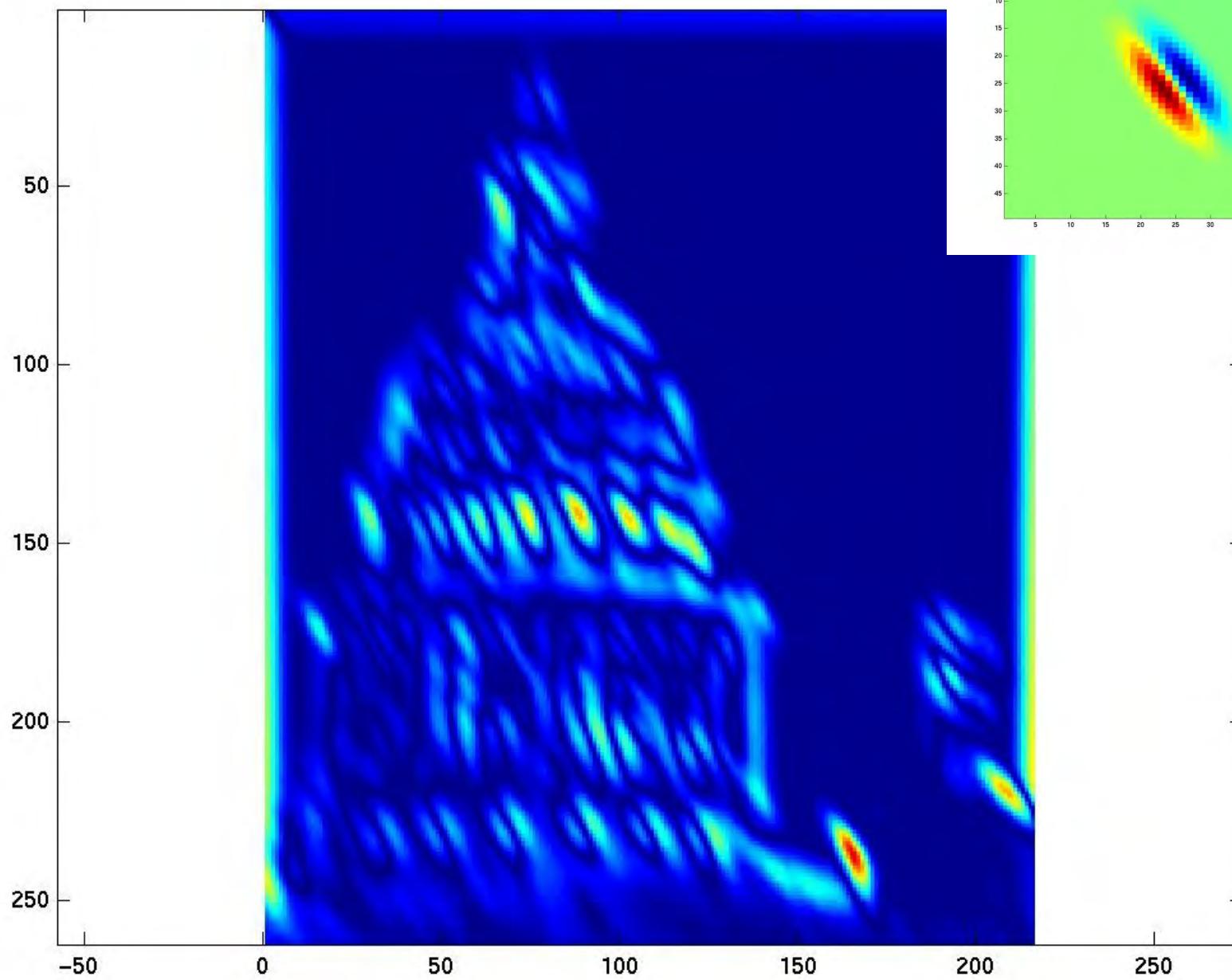
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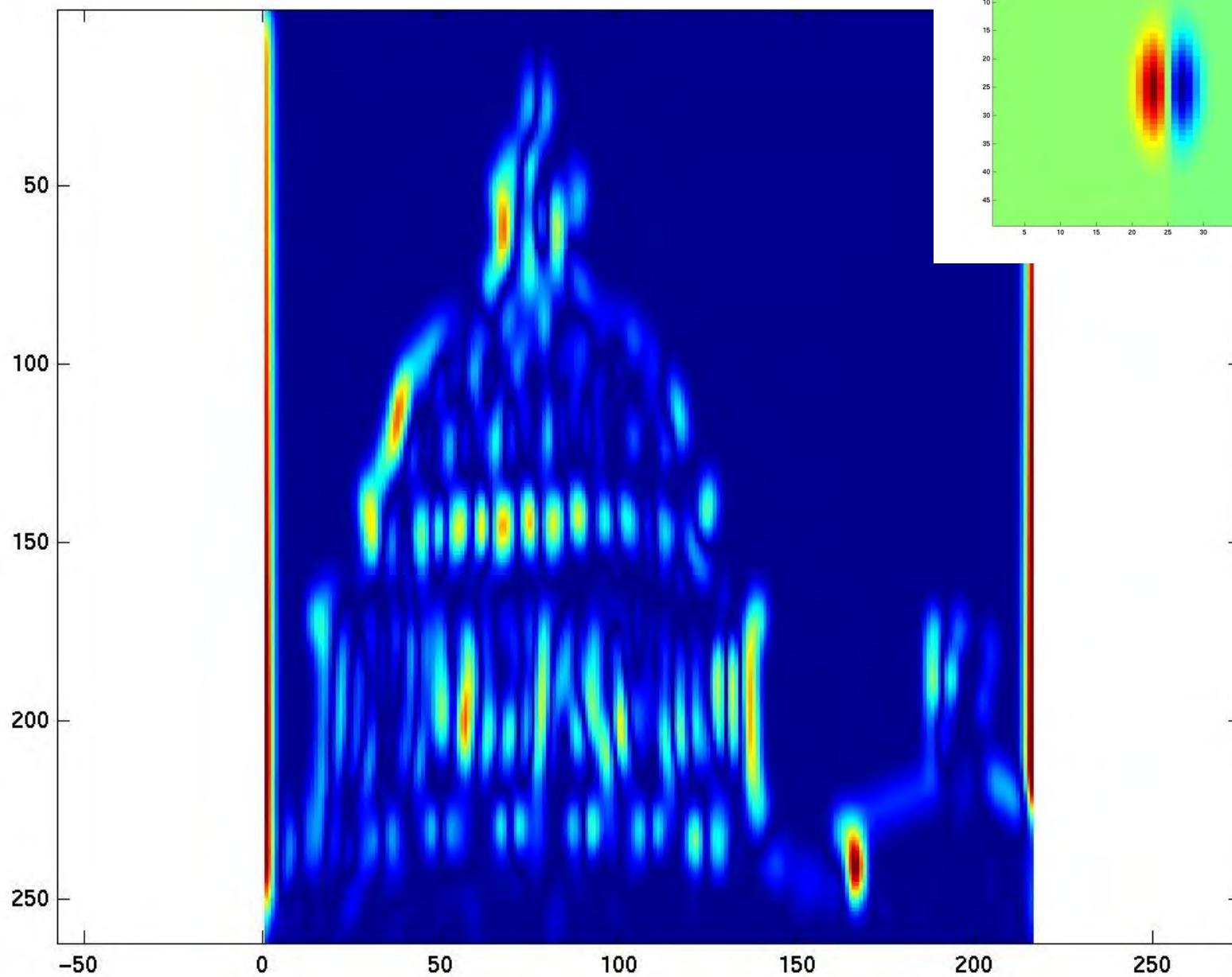
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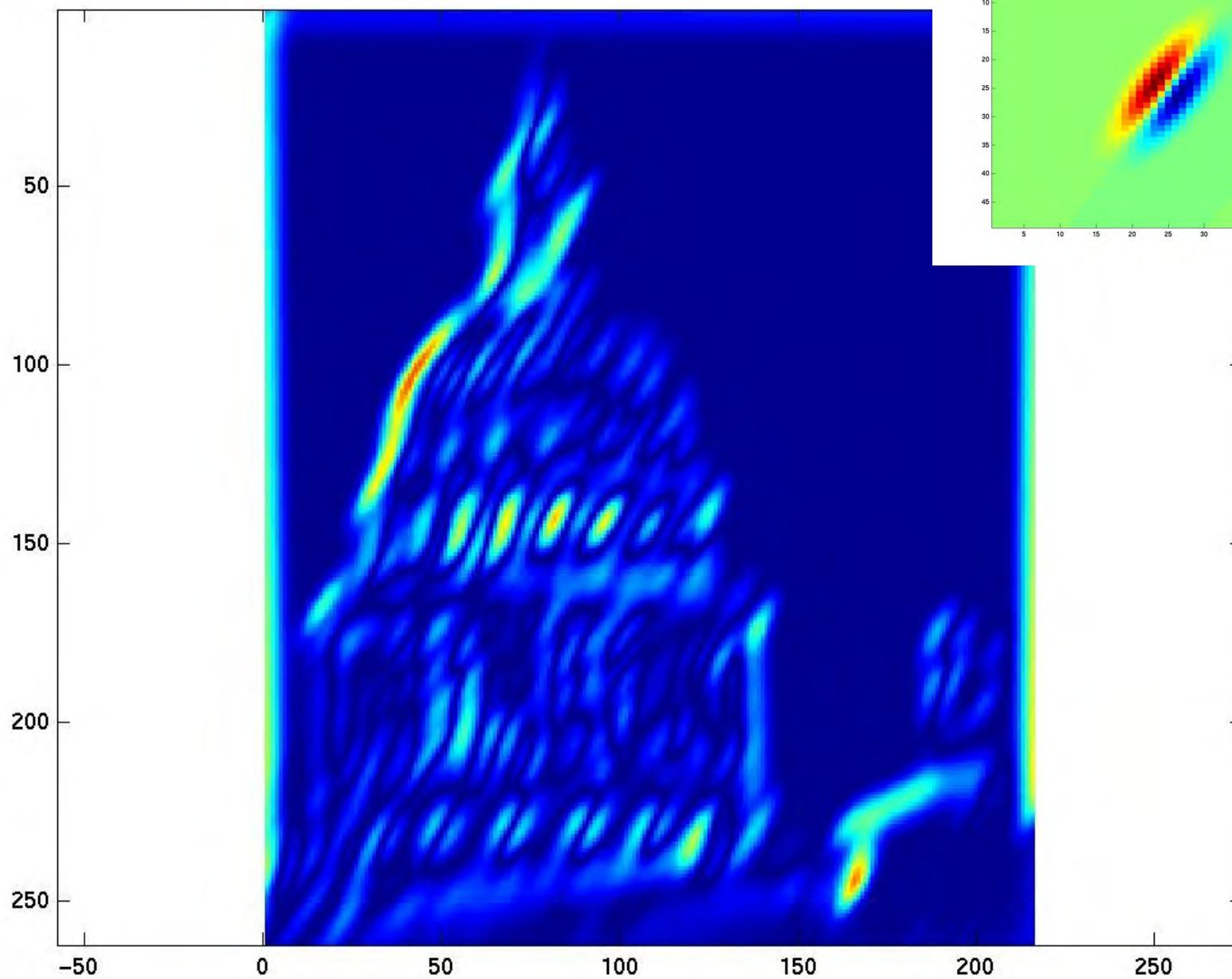
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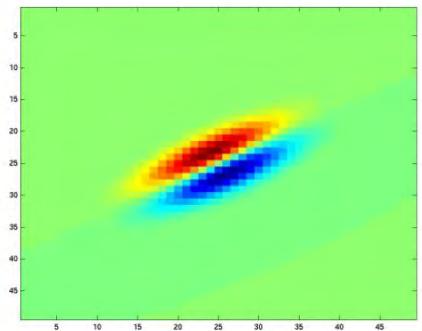
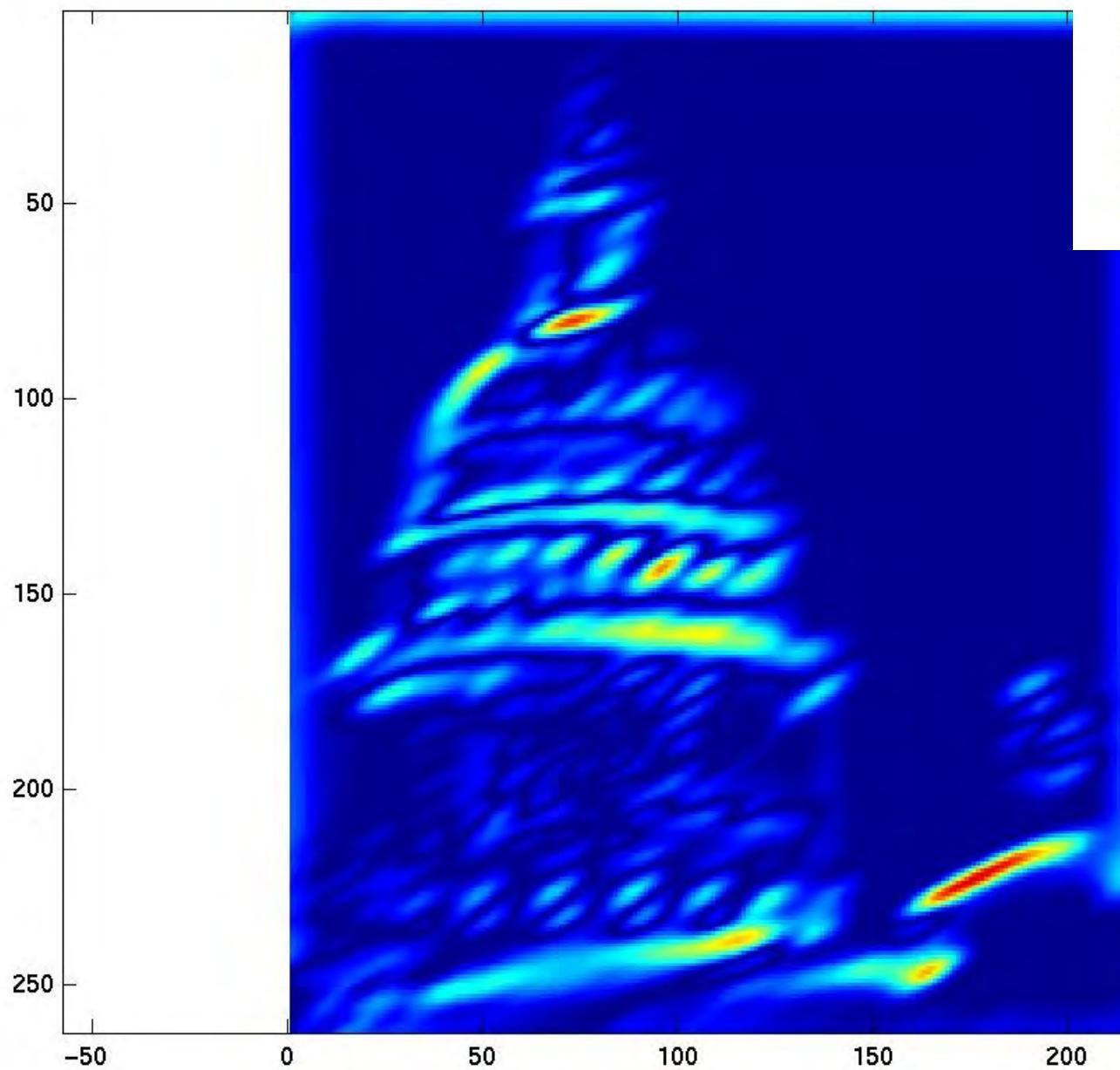
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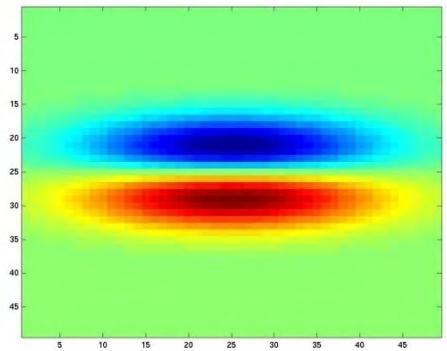
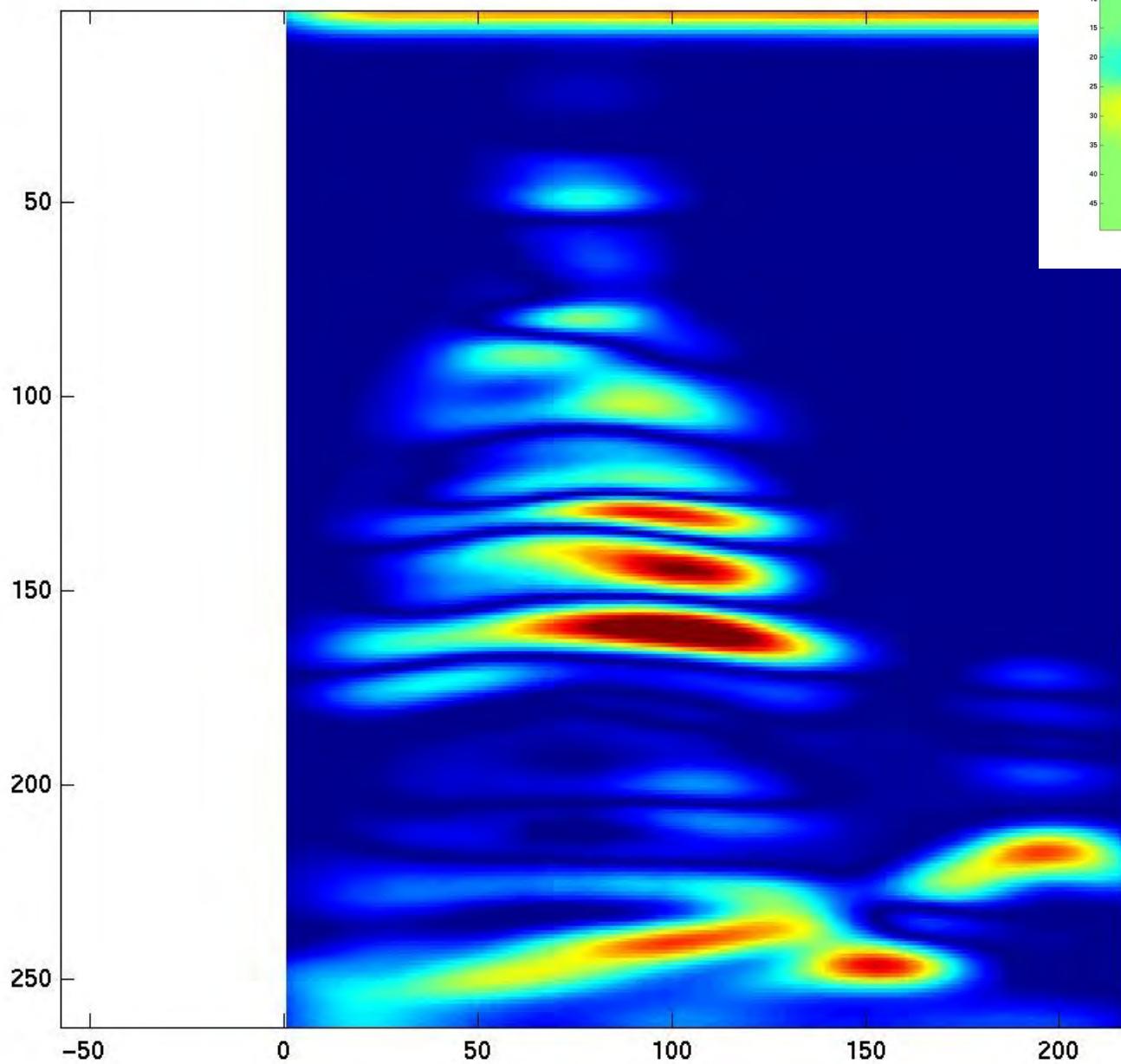
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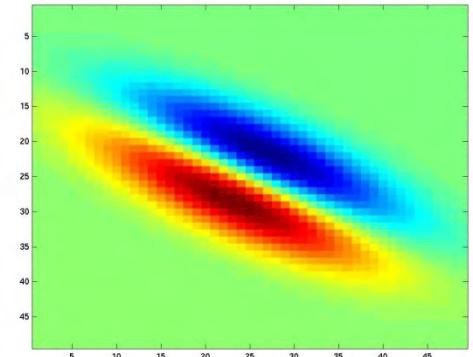
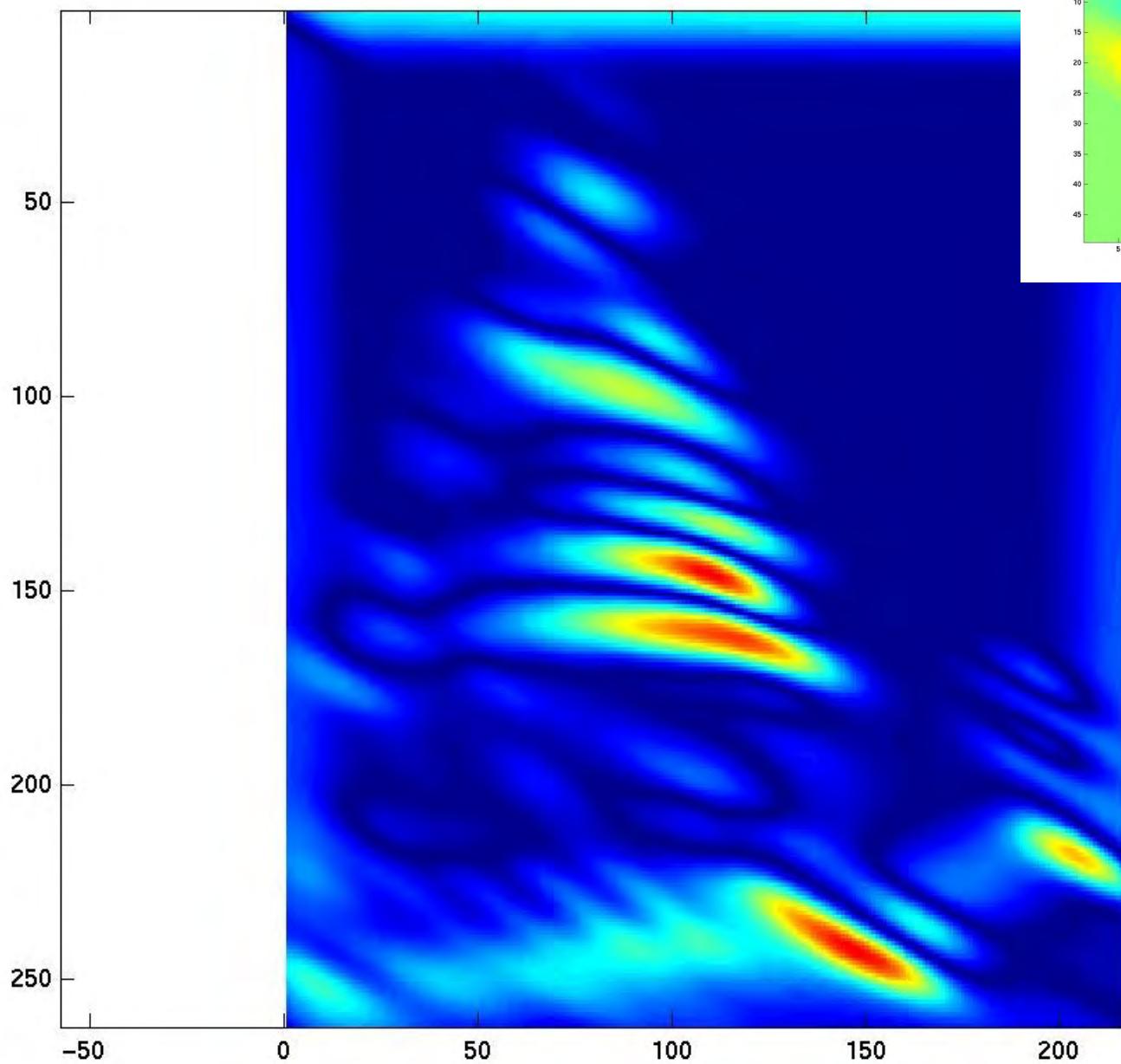
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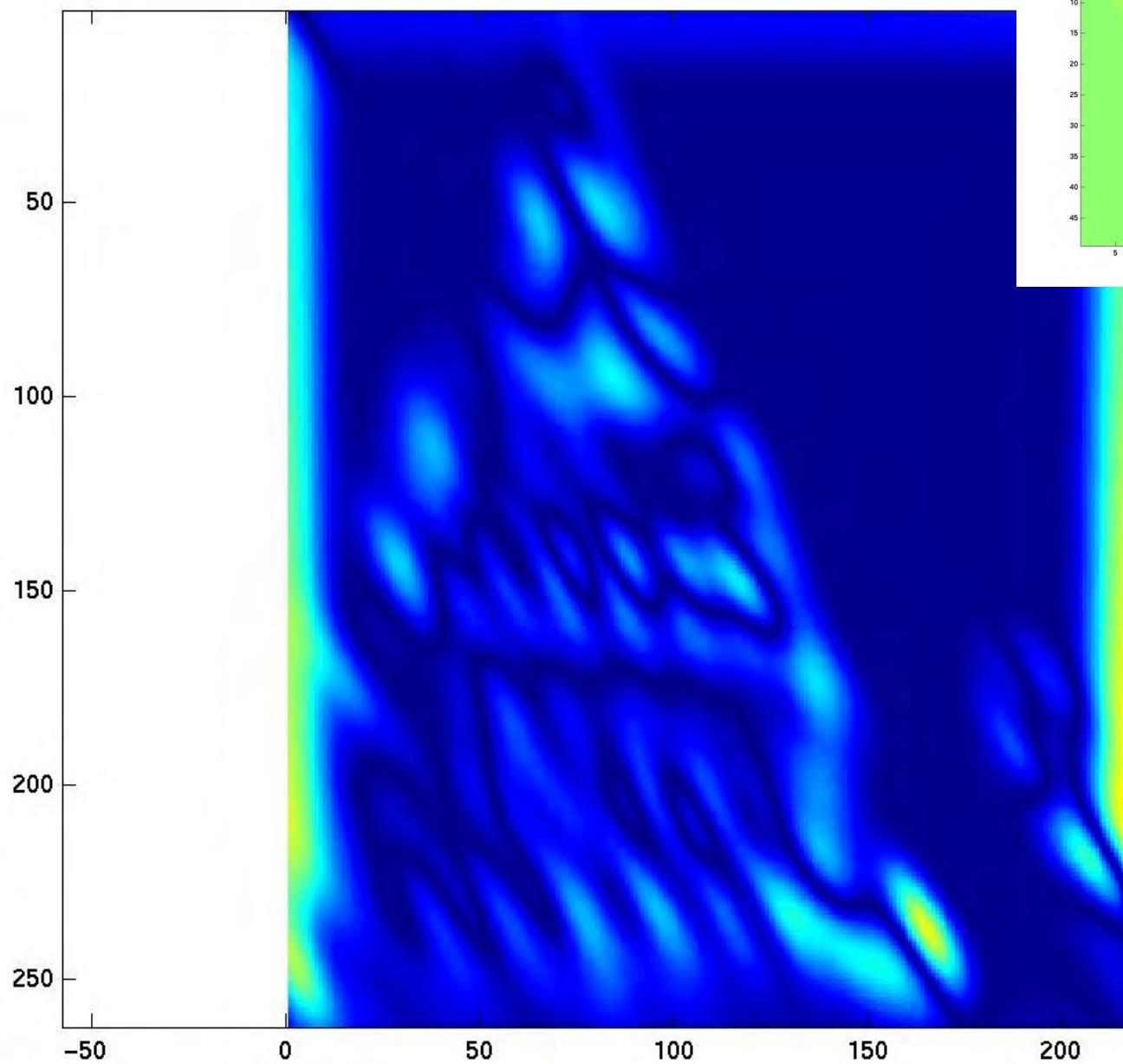
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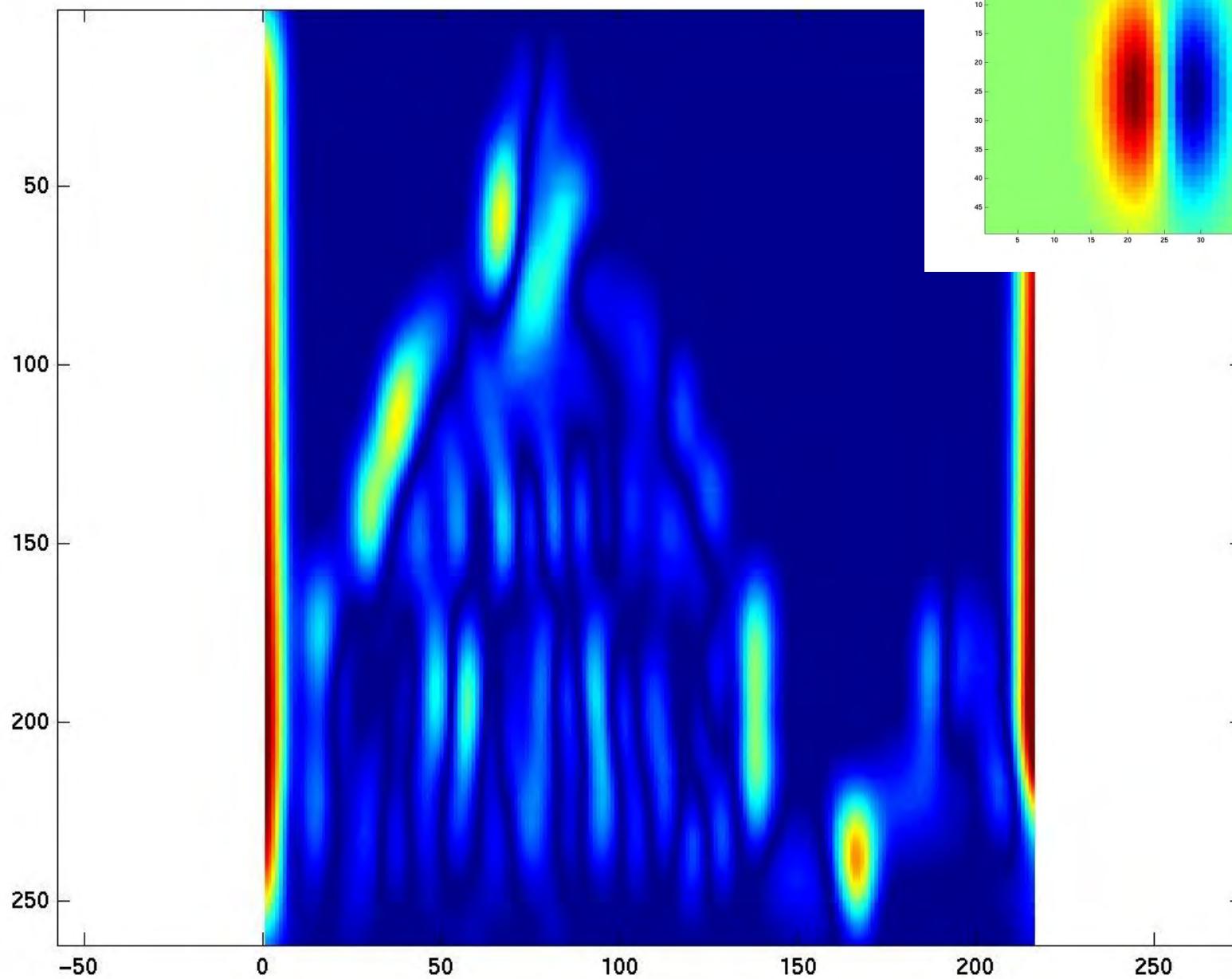
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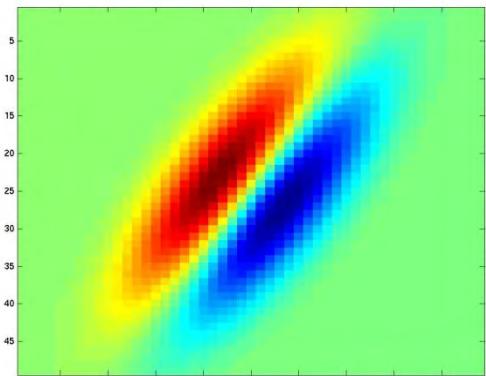
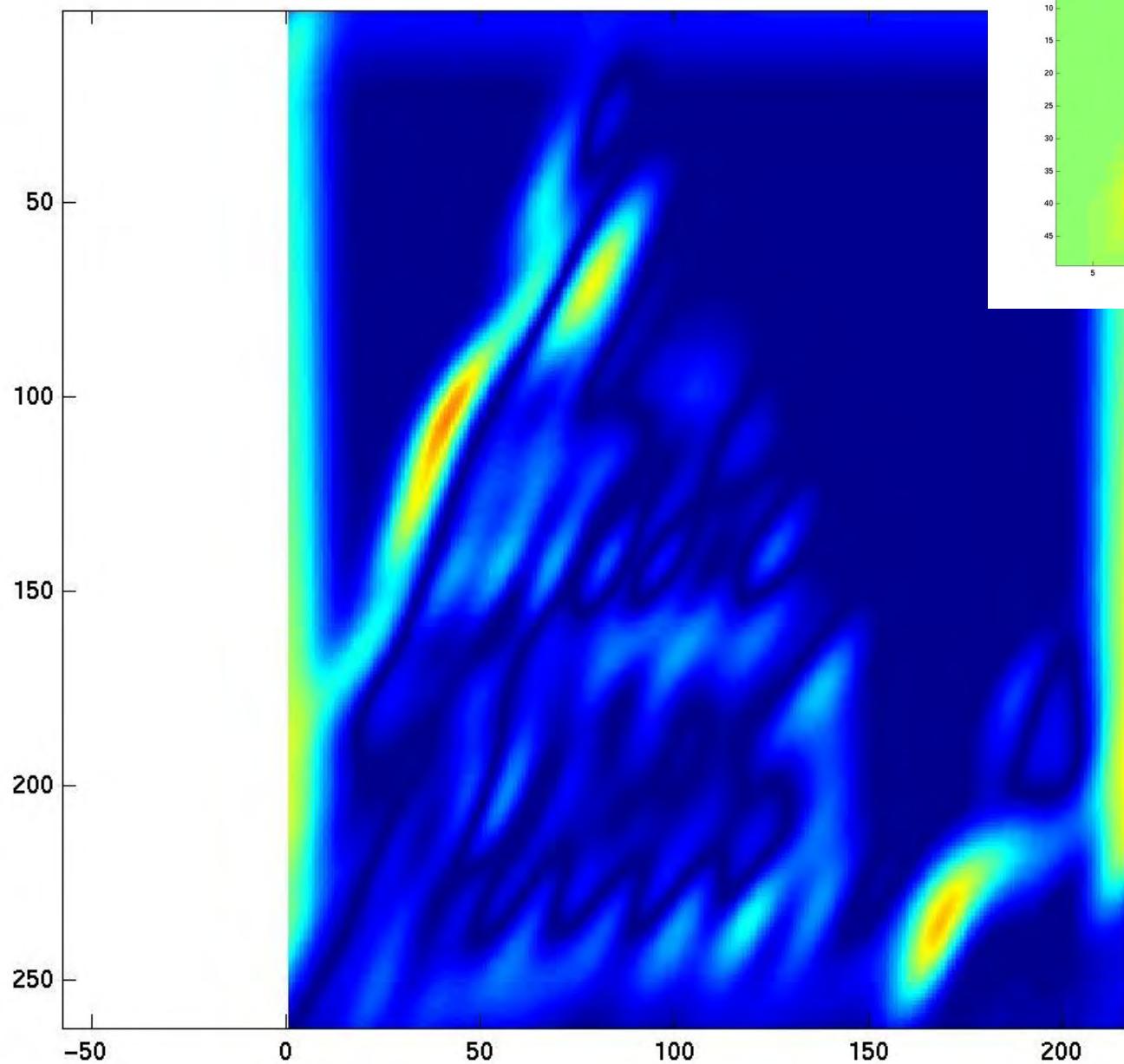
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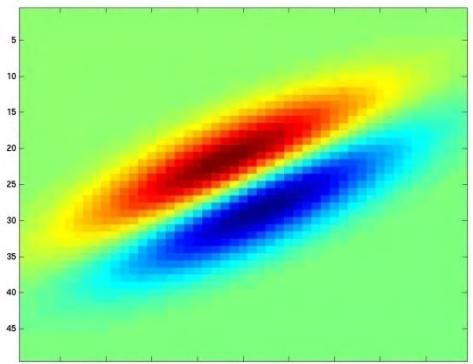
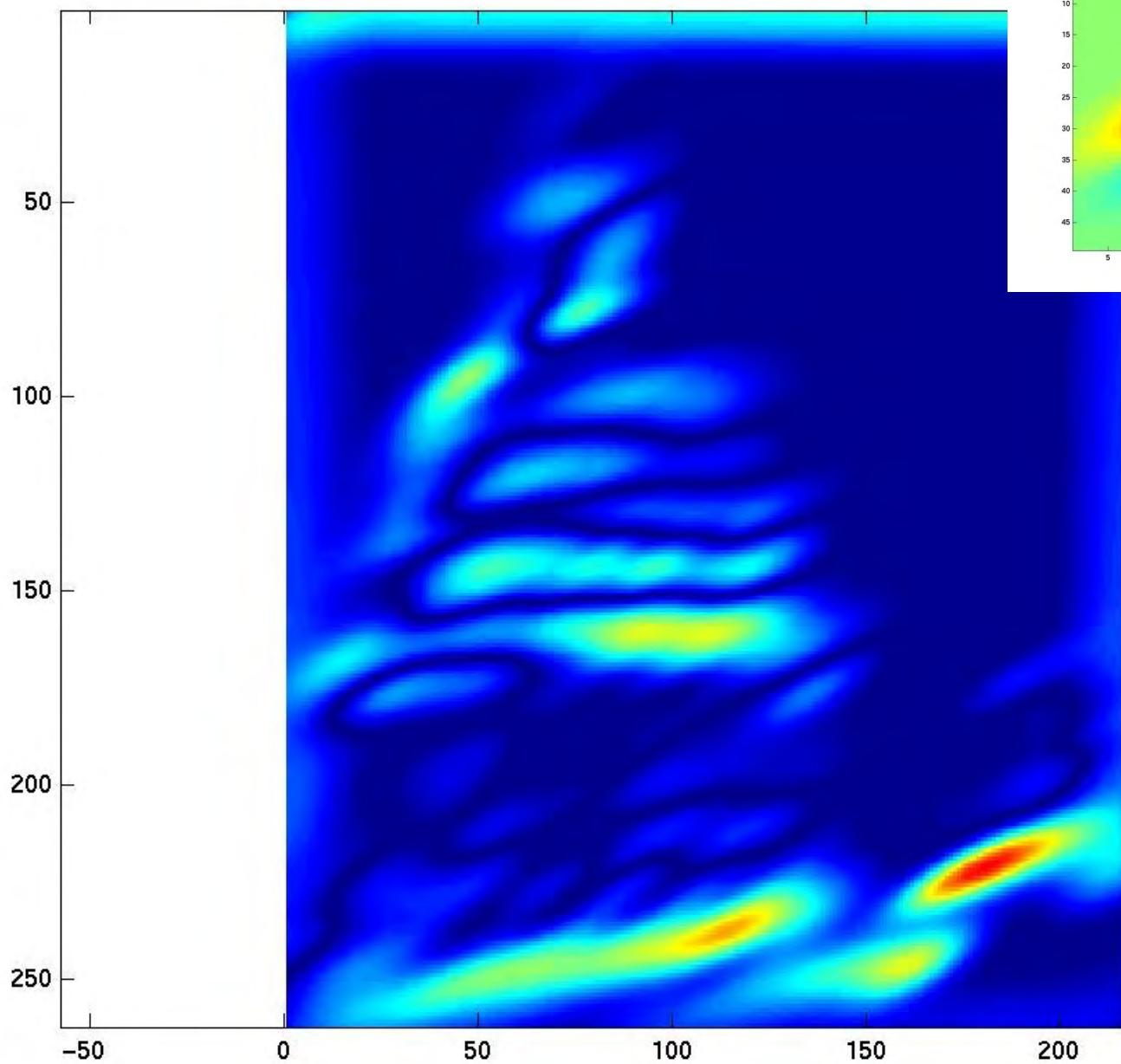
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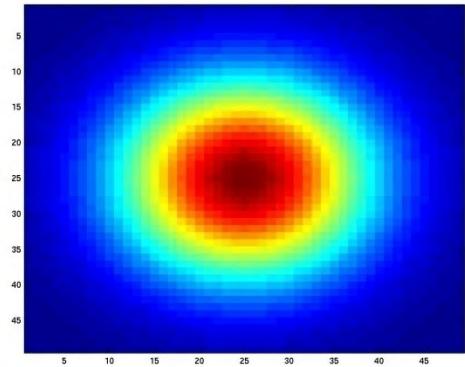
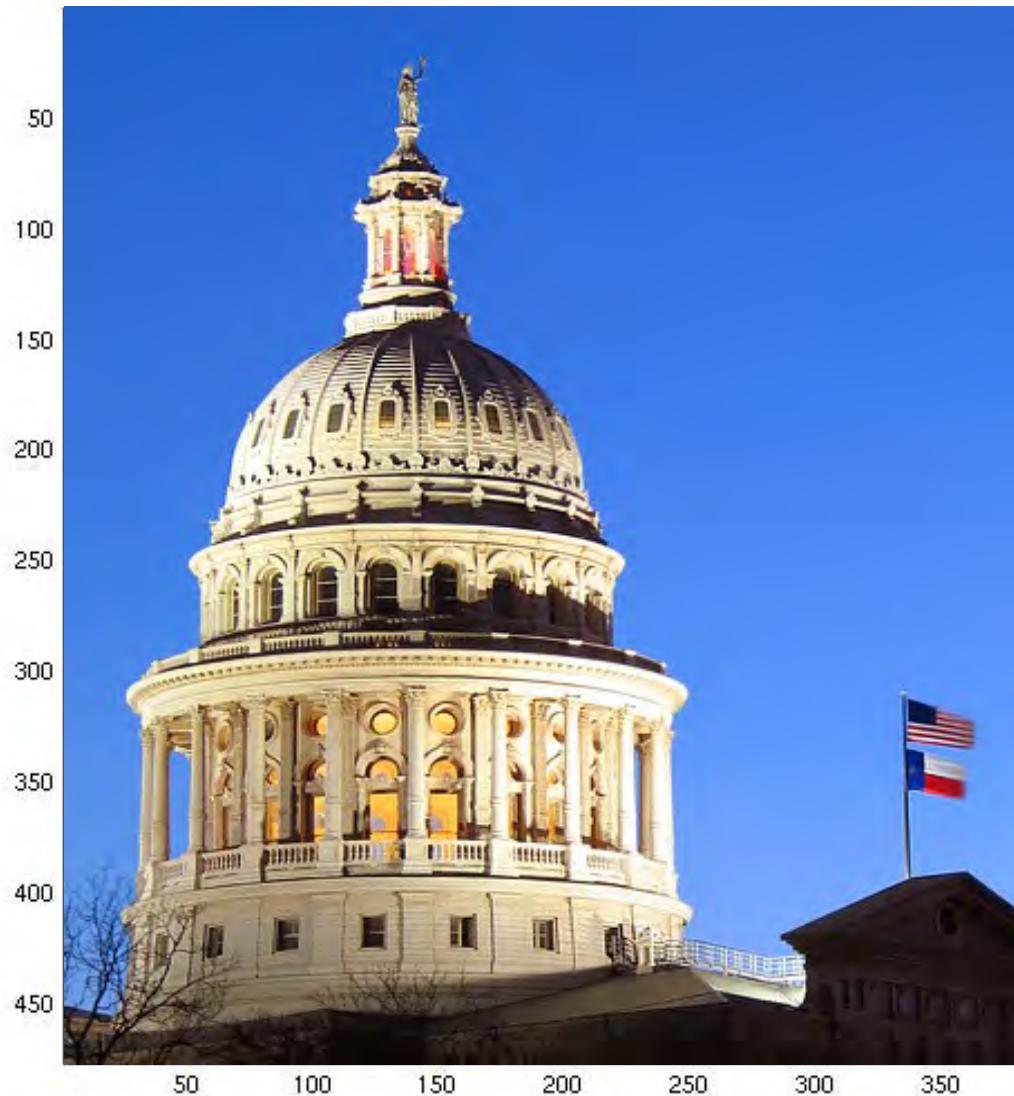
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Kristen Grauman



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Kristen Grauman



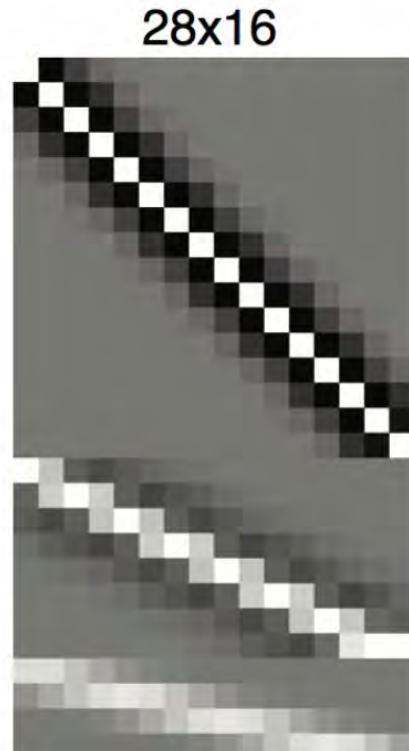
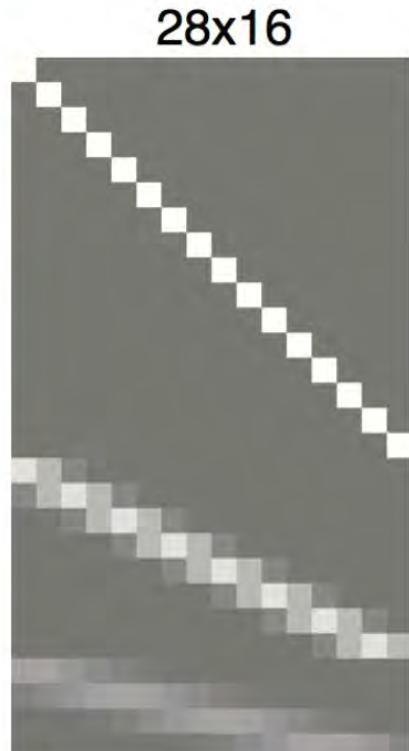
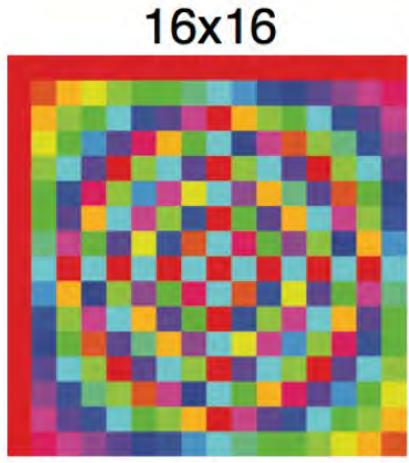
# Steerable pyramid applications



- Texture synthesis
- Noise removal
- Motion analysis
- Motion synthesis, motion magnification

# Linear Image Transforms

1297x256





# Image representation



- Pixels: great for spatial resolution, poor access to frequency
- Fourier transform: great for frequency, not for spatial information
- Pyramids/filter banks: balance between spatial and frequency information



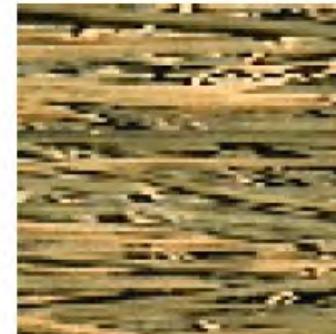
# Application: Representing Texture



Source: Forsyth



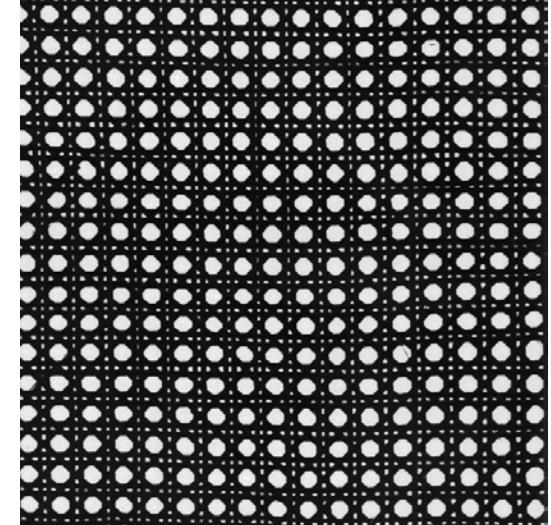
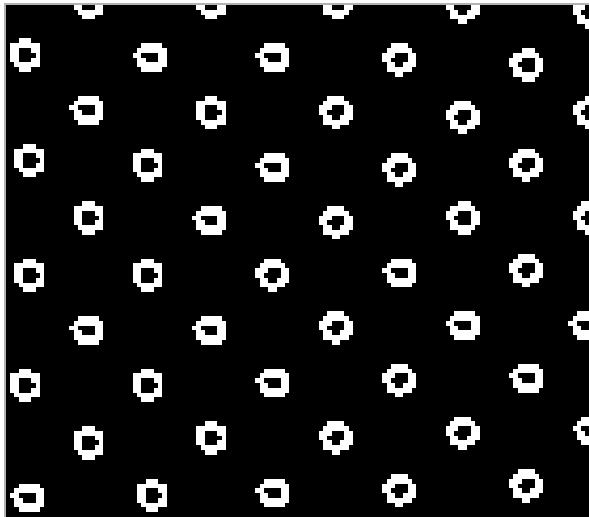
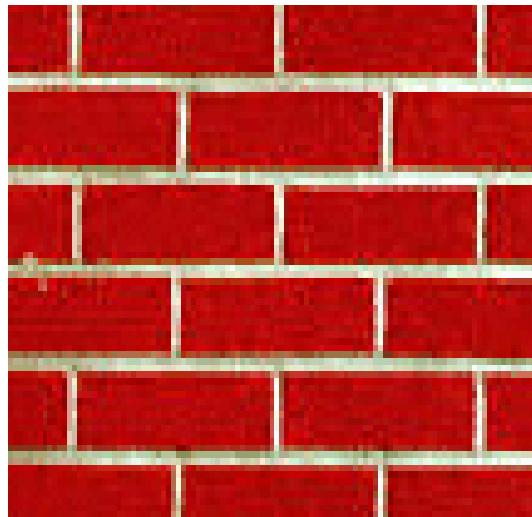
# Texture



What defines a texture?

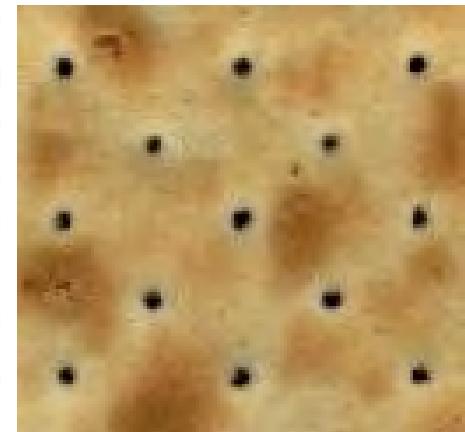
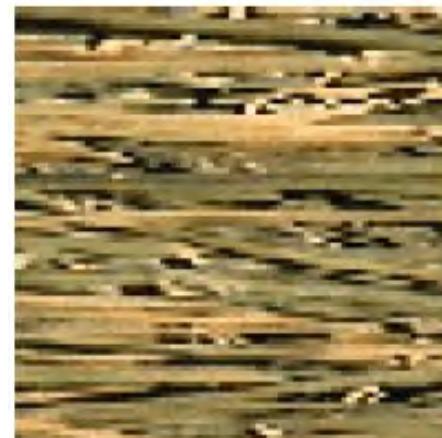


# Includes: more regular patterns



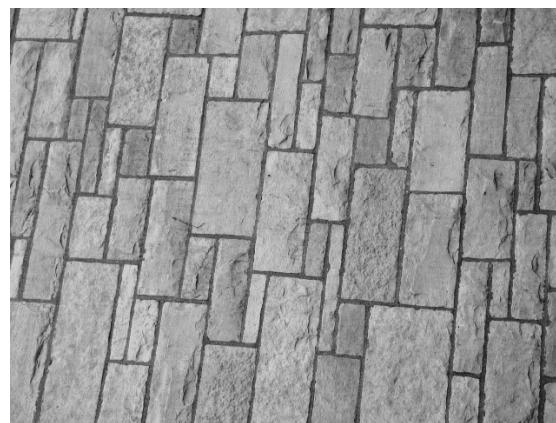
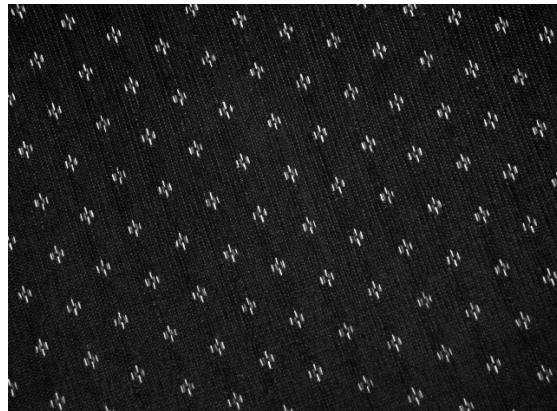
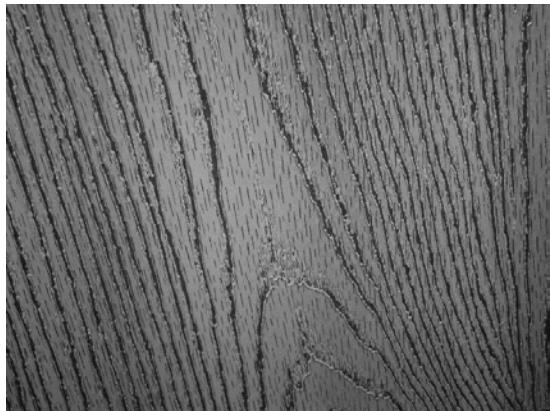


# Includes: more random patterns





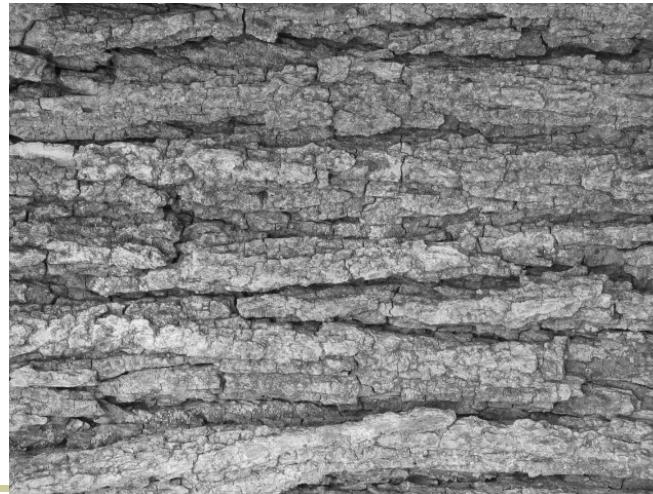
# Texture and Material



[http://www-cvr.ai.uiuc.edu/ponce\\_grp/data/texture\\_database/samples/](http://www-cvr.ai.uiuc.edu/ponce_grp/data/texture_database/samples/)



# Texture and Orientation





# Texture and Scale

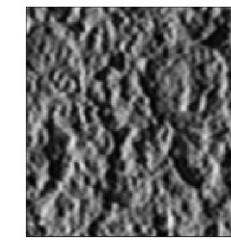
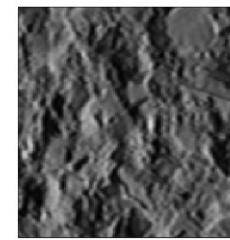
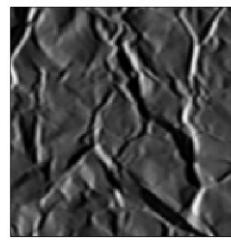
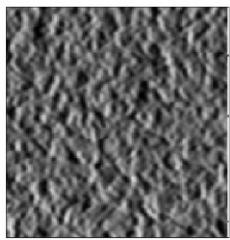
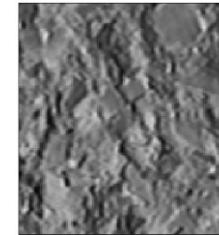
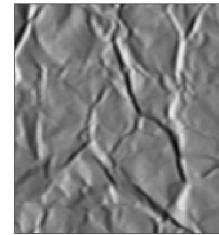
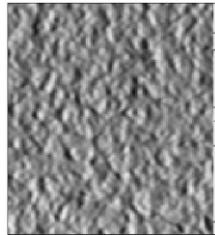




# Materials under different illumination and viewing directions



Different  
illumination  
and viewing  
directions



Plaster-a

Crumpled  
Paper

Concrete

Plaster-b  
(zoomed)



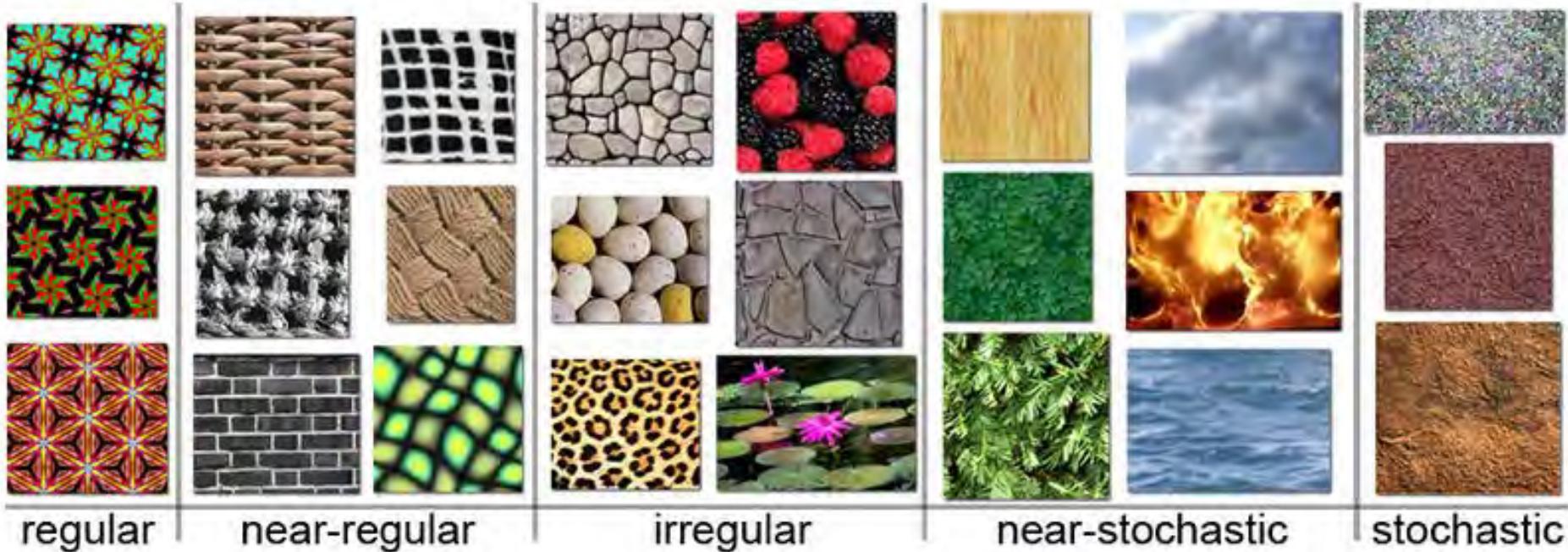
# What is texture?



- **Texture is a phenomenon that is widespread, easy to recognize, and hard to define.**
- Views of large numbers of small objects
- Regular or stochastic patterns caused by bumps, grooves, and/or markings
- Textures tend to show **repetition**: the same local patch appears again and again.



# Texture overview

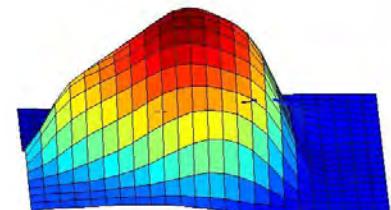
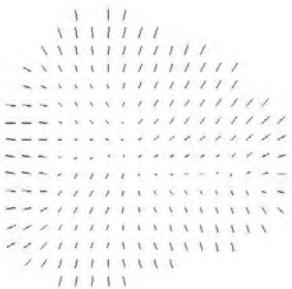
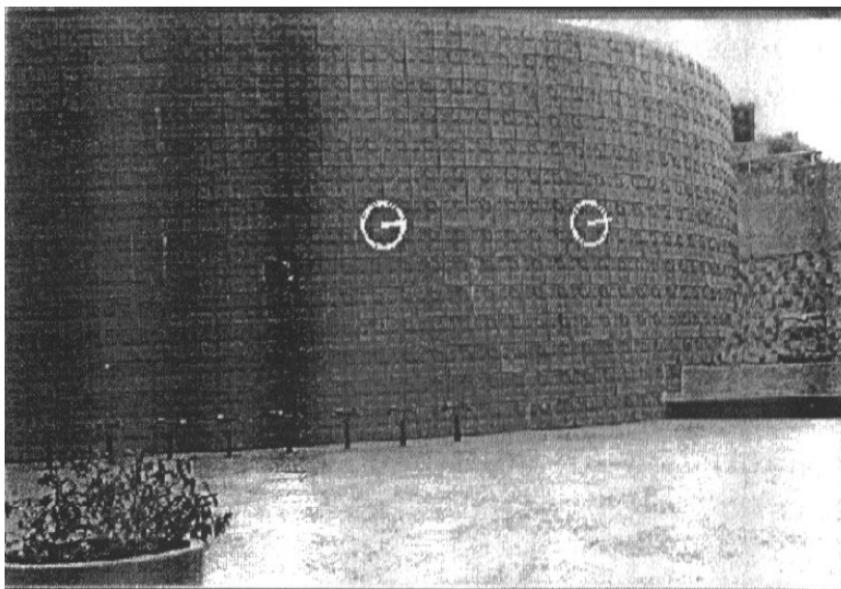




# Shape from texture

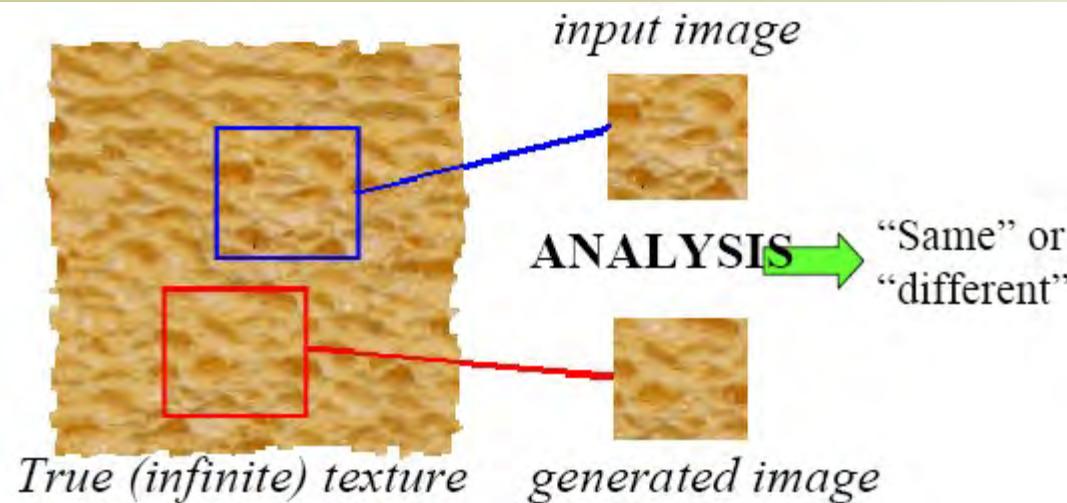


- Use deformation of texture from point to point to estimate surface shape

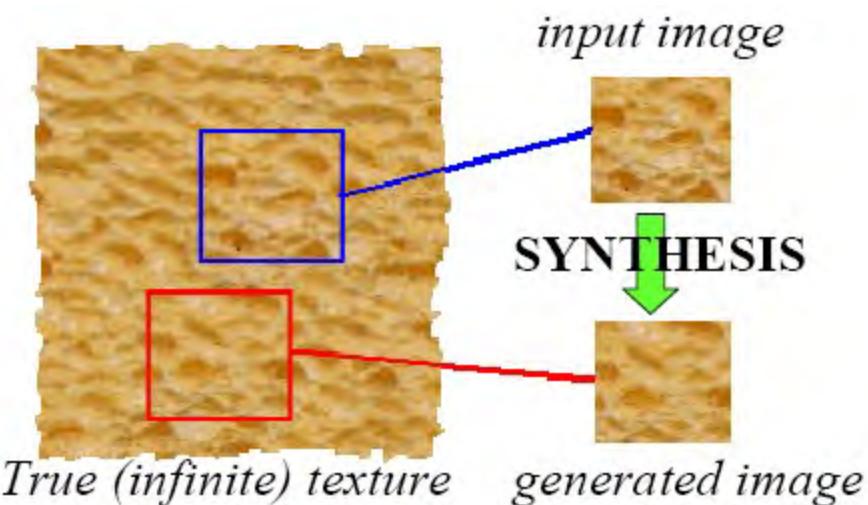




# Analysis vs. Synthesis



Why analyze  
texture?





# Texture-related tasks



## ■ Shape from texture

- Estimate surface orientation or shape from image texture

## ■ Segmentation/classification from texture cues

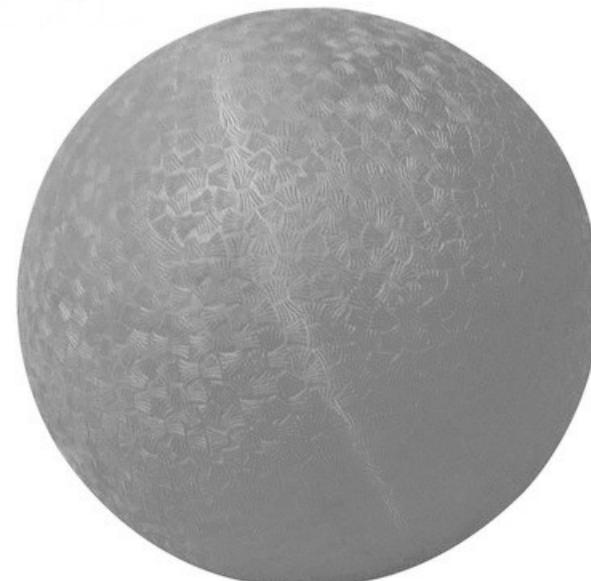
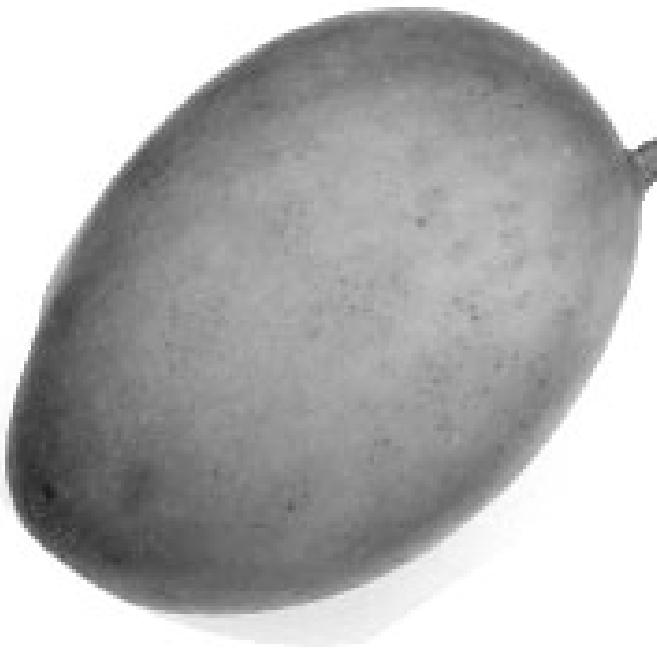
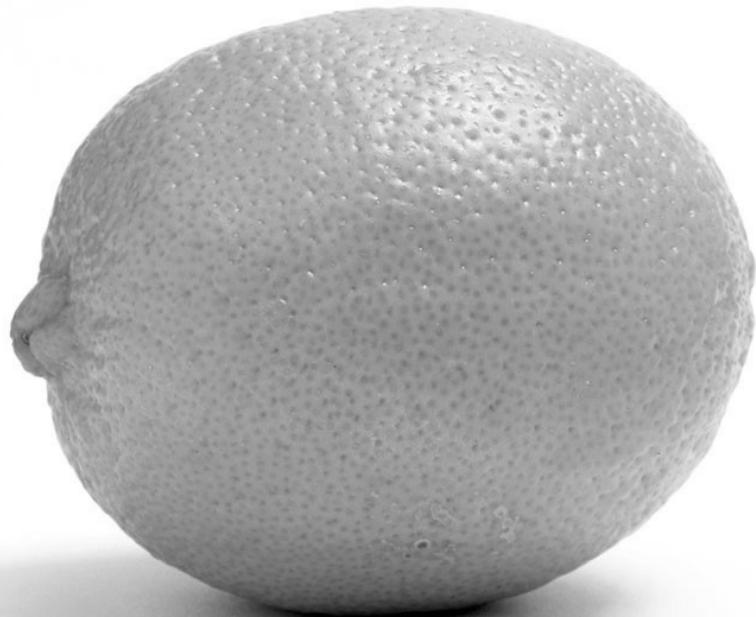
- Analyze, represent texture
- Group image regions with consistent texture

## ■ Synthesis

- Generate new texture patches/images given some examples

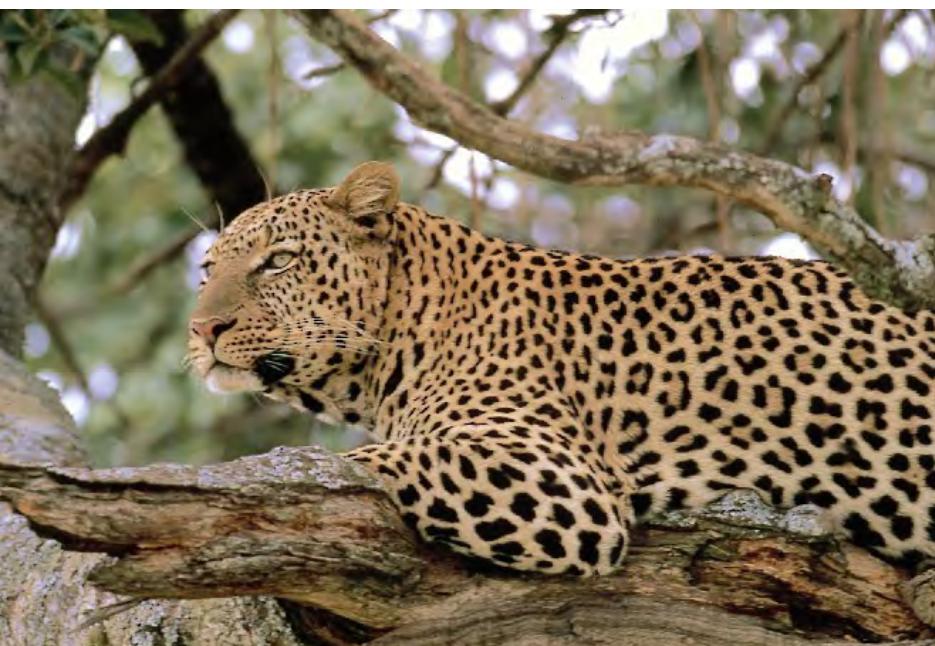


Slide credit:  
Kristen Grauman



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Slide credit:  
Kristen Grauman



Slide credit:  
Kristen Grauman

<http://animals.nationalgeographic.com/>



# Why analyze texture?



Importance to perception:

- Often indicative of a material's properties
- Can be important appearance cue, especially if shape is similar across objects
- Aim to distinguish between shape, boundaries, and texture

Technically:

- Representation-wise, we want a feature one step above “building blocks” of filters, edges.



# Psychophysics of texture

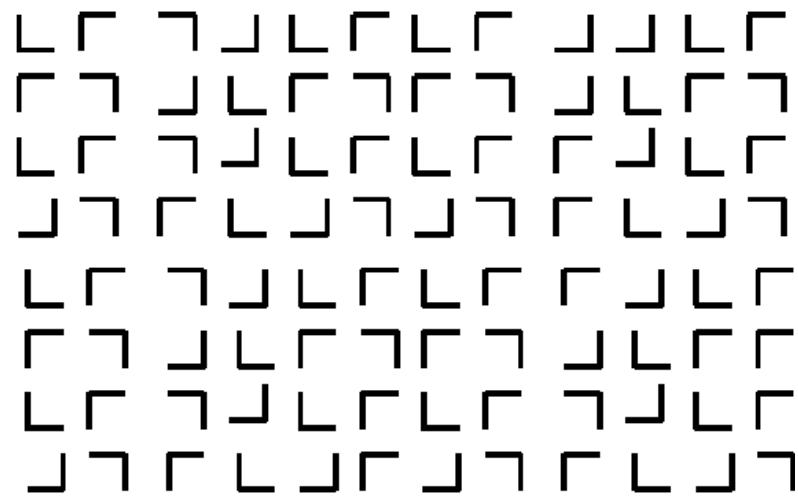


- Some textures distinguishable with *preattentive* perception— without scrutiny, eye movements [Julesz 1975]

Same or different?



南  
京  
大  
学  
校  
歌





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# Capturing the local patterns with image measurements



[Bergen &  
Adelson,  
*Nature* 1988]

Scale of  
patterns  
influences  
discriminability

Size-tuned  
linear filters



# Texture representation



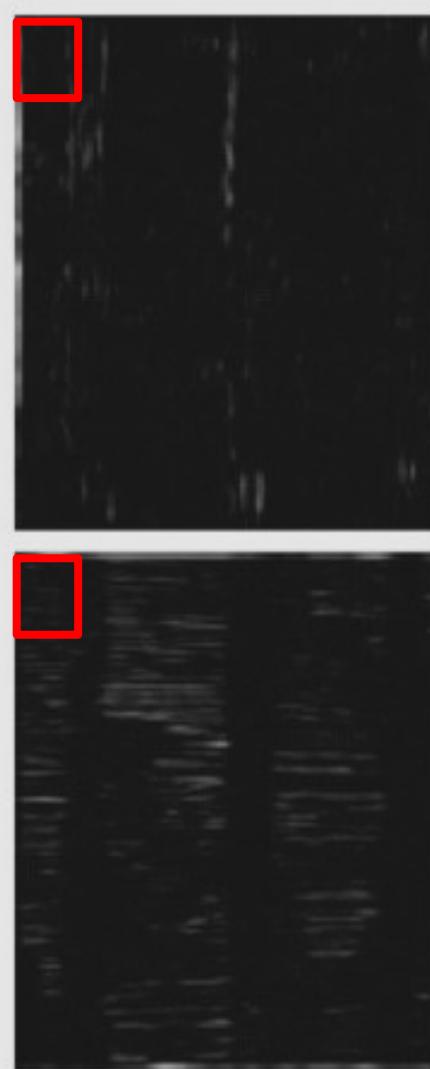
- Textures are made up of repeated local patterns, so:
  - Find the patterns
    - Use filters that look like patterns (spots, bars, raw patches...)
    - Consider magnitude of response
  - Describe their statistics within each local window, e.g.,
    - Mean, standard deviation
    - Histogram
    - Histogram of “prototypical” feature occurrences



# Texture representation: example



original image



derivative filter  
responses, squared

	<u>mean <math>d/dx</math> value</u>	<u>mean <math>d/dy</math> value</u>
Win. #1	4	10
⋮	⋮	⋮
⋮	⋮	⋮
⋮	⋮	⋮

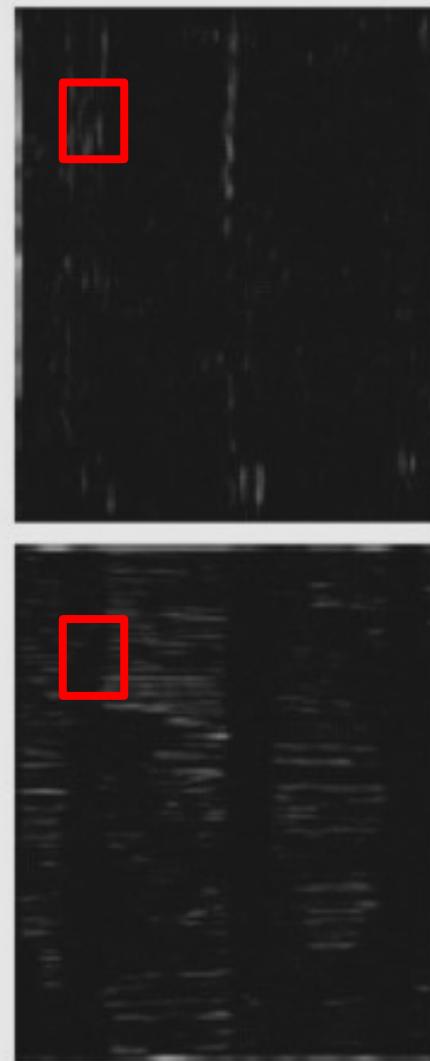
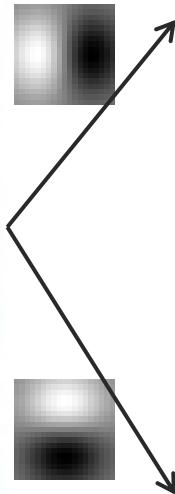
⋮  
statistics to summarize  
patterns in small  
windows



# Texture representation: example



original image



derivative filter  
responses, squared

	<u>mean <math>d/dx</math> value</u>	<u>mean <math>d/dy</math> value</u>
Win. #1	4	10
Win.#2	18	7

⋮

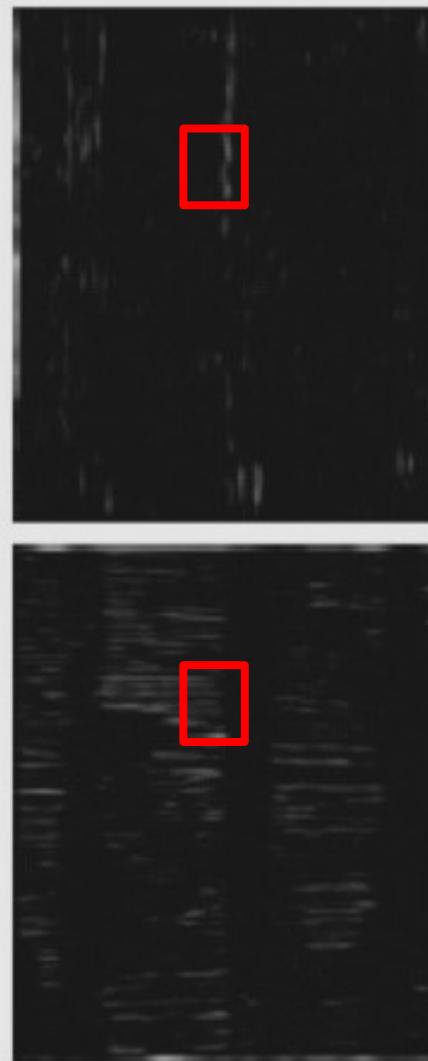
statistics to summarize  
patterns in small  
windows



# Texture representation: example



original image



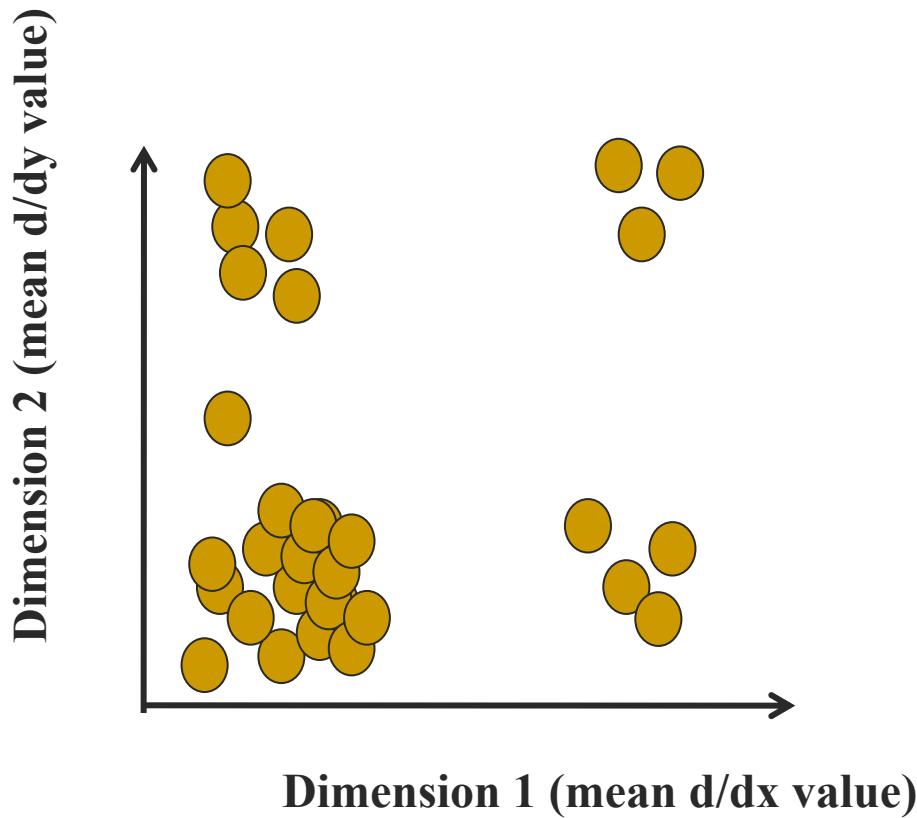
derivative filter  
responses, squared

	<u>mean <math>d/dx</math> value</u>	<u>mean <math>d/dy</math> value</u>
Win. #1	4	10
Win.#2	18	7
Win.#9	20	20
⋮	⋮	⋮

statistics to summarize  
patterns in small  
windows



# Texture representation: example



	<u>mean <math>d/dx</math> value</u>	<u>mean <math>d/dy</math> value</u>
Win. #1	4	10
Win.#2	18	7
Win.#9	20	20
⋮	⋮	⋮

statistics to summarize  
patterns in small  
windows



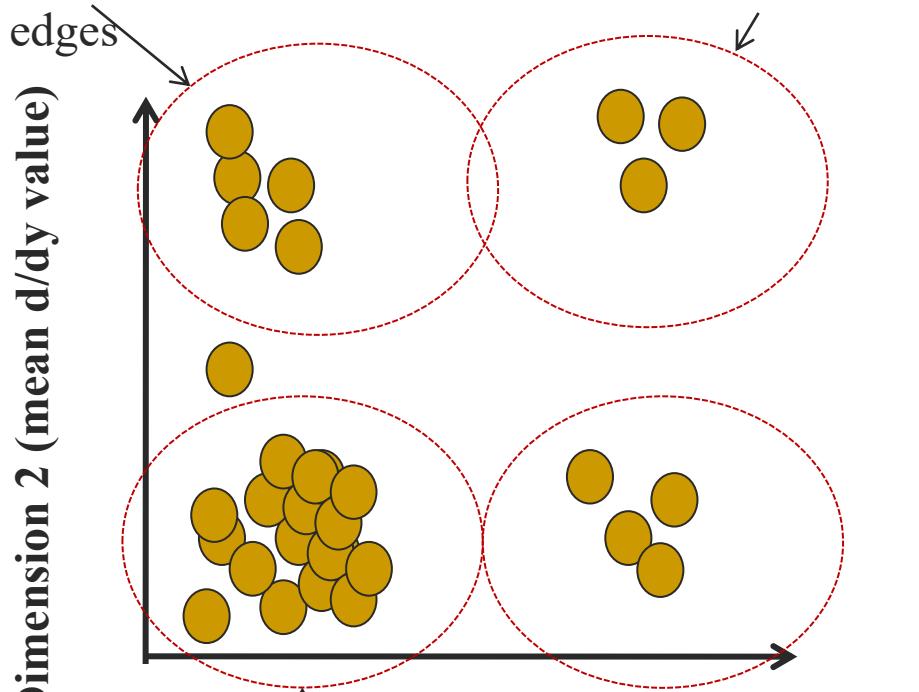
# Texture representation: example



Windows with

primarily horizontal  
edges

Both



Windows with  
small gradient in  
both directions

Windows with  
primarily vertical  
edges

statistics to summarize  
patterns in small  
windows

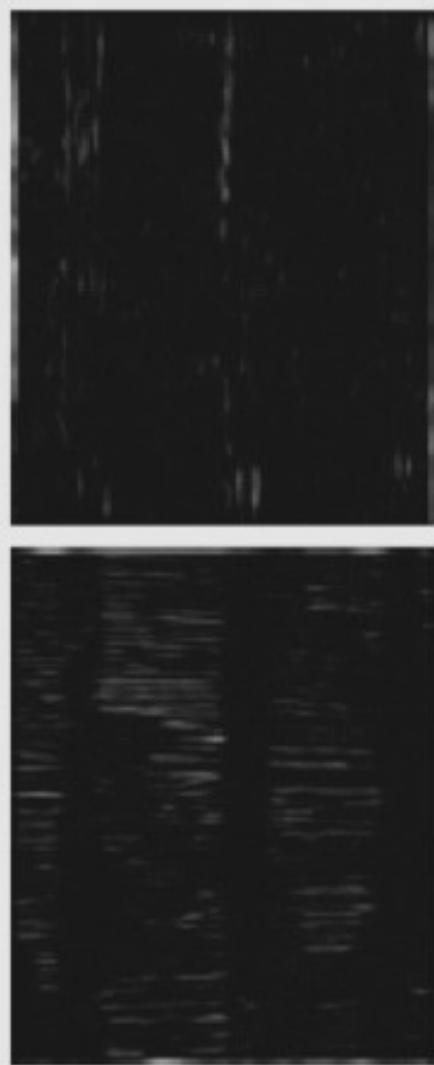
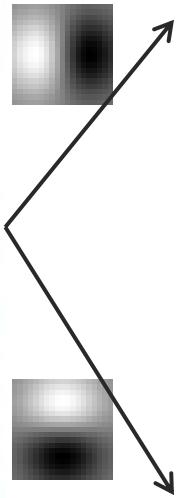
	<u>mean</u> <u><math>d/dx</math></u> <u>value</u>	<u>mean</u> <u><math>d/dy</math></u> <u>value</u>
Win. #1	4	10
Win.#2	18	7
Win.#9	20	20
⋮	⋮	⋮



# Texture representation: example



original image



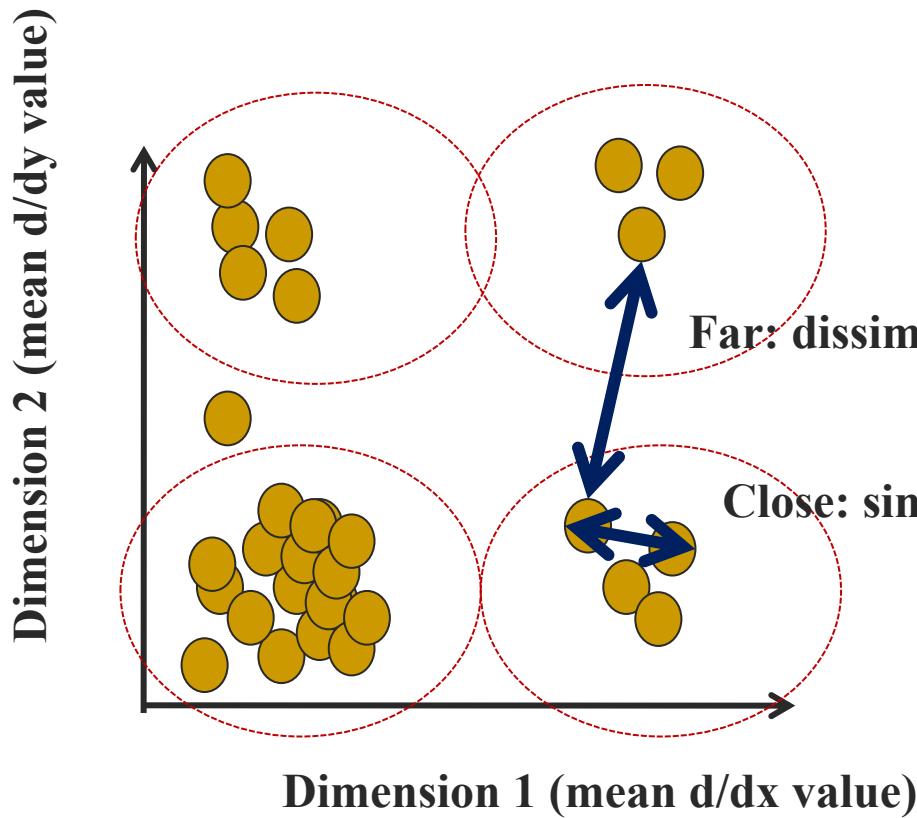
derivative filter  
responses, squared



visualization of the  
assignment to texture  
“types”



# Texture representation: example

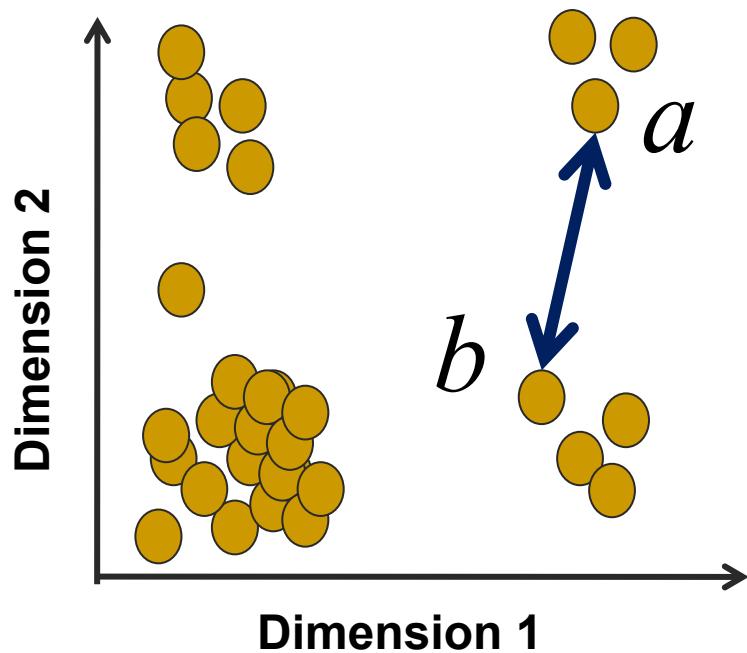


	<u>mean d/dx value</u>	<u>mean d/dy value</u>
Win. #1	4	10
Win. #2	18	7
Win. #9	20	20
⋮	⋮	⋮

statistics to summarize  
patterns in small  
windows



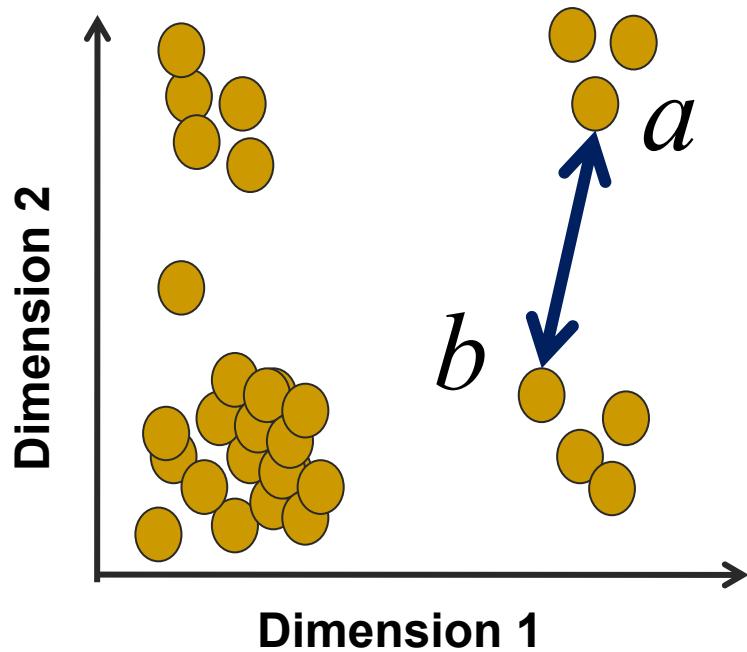
# Texture representation: example



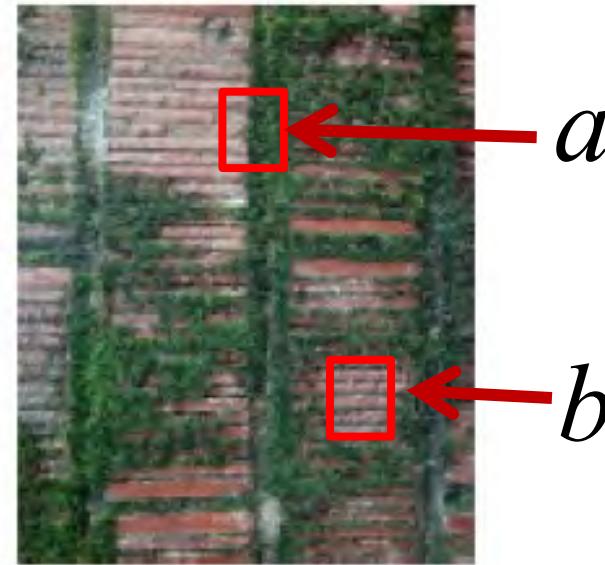
$$D(a,b) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$$



# Texture representation: example



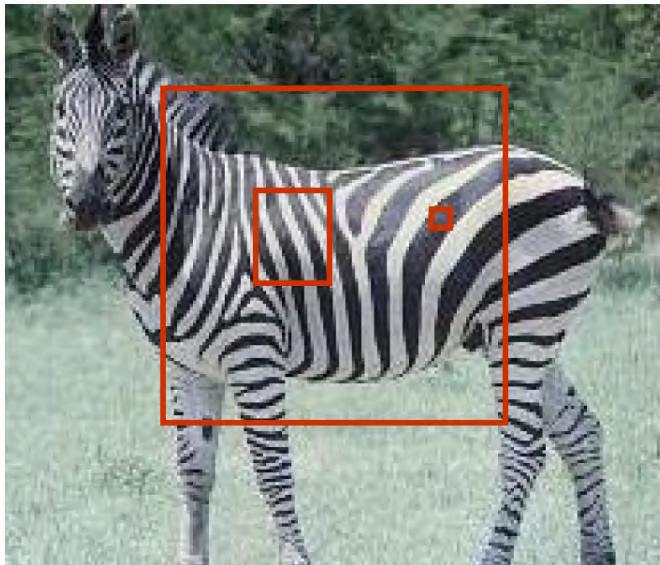
Distance reveals how dissimilar texture from window a is from texture in window b.





# Texture representation: window scale

- We're assuming we know the relevant window size for which we collect these statistics.



Possible to perform **scale selection** by looking for window scale where texture description not changing.



# Filter banks

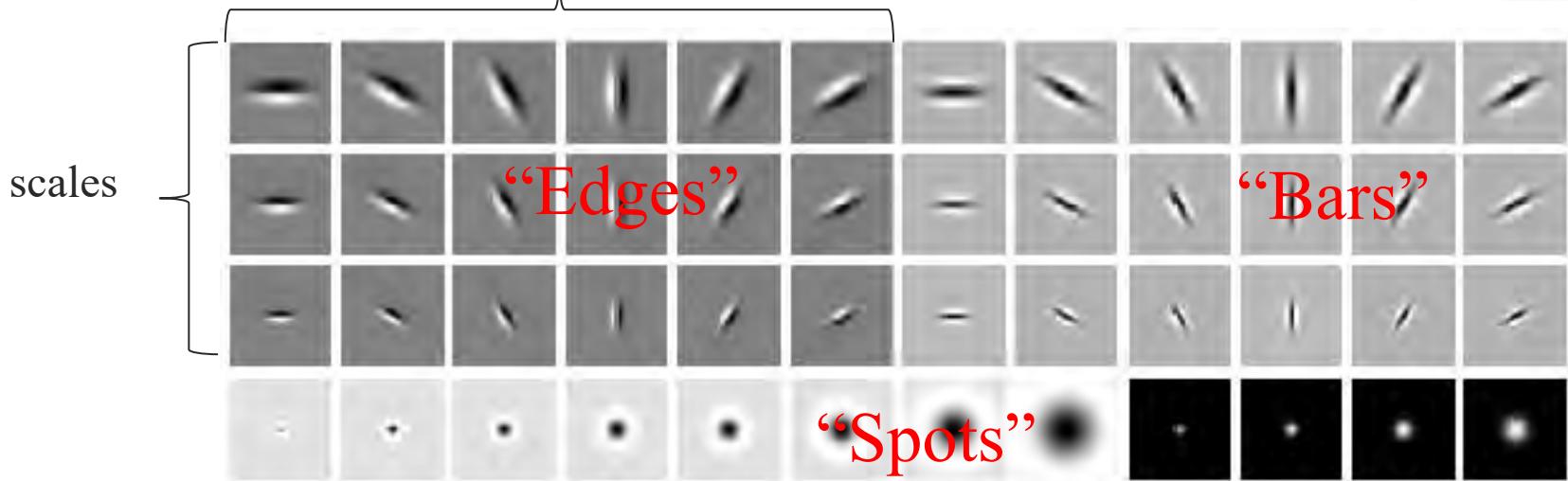


- Our previous example used two filters, and resulted in a 2-dimensional feature vector to describe texture in a window.
  - x and y derivatives revealed something about local structure.
- We can generalize to apply a collection of multiple ( $d$ ) filters: a “filter bank”
- Then our feature vectors will be  $d$ -dimensional.
  - still can think of nearness, farness in feature space



# Filter banks

orientations



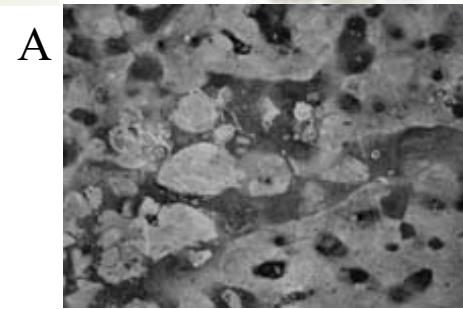
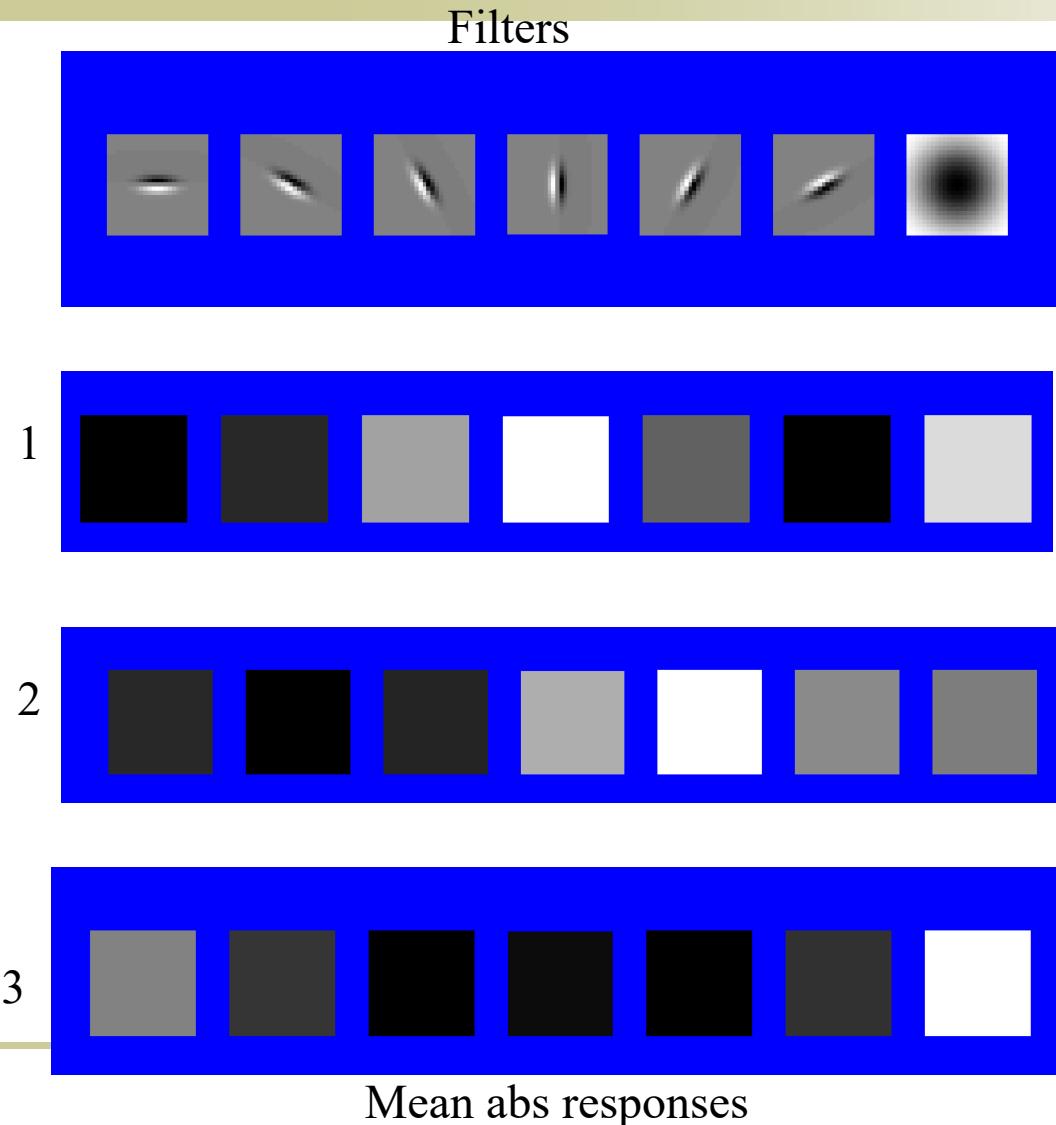
- What filters to put in the bank?
  - Typically we want a combination of scales and orientations, different types of patterns.

Matlab code available for these examples:

<http://www.robots.ox.ac.uk/~vgg/research/texclass/filters.html>

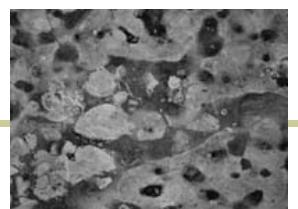
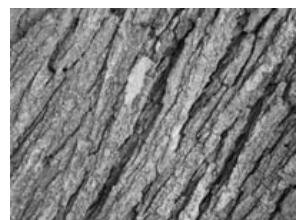


# You try: Can you match the texture to the response?

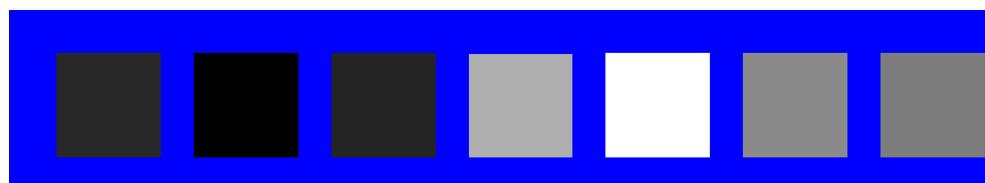
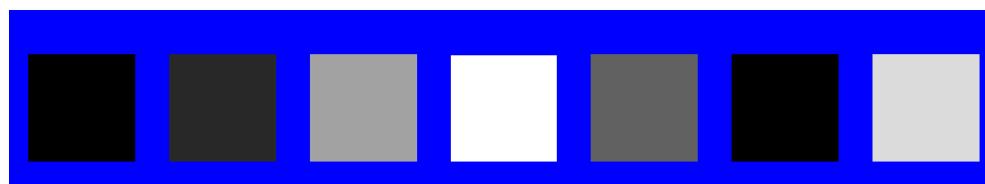
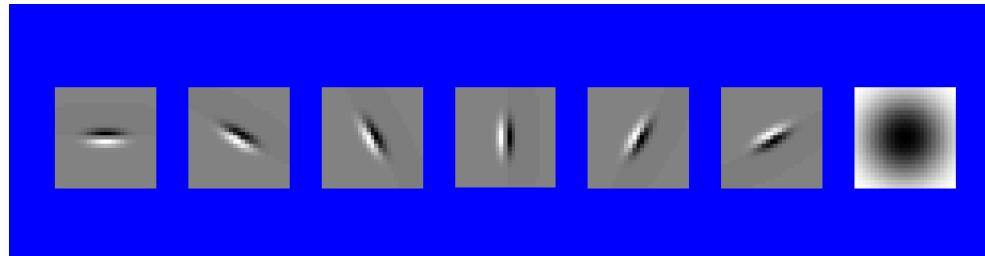




# Representing texture by mean abs response

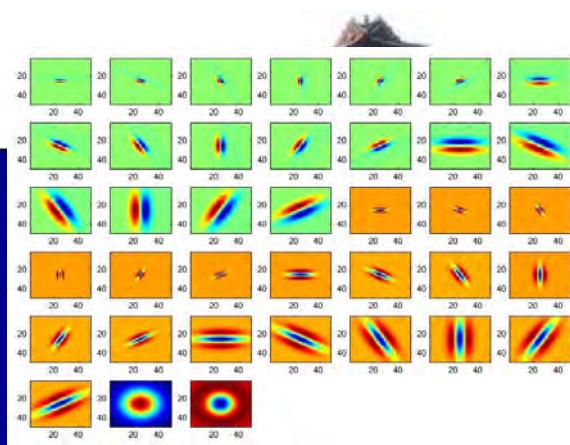
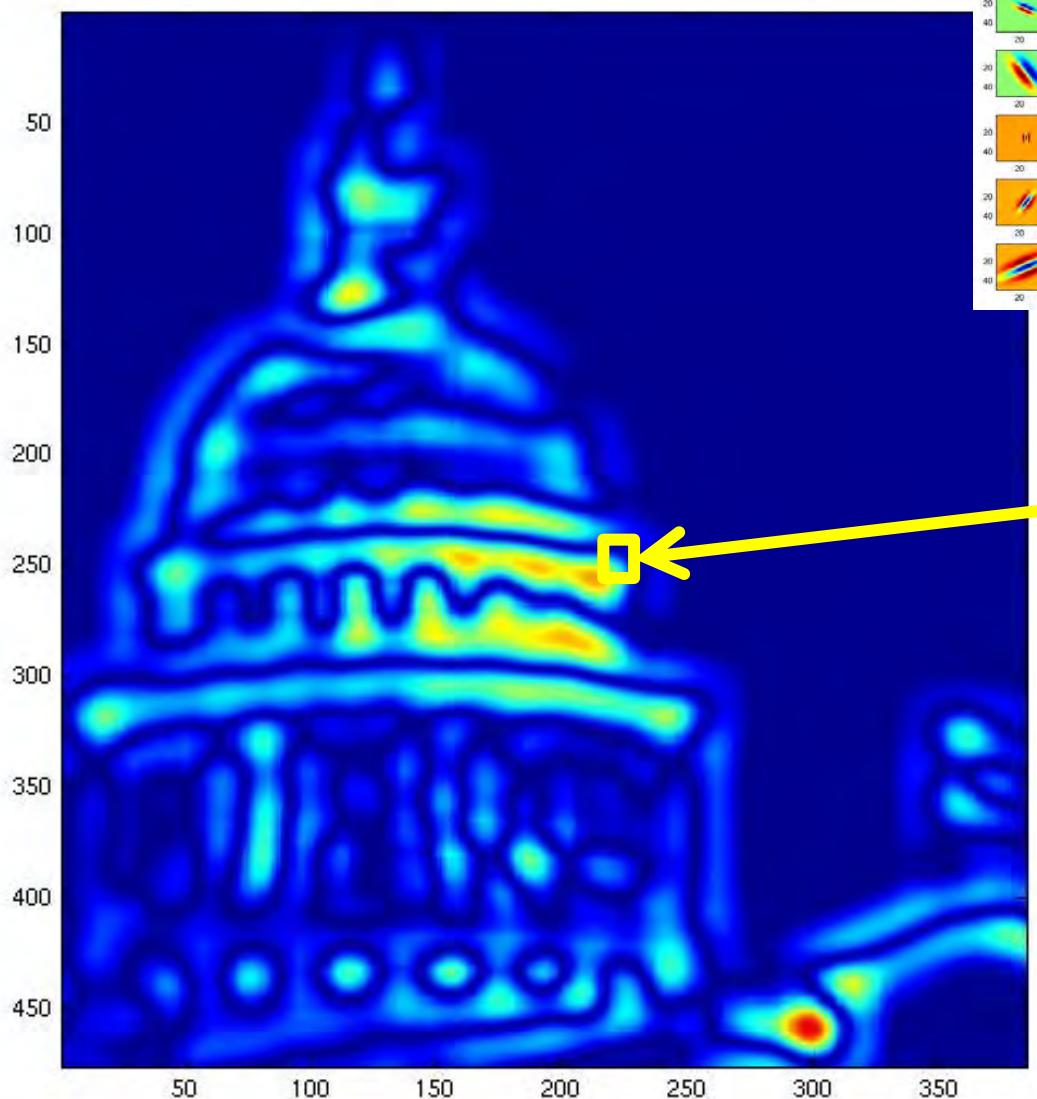


Filters



Mean abs responses

Derek Hoiem



$[r_1, r_2, \dots, r_{38}]$

We can form a feature vector from the list of responses at each pixel.

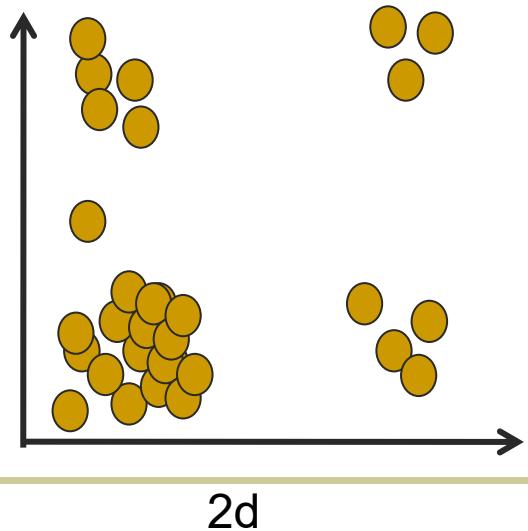


# $d$ -dimensional features



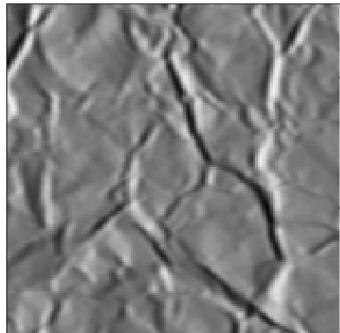
$$D(a, b) = \sqrt{\sum_{i=1}^d (a_i - b_i)^2}$$

Euclidean distance ( $L_2$ )





# Texture Recognition



?



Felt?

Polyester?

Terrycloth?

Rough Plaster?

Leather?

Plaster?

Concrete?

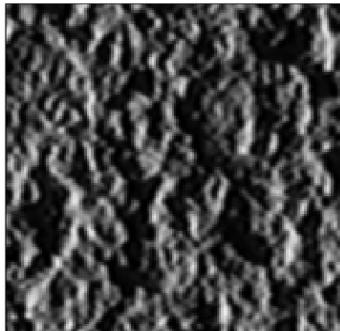
Crumpled Paper?

Sponge?

Limestone?

Brick?

⋮



?



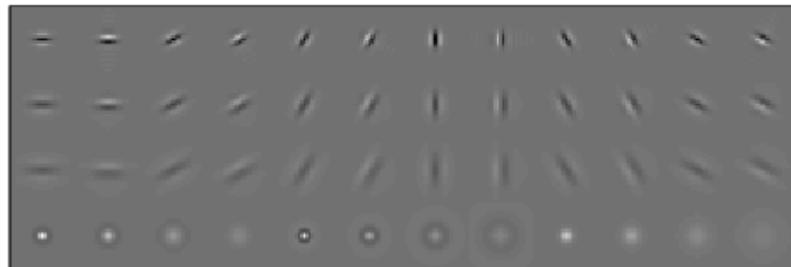


# 2D Textons



- Goal: find canonical local features in a texture;

- 1) Filter image with linear filters:

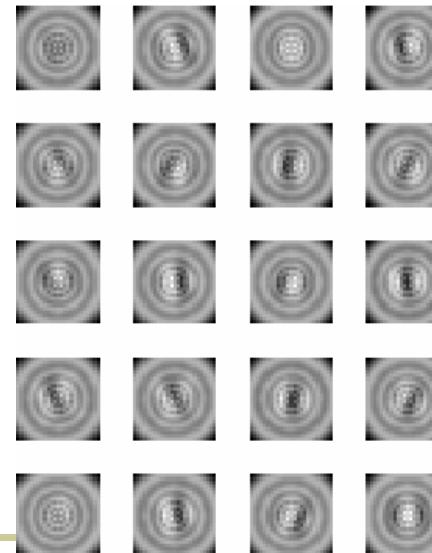
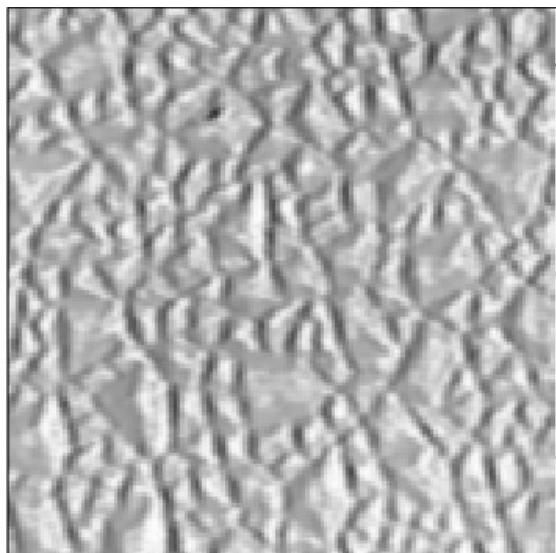
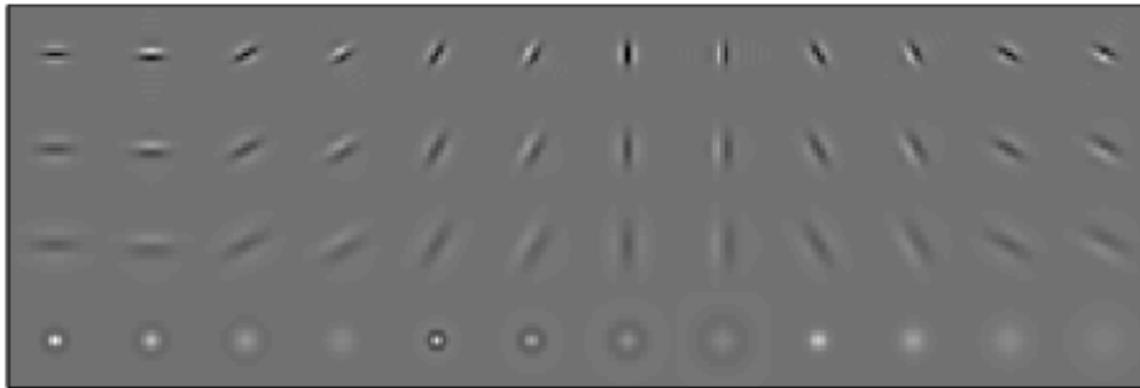


- 2) Vector quantization (k-means) on filter outputs;
- 3) Quantization centers are the textons.

- Spatial distribution of textons defines the texture;



# 2D Textons (cont'd)

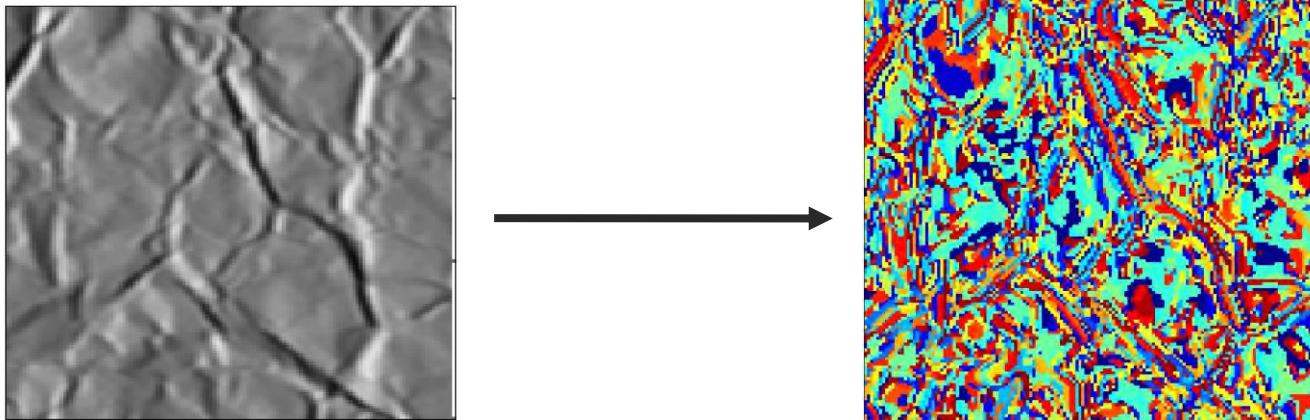




# Texton Labeling



- Each pixel labeled to texton  $i$  (1 to K) which is *most similar in appearance*;
- Similarity measured by the Euclidean distance between the filter responses;





# Material Representation



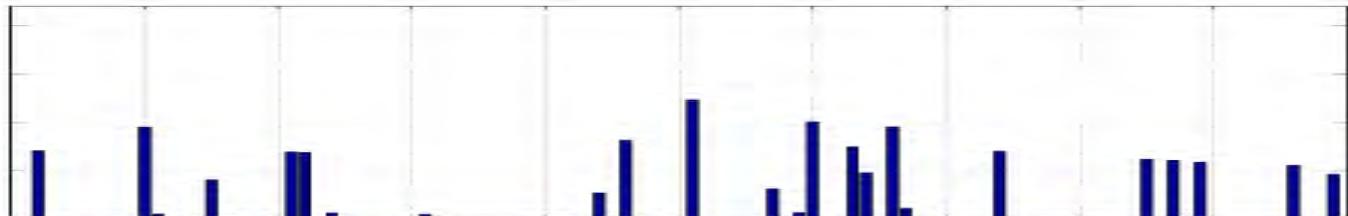
- Each material is now represented as a spatial arrangement of symbols from the texton vocabulary;
- Recognition: ignore spatial arrangement, use histogram ( $K=100$ );



# Histogram Models for Recognition (Leung & Malik, 1999)



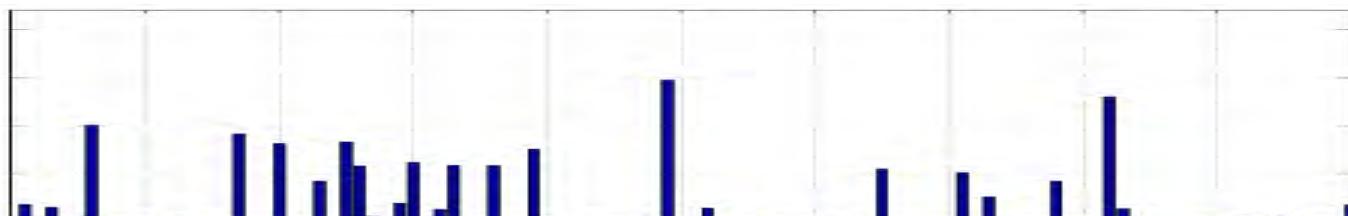
Rough Plastic



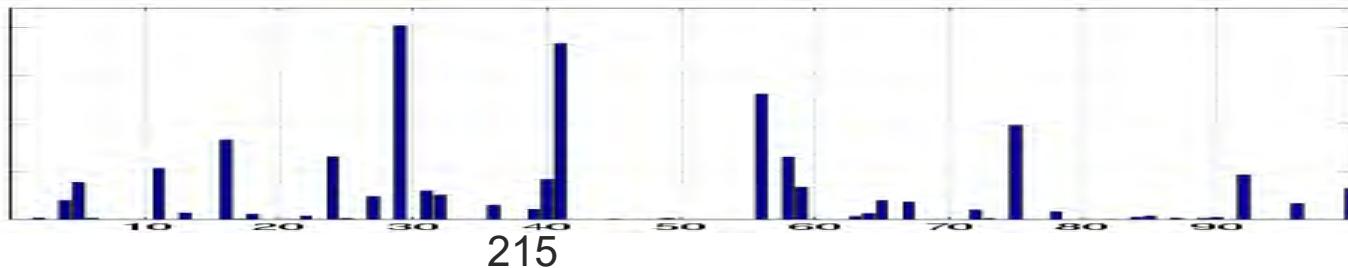
Pebbles



Plaster-b



Terrycloth





# Similarity of materials



- Similarity between histograms measured using chi-square difference:

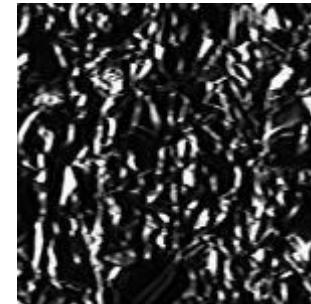
$$\chi^2(h_1, h_2) = \sum_{n=1}^N \frac{(h_1(n) - h_2(n))^2}{h_1(n) + h_2(n)}$$



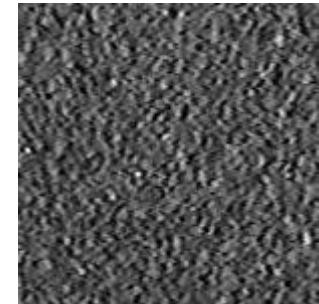
# Similarity Matrix



1



# Aluminum Foil



$$e_{ij} = \text{Similarity}(\text{material} = i, \text{sample} = j)$$



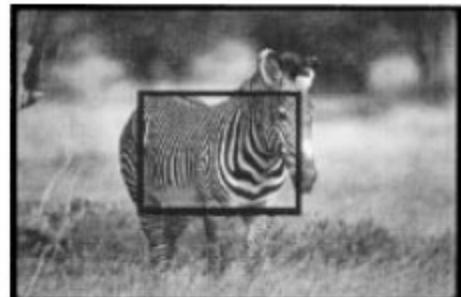
# Example uses of texture in vision: analysis



# Classifying materials, “stuff”



Figure by Varma &  
Zisserman



(a)



1) 130066



2) 130070



3) 130068



4) 130051



5) 130038

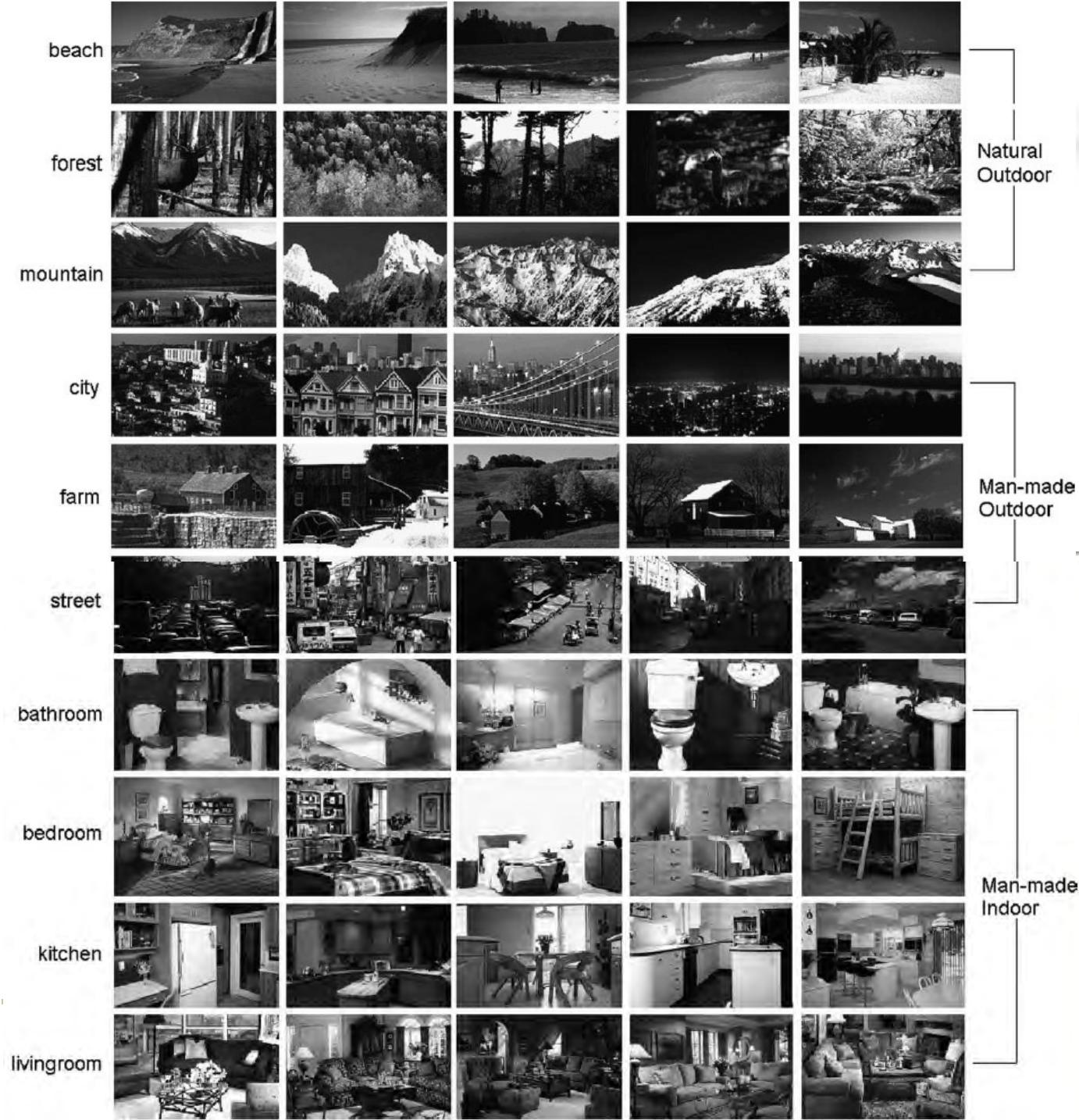


6) 130039

## Texture features for image retrieval



Y. Rubner, C. Tomasi, and L. J. Guibas. The earth mover's distance as a metric for image retrieval. *International Journal of Computer Vision*, 40(2):99-121, November 2000,



# Characterizing scene categories by texture

L. W. Renninger and  
J. Malik. When is  
scene identification  
just texture  
recognition? Vision  
Research 44 (2004)  
2301–2311



Segmenting  
aerial imagery  
by textures



# Texture-related tasks



## ■ Shape from texture

- Estimate surface orientation or shape from image texture

## ■ Segmentation/classification from texture cues

- Analyze, represent texture
- Group image regions with consistent texture

## ■ Synthesis

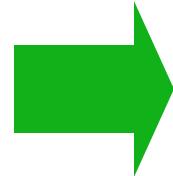
- Generate new texture patches/images given some examples



# Texture synthesis



- Goal: create new samples of a given texture
- Many applications: virtual environments, hole-filling, texturing surfaces

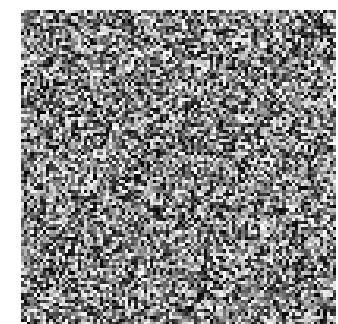
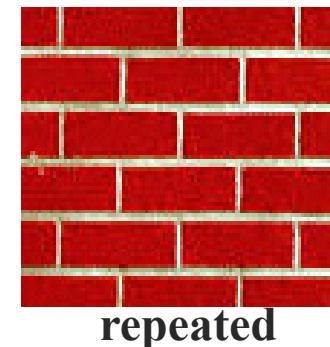




# The Challenge

- Need to model the whole spectrum: from repeated to stochastic texture

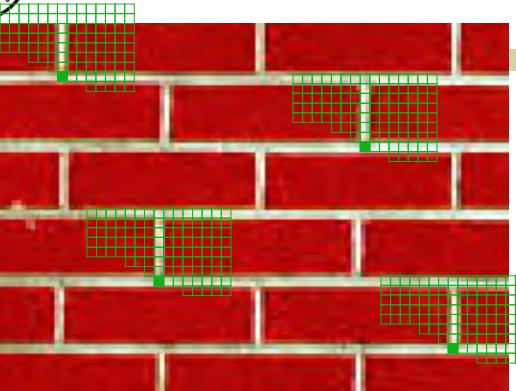
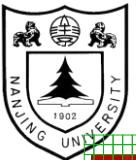
Alexei A. Efros and Thomas K. Leung, “Texture Synthesis by Non-parametric Sampling,” Proc. International Conference on Computer Vision (ICCV), 1999.



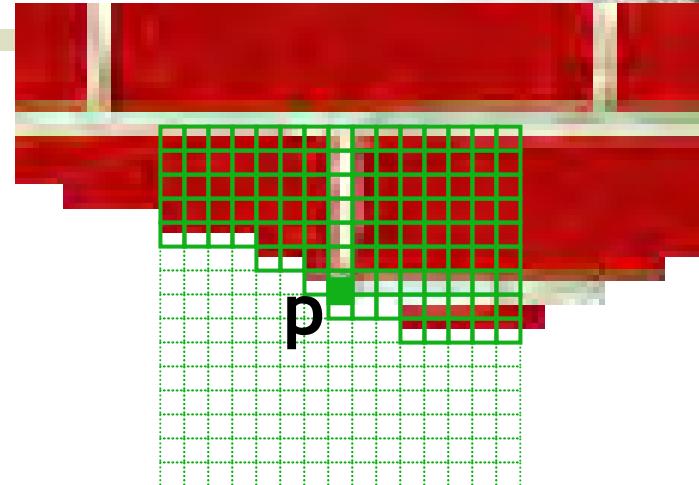
stochastic



Both?



input image

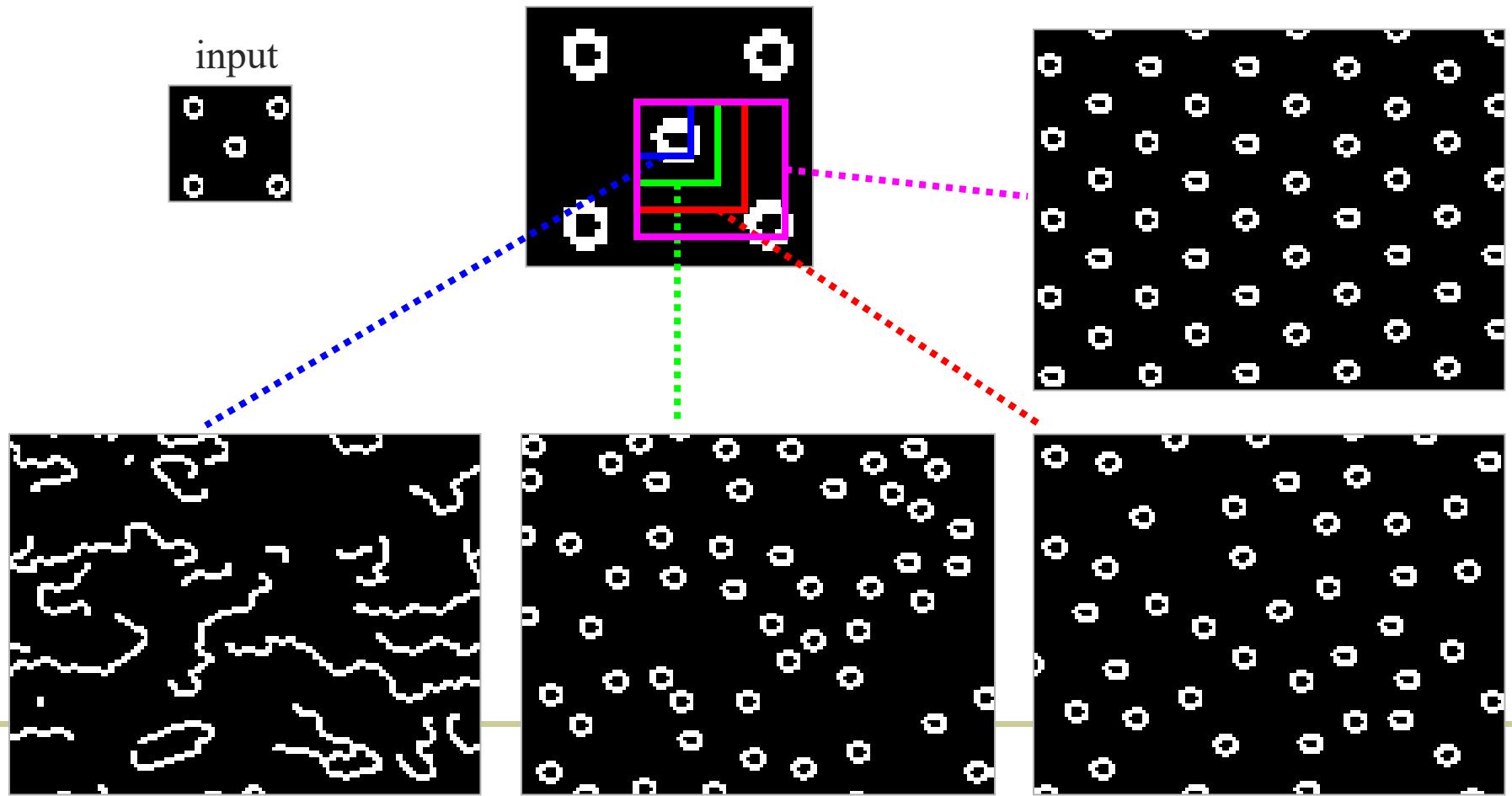


synthesized image

- What is  $P(x|\text{neighborhood of pixels around } x)$  ?
- Find all the windows in the image that match the neighborhood
- To synthesize  $x$ 
  - pick one matching window at random
  - assign  $x$  to be the center pixel of that window

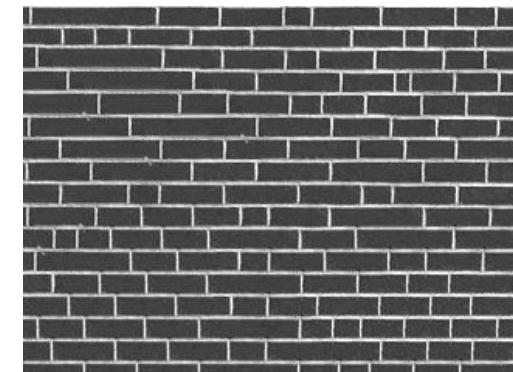
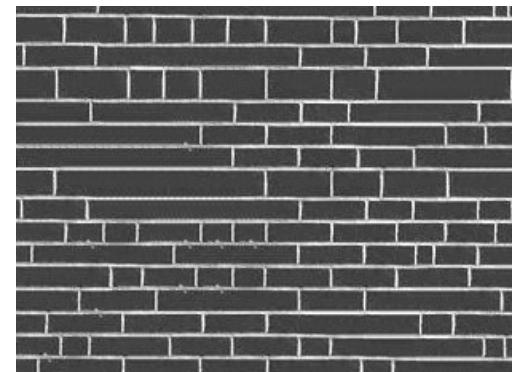
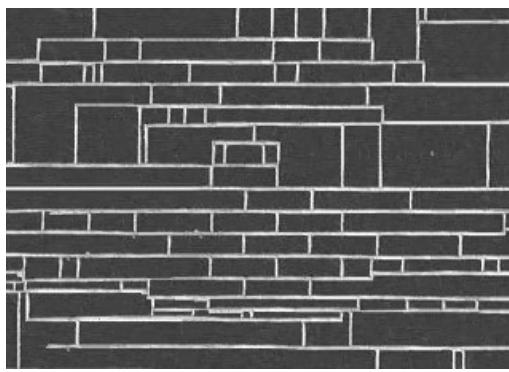
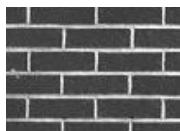
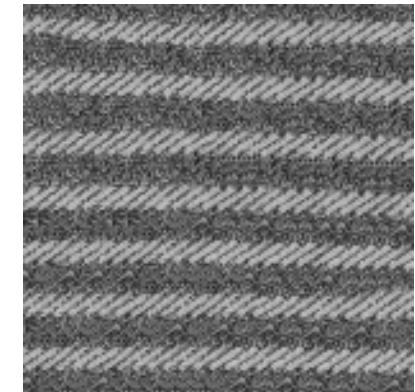
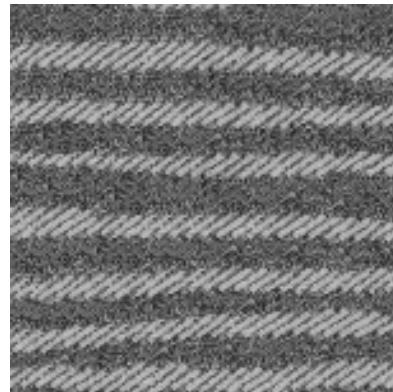
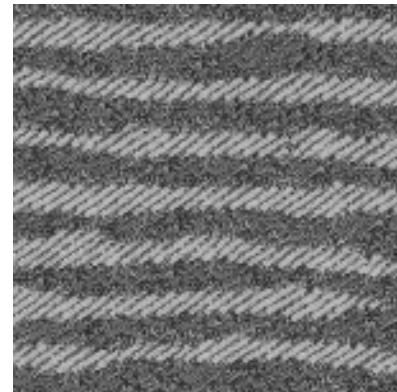
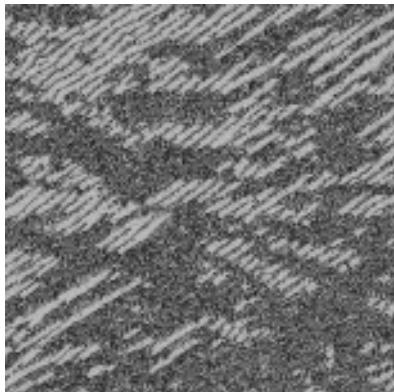


# Neighborhood Window





# Varying Window Size



Increasing window size

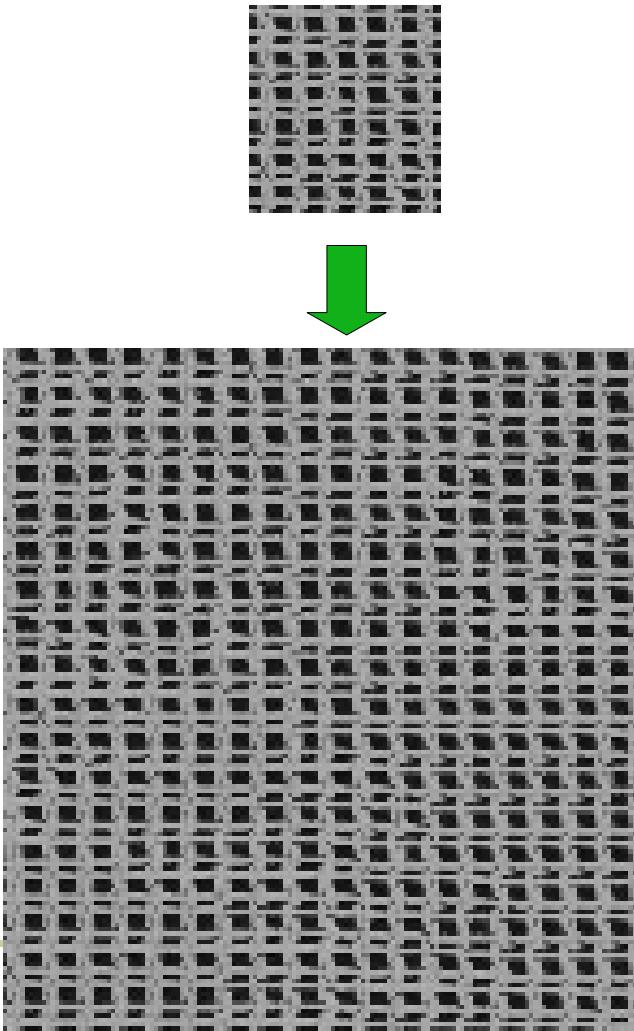




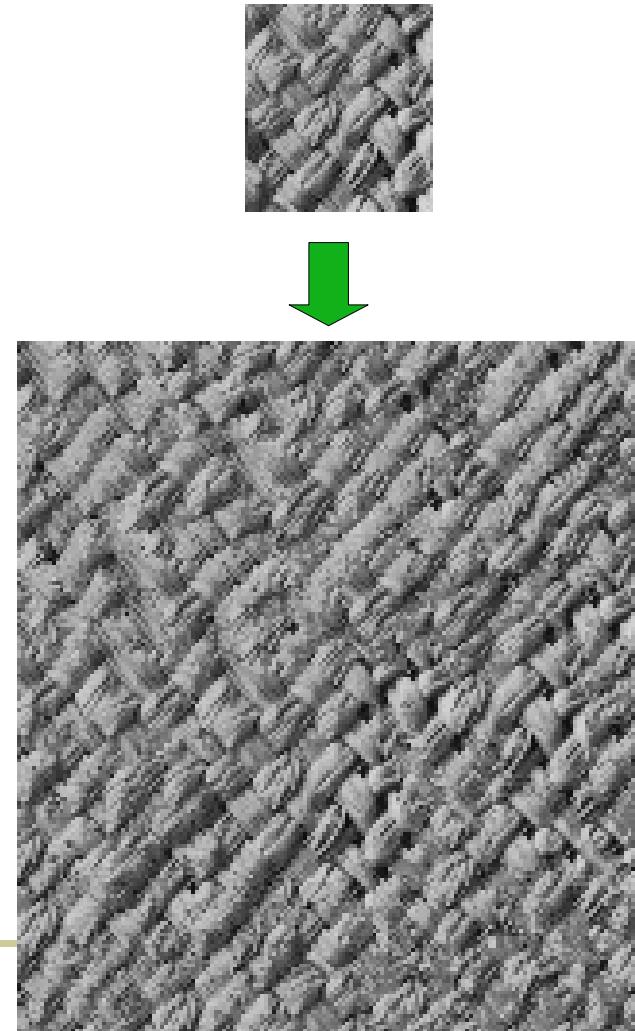
# Synthesis results



french canvas



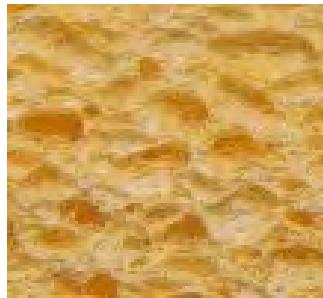
rafia weave



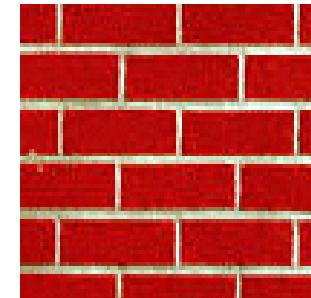
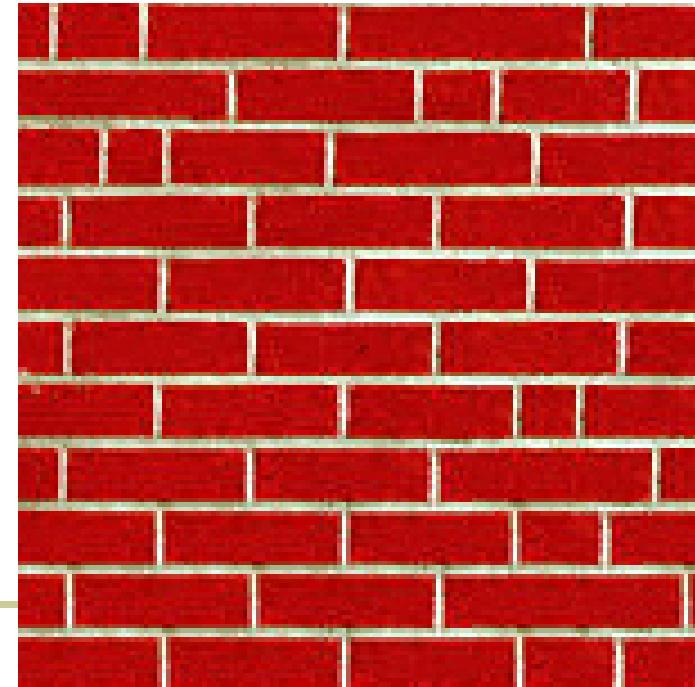


# Synthesis results

white bread



brick wall





# Synthesis results

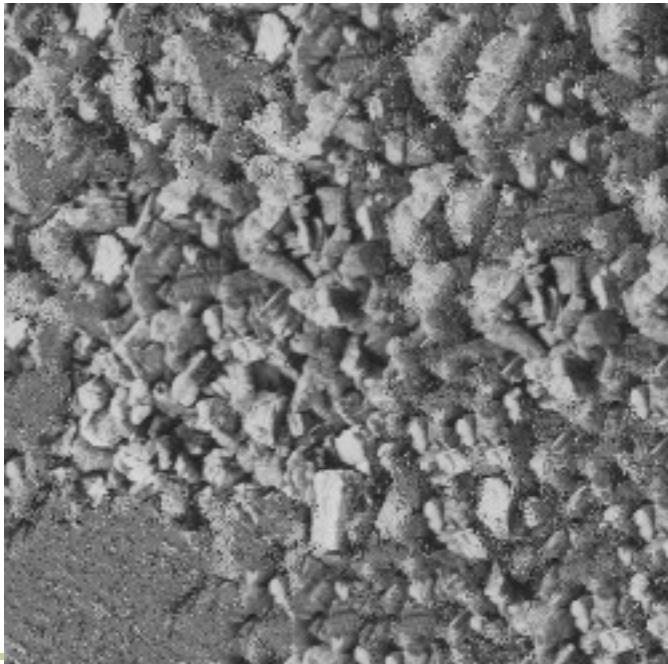


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# Failure Cases



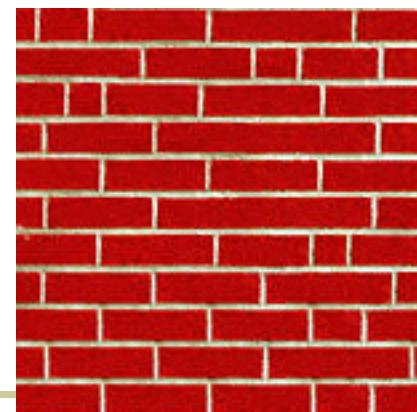
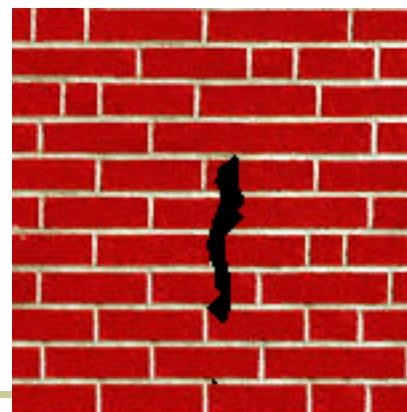
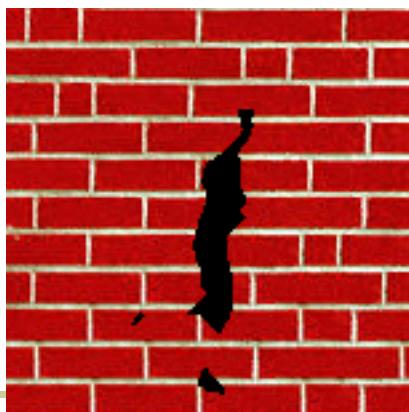
**Growing garbage**



**Verbatim copying**

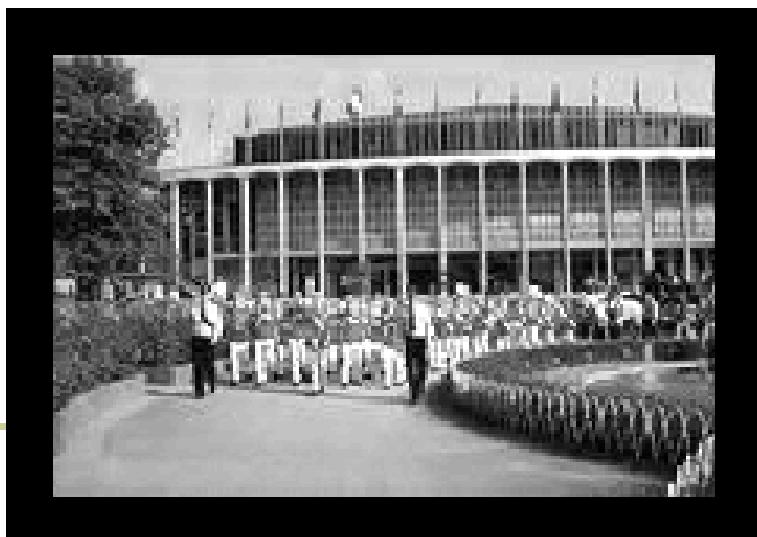
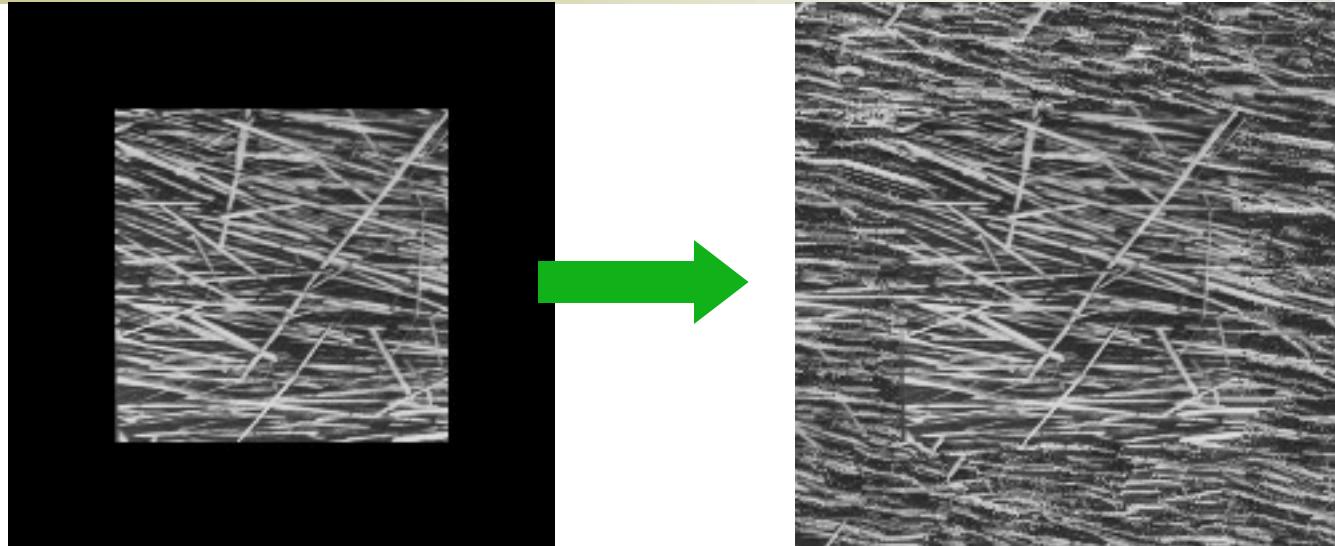


# Hole Filling





# Extrapolation





# Summary



- Template, Pyramid, and Texture
  - Template matching (SSD or Normxcorr2)
    - SSD can be done with linear filters, is sensitive to overall intensity
  - Gaussian pyramid
    - Coarse-to-fine search, multi-scale detection
  - Laplacian pyramid
    - More compact image representation
    - Can be used for compositing in graphics
  - Steerable pyramid
    - Filter banks for representing texture