



# 计算机视觉表征与识别

## Chapter 7: Interest Points: Detector

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# Today's Class



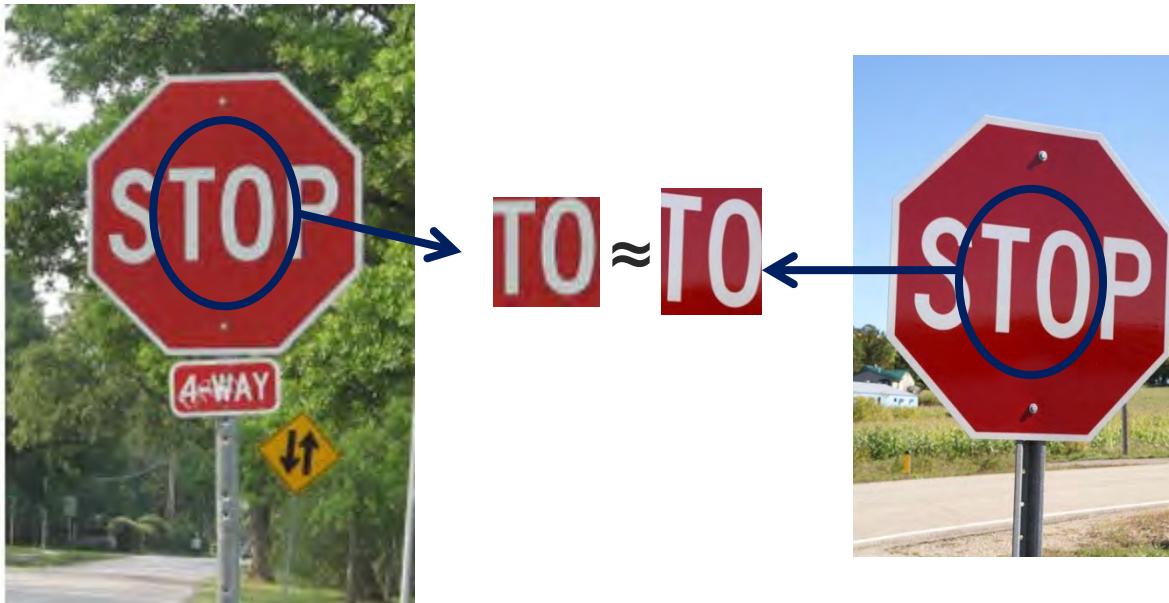
- **Introduction to correspondence and alignment**
- **Overview of interest points**
  - Matching pipeline
  - Repeatable & Distinctive
- **Keypoint Localization**
  - Harris detector
  - Hessian detector
- **Scale invariant region selection**
  - Automatic scale selection
  - Laplacian of Gaussian (LoG) & Difference of Gaussian (DoG)
  - Combinations: Harris-Laplacian & Hessian-Laplacian



# Correspondence and alignment



**Correspondence:** matching points, patches, edges, or regions across images





# Correspondence and alignment



**Alignment:** solving the transformation that makes two things match better

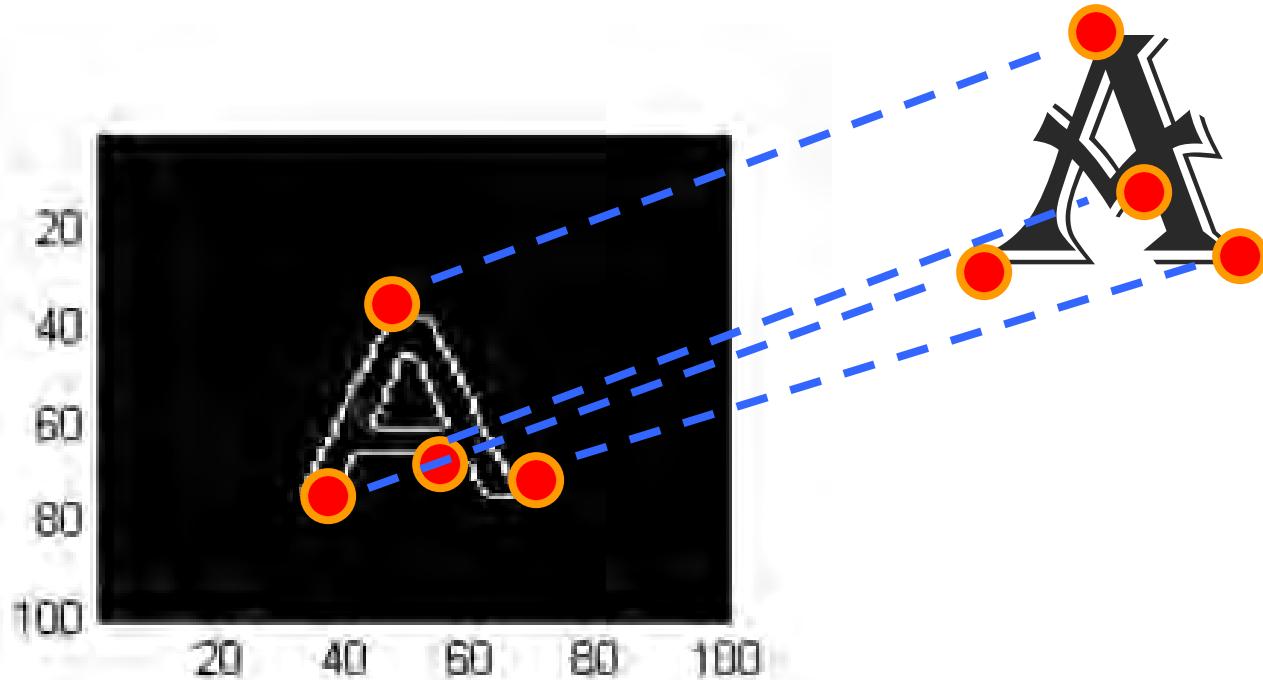


T





# Example: fitting an 2D shape template

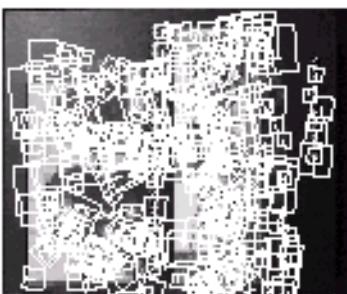
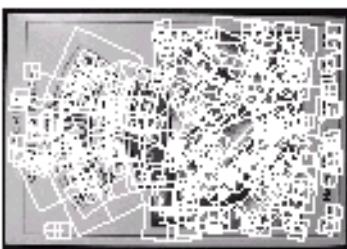




# Planar object instance recognition



Database of planar objects



Instance recognition





# 3D object recognition



Database of 3D objects

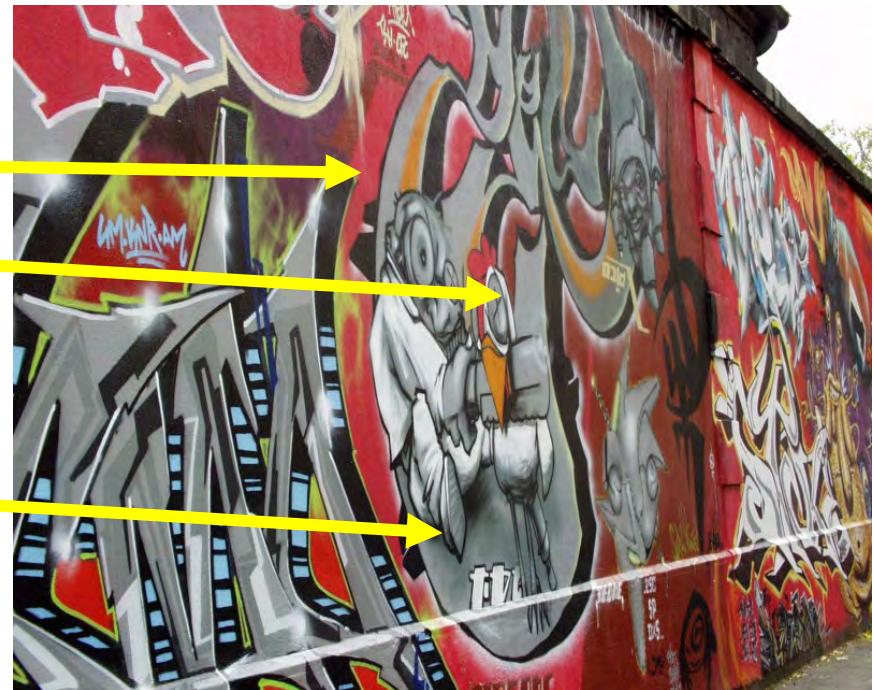
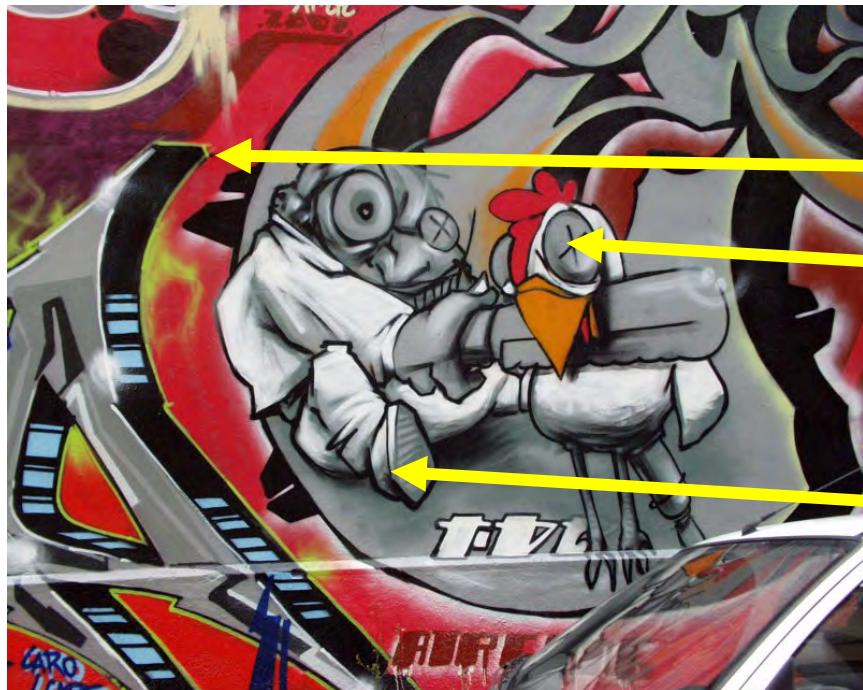


3D objects recognition



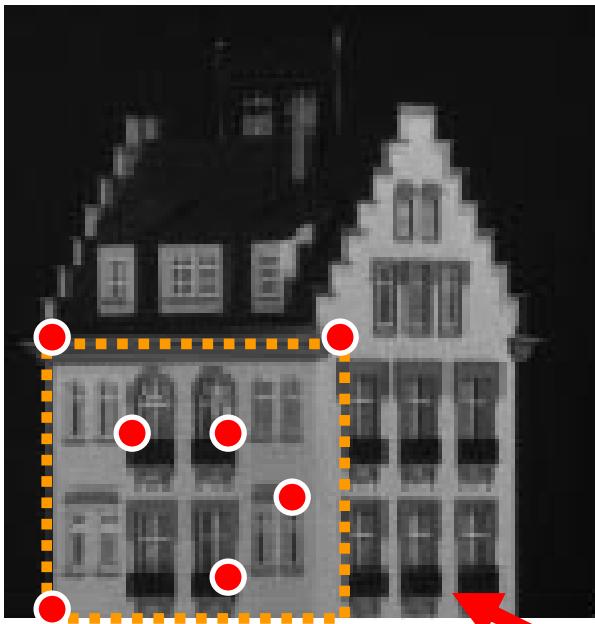
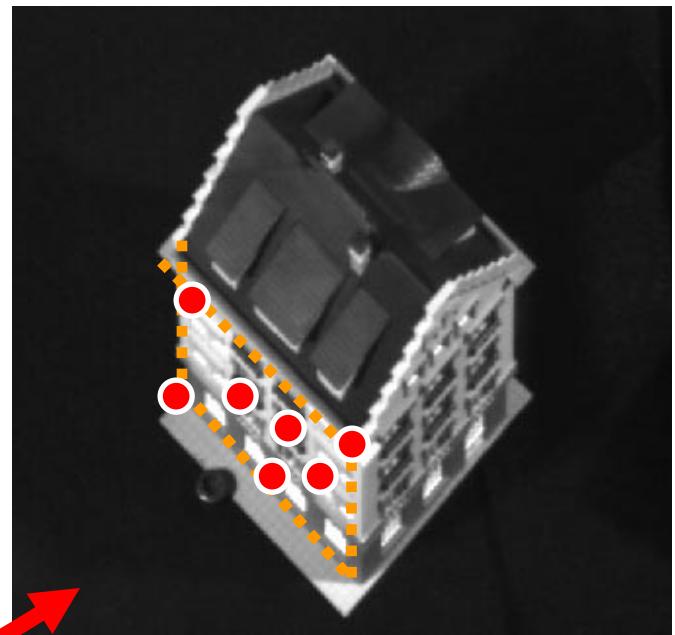


# Example: Image matching





# Example: Estimating an homographic transformation

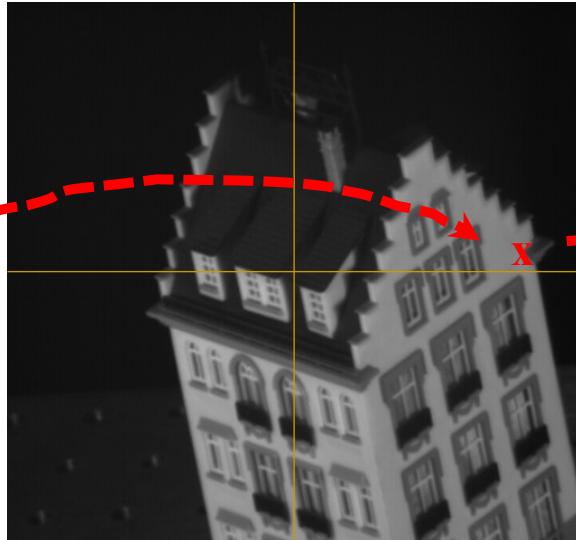
 $H$ 



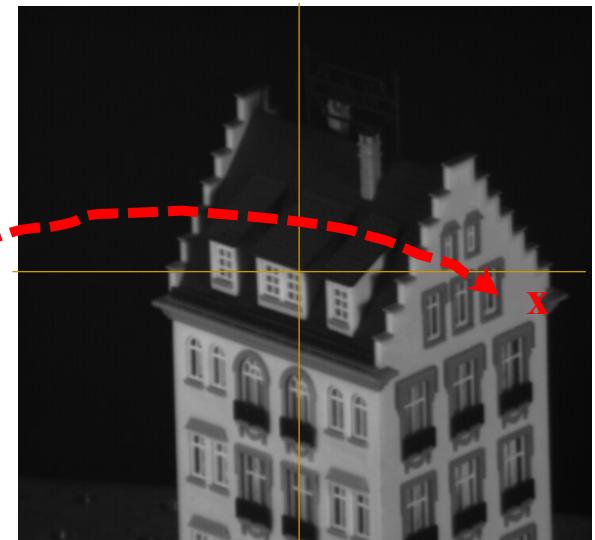
# Example: tracking points



frame 0



frame 22



frame 49



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# This class: interest points



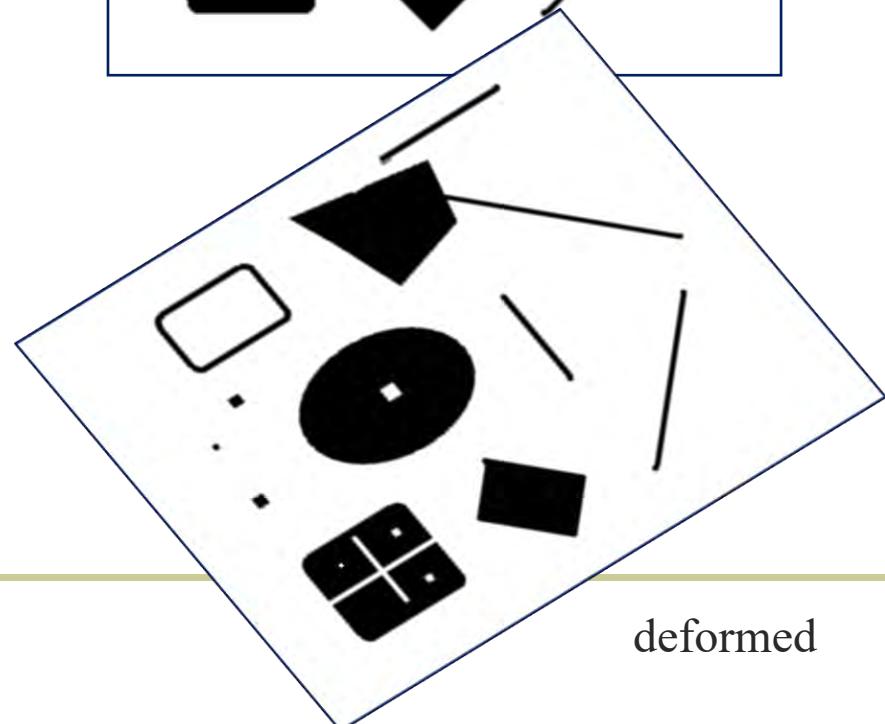
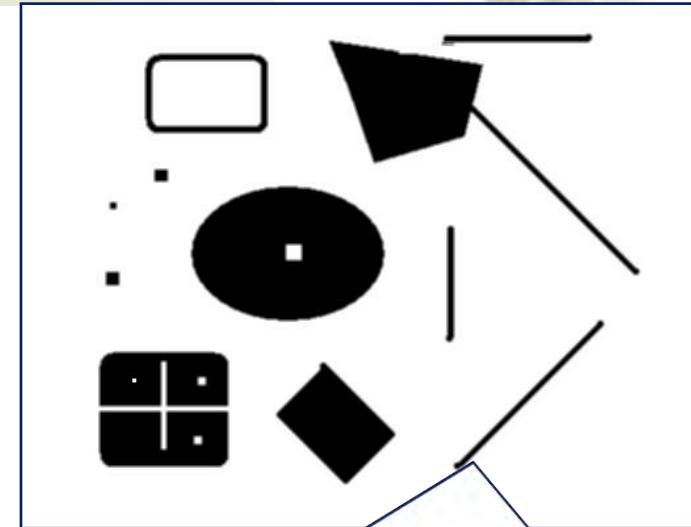
- Note: “interest points” = “keypoints”, also sometimes called “local features”
- Many applications
  - tracking: which points are good to track?
  - recognition: find patches likely to tell us something about object category
  - 3D reconstruction: find correspondences across different views



# This class: interest points



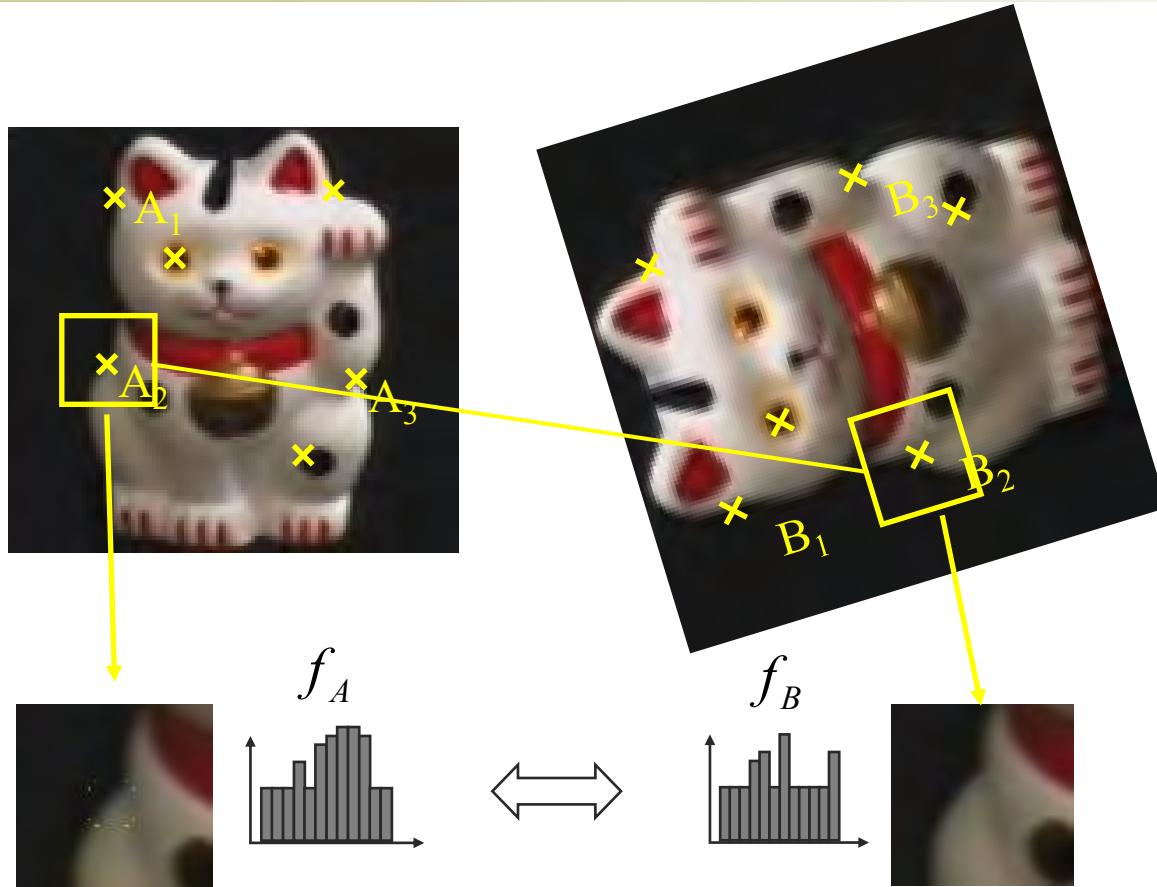
- Suppose you have to click on some point, go away and come back after I deform the image, and click on the same points again.
  - Which points would you choose?



deformed



# Overview of Keypoint Matching



1. Find a set of distinctive keypoints
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors



# Goals for Keypoints



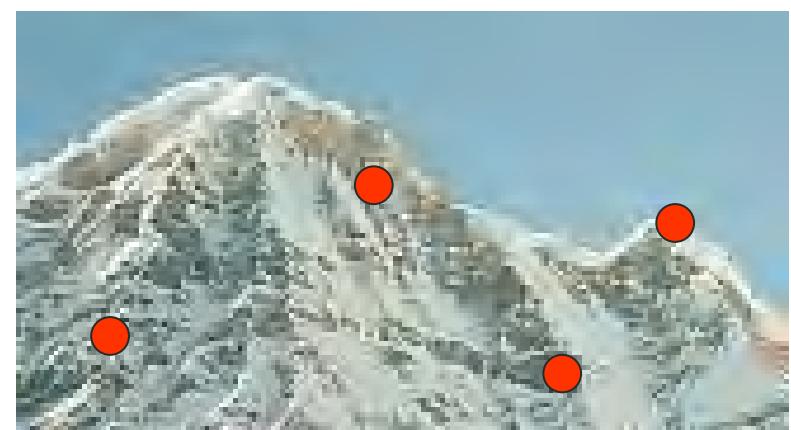
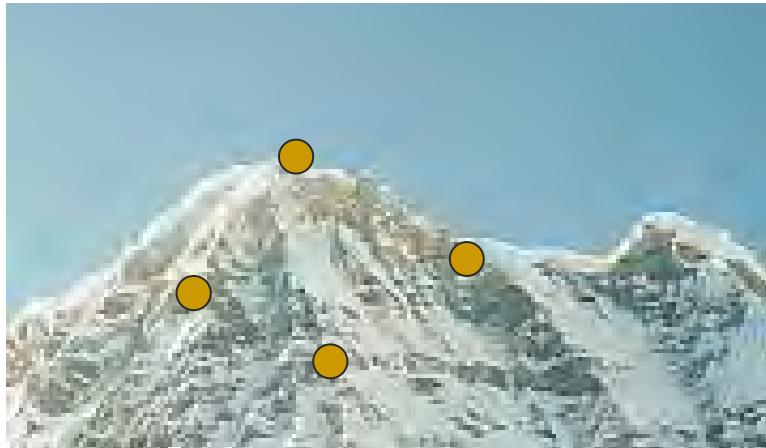
Detect points that are *repeatable* and *distinctive*



# Goal: interest operator repeatability



- We want to detect (at least some of) the same points in both images.



No chance to find true matches!

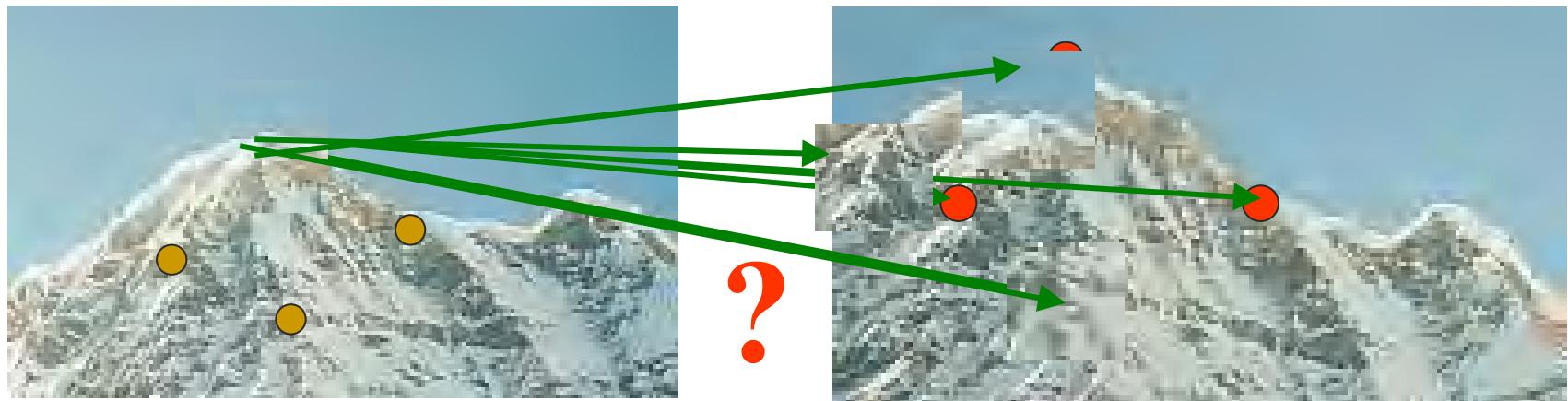
- Yet we have to be able to run the detection procedure *independently* per image.



# Goal: descriptor distinctiveness



- We want to be able to reliably determine which point goes with which.



- Must provide some invariance to geometric and photometric differences between the two views.



# Local features: desired properties

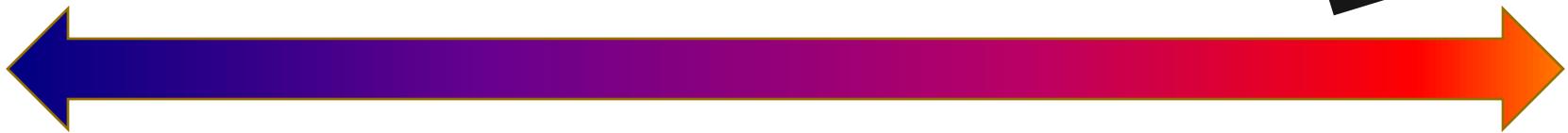


- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Distinctiveness
  - Each feature has a distinctive description
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion



# Key trade-offs

## Detection



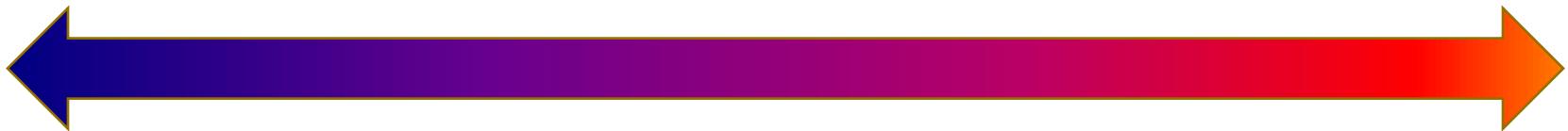
### More Repeatable

- Robust detection
- Precise localization

### More Points

- Robust to occlusion
- Works with less texture

## Description



### More Distinctive

- Minimize wrong matches

### More Flexible

- Robust to expected variations
- Maximize correct matches



# Choosing interest points



Where would you tell  
your friend to meet  
you?



Corner detection



# Choosing interest points



Where would you tell  
your friend to meet  
you?



Blob (valley/peak) detection



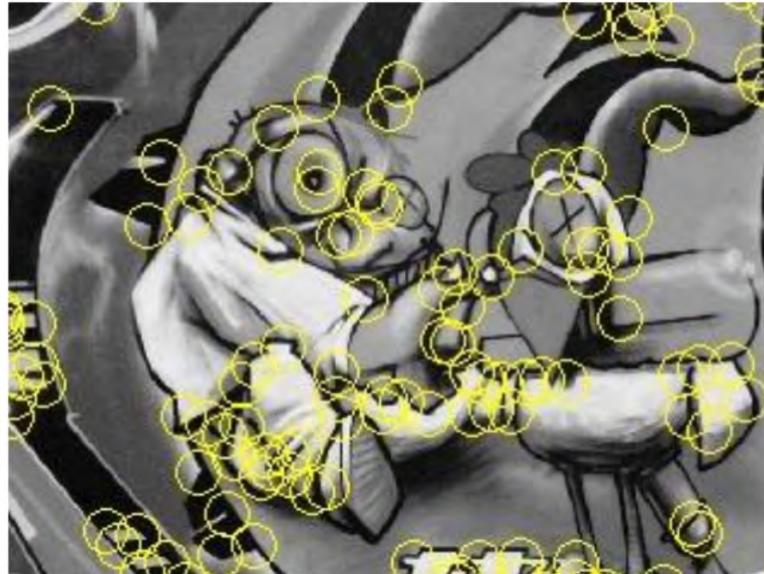
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# Keypoint Localization



- Goals:
    - Repeatable detection
    - Precise localization
    - Interesting content
- ⇒ *Look for two-dimensional signal changes*

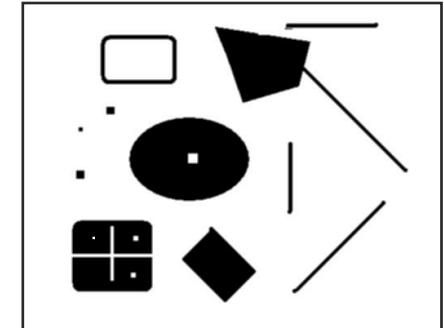


# Harris Detector [Harris88]



## ■ Second moment matrix

$$\mu(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$



*Intuition:* Search for local neighborhoods where the image content has two main directions (eigenvectors).

**C.Harris and M.Stephens. "A Combined Corner and Edge Detector."**  
***Proceedings of the 4th Alvey Vision Conference, 1988.***



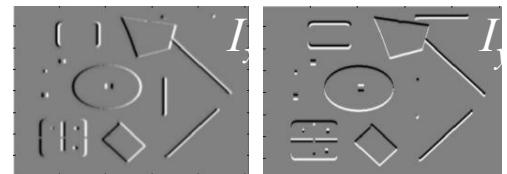
# Harris Detector [Harris88]



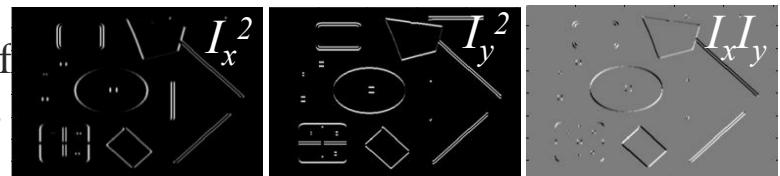
## ■ Second moment matrix

$$\mu(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

1. Image derivatives  
(optionally, blur first)



2. Square of derivatives



3. Gaussian filter  $g(\sigma_D)$



$$\det M = \lambda_1 \lambda_2$$

$$\text{trace } M = \lambda_1 + \lambda_2$$

4. Cornerness function – both eigenvalues are strong

$$har = \det[\mu(\sigma_I, \sigma_D)] - \alpha [\text{trace}(\mu(\sigma_I, \sigma_D))^2] =$$

$$g(I_x^2)g(I_y^2) - [g(I_x I_y)]^2 - \alpha[g(I_x^2) + g(I_y^2)]^2$$

5. Non-maxima suppression

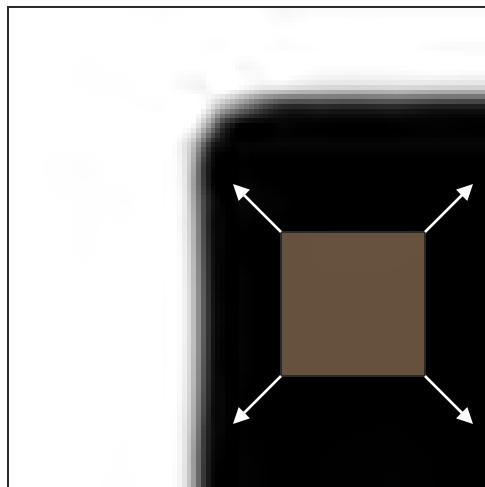




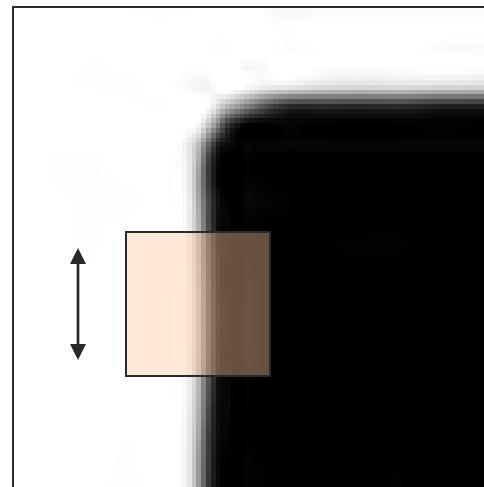
# Corners as distinctive interest points



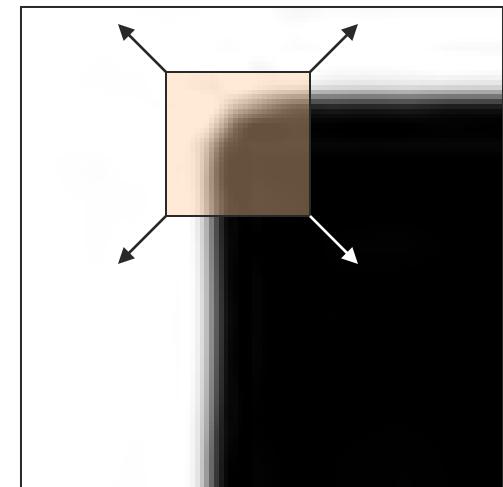
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give a *large change* in intensity



“flat” region:  
no change in  
all directions



“edge”:  
no change  
along the edge  
direction



“corner”:  
significant  
change in all  
directions



# Error function



Change of intensity for the shift  $[u, v]$ :

$$E(u, v) = \sum_{x, y} w(x, y)[I(x + u, y + v) - I(x, y)]$$

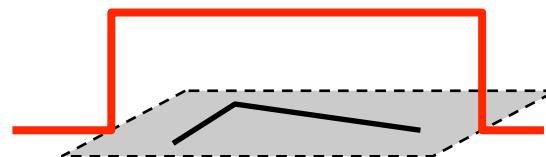
↑  
Error  
function

↑  
Window  
function

↑  
Shifted  
intensity

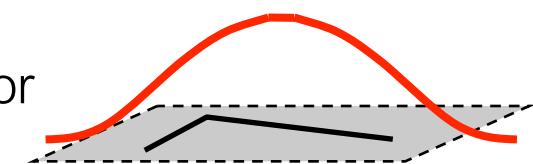
↑  
Intensity

Window function  $w(x, y) =$



1 in window, 0 outside

or



Gaussian



# Error function approximation



Change of intensity for the shift  $[u, v]$ :

$$E(u, v) = \sum_{x,y} w(x, y) [I(x + u, y + v) - I(x, y)]$$

First-order Taylor expansion of  $I(x, y)$  about  $(0, 0)$   
(bilinear approximation for small shifts)



# Bilinear approximation



For small shifts  $[u, v]$  we have a ‘bilinear approximation’:

Change in  
appearance for a  
shift  $[u, v]$

$$E(u, v) \cong [u, v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

where  $M$  is a  $2 \times 2$  matrix computed from image derivatives:

‘second moment’ matrix  
‘structure tensor’

$$M = \sum_{x,y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



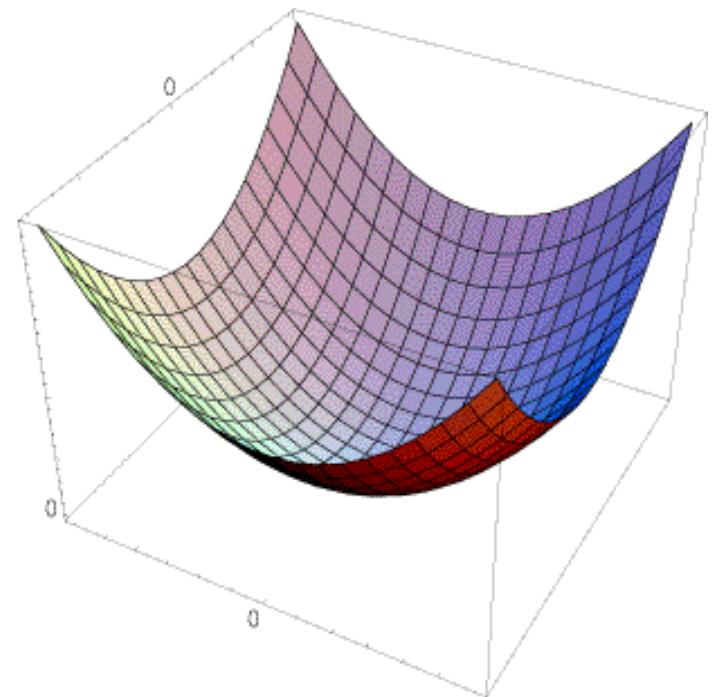
# Visualization of a quadratic



The surface  $E(u,v)$  is locally approximated by a quadratic form

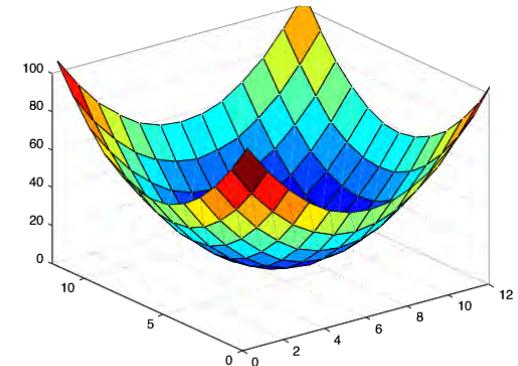
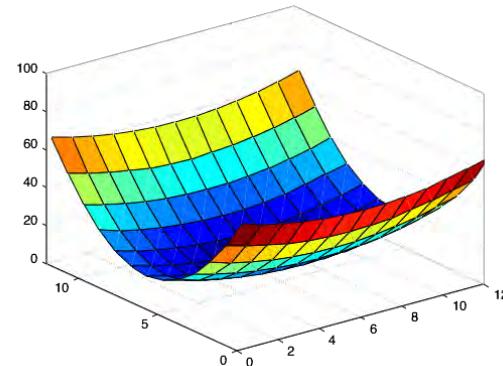
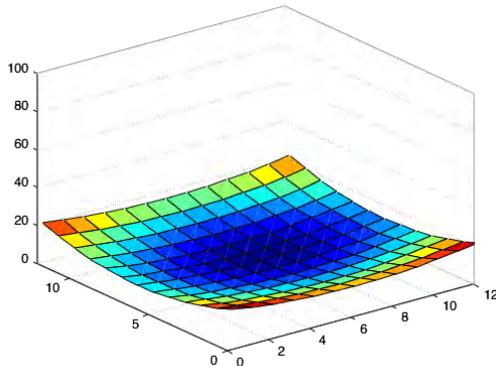
$$E(u,v) \approx [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$





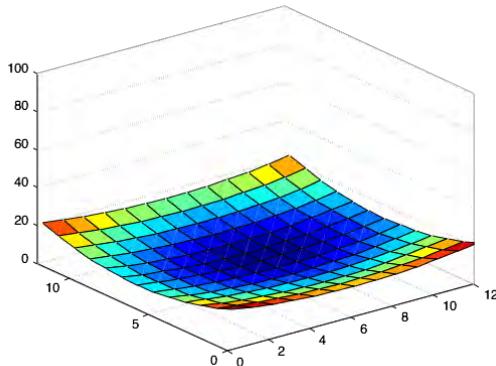
*Which error surface indicates a good image feature?*



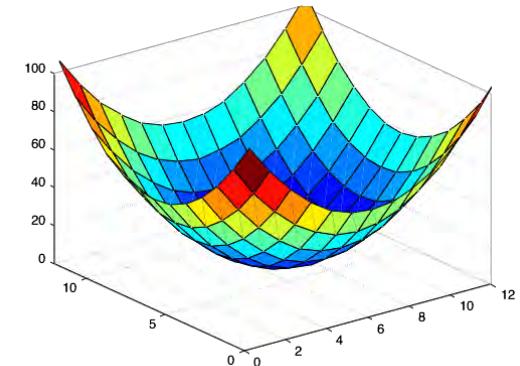
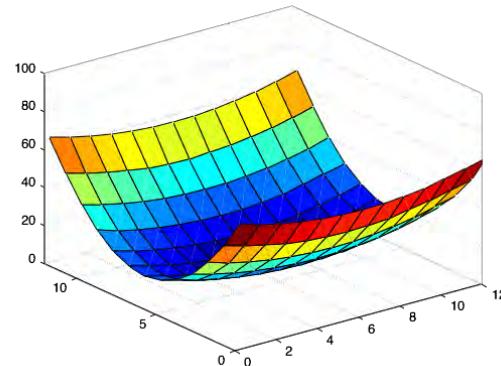
*What kind of image patch do these surfaces represent?*



## Which error surface indicates a good image feature?

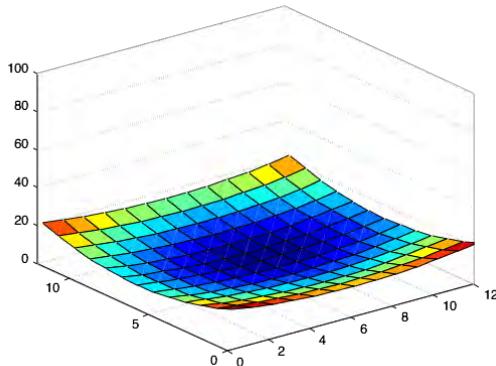


flat

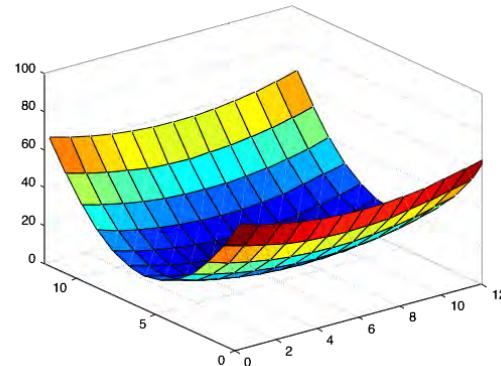




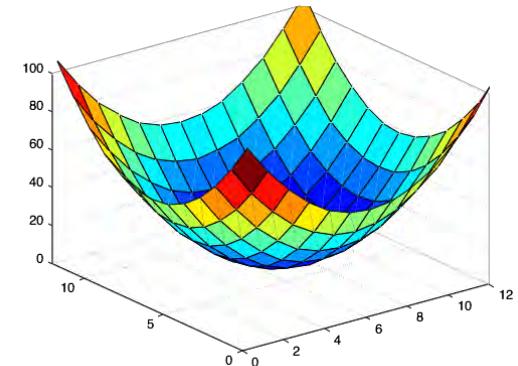
## Which error surface indicates a good image feature?



flat

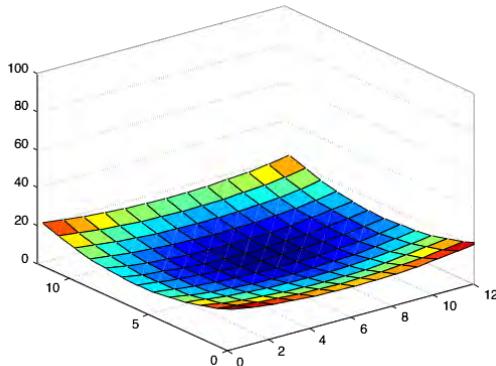


edge  
'line'

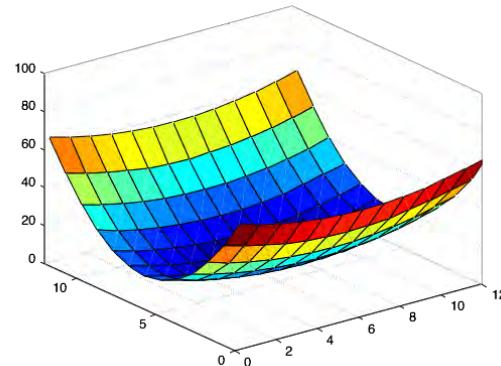




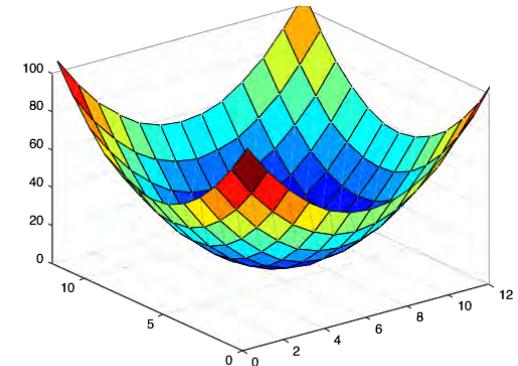
## Which error surface indicates a good image feature?



flat



edge  
'line'



corner  
'dot'



# Visualization as an ellipse



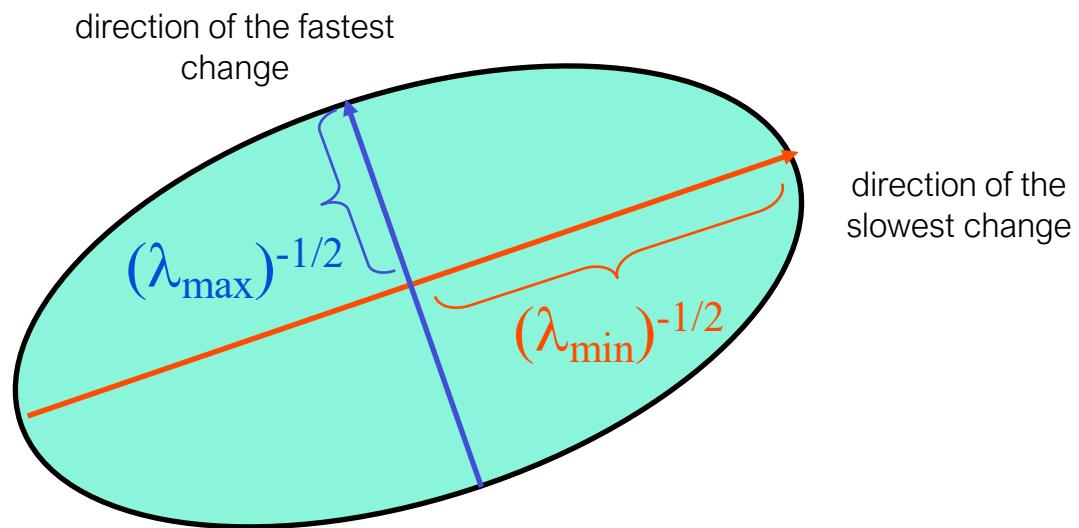
Since  $M$  is symmetric, we have

$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

We can visualize  $M$  as an ellipse with axis lengths determined by the eigenvalues and orientation determined by  $R$

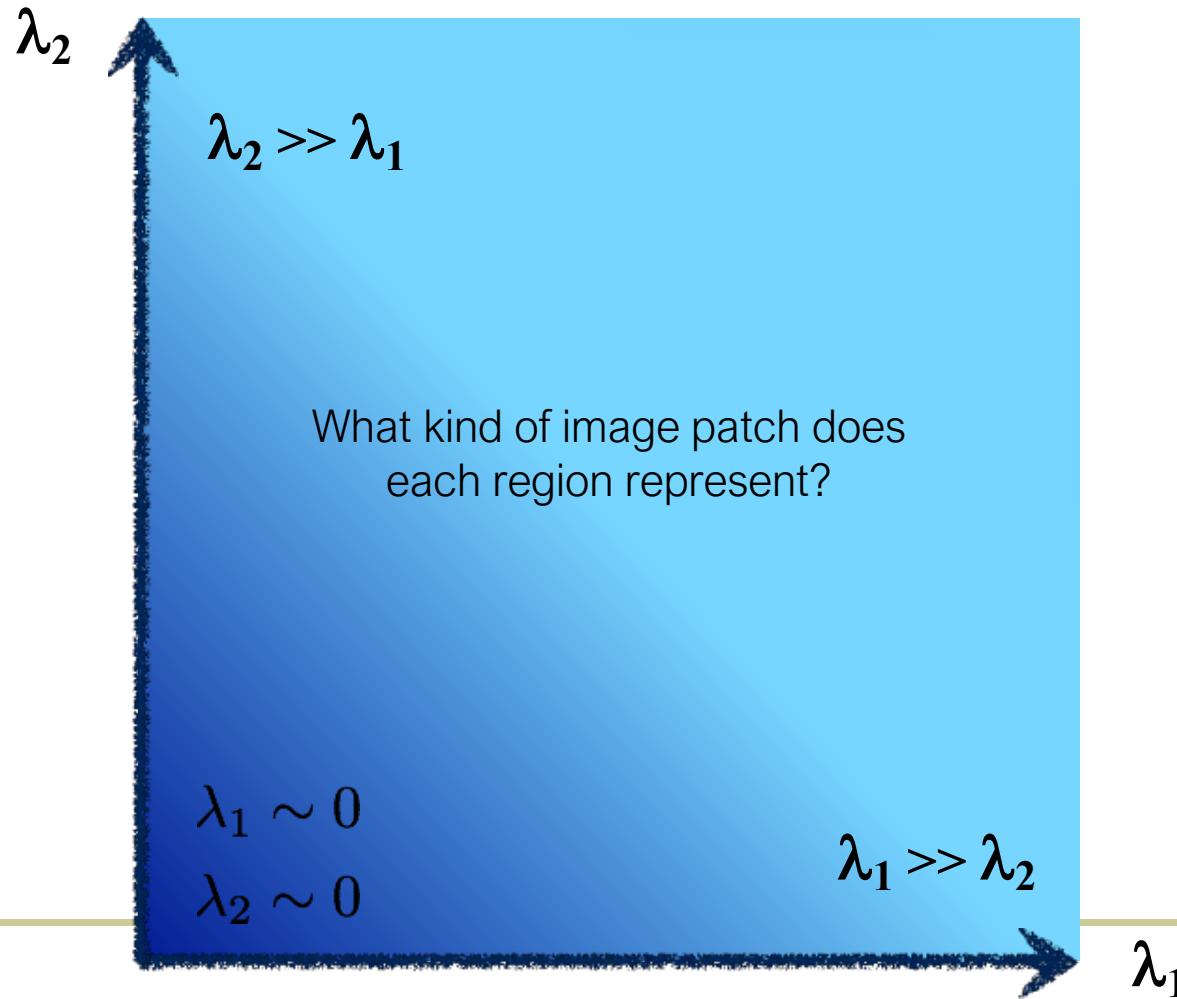
Ellipse equation:

$$[u \ v] M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$



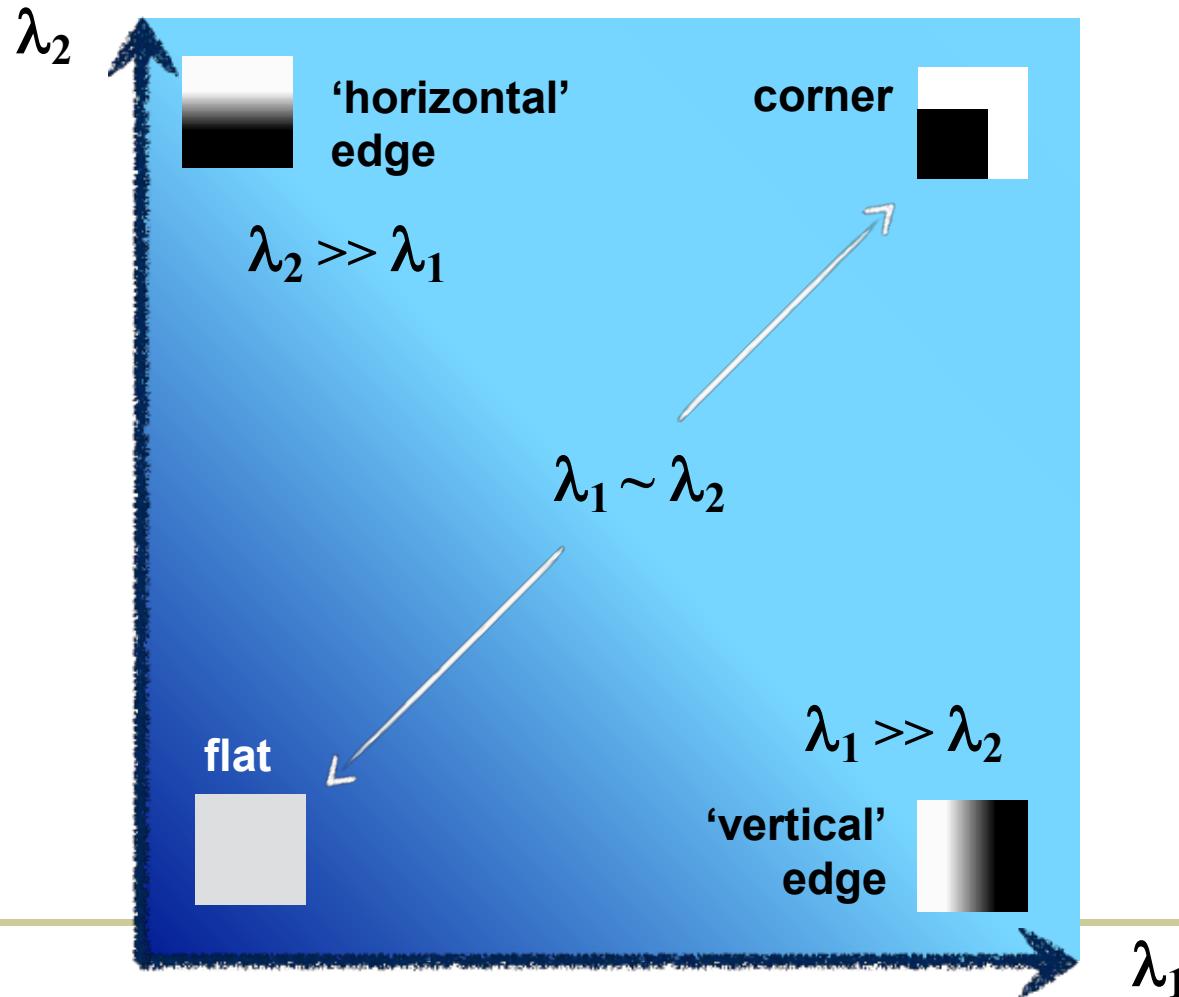


# interpreting eigenvalues



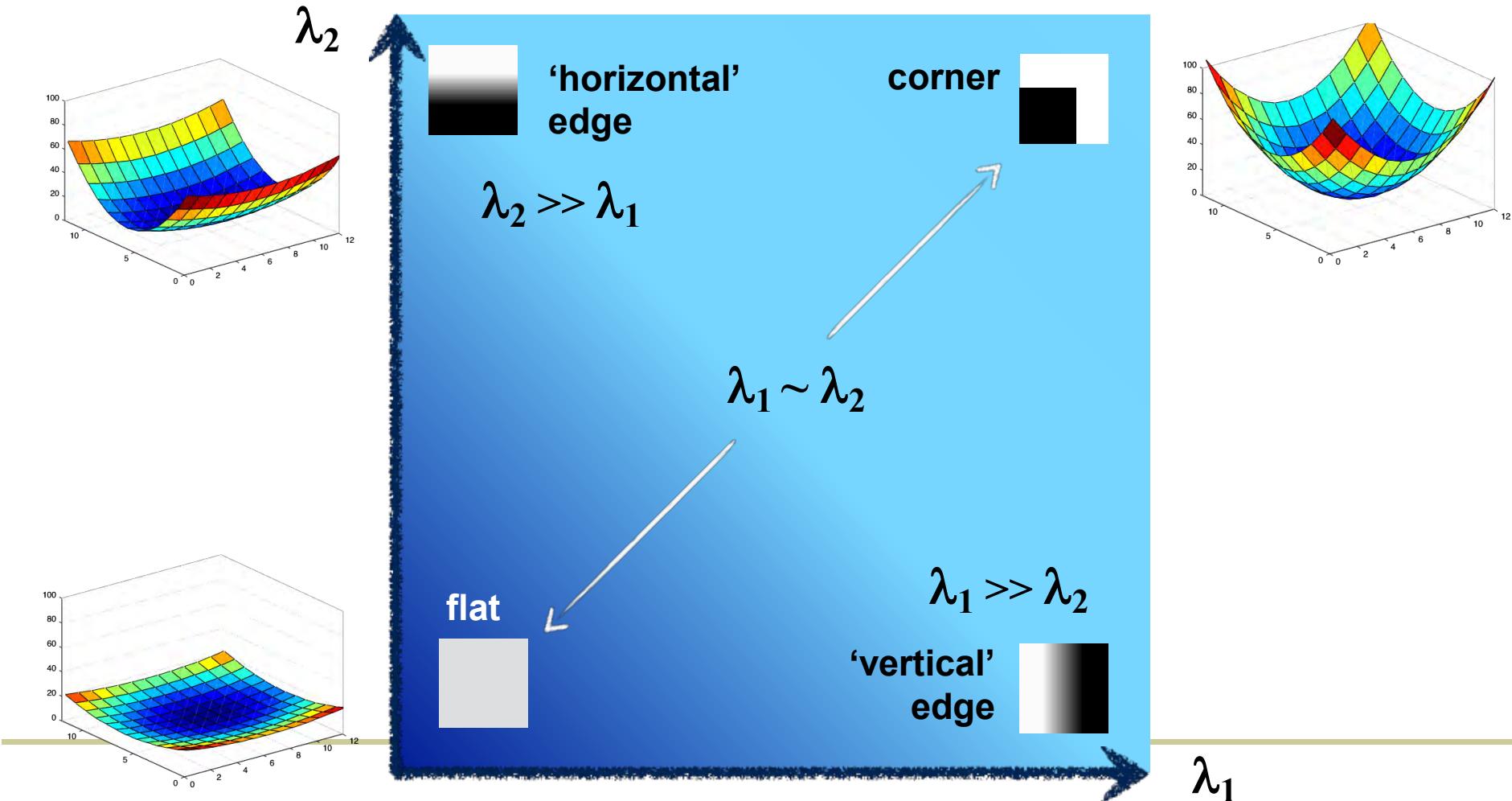


# interpreting eigenvalues



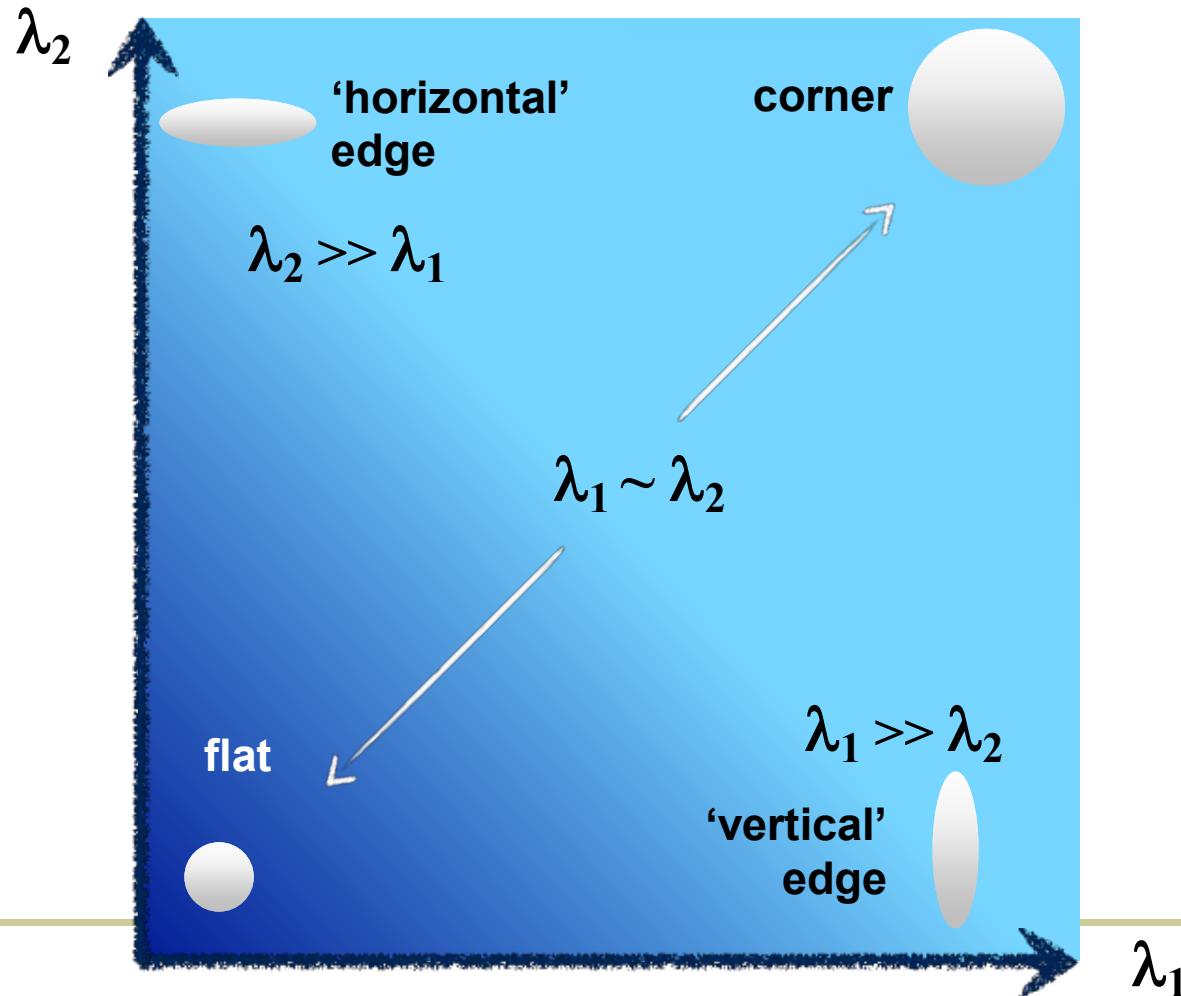


# interpreting eigenvalues





# interpreting eigenvalues



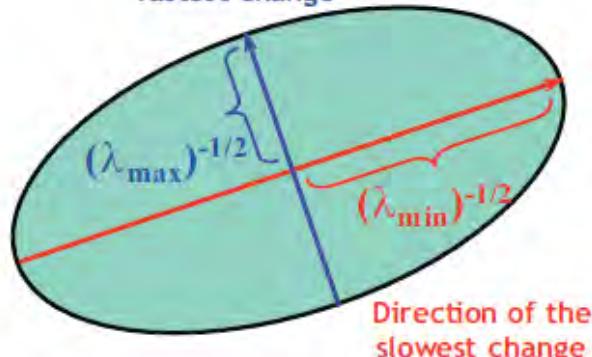


# Explanation of Harris Criterion



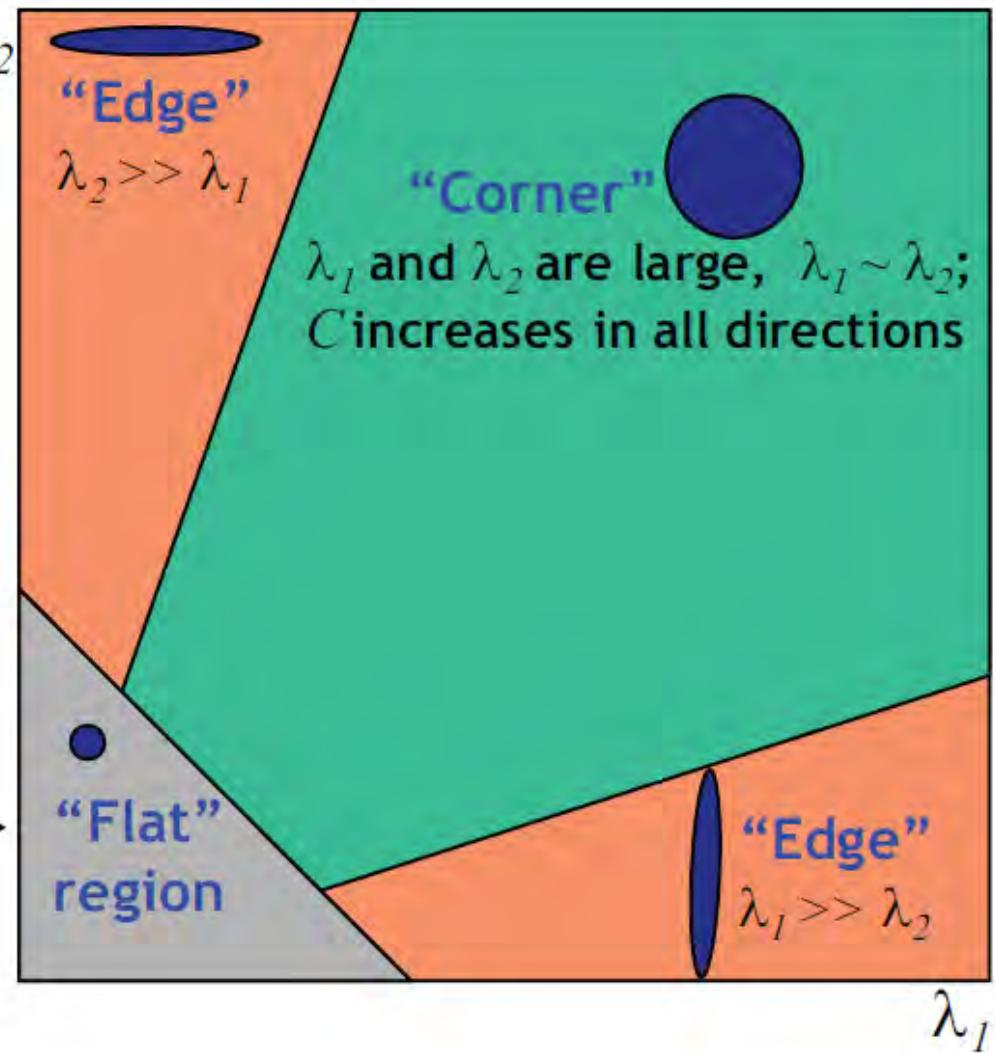
$$C = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

Direction of the  
fastest change



Direction of the  
slowest change

$\lambda_1$  and  $\lambda_2$  are small;  
 $C$  is almost constant  
in all directions





# Harris Detector: Criteria



$$M = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

1. Want large eigenvalues, and small ratio  $\frac{\lambda_1}{\lambda_2} < t$

2. We know

$$\det M = \lambda_1 \lambda_2$$

$$\text{trace } M = \lambda_1 + \lambda_2$$

3. Leads to

$$\det M - k \cdot \text{trace}^2(M) > t$$

( $k$  :empirical constant,  $k = 0.04\text{-}0.06$ )

Nice brief derivation on wikipedia



# Harris Detector: Criteria



Harris & Stephens (1988)

$$R = \det(M) - \kappa \text{trace}^2(M)$$

Kanade & Tomasi (1994)

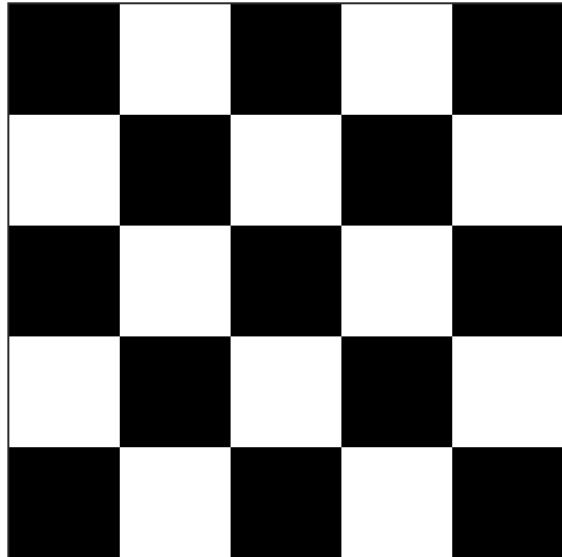
$$R = \min(\lambda_1, \lambda_2)$$

Nobel (1998)

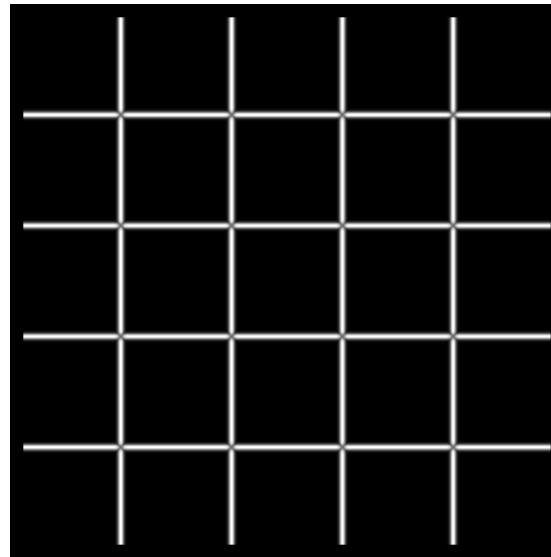
$$R = \frac{\det(M)}{\text{trace}(M) + \epsilon}$$



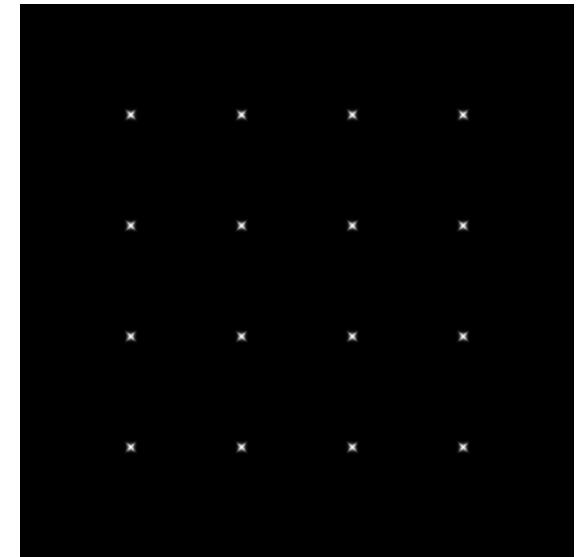
# Harris Detector: Criteria



$I$



$\lambda_{\max}$



$\lambda_{\min}$

Yet another option:

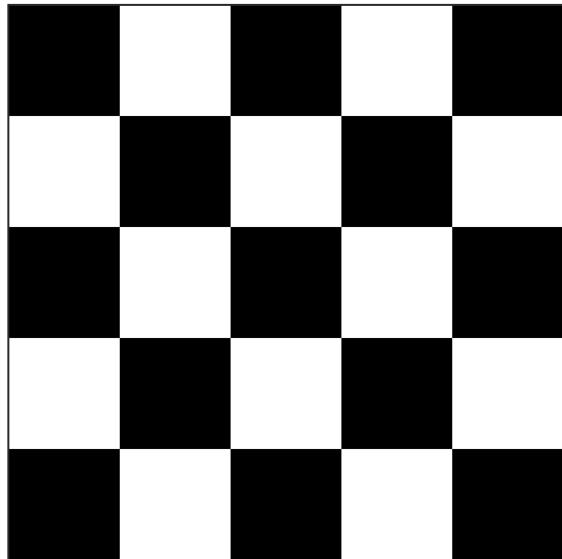
$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

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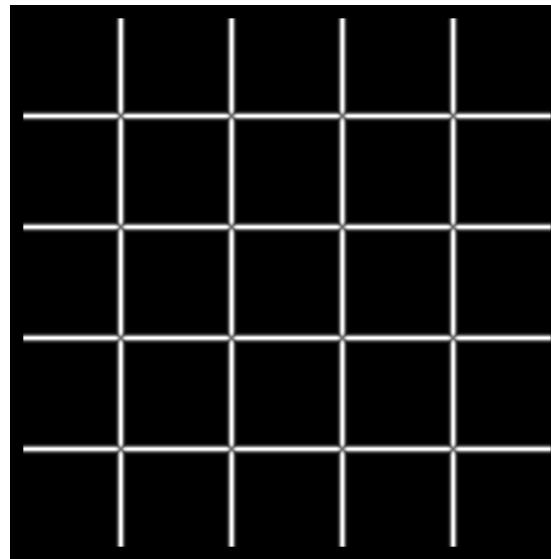
How do you write this equivalently  
using determinant and trace?



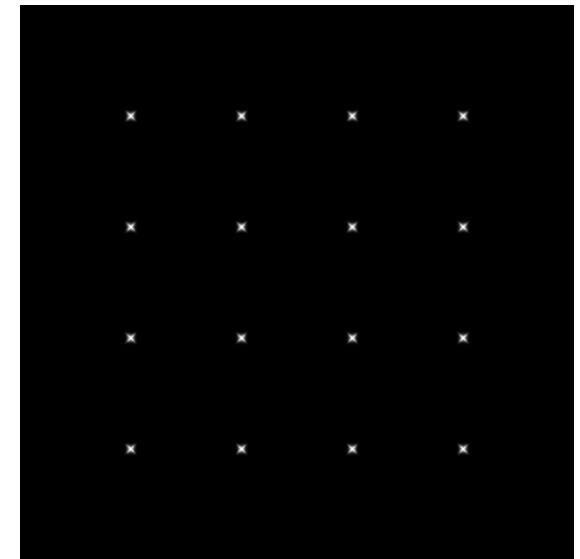
# Harris Detector: Criteria



$I$



$\lambda_{\max}$



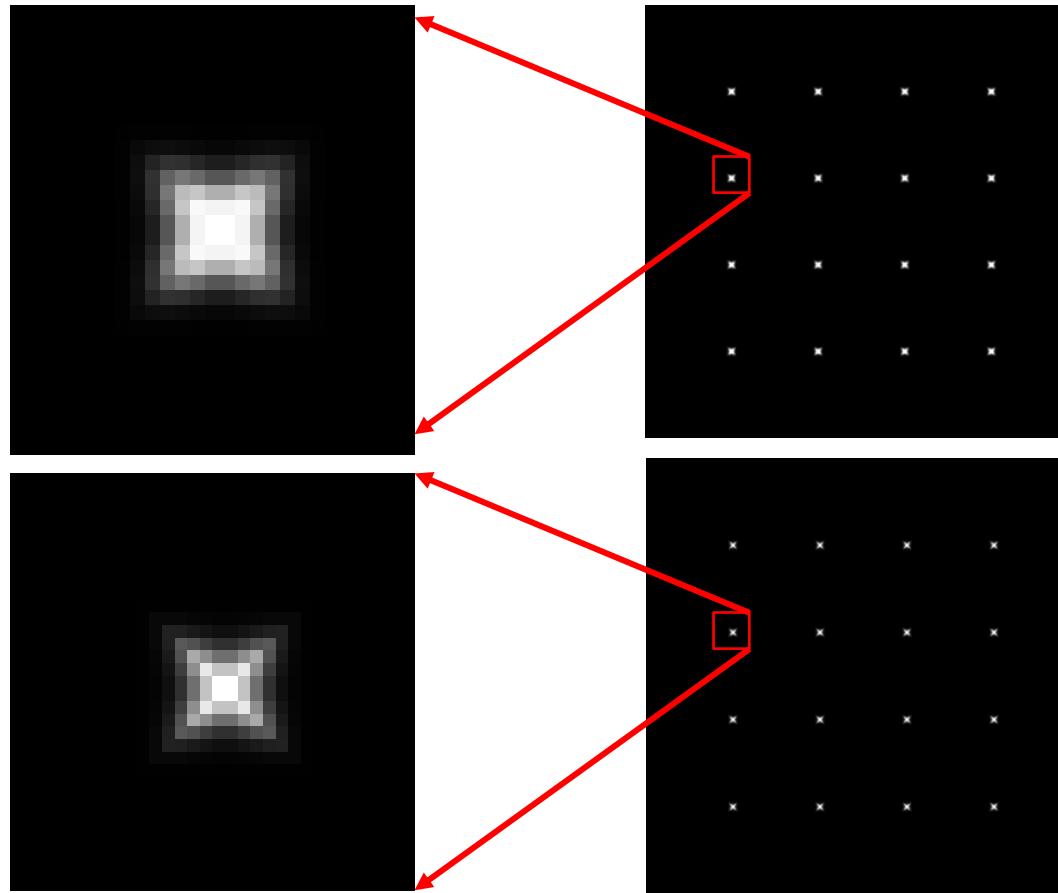
$\lambda_{\min}$

Yet another option:

$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} = \frac{\text{determinant}(H)}{\text{trace}(H)}$$



# Different criteria

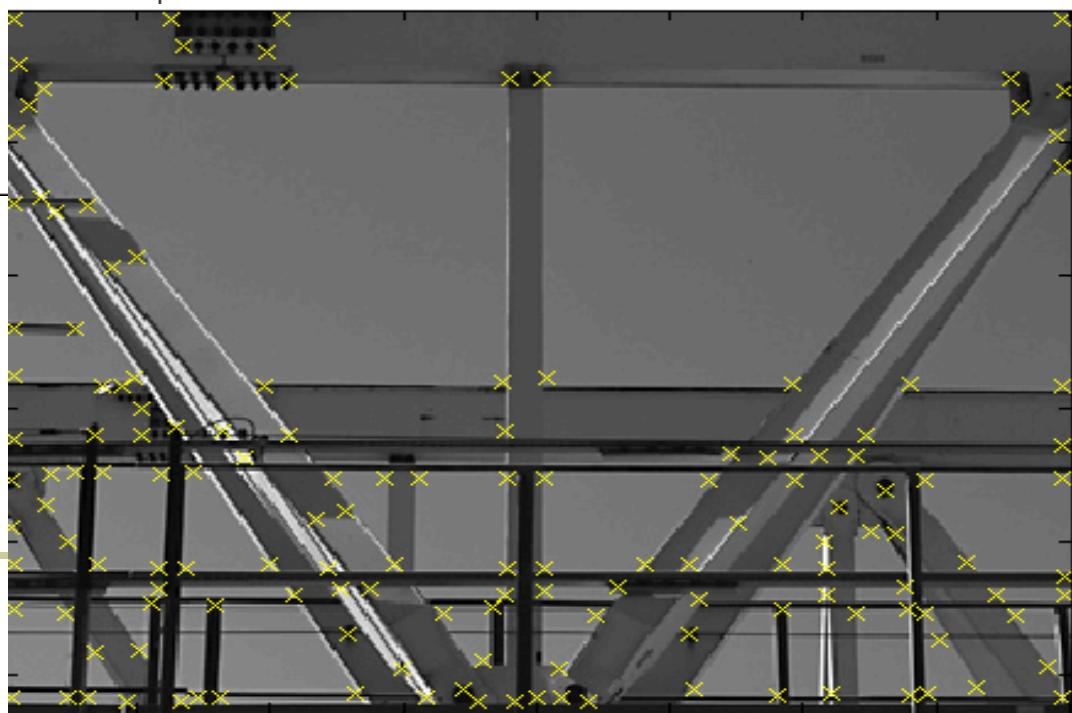
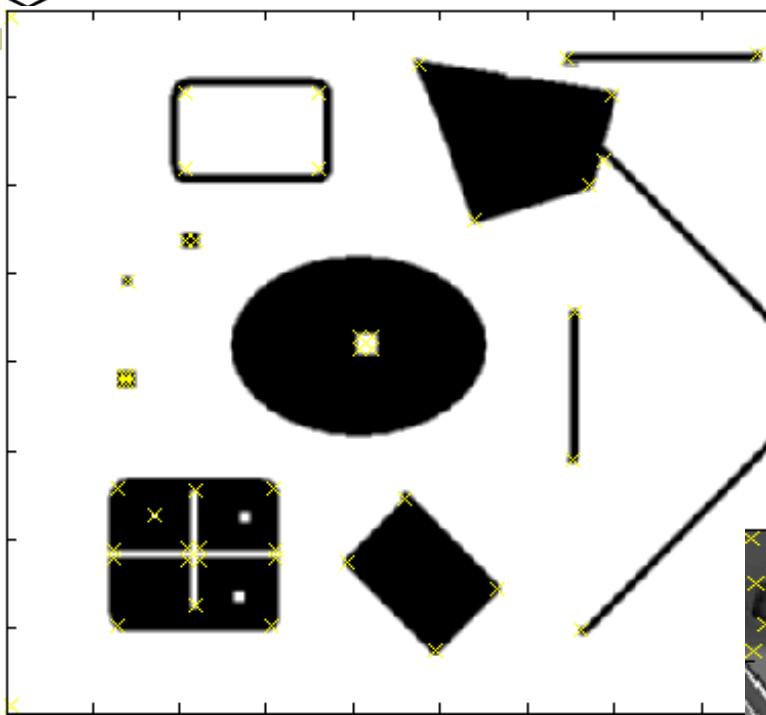


Harris criterion

$\lambda_{\min}$



# Harris Detector – Responses [Harris88]



*Effect:* A very precise corner detector.



# Harris Detector - Responses [Harris88]



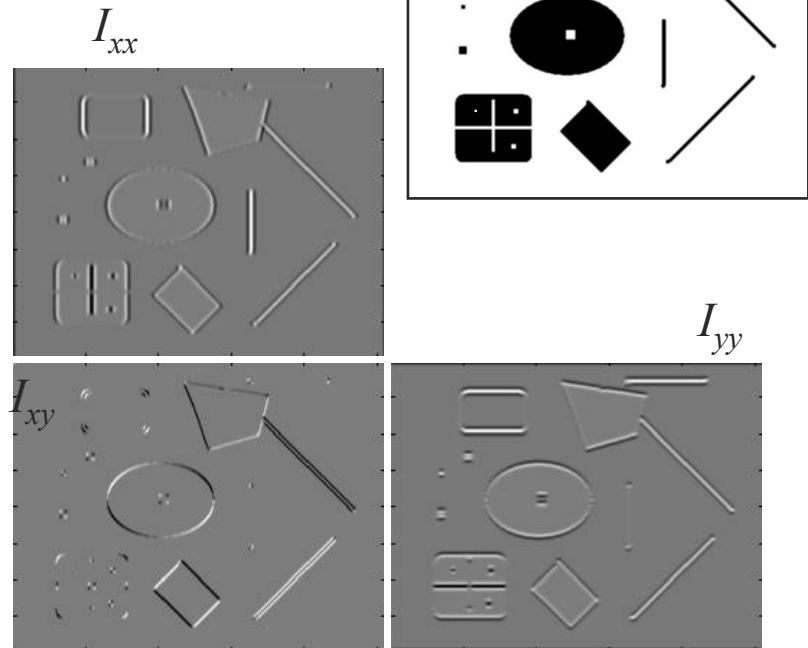


# Hessian Detector [Beaudet78]



## ■ Hessian determinant

$$Hessian(I) = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}$$



***Intuition:*** Search for strong curvature in two orthogonal directions



# Hessian Detector [Beaudet78]



## ■ Hessian determinant

$$Hessian(x, \sigma) = \begin{bmatrix} I_{xx}(x, \sigma) & I_{xy}(x, \sigma) \\ I_{xy}(x, \sigma) & I_{yy}(x, \sigma) \end{bmatrix}$$

$$\det M = \lambda_1 \lambda_2$$

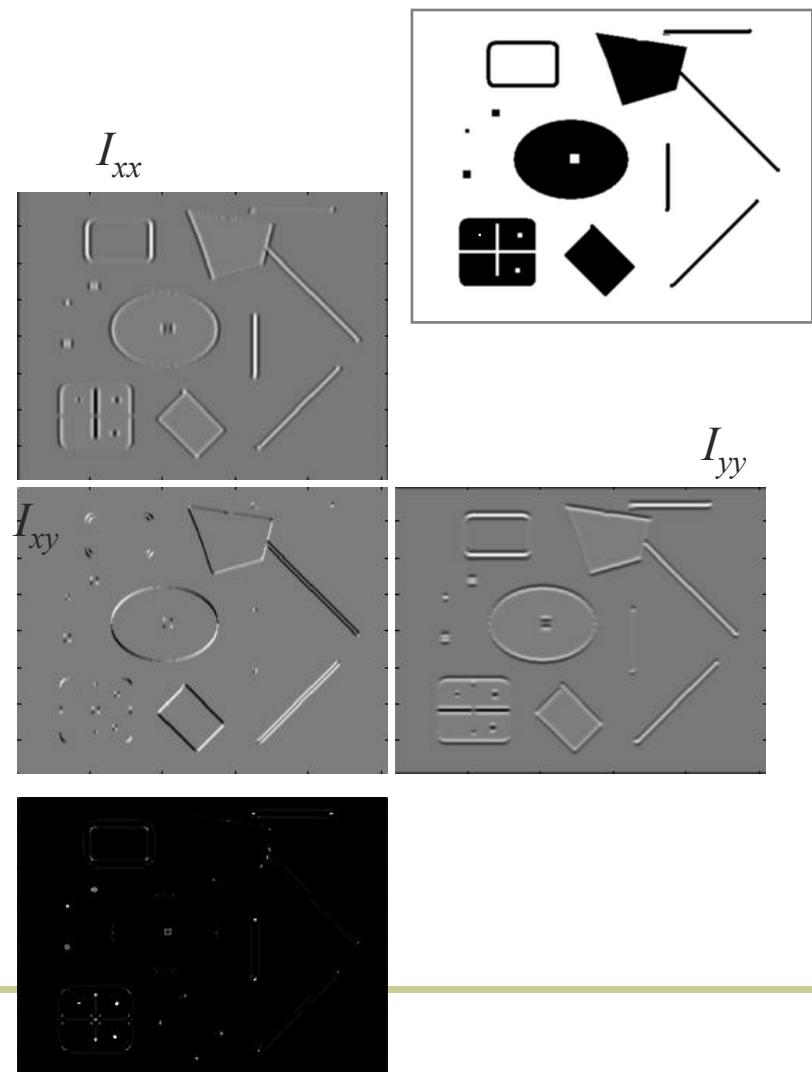
$$\text{trace } M = \lambda_1 + \lambda_2$$

Find maxima of determinant

$$\det(Hessian(x)) = I_{xx}(x)I_{yy}(x) - I_{xy}^2(x)$$

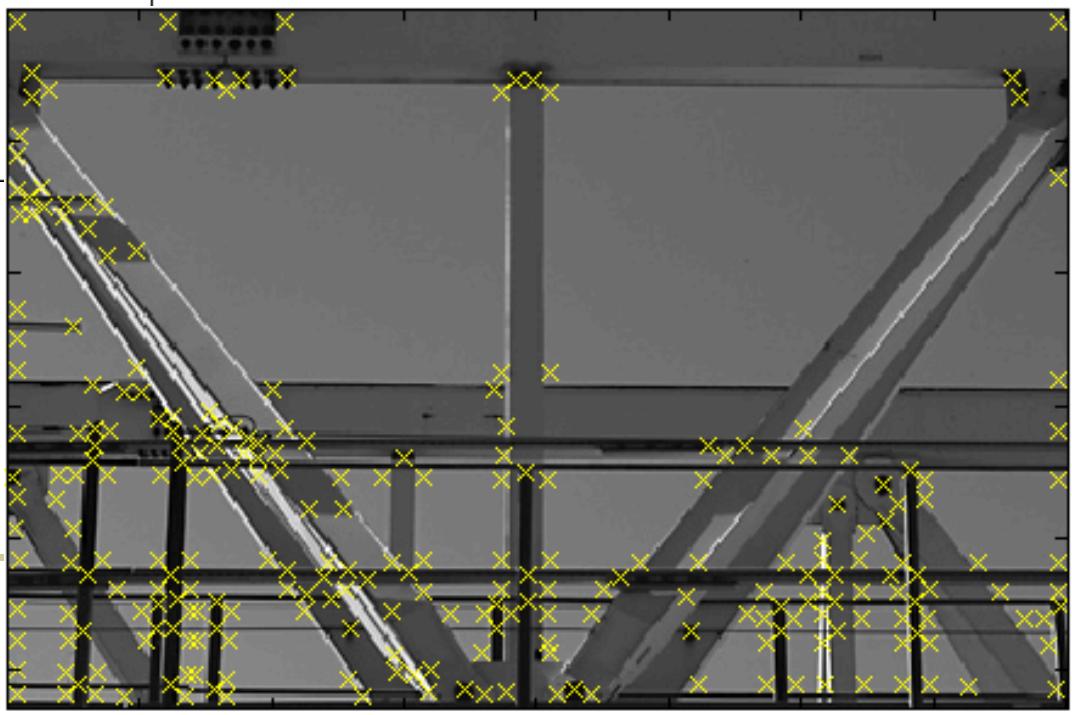
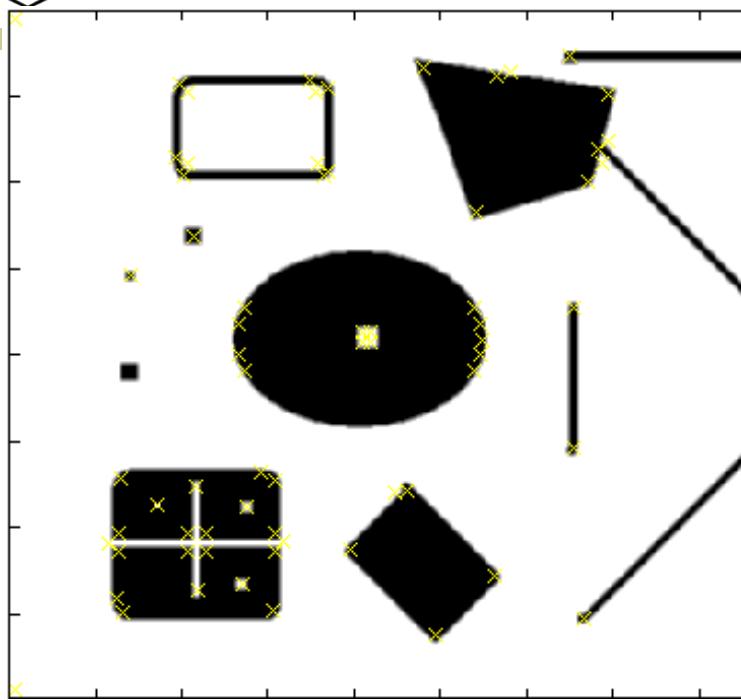
In Matlab:

$$I_{xx}.*I_{yy} - (I_{xy})^2$$





# Hessian Detector – Responses [Beaudet78]



*Effect:* Responses mainly on corners and strongly textured areas.

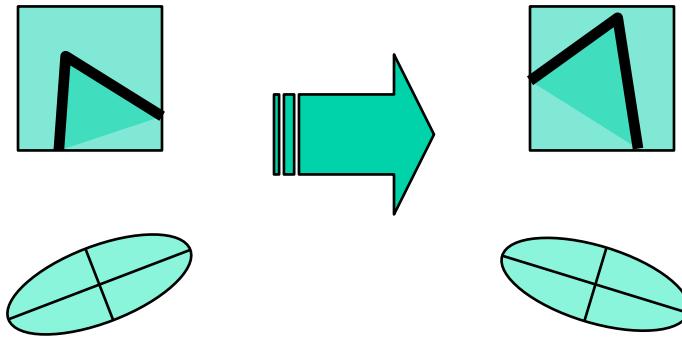


# Hessian Detector – Responses [Beaudet78]





# Harris corner response is invariant to rotation



Ellipse rotates but its shape  
**(eigenvalues)** remains the same

**Corner response R is invariant to image rotation**



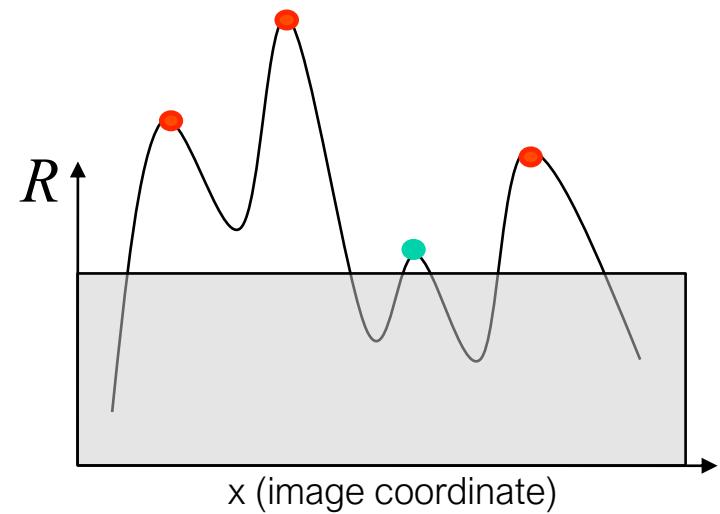
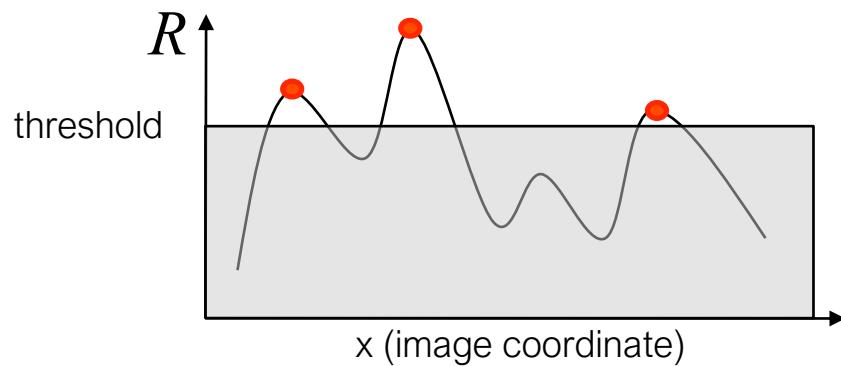
# Harris corner response is invariant to intensity changes



Partial invariance to *affine intensity* change

Only derivatives are used => invariance to intensity shift  $I \rightarrow I + b$

Intensity scale:  $I \rightarrow a I$

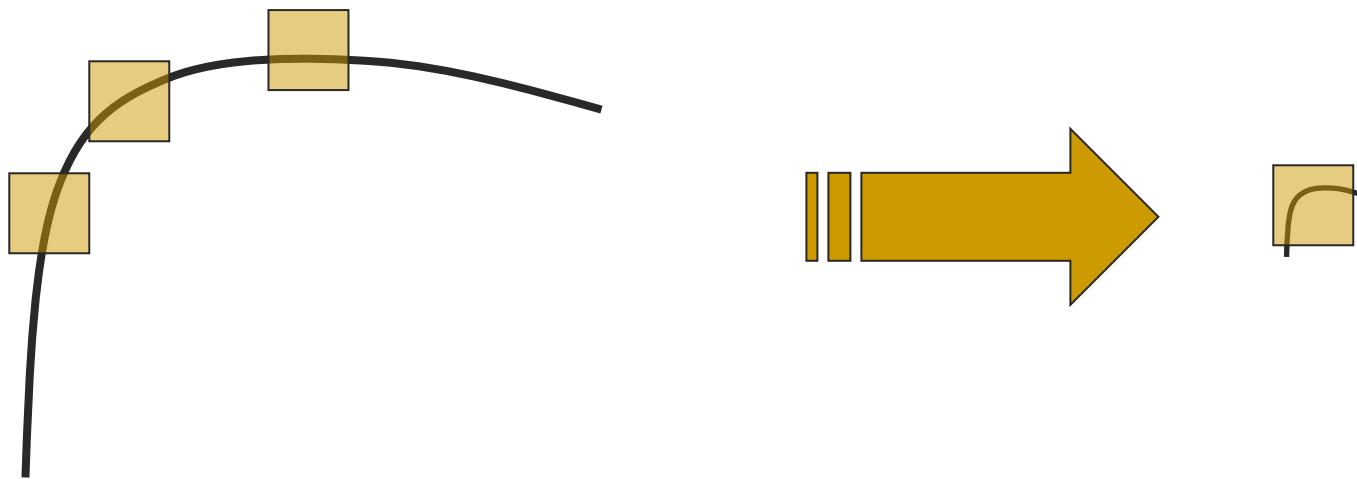




# Scale invariance?



- Scale invariant? No



All points will be  
classified as **edges**

**Corner !**



# Today's Class



- Introduction to correspondence and alignment
- Overview of interest points
  - Matching pipeline
  - Repeatable & Distinctive
- Keypoint Localization
  - Harris detector
  - Hessian detector
- Scale invariant region selection
  - Automatic scale selection
  - Laplacian of Gaussian (LoG) & Difference of Gaussian (DoG)
  - Combinations: Harris-Laplacian & Hessian-Laplacian



# From points to regions



- The Harris and Hessian operators define interest points.

- Precise localization
  - High repeatability



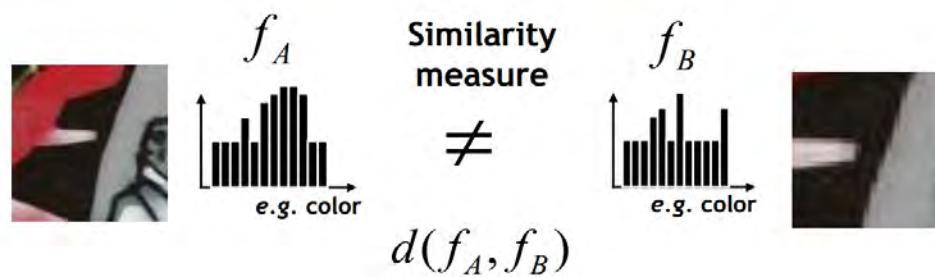
- In order to compare those points, we need to compute a descriptor over a region.
  - How can we define such a region in a scale invariant manner?
- *I.e. how can we detect scale invariant interest regions?*



# Naïve approach: exhaustive search



- Multi-scale procedure
  - Compare descriptors while varying the patch size

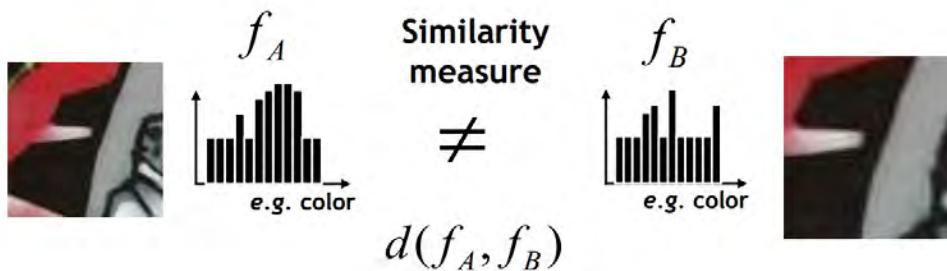




# Naïve approach: exhaustive search



- Multi-scale procedure
  - Compare descriptors while varying the patch size

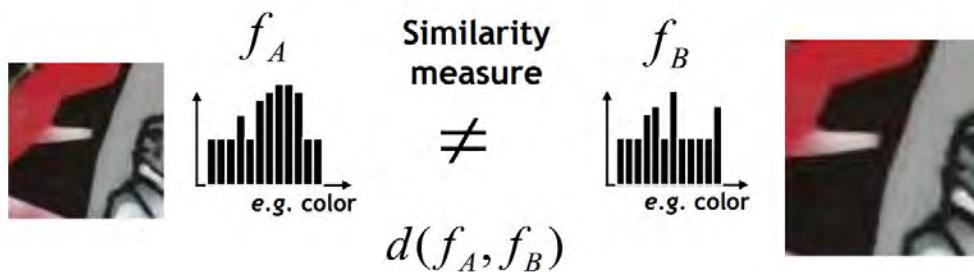




# Naïve approach: exhaustive search



- Multi-scale procedure
  - Compare descriptors while varying the patch size

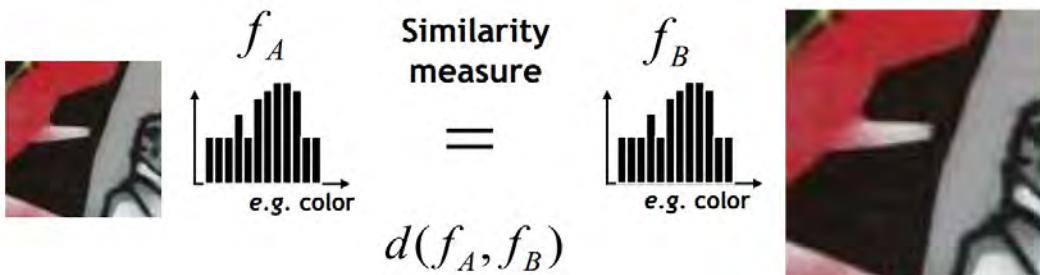




# Naïve approach: exhaustive search



- Multi-scale procedure
  - Compare descriptors while varying the patch size

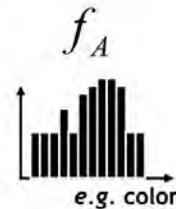




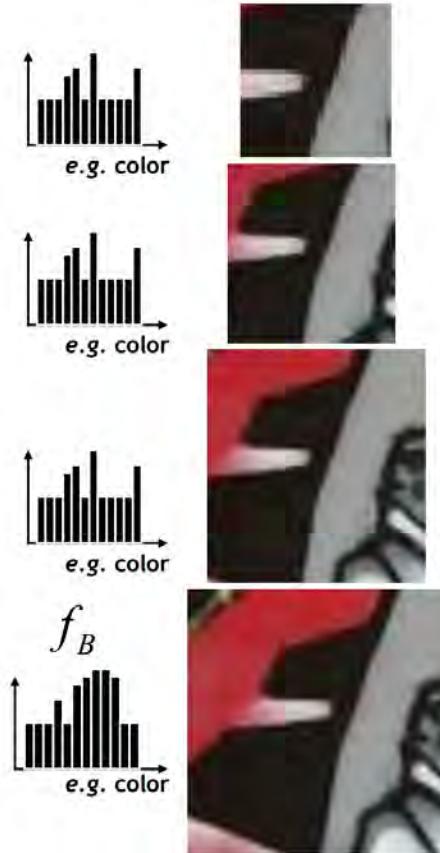
# Naïve approach: exhaustive search



- Comparing descriptors while varying the patch size
  - Computationally inefficient
  - Inefficient but possible for matching
  - Prohibitive for retrieval in large databases
  - Prohibitive for recognition



$$\text{Similarity measure} = d(f_A, f_B)$$

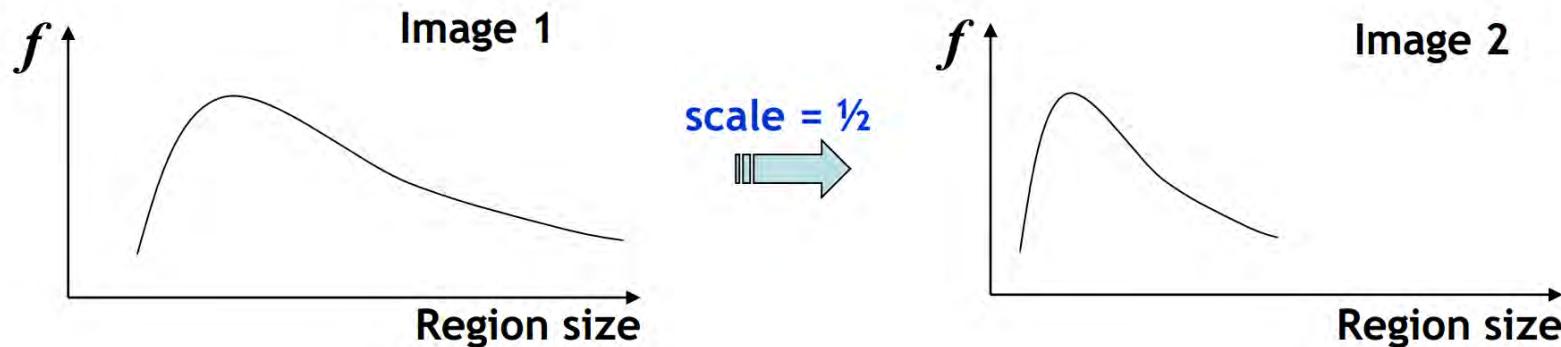




# Automatic scale selection



- Solution:
  - Design a function on the region, which is “scale invariant” (*the same for corresponding regions, even if they are at different scales*)  
**Example:** average intensity. For corresponding regions (even of different sizes) it will be the same.
  - For a point in one image, we can consider it as a function of region size (patch width)



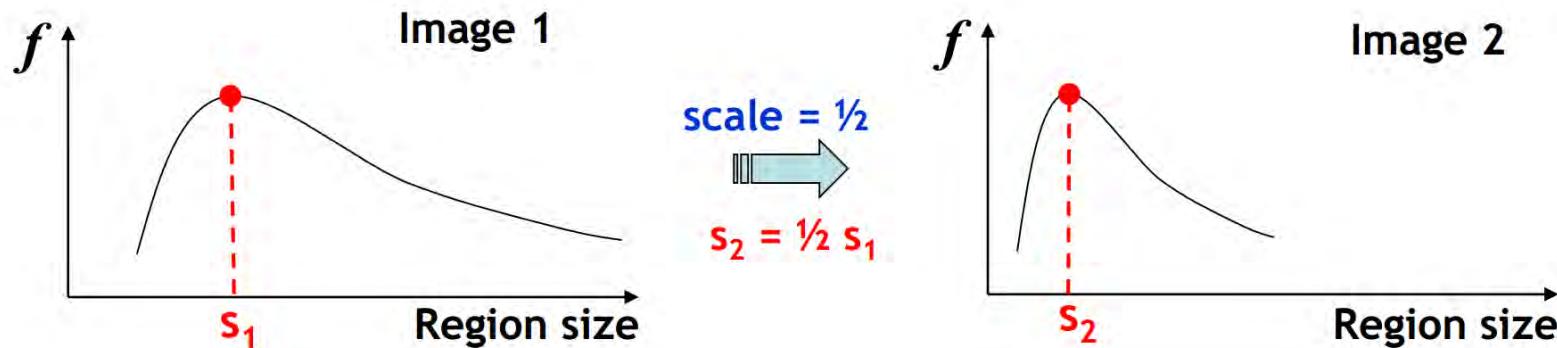


# Automatic scale selection



- Common approach:
  - Take a local maximum of this function.
  - Observation: region size for which the maximum is achieved should be *invariant* to image scale.

**Important: this scale invariant region size is found in each image independently!**

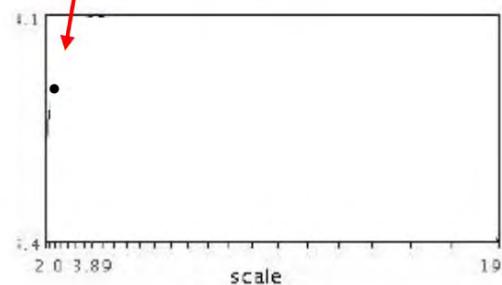




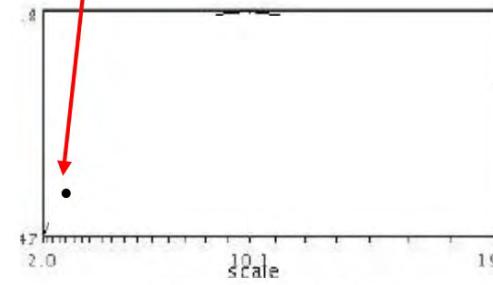
# Automatic scale selection



- Function responses for increasing scale (scale signature)



$$f(I_{i_1 \dots i_m}(x, \sigma))$$



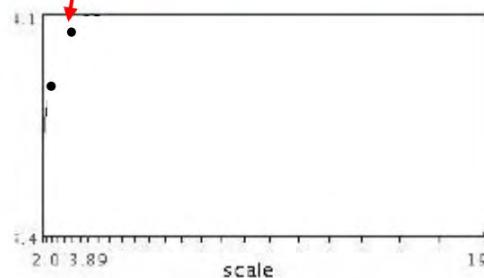
$$f(I_{i_1 \dots i_m}(x', \sigma))$$



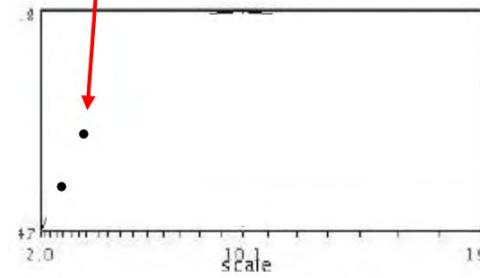
# Automatic scale selection



- Function responses for increasing scale (scale signature)



$$f(I_{i_1\dots i_m}(x, \sigma))$$



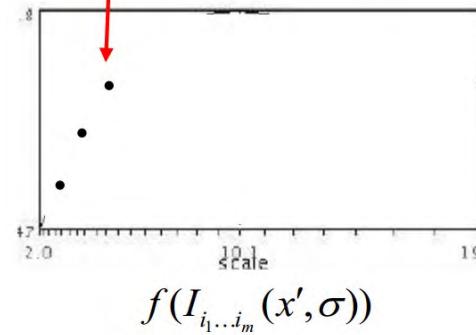
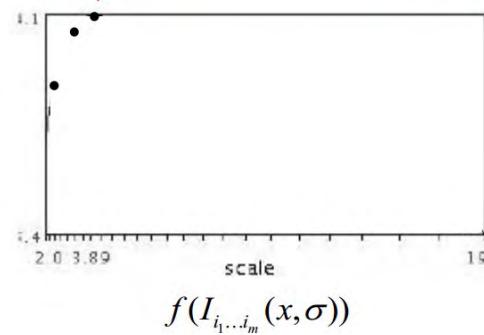
$$f(I_{i_1\dots i_m}(x', \sigma))$$



# Automatic scale selection



- Function responses for increasing scale (scale signature)

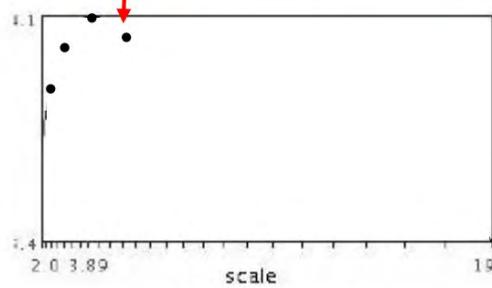
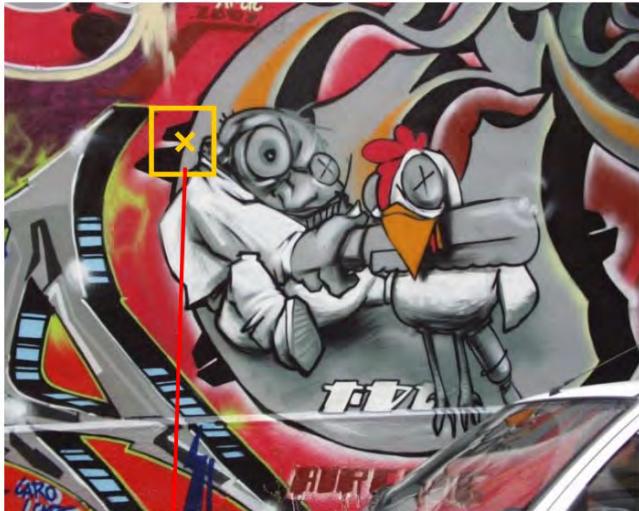




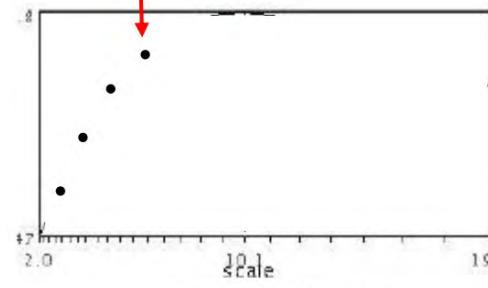
# Automatic scale selection



- Function responses for increasing scale (scale signature)



$$f(I_{i_1 \dots i_m}(x, \sigma))$$



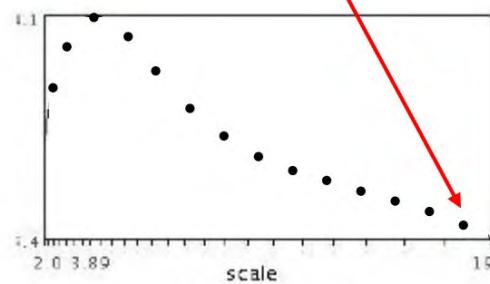
$$f(I_{i_1 \dots i_m}(x', \sigma))$$



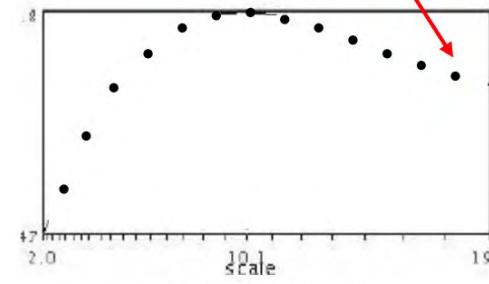
# Automatic scale selection



- Function responses for increasing scale (scale signature)



$$f(I_{i_1 \dots i_m}(x, \sigma))$$



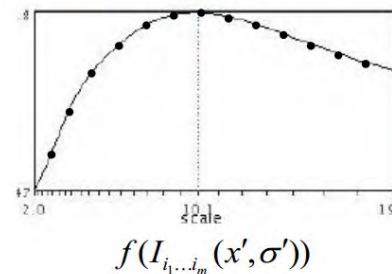
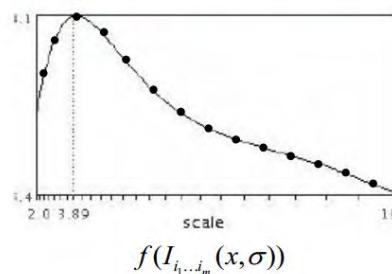
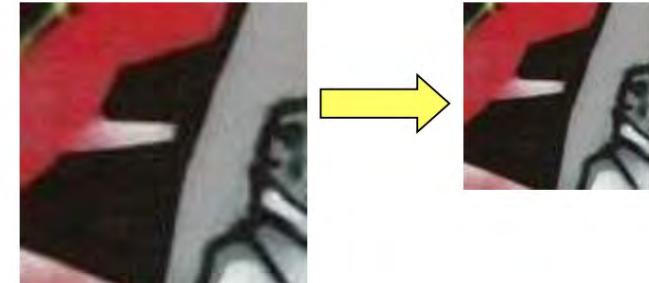
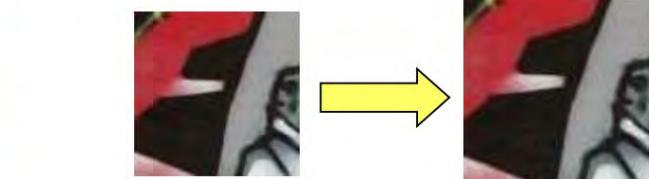
$$f(I_{i_1 \dots i_m}(x', \sigma))$$



# Automatic scale selection

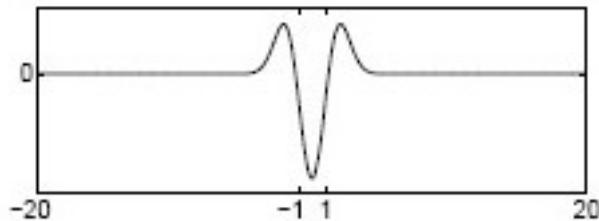


- Normalize: Rescale to fixed size

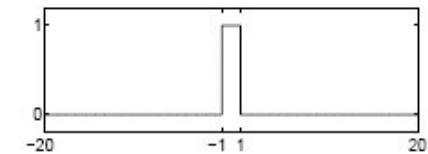
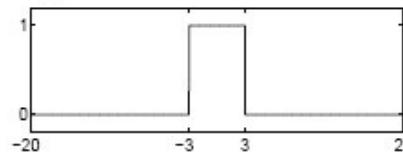
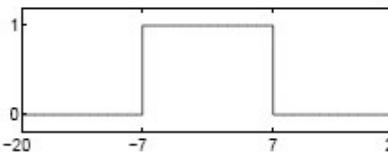
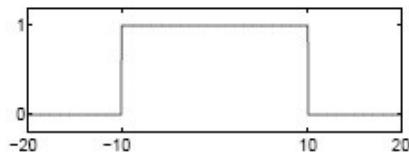




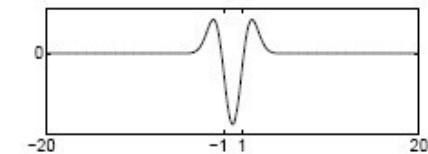
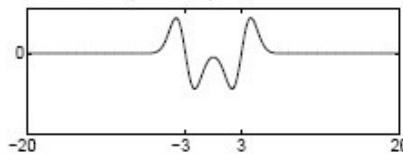
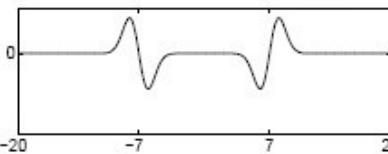
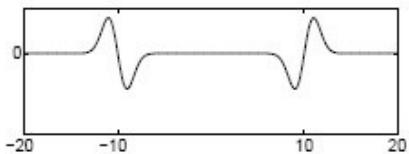
# What can be the “signature” function?



Original signal



Convolved with Laplacian ( $\sigma = 1$ )



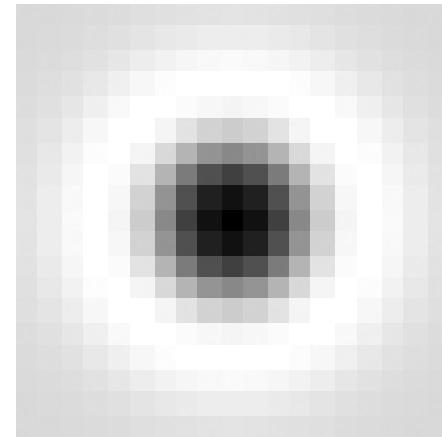
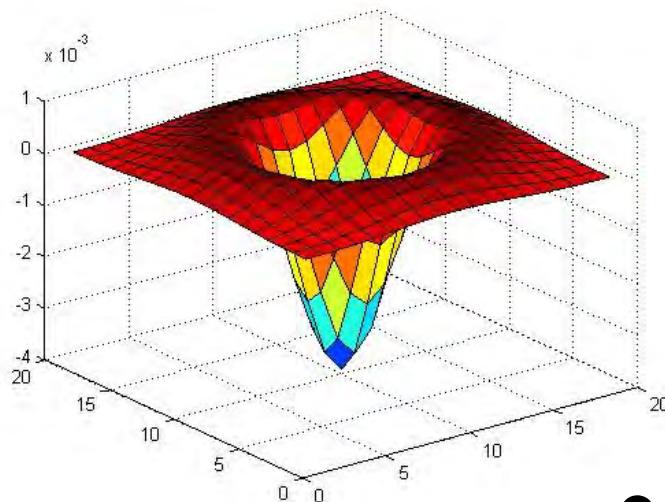
Highest response when the signal has the same **characteristic scale** as the filter



# Blob detection in 2D



- Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D



$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

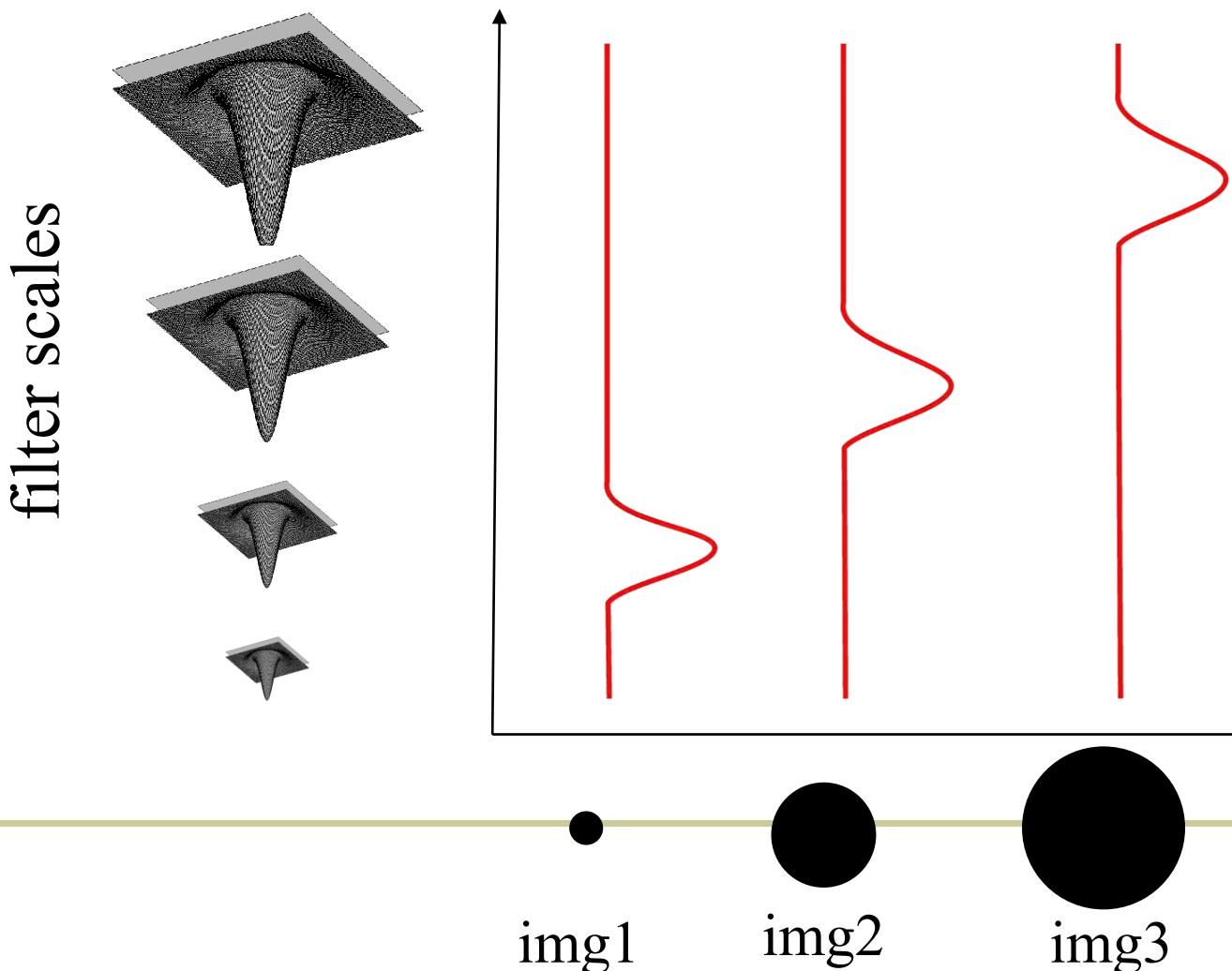


# Blob detection in 2D



- Laplacian-of-Gaussian = “blob” detector

$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

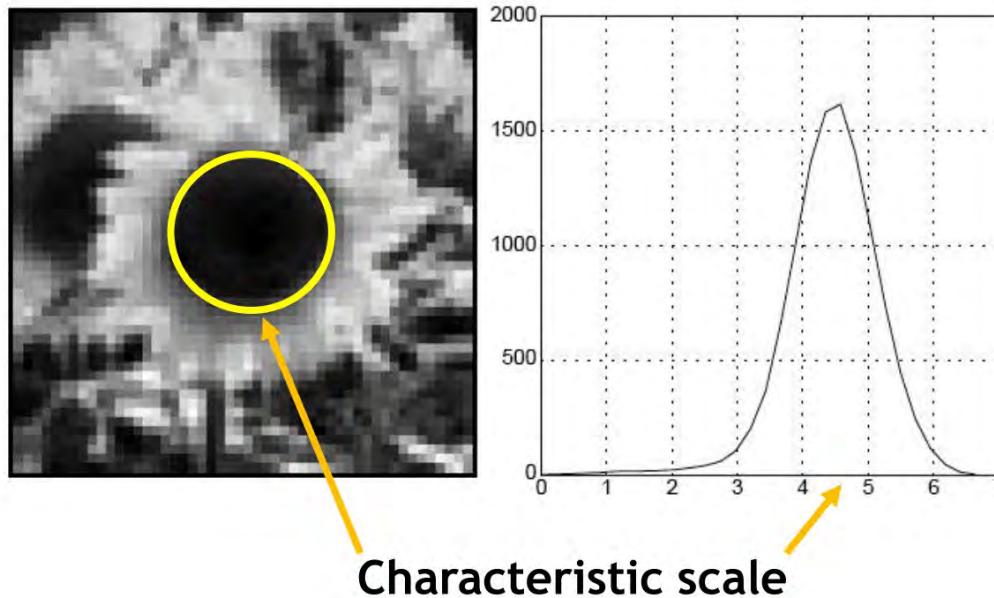




# Characteristic scale



- We define the *characteristic scale* as the scale that produces peak of Laplacian response



T. Lindeberg (1998). ["Feature detection with automatic scale selection."](#)  
*International Journal of Computer Vision* 30 (2): pp 77--116.



# Example

Original image at  
 $\frac{3}{4}$  the size



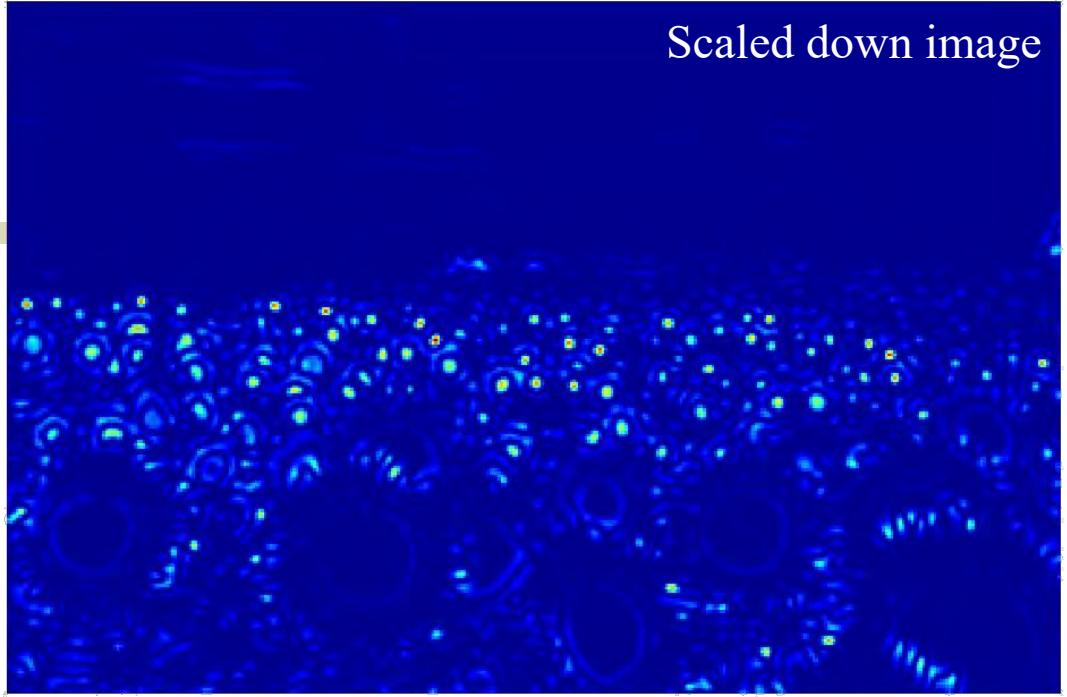
What happened  
when you applied  
different Laplacian  
filters?



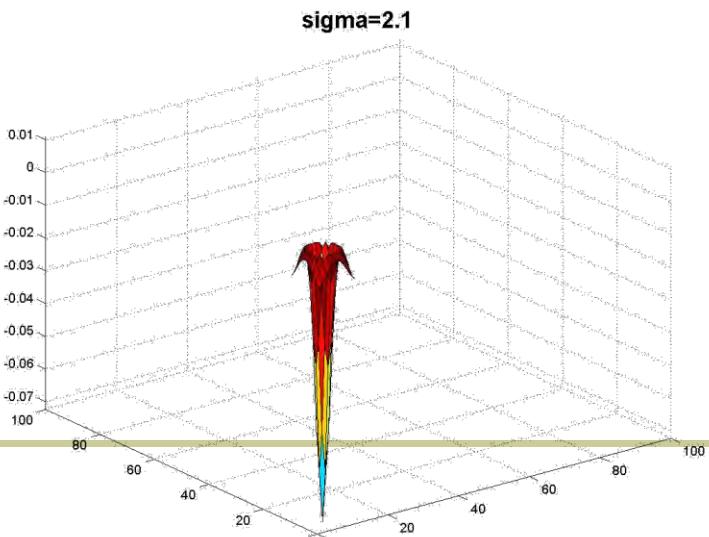


Original image at  
3/4 the size

Scaled down image



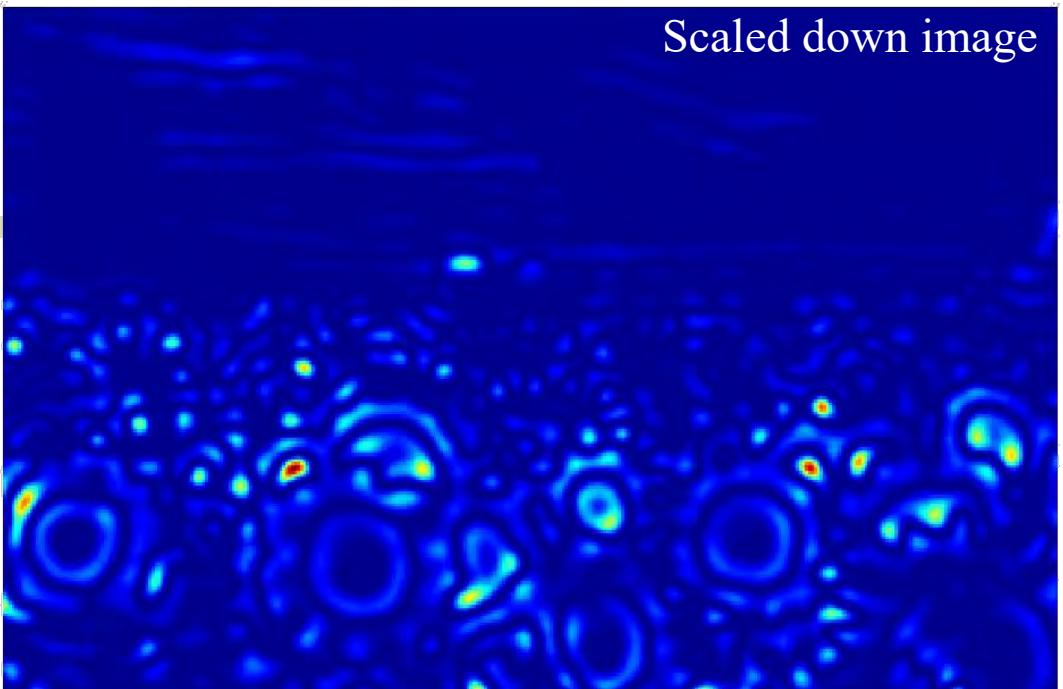
Original ima



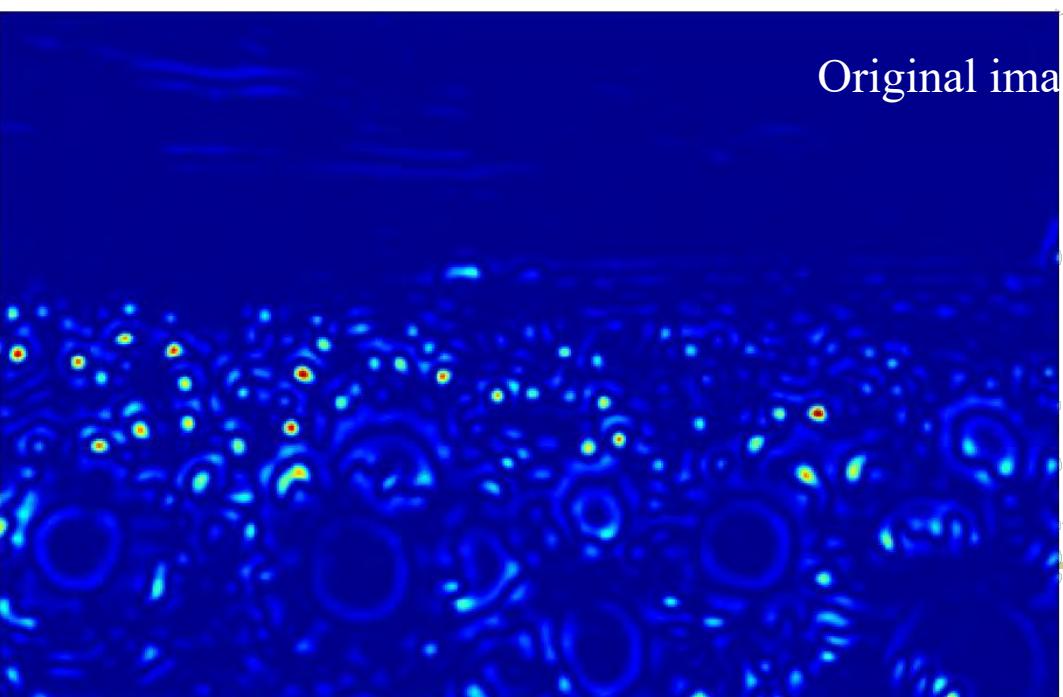
Slide credit: Kristen Grauman



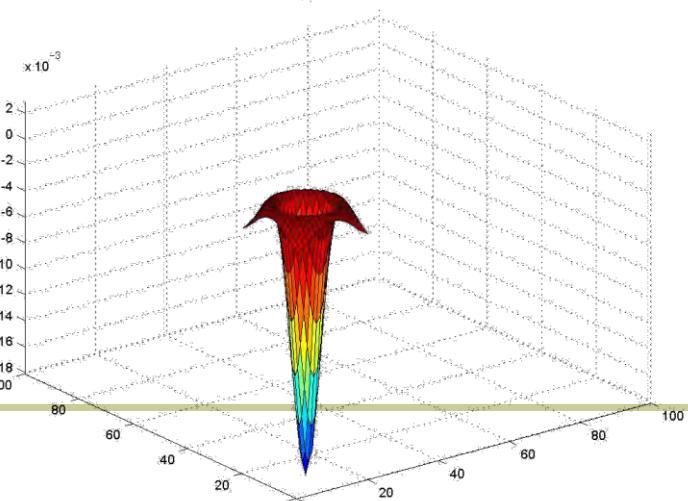
Scaled down image



Original ima



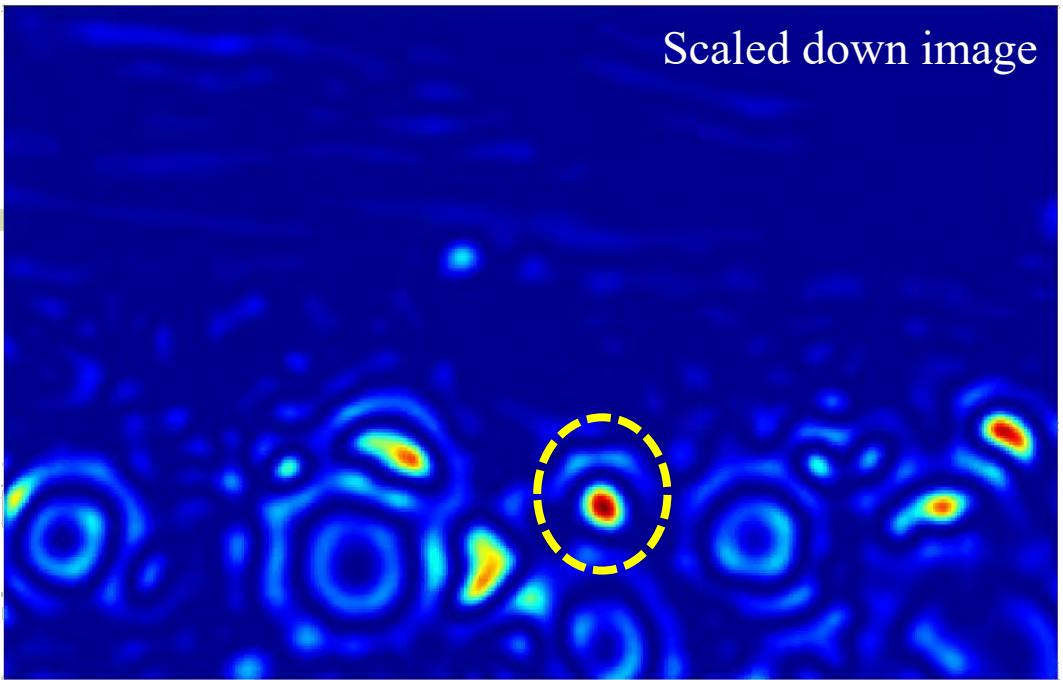
**sigma=4.2**



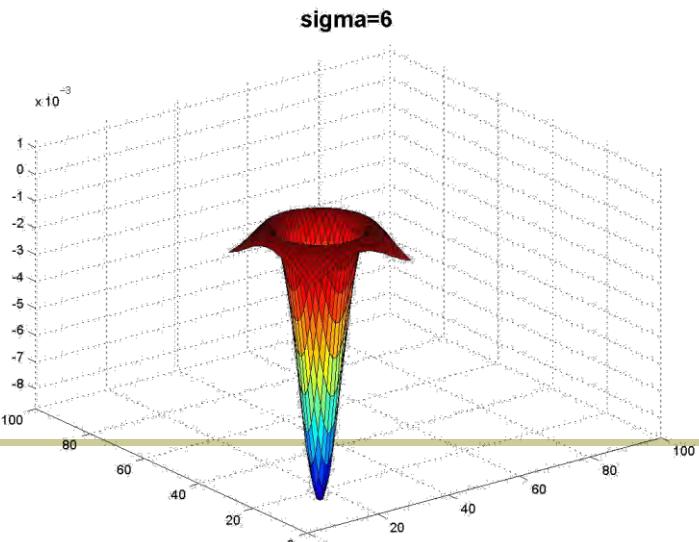
Slide credit: Kristen Grauman



Scaled down image



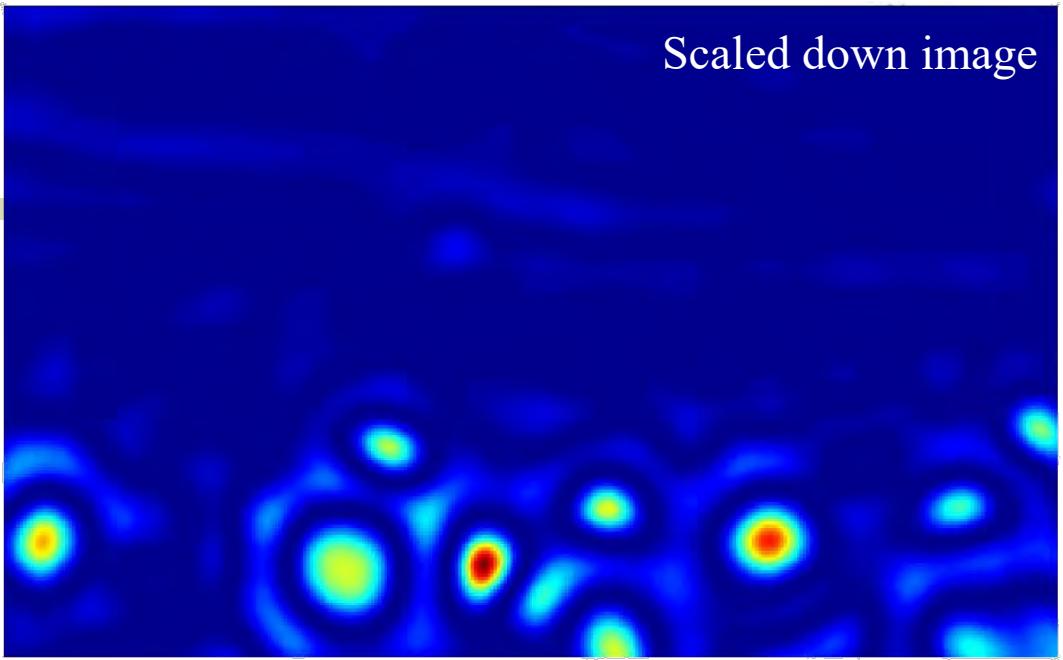
Original ima



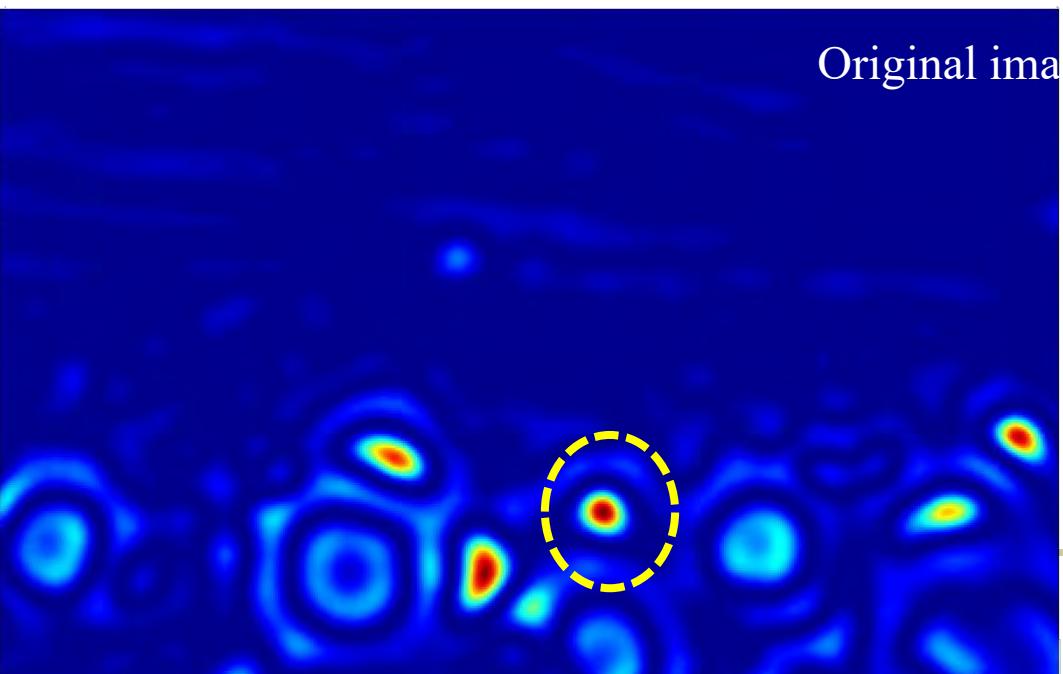
Slide credit: Kristen Grauman



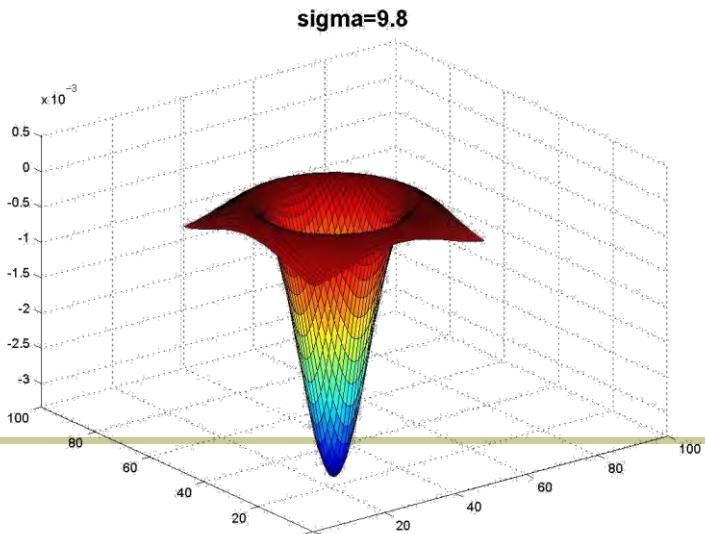
Scaled down image



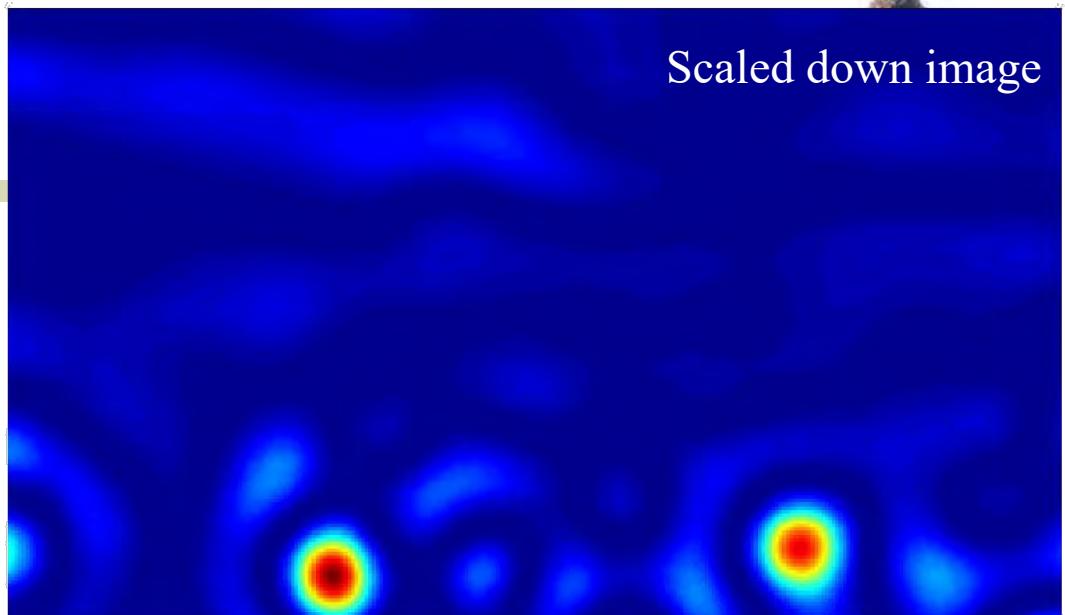
Original ima



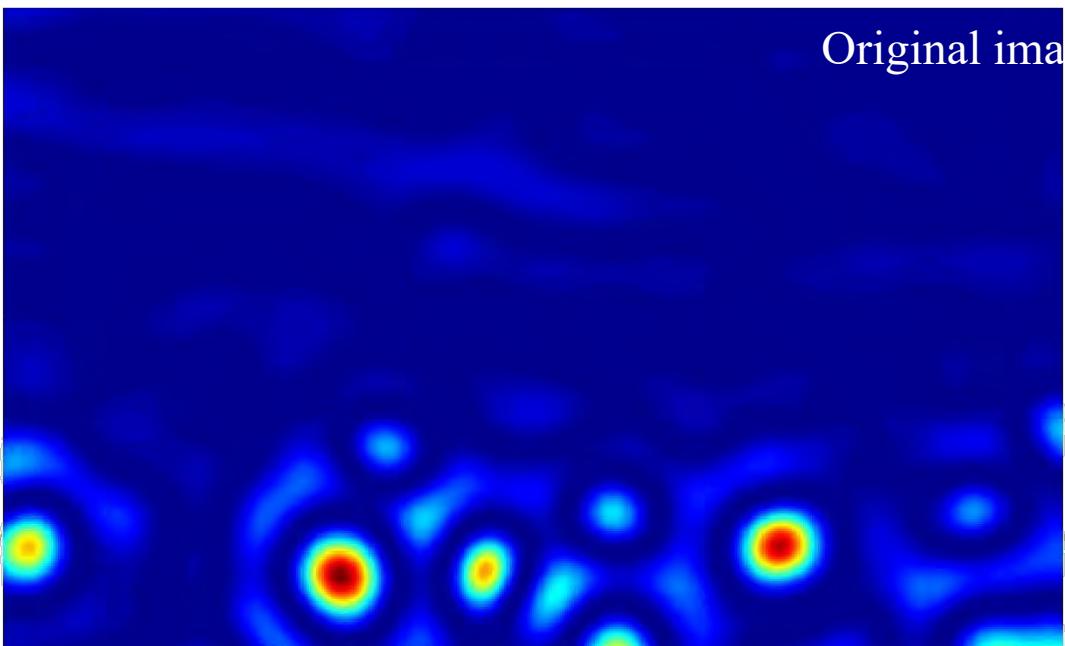
$\sigma = 9.8$



Slide credit: Kristen Grauman

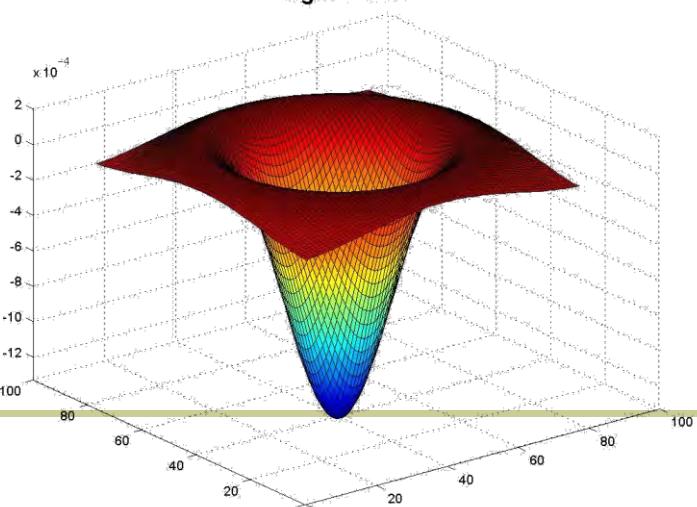


Scaled down image



Original ima

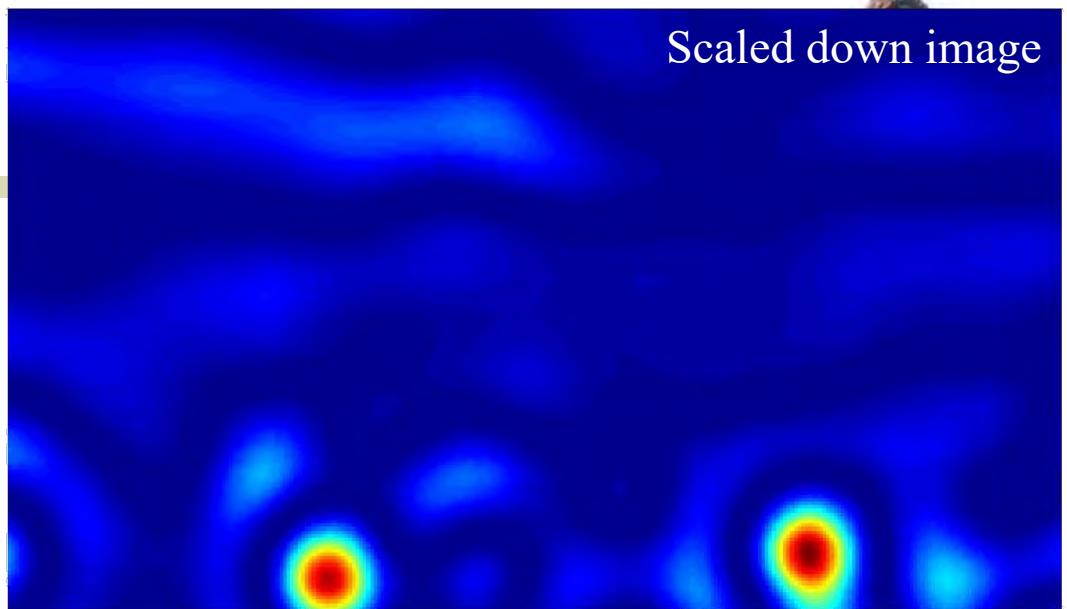
**sigma=15.5**



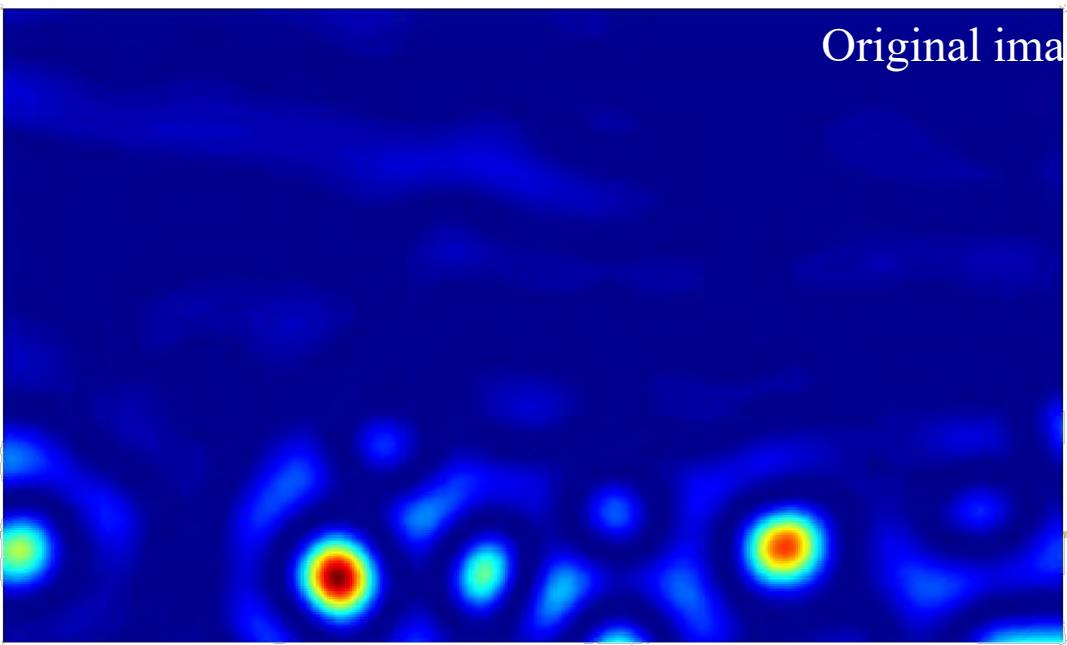
Slide credit: Kristen Grauman



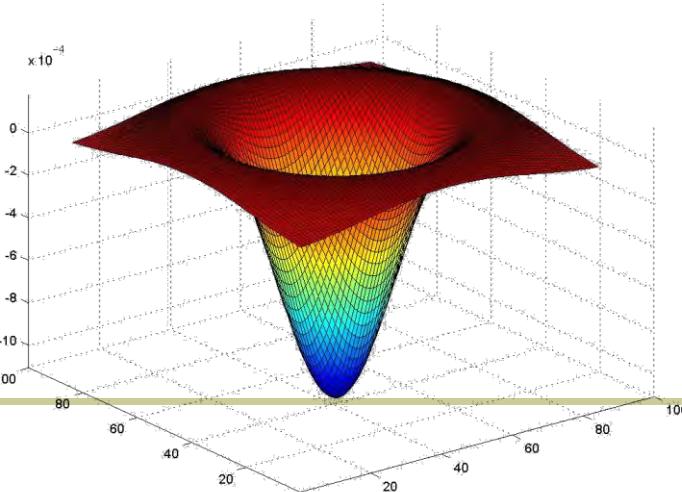
Scaled down image



Original ima



$\sigma = 17$



Slide credit: Kristen Grauman



# Optimal scale



2.1

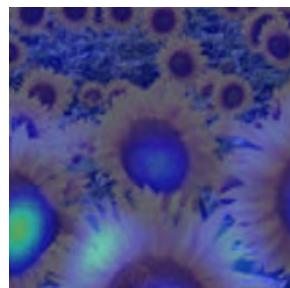
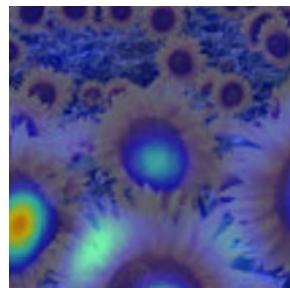
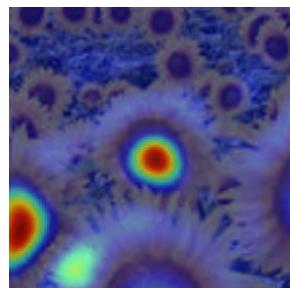
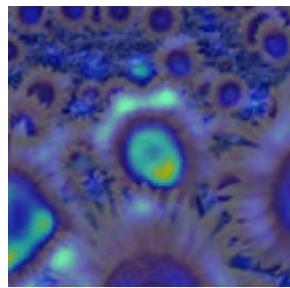
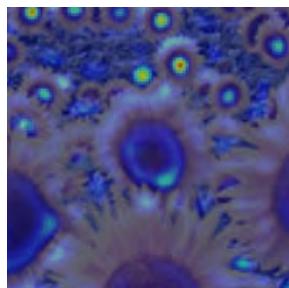
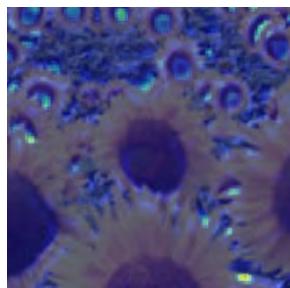
4.2

6.0

9.8

15.5

17.0



Full size image

2.1

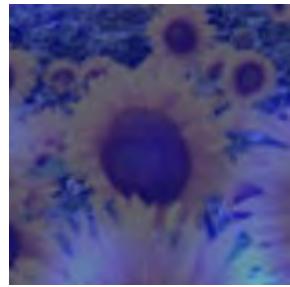
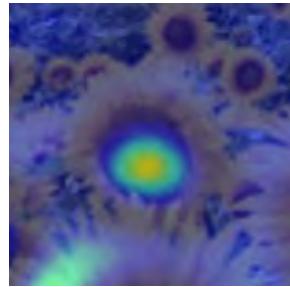
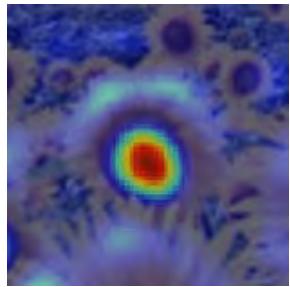
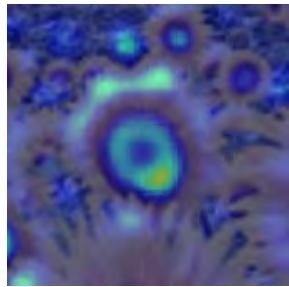
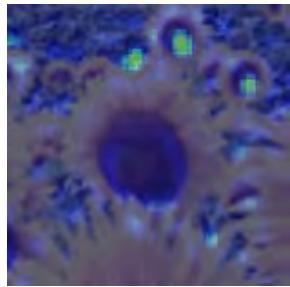
4.2

6.0

9.8

15.5

17.0



3/4 size image



# Optimal scale



2.1

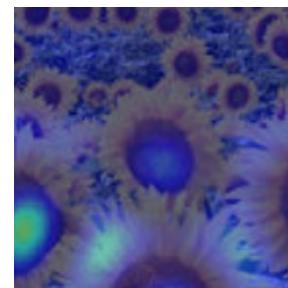
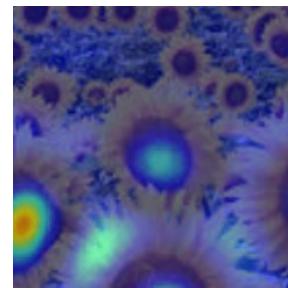
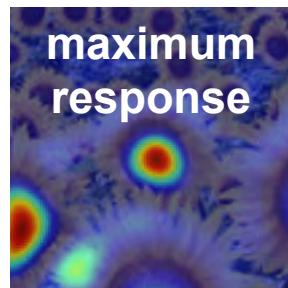
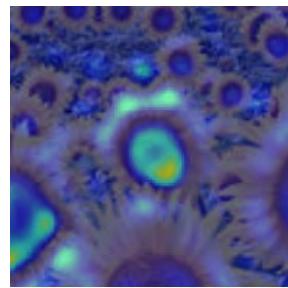
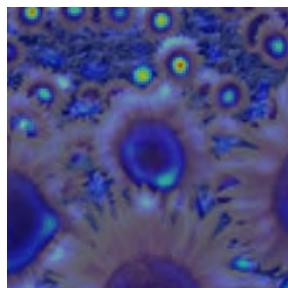
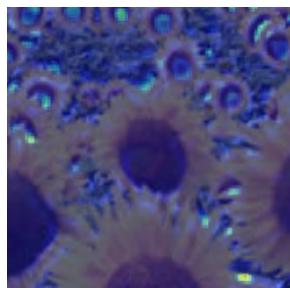
4.2

6.0

9.8

15.5

17.0



Full size image

2.1

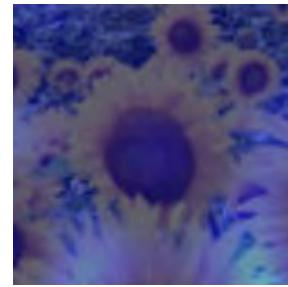
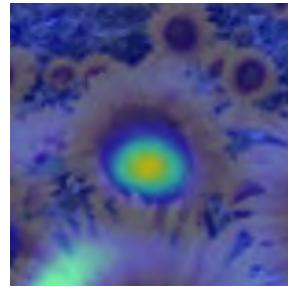
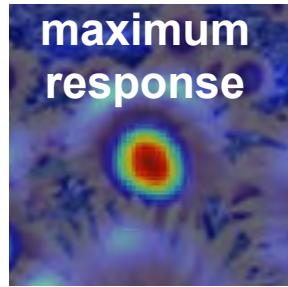
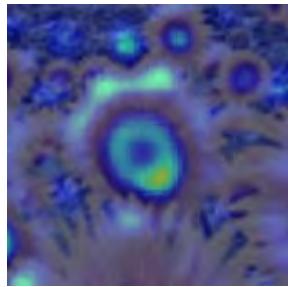
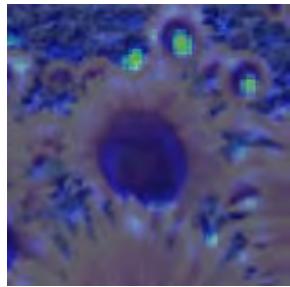
4.2

6.0

9.8

15.5

17.0



3/4 size image

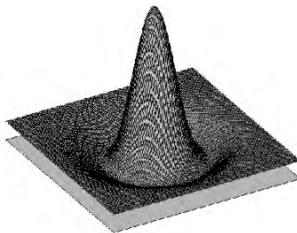


# Laplacian-of-Gaussian (LoG)



- **Interest points:**

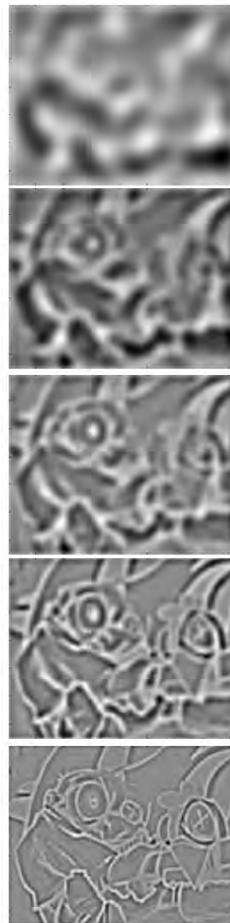
- Local maxima in scale space of Laplacian-of-Gaussian



$$L_{xx}(\sigma) + L_{yy}(\sigma) \rightarrow \sigma^3$$

Arrows point from the equation to the following levels of the scale space pyramid:

- $\sigma^5$ : Top level, very blurry image.
- $\sigma^4$ : Second level, slightly more detailed.
- $\sigma^3$ : Third level, showing some local structures.
- $\sigma^2$ : Fourth level, more detailed structures.
- $\sigma$ : Bottom level, highly detailed edge map.



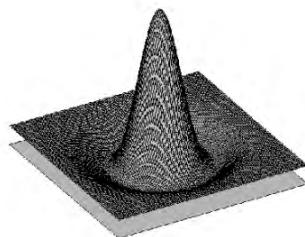


# Laplacian-of-Gaussian (LoG)



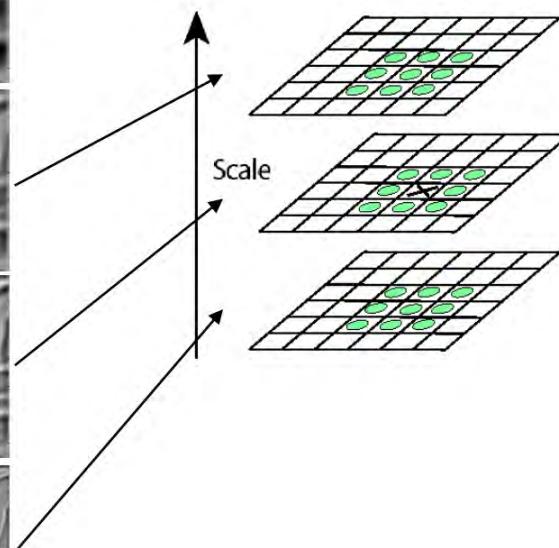
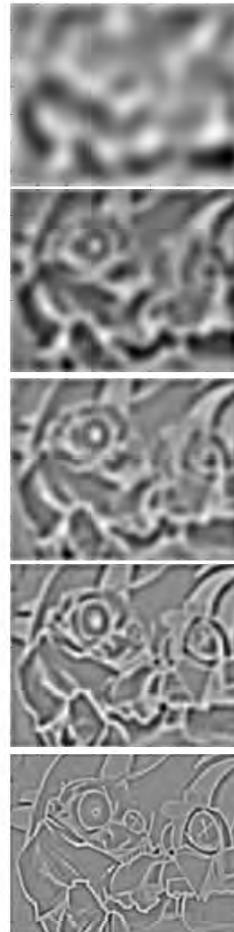
- Interest points:

- Local maxima in scale space of Laplacian-of-Gaussian



$$L_{xx}(\sigma) + L_{yy}(\sigma) \rightarrow \sigma^3$$

Diagram illustrating the computation of the Laplacian of Gaussian (LoG) filter. It shows a 3D surface plot of a Gaussian peak, which is then convolved with a Laplacian kernel. The resulting output is shown as a series of grayscale images decreasing in scale from  $\sigma^5$  at the top to  $\sigma$  at the bottom. The equation  $L_{xx}(\sigma) + L_{yy}(\sigma) \rightarrow \sigma^3$  indicates the relationship between the second derivatives and the resulting scale factor.



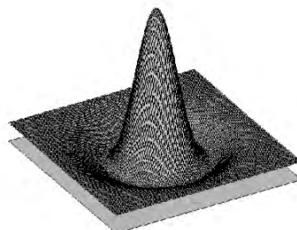


# Laplacian-of-Gaussian (LoG)



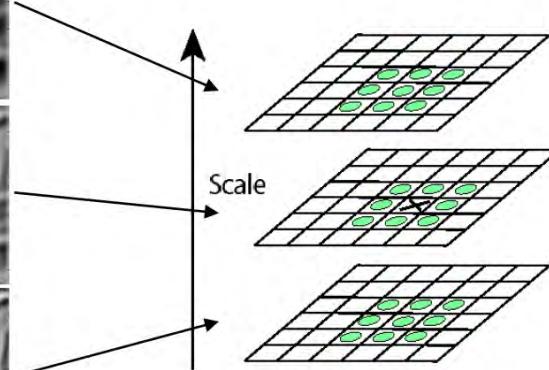
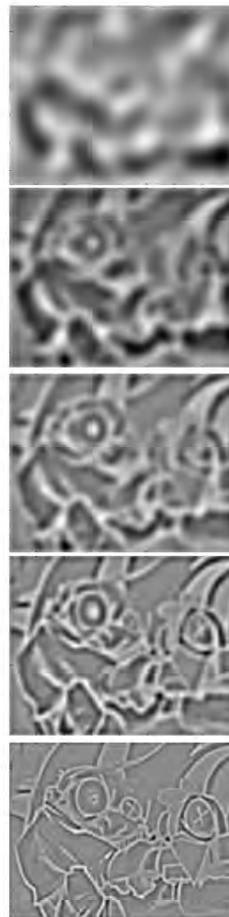
- **Interest points:**

- Local maxima in scale space of Laplacian-of-Gaussian



$$L_{xx}(\sigma) + L_{yy}(\sigma) \rightarrow \sigma^3$$
$$\sigma^2$$
$$\sigma$$
$$\sigma^4$$
$$\sigma^5$$

A diagram illustrating the computation of the Laplacian-of-Gaussian (LoG) filter. It shows a sequence of five grayscale images representing the filter response at increasing scales ( $\sigma$ ). Arrows point from the equation  $L_{xx}(\sigma) + L_{yy}(\sigma) \rightarrow \sigma^3$  to the first image, and arrows point from the labels  $\sigma^2, \sigma, \sigma^4, \sigma^5$  to the subsequent images, indicating the progression of scale.



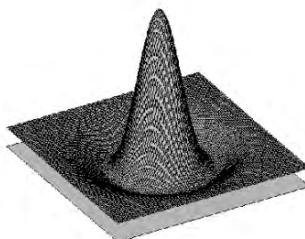


# Laplacian-of-Gaussian (LoG)



- **Interest points:**

- Local maxima in scale space of Laplacian-of-Gaussian



$$L_{xx}(\sigma) + L_{yy}(\sigma) \rightarrow \sigma^3$$

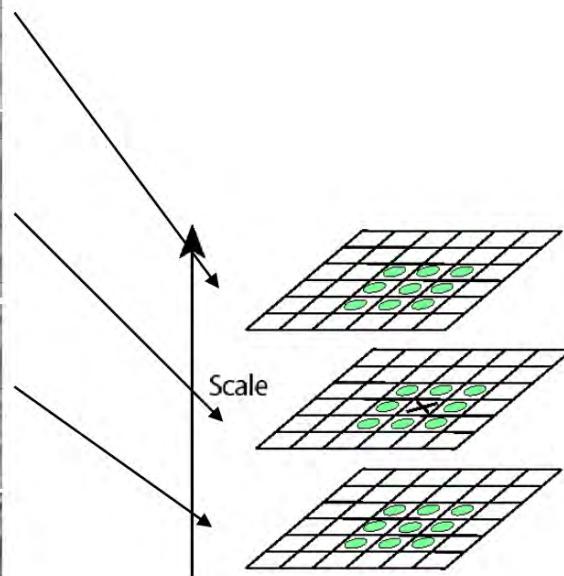
$\sigma^2$

$\sigma$

$\sigma^4$

$\sigma^5$

A diagram illustrating the computation of the Laplacian of a Gaussian (LoG) filter. It shows a central equation  $L_{xx}(\sigma) + L_{yy}(\sigma) \rightarrow \sigma^3$  with arrows pointing from it to higher powers of  $\sigma$ :  $\sigma^2$ ,  $\sigma$ ,  $\sigma^4$ , and  $\sigma^5$ .



⇒ List of  $(x, y, \sigma)$



# LoG detector: workflow





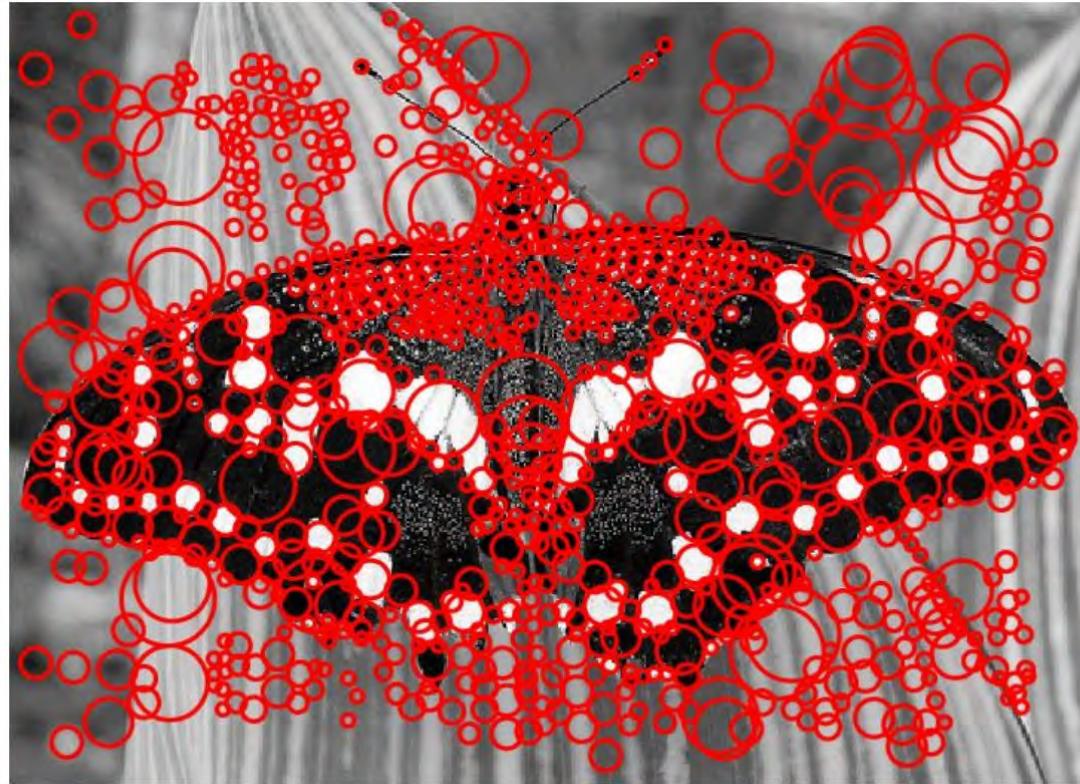
# LoG detector: workflow



$\sigma = 11.9912$



# LoG detector: workflow





# Technical detail



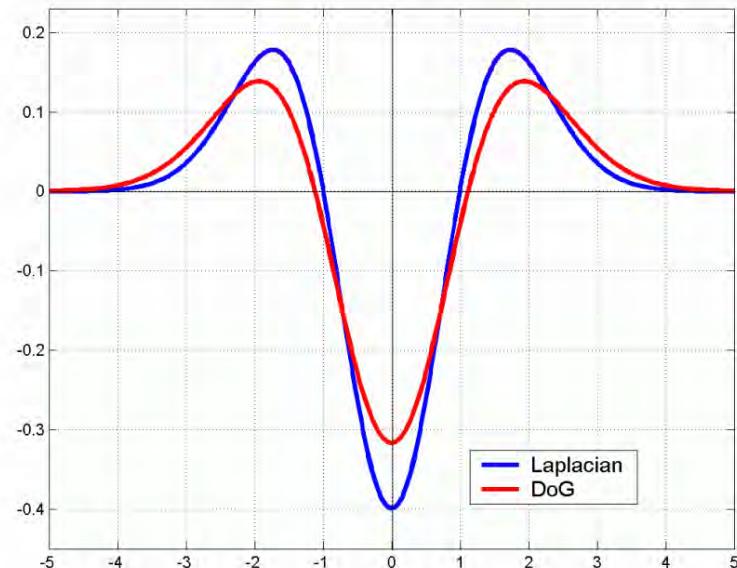
- We can efficiently approximate the Laplacian with a difference of Gaussians:

$$L = \sigma^2 (G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma))$$

(Laplacian)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)

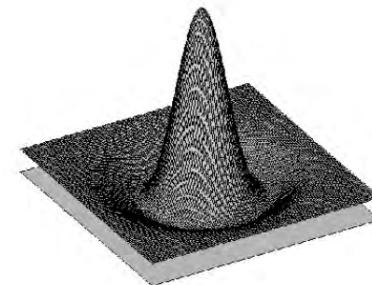




# Difference-of-Gaussian(DoG)



- Difference of Gaussians as approximation of the LoG
  - This is used e.g. in Lowe's SIFT pipeline for feature detection.
- Advantages
  - No need to compute 2<sup>nd</sup> derivatives
  - Gaussians are computed anyway, e.g. in a Gaussian pyramid.

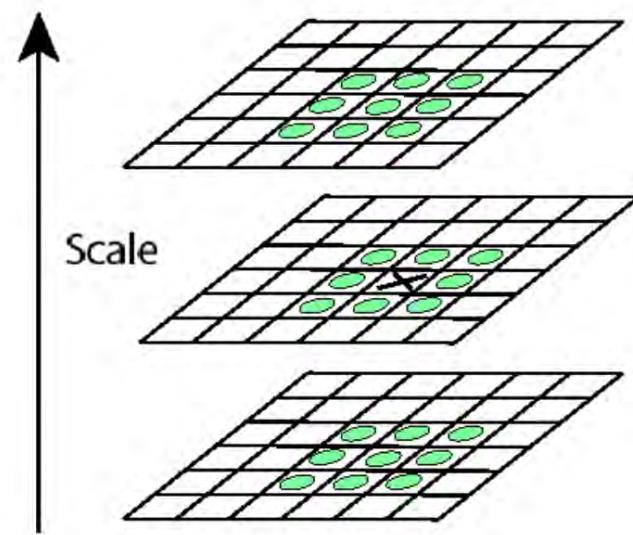




# Keypoint localization with DoG



- Detect maxima of difference-of-Gaussian (DoG) in scale space
- Then reject points with low contrast (threshold)
- Eliminate edge responses



Candidate keypoints:  
list of  $(x, y, \sigma)$



# DoG: Efficient implementation



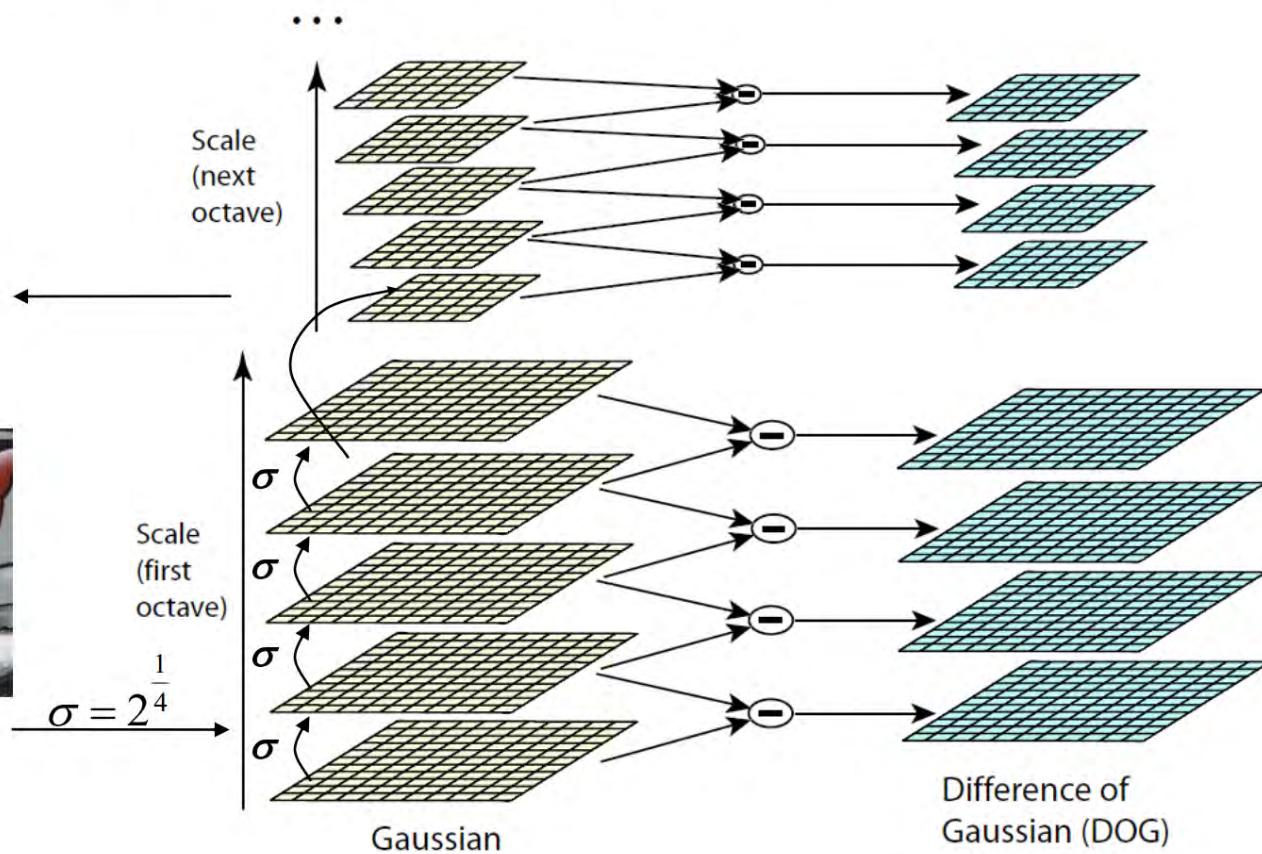
- Computation in Gaussian scale pyramid



*Sampling with  
step  $\sigma^4 = 2$*



*Original image*





# Results: Lowe's DoG





# Example of Keypoint Detection



(a)



(b)



(c)



(d)

**(a) 233x189 image**

**(b) 832 DoG extrema**

**(c) 729 left after peak value threshold**

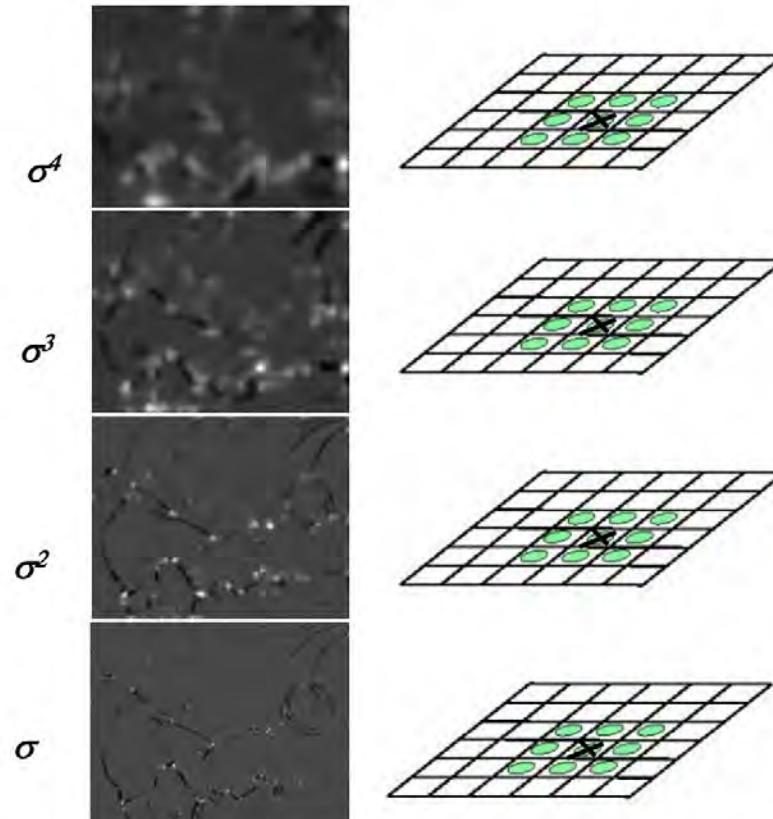
**(d) 536 left after testing ratio of principle curvatures (removing edge responses)**



# Harris-Laplace [Mikolajczyk '01]



## 1. Initialization: Multiscale Harris corner detection



Computing Harris function

Detecting local maxima

Slide adapted from Krystian Mikolajczyk

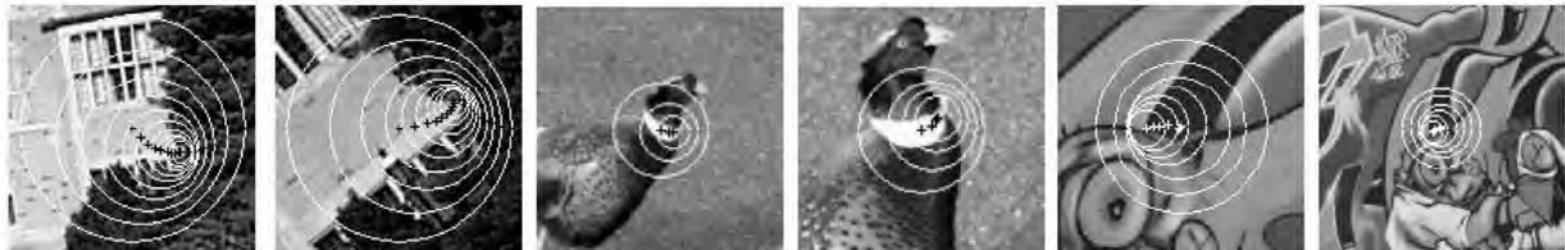


# Harris-Laplace [Mikolajczyk '01]



1. Initialization: Multiscale Harris corner detection
2. Scale selection based on Laplacian  
(same procedure with Hessian  $\Rightarrow$  Hessian-Laplace)

Harris points



Harris-Laplace points



# Summary: Scale Invariant Detection

- **Given:** Two images of the same scene with a large *scale difference* between them.
- **Goal:** Find *the same* interest points *independently* in each image.
- **Solution:** Search for *maxima* of suitable functions in *scale* and in *space* (over the image).
- Two strategies
  - Laplacian-of-Gaussian (LoG)
  - Difference-of-Gaussian (DoG) as a fast approximation
  - *These can be used either on their own, or in combinations with single-scale keypoint detectors (Harris, Hessian).*



# Summary



- Introduction to correspondence and alignment
- Overview of interest points
  - Matching pipeline
  - Repeatable & Distinctive
- Keypoint Localization
  - Harris detector
  - Hessian detector
- Scale invariant region selection
  - Automatic scale selection
  - Laplacian of Gaussian (LoG) & Difference of Gaussian (DoG)
  - Combinations: Harris-Laplacian & Hessian-Laplacian



# You Can Try It at Home



- For most local feature detectors, executables are available online:
- <http://robots.ox.ac.uk/~vgg/research/affine>
- <http://www.cs.ubc.ca/~lowe/keypoints/>
- <http://www.vision.ee.ethz.ch/~surf>
- <http://homes.esat.kuleuven.be/~ncorneli/gpusurf/>

# Affine Covariant Features



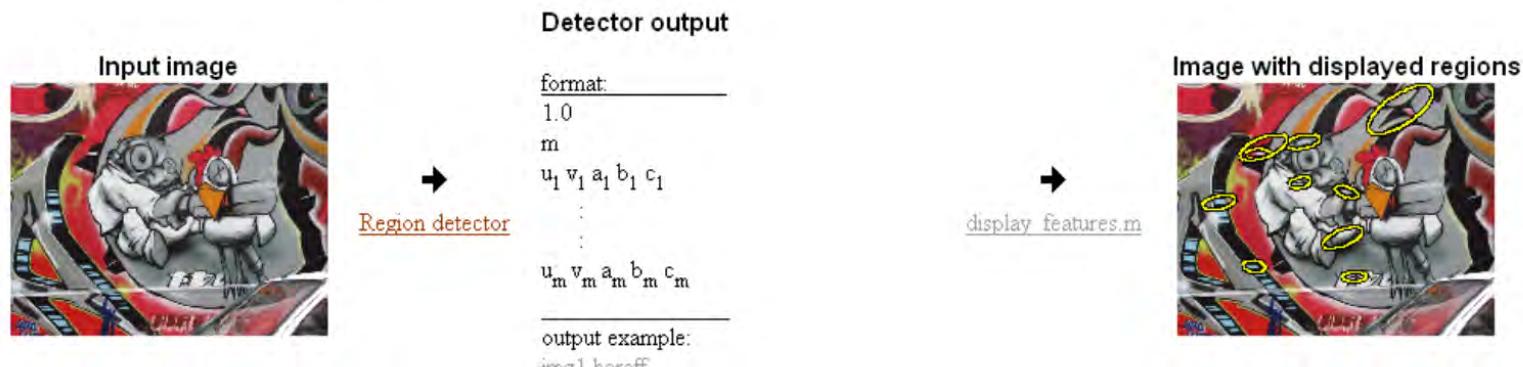
KATHOLIEKE UNIVERSITEIT  
**LEUVEN**

**INRIA**  
RHÔNE ALPES



Collaborative work between the Visual Geometry Group, Katholieke Universiteit Leuven, Inria Rhône-Alpes and the Center for Machine Perception.

## Affine Covariant Region Detectors



### Parameters defining an affine region

$u, v, a, b, c$  in  $a(x-u) + 2b(x-u)(y-v) + c(y-v)^2 = 1$   
with  $(0,0)$  at image top left corner

### Code

- provided by the authors, see [publications](#) for details and links to authors web sites.

#### Linux binaries

[Harris-Affine & Hessian-Affine](#)

[MSER](#) - Maximally stable extremal regions (also Windows)

[IBR](#) - Intensity extrema based detector

[EBR](#) - Edge based detector

[Salient](#) region detector

#### Example of use

prompt>./h\_affine.ln -haraff -i [img1.ppm](#) -o img1.haraff -thres 1000 matlab>> [d](#)

prompt>./h\_affine.ln -hesaff -i [img1.ppm](#) -o img1.hesaff -thres 500 matlab>> [d](#)

prompt>./mser.ln -t 2 -es 2 -i [img1.ppm](#) -o img1.mser matlab>> [d](#)

prompt>./ibr.ln [img1.ppm](#) img1.ibr -scalefactor 1.0 matlab>> [d](#)

prompt>./ebr.ln [img1.ppm](#) img1.ebr matlab>> [d](#)

prompt>./salient.ln [img1.ppm](#) img1.sal matlab>> [d](#)

#### Displaying 1

matlab>> [d](#)

matlab>> [d](#)

matlab>> [d](#)

matlab>> [d](#)



# References



- Read David Lowe's SIFT paper
  - D. Lowe,  
Distinctive image features from scale-invariant keypoints,  
IJCV 60(2), pp. 91-110, 2004
- Good survey paper on Int. Pt. detectors and descriptors
  - T. Tuytelaars, K. Mikolajczyk, Local Invariant Feature Detectors: A Survey, Foundations and Trends in Computer Graphics and Vision, Vol. 3, No. 3, pp 177-280, 2008.
- Try the example code, binaries, and Matlab wrappers
  - Good starting point: Oxford interest point page  
<http://www.robots.ox.ac.uk/~vgg/research/affine/detectors.html#binaries>