TALENT COURSE 11
LEARNING FROM DATA: BAYESIAN METHODS AND MACHINE LEARNING

Lecture 12: More on priors and Maximum Entropy

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Ohio University

TU Darmstadt

ExtreMe Matter Institute

TALENT Course 11 is possible thanks to funding from the STFC

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Posterior

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Probability as degree of belief cf. frequentist view

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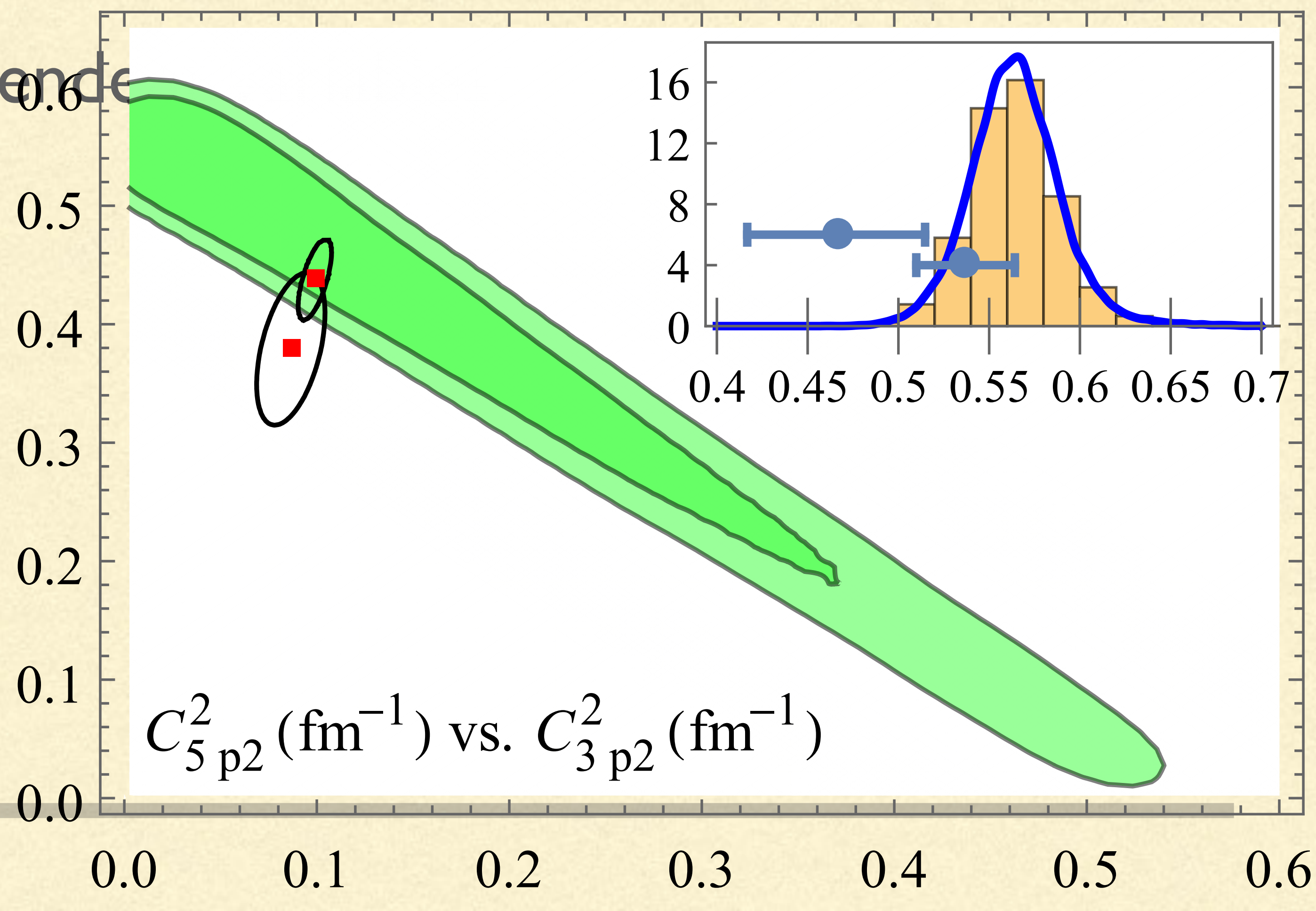

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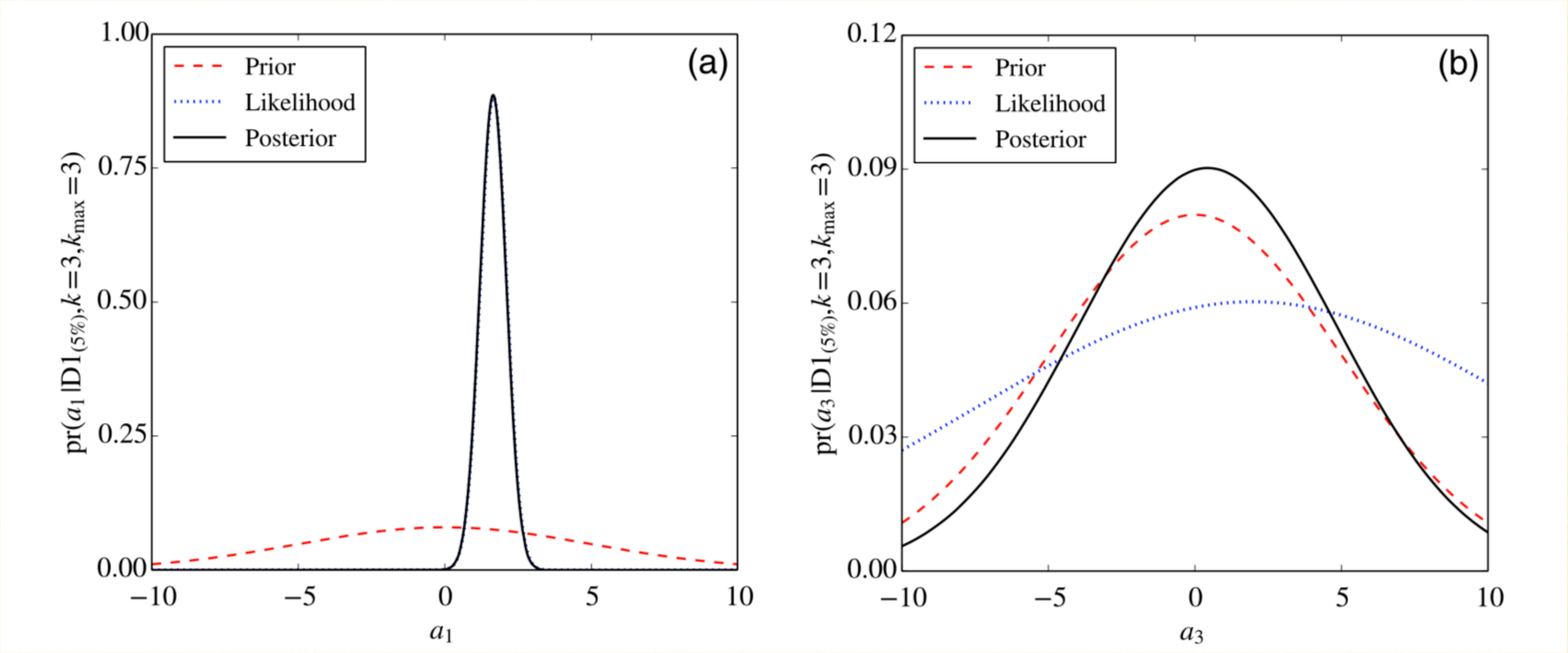
Should the likelihood dominate?

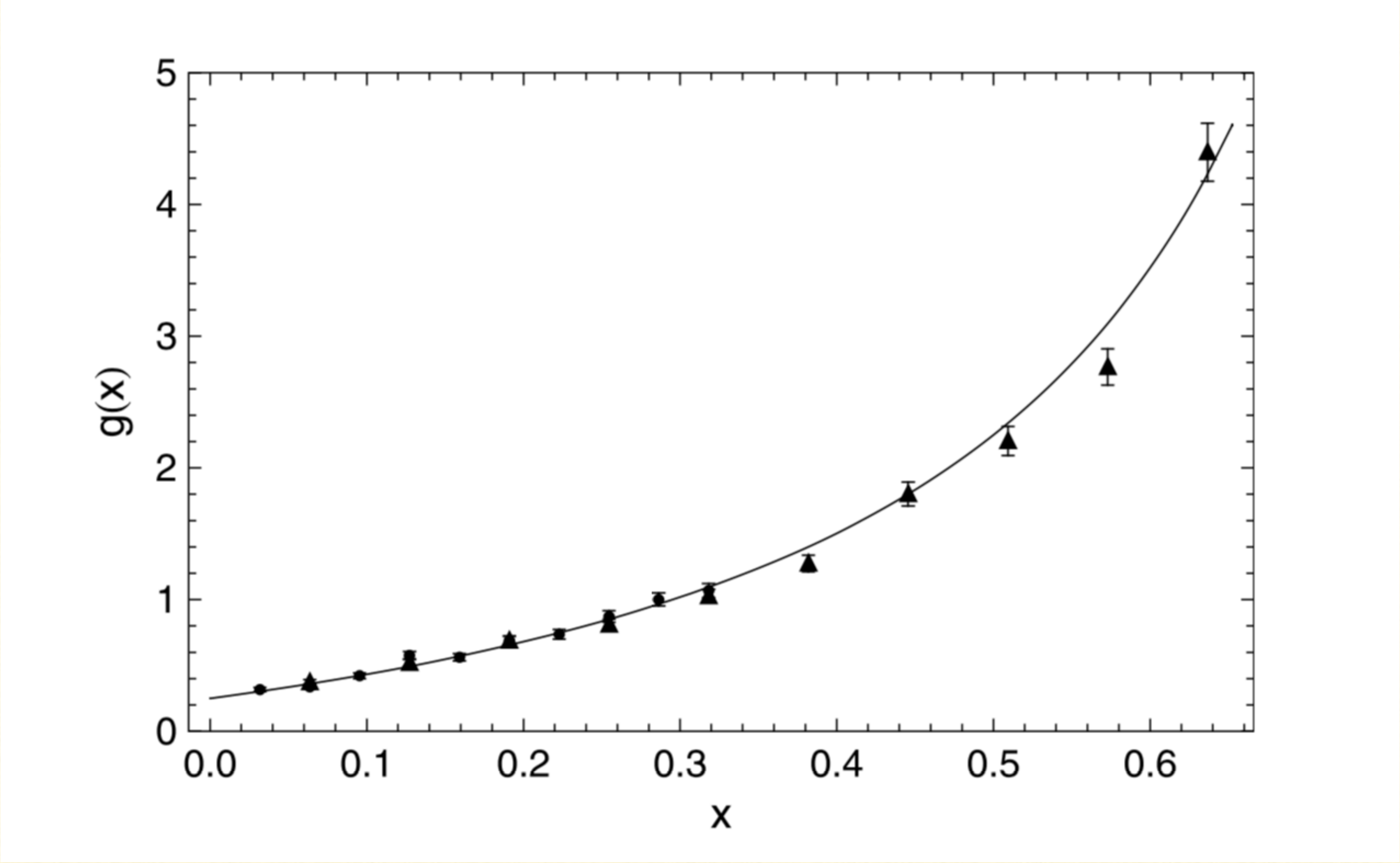
PRIOR VS. LIKELIHOOD

- Prior can only be assessed in context of likelihood
- "Robustness" analysis



BACK TO THE MINI-PROJECT

Schindler, DP (2009); Wesolowski, Klco, Furnstahl, DP, Thapaliya (2016)



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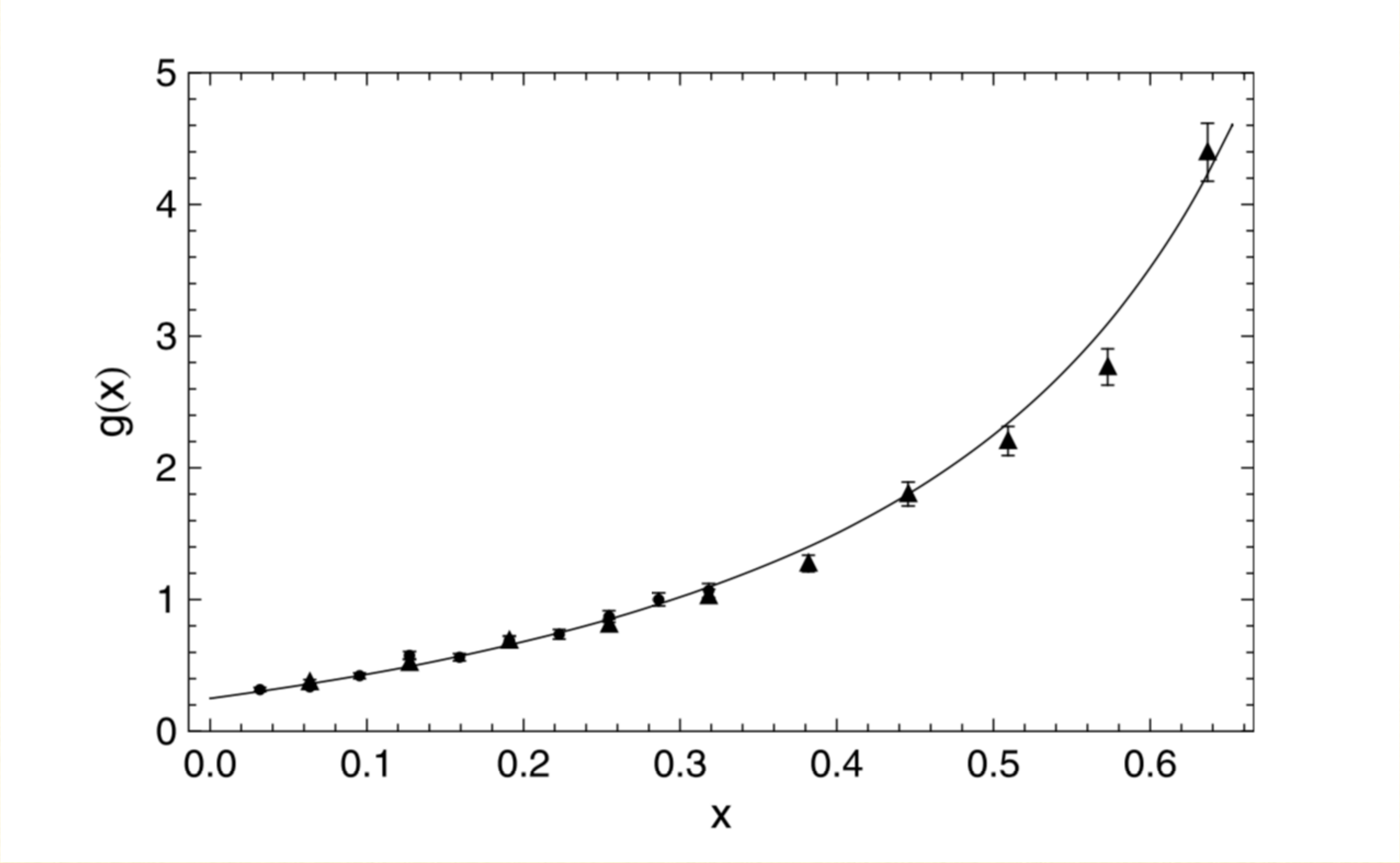

$$g(x) = 0.25 + 1.57x + 2.47x^2 + 1.29x^3 + \ldots$$

- Gaussian prior that encodes naturalness: $\mathrm{pr}(a_k \,|\, \bar{a}, I) \propto \exp\left(-\frac{a_k^2}{2\bar{a}^2}\right)$

RESULTS

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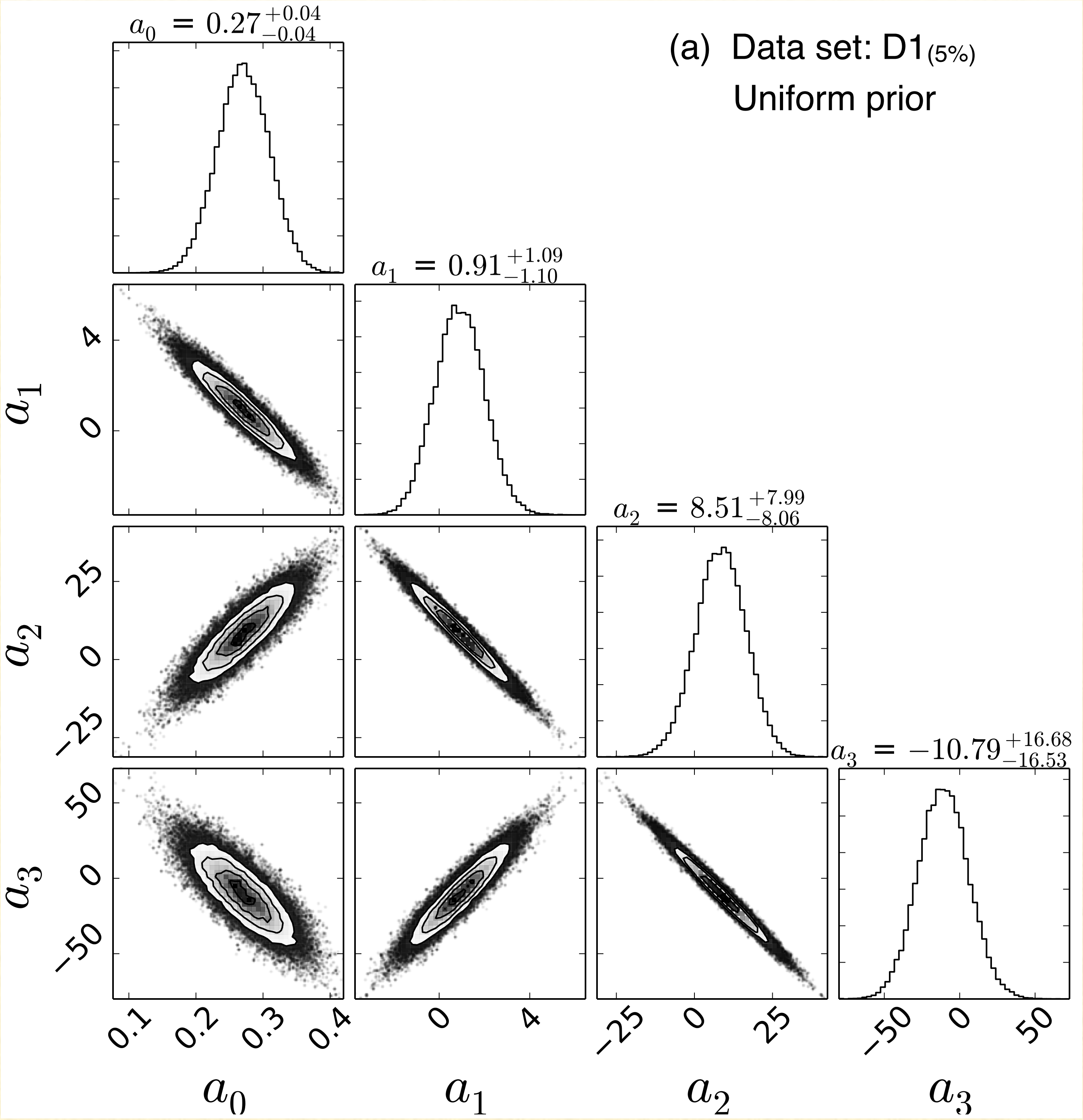

$a_0 = 0.27^{+0.04}_{-0.04}$
(a) Data set: D1(5%)
Uniform prior
$a_1 = 0.91^{+1.09}_{-1.10}$
$a_2 = 8.51^{+7.99}_{-8.06}$
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a_0
a_1
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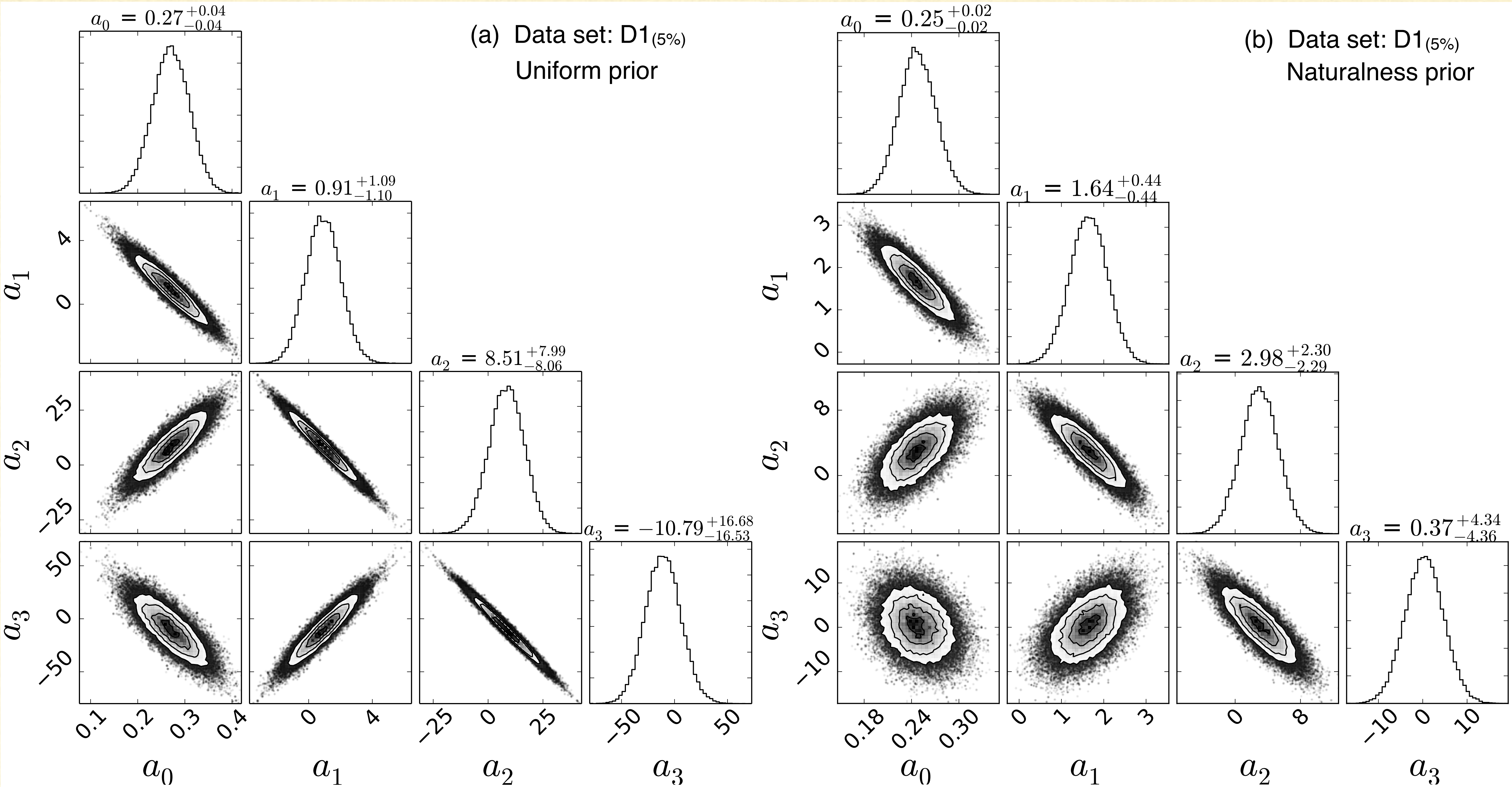

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(b) Data set: D1(5%)
Naturalness prior
$a_0 = 0.25^{+0.02}_{-0.02}$
$a_1 = 1.64^{+0.44}_{-0.44}$
$a_2 = 2.98^{+2.30}_{-2.29}$
$a_3 = 0.37^{+4.34}_{-4.36}$
a_0
a_1
a_2
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RESULTS

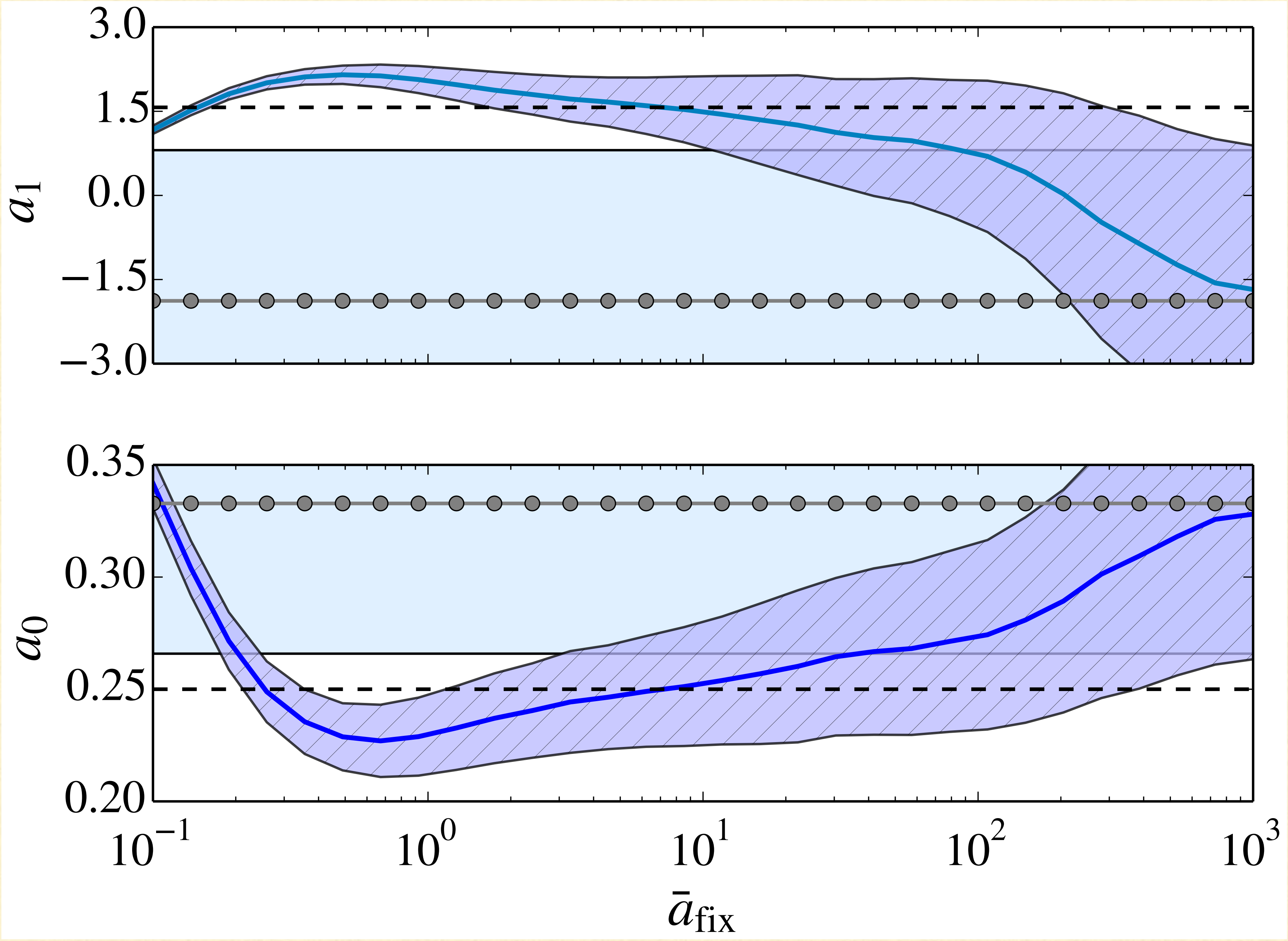
Table 3
Fit results for Bayesian approach with $R = 1$, $x_{\max} = 1/\pi$ and $c = 0.05$

M	$\log[\mathrm{pr}(\langle \mathbf{a} \rangle \| D_1, M, R)]$	a_0	a_1	a_2
2	12.00	0.228 ± 0.018	2.06 ± 0.25	1.60 ± 0.78
3	11.25	0.230 ± 0.018	2.04 ± 0.25	1.50 ± 0.79
4	10.35	0.230 ± 0.018	2.04 ± 0.25	1.49 ± 0.80
5	9.43	0.230 ± 0.018	2.04 ± 0.25	1.49 ± 0.80
6	8.51	0.230 ± 0.018	2.04 ± 0.25	1.49 ± 0.80
7	7.60	0.230 ± 0.018	2.04 ± 0.25	1.49 ± 0.80

Table 4
Fit results for Bayesian approach with $R = 5$, $x_{\max} = 1/\pi$ and $c = 0.05$

M	$\log[\mathrm{pr}(\langle \mathbf{a} \rangle \| D_1, M, R)]$	a_0	a_1	a_2
2	9.62	0.248 ± 0.023	1.63 ± 0.39	3.15 ± 1.27
3	7.10	0.247 ± 0.024	1.65 ± 0.45	2.98 ± 2.32
4	4.57	0.247 ± 0.024	1.64 ± 0.46	2.98 ± 2.39
5	2.04	0.247 ± 0.024	1.64 ± 0.46	2.98 ± 2.39
6	−0.488	0.247 ± 0.024	1.64 ± 0.46	2.98 ± 2.39
7	−3.02	0.247 ± 0.024	1.64 ± 0.46	2.98 ± 2.39

RELAXATION PLOT



FIVE LEVELS OF PRIOR

https://github.com/stan-dev/stan/wiki/Prior-Choice-Recommendations

- Flat prior
- Super-vague, but proper, prior: N(0,1,000,000)
- (Very) weakly informative prior: N (0,10)
- Weak informative prior: N(0,1): enough information to regularize
- Specific informative prior: N(0.4,.0.2)

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- Specific informative prior: N(0.4,.0.2)
- If you have to choose: weakly informative better than fully informative.
- Tails: maybe use t-distributions rather than Gaussians
- When using informative priors, be explicit about every choice!
- Conjugate priors

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$$Q[f; \{\lambda_j\}] = S[f] + \sum_{j=0}^{N} \lambda_j \left(\mu_j - \int dx\, x^j f(x) \right)$$

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Moment conditions: $\{\mu_j : j=0, \ldots, N\}$ must be completely monotonic

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- Choice of α: classic maximum entropy, historic maximum entropy, Bryan's method

IMAGE RECONSTRUCTION

Grotenhus, Masters' thesis

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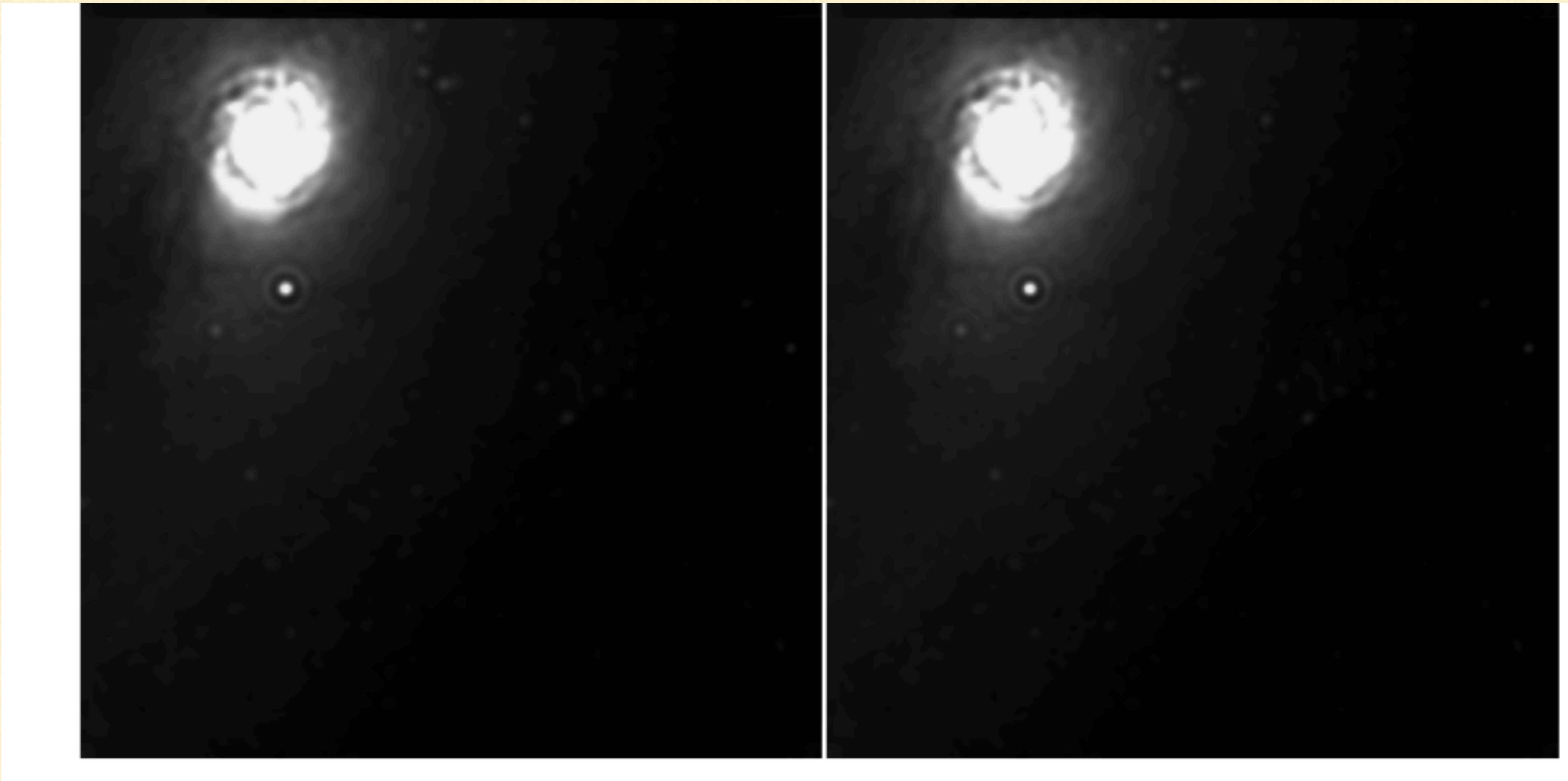

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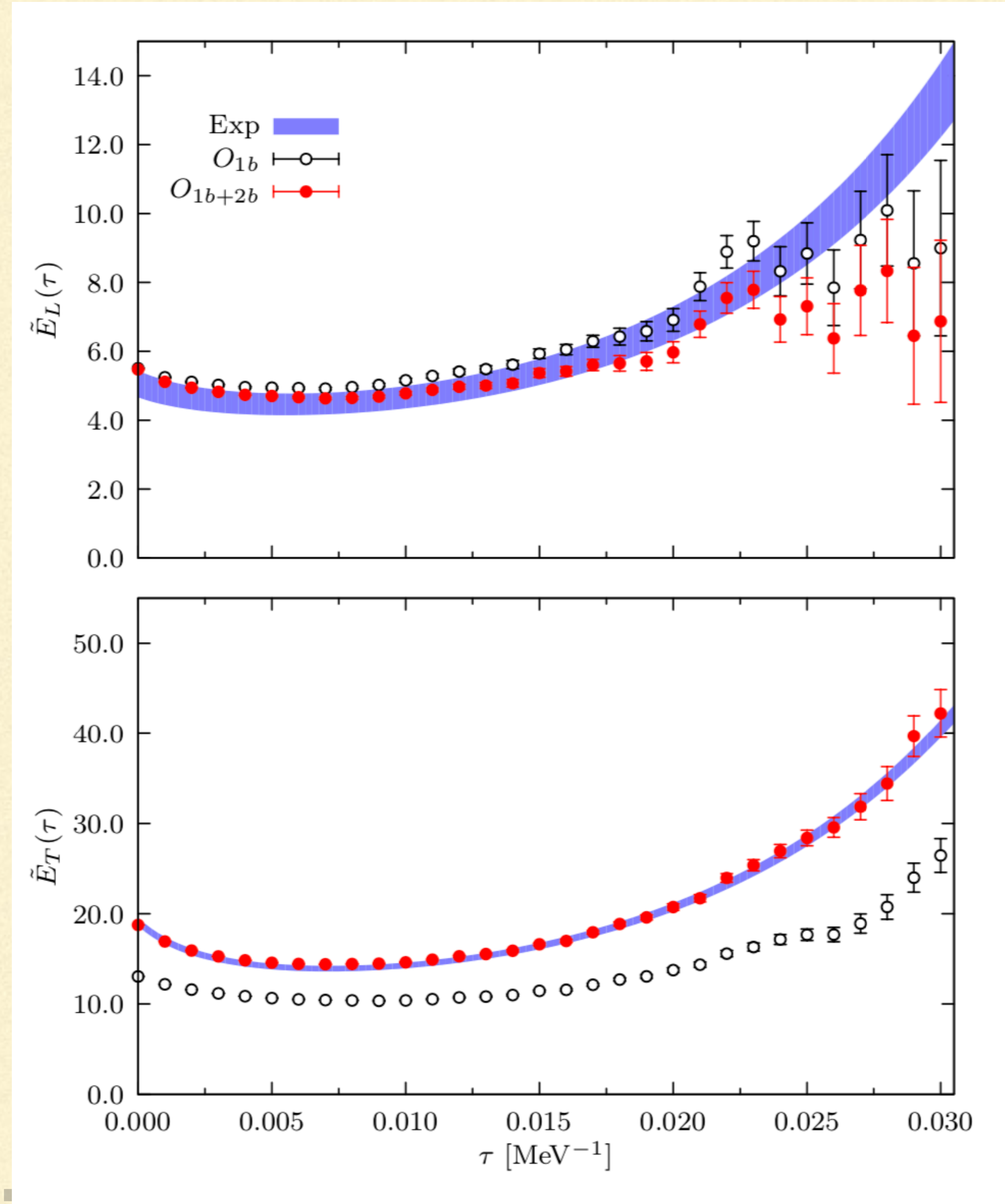
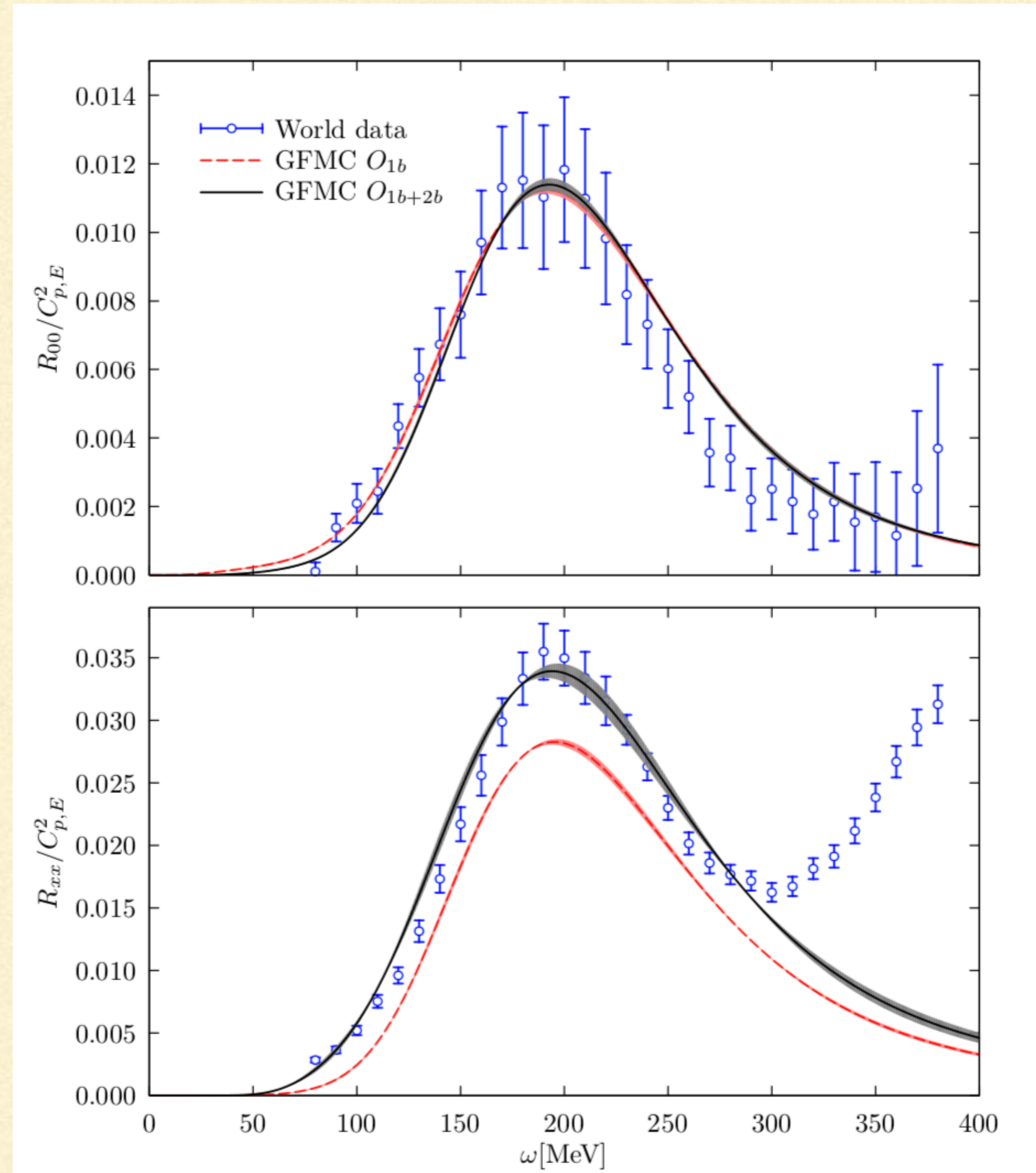
- At large positive frequencies the kernel is exponentially small, so large ω features of R(ω) depend on subtle features of E(τ).

RESULTS



Exp
O_{1b}
O_{1b+2b}
$\tilde{E}_L(\tau)$
$\tilde{E}_T(\tau)$
τ [MeV^{-1}]



World data
GFMC O_{1b}
GFMC O_{1b+2b}
$R_{00}/C^2_{p,E}$
$R_{xx}/C^2_{p,E}$
ω[MeV]