

Inverse kinematics

How to make a physicist's life difficult

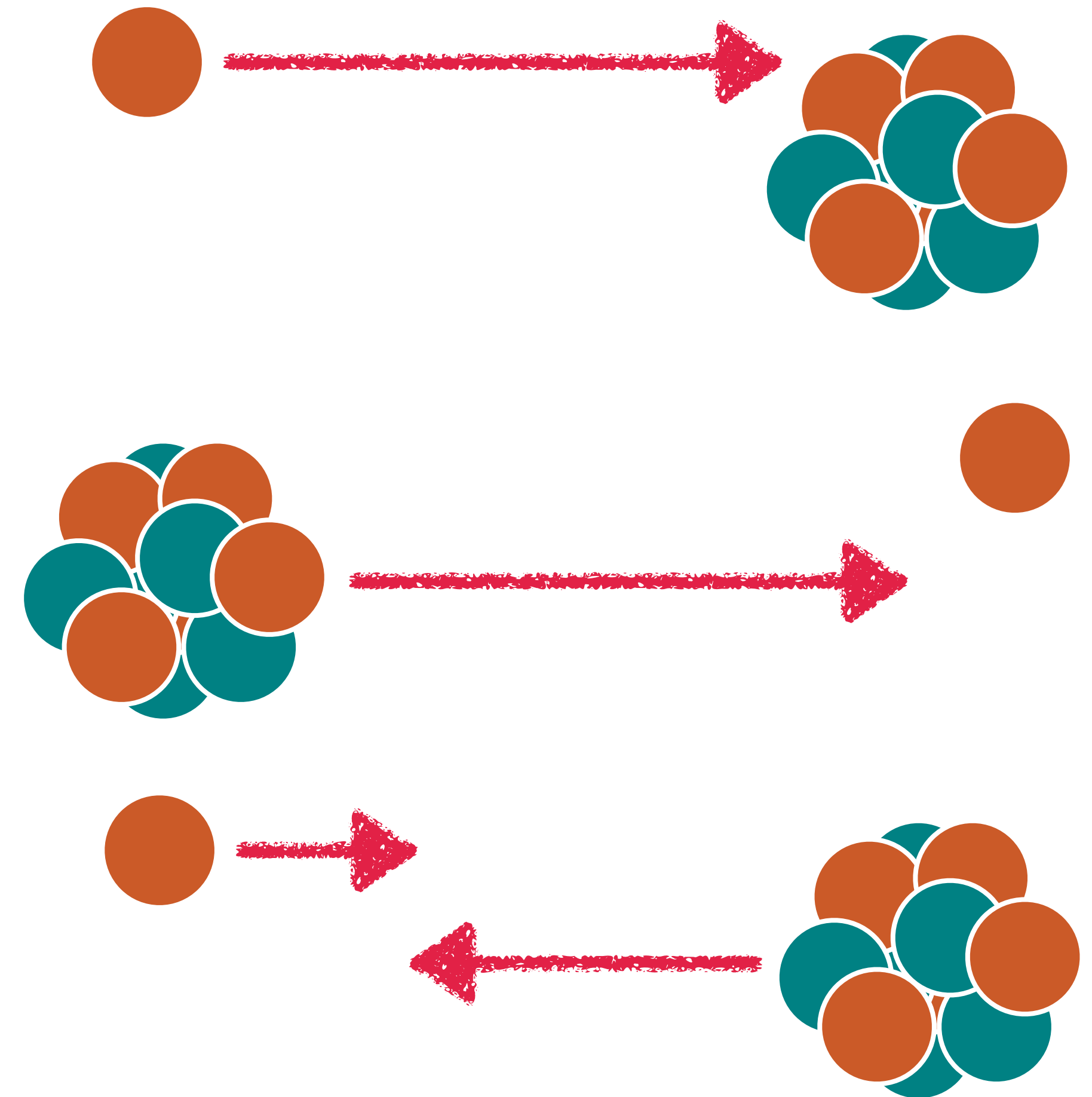
D. Bazin

**National Superconducting Cyclotron Laboratory
Facility for Rare Isotope Beams
Michigan State University**

What is inverse kinematics?

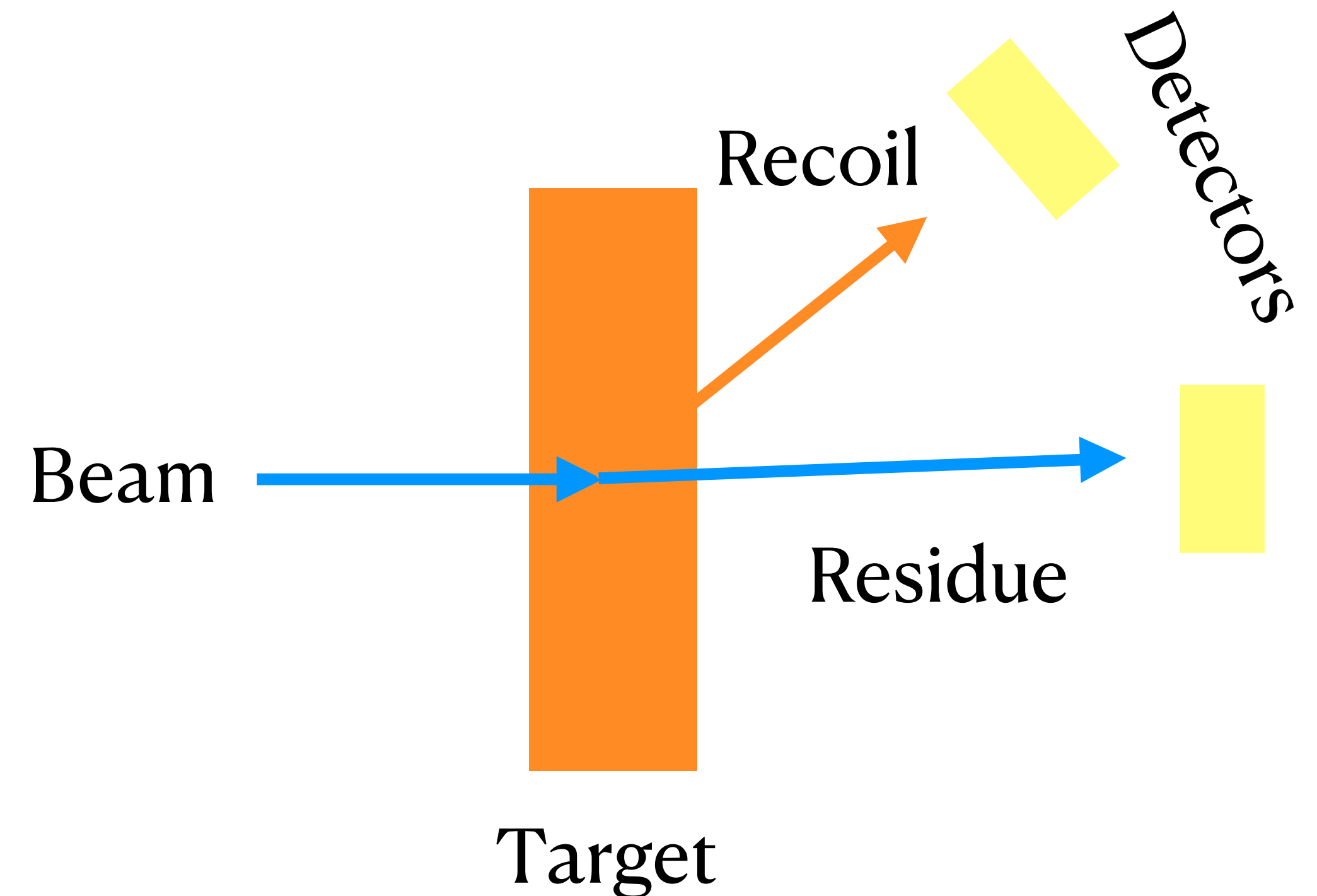
When reaction partners are not the same size

- Normal kinematics
 - Light partner is the beam, heavy one is the target
 - 5 MeV proton on ^{40}Ca
- Inverse kinematics
 - Heavy partner is the beam, light one is the target
 - 5 MeV/u ^{40}Ca on proton
- Center-of-mass
 - It's the same exact reaction!



Recoils and residues in inverse kinematics

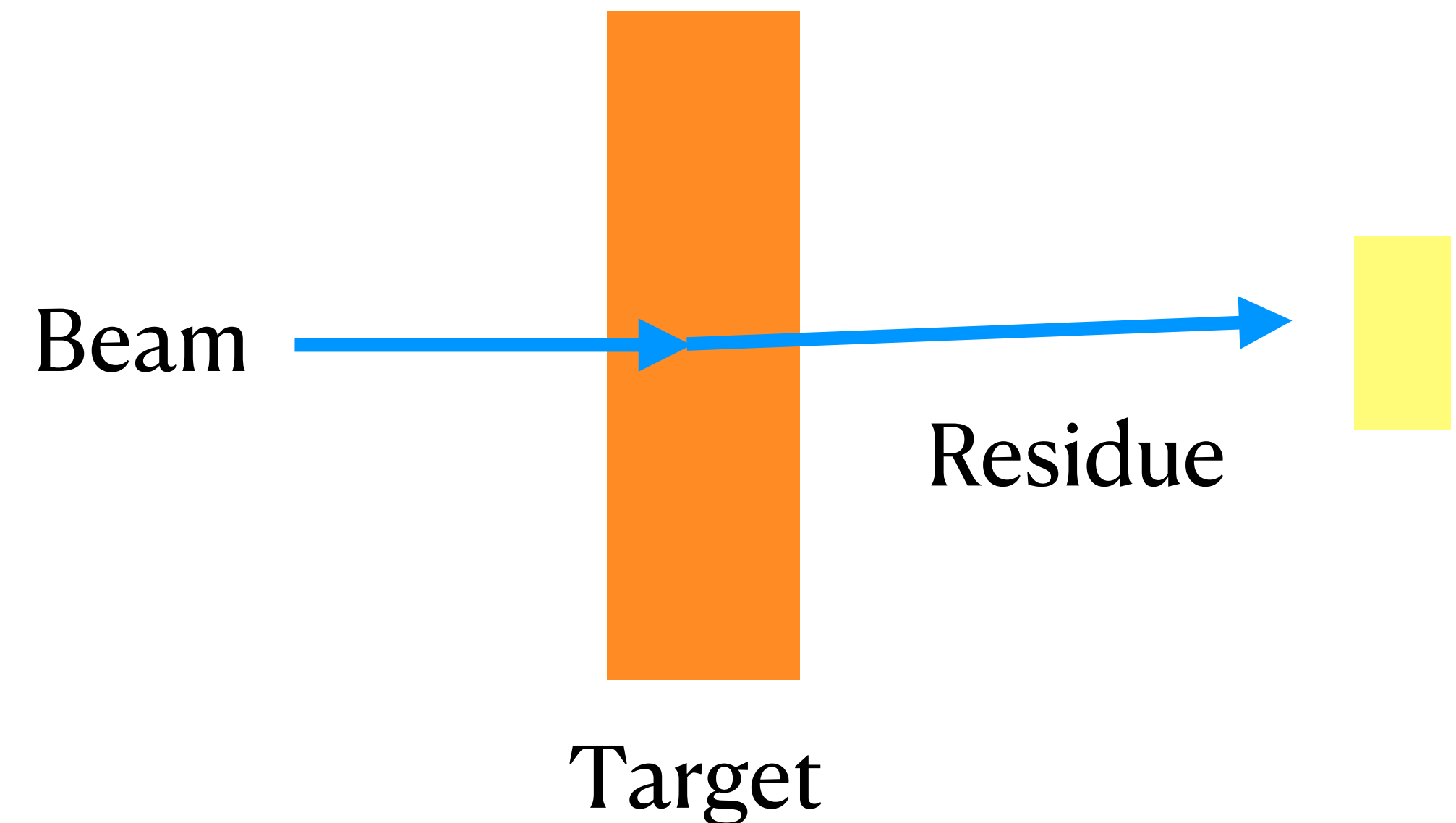
- Residue(s): beam-like particle(s) after reaction
 - Carry the momentum of the beam
 - Cover small solid angle range
 - Reduced kinematical information
- Recoil(s): target-like particle(s) after reaction
 - Start from being at rest
 - Cover large solid angle range
 - Extensive kinematical information



Detect the residue(s)

Inverse kinematics helps!

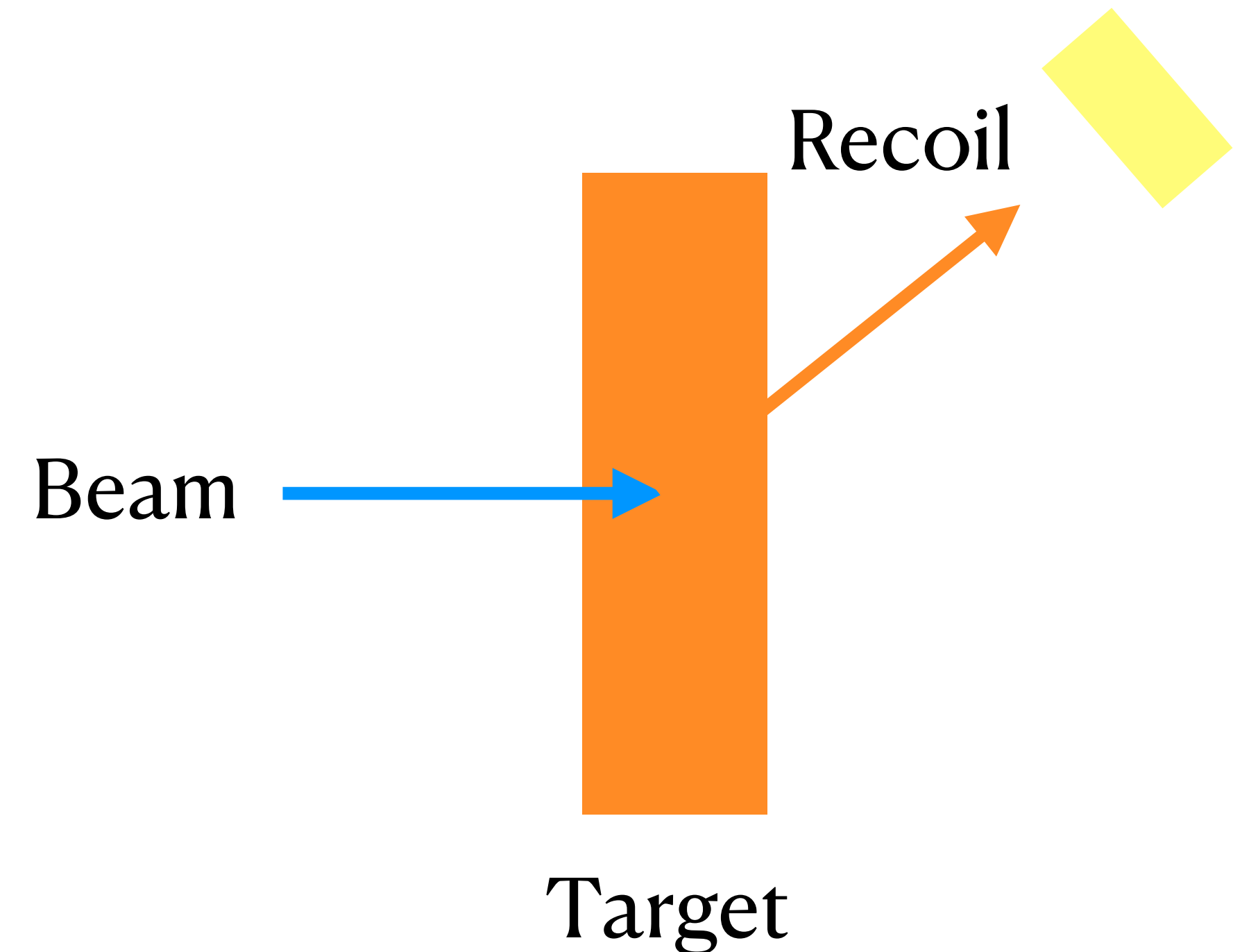
- Residue can easily escape target
- Target thickness can be large
- Small solid angle coverage needed
- High luminosity
- Example: knockout reactions on p, ^9Be or ^{12}C
 - Reaction properties extracted from measurements on the residue (γ -rays, parallel momentum,...)



Detect the recoil(s)

Inverse kinematics hurts!

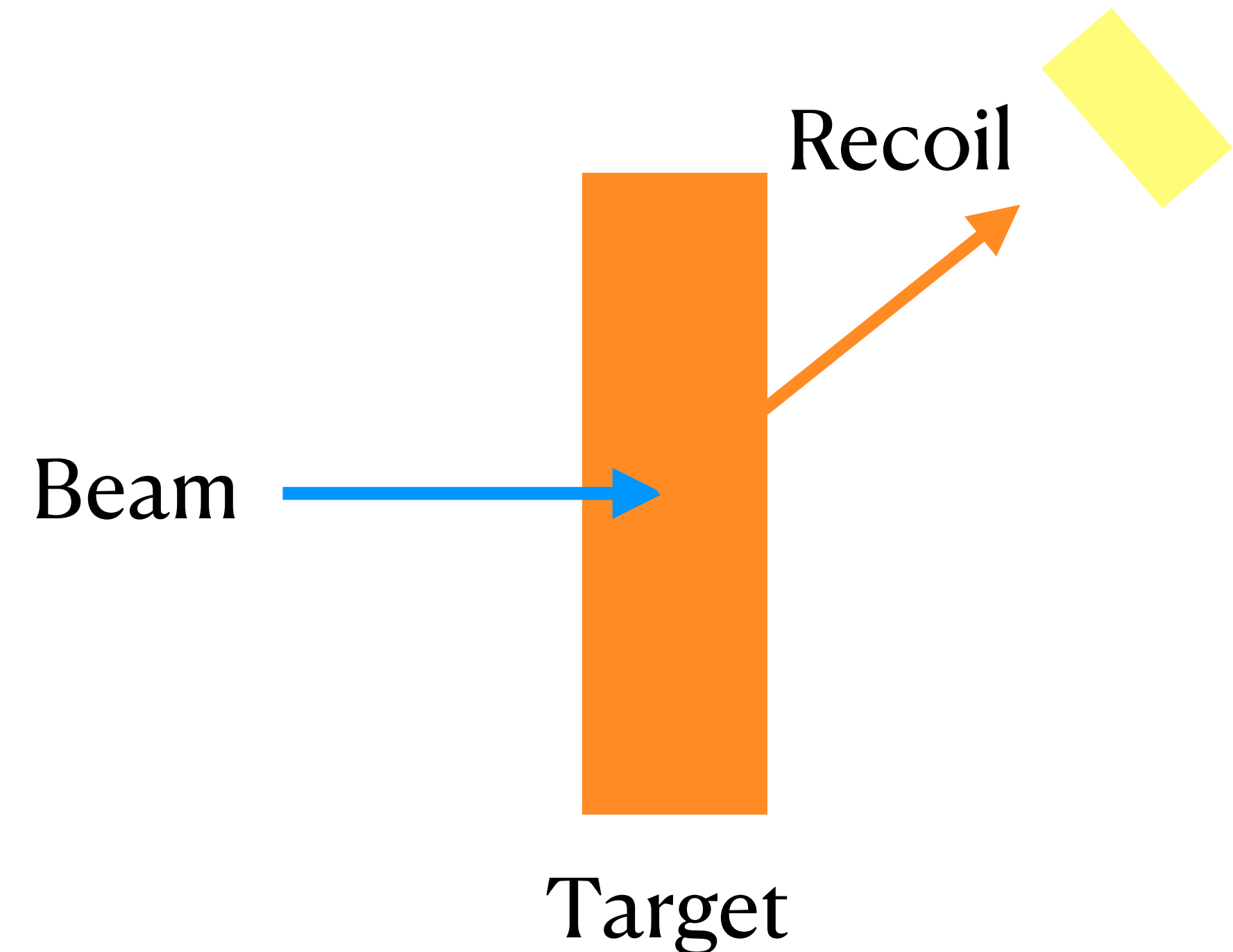
- Recoil doesn't have much energy to escape the target
- Target thickness cannot be too large
- Large solid angle coverage needed
- Low luminosity
- Example: (d,p) transfer reactions
 - Reaction properties extracted from measurements on the recoil (energy, scattering angle,...)



Passive target setup

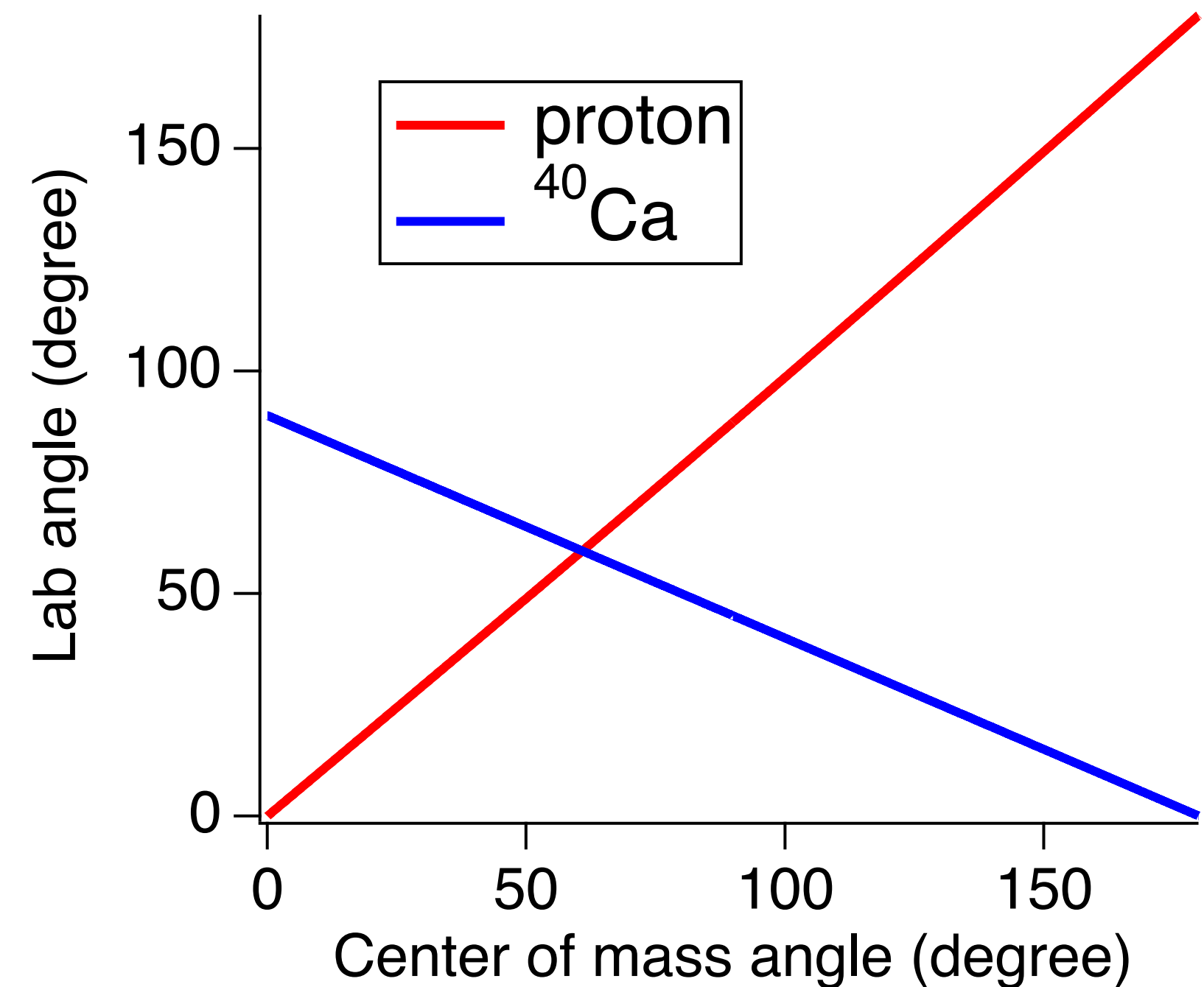
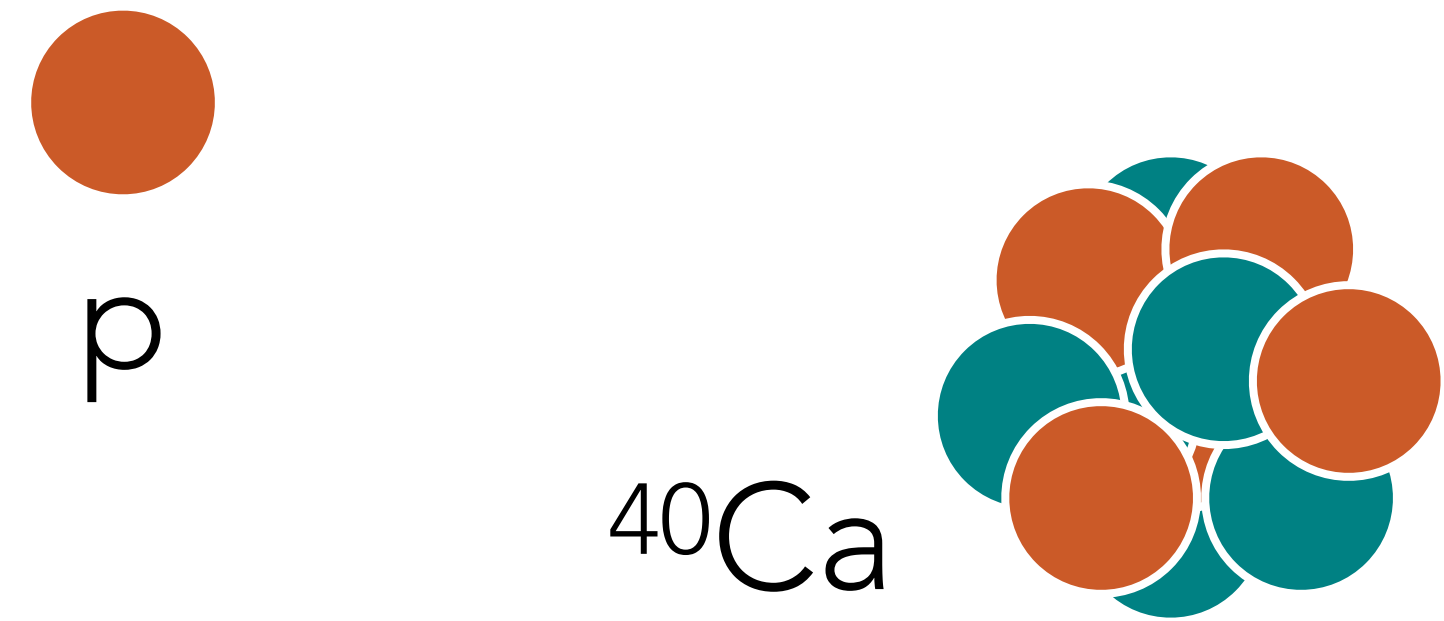
A comparison between normal and inverse kinematics

- Depth at which reaction takes place is unknown
- Exact energy at which reaction takes place is unknown
- Recoiling particles have to escape target in order to be detected
- Energy lost in target material before escaping is not measured by detectors



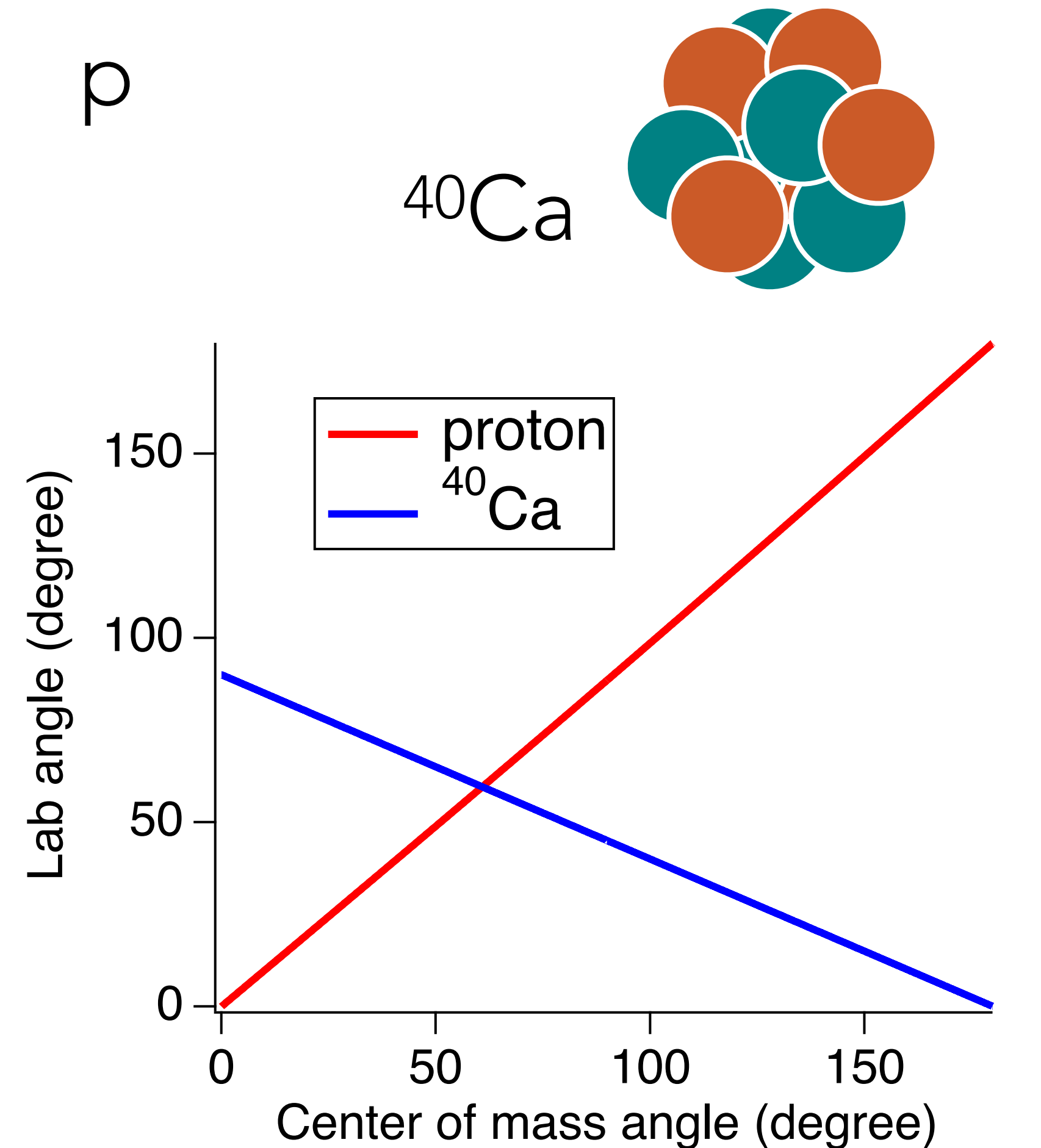
Normal kinematics

- Elastic scattering of proton on ^{40}Ca at 10 MeV
- Energy of outgoing proton almost constant
- Easily escapes the target at almost all angles
- ^{40}Ca has very low recoil energy and stays inside target



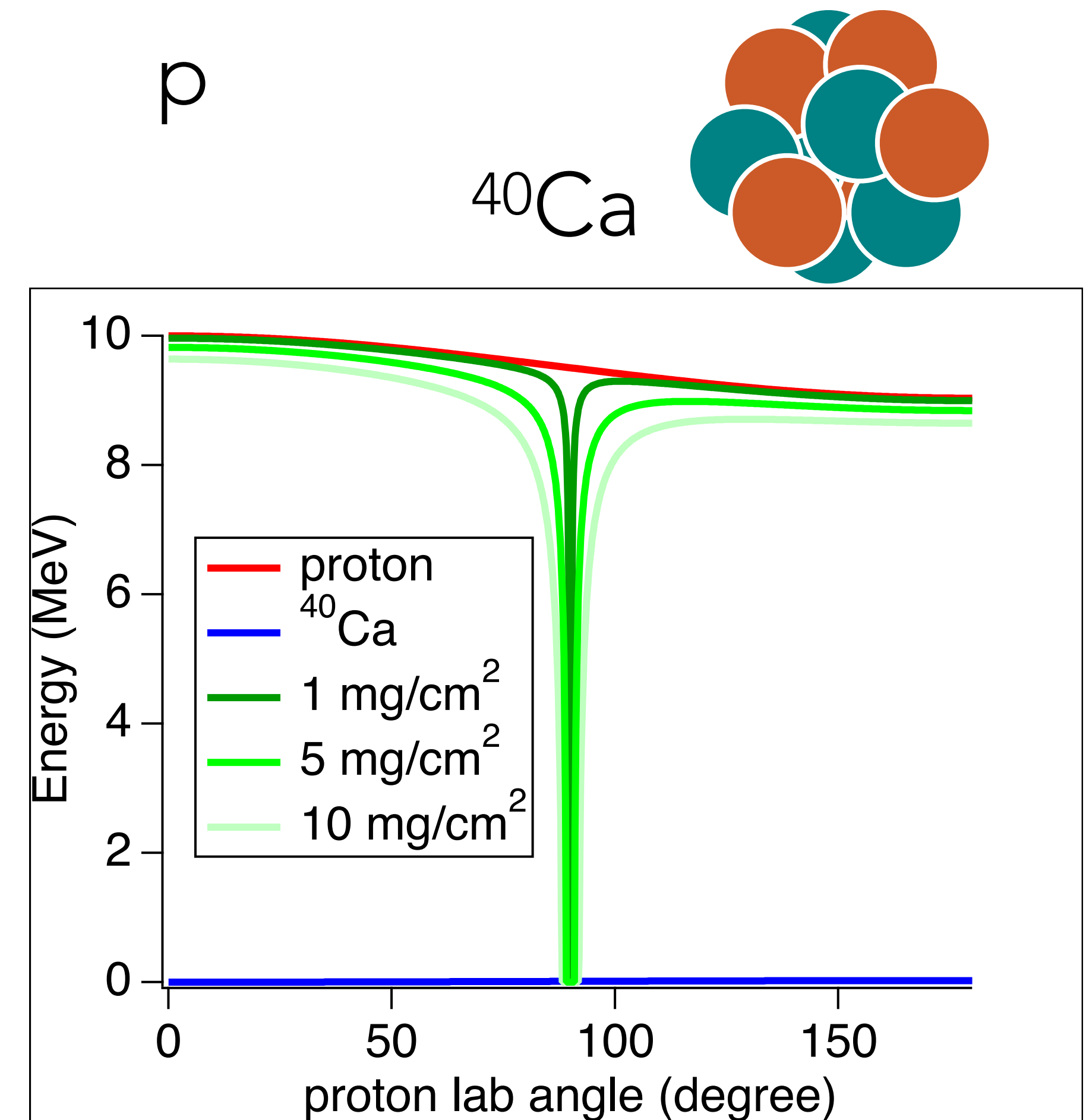
Normal kinematics

- Elastic scattering of proton on ^{40}Ca at 10 MeV
- Energy of outgoing proton almost constant
- Easily escapes the target at almost all angles
- ^{40}Ca has very low recoil energy and stays inside target



Normal kinematics

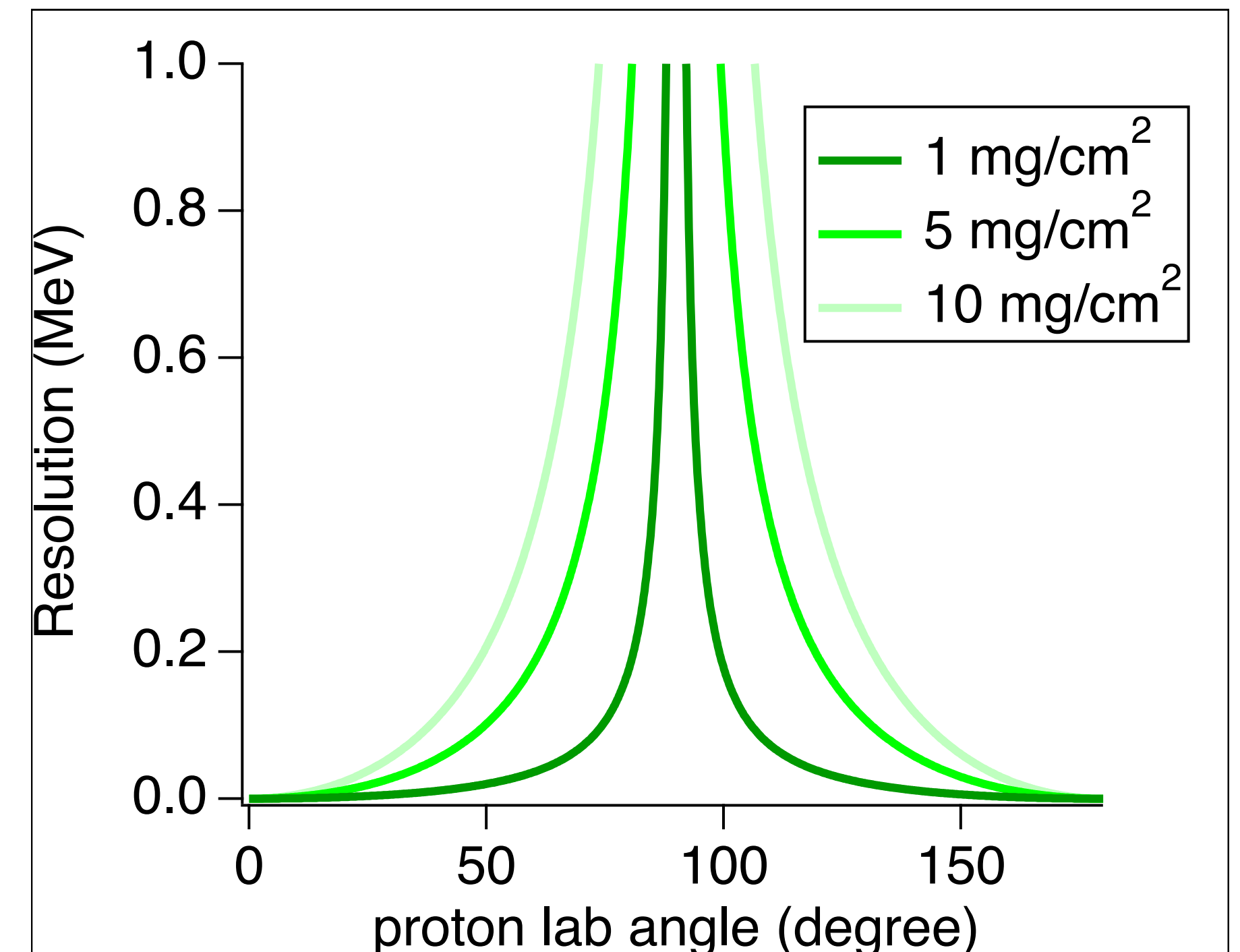
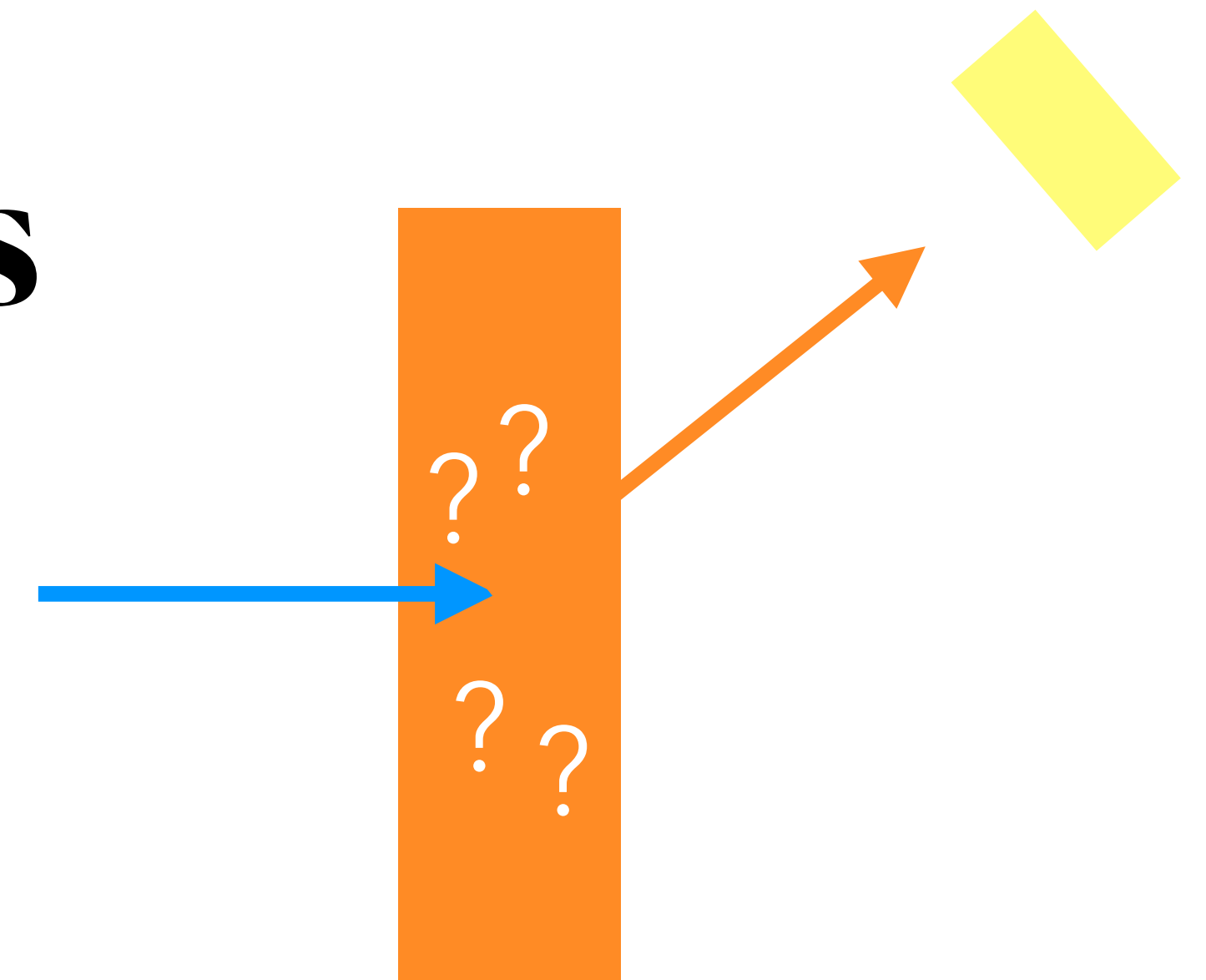
- Elastic scattering of proton on ^{40}Ca at 10 MeV
- Energy of outgoing proton almost constant
- Easily escapes the target at almost all angles
- ^{40}Ca has very low recoil energy and stays inside target



Normal kinematics

Resolution

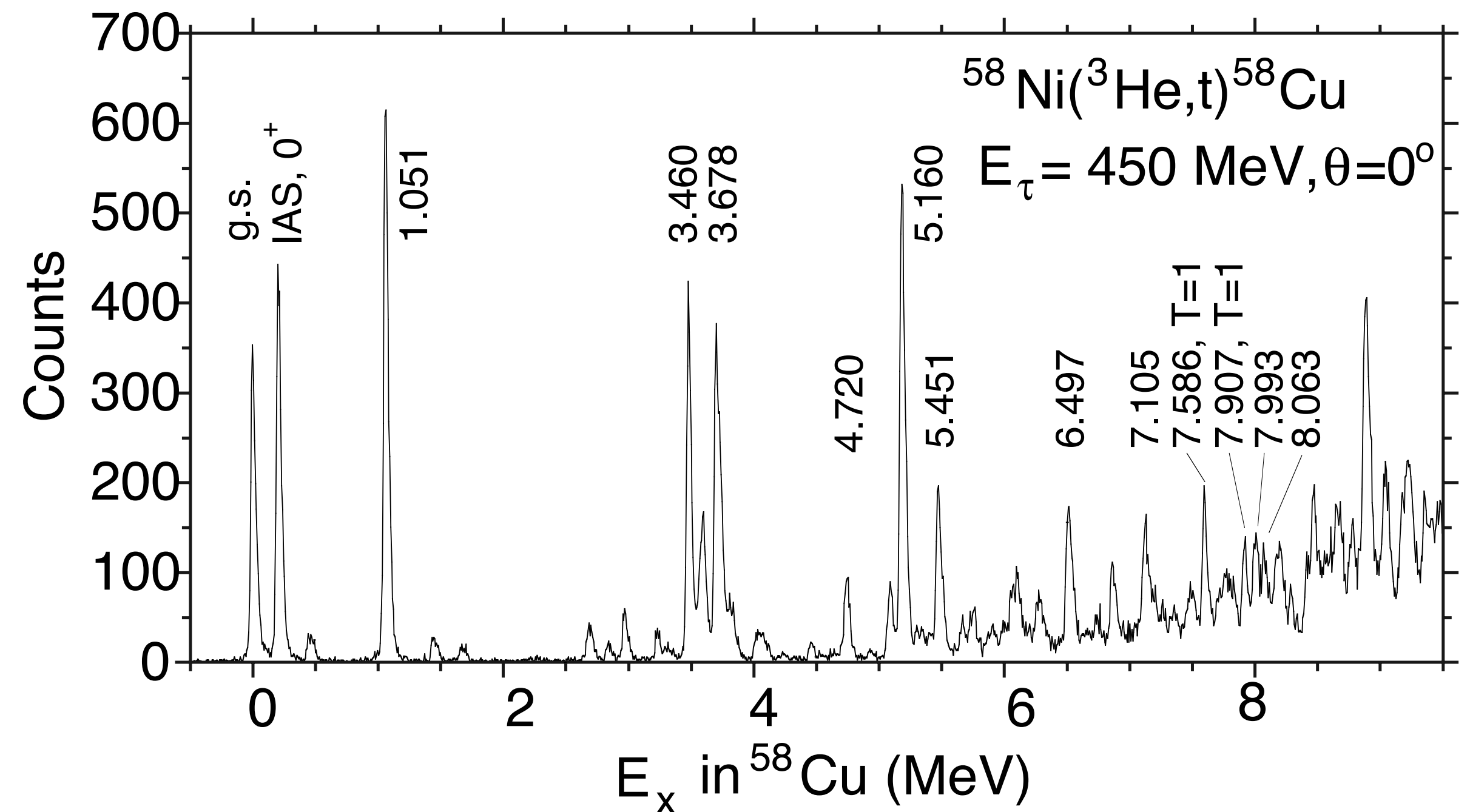
- Resolution from energy difference assuming reaction at entrance or exit
- Kinematical properties of proton not very much affected
- Resolution due to target thickness stays under control in maximum cross section region (from 0° to $\sim 60^\circ$)



Example of experiment

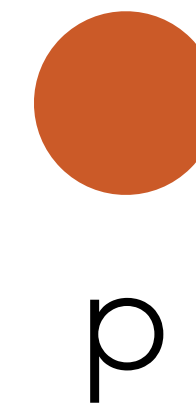
Charge-exchange ($^3\text{He},t$) on ^{58}Ni target

- State of the art direct kinematics
- $^{58}\text{Ni}(^3\text{He},t)^{58}\text{Cu}$ charge exchange at 150 MeV/u
- Grand Raiden spectrometer (RCNP Osaka, Japan)
- Resolution: 50 keV FWHM
- Excitation energies from 0 to 10 MeV in ^{58}Cu

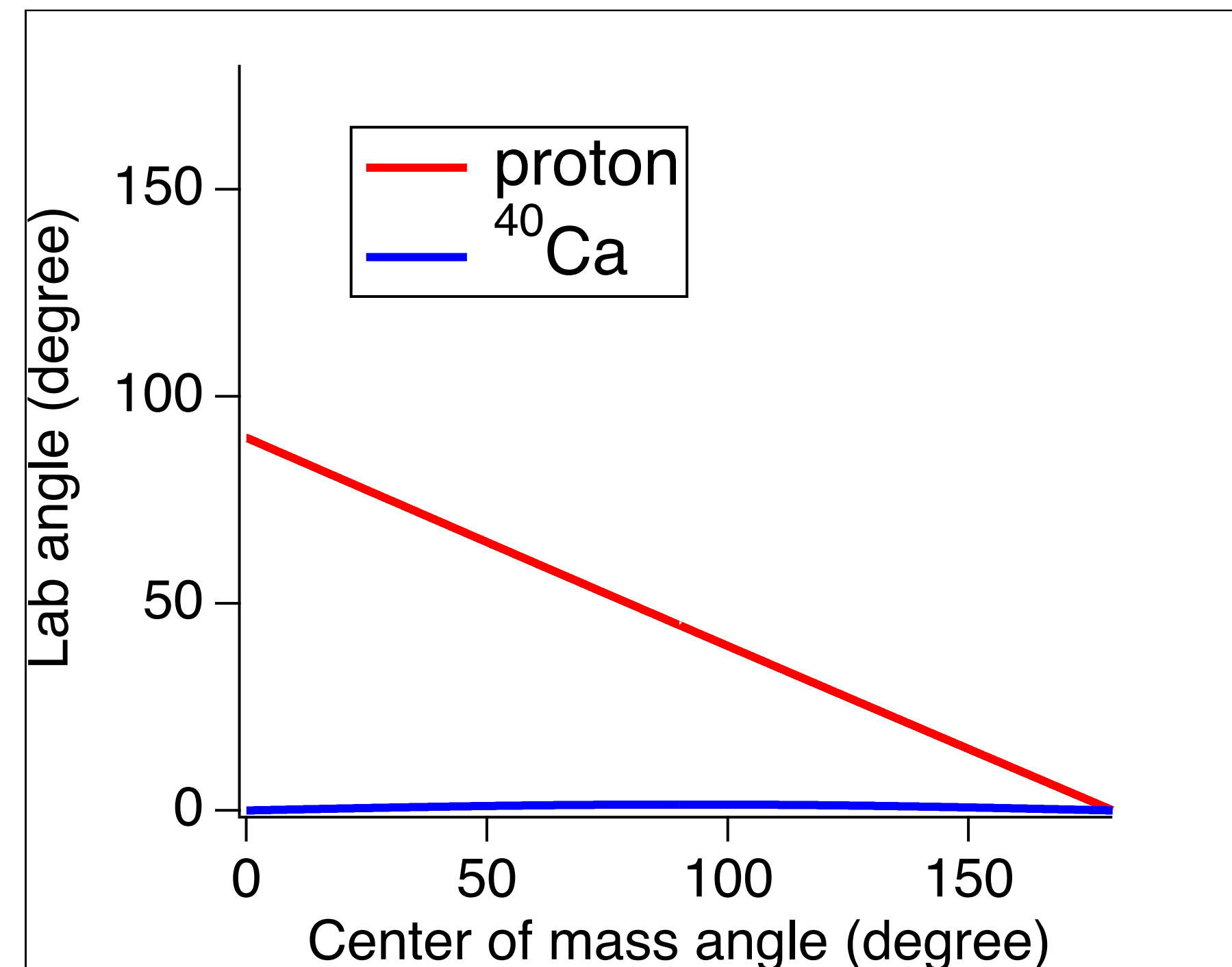


Y. Fujita et al., Eur. Phys. J. A 13, 411–418 (2002)

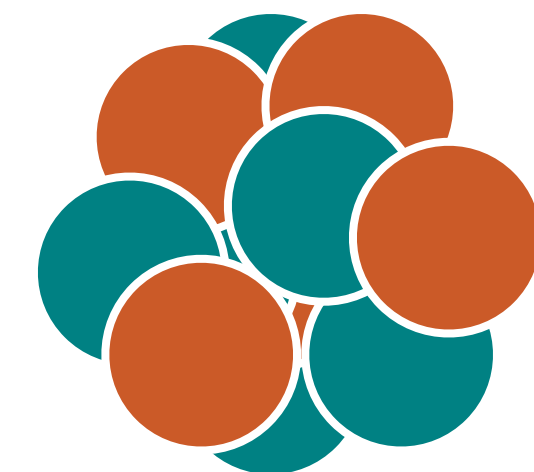
Inverse kinematics



- Beam and target roles are reversed
 - Beam now carries most of the center of mass motion
 - ^{40}Ca deflection angle very small
 - Kinematical properties of reaction can only be extracted from proton
- Energies drastically different
 - Proton energy varies from 0 to 40 MeV
 - Maximum cross section region is cut off (close to 90° in lab)



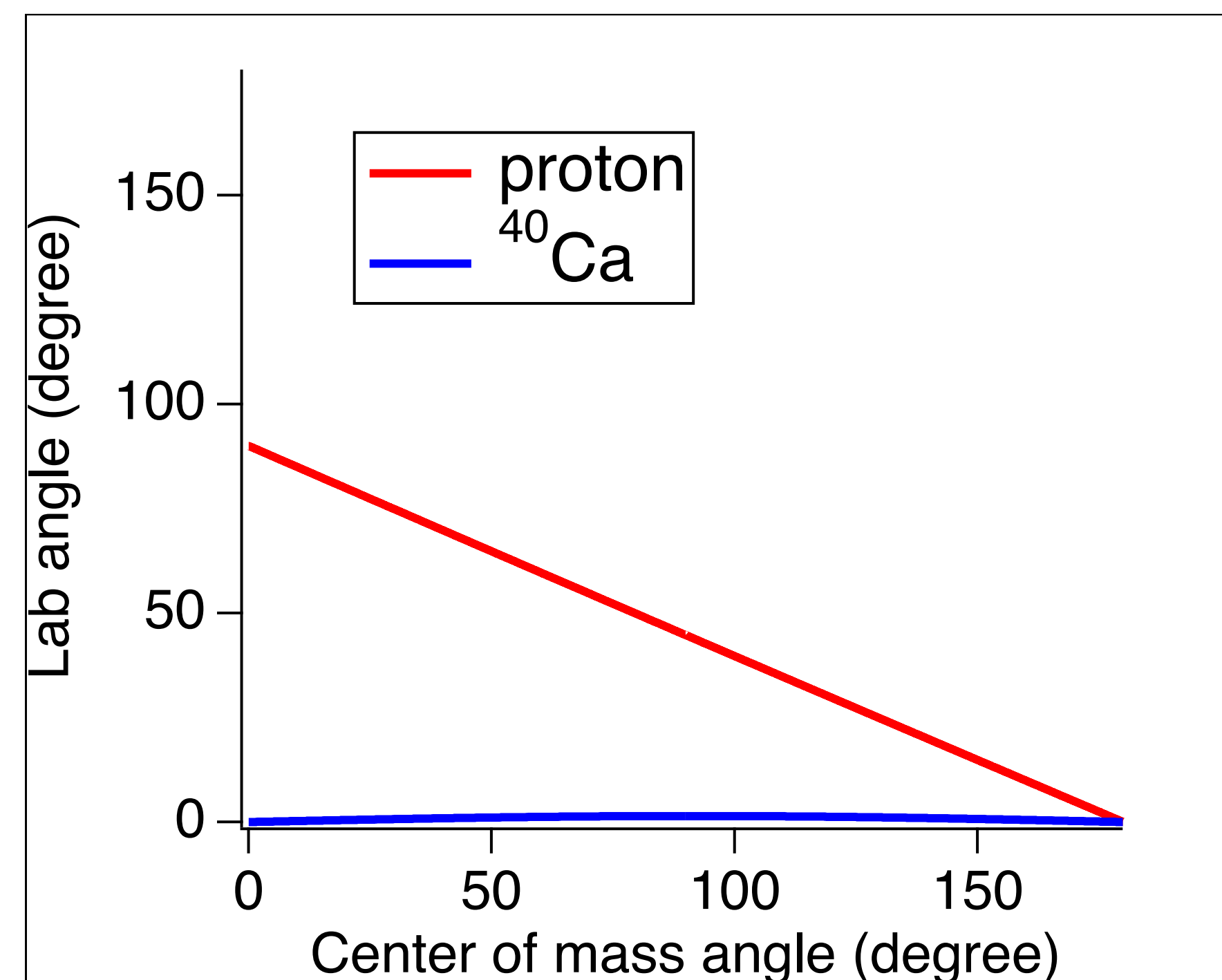
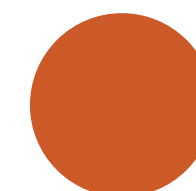
Inverse kinematics



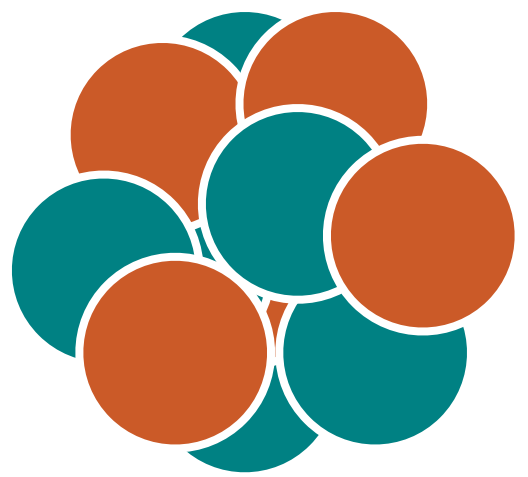
^{40}Ca

- Beam and target roles are reversed
 - Beam now carries most of the center of mass motion
 - ^{40}Ca deflection angle very small
 - Kinematical properties of reaction can only be extracted from proton
- Energies drastically different
 - Proton energy varies from 0 to 40 MeV
 - Maximum cross section region is cut off (close to 90° in lab)

p



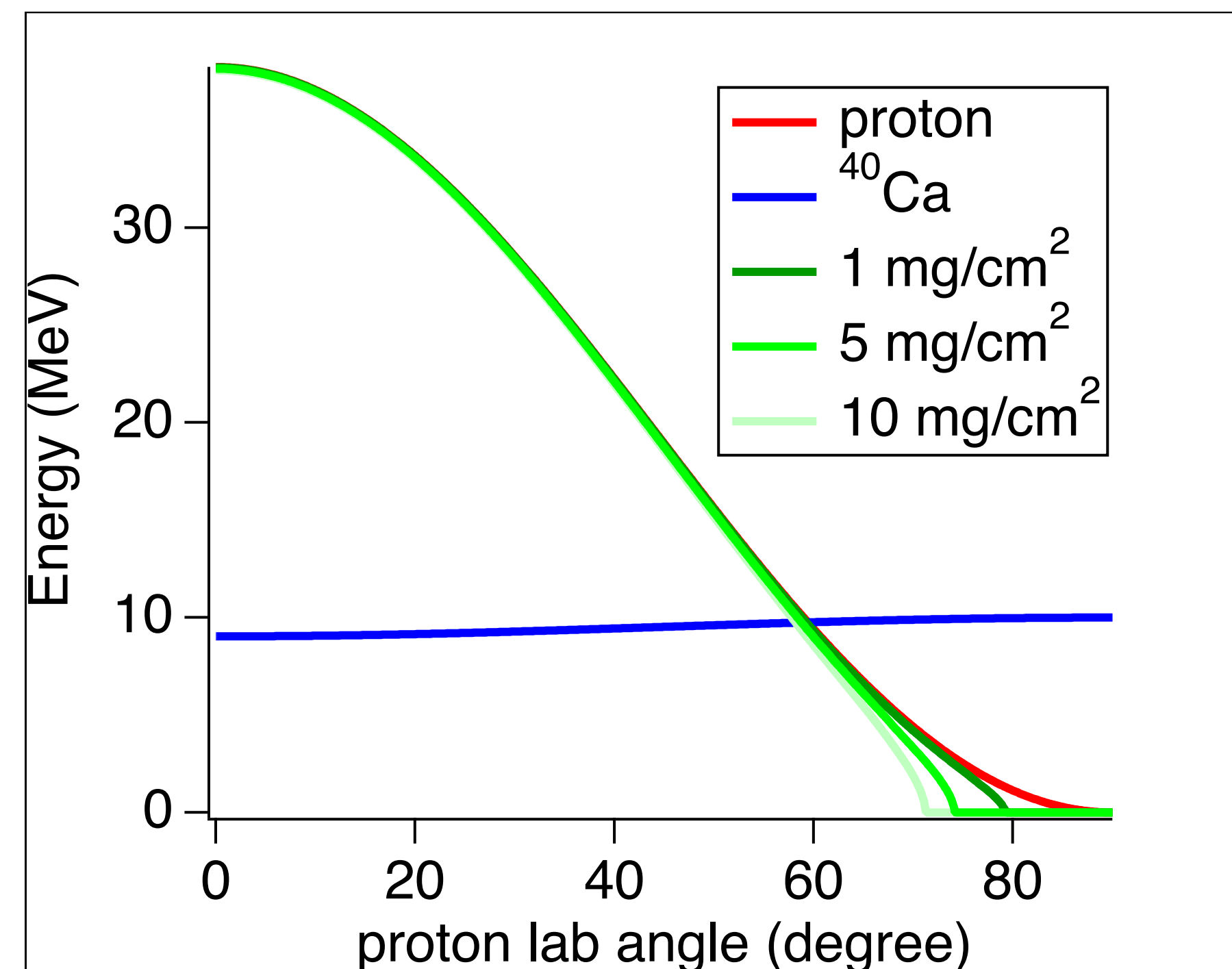
Inverse kinematics



^{40}Ca

- Beam and target roles are reversed
 - Beam now carries most of the center of mass motion
 - ^{40}Ca deflection angle very small
 - Kinematical properties of reaction can only be extracted from proton
- Energies drastically different
 - Proton energy varies from 0 to 40 MeV
 - Maximum cross section region is cut off (close to 90° in lab)

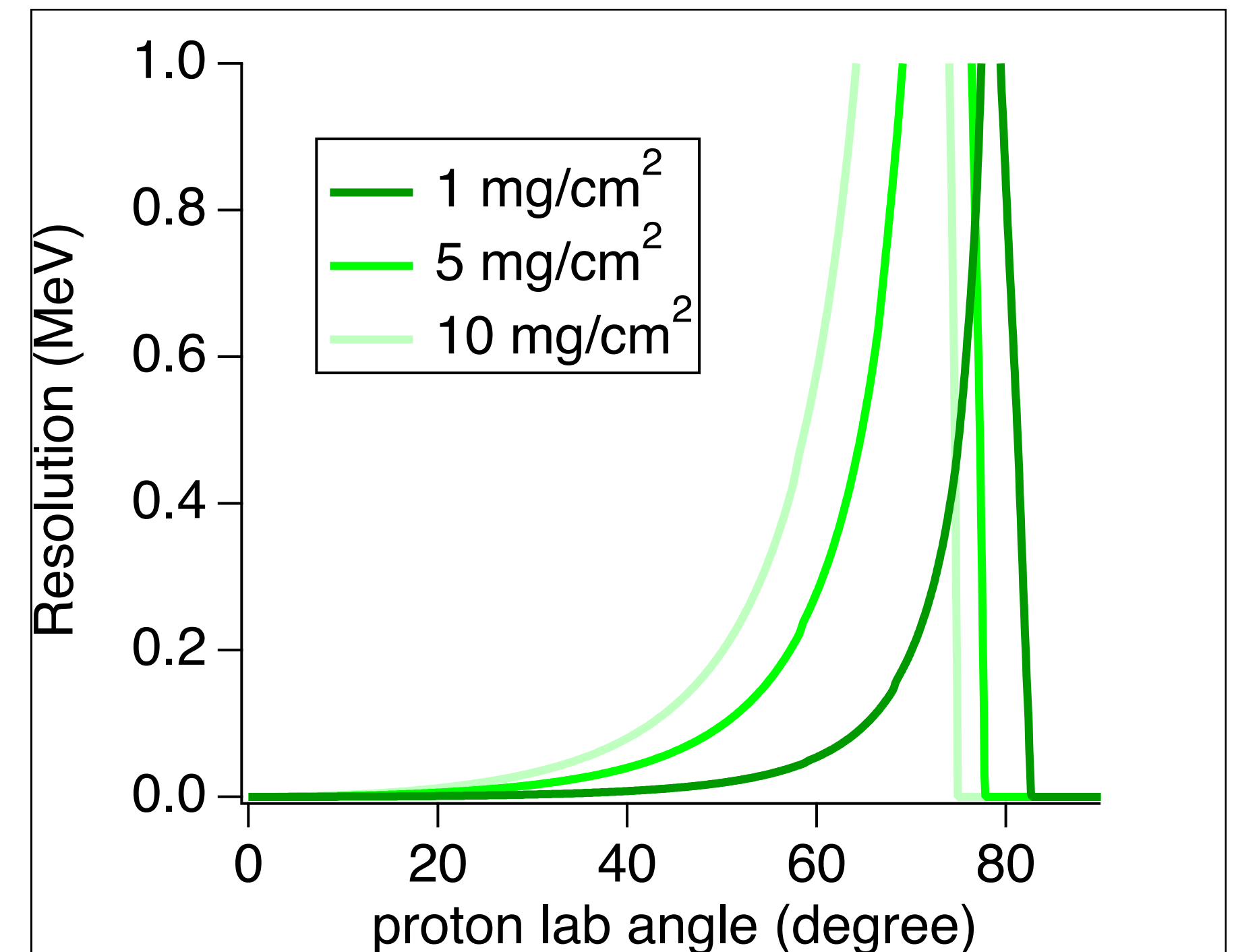
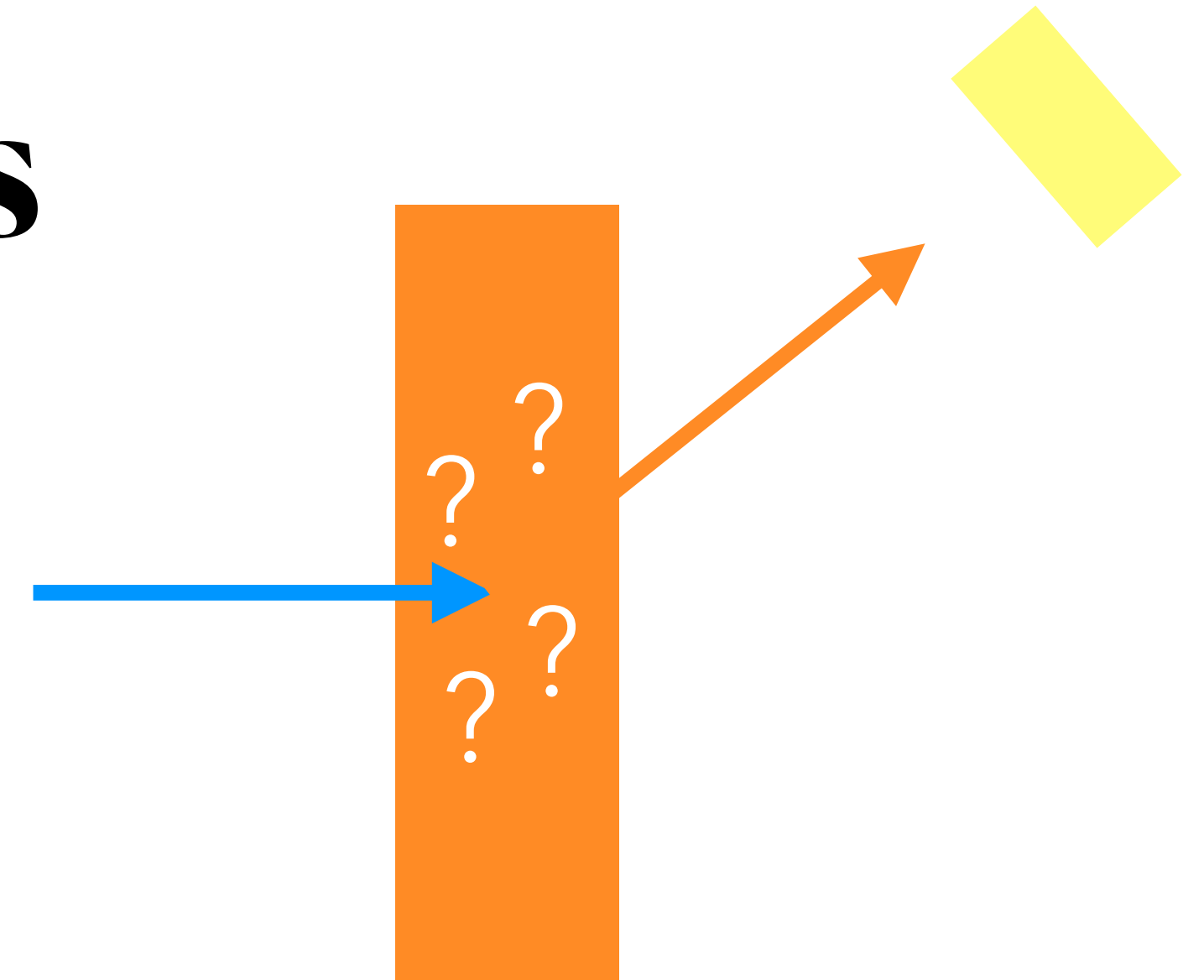
p



Inverse kinematics

Resolution

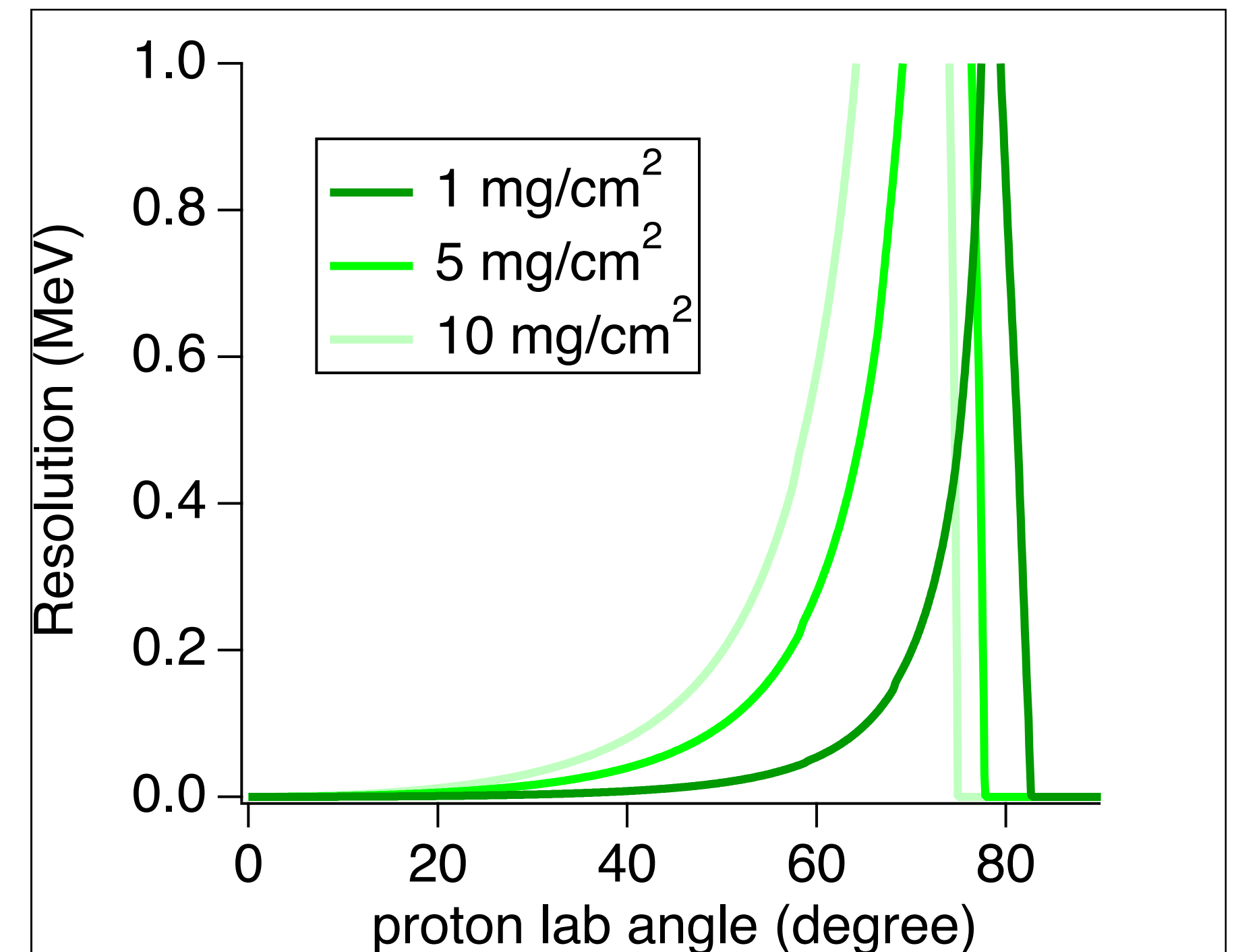
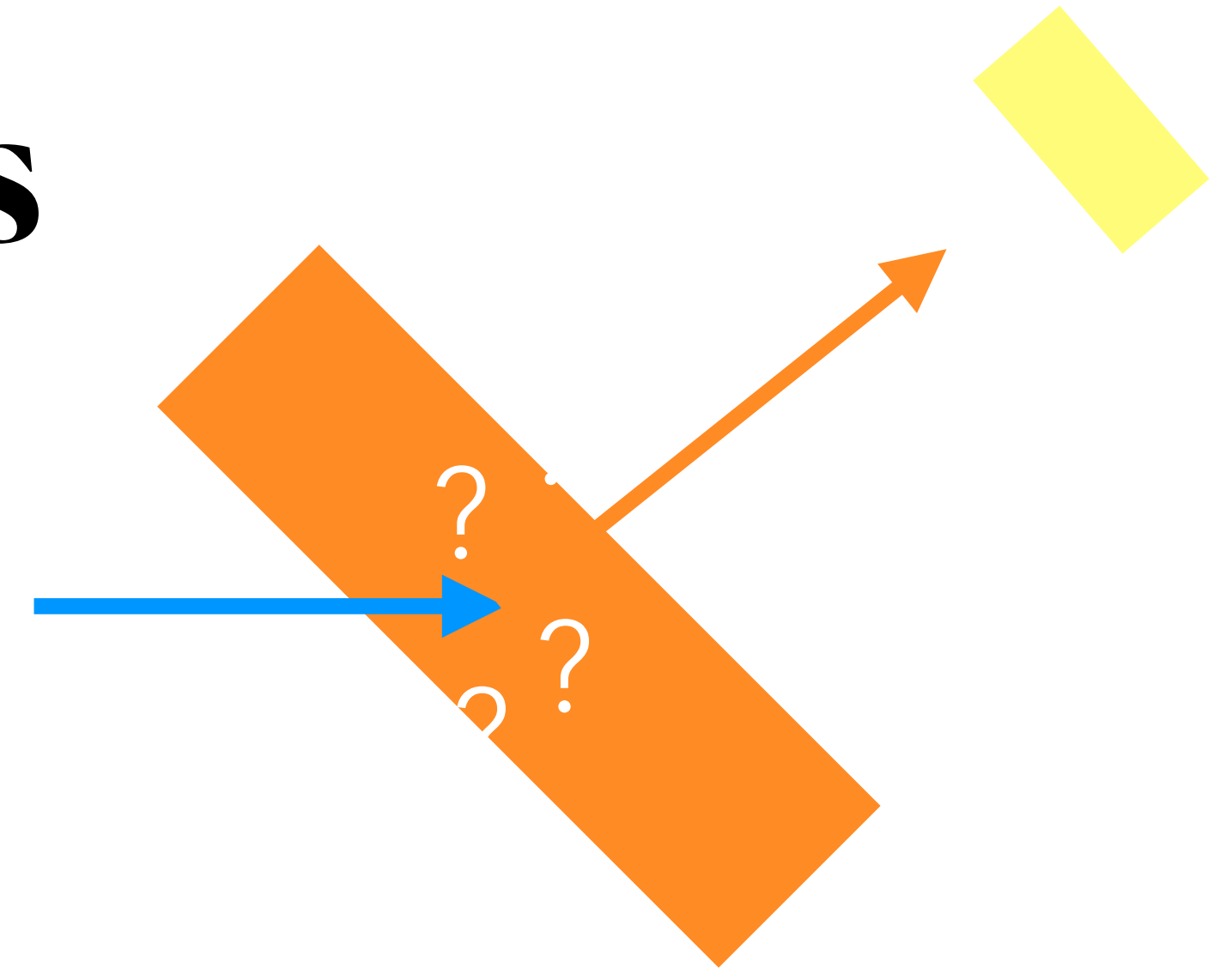
- Resolution due to unknown reaction site
- Worst close to the region of largest cross section
- Clever trick: tilt target by 45°
- Some improvement, but still not as good as direct kinematics
- Similar resolution only achieved by reducing target thickness



Inverse kinematics

Resolution

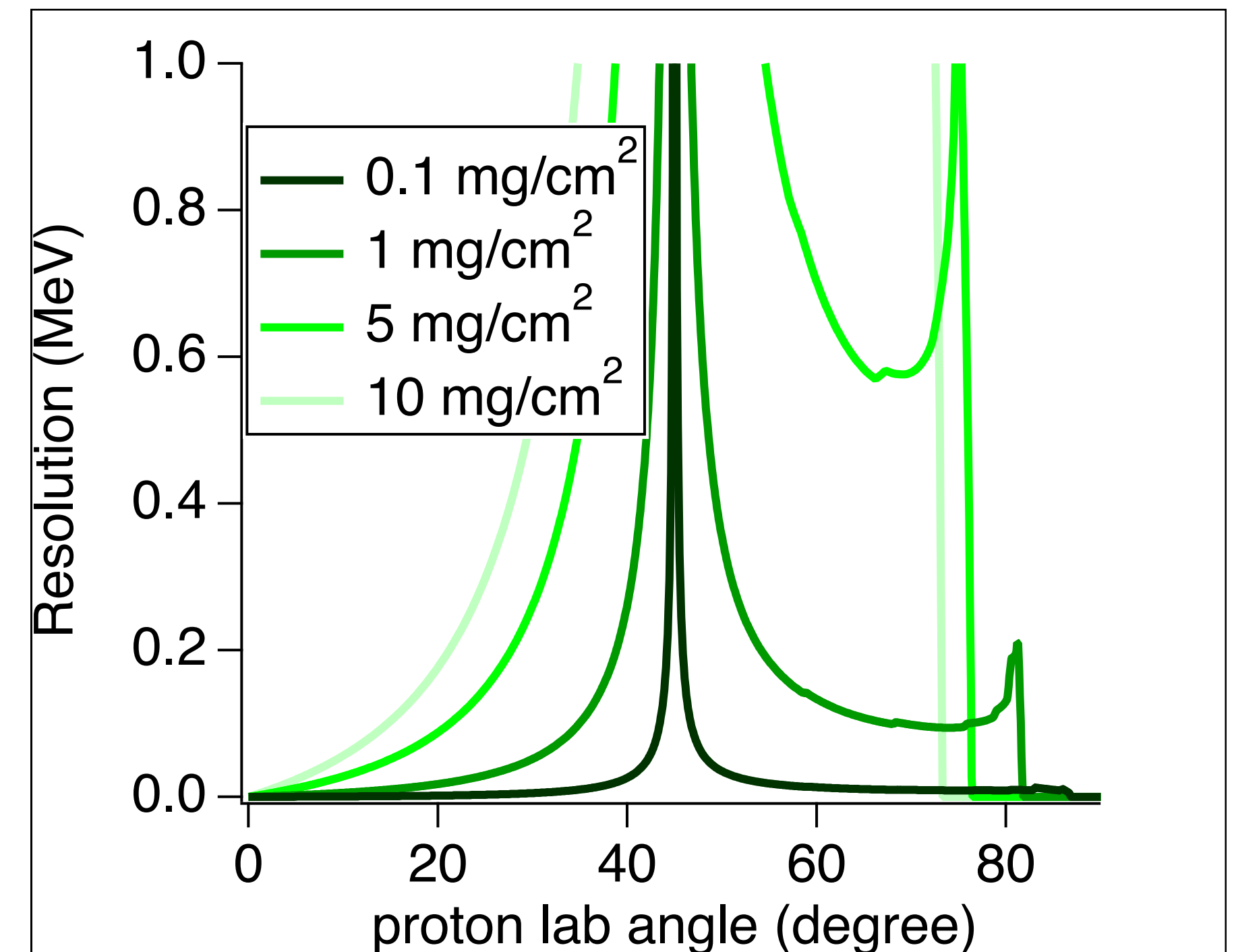
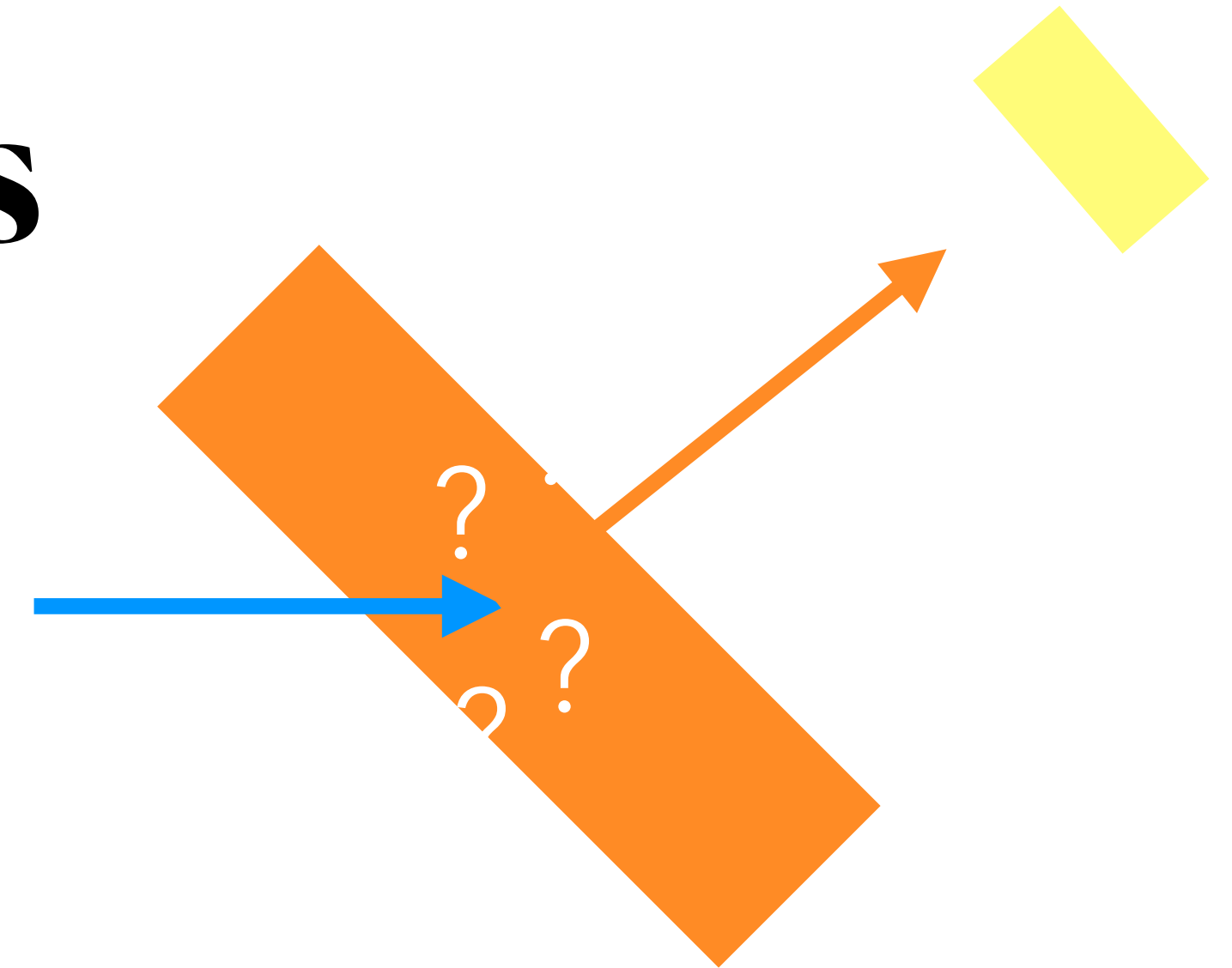
- Resolution due to unknown reaction site
- Worst close to the region of largest cross section
- Clever trick: tilt target by 45°
- Some improvement, but still not as good as direct kinematics
- Similar resolution only achieved by reducing target thickness



Inverse kinematics

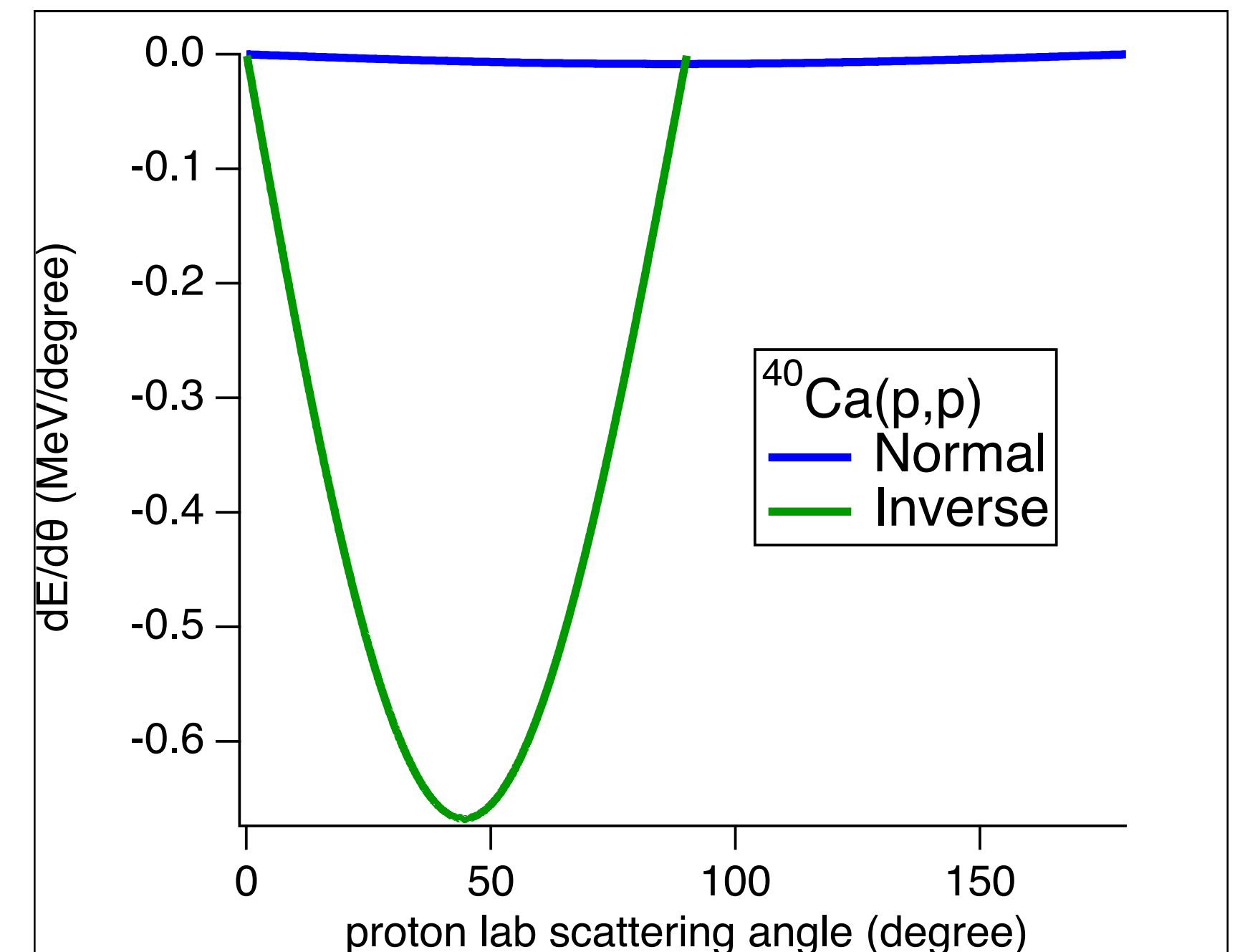
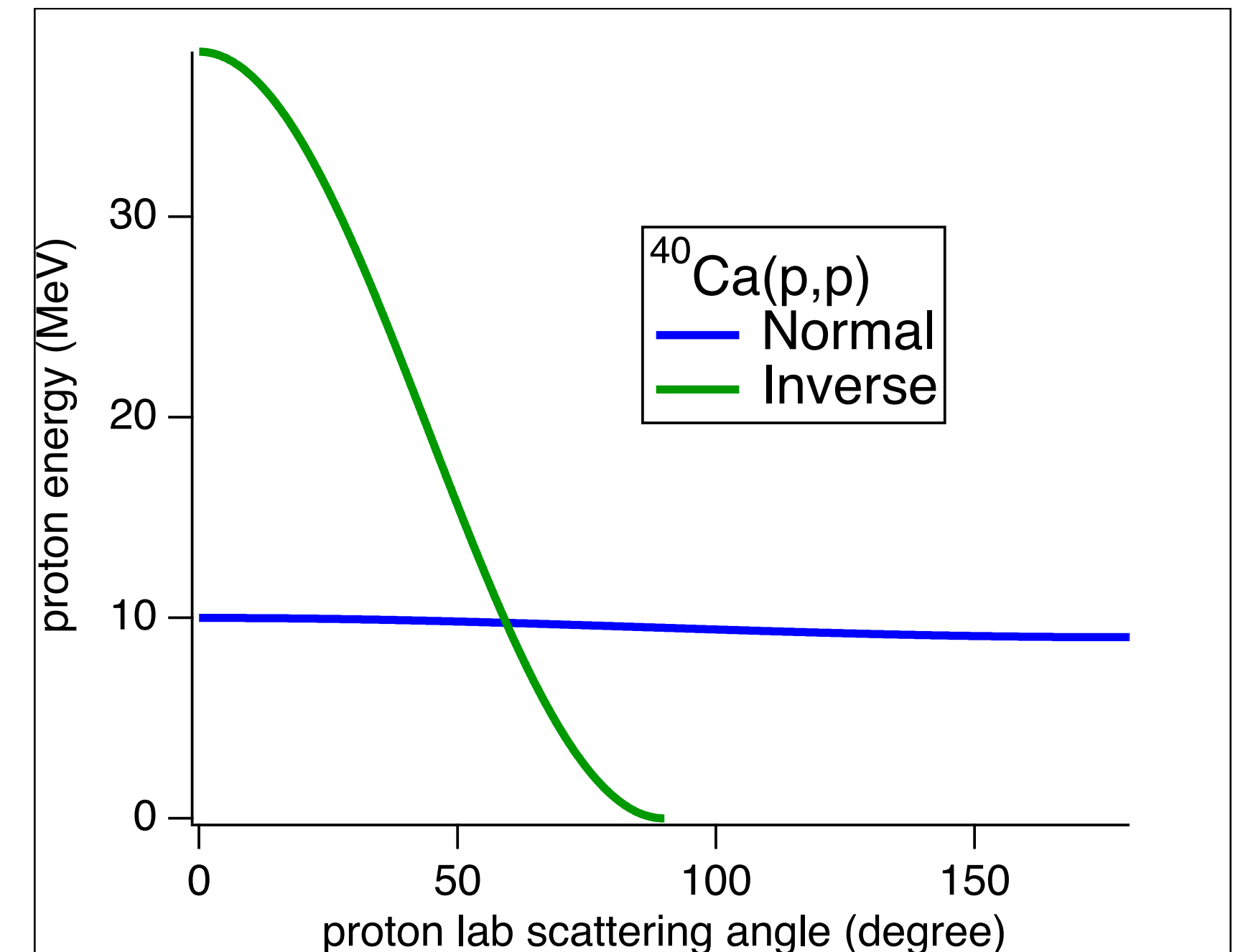
Resolution

- Resolution due to unknown reaction site
- Worst close to the region of largest cross section
- Clever trick: tilt target by 45°
- Some improvement, but still not as good as direct kinematics
- Similar resolution only achieved by reducing target thickness



It gets worse...

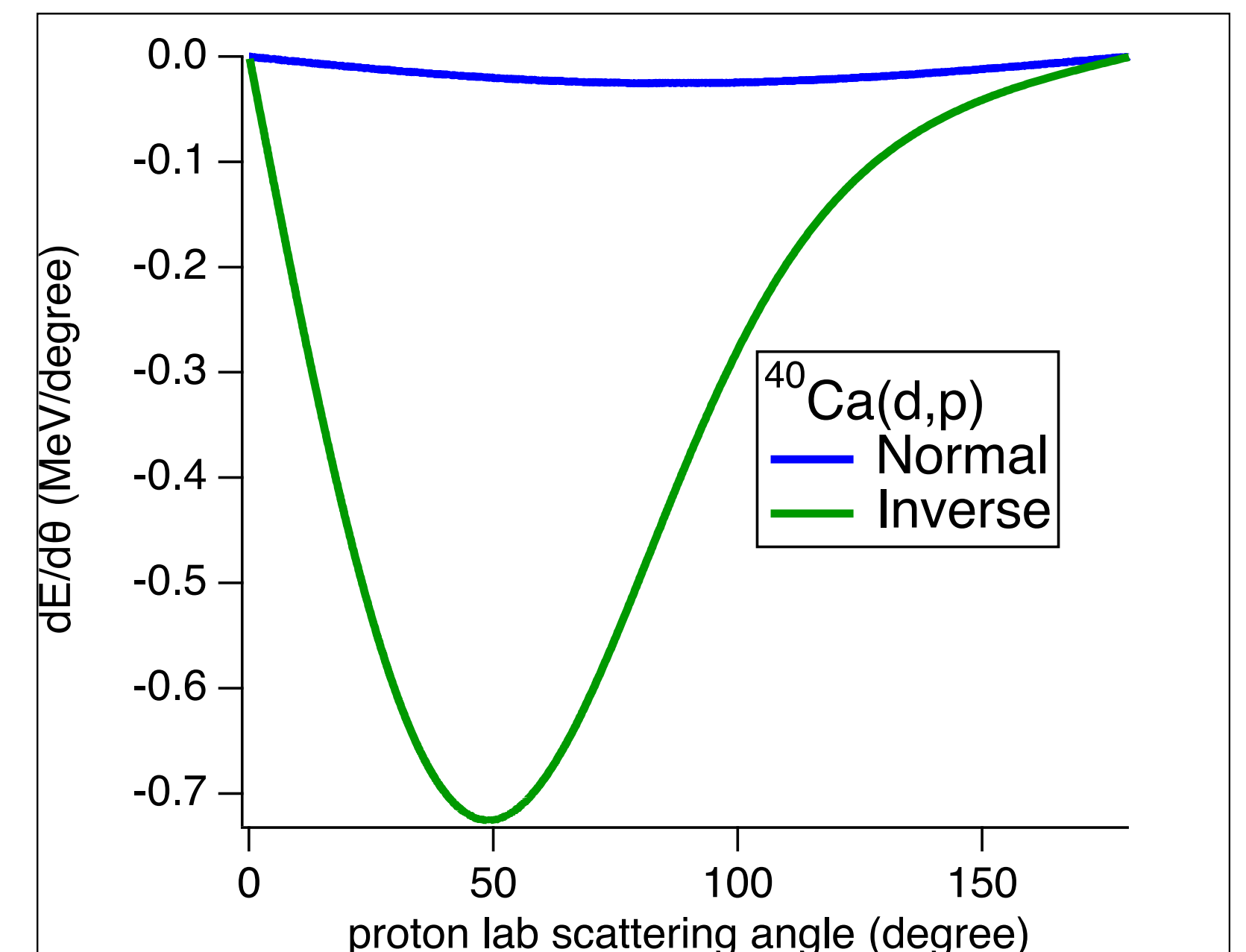
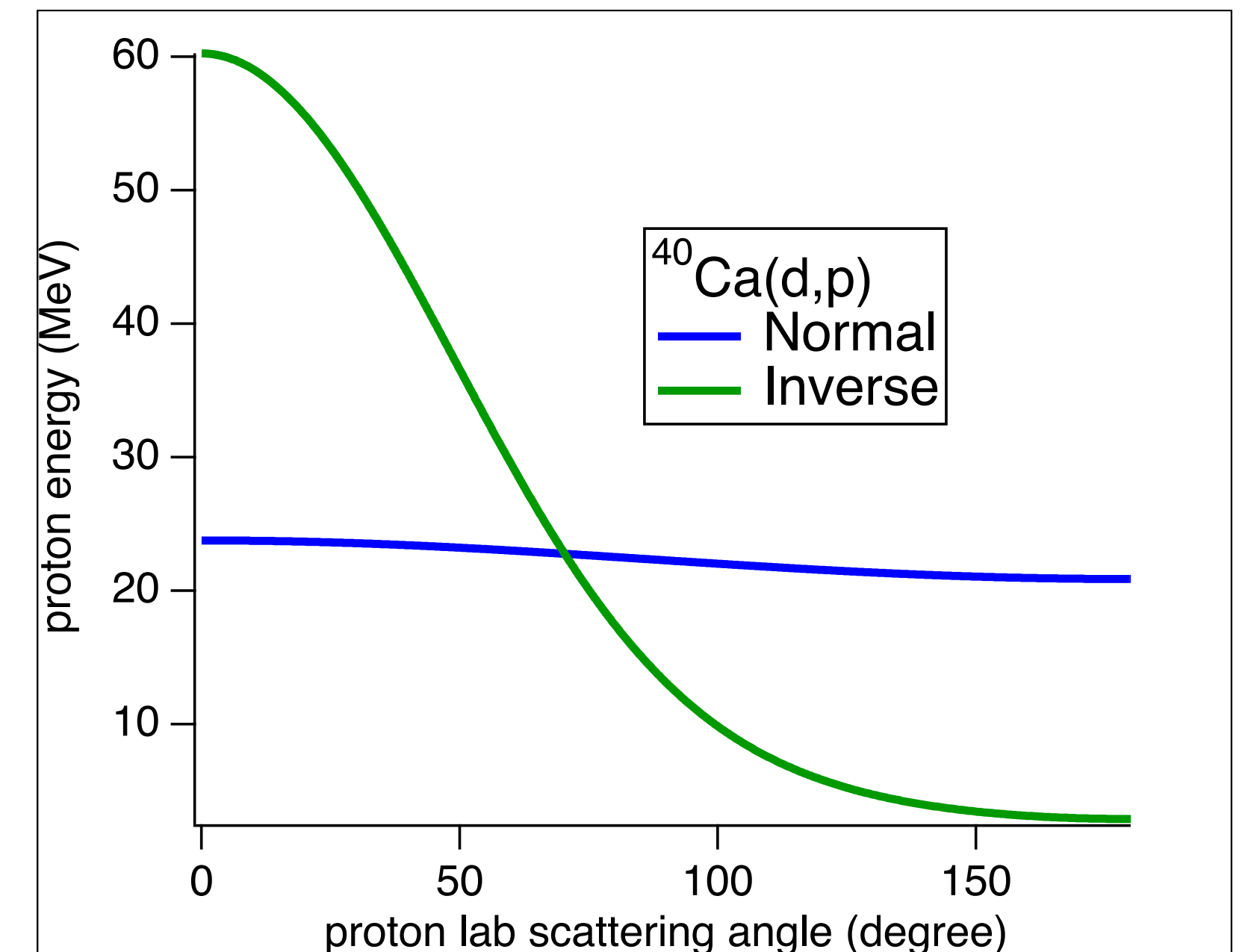
- Angular dependence of energy
 - Kinematical corrections are strongly dependent on scattering angle
 - Energy resolution highly constrained by angular resolution (derivative ~ 2 orders of magnitude larger)
- Three factors against inverse kinematics
 - Reaction site unknown
 - Angular dependence of energy
 - Beam intensities



A weird kinematics...

(d,p) transfer in inverse kinematics

- Consider $^{40}\text{Ca}(\text{d},\text{p})^{41}\text{Ca}$ at 10 MeV/u
- Similar differences between normal and inverse kinematics
- Proton emitted backwards in inverse kinematics
- Lowest proton energy is 2.9 MeV

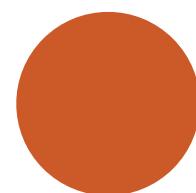


A weird kinematics...

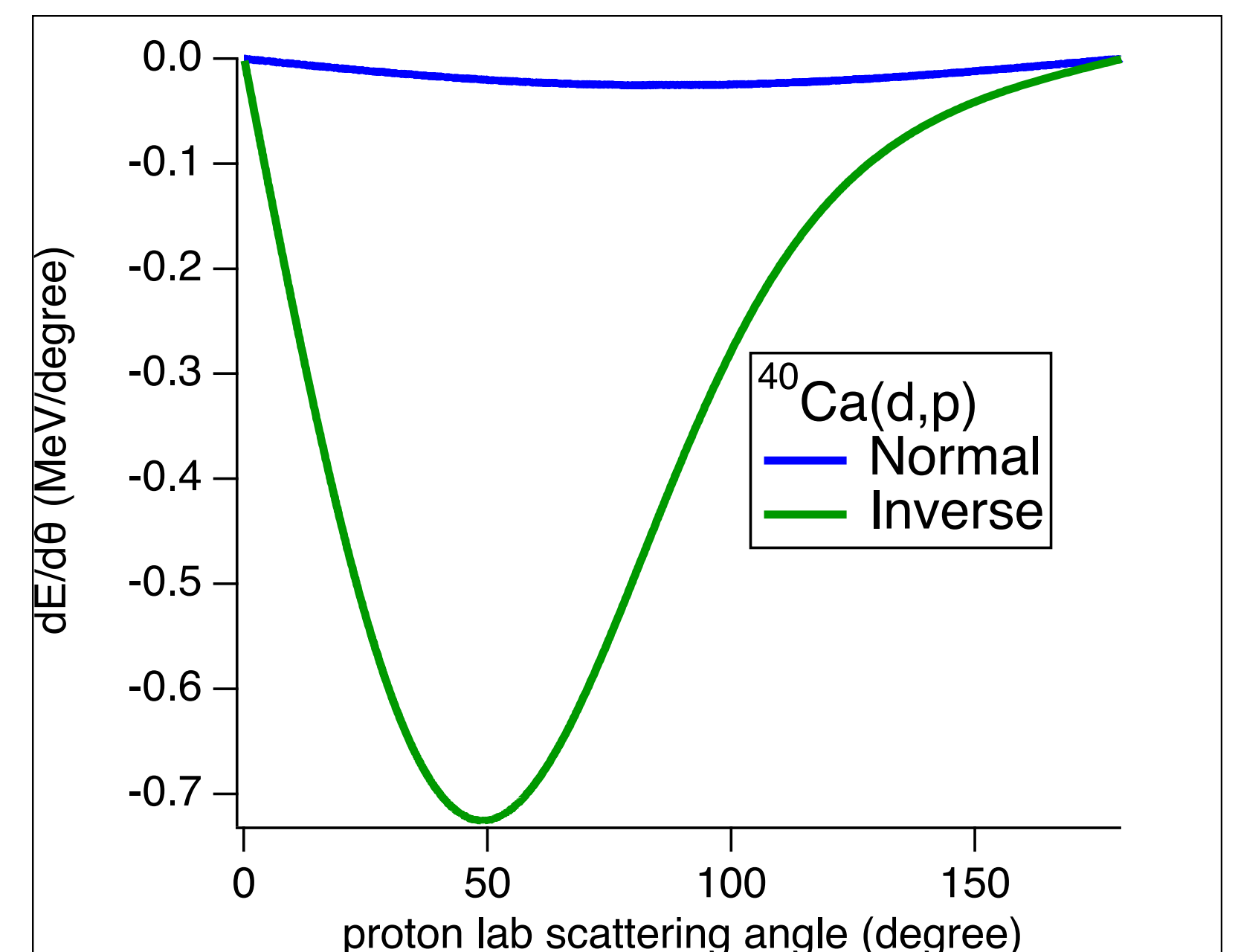
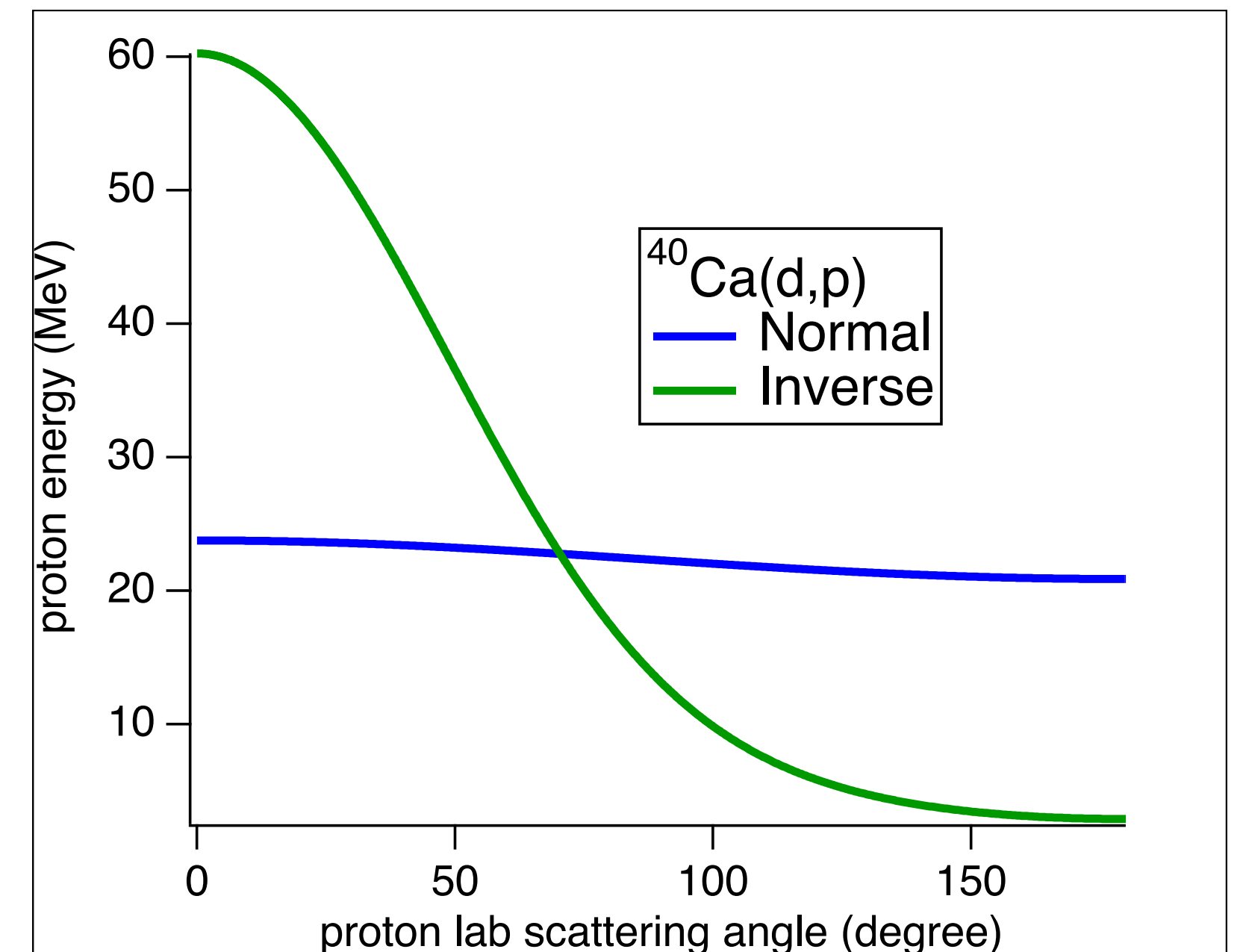
(d,p) transfer in inverse kinematics

- Consider $^{40}\text{Ca}(\text{d},\text{p})^{41}\text{Ca}$ at 10 MeV/u
- Similar differences between normal and inverse kinematics
- Proton emitted backwards in inverse kinematics
- Lowest proton energy is 2.9 MeV

^{40}Ca



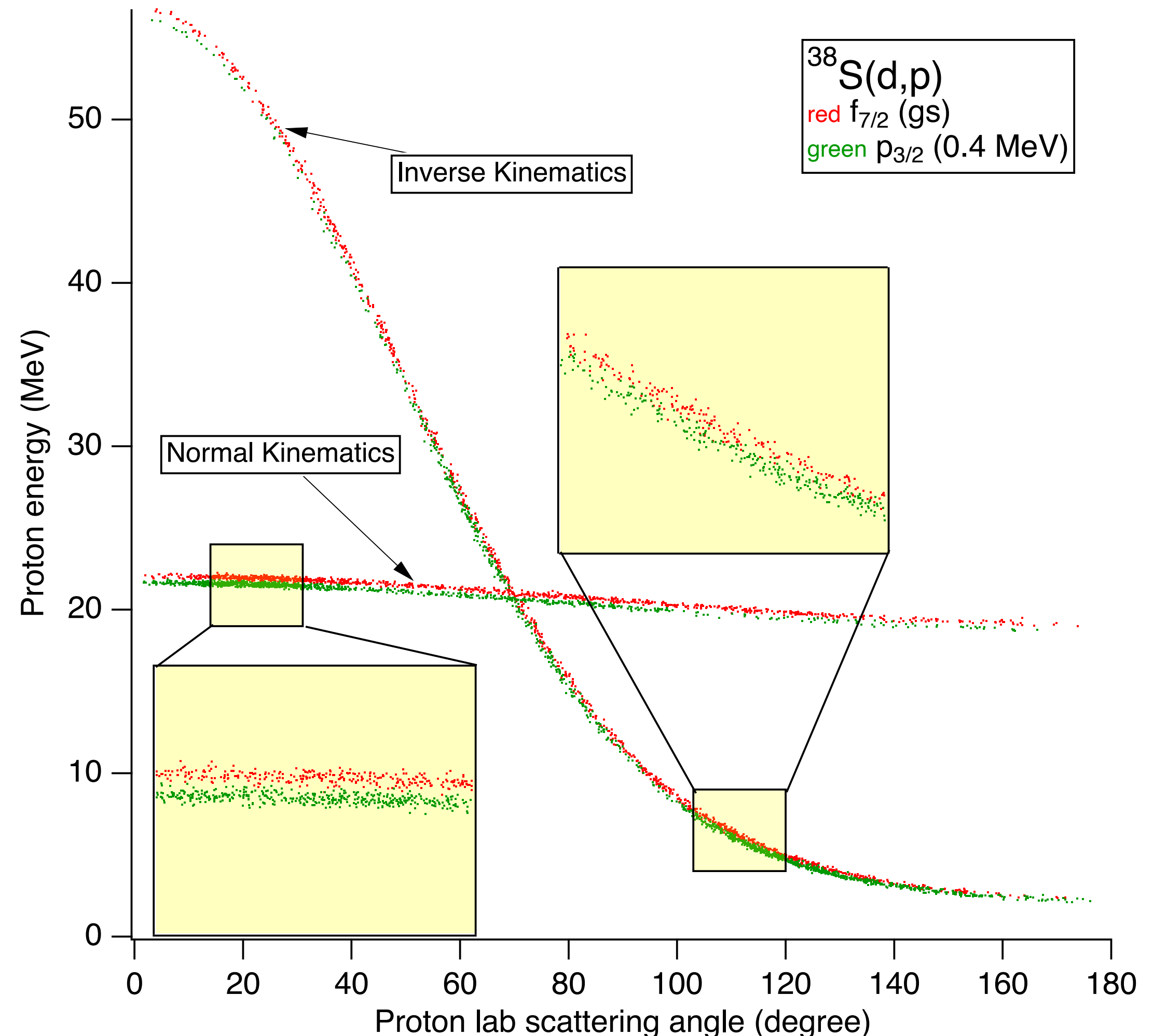
d



Inverse kinematics (d,p) reaction

A simulation

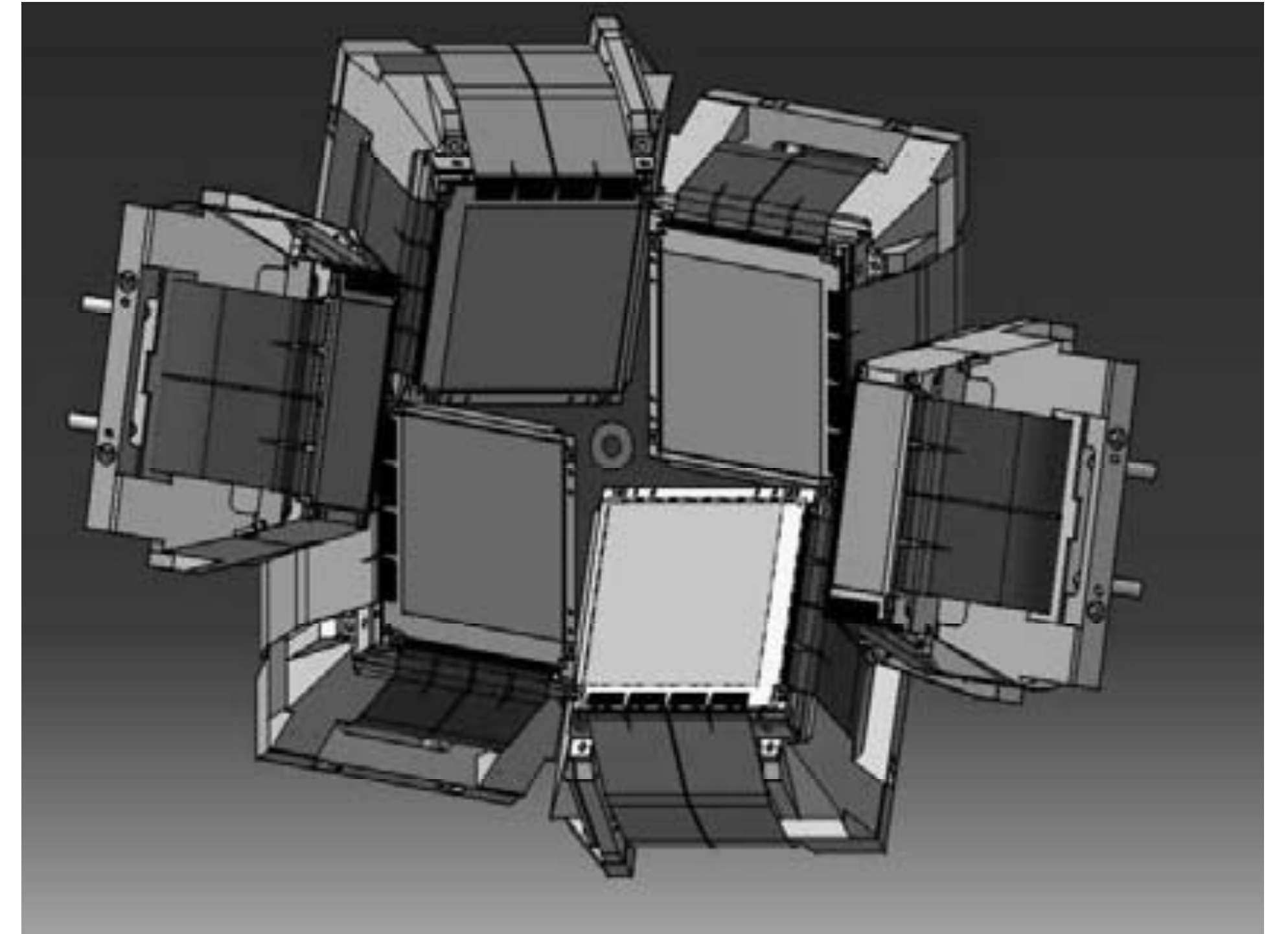
- Two final states populated in ^{39}S
- 400 keV between them
- Energy and angular resolutions the same for both normal and inverse
 - 200 keV in energy, 1° in scattering angle
- States easily separated in normal kinematics, not in inverse
- Much better angular resolution required in inverse kinematics



Passive target setups

Inverse kinematics is a major experimental challenge

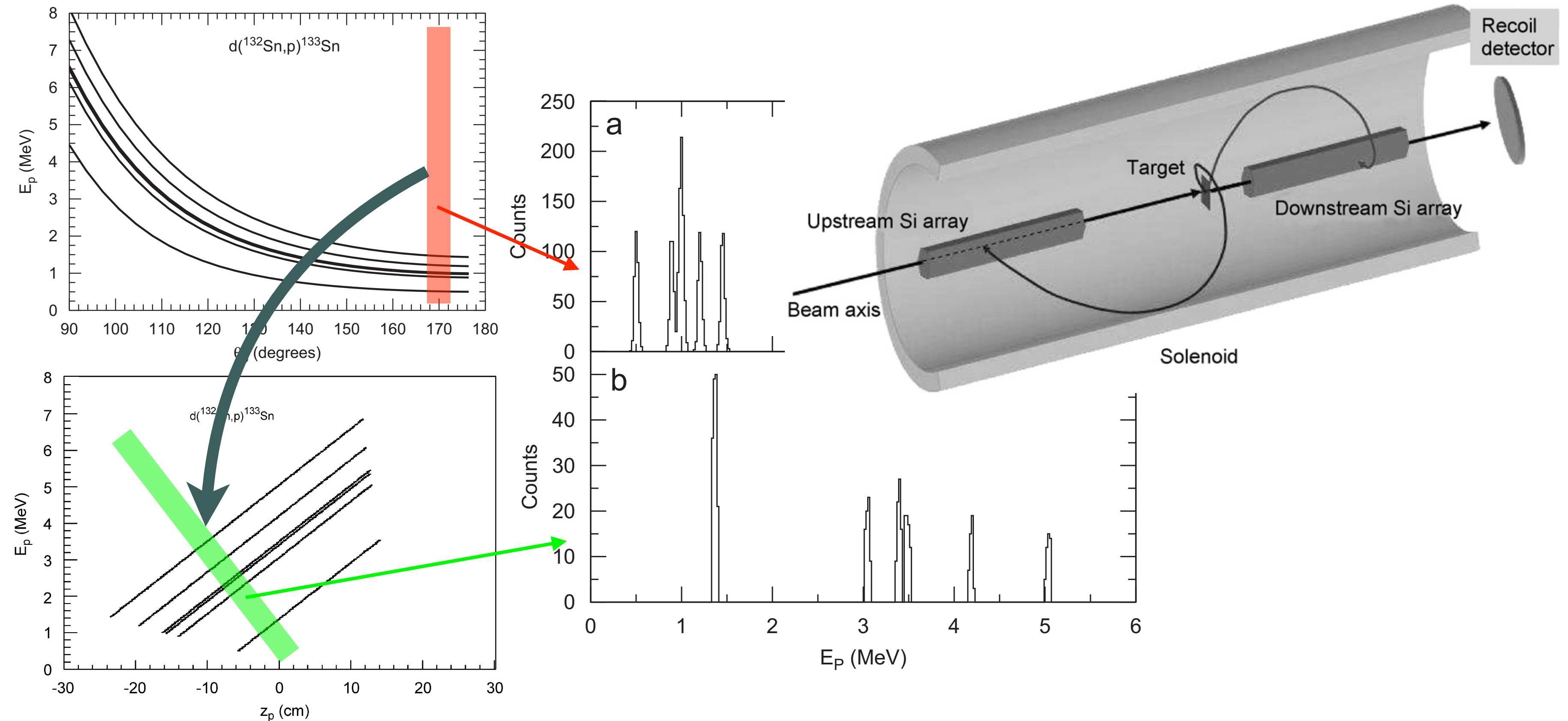
- Two compromises to deal with
 - Target thickness vs energy resolution
 - Solid angle coverage vs angular resolution
 - Cost vs two previous items
- Low radioactive beam intensities
- Luminosity and resolutions are limited
- Pushing forward these limitations requires new detector concepts



HELIOS: solving the inverse kinematics

Solenoid spectrometer directly measures center-of-mass energies

- Large angular acceptance within solenoid boundaries
- Compromise between target thickness and energy resolution still present

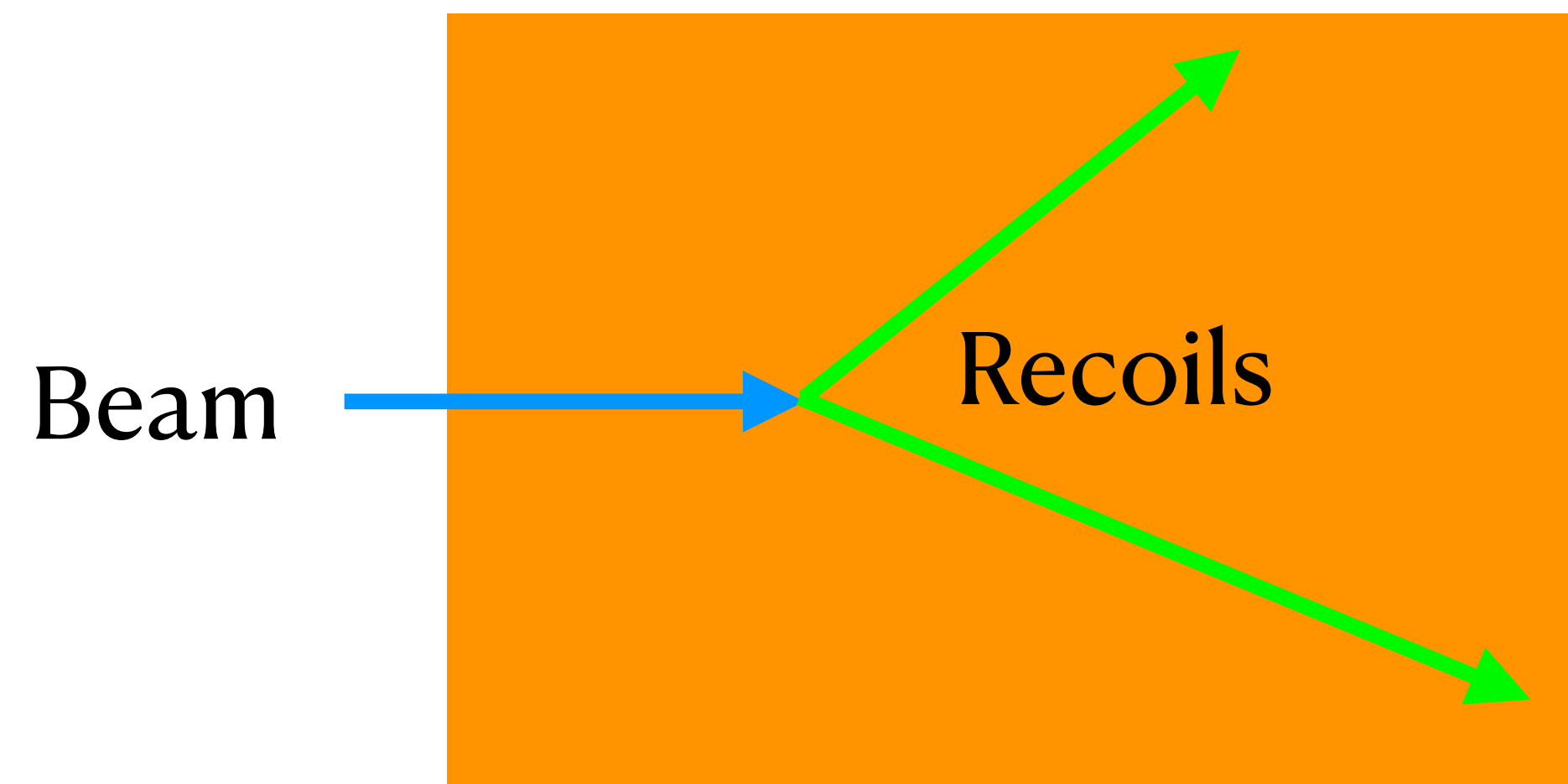


A. Wuosmaa et al., NIMA 507, 1290 (2007)

The promise of active targets

Erasing both compromises

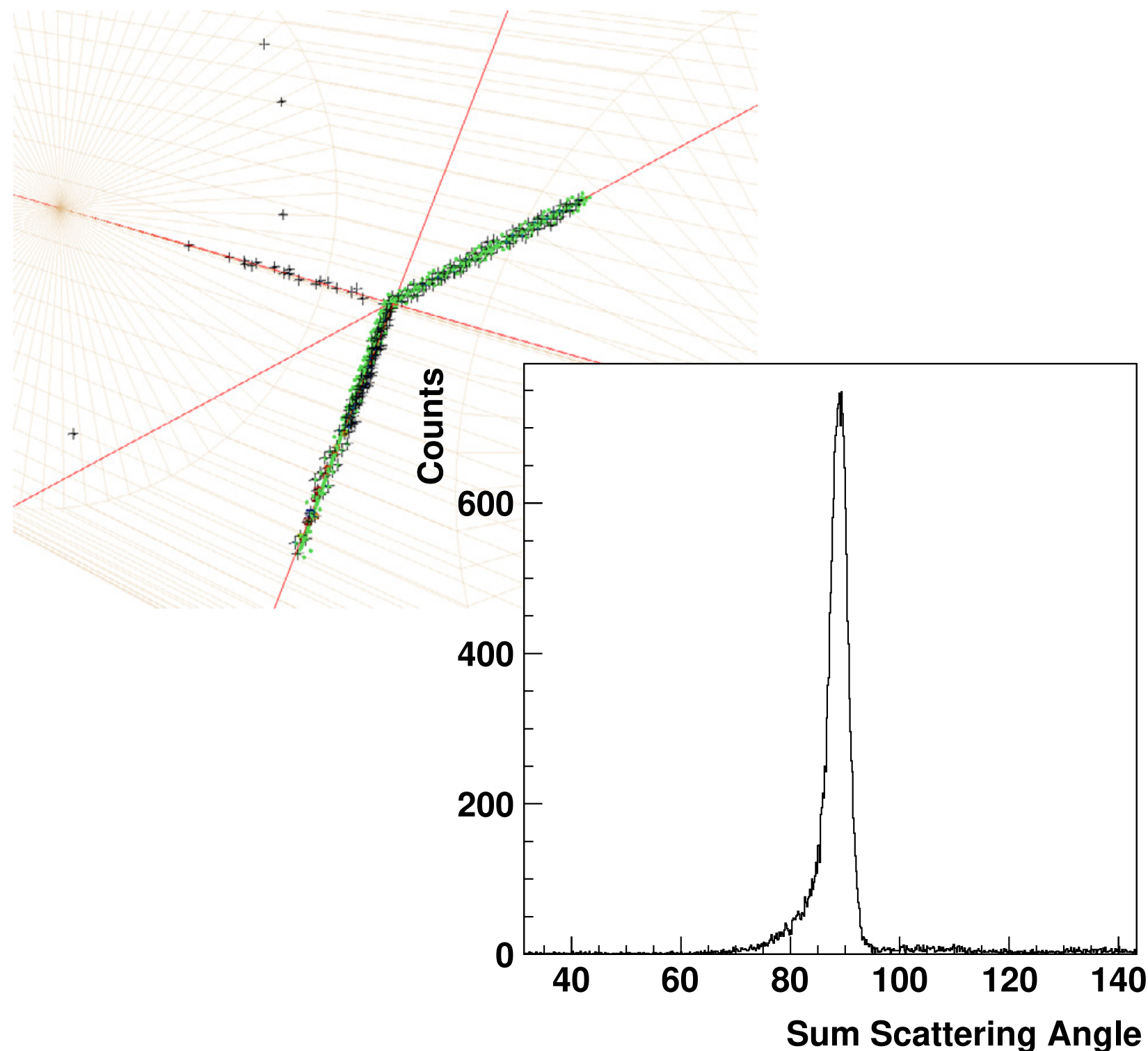
- Target thickness not limited by energy resolution
 - Vertex and energy of reaction known
- Solid angle coverage not limited by angular resolution and/or cost
 - Detecting recoils inside target maximizes angular coverage
- Inverse kinematics requirements
 - Need angular resolutions $< 1^\circ$
 - Need energy resolutions < 200 keV



Target = Detector

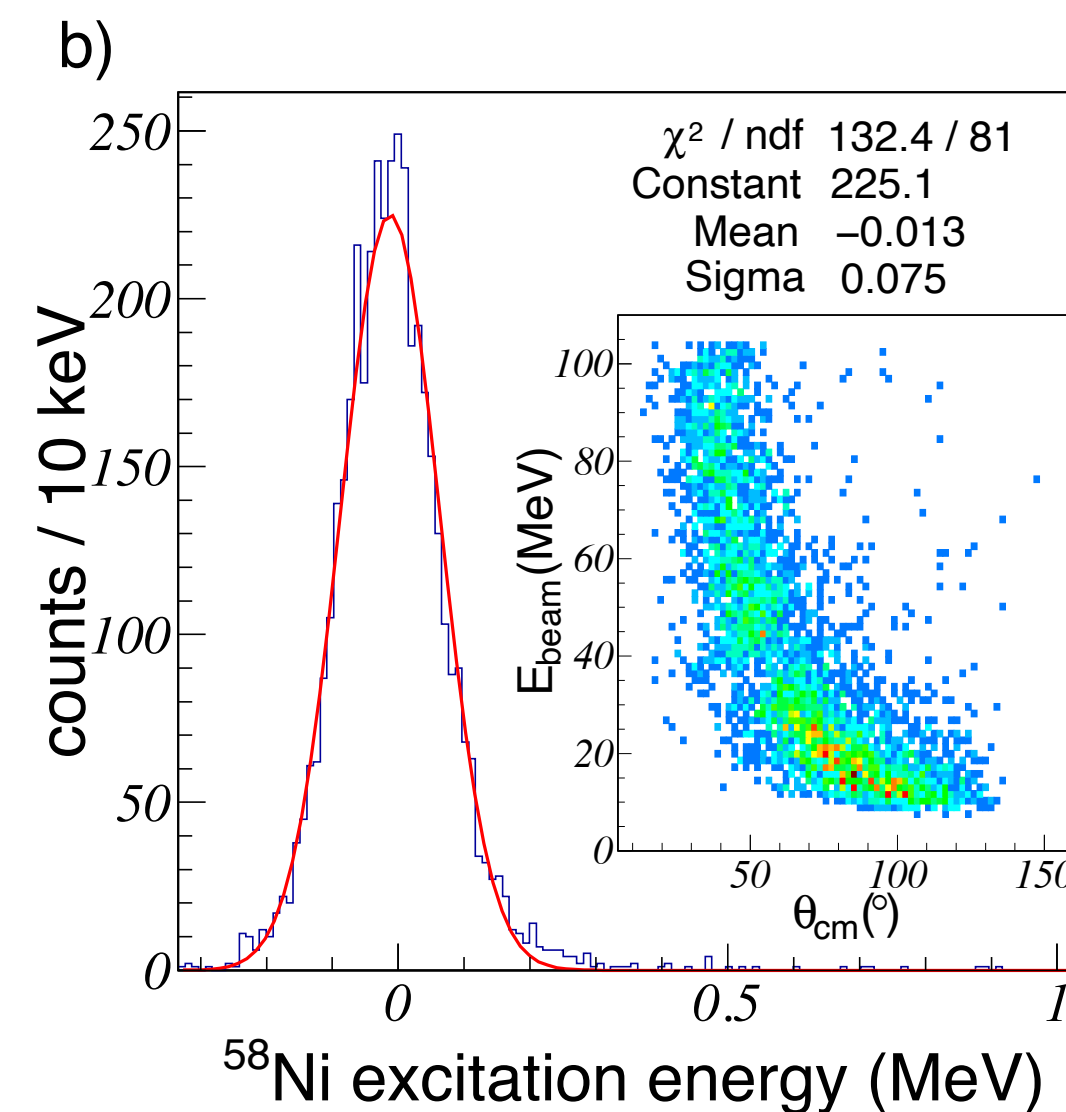
First resolution results

1.1° FWHM resolution From the AT-TPC and ACTAR TPC
from $^4\text{He}+^4\text{He}$ sum angle



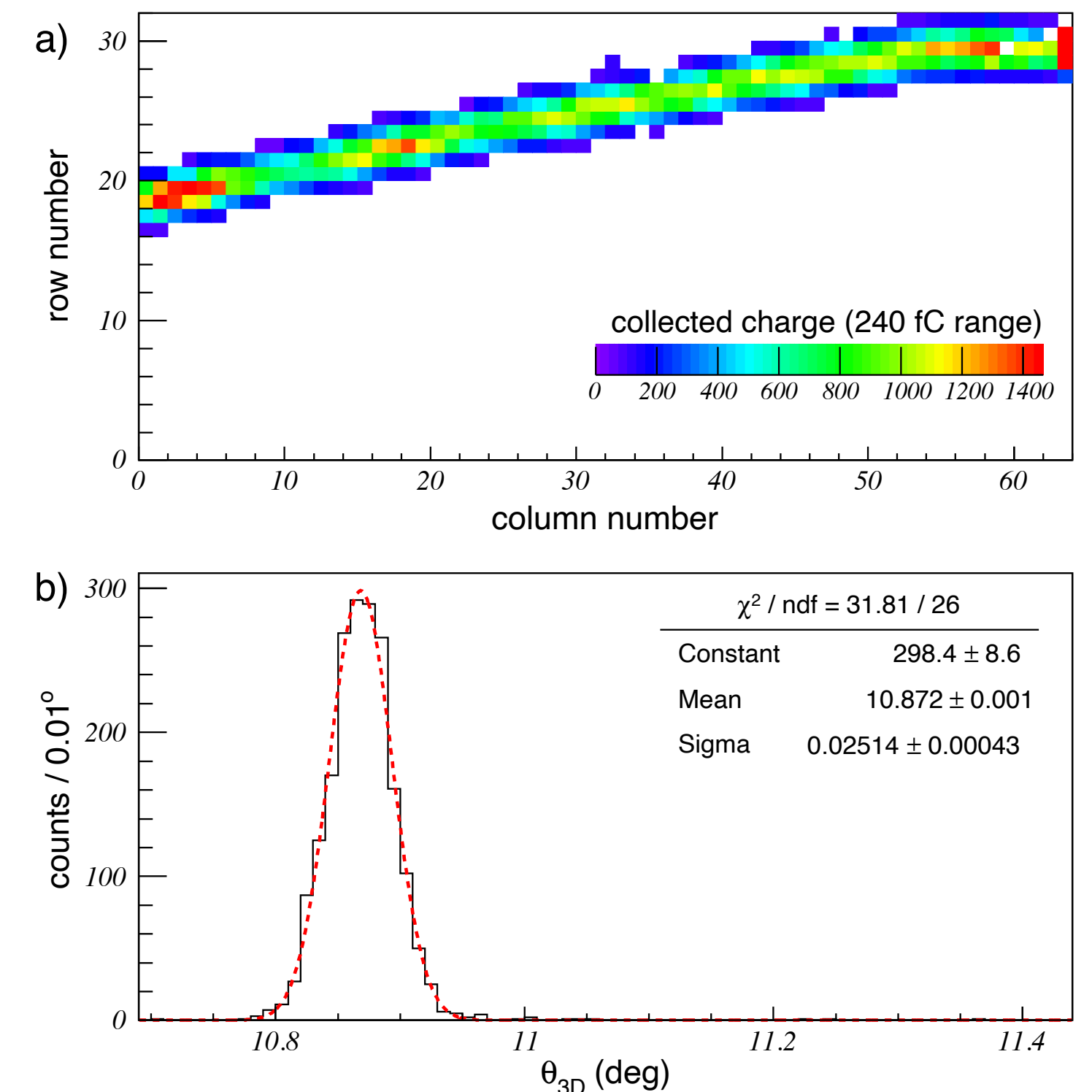
Y. Ayyad et al., NIMA 880, 166 (2018)

175 keV energy resolution
from ^{58}Ni scattering on p



T. Roger et al., NIMA 895, 126 (2018)

0.06° FWHM resolution
from laser beam test



Concluding remarks

Dealing with inverse kinematics

- Inverse kinematics is unavoidable when using reactions on radioactive isotopes
- It helps to boost the luminosity when detecting the residue only at high energy
- It is a big challenge when recoils have to be detected
- Passive target setups have to compromise luminosity with energy and angular resolutions
- Solenoidal spectrometer such as HELIOS elegantly solves the kinematics issue
- Active target promise to avoid luminosity compromises while providing enough resolution to meet inverse kinematics challenge