## Relevant IMSRG equations

Normal-ordered operators:

$$E_0 = \sum_{i} n_i t_{ii} + \frac{1}{2} \sum_{ij} n_i n_j V_{ijij} + \frac{1}{6} n_i n_j n_k V_{ijkijk}^{(3)}$$
 (1)

$$f_{pq} = t_{pq} + \sum_{i} n_i V_{piqi} + \frac{1}{2} \sum_{ij} n_i n_j V_{pijqij}^{(3)}$$
 (2)

$$\Gamma_{pqrs} = V_{pqrs} + \sum_{i} n_i V_{pqirsi}^{(3)} \tag{3}$$

$$W_{pqrstu} = V_{pqrstu}^{(3)} \tag{4}$$

IMSRG flow equations. Note that here we do not follow the CC/MBPT convetion that ijk are hole indices and abc are particle indices. Instead, abc are used for interal (contracted) indices which are summed over, and ijk are external indices.

$$\frac{d}{ds}E_0 = \sum_{ab} n_a \bar{n}_b \left( \eta_{ab} f_{ba} - f_{ab} \eta_{ba} \right) + \frac{1}{4} \sum_{abcd} n_a n_b \bar{n}_c \bar{n}_d \left( \eta_{abcd} \Gamma_{cdab} - \Gamma_{abcd} \eta_{cdab} \right)$$
(5)

$$\frac{d}{ds}f_{ij} = \sum_{a} (\eta_{ia}f_{aj} - f_{ia}\eta_{aj}) + \sum_{ab} (n_a\bar{n}_b - \bar{n}_a n_b) (\eta_{ab}\Gamma_{biaj} - f_{ab}\eta_{biaj}) 
+ \frac{1}{2} \sum_{abc} (n_a n_b\bar{n}_c + \bar{n}_a\bar{n}_b n_c) (\eta_{ciab}\Gamma_{abcj} - \Gamma_{ciab}\eta_{abcj})$$
(6)

$$\begin{split} \frac{d}{ds}\Gamma_{ijkl} &= \sum_{a} \left[ \left( 1 - P_{ij} \left( \eta_{ia} \Gamma_{ajkl} - f_{ia} \eta_{ajkl} \right) - \left( 1 - P_{kl} \right) \left( \eta_{ak} \Gamma_{kjal} - f_{ak} \eta_{ijal} \right) \right] \\ &+ \frac{1}{2} \sum_{ab} \left( \bar{n}_a \bar{n}_b - n_a n_b \right) \left( \eta_{ijab} \Gamma_{abkl} - \Gamma_{ijab} \eta_{abkl} \right) \\ &+ \left( 1 - P_{ij} \right) \left( 1 - P_{kl} \right) \sum_{ab} \left( n_a \bar{n}_b - \bar{n}_a n_b \right) \eta_{aibk} \Gamma_{bjal} \end{split}$$

ab (7)

The White generator:

$$\eta_{ai}^{\text{Wh}} = \frac{f_{ai}}{\Delta_{ai}} - h.c. \tag{8}$$

$$\eta_{abij}^{\text{Wh}} = \frac{\Gamma_{abij}}{\Delta_{abij}} - h.c. \tag{9}$$

Here we use the Moller-Plesset energy denominators. Other choices are possible, but these will suffice for our purposes.

$$\Delta_{ai} = f_{aa} - f_{ii} \tag{10}$$

$$\Delta_{abij} = f_{aa} + f_{bb} - f_{ii} - f_{jj} \tag{11}$$

For nuclear matter, momentum conservation restricts one-body operators to be diagonal. This allows for some simplification of flow equations:

$$\frac{d}{ds}E_0 = \frac{1}{4} \sum_{abcd} n_a n_b \bar{n}_c \bar{n}_d \left( \eta_{abcd} \Gamma_{cdab} - \Gamma_{abcd} \eta_{cdab} \right) \tag{12}$$

$$\frac{d}{ds}f_{ij} = \frac{1}{2} \sum_{abc} (n_a n_b \bar{n}_c + \bar{n}_a \bar{n}_b n_c) \left( \eta_{ciab} \Gamma_{abcj} - \Gamma_{ciab} \eta_{abcj} \right)$$
(13)

$$\frac{d}{ds}\Gamma_{ijkl} = (A_{ii} + A_{jj} - A_{kk} - A_{ll})B_{ijkl} - (B_{ii} + B_{jj} - B_{kk} - B_{ll})A_{ijkl} 
+ \frac{1}{2} \sum_{ab} (\bar{n}_a \bar{n}_b - n_a n_b) (\eta_{ijab} \Gamma_{abkl} - \Gamma_{ijab} \eta_{abkl}) 
+ (1 - P_{ij})(1 - P_{kl}) \sum_{ab} (n_a \bar{n}_b - \bar{n}_a n_b) \eta_{aibk} \Gamma_{bjal}$$
(14)

## Benchmarking the pairing model

Pairing model with 4 particles, in 4 doubly degenerate levels, for  $\delta=1$  and g=+0.5

Solving the IMSRG flow equation with a simple Euler step method with step size ds = 0.1.  $E_0$  is the zero-body piece of the flowing Hamiltonian H(s). EMBPT2 is the second order MBPT energy using H(s), and dE/ds is the zero body part of  $[\eta(s), H(s)]$ .

s	$E_0$	EMBPT2	dE/ds
0.0	1.50000	-0.0623932	0.0000000
0.1	1.48752	-0.0531358	-0.1247860
0.2	1.47689	-0.0453987	-0.1062720
0.3	1.46781	-0.0388940	-0.0907975
0.4	1.46004	-0.0333983	-0.0777880
0.5	1.45336	-0.0287359	-0.0667967

The same numbers as before, but now the  $[\eta_{2b}, H_{2b}]_{2b}$  particle-hole commutator term is omitted.

s	$E_0$	EMBPT2	dE/ds
0	1.5	-0.0623932	0
0.1	1.48752	-0.0531358	-0.124786
0.2	1.47689	-0.0453551	-0.106272
0.3	1.46782	-0.0387878	-0.0907102
0.4	1.46007	-0.0332251	-0.0775756
0.5	1.45342	-0.0284994	-0.0664503
0.6	1.44772	-0.0244746	-0.0569988
0.7	1.44283	-0.0210395	-0.0489492