

## Relevant IMSRG equations

Normal-ordered operators:

$$E_0 = \sum_i n_i t_{ii} + \frac{1}{2} \sum_{ij} n_i n_j V_{ijij} + \frac{1}{6} n_i n_j n_k V_{ijkijk}^{(3)} \quad (1)$$

$$f_{pq} = t_{pq} + \sum_i n_i V_{piqi} + \frac{1}{2} \sum_{ij} n_i n_j V_{pijqij}^{(3)} \quad (2)$$

$$\Gamma_{pqrs} = V_{pqrs} + \sum_i n_i V_{pqirsi}^{(3)} \quad (3)$$

$$W_{pqrstu} = V_{pqrstu}^{(3)} \quad (4)$$

IMSRG flow equations:

$$\frac{d}{ds} E_0 = \sum_{ab} n_a \bar{n}_b (\eta_{ab} f_{ba} - f_{ab} \eta_{ba}) + \frac{1}{4} \sum_{abcd} n_a n_b \bar{n}_c \bar{n}_d (\eta_{abcd} \Gamma_{cdab} - \Gamma_{abcd} \eta_{cdab}) \quad (5)$$

$$\begin{aligned} \frac{d}{ds} f_{ij} = & \sum_a (\eta_{ia} f_{aj} - f_{ia} \eta_{aj}) + \sum_{ab} (n_a \bar{n}_b - \bar{n}_a n_b) (\eta_{ab} \Gamma_{biaj} - f_{ab} \eta_{biaj}) \\ & + \frac{1}{2} \sum_{abc} (n_a n_b \bar{n}_c + \bar{n}_a \bar{n}_b n_c) (\eta_{ciab} \Gamma_{abcj} - \Gamma_{ciab} \eta_{abcj}) \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{d}{ds} \Gamma_{ijkl} = & \sum_a [(1 - P_{ij}) (\eta_{ia} \Gamma_{ajkl} - f_{ia} \eta_{ajkl}) - (1 - P_{kl}) (\eta_{ak} \Gamma_{kjal} - f_{ak} \eta_{kjal})] \\ & + \frac{1}{2} \sum_{ab} (\bar{n}_a \bar{n}_b - n_a n_b) (\eta_{ijab} \Gamma_{abkl} - \Gamma_{ijab} \eta_{abkl}) \\ & + (1 - P_{ij})(1 - P_{kl}) \sum_{ab} (n_a \bar{n}_b - \bar{n}_a n_b) \eta_{aibk} \Gamma_{bjal} \end{aligned} \quad (7)$$

The White generator:

$$\eta_{ai}^{\text{Wh}} = \frac{f_{ai}}{\Delta_{ai}} \quad (8)$$

$$\eta_{abij}^{\text{Wh}} = \frac{\Gamma_{abij}}{\Delta_{abij}} \quad (9)$$

Energy denominators:

$$\Delta_{ai} = f_{aa} - f_{ii} \quad (10)$$

$$\Delta_{abij} = f_{aa} + f_{bb} - f_{ii} - f_{jj} \quad (11)$$

Simplification of flow equation for nuclear matter:

$$\frac{d}{ds}E_0 = \frac{1}{4} \sum_{abcd} n_a n_b \bar{n}_c \bar{n}_d (\eta_{abcd} \Gamma_{cdab} - \Gamma_{abcd} \eta_{cdab}) \quad (12)$$

$$\frac{d}{ds}f_{ij} = \frac{1}{2} \sum_{abc} (n_a n_b \bar{n}_c + \bar{n}_a \bar{n}_b n_c) (\eta_{ciab} \Gamma_{abcj} - \Gamma_{ciab} \eta_{abcj}) \quad (13)$$

$$\begin{aligned} \frac{d}{ds}\Gamma_{ijkl} &= (A_{ii} + A_{jj} - A_{kk} - A_{ll})B_{ijkl} - (B_{ii} + B_{jj} - B_{kk} - B_{ll})A_{ijkl} \\ &+ \frac{1}{2} \sum_{ab} (\bar{n}_a \bar{n}_b - n_a n_b) (\eta_{ijab} \Gamma_{abkl} - \Gamma_{ijab} \eta_{abkl}) \\ &+ (1 - P_{ij})(1 - P_{kl}) \sum_{ab} (n_a \bar{n}_b - \bar{n}_a n_b) \eta_{aibk} \Gamma_{bjal} \end{aligned} \quad (14)$$

## Benchmarking the pairing model

Pairing model with 4 particles, in 4 doubly degenerate levels, for  $\delta = 1$  and  $g = +0.5$

Solving the IMSRG flow equation with a simple Euler step method with step size  $ds = 0.1$ .  $E_0$  is the zero-body piece of the flowing Hamiltonian  $H(s)$ . EMBPT2 is the second order MBPT energy using  $H(s)$ , and  $dE/ds$  is the zero body part of  $[\eta(s), H(s)]$ .

| $s$ | $E_0$   | EMBPT2     | $dE/ds$    |
|-----|---------|------------|------------|
| 0.0 | 1.50000 | -0.0623932 | 0.0000000  |
| 0.1 | 1.48752 | -0.0531358 | -0.1247860 |
| 0.2 | 1.47689 | -0.0453987 | -0.1062720 |
| 0.3 | 1.46781 | -0.0388940 | -0.0907975 |
| 0.4 | 1.46004 | -0.0333983 | -0.0777880 |
| 0.5 | 1.45336 | -0.0287359 | -0.0667967 |

The same numbers as before, but now the  $[\eta_2, H_2]_2$  particle-hole commutator term is omitted.

| $s$ | $E_0$   | EMBPT2     | $dE/ds$    |
|-----|---------|------------|------------|
| 0   | 1.5     | -0.0623932 | 0          |
| 0.1 | 1.48752 | -0.0531358 | -0.124786  |
| 0.2 | 1.47689 | -0.0453551 | -0.106272  |
| 0.3 | 1.46782 | -0.0387878 | -0.0907102 |
| 0.4 | 1.46007 | -0.0332251 | -0.0775756 |
| 0.5 | 1.45342 | -0.0284994 | -0.0664503 |
| 0.6 | 1.44772 | -0.0244746 | -0.0569988 |
| 0.7 | 1.44283 | -0.0210395 | -0.0489492 |

These are the same as before, but with the painful  $[\eta_{2b}, H_{2b}]_{2b}$  particle-hole term omitted.

| $s$ | $E0$    | EMBPT2     | $dE/ds$    |
|-----|---------|------------|------------|
| 0.0 | 1.50000 | -0.0623932 | 0          |
| 0.1 | 1.48752 | -0.0531358 | -0.124786  |
| 0.2 | 1.47689 | -0.0453551 | -0.106272  |
| 0.3 | 1.46782 | -0.0387878 | -0.0907102 |
| 0.4 | 1.46007 | -0.0332251 | -0.0775756 |
| 0.5 | 1.45342 | -0.0284994 | -0.0664503 |