

HANK WITH ENDOGENOUS RISK

1 Model

We consider a *closed* economy with heterogeneous agents and *flexible prices* and *sticky wages*.

Time is discrete and indexed by t . There is a continuum of households indexed by i .

Firms. A representative firm hires labor, N_t , to produce goods, with the production function

$$Y_t = \Gamma_t N_t. \quad (1)$$

where Γ_t is the exogenous technology level. Profits are

$$\Pi_t = P_t Y_t - W_t N_t. \quad (2)$$

where P_t is the price level and W_t is the wage level. The first order condition for labor implies that the real wage is exogenous

$$w_t \equiv W_t / P_t = \Gamma_t. \quad (3)$$

Inflation rates for wages and price are given by

$$\pi_t^w \equiv W_t / W_{t-1} - 1 \quad (4)$$

$$\pi_t \equiv \frac{P_t}{P_{t-1}} - 1 = \frac{W_t / \Gamma_t}{W_{t-1} / \Gamma_{t-1}} - 1 = \frac{1 + \pi_t^w}{\Gamma_t / \Gamma_{t-1}} - 1. \quad (5)$$

Households. Households are *ex post* heterogeneous in terms of their time-varying stochastic productivity, captured by e_{it} and u_{it} , and their (end-of-period) savings, a_{it-1} . The distribution of households over idiosyncratic states is denoted \underline{D}_t before shocks are realized

and D_t afterwards. Households supply labor, ℓ_{it} , chosen by a union, and choose consumption, c_{it} , on their own. Aggregate post-tax income is $Z_t \equiv w_t N_t - T_t$, where w_t is the real wage, N_t is employment, and T_t are taxes. The idiosyncratic income factor is

$$z_{it} = \frac{e_{it}^{1-\theta}}{\mathbb{E} \left[e_{it}^{1-\theta} \right]} \Delta_t \left(\bar{\phi} + u_{it} \left(\underline{\phi} - \bar{\phi} \right) \right),$$

where assumptions are made so $\mathbb{E} [z_{it}] = 1$. Households are not allowed to borrow. The return on savings from period $t - 1$ to t is r_{t-1} .

The household problem is

$$\begin{aligned} v_t(u_{it}, e_{it}, a_{it-1}) &= \max_{c_t} \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_{it}^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t [v_{t+1}(e_{it+1}, u_{it+1}, a_{it})] \\ \text{s.t. } a_{it} + c_{it} &= (1 + r_{t-1})a_{it-1} + Z_t z_{it} \\ z_{it} &= \frac{e_{it}^{1-\theta}}{\mathbb{E} \left[e_{it}^{1-\theta} \right]} \Delta_t \left(\bar{\phi} + u_{it} \left(\underline{\phi} - \bar{\phi} \right) \right) \\ \log e_{it+1} &= \rho_z \log e_{it} + \psi_{it+1}, \psi_{it} \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \mathbb{E} [e_{it}] = 1 \\ u_{it+1} &= \begin{cases} 1 & \text{with prob. } \delta_{t+1} \\ 0 & \text{else} \end{cases} \\ a_{it} &\geq 0, \end{aligned} \tag{6}$$

where β is the discount factor, σ is the inverse elasticity of substitution, φ controls the disutility of supplying labor and ν is the inverse of the Frish elasticity.

To ensure $\mathbb{E} [z_{it}] = 1$, we choose

$$\Delta_t = \left(\frac{Z_t}{Z_{ss}} \right)^{\gamma-1} \tag{7}$$

$$\delta_t = \frac{\bar{\phi} - \left(\frac{Z_t}{Z_{ss}} \right)^{1-\gamma}}{\bar{\phi} - \underline{\phi}}. \tag{8}$$

We assume γ is such that we always have $\delta_t \in (0, 1)$.

Aggregate quantities are

$$A_t^{hh} = \int a_t^* (z_{it}, a_{it-1}) d\mathbf{D}_t \quad (9)$$

$$N_t^{hh} = \int \ell_t^* (z_{it}, a_{it-1}) z_{it} d\mathbf{D}_t \quad (10)$$

$$C_t^{hh} = \int c_t^* (z_{it}, a_{it-1}) d\mathbf{D}_t. \quad (11)$$

Union. A union chooses the labor supply of each household and sets wages. Each household is chosen to supply the same amount of labor,

$$\ell_{it} = N_t^{hh}. \quad (12)$$

Unspecified adjustment costs imply a *New Keynesian Wage Philips Curve*,

$$\pi_t^w (1 + \pi_t^w) = \kappa \left(\frac{\varphi N_t^\nu}{(C_t^*)^{-\sigma} (1 - \theta) Z_t / N_t} - 1 \right) + \beta \pi_{t+1}^w (1 + \pi_{t+1}^w),$$

where $C_t^* = (\mathbb{E} [c_{it}^{-\sigma} z_{it}])^{-\frac{1}{\sigma}}$.

Central bank. The central bank either follows a standard Taylor rule,

$$1 + i_t = (1 + r_{ss}) (1 + \pi_t)^{\phi_\pi}, \quad (13)$$

where i_t is the nominal return from period t to period $t + 1$ and ϕ_π is the Taylor coefficient. Or a real rate rule where

$$1 + i_t = (1 + r_{ss}) (1 + \pi_{t+1}). \quad (14)$$

The *ex ante* real interest rate is

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}}. \quad (15)$$

Government. The government chooses consumption, G_t , and finances it with either taxes, T_t , or real bonds, B_t . The budget constraint is

$$B_t = (1 + r_{t-1}) B_{t-1} + G_t - T_t. \quad (16)$$

We assume the debt rule

$$B_t = B_{ss} + \phi_B (B_{t-1} - B_{ss} + G_t - G_{ss}). \quad (17)$$

Market clearing. Market clearing implies

1. Asset market: $B_t = A_t^{hh}$
2. Labor market: $N_t = N_t^{hh}$
3. Goods market: $Y_t = C_t^{hh} + G_t$

2 Questions

Code is provided as a starting point for solving the model for a baseline choice of parameters.

I. Technical questions

- a) Define the *stationary equilibrium* in words and equations.
- b) Define the *transition path* to a government consumption shock in words and equations.
- c) Describe the model as a *directed acyclical graph* (DAG).

In the code the *aggregate inputs* to the household problem are Z_t , Δ_t and δ_t

- d) Could you reduce the number of *inputs* to the household problem?
- e) Assume i_t is considered an *unknown*. What *target* would you add?
- f) Extend the code to allow for a lump-sum transfer (which is zero in steady state) from the government to households such that

$$\begin{aligned} a_{it} + c_{it} &= (1 + r_{t-1})a_{it-1} + Z_t z_{it} + \omega_t \\ B_t &= (1 + r_{t-1})B_{t-1} + G_t + \omega_t - T_t \end{aligned}$$

II. Intertemporal marginal propensity to consume Assume we are in the stationary equilibrium. The consumption function can be written as

$$C_t^{hh} = \mathcal{C}_t(\{Z_t\}, \{\Delta_t\}, \{\delta_t\}, \{\omega_t\}) \quad (18)$$

We define the following matrices:

\mathbf{M} has entries $[M]_{ts} = \frac{\partial \mathcal{C}_t}{\partial Z_s}$

\mathbf{M}_Δ has entries $[M_\Delta]_{ts} = \frac{\partial \mathcal{C}_t}{\partial \Delta_s}$

\mathbf{M}_δ has entries $[M_\delta]_{ts} = \frac{\partial \mathcal{C}_t}{\partial \delta_s}$

\mathbf{M}_ω has entries $[M]_{ts} = \frac{\partial \mathcal{C}_t}{\partial \omega_s}$

a) Plot and discuss the difference between \mathbf{M} and \mathbf{M}_ω

b) Show that we have

$$\mathbf{M}_\Delta = Z_{ss} \mathbf{M} \quad (19)$$

c) Show that for $\omega_t = 0, \forall t$, we have

$$\begin{aligned} d\mathbf{C}^{hh} &= (\gamma \mathbf{M} - (1 - \gamma) \chi \mathbf{M}_\delta) d\mathbf{Z} \\ &= (\mathbf{M} + (\gamma - 1) \mathbf{M} - (1 - \gamma) \chi \mathbf{M}_\delta) d\mathbf{Z} \end{aligned} \quad (20)$$

where

$$\chi = \frac{Z_{ss}^{\gamma-1}}{\phi - \underline{\phi}}$$

and $d\mathbf{X} = [X_0 - X_{ss}, X_1 - X_{ss}, \dots]$.

III. Fiscal shock Assume that a path of government consumption is announced such that $dG_t = 0.01 \cdot 0.76^t$.

a) Decompose what drives the response of consumption.

b) Explain the transmission mechanism.

Define the (cumulative) fiscal multiplier as

$$\mathcal{M} = \frac{\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} (Y_t - Y_{ss})}{\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} (T_t - T_{ss})}$$

c) Calculate the fiscal multiplier.

d) Discuss how the fiscal multiplier depend on γ and the amount of liquidity. Explain your results.

IV. Explorations

- a) Broaden the discussion of what determines the size of the fiscal multiplier in the model.
- b) Extend the model to allow for *endogenous* persistent risk

$$\Pr [u_{it+1} = 1 | u_{it} = 0] = \delta_{t+1}$$

$$\Pr [u_{it+1} = 0 | u_{it} = 0] = 1 - \delta_{t+1}$$

$$\Pr [u_{it+1} = 1 | u_{it} = 1] = (1 - \lambda) + \lambda \delta_{t+1}$$

$$\Pr [u_{it+1} = 0 | u_{it} = 1] = (1 - \delta_{t+1})\lambda$$