HANK WITH ENDOGENOUS RISK

1 Model

We consider a *closed* economy with heterogeneous agents and *flexible prices* and *sticky wages*.

Time is discrete and indexed by t. There is a continuum of households indexed by i.

Firms. A representative firm hires labor, N_t, to produce goods, with the production function

$$Y_t = \Gamma_t N_t. \tag{1}$$

where Γ_t is the exogenous technology level. Profits are

$$\Pi_t = P_t Y_t - W_t N_t. \tag{2}$$

where P_t is the price level and W_t is the wage level. The first order condition for labor implies that the real wage is exogenous

$$w_t \equiv W_t / P_t = \Gamma_t. \tag{3}$$

Inflation rates for wages and price are given by

$$\pi_t^w \equiv W_t / W_{t-1} - 1 \tag{4}$$

$$\pi_t \equiv \frac{P_t}{P_{t-1}} - 1 = \frac{W_t / \Gamma_t}{W_{t-1} / \Gamma_{t-1}} - 1 = \frac{1 + \pi_t^w}{\Gamma_t / \Gamma_{t-1}} - 1. \tag{5}$$

Households. Households are *ex post* heterogeneous in terms of their time-varying stochastic productivity, captured by e_{it} and u_{it}, and their (end-of-period) savings, a_{it-1}. The distribution of households over idiosyncratic states is denoted $\underline{\boldsymbol{D}}_t$ before shocks are realized and $\boldsymbol{D}_t$ afterwards. Households supply labor, ℓ_{it}, chosen by a union, and choose consumption, c_{it}, on their own. Aggregate post-tax income is $Z_t \equiv w_t N_t - T_t$, where w_t is the real wage, N_t is employment, and T_t are taxes. The idiosyncratic income factor is

$$z_{it} = \frac{e_{it}^{1-\theta}}{\mathbb{E}\left[e_{it}^{1-\theta}\right]} \Delta_t \left(\overline{\phi} + u_{it}\left(\underline{\phi} - \overline{\phi}\right)\right),$$

where assumptions are made so $\mathbb{E}\left[z_{it}\right] = 1$. Households are not allowed to borrow. The return on savings from period $t-1$ to t is r_{t-1}.

The household problem is

$$\begin{aligned}
v_t(u_{it}, e_{it}, a_{it-1}) &= \max_{c_t} \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_{it}^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t \left[v_{t+1}(e_{it+1}, u_{it+1}, a_{it})\right] \qquad (6) \\
\text{s.t. } a_{it} + c_{it} &= (1 + r_{t-1}) a_{it-1} + Z_t z_{it} \\
z_{it} &= \frac{e_{it}^{1-\theta}}{\mathbb{E}\left[e_{it}^{1-\theta}\right]} \Delta_t \left(\overline{\phi} + u_{it}\left(\underline{\phi} - \overline{\phi}\right)\right) \\
\log e_{it+1} &= \rho_z \log e_{it} + \psi_{it+1} \;\; , \psi_{it} \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \;\; \mathbb{E}\left[e_{it}\right] = 1 \\
u_{it+1} &= \begin{cases} 1 & \text{with prob. } \delta_{t+1} \\ 0 & \text{else} \end{cases} \\
a_{it} &\geq 0,
\end{aligned}$$

where β is the discount factor, σ is the inverse elasticity of substitution, φ controls the disutility of supplying labor and ν is the inverse of the Frish elasticity.

To ensure $\mathbb{E}\left[z_{it}\right] = 1$, we choose

$$\Delta_t = \left(\frac{Z_t}{Z_{ss}}\right)^{\gamma-1} \qquad (7)$$

$$\delta_t = \frac{\overline{\phi} - \left(\frac{Z_t}{Z_{ss}}\right)^{1-\gamma}}{\overline{\phi} - \underline{\phi}}. \qquad (8)$$

We assume γ is such that we always have $\delta_t \in (0, 1)$.

Aggregate quantities are

$$A_t^{hh} = \int a_t^* \left(z_{it}, a_{it-1}\right) d\boldsymbol{D}_t \tag{9}$$

$$N_t^{hh} = \int \ell_t^* \left(z_{it}, a_{it-1}\right) z_{it} d\boldsymbol{D}_t \tag{10}$$

$$C_t^{hh} = \int c_t^* \left(z_{it}, a_{it-1}\right) d\boldsymbol{D}_t. \tag{11}$$

Union. A union chooses the labor supply of each household and sets wages. Each household is chosen to supply the same amount of labor,

$$\ell_{it} = N_t^{hh}. \tag{12}$$

Unspecified adjustment costs imply a *New Keynesian Wage Philips Curve*,

$$\pi_t^w(1+\pi_t^w) = \kappa \left(\frac{\varphi N_t^{\nu}}{\left(C_t^*\right)^{-\sigma} (1-\theta) Z_t / N_t} - 1 \right) + \beta \pi_{t+1}^w \left(1 + \pi_{t+1}^w\right),$$

where $C_t^* = \left(\mathbb{E}\left[c_{it}^{-\sigma} z_{it}\right]\right)^{-\frac{1}{\sigma}}$.

Central bank. The central bank either follows a standard Taylor rule,

$$1 + i_t = \left(1 + r_{ss}\right)\left(1 + \pi_t\right)^{\phi_\pi}, \tag{13}$$

where i_t is the nominal return from period t to period $t+1$ and ϕ_π is the Taylor coefficient. Or a real rate rule where

$$1 + i_t = (1 + r_{ss})(1 + \pi_{t+1}). \tag{14}$$

The *ex ante* real interest rate is

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}}. \tag{15}$$

Government. The government chooses consumption, G_t, and finances it with either taxes, T_t, or real bonds, B_t. The budget constraint is

$$B_t = (1 + r_{t-1}) B_{t-1} + G_t - T_t. \tag{16}$$

We assume the debt rule

$$B_t = B_{ss} + \phi_B \left(B_{t-1} - B_{ss} + G_t - G_{ss}\right). \tag{17}$$

Market clearing. Market clearing implies

1. Asset market: $B_t = A_t^{hh}$
2. Labor market: $N_t = N_t^{hh}$
3. Goods market: $Y_t = C_t^{hh} + G_t$

2 Questions

Code is provided as a starting point for solving the model for a baseline choice of parameters.

I. Technical questions

a) Define the *stationary equilibrium* in words and equations.

b) Define the *transition path t*o a government consumption shock in words and equations.

c) Describe the model as a *directed acyclical graph* (DAG).

In the code the *aggregate inputs* to the household problem are Z_t, Δ_t and δ_t

d) Could you reduce the number of *inputs* to the household problem?

e) Assume i_t is considered an *unknown*. What *target* would you add?

f) Extend the code to allow for a lump-sum transfer (which is zero in steady state) from the government to households such that

$$\begin{aligned} a_{it} + c_{it} &= (1 + r_{t-1})a_{it-1} + Z_t z_{it} + \omega_t \\ B_t &= (1 + r_{t-1})B_{t-1} + G_t + \omega_t - T_t \end{aligned}$$

II. Intertemporal marginal propensity to consume Assume we are in the stationary equilibrium. The consumption function can be written as

$$C_t^{hh} = \mathcal{C}_t\left(\{Z_t\}, \{\Delta_t\}, \{\delta_t\}, \{\omega_t\}\right) \tag{18}$$

We define the following matrices:

M has entries $[M]_{ts} = \frac{\partial \mathcal{C}_t}{\partial Z_s}$
$\mathbf{M}_\Delta$ has entries $[M_\Delta]_{ts} = \frac{\partial \mathcal{C}_t}{\partial \Delta_s}$
$\mathbf{M}_\delta$ has entries $[M_\delta]_{ts} = \frac{\partial \mathcal{C}_t}{\partial \delta_s}$
$\mathbf{M}_\omega$ has entries $[M]_{ts} = \frac{\partial \mathcal{C}_t}{\partial \omega_s}$

a) Plot and discuss the difference between **M** and $\mathbf{M}_\omega$

b) Show that we have

$$\mathbf{M}_\Delta = Z_{ss}\mathbf{M} \tag{19}$$

c) Show that for $\omega_t = 0, \forall t$, we have

$$\begin{aligned} d\mathbf{C}^{hh} &= (\gamma\mathbf{M} - (1-\gamma)\chi\mathbf{M}_\delta)\, d\mathbf{Z} \\ &= (\mathbf{M} + (\gamma - 1)\mathbf{M} - (1-\gamma)\chi\mathbf{M}_\delta)\, d\mathbf{Z} \end{aligned} \tag{20}$$

where

$$\chi = \frac{Z_{ss}^{-1}}{\overline{\phi} - \underline{\phi}}$$

and $d\mathbf{X} = [X_0 - X_{ss}, X_1 - X_{ss}, \dots]$.

III. Fiscal shock Assume that a path of government consumption is announced such that $dG_t = 0.01 \cdot 0.76^t$.

a) Decompose what drives the response of consumption.

b) Explain the transmission mechanism.

Define the (cumulative) fiscal multiplier as

$$\mathcal{M} = \frac{\sum_{t=0}^{\infty}(1+r_{ss})^{-t}\,(Y_t - Y_{ss})}{\sum_{t=0}^{\infty}(1+r_{ss})^{-t}\,(T_t - T_{ss})}$$

c) Calculate the fiscal multiplier.

d) Discuss how the fiscal multiplier depend on γ and the amount of liquidity.
Explain you results.

IV. Explorations

a) Broaden the discussion of what determines the size of the fiscal multiplier in the model.

b) Extend the model to allow for *endogenous* persistent risk

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\begin{aligned}
\Pr\left[u_{it+1} = 1 | u_{it} = 0\right] &= \delta_{t+1} \\
\Pr\left[u_{it+1} = 0 | u_{it} = 0\right] &= 1 - \delta_{t+1} \\
\Pr\left[u_{it+1} = 1 | u_{it} = 1\right] &= (1 - \lambda) + \lambda \delta_{t+1} \\
\Pr\left[u_{it+1} = 0 | u_{it} = 0\right] &= (1 - \delta_{t+1}) \lambda
\end{aligned}
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