CENTER FOR
ECONOMIC
BEHAVIOR &
INEQUALITY

Introduction

Mini-Course: Heterogenous Agent Macro

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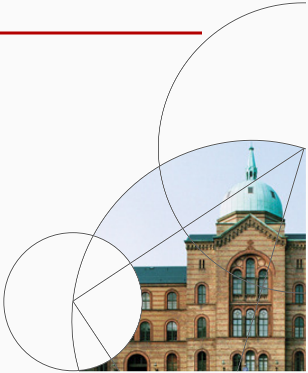

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Plan

1. Introduction

2. Programming in Python

Introduction

Introduction

- **Central economic topics:**
  1. Consumption-saving with risk and constraints (single-agent)
  2. Heterogeneous agents in general equilibrium models
     - 2.1 Long-run effects on aggregate outcomes
     - 2.2 Short-run effects on aggregate outcomes
     - 2.3 Drivers of inequality
- **History:**
  1. Heathcote et al. (2009), »Quantitative Macroeconomics with Heterogeneous Households«
  2. Kaplan and Violante (2018), »Microeconomic Heterogeneity and Macroeconomic Shocks«
  3. Cherrier et al. (2023), »Household Heterogeneity in Macroeconomic Models: A Historical Perspective«
  4. Auclert et. al. (2025), »Fiscal and Monetary Policy with Heterogeneous Agents«
- **Central technical method:** *Programming in Python*

Macroeconomic Models with Heterogeneous Agents

- **Model components:**
  1. Optimizing individual agents (households + firms)
  2. Idiosyncratic and aggregate risk
  3. Information flows (who knows what when $\Rightarrow$ often everything)
  4. Market clearing (Walras vs. search-and-match)
- **Insurance/markets:**
  Complete $\rightarrow$ idiosyncratic risk insured away $\sim$ representative agent
  Incomplete $\rightarrow$ agents need to *self-insure*
- **Heterogeneity:**
  Ex ante in preferences, abilities etc.
  Ex post after realization of idiosyncratic shocks
- **HANC:** Heterogeneous Agent *Neo-Classical* model
  (Aiyagari-Bewley-Hugget-Imrohoroglu or Standard Incomplete Markets model)
- **HANK:** Heterogeneous Agent *New Keynesian* model
  (i.e. include price and wage setting frictions)

Lectures

- **Topics:**
  1. Consumption-saving
  2. Stationary equilibrium
  3. Transitional dynamics
  4. HANK models
- **Teaching philosophy:**
  1. Go in depth - *from theory to implementation*
  2. Not a literature review - *key entrances to literature*
  3. Focus on tools - *easier to read economic insights*

Python

1. **Assumed knowledge:** Similar to my undergraduate course
   Introduction to Programming and Numerical Analysis
   Content: Python + VSCode + git
   Preparation: Video playlist (~10 hours at normal speed)
2. **Updated Python:** Install (or re-install) newest Anaconda
3. **Packages:** `pip install quantecon, EconModel, consav`
4. **GEMoodel tools:**
   4.1 Clone the `GEModelTools` repository
   4.2 Locate repository in command prompt
   4.3 Run `pip install -e .`
5. **Other repositories used in the course:**
   - EconModelNotebooks
   - ConsumptionSavingNotebooks
   - GEModelToolsNotebooks
   - MiniCourse-HetAMacro

Programming in Python

References (pointers)

- **Variables are references to an instance of an object**
- A **class** defines the **type** of an object
  - `.attribute`, state
  - `.method()`, action (incl. changing self)
- Inheritance (of methods) (`class Child(Parent)`)
- Arithmetic operators (e.g. `+`,`*`,`/`,`//`,`**`,`%`) combine objects
- = **assigns a reference** (*not a copy!*)

Question: What does `a` end up as? What if `a = [1,2,3]`?

```
a = np.array([1,2,3])
b = a
c = a[1:] # slicing
b[0] = 3 # indexing
c[0] = 3
```

Types and in-place operations

- **Atomic types:** `int`, `float`, `str`, `bool`, etc.
- **Containers** `list`, `tuple`, `dict`, `set`, `np.array`, etc.
- **Mutables** (e.g. `list`, `np.array`) can change in-place
    1. **In-place operators** (+=, `-=` etc.)
    2. **Slicing**: `x[:] = x + y`
- **Immutables** (e.g. atomic types and tuples) can never change

Questions: What does y end up as?

```
x = np.array([1,2,3])
y = x
x += 1
x[:] = x + 1
x = x + 1
```

Functions and scope

- **Functions are objects** (can e.g. be arguments in functions)
  Unlike in math:
  1. Can change its arguments (side-effects)
  2. Can call itself (recursion)
- Decorators change function behavior (e.g. @numba.njit)
- Variables can both be **local scope** (good) or **global scope** (bad)

Questions: What is the output?

```python
a = 1
def f(x):
    return x+a
print(f(1))
a = 2
print(f(1))
```

Computational tree and branches

- **Comparison** (`==`, `!=`, `<`, `<=`, `not`, `and`, `or` etc.)
- **Conditionals** (`if`, `elif`, `else`)
- **Loops** (`for`, `while`, `continue`, `break`)
- **Convergence** (tolerance in optimizer or root-finder/equation-solver)

Questions: How could this be implemented with a `while` loop?

```python
x = x0
for i in range(n):
    y = evaluate(x)
    if check(y): break
    x = update(x,y)
else:
    raise ValueError('did not converge')
```

Decimal numbers are not exact

- **Never use exactness for decimal numbers**
  - Order of computation matter
  - Best with numbers are around 1 (underflow and overflow)
- Division, exp, log etc. are (costly) approximations
- **Function approximation and interpolation often needed**

Questions: Which are `True` and which are `False`?

```
print(0.1 + 0.2 == 0.3)
print(0.5 + 0.5 == 1.0)
print(np.isclose(0.1+0.2,0.3))
print(np.isclose(1e-200*1e200*1e200*1e-200,1.0))
print(np.isinf(1e-200*(1e200*1e200)*1e-200))
print(np.isclose(1e200*(1e-200*1e-200)*1e200,0.0))
```

Pseudo random numbers

- **Only one seed** (randomness not assured across seeds)
- State of random number generator can be reset
- **Monte Carlo** simulation and integration
  1. Static alternative: Use **quadrature rules**
  2. Dynamic alternative: Discretize and derive **transition matrix**

Questions: What is z equal to?

```
rng = np.random.default_rng(123)
s = rng.bit_generator.state
x = rng.normal(size=5)
y = rng.normal(size=5)
rng.bit_generator.state = s
z = rng.normal(size=5)
```

Documentation and debugging

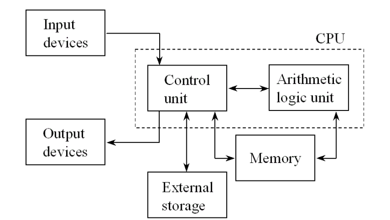
- **No code is self-explanatory** (for others, incl. future you)
- **Write documentation** (use *github-copilot*)
  1. The comments explain humans what the code does.
  2. The code makes the computer do what the comments say
- Important **design patterns**:
  1. Use namespaces (be aware of scope) and meaningful names
  2. No repetition of code-lines $\Rightarrow$ single-purpose functions/methods
  3. Use `assert` (also print and plot intermediate results)
  4. Use `try-except`
- **Run from top to bottom** (make shortcut)
  Replication: datacodestandard.org
- **Debugging** (see 02. Debugging.ipynb)
  1. Errors are (almost) always simple
  2. Go through code step-by-step (*manually* or *debugger*)

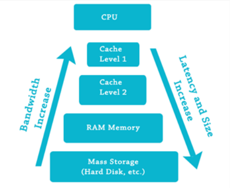
Other languages

- **High level languages:**
  1. **MATLAB**: Costly and not better.
  2. **R**: Better at statistics and data work, but not pure numerical work.
  3. **Julia**: Faster than Python (incl. numba), slower than C++. Smallish community.
- **Low level languages:**
  1. **C++:** State-of-the-art for fastest code.
  2. **Fortran**: No benefits relative to C++ (only legacy...).
- **Hardware:**
  1. CPU: Fastest cores.
  2. GPU: Many more cores, but specialized for e.g. linear algebra.
  3. TPU (Google): Even more specialized at AI (machine learning)

CPUs are complex

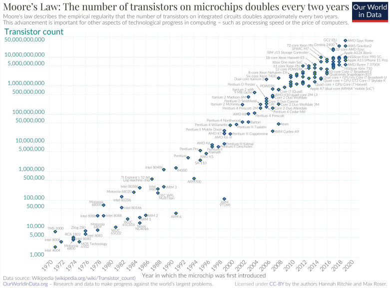
- **Instruction set** (assembly) is not just add, subtract, etc.
  1. Work on vectors (SIMD) $\Rightarrow$ *homogeneity is good*
  2. Out-of-order execution $\Rightarrow$ *predictability is good*
  3. Caching $\Rightarrow$ *latest read memory can be accessed quickly*

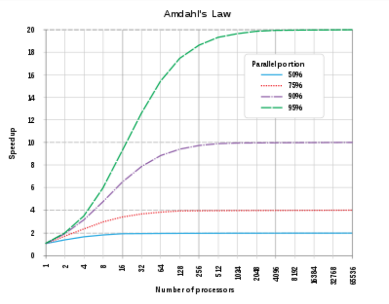




- **Compilers can optimize a lot** $\Rightarrow$ **use existing libraries**
- **Parallelisation:** *Start up costs*
  1. **Hardware:** Cores vs. CPUs vs. sockets vs computers
  2. **Software:** Shared memory (e.g. OpenMP) or not (e.g. MPI)

Moore vs. Amdahl





1. **Moore's law:** Exponential growth in computational power
    1.1 Originally: Faster cores (calculations per time unit)
    1.2 Now: More cores per CPU / per computer / per dollar
2. **Amdahl's law:** Sequential code becomes the bottleneck
    2.1 Time in tasks done in parallel $\rightarrow 0$
    2.2 95% done in parallel $\rightarrow$ max 20x speed-up

Need for speed (03. NeedForSpeed.ipynb)

- **Computation time vs programmer time**
  Use not-too-model-specific insights ⇒ *better algorithm*
- **Premature optimization is the root of all evil!**
  Use *line-profiler!*
  1. Use available code: Stand on the shoulder of giants
  2. In numpy: Use vectorization
  3. Else: Use numba
- **Automatic differentiation?** Use `JAX` (or `PyTorch`)
- **Faster still?** Implement bottleneck in C++ and call from Python

EconModelClass

- **Package:** EconModel
- **Purpose:**
  1. Make it easy to write well-structed code.
  2. Provide standard functionality for copying, saving and loading.
  3. Provide an easy interface to call numba JIT compilled functions.
  4. Provide an easy interface to call C++ functions.
- **Notebooks:**
  EconModelNotebooks\01. Using the EconModelClass.ipynb
  (not the C++ part)
- **Video:** Youtube - EconModel