HANK WITH ENDOGENOUS RISK

1 Model

We consider a *closed* economy with heterogeneous agents, *flexible prices* and *sticky wages*. Time is discrete and indexed by t. There is a continuum of households indexed by i.

Firms. A representative firm hires labor, N_t, to produce goods, with the production function

$$Y_t = \Gamma_t N_t. \tag{1}$$

where Γ_t is the exogenous technology level. Profits are

$$\Pi_t = P_t Y_t - W_t N_t. \tag{2}$$

where P_t is the price level and W_t is the wage level. The first order condition for labor implies that the real wage is exogenous

$$w_t \equiv W_t / P_t = \Gamma_t. \tag{3}$$

Inflation rates for wages and price are given by

$$\pi_t^w \equiv W_t / W_{t-1} - 1 \tag{4}$$

$$\pi_t \equiv \frac{P_t}{P_{t-1}} - 1 = \frac{W_t / \Gamma_t}{W_{t-1} / \Gamma_{t-1}} - 1 = \frac{1 + \pi_t^w}{\Gamma_t / \Gamma_{t-1}} - 1. \tag{5}$$

Perfect competition implies $\Pi_t = 0$.

Households. Households are *ex post* heterogeneous in terms of their time-varying stochastic productivity, captured by e_{it} and u_{it}, and their (end-of-period) savings, a_{it-1}. The distribution of households over idiosyncratic states is denoted $\underline{\boldsymbol{D}}_t$ before shocks are realized and $\boldsymbol{D}_t$ afterwards. Households supply labor, ℓ_{it}, chosen by a union, and choose consumption,

c_{it}, on their own. Aggregate post-tax income net of a lump-sum transfer is $Z_t \equiv w_t N_t - T_t$, where w_t is the real wage, N_t is employment, and T_t are taxes. The idiosyncratic income factor is

$$z_{it} = e_{it}\Delta_t \left(\overline{\phi} + u_{it}\left(\underline{\phi} - \overline{\phi}\right)\right),$$

where assumptions are made so $\mathbb{E}\left[z_{it}\right] = 1$. Households also receive a lump-sum transfer of ω_t. Households are not allowed to borrow. The return on savings from period $t-1$ to t is r_{t-1}.

The household problem is

$$\begin{aligned}
v_t(u_{it}, e_{it}, a_{it-1}) &= \max_{c_t} \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_{it}^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t\left[v_{t+1}(e_{it+1}, u_{it+1}, a_{it})\right] \qquad (6)\\
\text{s.t. } a_{it} + c_{it} &= (1+r_{t-1})a_{it-1} + y_{it}\\
y_{it} &= z_{it} + \omega_t\\
z_{it} &= e_{it}\Delta_t \left(\overline{\phi} + u_{it}\left(\underline{\phi} - \overline{\phi}\right)\right)\\
\log e_{it+1} &= \rho_z \log e_{it} + \psi_{it+1} \ \ , \psi_{it} \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \ \ \mathbb{E}\left[e_{it}\right] = 1\\
\Pr\left[u_{it+1} = 1 | u_{it} = 0\right] &= \delta_{t+1}\\
\Pr\left[u_{it+1} = 0 | u_{it} = 0\right] &= 1 - \delta_{t+1}\\
\Pr\left[u_{it+1} = 1 | u_{it} = 1\right] &= (1-\xi) + \xi\delta_{t+1}\\
\Pr\left[u_{it+1} = 0 | u_{it} = 0\right] &= (1-\delta_{t+1})\xi\\
a_{it} &\geq 0,
\end{aligned}$$

where β is the discount factor, σ is the inverse elasticity of substitution, φ controls the disutility of supplying labor and ν is the inverse of the Frisch elasticity. We assume

$$\delta_t = \frac{\overline{\phi} - \left(\frac{Z_t}{Z_{ss}}\right)^{1-\gamma}}{\overline{\phi} - \underline{\phi}}. \qquad (7)$$

and let the scaling factor Δ_t adjust to ensure $\mathbb{E}\left[z_{it}\right] = 1$. If $\xi = 1$ we have

$$\mathbb{E}\left[z_{it}\right] = 1 \Leftrightarrow \Delta_t \mathbb{E}\left[\left(\overline{\phi} + \delta_t\left(\underline{\phi} - \overline{\phi}\right)\right)\right] = 1 \Leftrightarrow \Delta_t = \left(\frac{Z_t}{Z_{ss}}\right)^{\gamma-1}$$

We assume γ is such that we always have $\delta_t \in (0,1)$.

Aggregate quantities are

$$A_t^{hh} = \int a_t^* \left(z_{it}, a_{it-1}\right) d\boldsymbol{D}_t \tag{8}$$

$$N_t^{hh} = \int \ell_t^* \left(z_{it}, a_{it-1}\right) z_{it} d\boldsymbol{D}_t \tag{9}$$

$$C_t^{hh} = \int c_t^* \left(z_{it}, a_{it-1}\right) d\boldsymbol{D}_t. \tag{10}$$

Union. A union chooses the labor supply of each household and sets wages. Each household is chosen to supply the same amount of labor,

$$\ell_{it} = N_t^{hh}. \tag{11}$$

Unspecified adjustment costs imply a *New Keynesian Wage Philips Curve*,

$$\pi_t^w(1+\pi_t^w) = \kappa \left(\frac{\varphi N_t^\nu}{\left(C_t^*\right)^{-\sigma} Z_t / N_t} - 1 \right) + \beta \pi_{t+1}^w \left(1 + \pi_{t+1}^w\right),$$

where $C_t^* = \left(\mathbb{E}\left[c_{it}^{-\sigma} z_{it}\right]\right)^{-\frac{1}{\sigma}}$.

Central bank. The central bank either follows a standard Taylor rule,

$$1 + i_t = \left(1 + r_{ss}\right)\left(1 + \pi_t\right)^{\phi_\pi}, \tag{12}$$

where i_t is the nominal return from period t to period $t+1$ and ϕ_π is the Taylor coefficient. Or a real rate rule where

$$1 + i_t = (1 + r_{ss})(1 + \pi_{t+1}). \tag{13}$$

The *ex ante* real interest rate is

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}}. \tag{14}$$

Government. The government chooses consumption, G_t, and finances it with either taxes, T_t, or real bonds, B_t. The budget constraint is

$$B_t = (1 + r_{t-1}) B_{t-1} + G_t + \omega_t - T_t. \tag{15}$$

We assume the debt rule

$$B_t = B_{ss} + \phi_B \left(B_{t-1} - B_{ss} + G_t - G_{ss}\right). \tag{16}$$

Market clearing. Market clearing implies

1. Asset market: $B_t = A_t^{hh}$
2. Labor market: $N_t = N_t^{hh}$
3. Goods market: $Y_t = C_t^{hh} + G_t$

2 Solution and Calibration

See provided notebook.

3 Questions

I. Intertemporal marginal propensities to consume. The consumption function can be written as

$$C_t^{hh} = \mathcal{C}_t\left(\{Z_t\}, \{\Delta_t\}, \{\delta_t\}, \{\omega_t\}\right) \tag{17}$$

We define the following matrices:

$\mathbf{M}$ has entries $[M]_{ts} = \frac{\partial \mathcal{C}_t}{\partial Z_s}$
$\mathbf{M}_\Delta$ has entries $[M_\Delta]_{ts} = \frac{\partial \mathcal{C}_t}{\partial \Delta_s}$
$\mathbf{M}_\delta$ has entries $[M_\delta]_{ts} = \frac{\partial \mathcal{C}_t}{\partial \delta_s}$
$\mathbf{M}_\omega$ has entries $[M_\omega]_{ts} = \frac{\partial \mathcal{C}_t}{\partial \omega_s}$

a) Discuss the difference between $\mathbf{M}$, $\mathbf{M}_\Delta$, $\mathbf{M}_\delta$, and$\mathbf{M}_\omega$

b) Verify analytically that

$$\mathbf{M}_\Delta = Z_{ss}\mathbf{M} \tag{18}$$

Use the notation $d\mathbf{X} = [X_0 - X_{ss}, X_1 - X_{ss}, \dots]$

d) Show analytically that with a real rate rule ($r_t = r_{ss}, \forall t$), no lump-sum transfer ($\omega_t = 0, \forall t$) and $\xi = 1$, the consumption sequence is given by

$$d\mathbf{C}^{hh} = \left(\gamma \mathbf{M} - (1-\gamma)\chi \mathbf{M}_\delta\right) d\mathbf{Z} \tag{19}$$

where $\chi \equiv \left(Z_{ss}\left(\overline{\phi} - \underline{\phi}\right)\right)^{-1}$.

II. Fiscal shock. Assume that a path of government consumption is announced such that $dG_t = 0.01 \cdot 0.80^t$.

a) Explain the transmission mechanism and what drives the response of consumption.

Define the (cumulative) fiscal multiplier as

$$\mathcal{M} = \frac{\sum_{t=0}^{\infty}(1+r_{ss})^{-t}\left(Y_t - Y_{ss}\right)}{\sum_{t=0}^{\infty}(1+r_{ss})^{-t}\left(T_t - T_{ss}\right)}$$

b) Discuss how the fiscal multiplier depend on i) B_{ss}/Y_{ss}, ii) γ and iii) ξ.

c) Broaden the discussion of what determines the fiscal multiplier in the model in your own choice of direction.