CENTER FOR
ECONOMIC
BEHAVIOR &
INEQUALITY

HANK models

Mini-Course: Heterogenous Agent Macro

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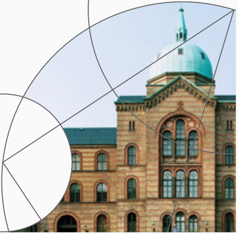

Introduction

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- **Today:** HANK - Heterogeneous Agent *New Keynesian* Model
  - Analytical insights (»opening the black box«)
    1. Zero-liquidity (Werning, 2015)
    2. Intertemporal Keynesian Cross (IKC) (Aucler et. al, 2023)
  - Sticky prices and sticky wages in practice (Kaplan, Moll, Violante, 2018)
  - Search-and-match labor market (Broer et. al., 2024)

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- **GEModelTools:**
  1. HANK-sticky-prices
  2. HANK-sticky-wages
  3. HANK-SAM
  4. I-HANK (not covered)
  5. HANK-two-asset (not covered)

Plan

1. Introduction
2. Zero liquidity
3. Sticky prices
4. Sticky wages
5. IKC
6. HANK-SAM
7. Summary

Zero liquidity

Households

1. Preferences: $\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[\frac{c_t^{1-\sigma}}{1-\sigma} \right]$
2. Idiosyncratic productivity, $s_t \sim \mathcal{S}$, which follows a Markov process
3. Risk-less bonds, b_{t-1}, with a real gross return of R_{t-1}
4. Income: $\gamma(s_t, Y_t)$ such that $Y_t = \int \gamma(s_t, Y_t) d\boldsymbol{D}_t$
5. Budget constraint, $c_t + b_t \leq \gamma(z_t, Y_t) + R_{t-1} b_{t-1}$
6. Borrowing constraint: $b_t \geq 0$
7. Optimal policy functions: $c_t^*(s_t, b_{t-1})$ and $b_t^*(s_t, b_{t-1})$.
8. Unconstrained, $b_t > 0$:

$$c_t^*(s_t, b_{t-1})^{-\sigma} = \beta R_t \mathbb{E}_t [c_t^*(s_{t+1}, b_t)^{-\sigma}]$$

9. Constrained, $b_t = 0$:

$$c_t^*(s_t, b_{t-1})^{-\sigma} > \beta R_t \mathbb{E}_t [c_t^*(s_{t+1}, b_t)^{-\sigma}]$$

Market clearing

- **Market clearing:**
  1. Goods:
  $$Y_t = C_t^{hh} = \int c_t^*(s_t, b_{t-1}) d\boldsymbol{D}_t$$
  2. Assets:
  $$B_t = B_t^{hh} \int b_t^*(s_t, b_{t-1}) d\boldsymbol{D}_t$$
- **Vanishing liquidity**, $B_t \to 0$ (equilibrium section rule):
  An infinitesimal increase in R_t in any given period makes at least one household willing to save more, i.e. buy more bonds.
  1. At least *one* household is on its Euler-equation
  2. Everybody consumes their own income each period (*autarky*)

Marginal saver

- **Equilibrium condition:** For a given $\{Y_t\}_{t\geq 0}$, the unique equilibrium price path is $\{R_t^*\}_{t\geq 0}$, where R_t^* is given by the Euler-equation of the *marginal saver,*

$$R_t^* \equiv R_t^*(\{Y_t\}_{t\geq 0}) = \min_{s_t \in \mathcal{S}} \tilde{R}_t(s_t)$$

where

$$\tilde{R}_t(s_t) \equiv \tilde{R}_t(s_t, \{Y_t\}_{t\geq 0}) = \beta^{-1} \frac{\gamma(s_t, Y_t)^{-\sigma}}{\mathbb{E}_t[\gamma(s_{t+1}, Y_{t+1})^{-\sigma}]}.$$

- **Intuition:**
  1. $R_t > R_t^*$: Some households would like to save.
  2. $R_t < R_t^*$: The Euler-equation would not bind for any household.
- **Marginal saver:** $s_t^* \equiv s_t^*(\{Y_t\}_{t\geq 0}) = \arg\min_{s_t \in \mathcal{S}} \tilde{R}_t(s_t)$,

Equilibrium

- **Equilibrium path:** $\{C_t, R_t\}_{t\geq 0}$ must satisfy

$$\gamma(s_t^*, C_t)^{-\sigma} = \beta R_t \mathbb{E}_t^*[\gamma(s_{t+1}, C_{t+1})^{-\sigma}]$$

where $Y_t = C_t$ (market clearing) and $\mathbb{E}_t^*[\bullet] = \mathbb{E}_t[\bullet \mid s_t = s_t^*]$.

- **Amplification and propagation:**

$$\frac{d \log C_t}{d \log R_t}_{|d \log C_{t+1}=0} = \frac{-\sigma}{\varepsilon(s_t^*, C_t)}$$

$$\frac{d \log C_t}{d \log C_{t+1}}_{|d \log R_t=0} = \mathbb{E}_t^* \left[\frac{\gamma(s_{t+1}, C_{t+1})^{-\sigma}}{\mathbb{E}_t^* \left[(\gamma(s_{t+1}, C_{t+1}))^{-\sigma} \right]} \frac{\varepsilon(s_{t+1}, C_{t+1})}{\varepsilon(s_t^*, C_t)} \right]$$

where $\varepsilon(s_t, Y_t)$ is the elasticity of hh. income wrt. agg. income.

$$\varepsilon(s_t, Y_t) = \frac{\gamma_Y(s_t, Y_t) Y_t}{\gamma(s_t, Y_t)}$$

- **Neutrality of heterogeneity** if $\varepsilon(s_t, Y_t) = 1$

Example: Employed vs. unemployed

- **Employed:** $\overline{y}Y^{\gamma}, \gamma > 0$ (marginal saver)
- **Unemployed:** $\underline{y}Y^{\gamma}$, $\underline{y} < \overline{y}$
- **Unemployment risk**, $\lambda(Y)$: $Y = (1 - \lambda(Y))\,\overline{y}Y^{\gamma} + \lambda(Y)\,\underline{y}Y^{\gamma}$
- **Marginal saver** is employed with Euler-equation

$$\left(\overline{y}Y^{\gamma}\right)^{-\sigma} = \beta R_t \left[(1 - \lambda(Y_{t+1}))\left(\overline{y}Y_{t+1}^{\gamma}\right)^{-\sigma} + \lambda(Y_{t+1})\left(\underline{y}Y_{t+1}^{\gamma}\right)^{-\sigma}\right] \Leftrightarrow$$

$$Y_t^{-\sigma} = \tilde{\beta}(Y_{t+1}) R_t Y_{t+1}^{-\sigma}$$

where $\beta(Y_{t+1}) = \left(\beta\,(1 - \lambda(Y_{t+1})) + \lambda(Y_{t+1})\left(\underline{y}/\overline{y}\right)^{-\sigma}\right)^{\frac{1}{\gamma}}$

- **Equivalence:** If $\gamma = 1$ with $\frac{\partial \lambda(Y)}{\partial Y} = 0$
- **Counter-cyclical income risk**, $\gamma < 1$: $\frac{\partial \lambda(Y)}{\partial Y} < 0$
  1. Amplification: $\frac{d \log C_t}{d \log R_t}_{|d \log C_{t+1}=0} \uparrow$
  2. Propagation: $\frac{d \log C_t}{d \log C_{t+1}}_{|d \log R_t=0} \uparrow$ (because $\frac{\partial \beta(Y)}{\partial Y} < 0$)

Sticky prices

Overview

- **Households:**
  1. Differ by stochastic idiosyncratic productivity and savings
  2. Supply labor and choose consumption
  3. Subject to a borrowing constraint

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- **Central bank**: Set nominal interest rate

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- ***Static* problem for representative final good firm:**

$$\max_{y_{jt}\ \forall j} P_t Y_t - \int_0^1 p_{jt} y_{jt} dj \text{ s.t. } Y_t = \left(\int_0^1 y_{jt}^{\frac{1}{\mu}} dj \right)^{\mu}$$

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- **Demand curve** derived from FOC wrt. y_{jt}

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- **Note:** Zero profits (can be used to derive price index)

Derivation of demand curve

- **FOC wrt.** y_{jt}

$$
\begin{aligned}
0 &= P_t \mu \left(\int_0^1 y_{jt}^{\frac{1}{\mu}} \, dj \right)^{\mu - 1} \frac{1}{\mu} y_{jt}^{\frac{1}{\mu} - 1} - p_{jt} \Leftrightarrow \\
\frac{p_{jt}}{P_t} &= \left(\int_0^1 y_{jt}^{\frac{1}{\mu}} \, dj \right)^{\mu - 1} y_{jt}^{\frac{1-\mu}{\mu}} \Leftrightarrow \\
\left(\frac{p_{jt}}{P_t} \right)^{\frac{\mu}{\mu - 1}} &= \left(\int_0^1 y_{jt}^{\frac{1}{\mu}} \, dj \right)^{\mu} y_{jt}^{-1} \Leftrightarrow \\
y_{jt} &= \left(\frac{p_{jt}}{P_t} \right)^{-\frac{\mu}{\mu - 1}} Y_t
\end{aligned}
$$

Intermediary goods firms

- ***Dynamic*** **problem for intermediary goods firms:**

$$J_t(p_{jt-1}) = \max_{y_{jt},p_{jt},n_{jt}} \left\{ \frac{p_{jt}}{P_t} y_{jt} - w_t n_{jt} - \Omega(p_{jt}, p_{jt-1}) Y_t + \frac{J_{t+1}(p_{jt})}{1+r_{t+1}} \right\}$$

$$\text{s.t. } y_{jt} = \Gamma_t n_{jt}, \quad y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\frac{\mu}{\mu-1}} Y_t$$

$$\Omega(p_{jt}, p_{jt-1}) = \frac{\mu}{\mu-1} \frac{1}{2\kappa} \left[\log\left(\frac{p_{jt}}{p_{jt-1}} \right) \right]^2$$

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- **NKPC** derived from FOC wrt. p_{jt} and envelope condition:

$$\log(1+\pi_t) = \kappa \left(\frac{w_t}{\Gamma_t} - \frac{1}{\mu} \right) + \frac{Y_{t+1}}{Y_t} \frac{\log(1 + \pi_{t+1})}{1 + r_{t+1}}, \;\; \pi_t \equiv P_t / P_{t-1} - 1$$

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- **Implied production:** $Y_t = y_{jt}$, $N_t = n_{jt}$ (from symmetry)
- **Implied dividends:** $d_t = Y_t - w_t N_t - \frac{\mu}{\mu - 1} \frac{1}{2\kappa} \left[\log(1 + \pi_t) \right]^2 Y_t$

Derivation of NKPC

- **FOC wrt. p_{jt}:**

$$
\begin{aligned}
0 = & \left(1 - \frac{\mu}{\mu - 1}\right) \left(\frac{p_{jt}}{P_t}\right)^{-\frac{\mu}{\mu-1}} \frac{Y_t}{P_t} + \frac{\mu}{\mu - 1} \frac{w_t}{\Gamma_t} \left(\frac{p_{jt}}{P_t}\right)^{-\frac{\mu}{\mu-1}} \frac{Y_t}{p_{jt}} \\
& - \frac{\mu}{\mu - 1} \frac{1}{\kappa} \frac{\log\left(\frac{p_{jt}}{p_{jt-1}}\right)}{p_{jt}} Y_t + \frac{J'_{t+1}(p_{jt})}{1 + r_{t+1}}
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- **FOC + Envelope + Symmetry +** $\pi_t = P_t/P_{t-1} - 1$

$$0 = \left(1 - \frac{\mu}{\mu - 1}\right)\frac{Y_t}{P_t} + \frac{\mu}{\mu - 1}\frac{w_t}{\Gamma_t}\frac{Y_t}{P_t}$$
$$+ \frac{\mu}{\mu - 1}\frac{1}{\kappa}\log\left(1 + \pi_t\right)\frac{Y_t}{P_t} + \frac{\frac{\mu}{\mu-1}\frac{1}{\kappa}\log\left(1 + \pi_{t+1}\right)\frac{Y_{t+1}}{P_t}}{1 + r_{t+1}}$$

Central NKPC intution

$$\log(1+\pi_t) = \kappa\left(\frac{w_t}{Z_t} - \frac{1}{\mu}\right) + \frac{Y_{t+1}}{Y_t}\frac{\log(1+\pi_{t+1})}{1+r_{t+1}}$$

1. **Zero-inflation steady state:**
   $\pi_t = 0 \to w_t = \frac{\Gamma_t}{\mu} \to$ wage is mark-downed relative to productivity

(Note: Sometimes a β^{firm} is used instead of $\frac{1}{1+r_{t+1}}$)

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   Less pass-through from marginal costs, $\frac{w_t}{Z_t}$, to inflation, π_t

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4. **Dividends:** *Counter-cyclical* as wages increase more than prices

(Note: Sometimes a β^{firm} is used instead of $\frac{1}{1+r_{t+1}}$)

Households

- **Household problem**: Distribution, $\boldsymbol{D}_t$, over z_{it} and a_{it-1}

$$v_t(z_{it}, a_{it-1}) = \max_{c_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_{it}^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t \left[v_{t+1}(z_{it+1}, a_{it}) \right]$$

$$\text{s.t. } a_{it} = (1+r_t)a_{it-1} + (w_t \ell_{it} - \tau_t + d_t) z_{it} - c_{it} \geq \underline{a}$$

$$\log z_{it+1} = \rho_z \log z_{it} + \psi_{it+1} \;\;, \psi_{it} \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \, \mathbb{E}[z_{it}] = 1$$

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- **Dividends:** Distributed proportional to productivity (ad hoc)
- **Taxes:** Collected proportional to productivity (ad hoc)

Households

- **Household problem**: Distribution, $\boldsymbol{D}_t$, over z_{it} and a_{it-1}

$$v_t(z_{it}, a_{it-1}) = \max_{c_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_{it}^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t \left[v_{t+1}(z_{it+1}, a_{it}) \right]$$

$$\text{s.t. } a_{it} = (1+r_t)a_{it-1} + (w_t \ell_{it} - \tau_t + d_t) z_{it} - c_{it} \geq \underline{a}$$

$$\log z_{it+1} = \rho_z \log z_{it} + \psi_{it+1} \;\; , \psi_{it} \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \, \mathbb{E}[z_{it}] = 1$$

- **Dividends:** Distributed proportional to productivity (ad hoc)
- **Taxes:** Collected proportional to productivity (ad hoc)
- **Optimality conditions:**

$$\text{FOC wrt. } c_{it} : 0 = c_{it}^{-\sigma} - \beta \mathbb{E}_t \left[v_{a,t+1}(z_{it+1}, a_{it}) \right]$$

$$\text{FOC wrt. } \ell_{it} : 0 = w_t z_{it} \beta \mathbb{E}_t \left[v_{a,t+1}(z_{it+1}, a_{it}) \right] - \varphi \ell_{it}^{\nu}$$

$$\text{Envelope condition: } v_{a,t}(z_{it}, a_{it-1}) = (1+r_t) c_{it}^{-\sigma}$$

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- **Effective labor-supply:** $n_{it} = z_{it} \ell_{it}$

EGM I

- **Beginning-of-period value function:**

$$\underline{v}_{a,t}(z_{it-1}, a_{it-1}) = \mathbb{E}_t \left[v_{a,t}(z_{it}, a_{it-1}) \right] = \mathbb{E}_t \left[(1 + r_t) c_{it}^{-\sigma} \right]$$

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- **Endogenous grid method:** Vary z_t and a_t to find

$$\begin{aligned} c_{it} &= \left(\beta \underline{v}_{a,t+1}(z_{it}, a_{it}) \right)^{-\frac{1}{\sigma}} \\ \ell_{it} &= \left(\frac{w_t z_{it}}{\varphi} c_{it}^{-\sigma} \right)^{\frac{1}{\nu}} \\ m_{it} &= c_{it} + a_{it} - (w_t \ell_{it} - \tau_t + d_t) z_{it} \end{aligned}$$

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$$m_{it} = c_{it} + a_{it} - (w_t \ell_{it} - \tau_t + d_t) z_{it}$$

- **Consumption and labor supply:** Use linear interpolation to find

$$c^*(z_{it}, a_{it-1}) \text{ and } \ell^*(z_{it}, a_{it-1}) \text{ with } m_{it} = (1 + r_t) a_{it-1}$$

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- **Savings:** $a^*(z_{it}, a_{it-1}) = (1+r_t)a_{it-1} - c_{it}^* + (w_t \ell_{it}^* - \tau_t + d_t) z_{it}$

EGM II

- **Problem:** $a_t^*(z_{it}, a_{it-1}) < \underline{a}$ violate borrowing constraint

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$$
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  2. Set

  $$\ell_{it}^* = \frac{f(\ell_{it}^*)}{f'(\ell_{it}^*)} = \frac{f(\ell_{it}^*)}{1 - \left(\frac{w_t z_{it}}{\varphi}\right)^{\frac{1}{\nu}} \left(-\frac{\sigma}{\nu}\right) (c_{it}^*)^{-\frac{\sigma}{\nu}} w_t z_{it}}$$

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  3. Return to step 1

Government and central bank

- **Monetary policy:** Follow Taylor-rule:

$$i_t = i_t^* + \phi\pi_t + \phi^Y(Y_t - Y_{ss})$$

where i_t^* is a shock

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$$r_t = (1 + i_{t-1})/(1 + \pi_t) - 1$$

Government and central bank

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  where i_t^* is a shock

- **Fisher relationship:**

$$r_t = (1 + i_{t-1})/(1 + \pi_t) - 1$$

- **Government:** Choose τ_t to keep debt constant and finance exogenous public consumption

$$\tau_t = r_t B_{ss} + G_t$$

Market clearing

1. Assets: $B_{ss} = \int a_t^*(z_{it}, a_{it-1}) d\boldsymbol{D}_t$
2. Labor: $N_t = \int n_t^*(z_{it}, a_{it-1}) d\boldsymbol{D}_t$ (in effective units)
3. Goods: $Y_t = \int c_t^*(z_{it}, a_{it-1}) d\boldsymbol{D}_t + G_t + \frac{\mu}{\mu-1}\frac{1}{2\kappa}\left[\log\left(1+\pi_t\right)\right]^2 Y_t$

As an equation system

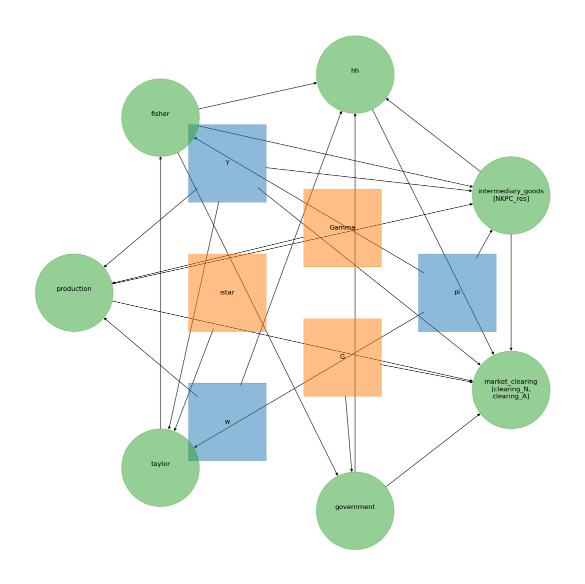
$$\boldsymbol{H}\left(\boldsymbol{\pi}, \boldsymbol{w}, \boldsymbol{Y}, \boldsymbol{i}^{*}, \boldsymbol{\Gamma}, \underline{\boldsymbol{D}}_0\right) = \mathbf{0}$$

$$\begin{bmatrix} \log(1+\pi_t) - \left[\kappa\left(\frac{w_t}{Z_t} - \frac{1}{\mu}\right) + \frac{Y_{t+1}}{Y_t}\frac{\log(1+\pi_{t+1}}{1+r_{t+1}})\right] \\ N_t - \int n_t^*(z_{it}, a_{it-1}) d\boldsymbol{D}_t \\ B_{ss} - \int a_t^*(z_{it}, a_{it-1}) d\boldsymbol{D}_t \end{bmatrix} = \mathbf{0}$$

The rest of the model is given by

$$\boldsymbol{X} = \boldsymbol{M}(\boldsymbol{\pi}, \boldsymbol{w}, \boldsymbol{Y}, \boldsymbol{i}^{*}, \boldsymbol{\Gamma})$$

As a DAG


hh
fisher
Y
intermediary_goods
[NKPC_res]
Gamma
production
istar
pi
G
market_clearing
[clearing_N,
clearing_A]
w
taylor
government

Steady state

- **Chosen:** B_{ss}, G_{ss}, r_{ss}
- **Analytically:**
  1. **Normalization:** $Z_{ss} = N_{ss} = 1$
  2. **Zero-inflation:** $\pi_{ss} = 0 \Rightarrow i_{ss} = i^*_{ss} = (1 + r_{ss})(1 + \pi_{ss}) - 1$
  3. **Firms:** $Y_{ss} = Z_{ss}N_{ss}$, $w_{ss} = \frac{Z_{ss}}{\mu}$ and $d_{ss} = Y_{ss} - w_{ss}N_{ss}$
  4. **Government:** $\tau_{ss} = r_{ss}B_{ss} + G_{ss}$
  5. **Assets:** $A_{ss} = B_{ss}$
- **Numerically:** Choose β and φ to get market clearing

Transmission mechanism to monetary policy shock

1. **Monetary policy shock:** $i_t^* \downarrow \Rightarrow i_t = i_t^* + \phi\pi_t \downarrow$
2. **Real interest rate:** $r_t = \frac{1+i_{t-1}}{1+\pi_t} \downarrow$
3. **Taxes:** $\tau_t = r_t B_{ss} \downarrow$
4. **Household consumption**, $C_t^{hh} \uparrow$, due to $r_t \downarrow$ and $\tau_t \downarrow$
5. **Firms production,** $Y_t\uparrow$, and **labor demand**, $N_t\uparrow$
6. **Inflation,** $\pi_t \uparrow$, and **wage**, $w_t\uparrow$ and **dividends**, $d_t\downarrow$
7. **Household labor supply**, $N_t^{hh}\uparrow$, due to $w_t \uparrow$ and $d_t \downarrow$,
   but dampened $\tau_t \downarrow$
8. **Nominal rate**, $i_t \uparrow$ due to $\pi_t \uparrow$ implying $r_t \uparrow$
9. **Household consumption**, $C_t^{hh}\uparrow$, due to $w_t \uparrow$
   but dampened by $d_t \downarrow$ and $r_t \uparrow$

Representative agent

- **Replace market clearing conditions with FOCs:**

$$C_t^{-\sigma} = \beta(1+r_{t+1})C_{t+1}^{-\sigma}$$
$$\varphi N_t^{\nu} = w_t C_t^{-\sigma}$$

- From resource constraint: $C_t = Y_t - G_t - \frac{\mu}{\mu-1}\frac{1}{2\kappa}\left[\log\left(1+\pi_t\right)\right]^2 Y_t$
- Ensure same steady state: $\beta^{RA} = \frac{1}{1+r_{ss}}$, $\varphi^{RA} = \frac{w_{ss}\left(C_{ss}^{hh}\right)^{-\sigma}}{\left(N_{ss}\right)^{\nu}}$
- **Intertemporal budget constraint:**

$$C_0 + \frac{C_1}{1+r_1} + \ldots = \left(1+r_0\right)A_{-1} + Y_0^{RA} + \frac{Y_1^{RA}}{1+r_1}\ldots$$

where $Y_t^{RA} = w_t N_t + d_t - \tau_t$ is household income

Monetary Policy According to HANK

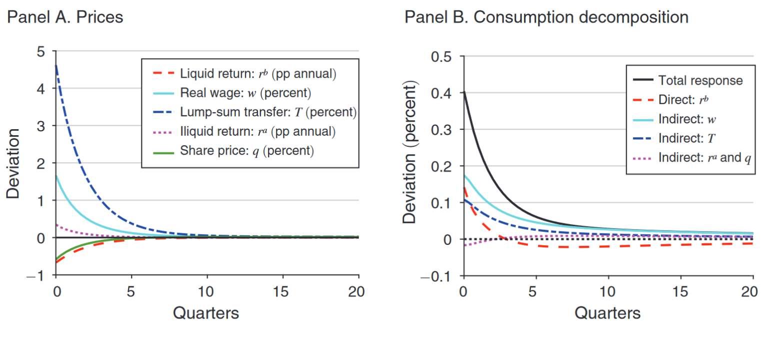


FIGURE 4. DIRECT AND INDIRECT EFFECTS OF MONETARY POLICY IN HANK

- **RANK:** Everything is due to substitution
- **HANK:** It is the indirect effects, which dominates

Source: Kaplan, Moll and Violante (2018)

Sticky wages

Households

- **Household problem:**

$$
\begin{aligned}
v_t(z_t, a_{t-1}) = \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_t^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t \left[v_{t+1}(z_{t+1}, a_t) \right] \\
\text{s.t. } a_t + c_t = (1 + r_t^a) a_{t-1} + (1 - \tau_t) w_t \ell_t z_t + \chi_t \\
\log z_{t+1} = \rho_z \log z_t + \psi_{t+1} \ , \psi_t \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \ \mathbb{E}[z_t] = 1 \\
a_t \geq 0
\end{aligned}
$$

- **Active decisions:** Consumption-saving, c_t (and a_t)
- **Union decision:** Labor supply, ℓ_t
- **Consumption function:** $C_t^{hh} = C^{hh} \left(\{ r_s^a, \tau_s, w_s, \ell_s, \chi_s \}_{s \geq 0} \right)$

Firms

- **Production and profits:**

$$
\begin{aligned}
Y_t &= \Gamma_t L_t \\
\Pi_t &= P_t Y_t - W_t L_t
\end{aligned}
$$

- **First order condition:**

$$\frac{\partial \Pi_t}{\partial L_t} = 0 \Leftrightarrow P_t \Gamma_t - W_t = 0 \Leftrightarrow w_t \equiv W_t / P_t = \Gamma_t$$

  Zero profits: $\Pi_t = 0$

- **Wage and price inflation:**

$$
\begin{aligned}
\pi_t^w &\equiv W_t / W_{t-1} - 1 \\
\pi_t &\equiv \frac{P_t}{P_{t-1}} - 1 = \frac{W_t / \Gamma_t}{W_{t-1} / \Gamma_{t-1}} - 1 = \frac{1 + \pi_t^w}{\Gamma_t / \Gamma_{t-1}} - 1
\end{aligned}
$$

Union

- Everybody works the same:

$$\ell_t = L_t^{hh}$$

- Unspecified *wage adjustment costs* imply a **New Keynesian Wage (Phillips) Curve** (NKWPC or NKWC)

$$\pi_t^w = \kappa \left(\varphi \left(L_t^{hh} \right)^{\nu} - \frac{1}{\mu} \left(1 - \tau_t \right) w_t \left(C_t^{hh} \right)^{-\sigma} \right) + \beta \pi_{t+1}^w$$

Government

- **Spending:** G_t
- **Tax bill:** T_t

$$T_t = \int \tau_t w_t \ell_t z_t d\boldsymbol{D}_t = \tau_t \Gamma_t L_t = \tau_t Y_t$$

- If **one-period bonds:**

$$B_t = (1 + r_t^b) B_{t-1} + G_t + \chi_t - T_t$$

- If **long-term bonds:** Geometrically declining payment stream of $1, \delta, \delta^2, \ldots$ for $\delta \in [0, 1]$. The bond price is q_t.

$$q_t (B_t - \delta B_{t-1}) = B_{t-1} + G_t + \chi_t - T_t$$

- Potential **tax-rule:**

$$\tau_t = \tau_{ss} + \omega q_{ss} \frac{B_{t-1} - B_{ss}}{Y_{ss}}$$

Central bank

- Standard **Taylor rule:**

$$1 + i_t = (1 + i_{t-1})^{\rho_i} \left((1 + r_{ss}) (1 + \pi_t)^{\phi_\pi} \right)^{1-\rho_i}$$

  Alternative: Real rate rule

$$1 + i_t = (1 + r_{ss})(1 + \pi_{t+1})$$

  Indeterminacy: Consider limit or assume future tightening

- **Fisher-equation:**

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}}$$

Arbitrage

1. One-period *real* bond, $q_t = 1$:

$$\begin{aligned} t > 0: \; r_t^b &= r_t^a = r_{t-1} \\ r_0^b &= r_0^a = 1 + r_{ss} \end{aligned}$$

2. or, one-period *nominal* bond, $q_t = 1$:

$$\begin{aligned} t > 0: \; r_t^b &= r_t^a = r_{t-1} \\ t > 0: \; r_0^b &= r_0^a = (1 + r_{ss})(1 + \pi_{ss}) / (1 + \pi_0) \end{aligned}$$

3. or, long-term *(real)* bonds:

$$\frac{1 + \delta q_{t+1}}{q_t} = 1 + r_t$$

$$1 + r_t^b = 1 + r_t^a = \frac{1 + \delta q_t}{q_{t-1}} = \begin{cases} \frac{1+\delta q_0}{q_{ss}} & \text{if } t = 0 \\ 1 + r_{t-1} & \text{else} \end{cases}$$

Market clearing

1. Asset market: $q_t B_t = A_t^{hh}$
2. Labor market: $L_t = L_t^{hh}$
3. Goods market: $Y_t = C_t^{hh} + G_t$

Equation system

Taylor-rule and long-term government debt:

$$
\begin{bmatrix}
w_t - \Gamma_t \\
Y_t - \Gamma_t L_t \\
1 + \pi_t - \frac{1+\pi_t^w}{\Gamma_t / \Gamma_{t-1}} \\
1 + i_t - (1 + i_{t-1})^{\rho_i} \left((1 + r_{ss}) (1 + \pi_t)^{\phi_\pi} \right)^{1-\rho_i} \\
1 + r_t - \frac{1+i_t}{1+\pi_{t+1}} \\
\frac{1+\delta q_{t+1}}{q_t} - (1 + r_t) \\
1 + r_t^a - \frac{1+\delta q_t}{q_{t-1}} \\
\tau_t - \left[\tau_{ss} + \omega q_{ss} \frac{B_{t-1} - B_{ss}}{Y_{ss}} \right] \\
q_t (B_t - \delta B_{t-1}) - [B_{t-1} + G_t + \chi_t - \tau_t Y_t] \\
q_t B_t - A_t^{hh} \\
\pi_t^w - \left[\kappa \left(\varphi \left(L_t^{hh} \right)^\nu - \frac{1}{\mu} (1 - \tau_t) w_t \left(C_t^{hh} \right)^{-\sigma} \right) + \beta \pi_{t+1}^w \right]
\end{bmatrix}
= \mathbf{0}
$$

Reduced equation system with ordered blocks

$$\boldsymbol{H}(\boldsymbol{\pi^w}, \boldsymbol{L}, \boldsymbol{G}, \chi, \boldsymbol{\Gamma}) = \begin{bmatrix} q_t B_t - A_t^{hh} \\ \pi_t^w - \left[\kappa\left(\varphi\left(L_t^{hh}\right)^{\nu} - \frac{1}{\mu}\left(1-\tau_t\right) w_t \left(C_t^{hh}\right)^{\text{-}\sigma}\right) + \beta \pi_{t+1}^W\right] \end{bmatrix} = \boldsymbol{0}$$

$$\begin{aligned}
\text{Production: } w_t &= \Gamma_t \\
Y_t &= \Gamma_t L_t \\
\pi_t &= \frac{1+\pi_t^w}{\Gamma_t/\Gamma_{t-1}} - 1 \\
\text{Central bank: } i_t &= \left(1+i_{t-1}\right)^{\rho_i}\left(\left(1+r_{ss}\right)\left(1+\pi_t\right)^{\phi_\pi}\right)^{1-\rho_i} - 1 \text{ (forwards)} \\
r_t &= \frac{1+i_t}{1+\pi_{t+1}} - 1 \\
\text{Mutual fund: } q_t &= \frac{1+\delta q_{t+1}}{1+r_t} \text{ (backwards)} \\
r_t^a &= \frac{1+\delta q_t}{q_{t-1}} - 1
\end{aligned}$$

$$\text{Government: } \begin{bmatrix} \tau_t \\ B_t \end{bmatrix} = \begin{bmatrix} \tau_{ss} + \omega q_{ss} \frac{B_{t-1}-B_{ss}}{Y_{ss}} \\ \frac{(1+\delta q_t)B_{t-1} + G_t + \chi_t - \tau_t Y_t}{q_t} \end{bmatrix} \text{ (forwards)}$$

DAG

hh
fisher
Y
intermediary_goods
[NKPC_res]
Gamma
production
istar
pi
G
market_clearing
[clearing_N,
clearing_A]
w
taylor
government

IKC

Simpler consumption function

- **Assumptions:**
  1. One-period real bond
  2. No lump-sum transfers, $\chi_t = 0$
  3. Real rate rule: $r_t = r_{ss}$
  4. Fiscal policy in terms of dG_t and dT_t satisfying IBC

$$\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} (dG_t - dT_t) = 0$$

- **Tax-bill:** $T_t = \tau_t w_t \int \ell_t z_t d\boldsymbol{D}_t = \tau_t \Gamma_t L_t = \tau_t Y_t$
- **Household income:** $(1 - \tau_t) w_t \ell_t z_t = \underbrace{(Y_t - T_t)}_{\equiv Z_t} z_t = Z_t z_t$
- **Consumption function:** Simplifies to

$$C_t^{hh} = C^{hh}(\{Y_s - T_s\}_{s \geq 0}) \Rightarrow \boldsymbol{C}^{hh} = C^{hh}(\boldsymbol{Y} - \boldsymbol{T}) = C^{hh}(\boldsymbol{Z})$$

Side-note: Two-equation version in $\boldsymbol{Y}$ and $\boldsymbol{r}$

$$\boldsymbol{Y} = \boldsymbol{G} + C^{hh}(\boldsymbol{r}, \boldsymbol{Y} - \boldsymbol{T})$$
$$\boldsymbol{r} = \mathcal{R}(\boldsymbol{Y}; \boldsymbol{G}, \boldsymbol{T})$$

- **First equation:** Goods market clearing
- **Second equation:**
    1. Government: $\boldsymbol{T}, \boldsymbol{Y} \to \boldsymbol{\tau}$
    2. Resource constraint: $\boldsymbol{G}, \boldsymbol{Y} \to \boldsymbol{C}$
    3. Firm behavior I: $\boldsymbol{\Gamma}, \boldsymbol{Y} \to \boldsymbol{L}, \boldsymbol{w}$
    4. NKWC: $\boldsymbol{L}, \boldsymbol{C}, \boldsymbol{w}, \boldsymbol{\tau} \to \boldsymbol{\pi}^{\boldsymbol{w}}$
    5. Firm behavior II: $\boldsymbol{\pi}^{\boldsymbol{w}}, \boldsymbol{\Gamma} \to \boldsymbol{\pi}$
    6. Central bank: $\boldsymbol{\pi} \to \boldsymbol{i}$
    7. Fisher: $\boldsymbol{i}, \boldsymbol{\pi} \to \boldsymbol{r}$
- **Heterogeneity does not enter** $\mathcal{R}(\boldsymbol{Y}; \boldsymbol{G}, \boldsymbol{T})$
- **Real rate rule:** *Inflation is a side-show*

Intertemporal Keynesian Cross

$$\boldsymbol{Y} = \boldsymbol{G} + C^{hh}(\boldsymbol{Y} - \boldsymbol{T})$$

- **Total differentiation:**

$$dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial C_t^{hh}}{\partial Z_s} dZ_s = dG_t + \sum_{s=0}^{\infty} \frac{\partial C_t^{hh}}{\partial Z_s} (dY_s - dT_s)$$

- **Intertemporal Keynesian Cross** in vector form

$$d\boldsymbol{Y} = d\boldsymbol{G} + \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) \Leftrightarrow$$
$$(\boldsymbol{I} - \boldsymbol{M})d\boldsymbol{Y} = d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T}$$

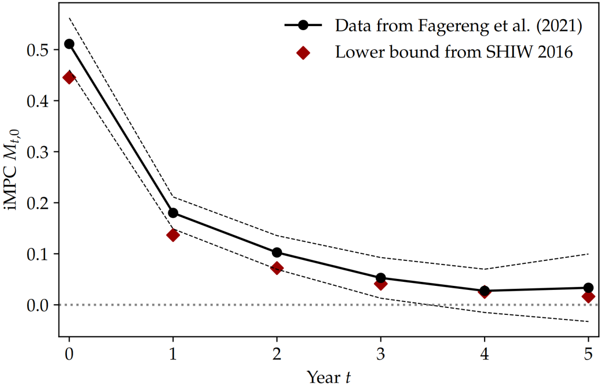
where $M_{t,s} = \frac{\partial C_t^{hh}}{\partial Z_s}$ encodes the entire *complexity*

iMPC matrix

$$\boldsymbol{M} = \begin{bmatrix} \frac{\partial C_0^{hh}}{\partial Z_0} & \frac{\partial C_0^{hh}}{\partial Z_1} & \cdots \\ \frac{\partial C_1^{hh}}{\partial Z_0} & \frac{\partial C_1^{hh}}{\partial Z_1} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

iMPCs in the data

Figure 1: iMPCs in the Norwegian and Italian data



Other columns: Druedahl et al. (2023) show in micro-data that consumption responds today to news about future income.

Perspective: Static Keynesian Cross

- **Old Keynesians:** Consumption only depends on current income

$$Y_t = G_t + C^{hh}(Y_t - T_t)$$

- **Total differentiate:**

$$\begin{aligned} dY_t &= dG_t + \frac{\partial C_t^{hh}}{\partial Z_t}(dY_t - dT_t) \\ &= dG_t + \text{mpc} \cdot (dY_t - dT_t) \end{aligned}$$

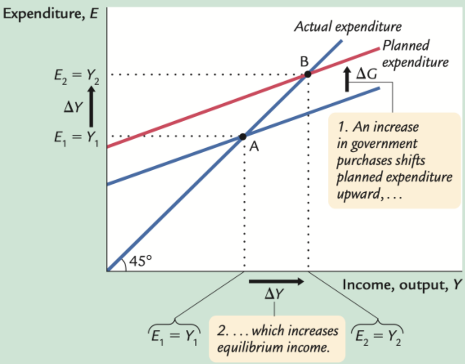
- **Solution**

$$dY_t = \frac{1}{1 - \text{mpc}} \left(dG_t - \text{mpc} \cdot dT_t\right)$$

from multiplier-process $1 + \text{mpc} + \text{mpc}^2 \cdots = \frac{1}{1-\text{mpc}}$

Static Keynesian Cross

figure 10-5



An Increase in Government Purchases in the Keynesian Cross An increase in government purchases of ΔG raises planned expenditure by that amount for any given level of income. The equilibrium moves from point A to point B, and income rises from Y_1 to Y_2. Note that the increase in income ΔY exceeds the increase in government purchases ΔG. Thus, fiscal policy has a multiplied effect on income.

NPV-vector

- **NPV-vector:** $\boldsymbol{q} \equiv [1, (1+r_{ss})^{-1}, (1+r_{ss})^{-2}, \ldots]'$
- **Government:** IBC holds

$$\sum_{t=0}^{\infty} (1+r_{ss})^{-t} (dG_t - dT_t) = 0 \Leftrightarrow$$

$$\boldsymbol{q}'(d\boldsymbol{G} - d\boldsymbol{T}) = 0$$

- **Households:** IBC holds

$$C_t^{hh} = A_t^{hh} = (1+r_{ss})A_{t-1}^{hh} + Z_t \Rightarrow$$

$$\sum_{t=0}^{\infty} (1+r_{ss})^{-t} C_t^{hh} = (1+r_{ss})A_{-1} + \sum_{t=0}^{\infty} (1+r_{ss})^{-t} Z_t \Rightarrow$$

$$\sum_{t=0}^{\infty} (1+r_{ss})^{-t} M_{t,s} = \frac{1}{(1+r)^s} \Rightarrow$$

$$\boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}' \Leftrightarrow \boldsymbol{q}'(\boldsymbol{I} - \boldsymbol{M}) = \boldsymbol{0}$$

Form of unique solution

- **Problem:** $(\boldsymbol{I}-\boldsymbol{M})^{-1}$ cannot exist because this leads to a contradiction

$$\boldsymbol{q}'(\boldsymbol{I}-\boldsymbol{M})(\boldsymbol{I}-\boldsymbol{M})^{-1} = \boldsymbol{0}(\boldsymbol{I}-\boldsymbol{M})^{-1} \Leftrightarrow$$
$$\boldsymbol{q}' = 0$$

- **Result:** If unique solution then on the form

$$d\boldsymbol{Y} = \mathcal{M}(d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T})$$
$$\mathcal{M} = (\boldsymbol{K}(\boldsymbol{I}-\boldsymbol{M}))^{-1}\boldsymbol{K}$$

- **Indeterminancy:** Still work-in-progress (Auclert et. al., 2023)

Response of consumption

$$
\begin{aligned}
d\boldsymbol{Y} &= d\boldsymbol{G} + \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) \Leftrightarrow \\
d\boldsymbol{Y} - d\boldsymbol{G} &= \boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T}) + \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{G}) \Leftrightarrow \\
(I - \boldsymbol{M})(d\boldsymbol{Y} - d\boldsymbol{G}) &= \boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T}) \Leftrightarrow \\
d\boldsymbol{Y} - d\boldsymbol{G} &= \mathcal{M}\boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T}) \Leftrightarrow \\
d\boldsymbol{C} &= \mathcal{M}\boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T})
\end{aligned}
$$

Fiscal multipliers

$$d\boldsymbol{Y} = d\boldsymbol{G} + \underbrace{\mathcal{M}\boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T})}_{d\boldsymbol{C}}$$

- **Balanced budget multiplier:**

$$d\boldsymbol{G} = d\boldsymbol{T} \Rightarrow d\boldsymbol{Y} = d\boldsymbol{G}, d\boldsymbol{C} = 0$$

  Note: Central that income and taxes affect household income proportionally in exactly the same way = no redistribution

- **Deficit multiplier:** $d\boldsymbol{G} \neq d\boldsymbol{T}$
  1. Larger effect of $d\boldsymbol{G}$ than $d\boldsymbol{T}$
  2. *Numerical results needed*

Fiscal multiplier

Impact-multiplier:

$$\frac{\partial Y_0}{\partial G_0}$$

Cumulative-multiplier:

$$\frac{\sum_{t=0}^{\infty}(1+r_{ss})^{-t}dY_t}{\sum_{t=0}^{\infty}(1+r_{ss})^{-t}dG_t}$$

Comparison with RA model

- **From lecture 1:** $\beta(1+r_{ss}) = 1$ implies

$$C_t = (1-\beta)\sum_{s=0}^{\infty}\beta^s Y_{t+s}^{hh} + r_{ss}a_{-1}$$

- The **iMPC-matrix** becomes

$$\boldsymbol{M}^{RA} = \begin{bmatrix} (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} = (1-\beta)\boldsymbol{1}\boldsymbol{q}'$$

- **Consumption response** is zero

$$\begin{aligned} d\boldsymbol{C}^{\boldsymbol{RA}} &= \mathcal{M}\boldsymbol{M}^{\boldsymbol{RA}}(d\boldsymbol{G} - d\boldsymbol{T}) \\ &= \mathcal{M}(1-\beta)\boldsymbol{1}\boldsymbol{q}'(d\boldsymbol{G} - d\boldsymbol{T}) \\ &= \boldsymbol{0} \Leftrightarrow d\boldsymbol{Y} = d\boldsymbol{G} \end{aligned}$$

Details on matrix formulation

$$(1-\beta)\mathbf{1}\boldsymbol{q}' = \begin{bmatrix} (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} 1 & (1+r_{ss})^{-1} & (1+r_{ss})^{-2} & \cdots \end{bmatrix}$$

$$= \begin{bmatrix} (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ (1-\beta) & (1-\beta) & (1-\beta) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} 1 & \beta & \beta^2 & \cdots \end{bmatrix}$$

$$= \begin{bmatrix} (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Comparison with TA model

- **Hand-to-Mouth (HtM) households:** λ share have $C_t = Y_t^{hh}$

$$\boldsymbol{M}^{TA} = (1-\lambda)\boldsymbol{M}^{RA} + \lambda \boldsymbol{I}$$

- **Intertemporal Keynesian Cross** becomes

$$(\boldsymbol{I} - \boldsymbol{M}^{TA})d\boldsymbol{Y} = d\boldsymbol{G} - \boldsymbol{M}^{TA}d\boldsymbol{T}$$
$$(\boldsymbol{I} - \boldsymbol{M}^{RA})d\boldsymbol{Y} = \underbrace{\frac{1}{1-\lambda}\left[d\boldsymbol{G} - \lambda d\boldsymbol{T}\right]}_{d\tilde{\boldsymbol{G}}_t} - \boldsymbol{M}^{RA}d\boldsymbol{T}$$

- **Same solution-form as RA:** $d\boldsymbol{Y} = d\tilde{\boldsymbol{G}}_t$

$$d\boldsymbol{Y} = d\tilde{\boldsymbol{G}}_t = d\boldsymbol{G}_t + \frac{\lambda}{1-\lambda}\left[d\boldsymbol{G} - d\boldsymbol{T}\right]$$

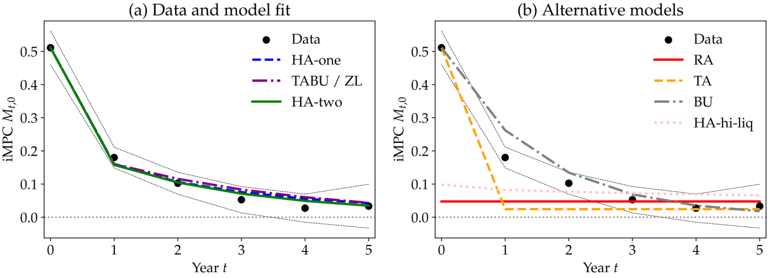
Cumulative multiplier still one

$$\frac{\boldsymbol{q}' d\boldsymbol{Y}}{\boldsymbol{q}' d\boldsymbol{G}} = \frac{\boldsymbol{q}' d\boldsymbol{G}_t + \frac{\lambda}{1-\lambda}\boldsymbol{q}' \left[d\boldsymbol{G} - d\boldsymbol{T}\right]}{\boldsymbol{q}' d\boldsymbol{G}}$$

$$= 1$$

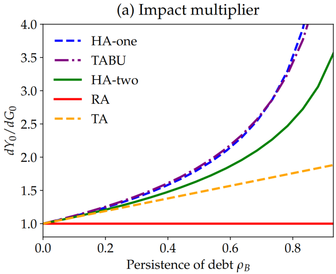
iMPCs in models

Figure 2: iMPCs in the Norwegian data and several models



Multipliers and debt-financing

Figure 5: Multipliers according to the IKC



Note. These figures assume a persistence of government spending equal to $\rho_G = 0.76$, and vary ρ_B in $dB_t = \rho_B(dB_{t-1} + dG_t)$. See section 7.1 for details on calibration choices.

Generalized IKC

- **Budget constraint** can be written with initial capital gain

$$a_t + c_t = (Y_t - T_t)z_t + \chi_t + \begin{cases} (1 + r_{t-1})a_{t-1} & \text{if } t > 0 \\ (1 + r_{ss} + \mathrm{cap}_0)a_{t-1} & \text{if } t = 0 \end{cases}$$

1. Real bond: $\mathrm{cap}_0 = 0$
2. Nominal bond:

$$\mathrm{cap}_0 = \frac{(1 + r_{ss})(1 + \pi_{ss})}{1 + \pi_0} - (1 + r_{ss})$$

3. Long-term bond:

$$\mathrm{cap}_0 = \frac{1 + \delta q_0}{q_{ss}} - (1 + r_{ss})$$

Generalized IKC

- **Consumption-function** $\boldsymbol{C}^{hh} = C^{hh}(\boldsymbol{r}, \boldsymbol{Y} - \boldsymbol{T}, \chi, \mathbf{cap}_0)$ implies

$$d\boldsymbol{C}^{hh} = \boldsymbol{M}^r d\boldsymbol{r} + \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) + \boldsymbol{M}^\chi d\boldsymbol{\chi} + \boldsymbol{m}^{\text{cap}}\text{cap}_0$$

  where

$$\boldsymbol{M}^r_{t,s} = \left[\frac{\partial C^{hh}_t}{\partial r_s}\right], \boldsymbol{M}^\chi_{t,s} = \left[\frac{\partial C^{hh}_t}{\partial \chi_s}\right], \boldsymbol{m}^{\text{cap}}_t = \left[\frac{\partial C^{hh}_t}{\partial \text{cap}_0}\right]$$

- Why are $\boldsymbol{M}^\chi$ and $\boldsymbol{M}$ different?

HANK-SAM

Household problem

$$v_t(\beta_i, u_{it}, a_{it-1}) = \max_{c_{it}, a_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta_i \mathbb{E}_t \left[v_{t+1}\left(\beta_i, u_{it+1}, a_{it}\right)\right]$$

$$\text{s.t.}\quad a_{it} + c_{it} = (1+r_t)a_{it-1} + (1-\tau_t)y_t(u_{it}) + \text{div}_t + \text{transfer}_t$$

$$a_{it} \geq 0$$

1. **Dividends and government transfers:** div_t and transfer_t
2. **Real wage**: w_t
3. **Income tax:** τ_t
4. **Separation rate** for employed: δ_t
5. **Job-finding rate** for unemployed: $\lambda_t^{u,s} s(u_{it-1})$
   (where $s(u_{it-1})$ is exogenous search effectiveness)
6. US-style duration-dependent **UI system:**
   a) High replacement rate $\overline{\phi}$, first $\overline{u}$ months
   b) Low replacement rate $\underline{\phi}$, after $\overline{u}$ months

Income process

- **Income** is

$$y_{it}(u_{it}) = w_{ss} \cdot \begin{cases} 1 & \text{if } u_{it} = 0 \\ \overline{\phi}\mathrm{UI}_{it} + (1 - \mathrm{UI}_{it})\underline{\phi} & \text{else} \end{cases}$$

where share of the month with UI is

$$\mathrm{UI}_{it} = \begin{cases} 0 & \text{if } u_{it} = 0 \\ 1 & \text{else if } u_{it} < \overline{u} \\ 0 & \text{else if } u_{it} > \overline{u} + 1 \\ \overline{u} - (u_{it} - 1) & \text{else} \end{cases}$$

- **Note:** *Hereby $\overline{u}$ becomes a continuous variables*

Transition probabilities

- **Beginning-of-period value function:**

$$\underline{v}_t\left(\beta_i, u_{it-1}, a_{it-1}\right) = \mathbb{E}\left[v_t(\beta_i, u_{it}, a_{it-1}) \,|\, u_{it-1}, a_{it-1}\right]$$

- **Grids:** $u_{it} \in \{0, 1, \ldots, \#_u - 1\}$ for $\#_u - 1$
- **Workers** with $u_{it-1} = 0$:

$$u_{it} = \begin{cases} 0 & \text{with } 1 - \delta_t \\ 1 & \text{with } \delta_t \end{cases}$$

- **Unemployed** with $u_{it-1} = 1$:

$$u_{it} = \begin{cases} 0 & \text{with } \lambda_t^{u,s} \mathrm{s}(u_{it-1}) \\ \min\left\{u_{it-1} + 1, \#_u - 1\right\} & \text{with } 1 - \lambda_t^{u,s} s(u_{it-1}) \end{cases}$$

Hiring and firing

- **Job value:**

$$V_t^j = p_t^x Z_t - w_{ss} + \beta^{\text{firm}} \mathbb{E}_t \left[(1 - \delta_{ss}) V_{t+1}^j \right]$$

- **Vacancy value:**

$$V_t^v = -\kappa + \lambda_t^v V_t^j + (1 - \lambda_t^v)(1 - \delta_{ss}) \beta^{\text{firm}} \mathbb{E}_t \left[V_{t+1}^v \right]$$

- **Free entry** implies

$$V_t^v = 0$$

Labor market dynamics

- **Labor market tightness** is given by

$$\theta_t = \frac{v_t}{S_t}$$

- **Cobb-Douglas matching function** implies:

$$\lambda_t^v = A\theta_t^{-\alpha}$$
$$\lambda_t^{u,s} = A\theta_t^{1-\alpha}$$

- **Law of motion for unemployment:**

$$u_t = u_{t-1} + \delta_t(1 - u_{t-1}) - \lambda_t^{u,s} S_t$$

Standard New Keynesian block

- **Intermediate goods price:** p_t^X
- Dixit-Stiglitz **demand curve** $\Rightarrow$ **Phillips curve** relating marginal cost, $MC_t = p_t^x$, and **final goods price inflation**, $\Pi_t = P_t/P_{t-1}$,

$$1 - \epsilon + \epsilon p_t^x = \phi\pi_t(1+\pi_t) - \phi\beta^{\text{firm}}\mathbb{E}_t\left[\pi_{t+1}(1+\pi_{t+1})\frac{Y_{t+1}}{Y_t}\right]$$

  with output $Y_t = Z_t(1-u_t)$
- **Flexible price limit:** $\phi \to 0$
- **Taylor rule:**

$$1 + i_t \quad = \quad (1+i_{ss})\left(\frac{1+\pi_t}{1+\pi_{ss}}\right)^{\delta_\pi}$$

Government

- **Unemployment insurance:** $\Phi_t = w_{ss}\left(\overline{\phi}\mathrm{UI}_t^{hh} + \underline{\phi}\left(u_t - \mathrm{UI}_t^{hh}\right)\right)$
- **Total expenses:** $X_t = \Phi_t + G_t + \mathrm{transfer}_t$
- **Total taxes:** $\mathrm{taxes}_t = \tau_t\left(\Phi_t + w_{ss}(1-u_t)\right)$
- **Government budget** is

$$q_t B_t = (1 + q_t\delta_q)B_{t-1} + X_t - \mathrm{taxes}_t$$

- **Tax rule:**

$$\tilde{\tau}_t = \frac{\left(1 + q_t\delta_q\right)B_{t-1} + X_t - q_{ss}B_{ss}}{\Phi_t + w_{ss}(1-u_t)}$$

$$\tau_t = \omega\tilde{\tau}_t + (1-\omega)\tau_{ss}$$

Equilibrium

1. **Financial markets:**

$$\frac{1+\delta_q q_{t+1}}{q_t} = \frac{1+i_t}{1+\pi_{t+1}}$$

$$1+r_t = \begin{cases} \frac{(1+\delta_q q_0)B_{-1}}{A_{-1}^{hh}} & \text{if } t=0 \\ \frac{1+i_{t-1}}{1+\pi_t} & \text{else} \end{cases}$$

2. **Market clearing:**

$$A_t^{hh} = q_t B_t$$

$$Y_t = C_t^{hh} + G_t$$

Summary

Summary

- **Today:** HANK models
  1. Some aggregate neutrality results - still distributional concerns
  2. Size of mechanisms are different - cash-flow effects important
  3. High MPC and precautionary saving become of central importance
- **Next:** *Exam*