Select, Schedule, and Route Foster Care Visitation

A Time-Space Network Approach

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Outline

1. Foster Care System in the US
2. Time-Space Network Approach
3. Problem Formulation
4. Computational Results
5. Takeaways

Outline

Foster Care System in the United States

- A **temporary** service for children who are unable to live with their biological parent(s)
  - Provide a stable family environment for children
  - Biological parents work with a social worker to stabilize their situation
- Over 400,000 children live in foster care settings[1]

https://www.bestmswprograms.com/top-10-ted-talks-for-social-workers/

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Foster Care Visitation

Foster children regularly meet with their biological parent(s)

- Maintain attachment
- Reduce sense of abandonment in children
- Increase motivation in parent(s)
- Increase the likelihood of timely family reunification

https://blog.adoptuskids.org/6-tips-for-foster-parent(s)-preparing-for-reunification/

Foster Care Visitation Scheduling Problem (FCVSP)

- A foster child might have siblings, either living in the same or different foster homes
- Children and parent(s) must attend court-ordered visits
- **Foster case.** An incident in the foster care system with one or more children and one or more biological parent(s)
- Foster care social workers are either:
  - Drivers
  - Supervisors (can also give a ride)
- Foster care visitations are either:
  - Supervised
  - Unsupervised

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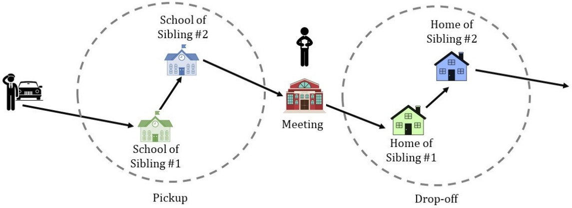
- There is a set of pre-determined locations:
  - Start garages for the case workers
  - Pickup and dropoff locations for the children
  - Meeting locations
- Children from different cases cannot be in the same vehicle
- During the meeting, the driver may pickup or dropoff another case
- All workers must take a lunch break of prespecified duration per day



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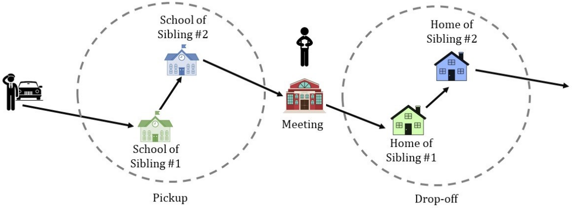

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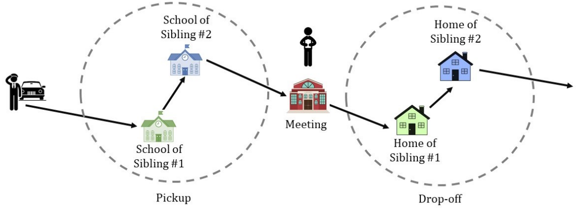

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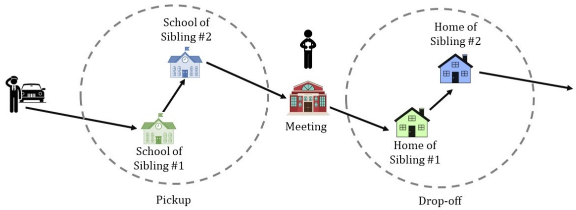

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- Each case is associated with two case events:
  - **Pickup event.** Pick up all children of a case from their origin locations, and immediately drop them off at meeting location
  - **Dropoff event.** Pick up all children of a case from meeting location, and immediately dropping them off at their final destination locations
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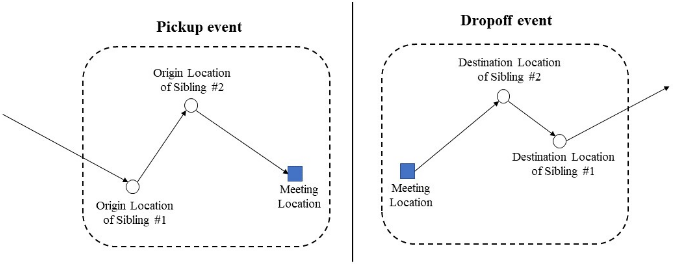

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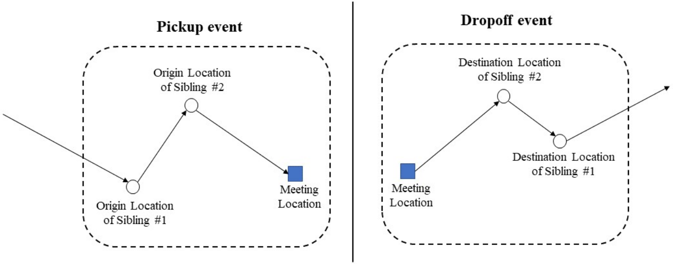

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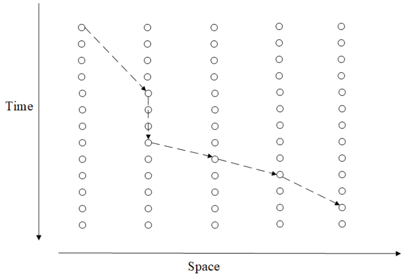
FCVSP Time-Space Network

- Integrate unique characteristics of the FCVSP into time-space network
  - VRP with Reward Collection, Team Orienteering, Site Dependent VRP, Skill VRP
- Construct an optimization formulation over the proposed network



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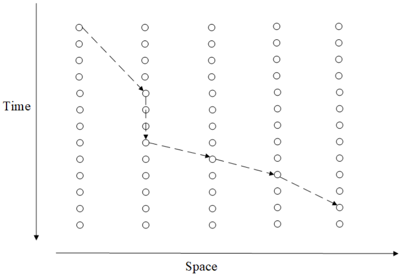

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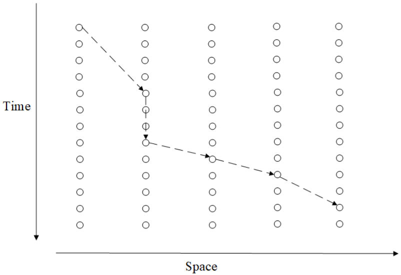

Time Lines in FCVSP Time-space Network

- **Garage.** Origin location for each worker
- **Arrival.** All case siblings arrive at the meeting location
- **Supervision.** An ongoing supervision
- **Departure.** Meeting is concluded and case siblings depart for drop off
- **Case complete.** Possible final destination of a case dropoff event
- **Lunch.** All worker lunch break locations
- **Sink.** End of all worker paths

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Connections between Nodes

- Given a tail node, a destination, and an activity duration, add arcs based on the following rules:
  - **Rule 1.** Head node should have a feasible time for that location
  - **Rule 2.** Time difference should be greater than or equal to the activity duration
    - Use arc contraction for idle time before or after an activity
  - **Rule 3.** If they belong to the same case, associated time windows should be the same
- Add ground arcs for supervision time lines, representing supervisor attendance

From \ To	Driver Garage	Supervisor Garage	Arrival	Supervision	Departure	Case Complete	Sink
Driver Garage	–	–	✓	–	–	–	✓
Supervisor Garage	–	–	✓	✓	–	–	✓
Arrival	–	–	✓*	✓	✓	–	✓
Supervision	–	–	✓*	✓*	✓	–	✓
Departure	–	–	–	–	–	✓†	–
Case Complete	–	–	✓*	✓*	✓*	–	✓
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Incoming Arcs to Arrival Time Lines

- Network Contraction:
  - Find all possible combinations of transporting a case in pickup event
  - Find all potential origins of paths to the case pickup event
  - Calculate length of all possible paths to the case arrival time line
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Arcs from Departure to Case Complete Time Lines

- Find shortest path between meeting location and case sibling final destination(s)
  - Find all possible combinations of transporting a case in dropoff event
  - Calculate length of all possible paths from case meeting location to final destination(s)
  - Select shortest path(s) as incoming arc(s) to case complete time line(s)
- Ensures all case siblings are transferred to their final destinations after the meeting
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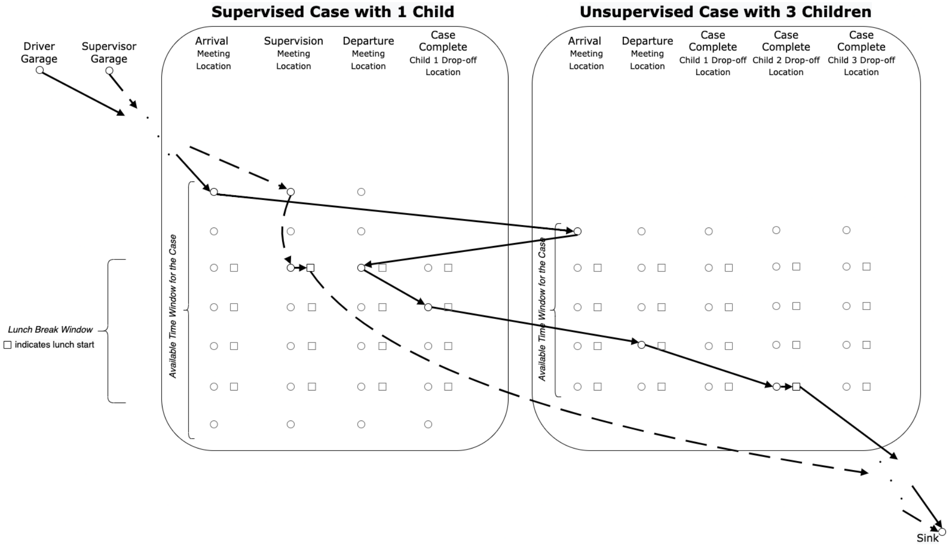
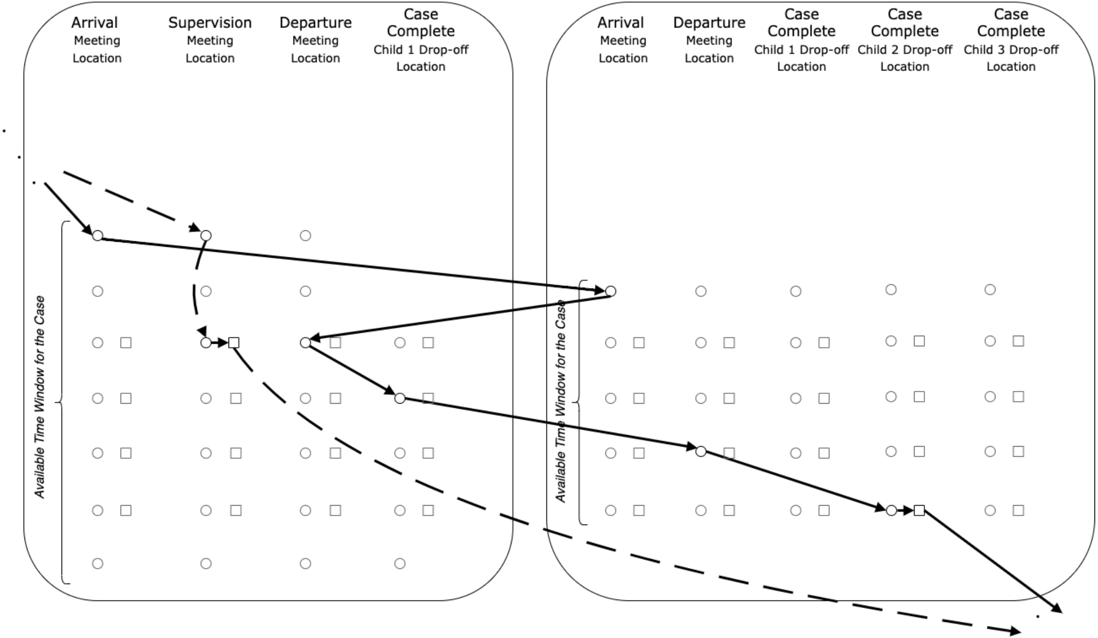
Illustrative Time-Space Network Structure for FCVSP




Arrival
Meeting
Location
Supervision
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Location
Departure
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Location
Case
Complete
Child 1 Drop-off
Location
Available Time Window for the Case
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Outline

Model Formulation: Sets, Parameters, and Variables

Notation	Description
$\mathcal{W}$	Set of case workers. It holds that $\mathcal{W} = \{\mathcal{W}^r \cup \mathcal{W}^s\}$
$\mathcal{W}^r$	Set of drivers
$\mathcal{W}^s$	Set of supervisors
$\mathcal{C}$	Set of cases. It holds that $\mathcal{C} = \{\mathcal{C}^u \cup \mathcal{C}^s\}$
$\mathcal{C}^u$	Set of unsupervised cases
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$\mathcal{N}$	Set of all nodes in the time-space network
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d_a	Travel time from node $i \in \mathcal{N}$ to node $j \in \mathcal{N}$, $i \neq j$
δ	Time interval used for discretization in the time-space network
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$\mathcal{W}$	Set of case workers. It holds that $\mathcal{W} = \{\mathcal{W}^r \cup \mathcal{W}^s\}$
$\mathcal{W}^r$	Set of drivers
$\mathcal{W}^s$	Set of supervisors
$\mathcal{C}$	Set of cases. It holds that $\mathcal{C} = \{\mathcal{C}^u \cup \mathcal{C}^s\}$
$\mathcal{C}^u$	Set of unsupervised cases
$\mathcal{C}^s$	Set of supervised cases
$\mathcal{N}$	Set of all nodes in the time-space network
$\mathcal{N}_g^w$	Set of nodes in garage time line belonging to worker $w \in \mathcal{W}$
$\mathcal{N}_a^c$	Set of nodes in arrival time line belonging to case $c \in \mathcal{C}$
$\mathcal{N}_s^c$	Set of nodes in supervision time line belonging to case $c \in \mathcal{C}^s$
$\mathcal{N}_d^c$	Set of nodes in departure time line belonging to case $c \in \mathcal{C}$
$\mathcal{N}_f^c$	Set of nodes in case complete time lines belonging to case $c \in \mathcal{C}$
$\mathcal{N}_l^w$	Set of nodes in lunch time line belonging to worker $w \in \mathcal{W}$
$\mathcal{N}_{sink}$	The sink node in the time-space network
$\mathcal{A}$	Set of arcs in the time-space network. Each arc emanates from $t(a) \in \mathcal{N}$ and enters $h(a) \in \mathcal{N}$
$\mathcal{A}^w$	Set of arcs belonging to worker $w \in \mathcal{W}$
$\mathcal{A}_v^c$	Set of ground arcs in supervision time lines belonging to case $c \in \mathcal{C}^s$
$\mathcal{O}^c$	Set of compatible ordered pairs of nodes in arrival and departure time lines belonging to case $c \in \mathcal{C}$
$\bar{\mathcal{O}}^c$	Set of compatible ordered triples of nodes in arrival and supervision time lines belonging to supervised case $c \in \mathcal{C}^s$
d_a	Travel time from node $i \in \mathcal{N}$ to node $j \in \mathcal{N}$, $i \neq j$
δ	Time interval used for discretization in the time-space network
α	A positive constant to penalize longer paths
x_a^w	$= \begin{cases} 1 & \text{if arc } e \in \mathcal{A}^w \text{ is traversed by worker } w \in \mathcal{W}, \\ 0 & \text{otherwise} \end{cases}$
z_c	$= \begin{cases} 1 & \text{if case } c \in \mathcal{C} \text{ is not visited}, \\ 0 & \text{otherwise} \end{cases}$

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Mathematical Modeling over FCVSP Time-Space Network

Minimize	Total number of unvisited cases, while favoring shorter paths	(1a)
$s.t.$	Each worker is dispatched once	(1b)
	All worker paths terminate at sink	(1c)
	Flow conservation is preserved at each node for each worker	(1d)
	Each case is visited at most once during the planning period	(1e)
	Dropoff event starts without delay if and only if pickup event ends	(1f)
	If a supervised meeting takes place, a supervisor must be present in meeting location	(1g)
	Departure and case complete time lines are visited if and only if arrival time line is visited	(1h)
	Supervision time line is visited if and only if arrival time line is visited	(1i)
	If a supervisor enters supervision time line, remains there for entire meeting duration	(1j)
	Each worker has a lunch break	(1k)
	Variable domains	(1l)

Mathematical Modeling over FCVSP Time-Space Network

Minimize	$\sum_{c \in \mathcal{C}} z_c + \alpha \sum_{w \in \mathcal{W}} \sum_{a \in \mathcal{A}^w} d_a x_a^w$	(1a)
$s.t.$	Each worker is dispatched once	(1b)
	All worker paths terminate at sink	(1c)
	Flow conservation is preserved at each node for each worker	(1d)
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Mathematical Modeling over FCVSP Time-Space Network

$$\text{Minimize} \quad \sum_{c \in \mathcal{C}} z_c + \alpha \sum_{w \in \mathcal{W}} \sum_{a \in \mathcal{A}^w} d_a x_a^w \tag{1a}$$

$$s.t. \quad \sum_{\substack{a \in \mathcal{A}^w \\ t(a) \in \mathcal{N}_g^w}} x_a^w = 1 \quad \forall\, w \in \mathcal{W} \tag{1b}$$

All worker paths terminate at sink (1c)

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$$\text{Minimize} \quad \sum_{c\in\mathcal{C}} z_c + \alpha \sum_{w\in\mathcal{W}} \sum_{a\in\mathcal{A}^w} d_a x_a^w \tag{1a}$$

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$$\sum_{\substack{a\in\mathcal{A}^w \\ t(a)=i}} x_a^w = \sum_{\substack{a\in\mathcal{A}^w \\ h(a)=i}} x_a^w \quad \forall\, i \in \mathcal{N} \setminus (\mathcal{N}_g^w \cup \mathcal{N}_{sink}), \forall\, w \in \mathcal{W} \tag{1d}$$

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$$\sum_{w \in \mathcal{W}} \sum_{\substack{a \in \mathcal{A}^w \\ t(a) \in \mathcal{N}_a^c}} x_a^w + z_c = 1 \quad \forall\, c \in \mathcal{C} \tag{1e}$$

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Variable domains (1l)

Mathematical Modeling over FCVSP Time-Space Network

$$\text{Minimize} \quad \sum_{c\in\mathcal{C}} z_c + \alpha \sum_{w\in\mathcal{W}} \sum_{a\in\mathcal{A}^w} d_a x_a^w \tag{1a}$$

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$$\sum_{w\in\mathcal{W}} \sum_{\substack{a\in\mathcal{A}^w \\ t(a)\in\mathcal{N}_a^c}} x_a^w + z_c = 1 \quad \forall\, c\in\mathcal{C} \tag{1e}$$

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Departure and case complete time lines are visited if and only if arrival time line is visited (1h)

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Mathematical Modeling over FCVSP Time-Space Network

$$\text{Minimize} \quad \sum_{c \in \mathcal{C}} z_c + \alpha \sum_{w \in \mathcal{W}} \sum_{a \in \mathcal{A}^w} d_a x_a^w \tag{1a}$$

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$$\sum_{w \in \mathcal{W}} \sum_{\substack{a \in \mathcal{A}^w \\ t(a) \in \mathcal{N}_a^c}} x_a^w + z_c = 1 \quad \forall\, c \in \mathcal{C} \tag{1e}$$

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$$\sum_{w \in \mathcal{W}} \sum_{\substack{a \in \mathcal{A}^w, \\ t(a) \in \mathcal{N}_a^c}} x_a^w = \sum_{w \in \mathcal{W}} \sum_{\substack{a \in \mathcal{A}^w, \\ t(a) \in \mathcal{N}_d^c}} x_a^w \leq 1 \quad \forall\, c \in \mathcal{C} \tag{1h}$$

Supervision time line is visited if and only if arrival time line is visited (1i)

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Mathematical Modeling over FCVSP Time-Space Network

$$
\begin{aligned}
\text{Minimize}\quad & \sum_{c\in\mathcal{C}} z_c + \alpha \sum_{w\in\mathcal{W}} \sum_{a\in\mathcal{A}^w} d_a x_a^w && \text{(1a)}\\
s.t.\quad & \sum_{\substack{a\in\mathcal{A}^w\\ t(a)\in\mathcal{N}_g^w}} x_a^w = 1 \quad \forall\, w\in\mathcal{W} && \text{(1b)}\\
& \sum_{w\in\mathcal{W}} \sum_{\substack{a\in\mathcal{A}^w\\ t(a)=\mathcal{N}_{sink}}} x_a^w = |\mathcal{W}| && \text{(1c)}\\
& \sum_{\substack{a\in\mathcal{A}^w\\ t(a)=i}} x_a^w = \sum_{\substack{a\in\mathcal{A}^w\\ h(a)=i}} x_a^w \quad \forall\, i\in\mathcal{N}\setminus(\mathcal{N}_g^w\cup\mathcal{N}_{sink}), \forall\, w\in\mathcal{W} && \text{(1d)}\\
& \sum_{w\in\mathcal{W}} \sum_{\substack{a\in\mathcal{A}^w\\ t(a)\in\mathcal{N}_a^c}} x_a^w + z_c = 1 \quad \forall\, c\in\mathcal{C} && \text{(1e)}\\
& \sum_{w\in\mathcal{W}} \sum_{\substack{a\in\mathcal{A}^w\\ t(a)=i}} x_a^w = \sum_{w\in\mathcal{W}} \sum_{\substack{a\in\mathcal{A}^w\\ t(a)=j}} x_a^w \le 1 \quad \forall\, c\in\mathcal{C}, \forall\,(i,j)\in\mathcal{O}^c && \text{(1f)}\\
& \sum_{w\in\mathcal{W}} \sum_{\substack{a\in\mathcal{A}^w\\ t(a)=i}} x_a^w = \sum_{w\in\mathcal{W}^s} \sum_{\substack{a\in\mathcal{A}^w\\ t(a)=j, h(a)=\hat{j}}} x_a^w \le 1 \quad \forall\, c\in\mathcal{C}^s, \forall\,(i,j,\hat{j})\in\bar{\mathcal{O}}^c && \text{(1g)}\\
& \sum_{w\in\mathcal{W}} \sum_{\substack{a\in\mathcal{A}^w,\\ t(a)\in\mathcal{N}_a^c}} x_a^w = \sum_{w\in\mathcal{W}} \sum_{\substack{a\in\mathcal{A}^w,\\ t(a)\in\mathcal{N}_d^c}} x_a^w \le 1 \quad \forall\, c\in\mathcal{C} && \text{(1h)}\\
& \sum_{w\in\mathcal{W}} \sum_{\substack{a\in\mathcal{A}^w,\\ t(a)\in\mathcal{N}_a^c}} x_a^w = \sum_{w\in\mathcal{W}} \sum_{\substack{a\in\mathcal{A}^w, a\notin\mathcal{A}_v^c\\ t(a)\in\mathcal{N}_s^c}} x_a^w \le 1 \quad \forall\, c\in\mathcal{C}^s && \text{(1i)}
\end{aligned}
$$

If a supervisor enters supervision time line, remains there for entire meeting duration (1j)

Each worker has a lunch break (1k)

Variable domains (1l)

Mathematical Modeling over FCVSP Time-Space Network

$$\text{Minimize} \quad \sum_{c \in \mathcal{C}} z_c + \alpha \sum_{w \in \mathcal{W}} \sum_{a \in \mathcal{A}^w} d_a x_a^w \tag{1a}$$

$$s.t. \quad \sum_{\substack{a \in \mathcal{A}^w \\ t(a) \in \mathcal{N}_g^w}} x_a^w = 1 \quad \forall\, w \in \mathcal{W} \tag{1b}$$

$$\sum_{w \in \mathcal{W}} \sum_{\substack{a \in \mathcal{A}^w \\ t(a) = \mathcal{N}_{sink}}} x_a^w = |\mathcal{W}| \tag{1c}$$

$$\sum_{\substack{a \in \mathcal{A}^w \\ t(a) = i}} x_a^w = \sum_{\substack{a \in \mathcal{A}^w \\ h(a) = i}} x_a^w \quad \forall\, i \in \mathcal{N} \setminus (\mathcal{N}_g^w \cup \mathcal{N}_{sink}), \forall\, w \in \mathcal{W} \tag{1d}$$

$$\sum_{w \in \mathcal{W}} \sum_{\substack{a \in \mathcal{A}^w \\ t(a) \in \mathcal{N}_a^c}} x_a^w + z_c = 1 \quad \forall\, c \in \mathcal{C} \tag{1e}$$

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$$\sum_{w \in \mathcal{W}} \sum_{\substack{a \in \mathcal{A}^w \\ t(a) = i}} x_a^w = \sum_{w \in \mathcal{W}^s} \sum_{\substack{a \in \mathcal{A}^w \\ t(a) = j, h(a) = \hat{j}}} x_a^w \leq 1 \quad \forall\, c \in \mathcal{C}^s, \forall\, (i,j,\hat{j}) \in \bar{\mathcal{O}}^c \tag{1g}$$

$$\sum_{w \in \mathcal{W}} \sum_{\substack{a \in \mathcal{A}^w, \\ t(a) \in \mathcal{N}_a^c}} x_a^w = \sum_{w \in \mathcal{W}} \sum_{\substack{a \in \mathcal{A}^w, \\ t(a) \in \mathcal{N}_d^c}} x_a^w \leq 1 \quad \forall\, c \in \mathcal{C} \tag{1h}$$

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Each worker has a lunch break (1k)

Variable domains (1l)

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$$\sum_{\substack{a\in\mathcal{A}^w\\ t(a)=i}} x_a^w = \sum_{\substack{a\in\mathcal{A}^w\\ h(a)=i}} x_a^w \quad \forall\, i\in\mathcal{N}\setminus(\mathcal{N}_g^w\cup\mathcal{N}_{sink}), \forall\, w\in\mathcal{W} \tag{1d}$$

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$$\sum_{\substack{a\in\mathcal{A}^w,\\ t(a)\in\mathcal{N}_g^w, h(a)\neq\mathcal{N}_{sink}}} x_a^w = \sum_{\substack{a\in\mathcal{A}^w,\\ t(a)\in\mathcal{N}_l^w}} x_a^w \quad \forall\, w\in\mathcal{W} \tag{1k}$$

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$$x_a^w \in \{0,1\} \quad \forall\, w \in \mathcal{W}, \forall\, a \in \mathcal{A}^w, \quad z_c \in \{0,1\} \quad \forall\, c \in \mathcal{C}. \tag{1l}$$

Outline

Dataset: Orange County, NY

- Meeting duration: 60, 90, 120, or 150 minutes
- Time interval: 15 and 30 minutes
- Number of cases: up to 80
- Number of workers: 5, 10, 15, 20
- Lunch break: 1 hour, sometime between 11:00 AM and 1:30 PM
- Persistence: 95% of total cases must be serviced by assigned workers

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Computational Insights

- With 30 minute time interval, all instances are solved to provable optimality in less than 12 minutes
- With 15 minute time interval, around 95% are solved to provable optimality in less than four hours
- A tradeoff exists between time interval duration and performance of model in terms of visiting more cases
- Selecting unsupervised cases is favored over supervised cases, as they require relatively less resources

# Cases	# Supervised cases	# Unsupervised cases	# Supervisors	# Drivers	Time Interval	# Scheduled Cases	# Scheduled Supervised Cases	# Scheduled Unsupervised Cases	Runtime (Seconds)
40	36	4	7	3	15 30	18 17	15 14	3 3	1311 80
60	55	5	10	5	15 30	32 31	28 27	4 4	7670 223
80	73	7	18	2	15 30	41 39	37 35	4 4	4170 302

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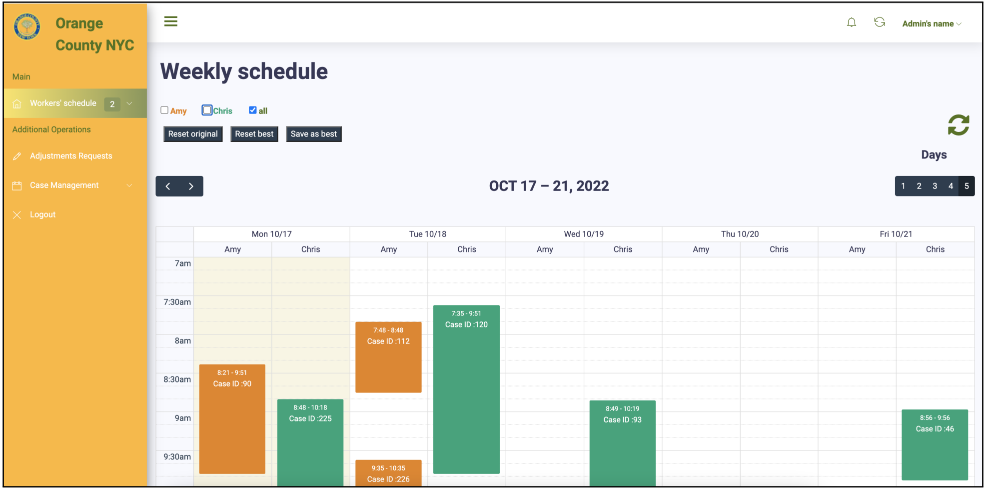
Scheduling Tool



Developed by Rizk Makroum

Outline

Takeaways

- Investigate operational challenges in foster care visitation scheduling problem
- Establish a novel optimization formulation over proposed time-space network:
  - Select foster cases for visits in a planning period
  - Schedule meetings with biological parent(s) within case time windows
  - Route social workers to transfer cases and monitor supervised case meetings
- Time-space network model validated with realistic data and practical performance

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Appreciation

Thank You

Questions & Comments