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Generalized Assignment Problem

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Introduction

The Generalized Assignment Problem (GAP) was first studied by Srinivasan and Thompson [94] to address a transportation problem. However, the term *generalized assignment problem* was officially introduced in [86]. GAP extends the model proposed in [21], which features agent-independent resource consumption. GAP aims to minimize the total cost of assignments while adhering to resource constraints:

$$\min \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{1a}$$

$$\text{subject to} \quad \sum_{j=1}^{n} a_{ij}x_{ij} \le b_i \quad i = 1, \ldots, m \tag{1b}$$

$$\sum_{i=1}^{m} x_{ij} = 1 \quad j = 1, \ldots, n \tag{1c}$$

$$x_{ij} \in \{0, 1\} \quad i = 1, \ldots, m; \quad j = 1, \ldots, n \tag{1d}$$

Here, c_{ij} denotes the cost of assigning task j to agent i. The parameter a_{ij} represents the resource used when task j is assigned to agent i, and b_i indicates the total available capacity of agent i. The binary variable x_{ij} indicates whether task j is assigned to agent i (1 if assigned, 0 otherwise).

The classical assignment problem, which establishes a one-to-one relationship between agents and tasks, can be solved in polynomial time [53]. However, GAP introduces an

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additional complexity by allowing an agent to handle multiple tasks, provided that each task is performed exactly once. This change makes the problem $\mathcal{NP}$-hard [31].

Extensions

GAP is extended in various ways to address more complex and realistic scenarios encountered in different fields. Each extension adds layers of complexity to the traditional GAP, allowing for more nuanced and effective solutions to problems in logistics, scheduling, and resource allocation. The following subsections provide an overview of the most notable extensions of GAP and their applications.

Earlier Generalizations

Generalized Assignment Problem is a special case of the *Weighted Assignment Problem* (WAP), where performance levels are incorporated, requiring each task to be executed by an agent at a specific performance level (see [87]). Another generalization that is more widely studied is the Multilevel Generalized Assignment Problem (MGAP). MGAP is first introduced in [33] addressing the *machine loading problem*. MGAP incorporates the concept of hierarchical levels where each level has its own set of constraints and objectives. Note that MGAP can be considered as a special case of WAP, where resource usages are not lower bounded [75].

$$\min \quad \sum_{l=1}^{L}\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ijl}x_{ijl} \tag{2a}$$

$$\text{Subject to} \quad \sum_{l=1}^{R}\sum_{j=1}^{n} a_{ijl}x_{ijl} \leq b_{il} \quad i = 1,\ldots,m \tag{2b}$$

$$\sum_{l=1}^{L}\sum_{i=1}^{m} x_{ijl} = 1 \quad j = 1,\ldots,n \tag{2c}$$

$$x_{ijl} \in \{0,1\} \quad l = 1,\ldots R, \quad i = 1,\ldots,m; \quad j = 1,\ldots,n \tag{2d}$$

Laguna et al. [52] employ an ejection chain-based tabu search algorithm, which introduces ejection moves that generate more powerful solutions without significantly increasing computational effort. Later, MGAP is addressed by French and Wilson [28], who propose two heuristic approaches derived from solution methods for GAP.

Exact approaches are extensively studied for the MGAP. Osorio and Laguna [77] exploit the fact that MGAP has knapsack constraints and introduce logic cuts that can be generated in linear time. In a different approach, Ceselli and Righini [15] present a branch-and-price algorithm. They propose an alternative formulation of MGAP, decomposing the problem into a master problem with set-partitioning constraints and a multiple-choice knapsack problem as the pricing problem. Their branch-and-price algorithm demonstrates significant performance improvements over the logic cuts introduced by Osorio and Laguna [77]. Later, Avella et al. [6] propose a branch-and-cut algorithm, similarly exploiting the intricate knapsack constraints in MGAP. They define an exact separation procedure for multiple-choice knapsack polytopes and introduce an iterative row generation procedure. This approach shows significant performance improvements over the branch-and-price algorithm by Ceselli and Righini [15].

Bottleneck Generalized Assignment Problem (BGAP)

The Bottleneck Generalized Assignment Problem (BGAP) extends GAP by incorporating a *minimax* objective. Two primary versions of BGAP are explored in the literature: Agent BGAP (ABGAP) and Task BGAP (TBGAP) (see [62, 63]). TBGAP focuses on minimizing the maximum assignment cost:

$$\min\{\max_{\substack{i=1,\ldots,m\\ j=1,\ldots,n}} \{c_{ij}x_{ij}\}\} \tag{3}$$

while ABGAP aims to minimize the maximum total cost assigned to each agent:

$$\min\{\max_{i=1,\ldots,m}\{\sum_{j=1}^{n} c_{ij}x_{ij}\}\} \tag{4}$$

Various relaxations for TBGAP are proposed by Martello and Toth [60]. Additionally, stochastic variants of BGAP are covered in the literature. For example, Fu et al. [30] study the problem under uncertain capacity, proposing robust optimization formulations for both TBGAP and ABGAP.

Dynamic Generalized Assignment Problem (DGAP)

The dynamic generalized assignment problem (DGAP) expands on the traditional GAP by considering the sequence in which agents carry out tasks. Introduced by Kogan and Shtub [49], this dynamic model includes due dates and task completion times and accounts for costs related to backlogging and inventory. This model provides a more comprehensive method for synchronizing task assignments with customer demand.

Kogan et al. [50] further develop the DGAP for a stochastic scenario where agents can take on multiple tasks, and tasks can be distributed among multiple agents. They establish its strong NP-hardness and simplify the problem to deterministic problems at discrete time points.

Moccia et al. [67] explore the application of DGAP in the operational management of container transshipment terminals, proposing column generation-based methods.

Finally, Kiraz and Topcuoglu [47] explore hyper-heuristics in the DGAP setting, proposing a hybrid hyper-heuristic combined with memory search algorithm.

Generalized Assignment Problem with Type II Special Ordered Sets

The Generalized Assignment Problem with Type II Special Ordered Sets (GAPS2) is an extension of GAP, where workload sharing with adjacent agents is permissible. Special ordered sets are introduced in [8]. A set of variables $\{x_1, \ldots, x_n\}$ is considered a special set of type 2 if at most two variables can be nonzero, and these nonzero variables are not consecutive.

The problem is first introduced by de Farias et al. [19]. They study a production scheduling generalized assignment problem with special ordered sets of type II. They derive three families of facet-defining valid inequalities that cut off all infeasible LP vertices. These inequalities are then used as cuts in a branch-and-cut scheme. Wilson [102] extends the heuristic algorithm for GAP in [100], achieving near-optimal solutions, with a 0.6% optimality gap for problems with known solutions and 1.5% gap for large-scale problems. French and Wilson [29] adapt the LP-based heuristic from [98], achieving superior results compared to [102], with an average gap of 0.1% from the lower bound.

Elastic Generalized Assignment Problem (EGAP)

The Elastic Generalized Assignment Problem (EGAP) extends the traditional GAP by allowing the relaxation of agent resource capacities. This relaxation permits capacity violations, which incur additional penalty costs in the objective function. EGAP is first studied by Brown and Graves [11] in the context of distributing refined petroleum products.

The model is initially introduced by Ronen [85]. The author introduces nonnegative undertime and overtime variables, each with hard lower and upper bounds, to indicate idle and overused resources, respectively. The author updates the objective function (1a) as follows:

$$\min \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{i=1}^{m}(d_i u_i + e_i v_i) \tag{5}$$

where u_i and v_i represent undertime and overtime variables, and d_i and e_i are the costs incurred for undertime and overtime, respectively. In the EGAP formulation, authors modify the capacity constraint (1b):

$$\sum_{j=1}^{n} a_{ij}x_{ij} + u_i - v_i = b_i \quad i = 1, \ldots, m \tag{6}$$

Ronen [85] proposes two heuristic algorithms that achieve solutions within a 1% gap of the linear relaxation. Additionally, special-purpose branch-and-bound algorithms are discussed in the literature (see [72,73]). Furthermore, Büther [12] introduces techniques to reduce the EGAP to the GAP, thereby allowing the utilization of efficient GAP solution approaches.

Stochastic Generalized Assignment Problem

The Stochastic Generalized Assignment Problem (SGAP) extends the classic GAP by incorporating uncertainties in costs, capacities, and demands, thus offering a more accurate model for real-world applications in logistics, project management, and resource allocation.

Dyer and Frieze [24] perform a probabilistic analysis, assuming all coefficients are independently drawn from a uniform distribution ($UNIF(0,1)$). They develop an algorithm that solves the GAP with high probability in polynomial time. Exploring further, Toktas et al. [97] address the GAP under uncertain capacities, investigating two methodologies to manage these uncertainties.

The uncertainties in stochastic models can affect various aspects of the problem, such as an uncertain number of jobs, demand, and resource consumption. Albareda-Sambola et al. [2] focus on scenarios where the set of jobs is uncertain. They model customer demand as random variables following a Bernoulli distribution and approach the problem using stochastic programming with recourse, employing a convex approximation of the recourse function. They introduce three versions of an exact algorithm. In a similar vein, Sarin et al. [88] propose a recourse formulation for the GAP with uncertain demand, developing a branch-and-price approach and exploring its extensions. Additionally, Yang and Chakraborty [110] examine the problem with uncertain resource consumption, reformulating the GAP with chance constraints and proposing a $(1+\alpha)$-approximation algorithm to address these uncertainties.

For specific applications, Şeker and Noyan [90] develop stochastic models for airport gate assignments that incorporate robustness measures to handle uncertainties in conflicts, idle times, and buffer times effectively.

Bi-criteria Generalized Assignment Problem (BiGAP)

Zhang and Ong [111] examine GAP from a multi-objective perspective. They introduce a linear programming (LP)-based heuristic to efficiently generate a good approximation of the nondominated frontier. Later, Subtil et al. [95] address the problem, proposing an integer-enhanced version of the NSGA-II for solving the GAP in a bi-objective setting with an equilibrium function. Both methods, however, only approximate the efficient frontier. Conversely, Al-Hasani et al. [1] propose two methodologies to find all nondominated points.

Generalized Quadratic Assignment Problem (GQAP)

The GQAP is more formally studied in [54]. The problem involves assigning m pieces of equipment to n facilities in a manner that minimizes the total cost of installation and transportation. The authors introduce a linearization approach and develop a branch-and-bound algorithm to solve this problem. The objective function they define is as follows:

$$\begin{aligned} \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \\ & + v\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{ik=1}^{n}\sum_{h=1}^{n} q_{ij}d_{kh}x_{ik}x_{jh} \qquad (7) \end{aligned}$$

where d_{kh} is the distance between facilities k and h, and q_{ij} is the volume transported from i to j. c_{ij} denotes the installation cost of equipment i at facility j.

The GQAP is particularly relevant in scenarios involving pairwise assignment costs, such as the facility location problem and the machine layout problem. Due to its quadratic nature, the GQAP is classified as strongly $\mathcal{NP}$-Hard [54], which necessitates the use of sophisticated solution methods. As a result, metaheuristics are extensively explored in the literature to develop efficient solutions (see [18, 25, 35, 37, 55, 61, 65, 109]).

Following [54], there are few exact approaches developed for GQAP. Hahn et al. [42] propose an algorithm that employs the level-1 RLT (Reformulation Linearization Technique) and a dual ascent procedure. Subsequent work by Pessoa et al. [79] define stronger Lagrangian relaxation and propose branch-and-bound algorithms based on previous work.

Sequencing Generalized Assignment Problem (SGAP)

The Sequencing Generalized Assignment Problem (SGAP) extends GAP by assigning heterogeneous workers to tasks over multiple periods, with the goal of minimizing production time. This includes constraints on worker availability and working/off-days, making it an NP-hard problem. SGAP can be viewed as a series of consecutive GAPs. The problem is first introduced by Moussavi et al. [70] in the context of worker allocation in assembly line systems. They demonstrate that exact solvers are inefficient for medium- to large-scale instances. Consequently, they propose two matheuristic approaches that decompose the problem into multiple subproblems, solving each sequentially. Continuing work by Moussavi et al. [69] introduces a greedy heuristic combined with a local search algorithm.

Multi-resource Generalized Assignment Problem

The multi-resource generalized assignment problem (MRGAP) is an extension of GAP, where each task requires varying amounts of multiple resources. MRGAP is first introduced by Gavish and Pirkul [32]. In MRGAP, the capacity constraint is modified to support multiple resource requirements, as shown below:

$$\sum_{j=1}^{n} a_{ijr} x_{ij} \le b_{ir} \quad i = 1, \ldots, m, \quad r = 1, \ldots, R \tag{8}$$

where a set of resources $r = 1, \ldots, R$ is introduced to the constraints.

While many problems can be modeled as GAP, multiple resource constraints are often necessary for effective modeling of real-life problems [51].

Gavish and Pirkul [32] introduce and compare Lagrangian relaxations of the problem. They propose heuristic approaches to repair these Lagrangian relaxations and design an exact algorithm by incorporating one of these heuristics along with a branch-and-bound procedure. Building on this, Mazzola and Wilcox [64] develop a hybrid heuristic by modifying the Gavish and Pirkul heuristic and integrating it into a more comprehensive hybrid approach.

Subsequent literature introduce various extensions to MRGAP. Shtub and Kogan [93] extended the problem to accommodate time-dependent demand and resource capacities. Meanwhile, Janak et al. [44] explore the NSF panel-assignment problem, optimizing the assignment of reviewers to proposals based on preferences and ensuring an even distribution of assignments among reviewers. Further extensions include *bottleneck* (see [10, 45]) and *bi-criteria* (see [46]).

Solution Approaches

Traditionally, GAP posed significant challenges for commercial solvers, particularly with large-scale instances, prompting the development of exact methods tailored specifically for these problems. As hardware limitations often hindered these exact solutions in dealing with very large datasets, heuristic approaches emerged to offer a practical balance between accuracy and computational feasibility. Recently, however, advancements in both commercial solvers and hardware capabilities have enabled these sophisticated commercial solvers to effectively tackle even the most daunting large-scale problems. For the interested reader, a comprehensive collection of solution approaches for GAP is detailed in Table 1.

It is common practice to evaluate different methods using the dataset described in [9], which comprises five types of instances: A, B,

Generalized Assignment Problem, Table 1 Solution Approaches for GAP

Approach	Methodology	References
Exact	Branch & Bound	[86], [27], [36], [13], [41], [81], [71]
	Branch & Price	[89]
	Branch & Cut & Price	[80]
	Cutting Plane	[5], [99]
	Polyhedral Analysis	[34], [20]
	Column Generation	[14]
Approximate	Variable Depth Search	[82], [3], [104], [105], [56]
	Very Large Neighborhood Search	[66], [40]
	Path Relinking & Ejection Chains	[83], [106], [108], [107], [4]
	Tabu Search	[76], [22], [43], [103],
	Genetic Algorithm	[16], [26], [23], [96], [91], [57]
	Simulated Annealing	[76]
	Ant System	[59]
	GRASP	[59]
	Artificial Bee Colony	[7]
	Bees Algorithm	[78]
	Greedy / Constructive	[84], [101]
	Neural Networks	[68]
	Subgradient	[48], [58], [39]
	Variable Fixing	[98]
	Approximation	[92], [17], [74]

C, D, and E. Types A and B are considered trivial and are often excluded from testing in most studies. In contrast, type D is recognized as the most challenging due to its tight knapsack constraints, where weights are strongly correlated with prices. Consequently, performance on type D instances is regarded as a reliable indicator of an algorithm's effectiveness on difficult problems. Posta et al. [81] report the best performance, solving type D instances up to a size of 20 $\times$ 100, constrained by the hardware available at the time.

However, these results are somewhat outdated. Using the latest version of GUROBI 11.03 [38] on a Linux operating system configured with an Intel i9-13900K 24-core processor and 64 GB DDR4 3200 MT/s RAM, solutions can be achieved with a gap of only 0.38% in less than 2 s for the largest problem instance available for type D, sized 80 $\times$ 1600. Similar results are obtained for types E and C as well. A more detailed examination of GUROBI's performance on these problems is provided in Table 2.

Generalized Assignment Problem, Table 2 GUROBI solver performance on the Generalized Assignment Problem (time limit of 7200 s)

Instance	Best solution	Gap at termination	Run time (s)		
			1%	0.05%	Total
c201600	18803		<1	<1	176
c401600	17145		<1	3	4267
c801600	16286	0.02%	<1	12	7200
d201600	97851	0.03%	<1	1	7200
d401600	97158	0.05%	1	5249	7200
d801600	97084	0.05%	1	6329	7200
e201600	180661		<1	2	12
e401600	178309		1	4	381
e801600	176834		3	52	480

Table 2 shows that commercial solvers such as GUROBI are sufficient even for large-scale problems. It is noteworthy that even in instances where it takes longer to find the optimal solution, GUROBI can find solutions very close to the best solution in just a few minutes. For example, in instance c401600, although proving optimality takes more than an hour, the second-best solution, with an objective function value of 17146—only 1 unit worse than the optimal solution—is found in 167 s. Similarly, in instance d801600, a solution with an objective function value of 97097—only 13 units worse than the optimal solution—is found in 404 s.

Conclusions

This chapter explores the Generalized Assignment Problem, a well-known problem in Operations Research, along with various extensions that are thoroughly examined in the literature. Section "Solution Approaches" provides an extensive review of solution approaches specifically designed for GAP. With advancements in modern hardware and commercial solvers, even the largest instances can be tackled effectively. Finding high-quality solutions has become relatively straightforward; however, proving optimality (particularly when dealing with tight knapsack constraints, such as those in types D and E) remains a computationally intensive challenge.

Considering these factors, future research should focus on developing dual bounds that help create more efficient methods for proving optimality. Additionally, more complex and challenging extensions of GAP can be studied. Prioritizing these areas of research can foster advancements in the field and enhance the practical applicability of GAP solutions in real-world contexts.

See also

- ▶ Bi-objective Assignment Problem
- ▶ Multidimensional Assignment Problem
- ▶ Quadratic Assignment Problem

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