# Geometry on the Sphere: Google's S2 Library

Octavian Procopiuc
tavi@google.com

# Library Overview

- C++ library, open source (Apache License 2)
- Designed and written by Eric Veach

- Basic representations of lat/lng points and 3d vectors
- Shapes on the unit sphere:
  - caps,
  - lat/lng rectangles,
  - polygons, polygonal lines.

- Hierarchical decomposition of the sphere into "cells".
- Ability to approximate regions using cells.

# S2 Cell Hierarchy

- Hierarchical division of the sphere.
- Goals:
  - Enough resolution for indexing geographic features
  - Compact representation of each cell
  - Fast methods for querying with arbitrary regions
  - All cells at a given level should have similar area.

- One solution: Quad-tree.

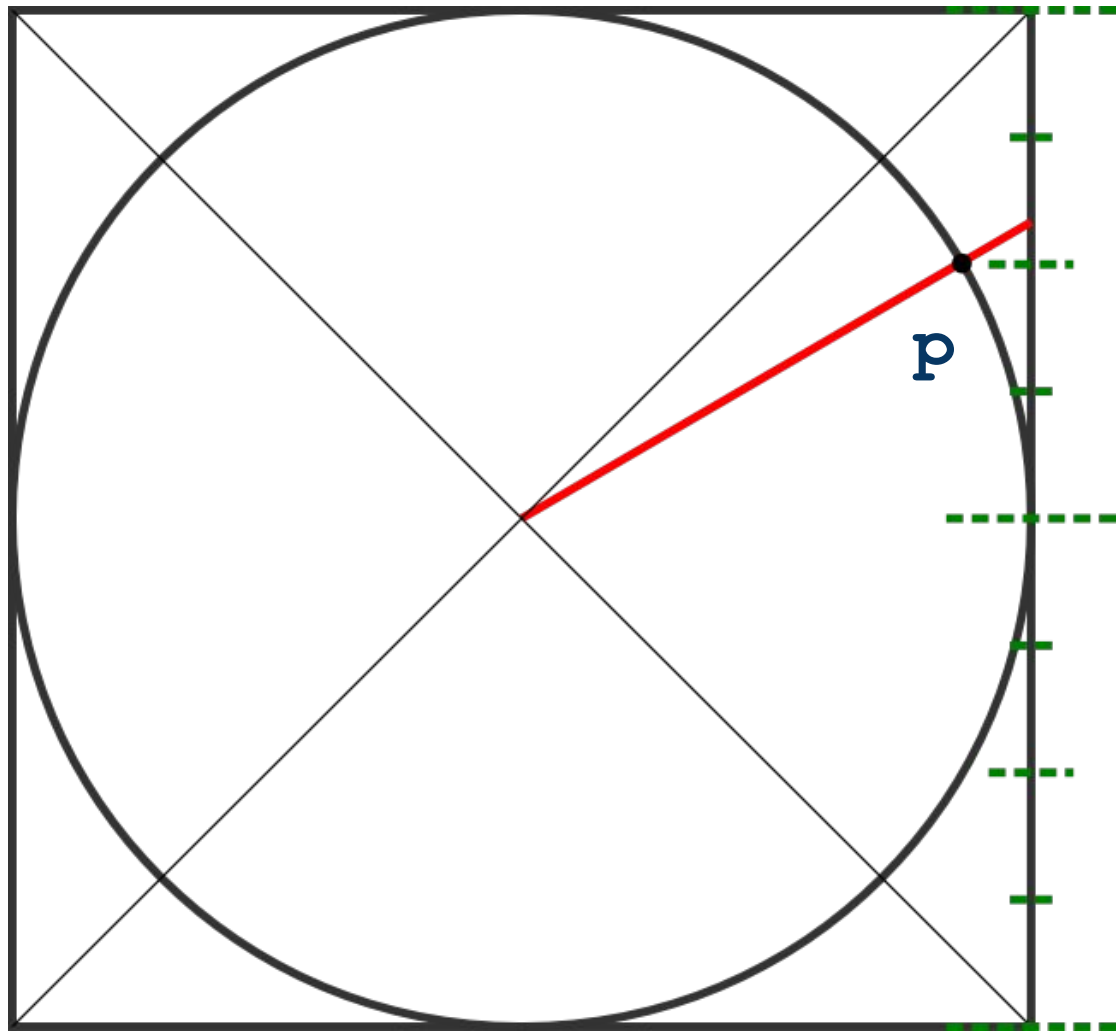
# S2 Cell Hierarchy - Construction

Overview:

- Enclose sphere in cube [-1,1] x [-1,1] x [-1,1]
- Project `p` on the cube
- Build a quad-tree on each cube face
- Find quad-tree cell that contains the projection of `p`

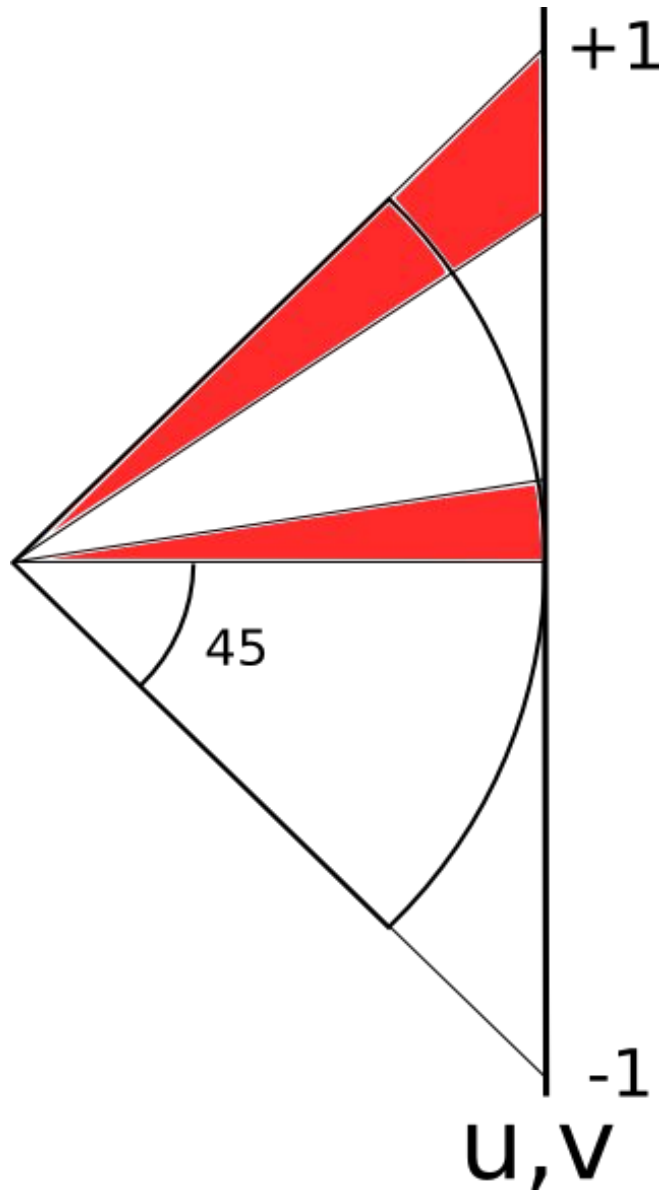
Step 1:

```
p=(lat,lng) => (x,y,z)
```



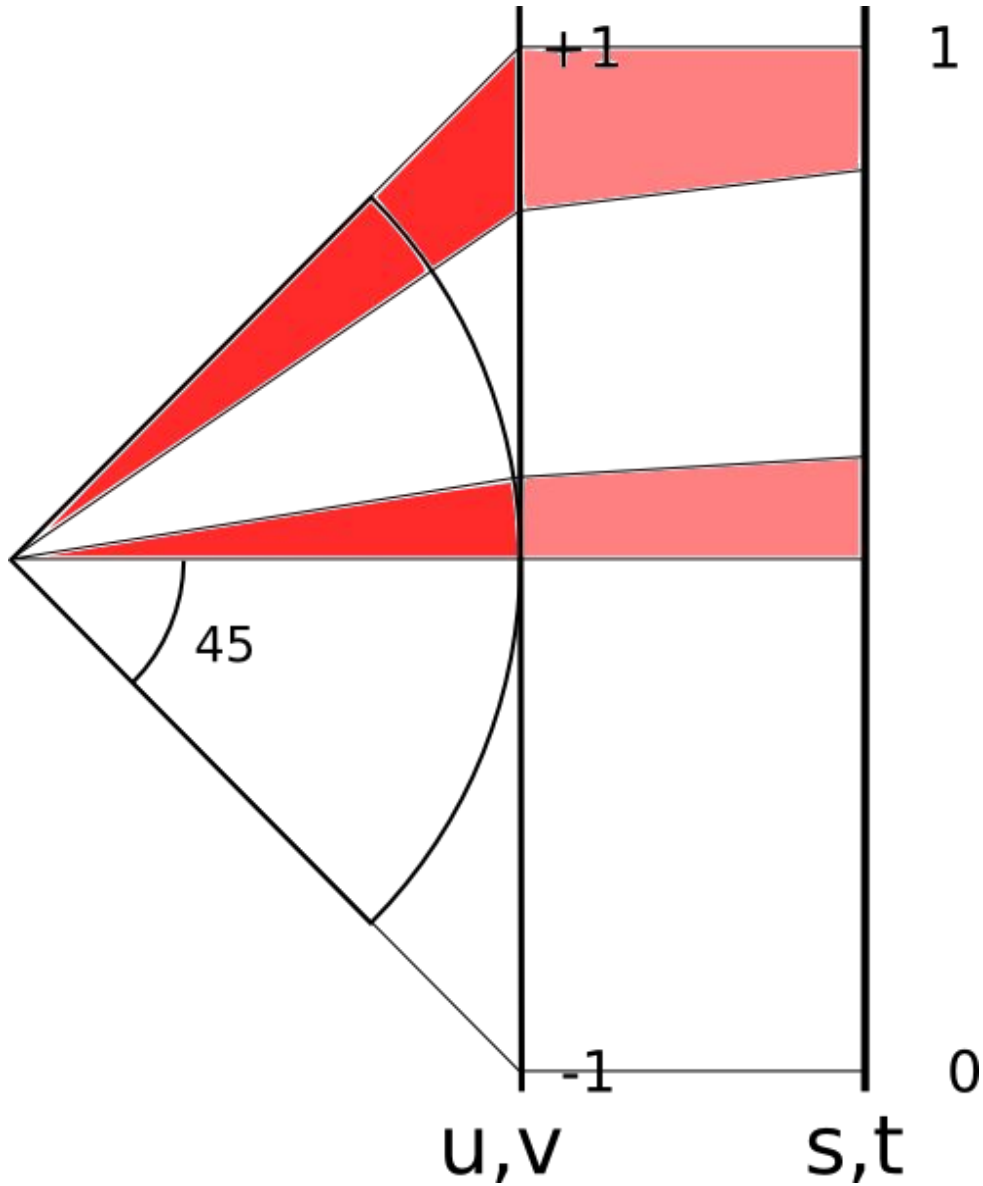
# S2 Cell Hierarchy - Construction

- Step 2:
  `(x,y,z) => (face,u,v)`
- Problem: same-area cells on the cube have different sizes on the sphere. Ratio of highest to lowest area: 5.2



# S2 Cell Hierarchy - Construction

- Solution: non-linear transform
  `(face,u,v) => (face,s,t)`
- s,t in [0,1]



# An Aside - Projection Trade-offs

- Choices for the `(u,v) => (s,t)` projection.
  - Linear: fast, but cell sizes vary widely
  - Tangent: uses atan() to make sizes more uniform; slow
  - **Quadratic**: much faster and almost as good as tangent.

| | Area Ratio | Cell -> Point | Point -> Cell |
|---|---|---|---|
| Linear | 5.20 | 0.087 | 0.085 |
| Tangent | 1.41 | 0.299 | 0.258 |
| Quadratic | 2.08 | 0.096 | 0.108 |

microseconds

# S2 Cell Hierarchy - Construction

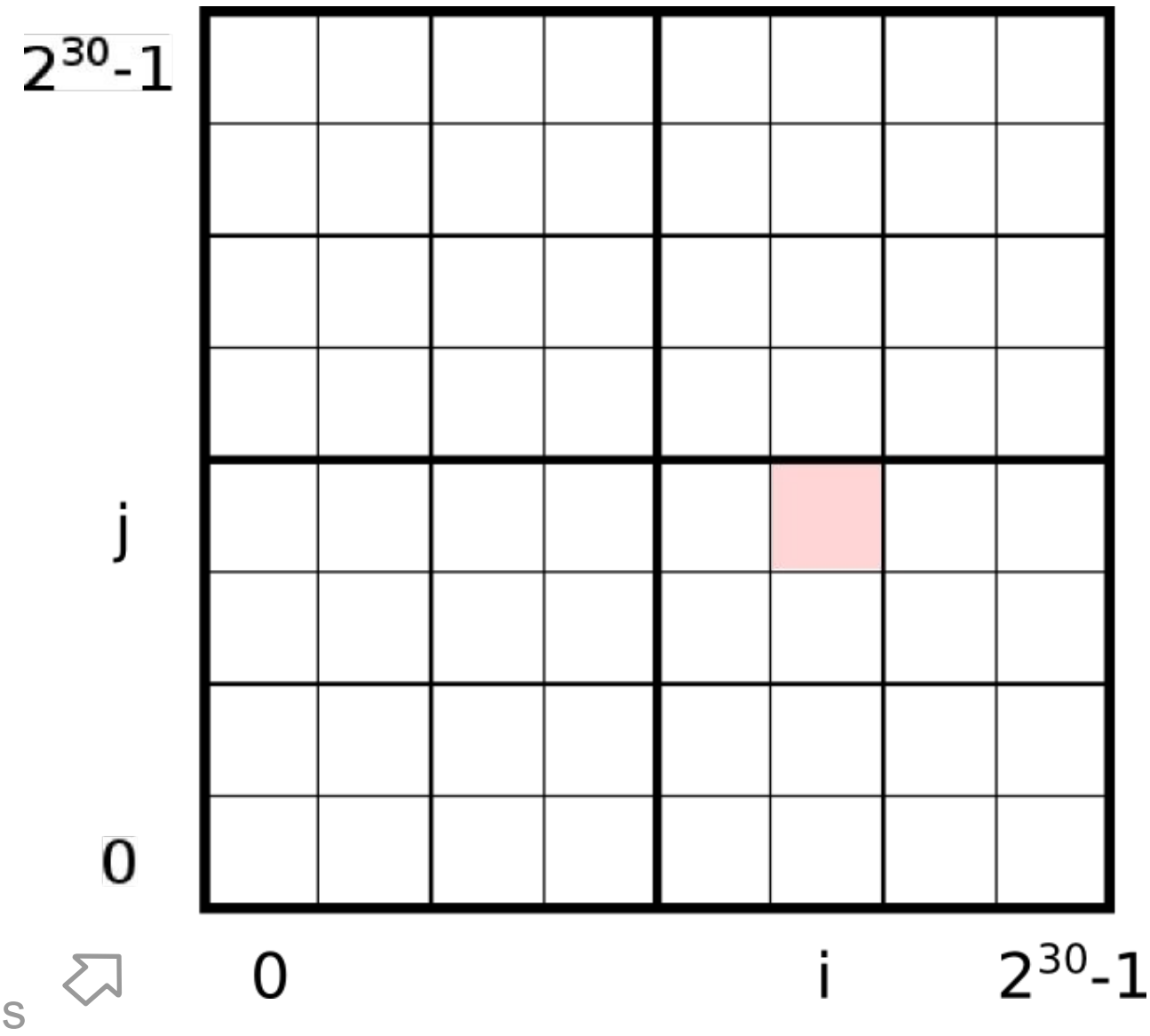
Story so far:

- `(lat,lng)`
- `(x,y,z)`
- `(face,u,v)`
- `(face,s,t)`

Discretize `(s,t)`

- `(face,i,j)`

Quad-tree cell: most significant bits of `i` and `j`



# S2 Cell Hierarchy - Construction

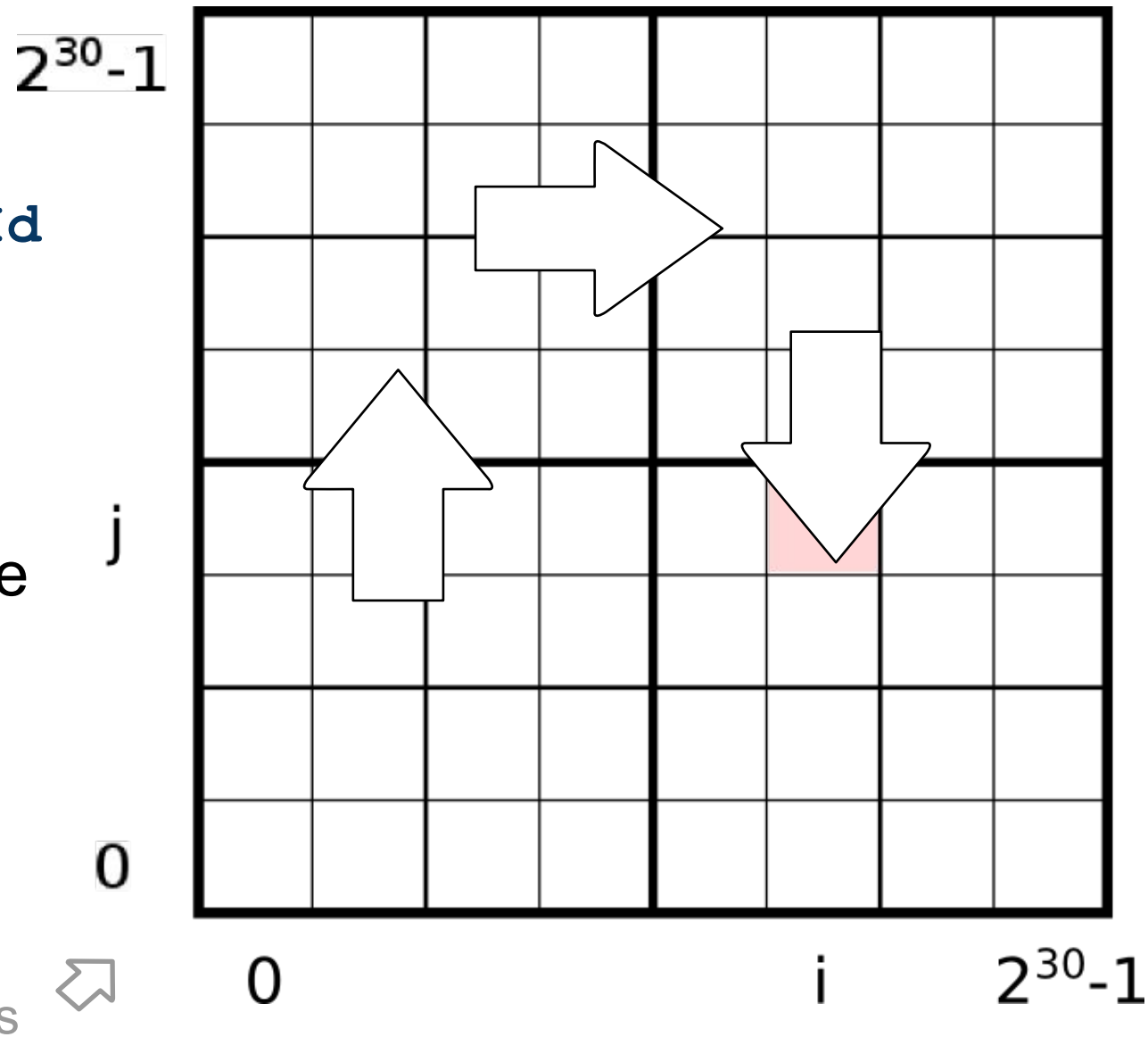
Last step:

`(face,i,j)=> S2CellId`

[ID is a 64-bit integer]

Enumerate cells along a Hilbert space-filling curve

- fast to encode and decode (bit flipping)
- preserves spatial locality



# S2 Cell Hierarchy - Construction

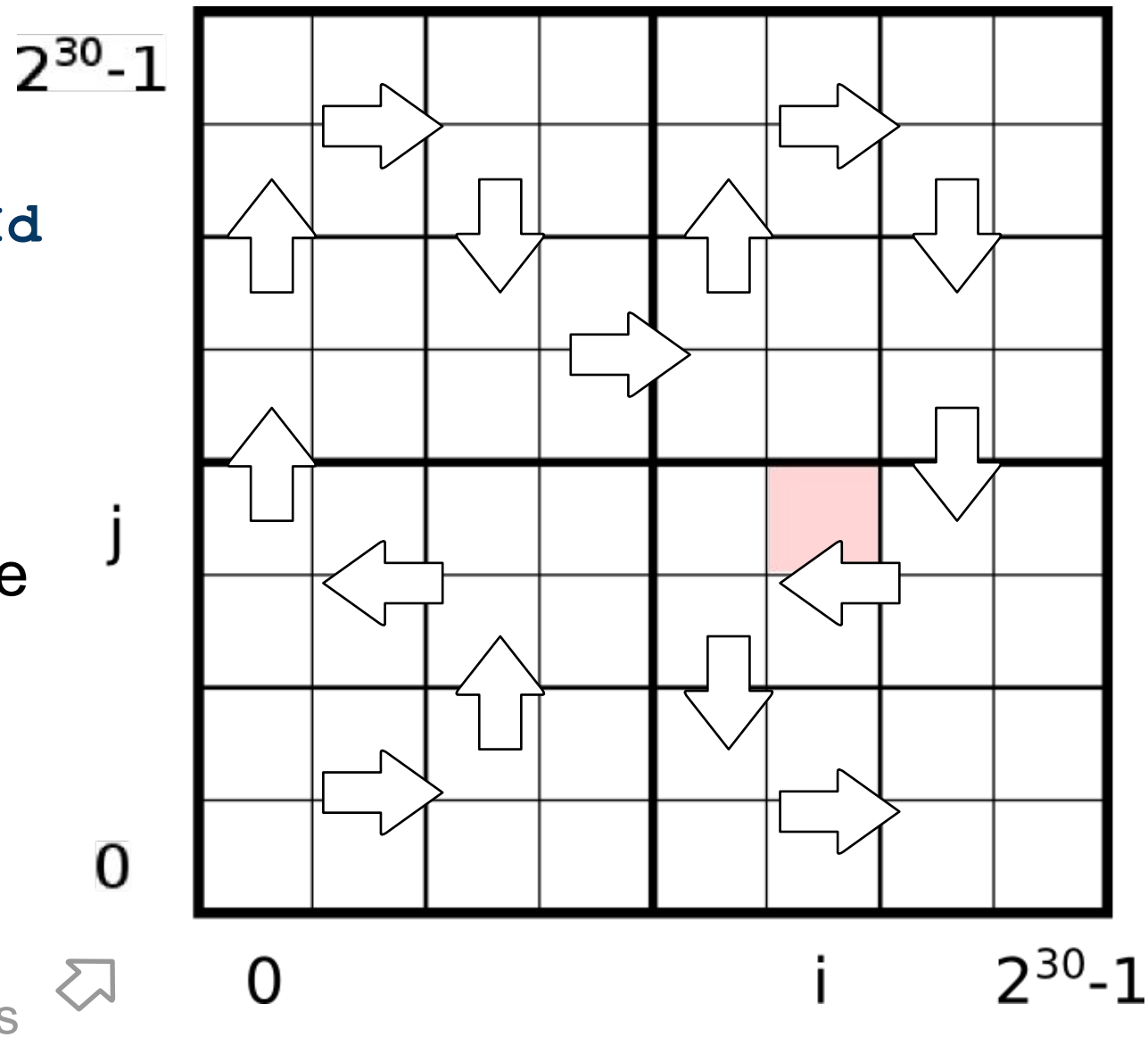
Last step:

```
(face,i,j)=> S2CellId
```

[ID is a 64-bit integer]

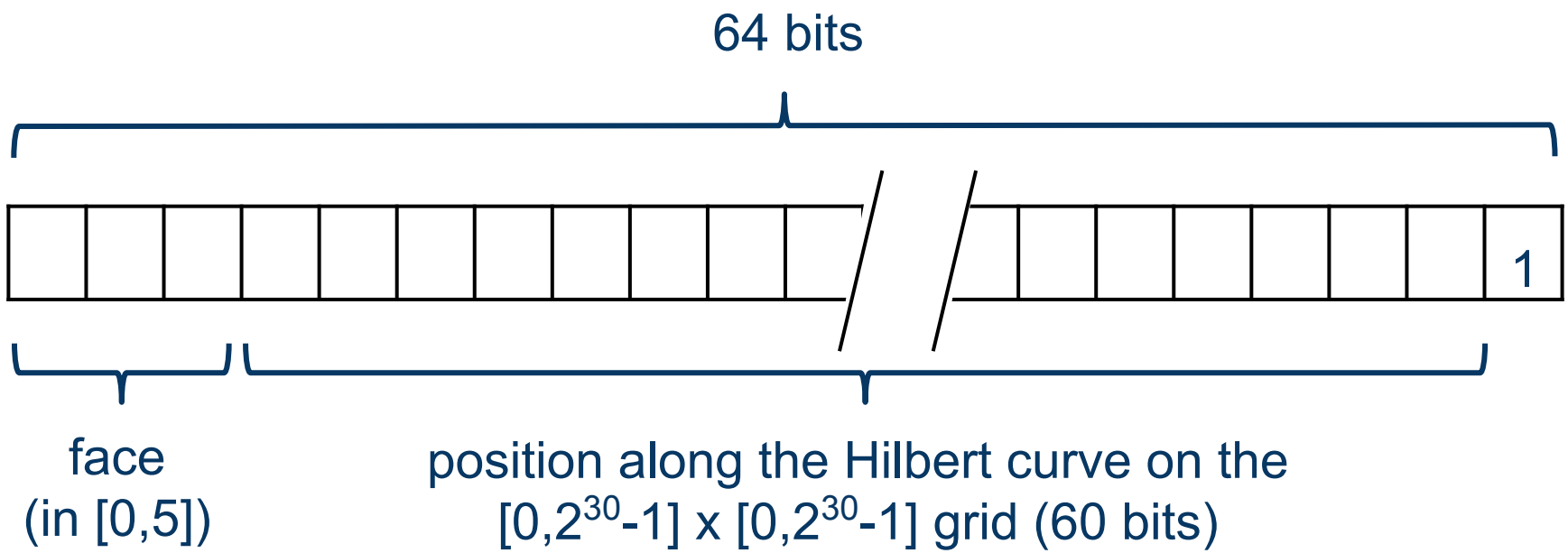
Enumerate cells along a Hilbert space-filling curve

- fast to encode and decode (bit flipping)
- preserves spatial locality



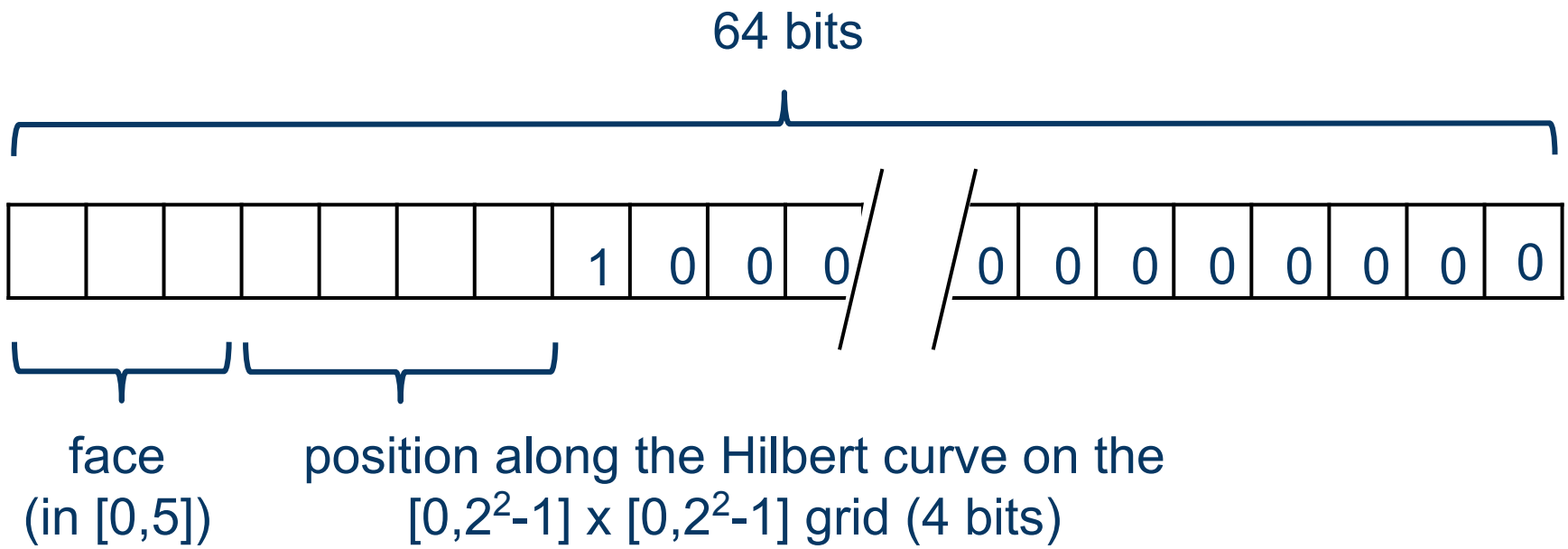
# S2 Cell Hierarchy - Construction

S2 Cell ID of a **leaf** cell (level 30):



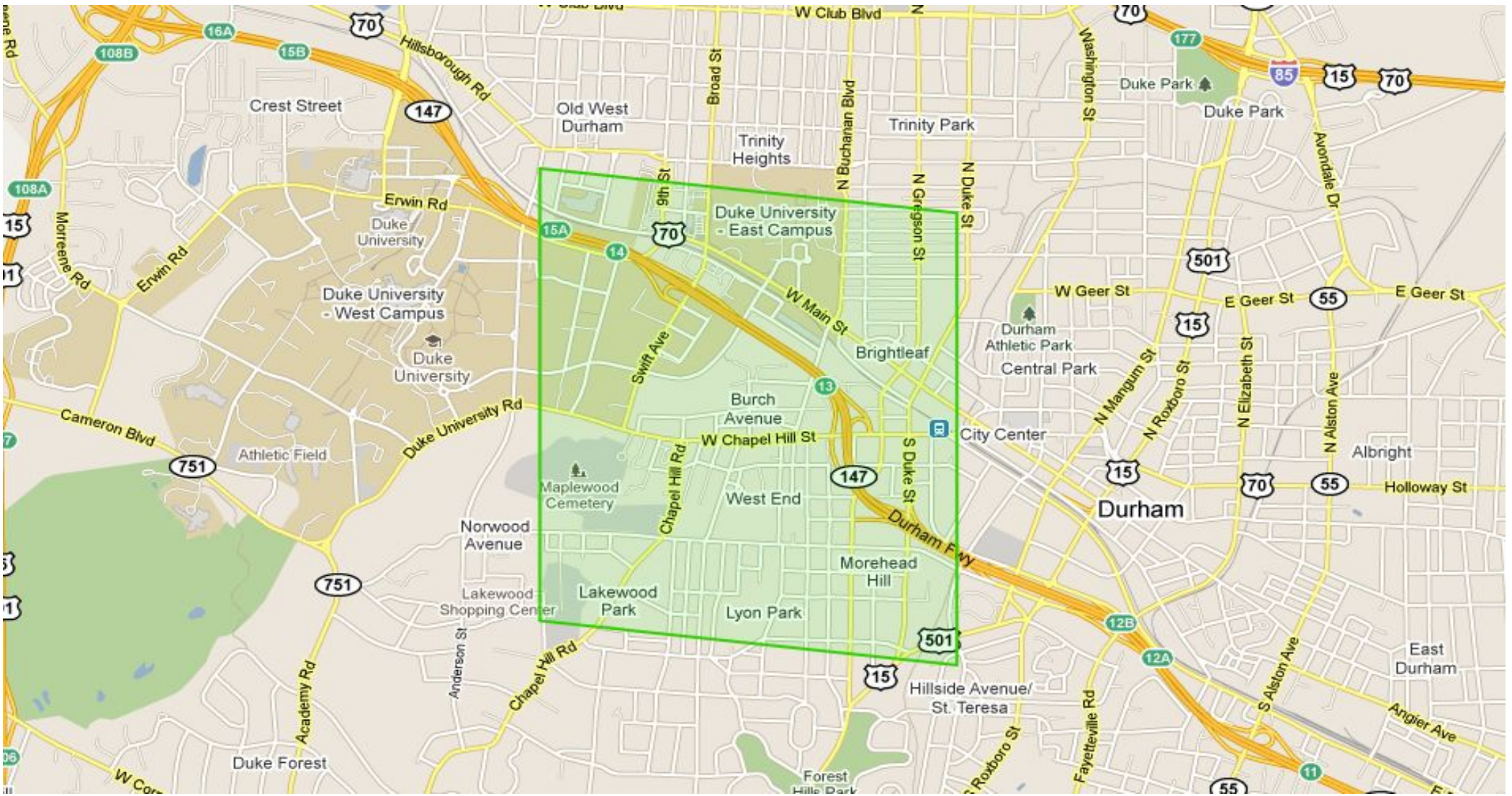
# S2 Cell Hierarchy - Construction

S2 Cell ID of a **level-2** cell:



# One S2 Cell

Id: 0x89ace41000000000 (0b1000100110101100111001000001000...), Level: 12



# S2 Cells - Stats

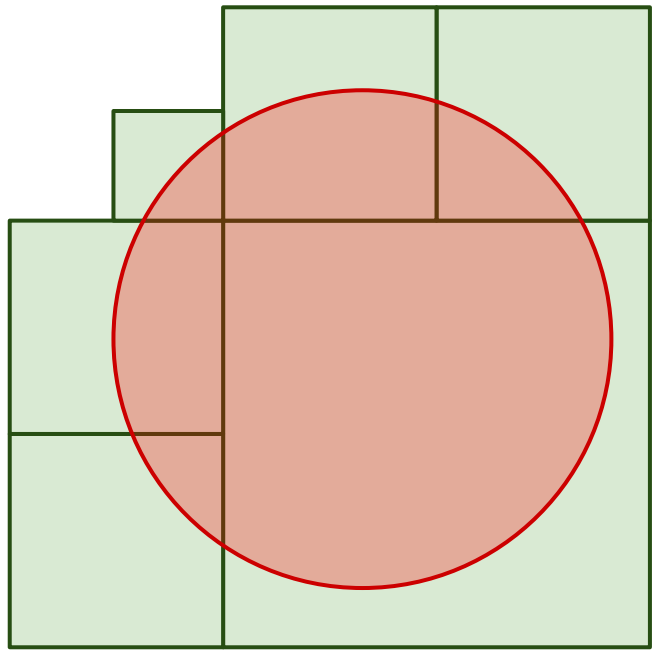
| Level | Min Area | Max Area |
|---|---|---|
| 0 | 85,011,012 $km^2$ | 85,011,012 $km^2$ |
| 1 | 21,252,753 $km^2$ | 21,252,753 $km^2$ |
| 12 | 3.31 $km^2$ | 6.38 $km^2$ |
| 30 | 0.48 $cm^2$ | 0.93 $cm^2$ |

smallest cell

Every $cm^2$ on Earth can be represented using a 64-bit integer.

# Approximating Regions Using S2 Cells

Given a region, find a (small) set of cells that cover it.



Parameters:

- max number of cells
- max cell level
- min cell level

| Max # cells | Median ratio (covering area / region area) | Worst ratio |
|---|---|---|
| 4 | 3.31 | 15.83 |
| 8 | 1.98 | 4.03 |
| 20 | 1.42 | 1.94 |
| 100 | 1.11 | 1.19 |

⇦ default

# What Else Is In the Library

- CCW: Given three points on the sphere, are they counter-clockwise?
  - Multiple implementations, with various tradeoffs.
- Polygons
  - Containment, intersection, union, difference, simplification, centroid computation, etc.
  - Serialization.
- Polygonal lines, Spherical caps.
- Extensive tests and micro-benchmarks.

# Other Similar Libraries

- Hierarchical Triangular Mesh (http://skyserver.org/HTM). The lat/lng <-> triangle id conversion is ~100 slower than the lat/lng <-> s2 cell id conversion.
- HEALPix (http://healpix.jpl.nasa.gov). Cell boundaries are not geodesics; structure is more complicated.
- COBE Quadrilateralized Spherical Cube (http://lambda.gsfc.nasa.gov/product/cobe/skymap_info_new.cfm). Similar decomposition of sphere. But does not use space-filling curve, edges are not geodesics, and projection is more complicated.

# The Code

**http://code.google.com/p/s2-geometry-library/**