

Package ‘NNS’

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Type Package

Title Nonlinear Nonparametric Statistics

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Description Nonlinear nonparametric statistics using partial moments. Partial moments are the elements of variance and asymptotically approximate the area of $f(x)$. These robust statistics provide the basis for nonlinear analysis while retaining linear equivalences. NNS offers: Numerical integration, Numerical differentiation, Clustering, Correlation, Dependence, Causal analysis, ANOVA, Regression, Classification, Seasonality, Autoregressive modelling, Normalization and Stochastic dominance. All routines based on: Viole, F. and Nawrocki, D. (2013), Nonlinear Nonparametric Statistics: Using Partial Moments (ISBN: 1490523995).

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LazyData TRUE

RoxygenNote 5.0.1

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VignetteBuilder knitr

R topics documented:

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Co.LPM

Co-Lower Partial Moment (Lower Left Quadrant 4)

Description

This function generates a co-lower partial moment for between two equal length variables for any degree or target.

Usage

```
Co.LPM(degree.x, degree.y, x, y, target.x, target.y)
```

Arguments

degree.x	integer; Degree for variable X. (degree.x = 0) is frequency, (degree.x = 1) is area.
degree.y	integer; Degree for variable Y. (degree.y = 0) is frequency, (degree.y = 1) is area.
x	a numeric vector.
y	a numeric vector of equal length to x.
target.x	numeric; Typically the mean of Variable X for classical statistics equivalences, but does not have to be. (Vectorized)
target.y	numeric; Typically the mean of Variable Y for classical statistics equivalences, but does not have to be. (Vectorized)

Value

Co-LPM of two variables

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
set.seed(123)
x<-rnorm(100); y<-rnorm(100)
Co.LPM(0,0,x,y,mean(x),mean(y))
```

Co.UPM	<i>Co-Upper Partial Moment (Upper Right Quadrant 1)</i>
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Description

This function generates a co-upper partial moment between two equal length variables for any degree or target.

Usage

```
Co.UPM(degree.x, degree.y, x, y, target.x, target.y)
```

Arguments

degree.x	integer; Degree for variable X. (degree.x = 0) is frequency, (degree.x = 1) is area.
degree.y	integer; Degree for variable Y. (degree.y = 0) is frequency, (degree.y = 1) is area.
x	a numeric vector.
y	a numeric vector of equal length to x.
target.x	numeric; Typically the mean of Variable X for classical statistics equivalences, but does not have to be. (Vectorized)
target.y	numeric; Typically the mean of Variable Y for classical statistics equivalences, but does not have to be. (Vectorized)

Value

Co-UPM of two variables

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
set.seed(123)
x<-rnorm(100); y<-rnorm(100)
Co.UPM(0,0,x,y,mean(x),mean(y))
```

D.LPM

*Divergent-Lower Partial Moment (Lower Right Quadrant 3)***Description**

This function generates a divergent lower partial moment between two equal length variables for any degree or target.

Usage

```
D.LPM(degree.x, degree.y, x, y, target.x, target.y)
```

Arguments

degree.x	integer; Degree for variable X. (degree.x = 0) is frequency, (degree.x = 1) is area.
degree.y	integer; Degree for variable Y. (degree.y = 0) is frequency, (degree.y = 1) is area.
x	a numeric vector.
y	a numeric vector of equal length to x.
target.x	numeric; Typically the mean of Variable X for classical statistics equivalences, but does not have to be. (Vectorized)
target.y	numeric; Typically the mean of Variable Y for classical statistics equivalences, but does not have to be. (Vectorized)

Value

Divergent LPM of two variables

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
set.seed(123)
x<-rnorm(100); y<-rnorm(100)
D.LPM(0,0,x,y,mean(x),mean(y))
```

D.UPM

Divergent-Upper Partial Moment (Upper Left Quadrant 2)

Description

This function generates a divergent upper partial moment between two equal length variables for any degree or target.

Usage

```
D.UPM(degree.x, degree.y, x, y, target.x, target.y)
```

Arguments

degree.x	integer; Degree for variable X. (degree.x = 0) is frequency, (degree.x = 1) is area.
degree.y	integer; Degree for variable Y. (degree.y = 0) is frequency, (degree.y = 1) is area.
x	a numeric vector.
y	a numeric vector of equal length to x.
target.x	numeric; Typically the mean of Variable X for classical statistics equivalences, but does not have to be. (Vectorized)
target.y	numeric; Typically the mean of Variable Y for classical statistics equivalences, but does not have to be. (Vectorized)

Value

Divergent UPM of two variables

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
set.seed(123)
x<-rnorm(100); y<-rnorm(100)
D.UPM(0,0,x,y,mean(x),mean(y))
```

dy.dx

*Partial Derivative dy/dx***Description**

Returns the numerical partial derivate of y wrt x for a point of interest.

Usage

```
dy.dx(x, y, order = NULL, stn = 0.99, eval.point = median(x),
      deriv.order = 1, h = 0.01, noise.reduction = "mean",
      deriv.method = "FS")
```

Arguments

x	a numeric vector.
y	a numeric vector.
order	integer; Controls the number of partial moment quadrant means. Defaults to (order=NULL) which generates a more accurate derivative for well specified cases.
stn	numeric [0,1]; Signal to noise parameter, sets the threshold of NNS.dep which reduces "order" when (order=NULL). Defaults to 0.99 to ensure high dependence for higher "order" and endpoint determination.
eval.point	numeric; x point to be evaluated. Defaults to (eval.point=median(x)). Set to (eval.points="overall") to find an overall partial derivative estimate.
deriv.order	numeric options: (1,2); 1 (default) For second derivative estimate of f(x), set (deriv.order=2).
h	numeric [0,...]; Percentage step used for finite step method. Defaults to h=.01 representing a 1 percent step from the value of the independent variable.
noise.reduction	the method of determing regression points options: ("mean","median","mode","off"); In low signal to noise situations, (noise.reduction="median") uses medians instead of means for partitions, while (noise.reduction="mode") uses modes instead of means for partitions. (noise.reduction="off") allows for maximum possible fit in NNS.reg . Default setting is (noise.reduction="mean").
deriv.method	method of derivative estimation "NNS","FS"; Determines the partial derivative from the coefficient of the NNS.reg output when (deriv.method="NNS") or generates a partial derivative using the finite step method (deriv.method="FS") (Default).

Value

Returns the value of the partial derivative estimate for the given order.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
x<-seq(0,2*pi,pi/100); y<-sin(x)
dy.dx(x,y,eval.point=1.75)
```

dy.d_	<i>Partial Derivative dy/d[wrt]</i>
-------	-------------------------------------

Description

Returns the numerical partial derivate of y with respect to [wrt] any regressor for a point of interest. Finite difference method is used with [NNS.reg](#) estimates as $f(x+h)$ and $f(x-h)$ values.

Usage

```
dy.d_(x, y, wrt, eval.points = "median", order = NULL, stn = 0.9,
      h = 0.1, n.best = 2, mixed = FALSE, plot = FALSE, norm = NULL,
      noise.reduction = "mean")
```

Arguments

x	a numeric matrix or data frame.
y	a numeric vector with compatible dimensions to x.
wrt	integer; Selects the regressor to differentiate with respect to.
eval.points	numeric or options: ("median","last"); Regressor points to be evaluated. Set to eval.points="median" to find partial derivatives at the median of every variable. Set to eval.points="last" to find partial derivatives at the last value of every variable.
order	integer; NNS.reg "order", defaults to NULL.
stn	numeric [0,1]; Signal to noise parameter, sets the threshold of NNS.dep which reduces "order" when (order=NULL). Defaults to 0.9 to ensure high dependence for higher "order" and endpoint determination.
h	numeric [0,...]; Percentage step used for finite step method. Defaults to h=.1 representing a 10 percent step from the value of the regressor.
n.best	integer; Sets the number of closest regression points to use in weighting. Defaults to 2.
mixed	logical; FALSE (default) If mixed derivative is to be evaluated, set (mixed=TRUE).
plot	logical; FALSE (default) Set to (plot=TRUE) to view plot.
norm	NULL (default) the method of normalization options: ("NNS","std"); Normalizes x between 0 and 1 for multivariate regression when set to (norm="std"), or normalizes x according to NNS.norm when set to (norm="NNS").
noise.reduction	the method of determing regression points options: ("mean","median","mode","off"); In low signal to noise situations, noise.reduction="median" uses medians instead of means for partitions, while noise.reduction="mode" uses modes instead of means for partitions. noise.reduction="off" allows for maximum possible fit in NNS.reg . Default setting is noise.reduction="mean".

Value

Returns the 1st derivative "First Derivative", 2nd derivative "Second Derivative", and mixed derivative "Mixed Derivative" (for two independent variables only).

Note

For known function testing and analysis, regressors should be transformed via [expand.grid](#) to fill the dimensions with (order="max"). Example provided below.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" <http://amzn.com/1490523995>

Examples

```
set.seed(123);x_1<-runif(100);x_2<-runif(100); y<-x_1^2*x_2^2
B=cbind(x_1,x_2)
## To find derivatives of y wrt 1st regressor
dy.d_(B,y,wrt=1,eval.points=c(.5,.5))

## Known function analysis
x_1<-seq(0,1,.1);x_2<-seq(0,1,.1)
B=expand.grid(x_1,x_2); y<-B[,1]^2*B[,2]^2
dy.d_(B,y,wrt=1,eval.points=c(.5,.5),order="max")
```

LPM

Lower Partial Moment

Description

This function generates a univariate lower partial moment for any degree or target.

Usage

```
LPM(degree, target, variable)
```

Arguments

degree	integer; (degree = 0) is frequency, (degree = 1) is area.
target	numeric; Typically set to mean, but does not have to be. (Vectorized)
variable	a numeric vector.

Value

LPM of variable

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
set.seed(123)
x<-rnorm(100)
LPM(0,mean(x),x)
```

LPM.VaR	<i>LPM VaR</i>
---------	----------------

Description

Generates a VaR based on the Lower Partial Moment ratio

Usage

```
LPM.VaR(percentile, degree, x)
```

Arguments

percentile	numeric [0,1]; The percentile for VaR.
degree	integer; (degree=0) for discrete distributions, (degree=1) for continuous distributions.
x	a numeric vector.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
set.seed(123)
x<-rnorm(100)
LPM.VaR(0.95,0,x)
```

NNS.ANOVA

NNS ANOVA

Description

Analysis of variance (ANOVA) based on lower partial moment CDFs for multiple variables. Returns a degree of certainty the samples belong to the same population, not a p-value.

Usage

```
NNS.ANOVA(control, treatment, confidence.interval = 0.95, pairwise = FALSE,
           plot = TRUE, binary = TRUE)
```

Arguments

control	a numeric vector, matrix or data frame.
treatment	NULL (default) a numeric vector, matrix or data frame.
confidence.interval	numeric [0,1]; The confidence interval surrounding the control mean when (binary=TRUE). Defaults to (confidence.interval=0.95).
pairwise	logical; FALSE (default) Returns pairwise certainty tests when set to pairwise=TRUE.
plot	logical; TRUE (default) Returns the boxplot of all variables along with grand mean identification. When (binary=TRUE), returns the boxplot of both variables along with grand mean identification and confidence interval thereof.
binary	logical; TRUE (default) Selects binary analysis between a control and treatment variable.

Value

For binary=FALSE returns the degree certainty the samples belong to the same population [0,1].

For binary=TRUE returns "Control Mean", "Treatment Mean", "Grand Mean", "Control CDF", "Treatment CDF", the certainty of the same population statistic "Certainty", the effect size of the treatment for a specified confidence interval with "Lower Bound Effect" and "Upper Bound Effect".

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
### Binary analysis and effect size
set.seed(123)
x<- rnorm(100); y<- rnorm(100)
NNS.ANOVA(control=x,treatment=y)

### Two variable analysis with no control variable
```

```

A<- cbind(x,y)
NNS.ANOVA(A)

### Multiple variable analysis with no control variable
set.seed(123)
x<- rnorm(100); y<- rnorm(100); z<- rnorm(100)
A<- cbind(x,y,z)
NNS.ANOVA(A)

```

NNS.ARMA

NNS ARMA

Description

Autoregressive model incorporating nonlinear regressions of component series.

Usage

```

NNS.ARMA(variable, h = 1, training.set = NULL, seasonal.factor = TRUE,
  negative.values = FALSE, method = "nonlin", dynamic = FALSE,
  plot = TRUE, seasonal.plot = TRUE, intervals = FALSE)

```

Arguments

variable	a numeric vector.
h	integer; 1 (default) Number of periods to forecast.
training.set	NULL (default) numeric; Sets the number of variable observations (variable[1:training.set]) to monitor performance of forecast over in-sample range.
seasonal.factor	logical or integer; TRUE (default) Automatically selects the best seasonal lag from the seasonality test. To use weighted average of all seasonal lags set to (seasonal.factor=FALSE). Otherwise, directly input known frequency integer lag to use, i.e. (seasonal.factor=12) for monthly data.
negative.values	logical; FALSE (default) If the variable can be negative, set to (negative.values=TRUE).
method	options:("lin","nonlin","both"); "nonlin" (default) To select the regression type of the component series, select (method="both") where both linear and nonlinear estimates are generated. To use a nonlinear regression, set to (method="nonlin"); to use a linear regression set to method="lin".
dynamic	logical; FALSE (default) To update the seasonal factor with each forecast point, set to (dynamic=TRUE). The default is (dynamic=FALSE) to retain the original seasonal factor from the inputted variable for all ensuing h.
plot	logical; TRUE (default) Returns the plot of all periods exhibiting seasonality and the variable level reference in upper panel. Lower panel returns original data and forecast.
seasonal.plot	logical; TRUE (default) Adds the seasonality plot above the forecast. Will be set to FALSE if no seasonality is detected or seasonal.factor is set to an integer value.
intervals	logical; FALSE (default) Plots the surrounding forecasts around the final estimate when (intervals=TRUE) and (seasonal.factor=FALSE). There are no other forecasts to plot when a single seasonal.factor is selected.

Value

Returns a vector of forecasts of length (h).

Note

(seasonal.factor=FALSE) can be a very computationally expensive exercise due to the number of seasonal periods detected.

Author(s)

Fred Violen, OVVO Financial Systems

References

Violen, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
## Nonlinear NNS.ARMA using AirPassengers monthly data and 12 period lag
NNS.ARMA(AirPassengers,h=45,training.set=100,seasonal.factor=12,method='nonlin')
```

NNS.caus	<i>NNS Causation</i>
----------	----------------------

Description

Returns the causality from observational data between two variables

Usage

```
NNS.caus(x, y, tau, plot = FALSE)
```

Arguments

x	a numeric vector.
y	a numeric vector.
tau	integer; Number of lagged observations to consider.
plot	logical; FALSE (default) Plots the raw variables, tau normalized, and cross-normalized variables.

Value

Returns the directional causation and quantity of association.

Author(s)

Fred Violen, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
## x clearly causes y...
set.seed(123)
x<-rnorm(100); y<-x^2
NNS.caus(x,y,1)
```

NNS.cor	<i>NNS Correlation</i>
---------	------------------------

Description

Returns the nonlinear correlation between two variables based on higher order partial moment matrices measured by frequency or area.

Usage

```
NNS.cor(x, y = NULL, order = 3, degree = ifelse(length(x) < 100, 0, 1))
```

Arguments

x	a numeric vector, matrix or data frame.
y	NULL (default) or a numeric vector with compatible dimensions to x.
order	integer; Controls the level of quadrant partitioning. Defaults to (order=3). Errors can generally be rectified by setting (order=1).
degree	integer; (degree = 0) is frequency based correlations, while (degree = 1) is for area based correlations. Defaults to (degree = 0) for smaller number of observations.

Value

Returns nonlinear correlation coefficient between two variables, or nonlinear correlation matrix for matrix input.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```

set.seed(123)
## Pairwise Correlation
x<-rnorm(100); y<-rnorm(100)
NNS.cor(x,y)

## Correlation Matrix
x<-rnorm(100); y<-rnorm(100); z<-rnorm(100)
B<-cbind(x,y,z)
NNS.cor(B)

```

NNS.cor.hd

*NNS Co-Partial Moments Higher Dimension Correlation***Description**

Determines higher dimension correlation coefficients based on degree 0 co-partial moments.

Usage

```
NNS.cor.hd(x, plot = FALSE, independence.overlay = FALSE)
```

Arguments

x	a numeric matrix or data frame.
plot	logical; FALSE (default) Generates a 3d scatter plot with regression points using plot3d .
independence.overlay	logical; FALSE (default) Creates and overlays independent Co.LPM and Co.UPM regions to visually reference the difference in dependence from the data.frame of variables being analyzed. Under independence, the light green and red shaded areas would be occupied by green and red data points respectively.

Value

Returns multivariate nonlinear correlation coefficient

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. (2016) "Beyond Correlation: Using the Elements of Variance for Conditional Means and Probabilities" <http://ssrn.com/abstract=2745308>.

Examples

```

set.seed(123)
x<-rnorm(1000); y<-rnorm(1000); z<-rnorm(1000)
A<-data.frame(x,y,z)
NNS.cor.hd(A,plot=TRUE,independence.overlay=TRUE)

```

NNS.dep

*NNS Dependence***Description**

Returns the dependence and nonlinear correlation between two variables based on higher order partial moment matrices measured by frequency or area.

Usage

```
NNS.dep(x, y = NULL, order = 3, degree = NULL, print.map = FALSE)
```

Arguments

x	a numeric vector, matrix or data frame.
y	NULL (default) or a numeric vector with compatible dimensions to x.
order	integer; Controls the level of quadrant partitioning. Defaults to (order=3). Errors can generally be rectified by setting (order=1). Will not partition further if less than 4 observations exist in a quadrant.
degree	integer; Defaults to NULL to allow number of observations to be "degree" determinant.
print.map	logical; FALSE (default) Plots quadrant means.

Value

Returns the bi-variate "Correlation" and "Dependence" or correlation / dependence matrix for matrix input.

Author(s)

Fred Violen, OVVO Financial Systems

References

Violen, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
set.seed(123)
x<-rnorm(100); y<-rnorm(100)
NNS.dep(x,y)

## Correlation / Dependence Matrix
x<-rnorm(100); y<-rnorm(100); z<-rnorm(100)
B<-cbind(x,y,z)
NNS.dep(B)
```

NNS.diff

NNS Numerical Differentiation

Description

Determines numerical derivative of a given function using projected secant lines on the y-axis. These projected points infer finite steps h , in the finite step method.

Usage

```
NNS.diff(f, point, h = 0.1, tol = 1e-10, print.trace = FALSE)
```

Arguments

<code>f</code>	an expression or call or a formula with no lhs.
<code>point</code>	numeric; Point to be evaluated for derivative of a given function <code>f</code> .
<code>h</code>	numeric [0,...]; Initial step for secant projection. Defaults to ($h=0.1$).
<code>tol</code>	numeric; Sets the tolerance for the stopping condition of the inferred h . Defaults to ($tol=1e-10$).
<code>print.trace</code>	logical; FALSE (default) Displays each iteration, lower y-intercept, upper y-intercept and inferred h .

Value

Returns a matrix of values, intercepts, derivatives, inferred step sizes for multiple methods of estimation.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
f<- function(x) sin(x)/x
NNS.diff(f,4.1)

g<- function(x) sin(x)
NNS.diff(g,1)
```

NNS.FSD

NNS FSD Test

Description

Bi-directional test of first degree stochastic dominance using lower partial moments.

Usage

NNS.FSD(x, y)

Arguments

x a numeric vector.

y a numeric vector.

Value

Returns one of the following FSD results: "X FSD Y", "Y FSD X", or "NO FSD EXISTS".

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. <http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817>.

Examples

```
set.seed(123)
x<-rnorm(100); y<-rnorm(100)
NNS.FSD(x,y)
```

NNS.FSD.uni

NNS FSD Test uni-directional

Description

Uni-directional test of first degree stochastic dominance using lower partial moments used in SD Efficient Set routine.

Usage

NNS.FSD.uni(x, y)

Arguments

`x` a numeric vector.
`y` a numeric vector.

Value

Returns (1) if "`X FSD Y`", else (0).

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. <http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817>.

Examples

```
set.seed(123)
x<-rnorm(100); y<-rnorm(100)
NNS.FSD.uni(x,y)
```

NNS.norm

NNS Normalization

Description

Normalizes a matrix of variables based on nonlinear scaling normalization method.

Usage

```
NNS.norm(A, chart.type = NULL, linear = F, order = NULL)
```

Arguments

`A` a numeric matrix or data frame.
`chart.type` options:("l","b"); NULL (default). Set (`chart.type="l"`) for line, (`chart.type="b"`) for boxplot.
`linear` logical; FALSE (default) Performs a linear scaling normalization, resulting in equal means for all variables.
`order` integer; NULL (default) Controls the [NNS.cor](#) "order" for number of partial moment quadrant partitions.

Value

Returns a [data.frame](#) of normalized values.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
set.seed(123)
x<-rnorm(100); y<-rnorm(100)
A<-cbind(x,y)
NNS.norm(A)
```

NNS.part

NNS Partition Map

Description

Creates partitions based on partial moment quadrant means, iteratively assigning identifications to observations based on those quadrants (unsupervised partitioning and hierarchical clustering method). Basis for correlation [NNS.cor](#), dependence [NNS.dep](#), regression [NNS.reg](#) routines.

Usage

```
NNS.part(x, y, Voronoi = FALSE, type = NULL, order = NULL, min.obs = 4,
  noise.reduction = "mean")
```

Arguments

x	a numeric vector.
y	a numeric vector with compatible dimensions to x.
Voronoi	logical; FALSE (default) Displays a Voronoi type diagram using partial moment quadrants.
type	NULL (default) Controls the partitioning basis. Set to (type="XONLY") for X-axis based partitioning. Defaults to NULL for both X and Y-axis partitioning.
order	integer; Number of partial moment quadrants to be generated. (order="max") will institute a perfect fit.
min.obs	integer; Reduces minimum number of necessary observations in a quadrant to 1 when (min.obs=1). In the instances where "regression.points" fail to be generated in the output, re-run partitioning with (min.obs=1) for the given (order=...). Defaults to (min.obs=4).
noise.reduction	the method of determining regression points options: ("mean","median","mode","off"); (noise.reduction="median") uses medians instead of means for partitions, while (noise.reduction="mode") uses modes instead of means for partitions. Defaults to (noise.reduction="mean"), while (noise.reduction="off") will partition quadrant to a single observation for a given (order=...).

Value

Returns both a [data.table](#) ("dt") of x and y observations with their partition assignment "NNS.ID" in the 3rd column; and the [data.table](#) of regression points ("regression.points") for that given (order=...). Also returns the "order" of the final partition given "min.obs" constraints.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
set.seed(123)
x<-rnorm(100); y<-rnorm(100)
NNS.part(x,y)

## Data.table of observations and partitions
NNS.part(x,y,order=1)$dt

## Regression points
NNS.part(x,y,order=1)$regression.points

## Voronoi style plot
NNS.part(x,y,Voronoi=TRUE)
```

NNS.reg

NNS Regression

Description

Generates a nonlinear regression based on partial moment quadrant means.

Usage

```
NNS.reg(x, y, order = NULL, stn = 0.99, type = NULL, point.est = NULL,
  location = "top", return.values = TRUE, plot = TRUE,
  plot.regions = FALSE, residual.plot = TRUE, threshold = 0,
  n.best = NULL, noise.reduction = "mean", norm = NULL, dist = "L2",
  multivariate.call = FALSE)
```

Arguments

<code>x</code>	a vector, matrix or data frame of variables of numeric or factor data types.
<code>y</code>	a numeric or factor vector with compatible dimensions to <code>x</code> .
<code>order</code>	integer; Controls the number of partial moment quadrant means. Users are encouraged to try different (<code>order=...</code>) integer settings with (<code>noise.reduction="off"</code>). (<code>order="max"</code>) will force a limit condition perfect fit.
<code>stn</code>	numeric [0,1]; Signal to noise parameter, sets the threshold of (<code>NNS.dep</code>) which reduces (" <code>order</code> ") when (<code>order=NULL</code>). Defaults to 0.99 to ensure high dependence for higher (" <code>order</code> ") and endpoint determination. (<code>noise.reduction="off"</code>) sets (<code>stn=0</code>) to allow for maximum fit.
<code>type</code>	NULL (default). To perform logistic regression, set to (<code>type = "LOGIT"</code>). To perform a classification, set to (<code>type = "CLASS"</code>).

point.est	a numeric or factor vector with compatible dimensions to x. Returns the fitted value \hat{y} for any value of x.
location	Sets the legend location within the plot, per the x and y co-ordinates used in base graphics legend .
return.values	logical; TRUE (default), set to FALSE in order to only display a regression plot and call values as needed.
plot	logical; TRUE (default) To plot regression.
plot.regions	logical; FALSE (default). Generates 3d regions associated with each regression point for multivariate regressions. Note, adds significant time to routine.
residual.plot	logical; TRUE (default) To plot \hat{y} and Y.
threshold	numeric [0,1]; threshold=0 (default) Sets the correlation threshold for independent variables.
n.best	integer; NULL (default) Sets the number of nearest regression points to use in weighting for multivariate regression at $2 * (\# \text{ of regressors})$. (n.best="all") will select and weight all generated regression points. Analogous to k in k Nearest Neighbors algorithm and different values are tested using cross-validation in NNS.stack .
noise.reduction	the method of determining regression points options: ("mean", "median", "mode", "off"); In low signal:noise situations, (noise.reduction="mean") uses means for NNS.dep restricted partitions, (noise.reduction="median") uses medians instead of means for NNS.dep restricted partitions, while (noise.reduction="mode") uses modes instead of means for NNS.dep restricted partitions. (noise.reduction="off") allows for maximum possible fit with a specific order.
norm	NULL (default) the method of normalization options: ("NNS", "std"); Normalizes x between 0 and 1 for multivariate regression when set to (norm="std"), or normalizes x according to NNS.norm when set to (norm="NNS").
dist	options:("L1", "L2") the method of distance calculation; Selects the distance calculation used. dist="L2" (default) selects the Euclidean distance and (dist="L1") selects the Manhattan distance.
multivariate.call	Internal parameter for multivariate regressions.

Value

UNIVARIATE regression returns the values: "Fitted" for only the fitted values, \hat{y} ; "Fitted.xy" for a data frame of x, y and \hat{y} ; "derivative" for the coefficient of the x and its applicable range; "partition" returns the x, y, "quadrant" assigned to the observation and "prior.quadrant"; "Point" returns the x point(s) being evaluated; "Point.est" for the predicted value generated; "Point" returns the x point(s) being evaluated; "regression.points" provides the points used in the regression equation for the given order of partitions; "R2" provides the goodness of fit; "MSE" returns the MSE between y and \hat{y} ; "Prediction.Accuracy" returns the correct rounded "Point.est" used in classifications versus the categorical y.

MULTIVARIATE regression returns the values: "Fitted" for only the fitted values of x; "Fitted.xy" for a data frame of y and fitted values; "RPM" provides the Regression Point Matrix, the points for each x used in the regression equation for the given order of partitions; "rhs.partitions" returns the partition points for each x; "partition" returns the "NNS.ID" assigned to the observation; "Point" returns the x point(s) being evaluated; "Point.est" returns the predicted value generated; "equation" returns the synthetic X^* dimension reduction equation.

Note

Please ensure `point.est` is of compatible dimensions to `x`, error message will ensue if not compatible.

Author(s)

Fred Violo, OVVO Financial Systems

References

Violo, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
set.seed(123)
x<-rnorm(100); y<-rnorm(100)
NNS.reg(x,y)

## Manual {order} selection
NNS.reg(x,y,order=2)

## Maximum {order} selection
NNS.reg(x,y,order='max')

## x-only partitioning (Univariate only)
NNS.reg(x,y,type="XONLY")

## Logistic Regression (Univariate only)
NNS.reg(x,y,type="LOGIT")

## For Multiple Regression:
x<-cbind(rnorm(100),rnorm(100),rnorm(100)); y<-rnorm(100)
NNS.reg(x,y,point.est=c(.25,.5,.75))

## For Multiple Regression based on Synthetic X* (Dimension Reduction):
x<-cbind(rnorm(100),rnorm(100),rnorm(100)); y<-rnorm(100)
NNS.reg(x,y,point.est=c(.25,.5,.75),type="CLASS")

## IRIS dataset example:
#Dimension Reduction:
NNS.reg(iris[,1:4],iris[,5],type="CLASS",order=5)
#Multiple Regression:
NNS.reg(iris[,1:4],iris[,5],order=2,noise.reduction='off')

## To call fitted values:
x<-rnorm(100); y<-rnorm(100)
NNS.reg(x,y)$Fitted

## To call partial derivative (univariate regression only):
x<-rnorm(100); y<-rnorm(100)
NNS.reg(x,y)$derivative
```

NNS.SD.Efficient.Set *NNS SD Efficient Set*

Description

Determines the set of stochastic dominant variables for various degrees.

Usage

```
NNS.SD.Efficient.Set(x, degree)
```

Arguments

x a numeric matrix or data frame.
degree numeric options: (1,2,3); Degree of stochastic dominance test from (1,2 or 3).

Value

Returns set of stochastic dominant variable names.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. <http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817>.

Examples

```
set.seed(123)
x<-rnorm(100); y<-rnorm(100); z<-rnorm(100)
x<-data.frame(x,y,z)
NNS.SD.Efficient.Set(x,1)
```

NNS.seas *NNS Seasonality Test*

Description

Seasonality test based on the coefficient of variance for the variable and lagged component series.
A result of 1 signifies no seasonality present.

Usage

```
NNS.seas(variable, plot = TRUE)
```

Arguments

variable	a numeric vector.
plot	logical; TRUE (default) Returns the plot of all periods exhibiting seasonality and the variable level reference.

Value

Returns a matrix of all periods exhibiting less coefficient of variance than the variable with "all.periods"; and the single period exhibiting the least coefficient of variance versus the variable with "best.period". If no seasonality is detected, NNS.seas will return ("No Seasonality Detected").

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
set.seed(123)
x<-rnorm(100)

## To call strongest period based on coefficient of variance:
NNS.seas(x)$best.period
```

NNS.SSD

NNS SSD Test

Description

Bi-directional test of second degree stochastic dominance using lower partial moments.

Usage

```
NNS.SSD(x, y)
```

Arguments

x	a numeric vector.
y	a numeric vector.

Value

Returns one of the following SSD results: "X SSD Y", "Y SSD X", or "NO SSD EXISTS".

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. <http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817>.

Examples

```
set.seed(123)
x<-rnorm(100); y<-rnorm(100)
NNS.SSD(x,y)
```

NNS.SSD.uni

NNS SSD Test uni-directional

Description

Uni-directional test of second degree stochastic dominance using lower partial moments used in SD Efficient Set routine.

Usage

```
NNS.SSD.uni(x, y)
```

Arguments

x	a numeric vector.
y	a numeric vector.

Value

Returns (1) if "X SSD Y", else (0).

Author(s)

Fred Viole, OVVO Financial Systems

Examples

```
set.seed(123)
x<-rnorm(100); y<-rnorm(100)
NNS.SSD.uni(x,y)
```

NNS.stack

*NNS stack***Description**

Prediction model using the predictions of the NNS base models ([NNS.reg](#) as features (i.e. meta-features) for the stacked model.

Usage

```
NNS.stack(IVs.train, DV.train, IVs.test = NULL, CV.size = 0.2,
  weight = "MSE", precision = "LOW", method = c(1, 2), threshold = 0,
  seed = 123)
```

Arguments

IVs.train	a vector, matrix or data frame of variables of numeric or factor data types.
DV.train	a numeric or factor vector with compatible dimensions to (IVs.train).
IVs.test	a vector, matrix or data frame of variables of numeric or factor data types.
CV.size	numeric [0,1]; Sets the cross-validation size if (IVs.test=NULL). Defaults to 0.2 for a 20 percent random sampling of the training set.
weight	options:("MSE","Features") method for selecting model output weight; Set (weight="MSE") for optimum parameters and weighting based on each base model's "MSE". (weight="Features") uses a weighting based on the number of features present, whereby logistic NNS.reg receives higher relative weights for more regressors. Defaults to "MSE".
precision	options:("LOW","HIGH"); 2 settings offered: "LOW" (Default), and "HIGH". "HIGH" is the limit condition of every observation as a regression point and uses a (norm="NNS") while (precision="LOW") uses a (norm="std") in NNS.reg . Errors/warnings can generally be reconciled with (precision="LOW").
method	numeric options:(1,2); Select the NNS method to include in stack. (method=1) selects NNS.reg ; (method=2) selects NNS.reg dimension reduction logistic regression. Defaults to method=c(1, 2), including both NNS regression methods in the stack.
threshold	numeric [0,1]; Sets the correlation threshold for independent variables in NNS.reg . Defaults to (threshold=0).
seed	numeric; 123 (default) Sets seed for CV sampling.

Value

Returns a vector of fitted values for the dependent variable test set for all models. "NNS.reg.n.best" returns the optimum "n.best" parameter for the [NNS.reg](#) multivariate regression. "NNS.logistic.order" returns the optimum "order" from the [NNS.reg](#) logistic regression. "reg" returns [NNS.reg](#) output, "logistic" returns [NNS.reg](#) logistic regression output, and "stack" returns the output of the stacked model.

Note

If character variables are used, transform them first to factors using `as.factor`, or `data.matrix` to ensure overall dataset is numeric. A multifunction `sapply` can also be applied to the overall dataset: `data<- sapply(data,function(x){as.factor(x);as.numeric(x)})`. Then run `NNS.stack` with transformed variables.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. (2016) "Classification Using NNS Clustering Analysis" <https://ssrn.com/abstract=2864711>

Examples

```
## Using 'iris' dataset where test set \code{IVs.test} is rows 141:150.
NNS.stack(iris[1:140,1:4],iris[1:140,5],IVs.test=iris[141:150,1:4])

## Selecting NNS.reg and dimension reduction techniques.
NNS.stack(iris[1:140,1:4],iris[1:140,5],iris[141:150,1:4],method=c(1,2))
```

NNS.term.matrix	<i>NNS Term Matrix</i>
-----------------	------------------------

Description

Generates a term matrix for text classification use in `NNS.reg`.

Usage

```
NNS.term.matrix(x, oos = NULL, names = FALSE)
```

Arguments

<code>x</code>	Text A two column dataset should be used. Concatenate text from original sources to comply with format. Also note the possibility of factors in "DV", so " <code>as.numeric(as.character(...))</code> " is used to avoid issues.
<code>oos</code>	Out-of-sample text dataset to be classified.
<code>names</code>	Column names for "IV" and "oos". Defaults to FALSE.

Value

Returns the text as independent variables "IV" and the classification as the dependent variable "DV". Out-of-sample independent variables are returned with "OOS".

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" <http://amzn.com/1490523995>

Examples

```
x<- data.frame(cbind(c("sunny","rainy"),c(1,-1)))
NNS.term.matrix(x)

### Concatenate Text with space separator, cbind with "DV"
x<- data.frame(cbind(c("sunny","rainy"),c("windy","cloudy"),c(1,-1)))
x<- data.frame(cbind(paste(x[,1],x[,2],sep=" "),as.numeric(as.character(x[,3]))))
NNS.term.matrix(x)

### NYT Example
## Not run:
require(RTextTools)
data(NYTimes)

### Concatenate Columns 3 and 4 containing text, with column 5 as DV
NYT=data.frame(cbind(paste(NYTimes[,3],NYTimes[,4],sep = " "),
                        as.numeric(as.character(NYTimes[,5]))))
NNS.term.matrix(NYT)
## End(Not run)
```

NNS.TSD

NNS TSD Test

Description

Bi-directional test of third degree stochastic dominance using lower partial moments.

Usage

```
NNS.TSD(x, y)
```

Arguments

x	a numeric vector.
y	a numeric vector.

Value

Returns one of the following TSD results: "X TSD Y", "Y TSD X", or "NO TSD EXISTS".

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." *Journal of Mathematical Finance*, 6, 105-126. <http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817>.

Examples

```
set.seed(123)
x<-rnorm(100); y<-rnorm(100)
NNS.TSD(x,y)
```

NNS.TSD.uni

NNS TSD Test uni-directional

Description

Uni-directional test of third degree stochastic dominance using lower partial moments used in SD Efficient Set routine.

Usage

```
NNS.TSD.uni(x, y)
```

Arguments

x	a numeric vector.
y	a numeric vector.

Value

Returns (1) if "X TSD Y", else (0).

Author(s)

Fred Viole, OVVO Financial Systems

Examples

```
set.seed(123)
x<-rnorm(100); y<-rnorm(100)
NNS.TSD.uni(x,y)
```

Uni.caus

NNS Causation uni-directional (INTERNAL CALL FOR [NNS.caus](#))

Description

Returns the uni-directional causality from observational data between two variables. Causation nets out the univariate effect.

Usage

```
Uni.caus(x, y, tau, plot = TRUE)
```

Arguments

x	a numeric vector.
y	a numeric vector.
tau	integer; Number of lagged observations to consider
plot	logical; TRUE (default) Plots the raw variables, tau normalized, and cross-normalized variables.

Author(s)

Fred Viole, OVVO Financial Systems

UPM

Upper Partial Moment

Description

This function generates a univariate upper partial moment for any degree or target.

Usage

```
UPM(degree, target, variable)
```

Arguments

degree	integer; (degree = 0) is frequency, (degree = 1) is area.
target	numeric; Typically set to mean, but does not have to be. (Vectorized)
variable	a numeric vector.

Value

UPM of variable

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<http://amzn.com/1490523995>

Examples

```
set.seed(123)
x<-rnorm(100)
UPM(0,mean(x),x)
```

UPM.VaR	<i>UPM VaR</i>
---------	----------------

Description

Generates an upside VaR based on the Upper Partial Moment ratio

Usage

```
UPM.VaR(percentile, degree, x)
```

Arguments

percentile	numeric [0,1]; The percentile for VaR.
degree	integer; (degree=0) for discrete distributions, (degree=1) for continuous distributions.
x	a numeric vector.

Examples

```
set.seed(123)
x<-rnorm(100)
UPM.VaR(0.95,0,x)
```

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