5 Way Rank Probabilities in R

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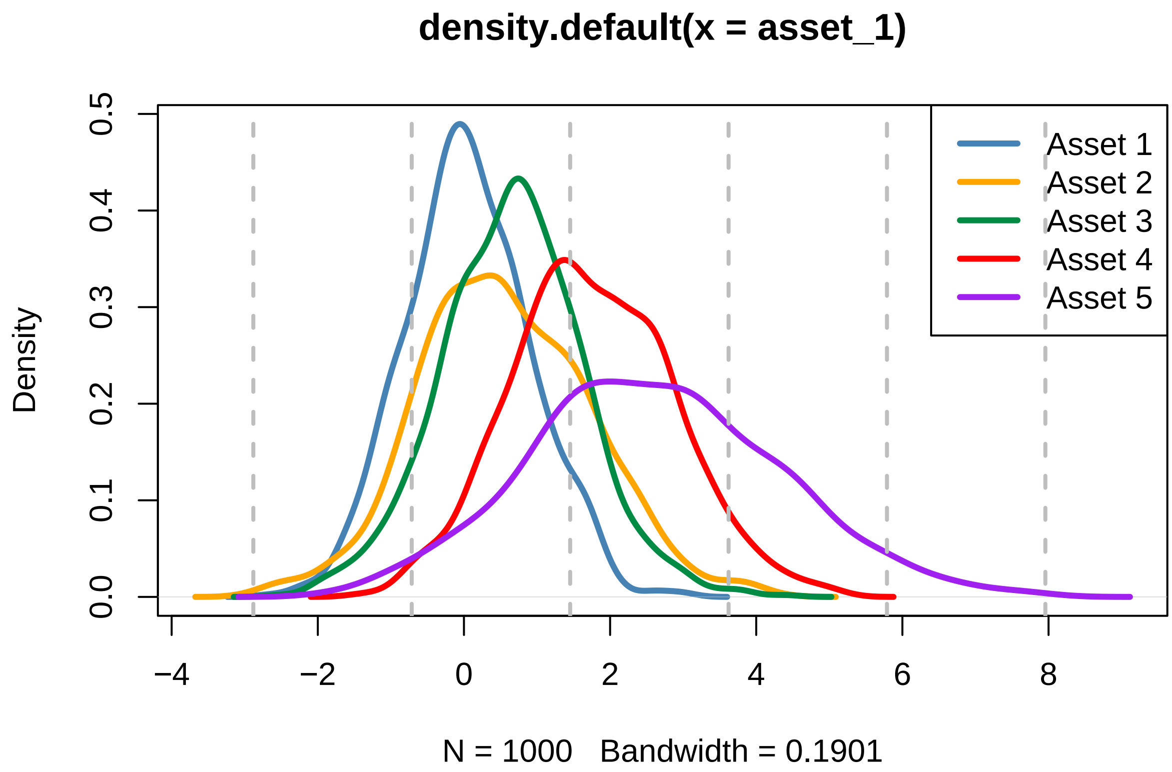
12/13/2021

Generate Data

```
library(data.table)
library(sn)
library(NNS)

set.seed(12345)
n = 1000
asset_1 = rsn(n, xi = -.5, omega = 1, alpha = 1)
asset_2 = rsn(n, xi = -.25, omega = 1.5, alpha = 1)
asset_3 = rsn(n, xi = 0, omega = 1.2, alpha = 1)
asset_4 = rsn(n, xi = 1, omega = 1.3, alpha = 1)
asset_5 = rsn(n, xi = 1.5, omega = 2, alpha = 1)

assets = setDT(list(asset_1, asset_2, asset_3, asset_4, asset_5))
colnames(assets) = paste0("Asset_",1:5)

# Grid values
cutpoints = seq(min(assets), max(assets), length.out = 6)
```

Plot

Grey dashed lines are the target value all `UPM` calculations are based off of for each density and then averaged. This is analogous to the shifted distribution $f^{\rightarrow a}(.)$.



Generate Probabilities

`rank_5` based on relative shifted densities measured from several lattice points

```
# UPM from min value
upms = apply(assets, 2, function(x) UPM(1, cutpoints, x))
upms = cbind(upms, rowSums(upms))
upms = upms/upms[,ncol(upms)]
upms = upms[complete.cases(upms),-ncol(upms)]

rank_5_probabilities = colMeans(upms)
rank_5_probabilities
```

```
##    Asset_1    Asset_2    Asset_3    Asset_4    Asset_5
## 0.04909595 0.07822471 0.07570879 0.16709926 0.62987130
```

MC Verification

Sample each asset without replacement based on its winning probability. Each column represents a race, and the row represents the asset's finishing order.

```
replicates = 1000
races = replicate(replicates, sample(paste0("Asset_",1:5), 5, replace = F,
                                    prob = rank_5_probabilities))
rownames(races) = paste0("Rank_",5:1)
head(races[, 1:10])
```

```
##        [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## Rank_5 "Asset_5" "Asset_4" "Asset_5" "Asset_5" "Asset_3" "Asset_5" "Asset_2"
## Rank_4 "Asset_1" "Asset_3" "Asset_1" "Asset_4" "Asset_5" "Asset_2" "Asset_5"
## Rank_3 "Asset_2" "Asset_5" "Asset_4" "Asset_3" "Asset_4" "Asset_4" "Asset_3"
## Rank_2 "Asset_3" "Asset_2" "Asset_2" "Asset_2" "Asset_1" "Asset_1" "Asset_4"
## Rank_1 "Asset_4" "Asset_1" "Asset_3" "Asset_1" "Asset_2" "Asset_3" "Asset_1"
##        [,8]      [,9]      [,10]
## Rank_5 "Asset_2" "Asset_5" "Asset_5"
## Rank_4 "Asset_5" "Asset_2" "Asset_2"
## Rank_3 "Asset_3" "Asset_3" "Asset_3"
## Rank_2 "Asset_4" "Asset_1" "Asset_4"
## Rank_1 "Asset_1" "Asset_4" "Asset_1"
```

```
Results = matrix(NA, 5,5)

for(i in 1:5){
  for(j in 1:5){
    Results[i,(6-j)] = sum(races[j,]==colnames(assets)[i])/replicates
  }
}

colnames(Results) = paste0("Rank_", 1:5)
rownames(Results) = paste0("Asset_", 1:5)
Results
```

```
##         Rank_1 Rank_2 Rank_3 Rank_4 Rank_5
## Asset_1  0.449  0.259  0.156  0.094  0.042
## Asset_2  0.232  0.290  0.241  0.160  0.077
## Asset_3  0.243  0.276  0.270  0.135  0.076
## Asset_4  0.076  0.161  0.265  0.328  0.170
## Asset_5  0.000  0.014  0.068  0.283  0.635
```

```
# sanity check
rowSums(Results)
```

```
## Asset_1 Asset_2 Asset_3 Asset_4 Asset_5
##       1       1       1       1       1
```

```
colSums(Results)
```

```
## Rank_1 Rank_2 Rank_3 Rank_4 Rank_5
##      1      1      1      1      1
```

Ranks from your notebook with $\rho = 0$.

```
[10] a2l.to_ltx(RP)
```

```
\begin{tabular}{lrrrrr}
\toprule
     & Rank 1 & Rank 2 & Rank 3 & Rank 4 & Rank 5 \\\n\midrule
 Asset 1 &  0.37 &  0.33 &  0.20 &  0.08 &  0.02\\
 Asset 2 &  0.32 &  0.24 &  0.21 &  0.15 &  0.08\\
 Asset 3 &  0.20 &  0.26 &  0.28 &  0.19 &  0.07\\
 Asset 4 &  0.04 &  0.10 &  0.19 &  0.37 &  0.31\\
 Asset 5 &  0.07 &  0.08 &  0.12 &  0.21 &  0.52\\
\bottomrule
\end{tabular}
```

Sampling with Dependence

Alternatively we can just infer the probabilities from the ranks of the tuple elements themselves…

```
empirical_samples = cbind(assets,
                  t(apply(assets, 1, function(x) colnames(assets)[order(x, decreasing = F)])))

colnames(empirical_samples) = c(paste0("Asset_", 1:5),
                               paste0("Rank_", 1:5))

empirical_samples
```

```
##           Asset_1    Asset_2     Asset_3    Asset_4    Asset_5  Rank_1  Rank_2
##    1:  0.41569963  1.5362199  0.72866722  0.3777486  0.4843671 Asset_4 Asset_1
##    2: -0.74338181  1.5266611 -0.41525272  2.5461900  0.1822763 Asset_1 Asset_3
##    3: -1.35706186  1.2815910  0.96831269  0.7305048 -0.3425123 Asset_1 Asset_5
##    4: -0.24974470  0.4594755 -0.10040605  0.6829659 -0.8650106 Asset_5 Asset_1
##    5: -0.94912754 -0.5656314  1.52439859  2.0519467  2.2742939 Asset_1 Asset_2
##   ---
##  996:  0.30423320  1.8321389 -0.04146175  2.2884961  5.1382054 Asset_3 Asset_1
##  997: -0.01683564  1.6197419  0.83305115 -0.2918981  2.2538016 Asset_4 Asset_1
##  998:  0.95846702  0.1406389  0.57545219  1.9620473  4.0652914 Asset_2 Asset_3
##  999: -0.32830364  1.0182948  1.14756094  1.5355784  4.8748741 Asset_1 Asset_2
## 1000: -0.91661260  0.2310668  0.51263975  1.3483795  3.7713607 Asset_1 Asset_2
##        Rank_3  Rank_4  Rank_5
##    1: Asset_5 Asset_3 Asset_2
##    2: Asset_5 Asset_2 Asset_4
##    3: Asset_4 Asset_3 Asset_2
##    4: Asset_3 Asset_2 Asset_4
##    5: Asset_3 Asset_4 Asset_5
##   ---
##  996: Asset_2 Asset_4 Asset_5
##  997: Asset_3 Asset_2 Asset_5
##  998: Asset_1 Asset_4 Asset_5
##  999: Asset_3 Asset_4 Asset_5
## 1000: Asset_3 Asset_4 Asset_5
```

```r
Results = matrix(NA, 5,5)

for(i in 1:5){
  for(j in 6:10){
    Results[i,(j-5)] = sum(empirical_samples[, .SD, .SDcols = j]==colnames(assets)[i]) / n
  }
}

colnames(Results) = paste0("Rank_", 1:5)
rownames(Results) = paste0("Asset_", 1:5)
Results
```

```
##         Rank_1 Rank_2 Rank_3 Rank_4 Rank_5
## Asset_1  0.451  0.302  0.174  0.063  0.010
## Asset_2  0.282  0.253  0.236  0.166  0.063
## Asset_3  0.190  0.281  0.305  0.167  0.057
## Asset_4  0.041  0.100  0.186  0.387  0.286
## Asset_5  0.036  0.064  0.099  0.217  0.584
```

Increasing the empirical samples via `meboot`

If we have small samples, we can increase them using the maximum entropy bootstrap which preserves the dependence structure within time series.

We started off with 1,000 observations, let's make it 10,000 using $\rho = 0.95$ for each bootstrap replicate.

```r
bootstraps = do.call(cbind, lapply(assets, function(x)
  as.vector(NNS.meboot(unlist(x), reps = 10, rho = .95)$replicates)))


bootstrap_samples = cbind(bootstraps,
                 t(apply(bootstraps, 1, function(x) colnames(bootstraps)[order(x, decreasing = F)])))

bootstrap_samples = data.table(bootstrap_samples)

colnames(bootstrap_samples) = c(paste0("Asset_", 1:5),
                                paste0("Rank_", 1:5))

bootstrap_samples
```

```
##          Asset_1   Asset_2   Asset_3   Asset_4  Asset_5  Rank_1  Rank_2  Rank_3
##     1:   0.04583  1.411913  1.299483  0.841006 0.699054 Asset_1 Asset_5 Asset_4
##     2: -1.042426  0.186548 -0.622692  2.900695  1.90915 Asset_1 Asset_3 Asset_2
##     3: -1.442048  1.241001  0.821467  1.173776 0.549658 Asset_1 Asset_5 Asset_3
##     4: -0.564878  0.100575 -0.042659  1.561435 0.173731 Asset_1 Asset_3 Asset_2
##     5:   -1.3208 -0.842794  1.817027  1.766772 2.697114 Asset_1 Asset_2 Asset_4
##   ---
##  9996:   0.28159  1.759054  0.710106  2.216156 5.676325 Asset_1 Asset_3 Asset_2
##  9997: -0.047135  1.538926   1.72411 -0.328612 2.645264 Asset_4 Asset_1 Asset_2
##  9998:  1.343108  0.592453  0.953663  1.623247 3.618084 Asset_2 Asset_3 Asset_1
##  9999: -0.104326  1.144804  1.454609  2.297013 4.583305 Asset_1 Asset_2 Asset_3
## 10000:  -0.84067   0.72057  0.703523  1.416391 3.064384 Asset_1 Asset_3 Asset_2
```

```
##         Rank_4  Rank_5
##     1: Asset_3 Asset_2
##     2: Asset_5 Asset_4
##     3: Asset_4 Asset_2
##     4: Asset_5 Asset_4
##     5: Asset_3 Asset_5
##    ---
##  9996: Asset_4 Asset_5
##  9997: Asset_3 Asset_5
##  9998: Asset_4 Asset_5
##  9999: Asset_4 Asset_5
## 10000: Asset_4 Asset_5
```

```r
Results = matrix(NA, 5,5)

for(i in 1:5){
  for(j in 6:10){
    Results[i,(j-5)] = sum(bootstrap_samples[, .SD, .SDcols = j]==colnames(bootstraps)[i]) /
      nrow(bootstraps)
  }
}

colnames(Results) = paste0("Rank_", 1:5)
rownames(Results) = paste0("Asset_", 1:5)
Results
```

```
##         Rank_1 Rank_2 Rank_3 Rank_4 Rank_5
## Asset_1 0.4384 0.2888 0.1883 0.0699 0.0146
## Asset_2 0.2692 0.2579 0.2295 0.1708 0.0726
## Asset_3 0.1963 0.2736 0.2922 0.1658 0.0721
## Asset_4 0.0447 0.1038 0.1893 0.3876 0.2746
## Asset_5 0.0514 0.0759 0.1007 0.2059 0.5661
```

Expanding the number of bootstraps, here's 1e6 replicates:

```
bootstraps2 = do.call(cbind, lapply(assets, function(x)
  as.vector(NNS.meboot(unlist(x), reps = 1000, rho = .95)$replicates)))


bootstrap_samples2 = cbind(bootstraps2,
                  t(apply(bootstraps2, 1, function(x) colnames(bootstraps2)[order(x, decreasing = F)])))

bootstrap_samples2 = data.table(bootstrap_samples2)


Results = matrix(NA, 5,5)

for(i in 1:5){
  for(j in 6:10){
    Results[i,(j-5)] = sum(bootstrap_samples2[, .SD, .SDcols = j]==colnames(bootstraps2)[i]) /
      nrow(bootstraps2)
  }
}

colnames(Results) = paste0("Rank_", 1:5)
rownames(Results) = paste0("Asset_", 1:5)
Results
```

```
##           Rank_1   Rank_2   Rank_3   Rank_4   Rank_5
## Asset_1 0.447808 0.296088 0.180374 0.063969 0.011761
## Asset_2 0.276346 0.248454 0.242835 0.170901 0.061464
## Asset_3 0.195724 0.284315 0.281298 0.174721 0.063942
## Asset_4 0.037285 0.099068 0.203100 0.372655 0.287892
## Asset_5 0.042837 0.072075 0.092393 0.217754 0.574941
```

References:

- Vinod, Hrishikesh D. and Viole, Fred, *Arbitrary Spearman's Rank Correlations in Maximum Entropy Bootstrap and Improved Monte Carlo Simulations* (June 7, 2020). Available at SSRN: <http://dx.doi.org/10.2139/ssrn.3621614>
- Cotton, P. Python notebook: https://github.com/microprediction/m6/blob/main/notebook_examples/five_way_rank_example.ipynb