

Package ‘NNS’

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Type Package

Title Nonlinear Nonparametric Statistics

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Description Nonlinear nonparametric statistics using partial moments. Partial moments are the elements of variance and asymptotically approximate the area of $f(x)$. These robust statistics provide the basis for nonlinear analysis while retaining linear equivalences. NNS offers: Numerical integration, Numerical differentiation, Clustering, Correlation, Dependence, Causal analysis, ANOVA, Regression, Classification, Seasonality, Autoregressive modeling, Normalization and Stochastic dominance. All routines based on: Viole, F. and Nawrocki, D. (2013), Nonlinear Nonparametric Statistics: Using Partial Moments (ISBN: 1490523995).

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BugReports <https://github.com/OVVO-Financial/NNS/issues>

LazyData TRUE

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Depends R (>= 3.3.0), doParallel

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Suggests knitr, rmarkdown

VignetteBuilder knitr

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Co.LPM

Co-Lower Partial Moment (Lower Left Quadrant 4)

Description

This function generates a co-lower partial moment for between two equal length variables for any degree or target.

Usage

```
Co.LPM(degree.x, degree.y, x, y, target.x = mean(x),
       target.y = mean(y))
```

Arguments

degree.x	integer; Degree for variable X. (degree.x = 0) is frequency, (degree.x = 1) is area.
degree.y	integer; Degree for variable Y. (degree.y = 0) is frequency, (degree.y = 1) is area.
x	a numeric vector.

y	a numeric vector of equal length to x.
target.x	numeric; Typically the mean of Variable X for classical statistics equivalences, but does not have to be. (Vectorized)
target.y	numeric; Typically the mean of Variable Y for classical statistics equivalences, but does not have to be. (Vectorized)

Value

Co-LPM of two variables

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
Co.LPM(0, 0, x, y, mean(x), mean(y))
```

Co.UPM

Co-Upper Partial Moment (Upper Right Quadrant 1)

Description

This function generates a co-upper partial moment between two equal length variables for any degree or target.

Usage

```
Co.UPM(degree.x, degree.y, x, y, target.x = mean(x),
       target.y = mean(y))
```

Arguments

degree.x	integer; Degree for variable X. (degree.x = 0) is frequency, (degree.x = 1) is area.
degree.y	integer; Degree for variable Y. (degree.y = 0) is frequency, (degree.y = 1) is area.
x	a numeric vector.
y	a numeric vector of equal length to x.
target.x	numeric; Typically the mean of Variable X for classical statistics equivalences, but does not have to be. (Vectorized)
target.y	numeric; Typically the mean of Variable Y for classical statistics equivalences, but does not have to be. (Vectorized)

Value

Co-UPM of two variables

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
Co.UPM(0,0,x,y,mean(x),mean(y))
```

D.LPM

Divergent-Lower Partial Moment (Lower Right Quadrant 3)

Description

This function generates a divergent lower partial moment between two equal length variables for any degree or target.

Usage

```
D.LPM(degree.x, degree.y, x, y, target.x = mean(x), target.y = mean(y))
```

Arguments

degree.x	integer; Degree for variable X. (degree.x = 0) is frequency, (degree.x = 1) is area.
degree.y	integer; Degree for variable Y. (degree.y = 0) is frequency, (degree.y = 1) is area.
x	a numeric vector.
y	a numeric vector of equal length to x.
target.x	numeric; Typically the mean of Variable X for classical statistics equivalences, but does not have to be. (Vectorized)
target.y	numeric; Typically the mean of Variable Y for classical statistics equivalences, but does not have to be. (Vectorized)

Value

Divergent LPM of two variables

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
D.LPM(0, 0, x, y, mean(x), mean(y))
```

D.UPM

Divergent-Upper Partial Moment (Upper Left Quadrant 2)

Description

This function generates a divergent upper partial moment between two equal length variables for any degree or target.

Usage

```
D.UPM(degree.x, degree.y, x, y, target.x = mean(x), target.y = mean(y))
```

Arguments

degree.x	integer; Degree for variable X. (degree.x = 0) is frequency, (degree.x = 1) is area.
degree.y	integer; Degree for variable Y. (degree.y = 0) is frequency, (degree.y = 1) is area.
x	a numeric vector.
y	a numeric vector of equal length to x.
target.x	numeric; Typically the mean of Variable X for classical statistics equivalences, but does not have to be. (Vectorized)
target.y	numeric; Typically the mean of Variable Y for classical statistics equivalences, but does not have to be. (Vectorized)

Value

Divergent UPM of two variables

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
D.UPM(0, 0, x, y, mean(x), mean(y))
```

dy.dx

*Partial Derivative dy/dx***Description**

Returns the numerical partial derivate of y wrt x for a point of interest.

Usage

```
dy.dx(x, y, order = NULL, stn = 0.99, eval.point = median(x),
      deriv.order = 1, h = 0.05, noise.reduction = "mean",
      deriv.method = "FS")
```

Arguments

x	a numeric vector.
y	a numeric vector.
order	integer; Controls the number of partial moment quadrant means. Defaults to (order = NULL) which generates a more accurate derivative for well specified cases.
stn	numeric [0, 1]; Signal to noise parameter, sets the threshold of NNS.dep which reduces "order" when (order = NULL). Defaults to 0.99 to ensure high dependence for higher "order" and endpoint determination.
eval.point	numeric; x point to be evaluated. Defaults to (eval.point = median(x)). Set to (eval.point = "overall") to find an overall partial derivative estimate.
deriv.order	numeric options: (1, 2); 1 (default) for first derivative. For second derivative estimate of f(x), set (deriv.order = 2).
h	numeric [0, ...]; Percentage step used for finite step method. Defaults to h = .05 representing a 5 percent step from the value of the independent variable.
noise.reduction	the method of determing regression points options: ("mean", "median", "mode", "off"); In low signal to noise situations, (noise.reduction = "median") uses medians instead of means for partitions, while (noise.reduction = "mode") uses modes instead of means for partitions. (noise.reduction = "off") allows for maximum possible fit in NNS.reg . Default setting is (noise.reduction = "mean").
deriv.method	method of derivative estimation, options: ("NNS", "FS"); Determines the partial derivative from the coefficient of the NNS.reg output when (deriv.method = "NNS") or generates a partial derivative using the finite step method (deriv.method = "FS") (Default).

Value

Returns the value of the partial derivative estimate for the given order.

Note

If a vector of derivatives is required, ensure (deriv.method = "FS").

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" <https://www.amazon.com/dp/1490523995>

Vinod, H. and Viole, F. (2017) "Nonparametric Regression Using Clusters" <https://link.springer.com/article/10.1007/s10614-017-9713-5>

Examples

```
## Not run:
x <- seq(0, 2 * pi, pi / 100) ; y <- sin(x)
dy.dx(x, y, eval.point = 1.75)

# Vector of derivatives
dy.dx(x, y, eval.point = c(1.75, 2.5), deriv.method = "FS")
## End(Not run)
```

dy.d_	<i>Partial Derivative $dy/d_{[wrt]}$</i>
-------	---

Description

Returns the numerical partial derivate of y with respect to [wrt] any regressor for a point of interest. Finite difference method is used with [NNS.reg](#) estimates as $f(x + h)$ and $f(x - h)$ values.

Usage

```
dy.d_(x, y, wrt, eval.points = "median", folds = 5, mixed = FALSE,
      plot = FALSE, messages = TRUE)
```

Arguments

x	a numeric matrix or data frame.
y	a numeric vector with compatible dimensions to x.
wrt	integer; Selects the regressor to differentiate with respect to.
eval.points	numeric or options: ("mean", "median", "last"); Regressor points to be evaluated. (eval.points = "median") (default) to find partial derivatives at the median of every variable. Set to (eval.points = "last") to find partial derivatives at the last value of every variable. Set to (eval.points="mean") to find partial derivatives at the mean value of every variable. Set to (eval.points = "all") to find partial derivatives at every observation.
folds	integer; 5 (default) Sets the number of folds in the NNS.stack procedure for optimal n.best parameter.
mixed	logical; FALSE (default) If mixed derivative is to be evaluated, set (mixed = TRUE).
plot	logical; FALSE (default) Set to (plot = TRUE) to view plot. Default setting is (noise.reduction = "mean").
messages	logical; TRUE (default) Prints status messages of cross-validation on n.best parameter for NNS.reg .

Value

Returns:

- `dy.d(...)$"First Derivative"` the 1st derivative
- `dy.d(...)$"Second Derivative"` the 2nd derivative
- `dy.d(...)$"Mixed Derivative"` the mixed derivative (for two independent variables only).

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
## Not run:
set.seed(123) ; x_1 <- runif(100) ; x_2 <- runif(100) ; y <- x_1 ^ 2 * x_2 ^ 2
B <- cbind(x_1, x_2)
## To find derivatives of y wrt 1st regressor
dy.d_(B, y, wrt = 1, eval.points = c(.5, .5))

## Known function analysis: [y = a ^ 2 * b ^ 2]
x_1 <- seq(0, 1, .1) ; x_2 <- seq(0, 1, .1)
B <- expand.grid(x_1, x_2) ; y <- B[, 1] ^ 2 * B[, 2] ^ 2
dy.d_(B, y, wrt = 1, eval.points = c(.5, .5))
## End(Not run)
```

LPM

Lower Partial Moment

Description

This function generates a univariate lower partial moment for any degree or target.

Usage

```
LPM(degree, target, variable)
```

Arguments

degree	integer; (degree = 0) is frequency, (degree = 1) is area.
target	numeric; Typically set to mean, but does not have to be. (Vectorized)
variable	a numeric vector.

Value

LPM of variable

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
set.seed(123)
x <- rnorm(100)
LPM(0, mean(x), x)
```

LPM.ratio

Lower Partial Moment RATIO

Description

This function generates a standardized univariate lower partial moment for any degree or target.

Usage

```
LPM.ratio(degree, target, variable)
```

Arguments

degree	integer; (degree = 0) is frequency, (degree = 1) is area.
target	numeric; Typically set to mean, but does not have to be. (Vectorized)
variable	a numeric vector.

Value

Standardized LPM of variable

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Viole, F. (2017) "Continuous CDFs and ANOVA with NNS" <https://ssrn.com/abstract=3007373>

Examples

```

set.seed(123)
x <- rnorm(100)
LPM.ratio(0, mean(x), x)

## Not run:
## Empirical CDF (degree = 0)
lpm_cdf <- LPM.ratio(0, sort(x), x)
plot(sort(x), lpm_cdf)

## Continuous CDF (degree = 1)
lpm_cdf_1 <- LPM.ratio(1, sort(x), x)
plot(sort(x), lpm_cdf_1)

## Joint CDF
x <- rnorm(5000) ; y <- rnorm(5000)
plot3d(x, y, Co.LPM(0, 0, sort(x), sort(y), x, y), col = "blue", xlab = "X", ylab = "Y",
zlab = "Probability", box = FALSE)

## End(Not run)

```

LPM.VaR

*LPM VaR***Description**

Generates a value at risk (VaR) based on the Lower Partial Moment ratio.

Usage

```
LPM.VaR(percentile, degree, x)
```

Arguments

percentile	numeric [0, 1]; The percentile for left-tail VaR.
degree	integer; (degree = 0) for discrete distributions, (degree = 1) for continuous distributions.
x	a numeric vector.

Value

Returns a numeric value representing the point at which "percentile" of the area of x is above.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
set.seed(123)
x <- rnorm(100)

## For 95th percentile VaR (left-tail)
LPM.VaR(0.95, 0, x)
```

NNS.ANOVA	<i>NNS ANOVA</i>
-----------	------------------

Description

Analysis of variance (ANOVA) based on lower partial moment CDFs for multiple variables. Returns a degree of certainty the difference in sample means is zero, not a p-value.

Usage

```
NNS.ANOVA(control, treatment, confidence.interval = 0.95,
           tails = "Both", pairwise = FALSE, plot = TRUE, binary = TRUE)
```

Arguments

control	a numeric vector, matrix or data frame.
treatment	NULL (default) a numeric vector, matrix or data frame.
confidence.interval	numeric [0, 1]; The confidence interval surrounding the control mean when (binary = TRUE). Defaults to (confidence.interval = 0.95).
tails	options: ("Left", "Right", "Both"). tails = "Both"(Default) Selects the tail of the distribution to determine effect size.
pairwise	logical; FALSE (default) Returns pairwise certainty tests when set to pairwise = TRUE.
plot	logical; TRUE (default) Returns the boxplot of all variables along with grand mean identification. When (binary = TRUE), returns the boxplot of both variables along with grand mean identification and confidence interval thereof.
binary	logical; TRUE (default) Selects binary analysis between a control and treatment variable.

Value

For (binary = FALSE) returns the degree certainty the difference in sample means is zero [0, 1].

For (binary = TRUE) returns:

- "Control Mean"
- "Treatment Mean"
- "Grand Mean"
- "Control CDF"
- "Treatment CDF"
- "Certainty" the certainty of the same population statistic
- "Lower Bound Effect" and "Upper Bound Effect" the effect size of the treatment for the specified confidence interval

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
### Binary analysis and effect size
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.ANOVA(control = x, treatment = y)

### Two variable analysis with no control variable
A <- cbind(x, y)
NNS.ANOVA(A)

### Multiple variable analysis with no control variable
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100) ; z <- rnorm(100)
A <- cbind(x, y, z)
NNS.ANOVA(A)
```

NNS.ARMA

NNS ARMA

Description

Autoregressive model incorporating nonlinear regressions of component series.

Usage

```
NNS.ARMA(variable, h = 1, training.set = NULL,
  seasonal.factor = TRUE, weights = NULL, best.periods = 2,
  negative.values = FALSE, method = "nonlin", dynamic = FALSE,
  plot = TRUE, seasonal.plot = TRUE, intervals = FALSE,
  ncores = NULL)
```

Arguments

<code>variable</code>	a numeric vector.
<code>h</code>	integer; 1 (default) Number of periods to forecast.
<code>training.set</code>	numeric; NULL (default) Sets the number of variable observations (<code>variable[1 : training.set]</code>) to monitor performance of forecast over in-sample range.
<code>seasonal.factor</code>	logical or integer(s); TRUE (default) Automatically selects the best seasonal lag from the seasonality test. To use weighted average of all seasonal lags set to (<code>seasonal.factor = FALSE</code>). Otherwise, directly input known frequency integer lag to use, i.e. (<code>seasonal.factor = 12</code>) for monthly data. Multiple frequency integers can also be used, i.e. (<code>seasonal.factor = c(12, 24, 36)</code>)

<code>weights</code>	numeric; NULL (default) sets the weights of the <code>seasonal.factor</code> vector when specified as integers. If (<code>weights = NULL</code>) each <code>seasonal.factor</code> is weighted on its NNS.seas result and number of observations it contains.
<code>best.periods</code>	integer; [2] (default) used in conjunction with (<code>seasonal.factor = FALSE</code>), uses the <code>best.periods</code> number of detected seasonal lags instead of ALL lags when (<code>seasonal.factor = FALSE</code>).
<code>negative.values</code>	logical; FALSE (default) If the variable can be negative, set to (<code>negative.values = TRUE</code>). If there are negative values within the variable, <code>negative.values</code> will automatically be detected.
<code>method</code>	options: ("lin", "nonlin", "both"); "nonlin" (default) To select the regression type of the component series, select (<code>method = "both"</code>) where both linear and nonlinear estimates are generated. To use a nonlinear regression, set to (<code>method = "nonlin"</code>); to use a linear regression set to (<code>method = "lin"</code>).
<code>dynamic</code>	logical; FALSE (default) To update the seasonal factor with each forecast point, set to (<code>dynamic = TRUE</code>). The default is (<code>dynamic = FALSE</code>) to retain the original seasonal factor from the inputted variable for all ensuing h.
<code>plot</code>	logical; TRUE (default) Returns the plot of all periods exhibiting seasonality and the variable level reference in upper panel. Lower panel returns original data and forecast.
<code>seasonal.plot</code>	logical; TRUE (default) Adds the seasonality plot above the forecast. Will be set to FALSE if no seasonality is detected or <code>seasonal.factor</code> is set to an integer value.
<code>intervals</code>	logical; FALSE (default) Plots the surrounding forecasts around the final estimate when (<code>intervals = TRUE</code>) and (<code>seasonal.factor = FALSE</code>). There are no other forecasts to plot when a single <code>seasonal.factor</code> is selected.
<code>ncores</code>	integer; value specifying the number of cores to be used in the parallelized procedure. If NULL (default), the number of cores to be used is equal to half the number of cores of the machine - 1.

Value

Returns a vector of forecasts of length (h).

Note

For monthly data series, increased accuracy may be realized from forcing seasonal factors to multiples of 12. For example, if the best periods reported are: {37, 47, 71, 73} use (`seasonal.factor = c(36, 48, 72)`). (`seasonal.factor = FALSE`) can be a very computationally expensive exercise due to the number of seasonal periods detected.

If error encountered when (`seasonal.factor = TRUE`):

"NaNs produced Error in seq.default(length(variable)+1, 1, -lag[i]) : wrong sign in 'by' argument"
use the combination of (`seasonal.factor = FALSE`, `best.periods = 1`).

Author(s)

Fred Viole, OVVO Financial Systems

References

- Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" <https://www.amazon.com/dp/1490523995>
- Viole, F. (2019) "Forecasting Using NNS" <https://ssrn.com/abstract=3382300>

Examples

```
## Nonlinear NNS.ARMA using AirPassengers monthly data and 12 period lag
## Not run:
NNS.ARMA(AirPassengers, h = 45, training.set = 100, seasonal.factor = 12, method = "nonlin")

## Linear NNS.ARMA using AirPassengers monthly data and 12, 24, and 36 period lags
NNS.ARMA(AirPassengers, h = 45, training.set = 120, seasonal.factor = c(12, 24, 36), method = "lin")

## Nonlinear NNS.ARMA using AirPassengers monthly data and 2 best periods lag
NNS.ARMA(AirPassengers, h = 45, training.set = 120, seasonal.factor = FALSE, best.periods = 2)
## End(Not run)
```

NNS.ARMA.optim	<i>NNS ARMA Optimizer</i>
----------------	---------------------------

Description

Wrapper function for optimizing any combination of a given seasonal.factor vector in [NNS.ARMA](#). Minimum sum of squared errors (forecast-actual) is used to determine optimum across all [NNS.ARMA](#) methods.

Usage

```
NNS.ARMA.optim(variable, training.set, seasonal.factor,
  negative.values = FALSE, obj.fn = expression(sum((predicted -
    actual)^2)), objective = "min", linear.approximation = TRUE,
  print.trace = TRUE, ncores = NULL, subcores = NULL)
```

Arguments

variable	a numeric vector.
training.set	numeric; NULL (default) Sets the number of variable observations
seasonal.factor	integers; Multiple frequency integers considered for NNS.ARMA model, i.e. (seasonal.factor = c(12, 24, 36))
negative.values	logical; FALSE (default) If the variable can be negative, set to (negative.values = TRUE).
obj.fn	expression; expression(sum((predicted - actual)^2)) (default) Sum of squared errors is the default objective function. Any expression() using the specific terms predicted and actual can be used.
objective	options: ("min", "max") "min" (default) Select whether to minimize or maximize the objective function obj.fn.

<code>linear.approximation</code>	logical; TRUE (default) Uses the best linear output from <code>NNS.reg</code> to generate a nonlinear and mixture regression for comparison. FALSE is a more exhaustive search over the objective space.
<code>print.trace</code>	logical; TRUE (default) Prints current iteration information. Suggested as backup in case of error, best parameters to that point still known and copyable!
<code>ncores</code>	integer; value specifying the number of cores to be used in the parallelized procedure. If NULL (default), the number of cores to be used is equal to half the number of cores of the machine.
<code>subcores</code>	integer; value specifying the number of cores to be used in the parallelized procedure in the subroutine <code>NNS.ARMA</code> . If NULL (default), the number of cores to be used is equal to half the number of cores of the machine - 1.

Value

Returns a list containing:

- `$period` a vector of optimal seasonal periods
- `$weights` the optimal weights of each seasonal period between an equal weight or NULL weighting
- `$obj.fn` the objective function value
- `$method` the method identifying which `NNS.ARMA` method was used.
- `$bias.shift` a numerical result of the overall bias of the optimum objective function result. To be added to the final result when using the `NNS.ARMA` with the derived parameters.

Note

- The number of combinations will grow prohibitively large, they should be kept as small as possible. `seasonal.factor` containing an element too large will result in an error. Please reduce the maximum `seasonal.factor`.
- If variable cannot logically assume negative values, then the `$bias.shift` must be limited to 0 via a `pmax(0, ...)` call.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
## Nonlinear NNS.ARMA period optimization using 2 yearly lags on AirPassengers monthly data
## Not run:
nns.optims <- NNS.ARMA.optim(AirPassengers[1:132], training.set = 120,
seasonal.factor = seq(12, 24, 6))

## Then use optimal parameters in NNS.ARMA to predict 12 periods in-sample.
## Note the {$bias.shift} usage in the {NNS.ARMA} function:
```

```

nns.estimates <- NNS.ARMA(AirPassengers, h = 12, training.set = 132,
seasonal.factor = nns.optims$periods, method = nns.optims$method) + nns.optims$bias.shift

## If variable cannot logically assume negative values
nns.estimates <- pmax(0, nns.estimates)

## End(Not run)

```

NNS.boost

NNS Boost

Description

Ensemble method for classification using the predictions of the NNS multivariate regression [NNS.reg](#) collected from uncorrelated feature combinations.

Usage

```

NNS.boost(IVs.train, DV.train, IVs.test, type = "CLASS",
  representative.sample = FALSE, depth = "max", n.best = NULL,
  learner.trials = NULL, epochs = NULL, CV.size = 0.25,
  ts.test = NULL, folds = 5, threshold = NULL,
  obj.fn = expression(mean(round(predicted) == as.numeric(actual))),
  objective = "max", extreme = FALSE, feature.importance = TRUE,
  status = TRUE, ncores = NULL)

```

Arguments

IVs.train	a matrix or data frame of variables of numeric or factor data types.
DV.train	a numeric or factor vector with compatible dimensions to (IVs.train).
IVs.test	a matrix or data frame of variables of numeric or factor data types with compatible dimensions to (IVs.train).
type	"CLASS" (default). To perform a classification, set to (type = "CLASS"), else for continuous DV set to (type = NULL).
representative.sample	logical; FALSE (default) Reduces observations of IVs.train to a set of representative observations per regressor.
depth	options: (integer, NULL, "max"); "max" (default) Specifies the order parameter in the NNS.reg routine, assigning a number of splits in the regressors. (depth = "max") will be significantly faster, but increase the variance of results.
n.best	integer; NULL (default) Sets the number of nearest regression points to use in weighting for multivariate regression at sqrt(# of regressors). Analogous to k in a k Nearest Neighbors algorithm. If NULL, determines the optimal clusters via the NNS.stack procedure.
learner.trials	integer; NULL (default) Sets the number of trials to obtain an accuracy threshold level. Number of observations in the training set is the default setting.
epochs	integer; 2*length(DV.train) (default) Total number of feature combinations to run.

CV.size	numeric [0, 1]; (CV.size = .25) (default) Sets the cross-validation size. Defaults to 0.25 for a 25 percent random sampling of the training set.
ts.test	integer; NULL (default) Sets the length of the test set for time-series data; typically 2*h parameter value from NNS.ARMA or double known periods to forecast.
folds	integer; 5 (default) Sets the number of folds in the NNS.stack procedure for optimal n.best parameter.
threshold	numeric; NULL (default) Sets the obj.fn threshold to keep feature combinations.
obj.fn	expression; expression(mean(round(predicted)==as.numeric(actual))) (default) Mean accuracy is the default objective function. Any expression() using the specific terms predicted and actual can be used.
objective	options: ("min", "max") "max" (default) Select whether to minimize or maximize the objective function obj.fn.
extreme	logical; FALSE (default) Uses the maximum (minimum) threshold obtained from the learner.trials, rather than the upper (lower) quintile level for maximization (minimization) objective.
feature.importance	logical; TRUE (default) Plots the frequency of features used in the final estimate.
status	logical; TRUE (default) Prints status update message in console.
ncores	integer; value specifying the number of cores to be used in the parallelized procedure. If NULL (default), the number of cores to be used is equal to the number of cores of the machine - 1.

Value

Returns a vector of fitted values for the dependent variable test set \$results, and the final feature loadings \$feature.weights.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. (2016) "Classification Using NNS Clustering Analysis" <https://ssrn.com/abstract=2864711>

Examples

```
## Using 'iris' dataset where test set [IVs.test] is 'iris' rows 141:150.
## Not run:
a <- NNS.boost(iris[1:140, 1:4], iris[1:140, 5],
IVs.test = iris[141:150, 1:4],
epochs = 100, learner.trials = 100)

## Test accuracy
mean(round(a$results) == as.numeric(iris[141:150, 5]))

## End(Not run)
```

NNS.caus

*NNS Causation***Description**

Returns the causality from observational data between two variables.

Usage

```
NNS.caus(x, y, factor.2.dummy = FALSE, tau = 0, plot = FALSE)
```

Arguments

x	a numeric vector, matrix or data frame.
y	NULL (default) or a numeric vector with compatible dimensions to x.
factor.2.dummy	logical; FALSE (default) Automatically augments variable matrix with numerical dummy variables based on the levels of factors. Includes dependent variable y.
tau	options: ("cs", "ts", integer); 0 (default) Number of lagged observations to consider (for time series data). Otherwise, set (tau = "cs") for cross-sectional data. (tau = "ts") automatically selects the lag of the time series data, while (tau = [integer]) specifies a time series lag.
plot	logical; FALSE (default) Plots the raw variables, tau normalized, and cross-normalized variables.

Value

Returns the directional causation ($x \rightarrow y$) or ($y \rightarrow x$) and net quantity of association. For causal matrix, directional causation is returned as ([column variable] \rightarrow [row variable]). Negative numbers represent causal direction attributed to [row variable].

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
## Not run:
## x causes y...
set.seed(123)
x <- rnorm(1000) ; y <- x ^ 2
NNS.caus(x, y, tau = "cs")

## AirPassengers
NNS.caus(1:length(AirPassengers), AirPassengers, tau = "ts")

## Causal matrix without per factor causation
```

```

NNS.caus(iris, tau = 0)

## Causal matrix with per factor causation
NNS.caus(iris, factor.2.dummy = TRUE, tau = 0)

## End(Not run)

```

NNS.cor

*NNS Correlation***Description**

Returns the nonlinear correlation between two variables based on higher order partial moment matrices measured by frequency or area.

Usage

```
NNS.cor(x, y = NULL, order = NULL, degree = NULL)
```

Arguments

x	a numeric vector, matrix or data frame.
y	NULL (default) or a numeric vector with compatible dimensions to x.
order	integer; Controls the level of quadrant partitioning. Defaults to (order = NULL). Errors can generally be rectified by setting (order = 1).
degree	integer; (degree = 0) is frequency based correlations, while (degree = 1) is for area based correlations. Defaults to (degree = 0) for smaller number of observations.

Value

Returns nonlinear correlation coefficient between two variables, or nonlinear correlation matrix for matrix input.

Note

p-values and confidence intervals can be obtained from sampling random permutations of y_p and running `NNS.dep(x, y_p)` to compare against a null hypothesis of 0 correlation or independence between x, y .

See [NNS.dep](#) for examples.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
## Not run:
set.seed(123)
## Pairwise Correlation
x <- rnorm(100) ; y <- rnorm(100)
NNS.cor(x, y)

## Correlation Matrix
x <- rnorm(100) ; y <- rnorm(100) ; z<-rnorm(100)
B <- cbind(x, y, z)
NNS.cor(B)
## End(Not run)
```

NNS.dep

*NNS Dependence***Description**

Returns the dependence and nonlinear correlation between two variables based on higher order partial moment matrices measured by frequency or area.

Usage

```
NNS.dep(x, y = NULL, order = NULL, degree = NULL,
        print.map = FALSE, ncores = NULL)
```

Arguments

x	a numeric vector, matrix or data frame.
y	NULL (default) or a numeric vector with compatible dimensions to x.
order	integer; Controls the level of quadrant partitioning. Defaults to (order = NULL). Errors can generally be rectified by setting (order = 1). Will not partition further if less than 4 observations exist in a quadrant.
degree	integer; Defaults to NULL to allow number of observations to be "degree" determinant.
print.map	logical; FALSE (default) Plots quadrant means.
ncores	integer; value specifying the number of cores to be used in the parallelized procedure. If NULL (default), the number of cores to be used is equal to the number of cores of the machine - 1.

Value

Returns the bi-variate "Correlation" and "Dependence" or correlation / dependence matrix for matrix input.

Note

p-values and confidence intervals can be obtained from sampling random permutations of y_p and running NNS.dep(x, y_p) to compare against a null hypothesis of 0 correlation or independence between x, y.

Author(s)

Fred Violo, OVVO Financial Systems

References

Violo, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
## Not run:
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.dep(x, y)

## Correlation / Dependence Matrix
x <- rnorm(100) ; y <- rnorm(100) ; z <- rnorm(100)
B <- cbind(x, y, z)
NNS.dep(B)

## p-values for [NNS.dep]
x <- seq(-5, 5, .1); y <- x^2 + rnorm(length(x))

nns_cor_dep <- NNS.dep(x, y, print.map = TRUE)
nns_cor_dep

## Create permutations of y
y_p <- replicate(1000, sample.int(length(y)))

## Generate new correlation and dependence measures on each new permutation of y
nns.mc <- apply(y_p, 2, function(g) NNS.dep(x, y[g]))

## Store results
cors <- unlist(lapply(nns.mc, "[", 1))
deps <- unlist(lapply(nns.mc, "[", 2))

## View results
hist(cors)
hist(deps)

## Left tailed correlation p-value
cor_p_value <- LPM(0, nns_cor_dep$Correlation, cors)
cor_p_value

## Right tailed correlation p-value
cor_p_value <- UPM(0, nns_cor_dep$Correlation, cors)
cor_p_value

## Confidence Intervals
## For 95th percentile VaR (both-tails) see [LPM.VaR] and [UPM.VaR]
## Lower CI
LPM.VaR(.975, 0, cors)
## Upper CI
UPM.VaR(.975, 0, cors)
```

```
## Left tailed dependence p-value
dep_p_value <- LPM(0, nns_cor_dep$Dependence, deps)
dep_p_value

## Right tailed dependence p-value
dep_p_value <- UPM(0, nns_cor_dep$Dependence, deps)
dep_p_value

## End(Not run)
```

NNS.dep.base	<i>NNS Dependence Base</i>
--------------	----------------------------

Description

Internal function for NNS dependence [NNS.dep](#) parallel instances.

Usage

```
NNS.dep.base(x, y = NULL, order = NULL, degree = NULL,
  print.map = FALSE)
```

Arguments

x	from NNS.part
y	from NNS.part
order	from NNS.part
degree	from NNS.part
print.map	from NNS.part

Value

Returns NNS dependence.

NNS.dep.hd	<i>NNS Co-Partial Moments Higher Dimension Dependence</i>
------------	---

Description

Determines higher dimension dependence coefficients based on degree 0 co-partial moments.

Usage

```
NNS.dep.hd(x, plot = FALSE, independence.overlay = FALSE)
```

Arguments

- `x` a numeric matrix or data frame.
- `plot` logical; FALSE (default) Generates a 3d scatter plot with regression points using [plot3d](#).
- `independence.overlay` logical; FALSE (default) Creates and overlays independent [Co.LPM](#) and [Co.UPM](#) regions to visually reference the difference in dependence from the data.frame of variables being analyzed. Under independence, the light green and red shaded areas would be occupied by green and red data points respectively.

Value

- `$actual.observations` Number of [Co.LPM](#) and [Co.UPM](#) observations.
- `$independent.null` Expected number of [Co.LPM](#) and [Co.UPM](#) observations under the null hypothesis of independence.
- `$Dependence` Multivariate nonlinear dependence coefficient [0,1]

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. (2016) "Beyond Correlation: Using the Elements of Variance for Conditional Means and Probabilities" <http://ssrn.com/abstract=2745308>.

Examples

```
set.seed(123)
x <- rnorm(1000) ; y <- rnorm(1000) ; z <- rnorm(1000)
A <- data.frame(x, y, z)
NNS.dep.hd(A, plot = TRUE, independence.overlay = TRUE)
```

NNS.diff

NNS Numerical Differentiation

Description

Determines numerical derivative of a given function using projected secant lines on the y-axis. These projected points infer finite steps h , in the finite step method.

Usage

```
NNS.diff(f, point, h = 0.1, tol = 1e-10, print.trace = FALSE)
```

Arguments

f	an expression or call or a formula with no lhs.
point	numeric; Point to be evaluated for derivative of a given function f.
h	numeric [0, ...]; Initial step for secant projection. Defaults to (h = 0.1).
tol	numeric; Sets the tolerance for the stopping condition of the inferred h. Defaults to (tol = 1e-10).
print.trace	logical; FALSE (default) Displays each iteration, lower y-intercept, upper y-intercept and inferred h.

Value

Returns a matrix of values, intercepts, derivatives, inferred step sizes for multiple methods of estimation.

Author(s)

Fred Violen, OVVO Financial Systems

References

Violen, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" <https://www.amazon.com/dp/1490523995>

Examples

```
f <- function(x) sin(x) / x
NNS.diff(f, 4.1)
```

NNS.distance	<i>NNS Distance</i>
--------------	---------------------

Description

Internal function for NNS multivariate regression [NNS.reg](#) parallel instances.

Usage

```
NNS.distance(rpm, dist.estimate, type, k)
```

Arguments

rpm	REGRESSION.POINT.MATRIX from NNS.reg
dist.estimate	Vector to generate distances from.
type	"L1", "L2" or "DTW"
k	n.best from NNS.reg

Value

Returns sum of weighted distances.

NNS.FSD	<i>NNS FSD Test</i>
---------	---------------------

Description

Bi-directional test of first degree stochastic dominance using lower partial moments.

Usage

```
NNS.FSD(x, y, type = "discrete")
```

Arguments

x	a numeric vector.
y	a numeric vector.
type	options: ("discrete", "continuous"); "discrete" (default) selects the type of CDF.

Value

Returns one of the following FSD results: "X FSD Y", "Y FSD X", or "NO FSD EXISTS".

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. <http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817>.

Viole, F. (2017) "A Note on Stochastic Dominance." <https://ssrn.com/abstract=3002675>.

Examples

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.FSD(x, y)
```

NNS.FSD.uni

NNS FSD Test uni-directional

Description

Uni-directional test of first degree stochastic dominance using lower partial moments used in SD Efficient Set routine.

Usage

```
NNS.FSD.uni(x, y, type = "discrete")
```

Arguments

x	a numeric vector.
y	a numeric vector.
type	options: ("discrete", "continuous"); "discrete" (default) selects the type of CDF.

Value

Returns (1) if "X FSD Y", else (0).

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. <http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817>.

Viole, F. (2017) "A Note on Stochastic Dominance." <https://ssrn.com/abstract=3002675>.

Examples

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.FSD.uni(x, y)
```

NNS.normNNS Normalization

Description

Normalizes a matrix of variables based on nonlinear scaling normalization method.

Usage

```
NNS.norm(A, linear = FALSE, chart.type = NULL, location = "topleft")
```

Arguments

A	a numeric matrix or data frame.
linear	logical; FALSE (default) Performs a linear scaling normalization, resulting in equal means for all variables.
chart.type	options: ("l", "b"); NULL (default). Set (chart.type = "l") for line, (chart.type = "b") for boxplot.
location	Sets the legend location within the plot, per the x and y co-ordinates used in base graphics legend .

Value

Returns a [data.frame](#) of normalized values.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
set.seed(123)
x <- rnorm(100) ; y<-rnorm(100)
A <- cbind(x, y)
NNS.norm(A)
```

NNS.part

*NNS Partition Map***Description**

Creates partitions based on partial moment quadrant means, iteratively assigning identifications to observations based on those quadrants (unsupervised partitional and hierarchial clustering method). Basis for correlation [NNS.cor](#), dependence [NNS.dep](#), regression [NNS.reg](#) routines.

Usage

```
NNS.part(x, y, Voronoi = FALSE, type = NULL, order = NULL,
  obs.req = 8, min.obs.stop = FALSE, noise.reduction = "mean")
```

Arguments

x	a numeric vector.
y	a numeric vector with compatible dimensions to x.
Voronoi	logical; FALSE (default) Displays a Voronoi type diagram using partial moment quadrants.
type	NULL (default) Controls the partitioning basis. Set to (type = "XONLY") for X-axis based partitioning. Defaults to NULL for both X and Y-axis partitioning.
order	integer; Number of partial moment quadrants to be generated. (order = "max") will institute a perfect fit.
obs.req	integer; (8 default) Required observations per cluster where quadrants will not be further partitioned if observations are not greater than the entered value. Reduces minimum number of necessary observations in a quadrant to 1 when (obs.req = 1).
min.obs.stop	logical; FALSE (default) Stopping condition where quadrants will not be further partitioned if a single cluster contains less than the entered value of obs.req.
noise.reduction	the method of determing regression points options: ("mean", "median", "mode", "off"); (noise.reduction = "median") uses medians instead of means for partitions, while (noise.reduction = "mode") uses modes instead of means for partitions. Defaults to (noise.reduction = "mean"), while (noise.reduction = "off") will partition quadrant to a single observation for a given (order = ...).

Value

Returns:

- "dt" a [data.table](#) of x and y observations with their partition assignment "quadrant" in the 3rd column and their prior partition assignment "prior.quadrant" in the 4th column.
- "regression.points" the [data.table](#) of regression points for that given (order = ...).
- "order" the order of the final partition given "min.obs.stop" stopping condition.

Author(s)

Fred Viole, OVVO Financial Systems

References

Violo, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.part(x, y)

## Data.table of observations and partitions
NNS.part(x, y, order = 1)$dt

## Regression points
NNS.part(x, y, order = 1)$regression.points

## Voronoi style plot
NNS.part(x, y, Voronoi = TRUE)

## Examine final counts by quadrant
DT <- NNS.part(x, y)$dt
DT[, counts := .N, by = quadrant]
DT
```

NNS.PDF

NNS PDF

Description

This function generates an empirical PDF using continuous CDFs from [LPM.ratio](#).

Usage

```
NNS.PDF(variable, degree = 1, target = NULL, bins = NULL,
        plot = TRUE)
```

Arguments

variable	a numeric vector.
degree	integer; (degree = 0) is frequency, (degree = 1) (default) is area.
target	a numeric range of values [a,b] where a < b. NULL (default) uses the variable observations.
bins	numeric; NULL (default) Selects number of observations as default bins.
plot	logical; plots PDF.

Value

Returns a data.table containing the intervals used and resulting PDF of the variable.

Author(s)

Fred Violo, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
set.seed(123)
x <- rnorm(100)
NNS.PDF(x)

## Custom target range
NNS.PDF(x, target = c(-5, 5))
```

NNS.reg

NNS Regression

Description

Generates a nonlinear regression based on partial moment quadrant means.

Usage

```
NNS.reg(x, y, factor.2.dummy = TRUE, order = NULL, stn = 0.96,
  dim.red.method = NULL, tau = NULL, type = NULL, point.est = NULL,
  location = "top", return.values = TRUE, plot = TRUE,
  plot.regions = FALSE, residual.plot = TRUE, std.errors = FALSE,
  confidence.interval = NULL, threshold = 0, n.best = NULL,
  noise.reduction = "mean", dist = "L2", ncores = NULL,
  multivariate.call = FALSE)
```

Arguments

x	a vector, matrix or data frame of variables of numeric or factor data types.
y	a numeric or factor vector with compatible dimensions to x.
factor.2.dummy	logical; TRUE (default) Automatically augments variable matrix with numerical dummy variables based on the levels of factors.
order	integer; Controls the number of partial moment quadrant means. Users are encouraged to try different (order = ...) integer settings with (noise.reduction = "off"). (order = "max") will force a limit condition perfect fit.
stn	numeric [0, 1]; Signal to noise parameter, sets the threshold of (NNS.dep) which reduces ("order") when (order = NULL). Defaults to 0.96 to ensure high dependence for higher ("order") and endpoint determination.
dim.red.method	options: ("cor", "NNS.dep", "NNS.caus", "all", NULL) method for determining synthetic X* coefficients. Selection of a method automatically engages the dimension reduction regression. The default is NULL for full multivariate regression. (dim.red.method = "NNS.dep") uses NNS.dep for nonlinear dependence weights, while (dim.red.method = "NNS.caus") uses NNS.caus for causal weights. (dim.red.method = "cor") uses standard linear correlation for weights. (dim.red.method = "all") averages all methods for further feature engineering.

tau	options("ts", NULL); NULL (default) To be used in conjunction with (dim.red.method = "NNS.caus") or (dim.red.method = "all"). If the regression is using time-series data, set (tau = "ts") for more accurate causal analysis.
type	NULL (default). To perform a classification, set to (type = "CLASS").
point.est	a numeric or factor vector with compatible dimensions to x. Returns the fitted value \hat{y} for any value of x.
location	Sets the legend location within the plot, per the x and y co-ordinates used in base graphics legend .
return.values	logical; TRUE (default), set to FALSE in order to only display a regression plot and call values as needed.
plot	logical; TRUE (default) To plot regression.
plot.regions	logical; FALSE (default). Generates 3d regions associated with each regression point for multivariate regressions. Note, adds significant time to routine.
residual.plot	logical; TRUE (default) To plot \hat{y} and Y.
std.errors	logical; FALSE (default) To provide standard errors of each linear segment in the "Fitted.xy" output.
confidence.interval	numeric [0, 1]; NULL (default) Plots the associated confidence interval with the estimate and reports the standard error for each individual segment.
threshold	numeric [0, 1]; (threshold = 0) (default) Sets the threshold for dimension reduction of independent variables when (dim.red.method) is not NULL.
n.best	integer; NULL (default) Sets the number of nearest regression points to use in weighting for multivariate regression at $\sqrt{\text{\# of regressors}}$. (n.best = "all") will select and weight all generated regression points. Analogous to k in a k Nearest Neighbors algorithm. Different values of n.best are tested using cross-validation in NNS.stack .
noise.reduction	the method of determining regression points options: ("mean", "median", "mode", "off"); In low signal:noise situations, (noise.reduction = "mean") uses means for NNS.dep restricted partitions, (noise.reduction = "median") uses medians instead of means for NNS.dep restricted partitions, while (noise.reduction = "mode") uses modes instead of means for NNS.dep restricted partitions. (noise.reduction = "off") allows for maximum possible fit with a specific order.
dist	options:("L1", "L2", "DTW", "FACTOR") the method of distance calculation; Selects the distance calculation used. dist = "L2" (default) selects the Euclidean distance and (dist = "L1") selects the Manhattan distance; (dist = "DTW") selects the dynamic time warping distance; (dist = "FACTOR") uses a frequency.
ncores	integer; value specifying the number of cores to be used in the parallelized procedure. If NULL (default), the number of cores to be used is equal to the number of cores of the machine - 1.
multivariate.call	Internal parameter for multivariate regressions.

Value

UNIVARIATE REGRESSION RETURNS THE FOLLOWING VALUES:

- "R2" provides the goodness of fit;

- "SE" returns the overall standard error of the estimate between y and \hat{y} ;
- "Prediction.Accuracy" returns the correct rounded "Point.est" used in classifications versus the categorical y ;
- "derivative" for the coefficient of the x and its applicable range;
- "Point.est" for the predicted value generated;
- "regression.points" provides the points used in the regression equation for the given order of partitions;
- "Fitted.xy" returns a [data.table](#) of x , y , \hat{y} , resid, NNS.ID, gradient;

MULTIVARIATE REGRESSION RETURNS THE FOLLOWING VALUES:

- "R2" provides the goodness of fit;
- "equation" returns the numerator of the synthetic X^* dimension reduction equation as a [data.table](#) consisting of regressor and its coefficient. Denominator is simply the length of all coefficients > 0 , returned in last row of equation [data.table](#).
- " $x.star$ " returns the synthetic X^* as a vector;
- "rhs.partitions" returns the partition points for each regressor x ;
- "RPM" provides the Regression Point Matrix, the points for each x used in the regression equation for the given order of partitions;
- "Point.est" returns the predicted value generated;
- "Fitted.xy" returns a [data.table](#) of x , y , \hat{y} , gradient, and NNS.ID.

Note

Please ensure `point.est` is of compatible dimensions to x , error message will ensue if not compatible.

Author(s)

Fred Viole, OVVO Financial Systems

References

- Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" <https://www.amazon.com/dp/1490523995>
- Vinod, H. and Viole, F. (2017) "Nonparametric Regression Using Clusters" <https://link.springer.com/article/10.1007/s10614-017-9713-5>
- Vinod, H. and Viole, F. (2018) "Clustering and Curve Fitting by Line Segments" <https://www.preprints.org/manuscript/201801.0090/v1>

Examples

```
## Not run:
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.reg(x, y)

## Manual {order} selection
NNS.reg(x, y, order = 2)

## Maximum {order} selection
```



```

NNS.reg(x, y, order = "max")

## x-only paritioning (Univariate only)
NNS.reg(x, y, type = "XONLY")

## For Multiple Regression:
x <- cbind(rnorm(100), rnorm(100), rnorm(100)) ; y <- rnorm(100)
NNS.reg(x, y, point.est = c(.25, .5, .75))

## For Multiple Regression based on Synthetic X* (Dimension Reduction):
x <- cbind(rnorm(100), rnorm(100), rnorm(100)) ; y <- rnorm(100)
NNS.reg(x, y, point.est = c(.25, .5, .75), dim.red.method = "cor")

## IRIS dataset examples:
# Dimension Reduction:
NNS.reg(iris[,1:4], iris[,5], dim.red.method = "cor", order = 5)

# Dimension Reduction using causal weights:
NNS.reg(iris[,1:4], iris[,5], dim.red.method = "NNS.caus", order = 5)

# Multiple Regression:
NNS.reg(iris[,1:4], iris[,5], order = 2, noise.reduction = "off")

# Classification:
NNS.reg(iris[,1:4], iris[,5], point.est = iris[1:10, 1:4], type = "CLASS")$Point.est

## To call fitted values:
x <- rnorm(100) ; y <- rnorm(100)
NNS.reg(x, y)$Fitted

## To call partial derivative (univariate regression only):
NNS.reg(x, y)$derivative
## End(Not run)

```

NNS.SD.efficient.set *NNS SD Efficient Set*

Description

Determines the set of stochastic dominant variables for various degrees.

Usage

```
NNS.SD.efficient.set(x, degree, type = "discrete")
```

Arguments

x	a numeric matrix or data frame.
degree	numeric options: (1, 2, 3); Degree of stochastic dominance test from (1, 2 or 3).
type	options: ("discrete", "continuous"); "discrete" (default) selects the type of CDF.

Value

Returns set of stochastic dominant variable names.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. <http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817>.

Viole, F. (2017) "A Note on Stochastic Dominance." <https://ssrn.com/abstract=3002675>.

Examples

```
set.seed(123)
x <- rnorm(100) ; y<-rnorm(100) ; z<-rnorm(100)
A <- cbind(x, y, z)
NNS.SD.efficient.set(A, 1)
```

NNS.seas	<i>NNS Seasonality Test</i>
----------	-----------------------------

Description

Seasonality test based on the coefficient of variation for the variable and lagged component series.
A result of 1 signifies no seasonality present.

Usage

```
NNS.seas(variable, modulo = NULL, mod.only = TRUE, plot = TRUE)
```

Arguments

variable	a numeric vector.
modulo	integer(s); NULL (default) Used to find the nearest multiple(s) in the reported seasonal period.
mod.only	logical; code TRUE (default) Limits the number of seasonal periods returned to the specified modulo.
plot	logical; TRUE (default) Returns the plot of all periods exhibiting seasonality and the variable level reference.

Value

Returns a matrix of all periods exhibiting less coefficient of variation than the variable with "all.periods"; and the single period exhibiting the least coefficient of variation versus the variable with "best.period"; as well as a vector of "periods" for easy call into [NNS.ARMA.optim](#). If no seasonality is detected, NNS.seas will return ("No Seasonality Detected").

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
set.seed(123)
x <- rnorm(100)

## To call strongest period based on coefficient of variation:
NNS.seas(x, plot = FALSE)$best.period

## Using modulus for logical seasonal inference:
NNS.seas(x, modulo = c(2,3,5,7), plot = FALSE)
```

NNS.SSD

NNS SSD Test

Description

Bi-directional test of second degree stochastic dominance using lower partial moments.

Usage

```
NNS.SSD(x, y)
```

Arguments

x	a numeric vector.
y	a numeric vector.

Value

Returns one of the following SSD results: "X SSD Y", "Y SSD X", or "NO SSD EXISTS".

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. <http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817>.

Examples

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.SSD(x, y)
```

NNS.SSD.uni

NNS SSD Test uni-directional

Description

Uni-directional test of second degree stochastic dominance using lower partial moments used in SD Efficient Set routine.

Usage

```
NNS.SSD.uni(x, y)
```

Arguments

x a numeric vector.
y a numeric vector.

Value

Returns (1) if "X SSD Y", else (0).

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. <http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817>.

Examples

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.SSD.uni(x, y)
```

NNS.stack

NNS Stack

Description

Prediction model using the predictions of the NNS base models [NNS.reg](#) as features (i.e. meta-features) for the stacked model.

Usage

```
NNS.stack(IVs.train, DV.train, IVs.test = NULL, type = NULL,
  obj.fn = expression(sum((predicted - actual)^2)), objective = "min",
  dist = "L2", CV.size = NULL, ts.test = NULL, folds = 5,
  order = NULL, norm = NULL, method = c(1, 2),
  dim.red.method = "cor", status = TRUE, ncores = NULL)
```

Arguments

<code>IVs.train</code>	a vector, matrix or data frame of variables of numeric or factor data types.
<code>DV.train</code>	a numeric or factor vector with compatible dimensions to (<code>IVs.train</code>).
<code>IVs.test</code>	a vector, matrix or data frame of variables of numeric or factor data types with compatible dimensions to (<code>IVs.train</code>).
<code>type</code>	NULL (default). To perform a classification, set to (<code>type = "CLASS"</code>).
<code>obj.fn</code>	expression; <code>expression(sum((predicted - actual)^2))</code> (default) Sum of squared errors is the default objective function. Any <code>expression()</code> using the specific terms <code>predicted</code> and <code>actual</code> can be used.
<code>objective</code>	options: ("min", "max") "min" (default) Select whether to minimize or maximize the objective function <code>obj.fn</code> .
<code>dist</code>	options: ("L1", "L2", "DTW", "FACTOR") the method of distance calculation; Selects the distance calculation used. <code>dist = "L2"</code> (default) selects the Euclidean distance and (<code>dist = "L1"</code>) selects the Manhattan distance; (<code>dist = "DTW"</code>) selects the dynamic time warping distance; (<code>dist = "FACTOR"</code>) uses a frequency.
<code>CV.size</code>	numeric [0, 1]; NULL (default) Sets the cross-validation size if (<code>IVs.test = NULL</code>). Defaults to 0.25 for a 25 percent random sampling of the training set under (<code>CV.size = NULL</code>).
<code>ts.test</code>	integer; NULL (default) Sets the length of the test set for time-series data; typically 2*h parameter value from NNS.ARMA or double known periods to forecast.
<code>folds</code>	integer; <code>folds = 5</code> (default) Select the number of cross-validation folds.
<code>order</code>	integer; NULL (default) Sets the order for NNS.reg , where (<code>order = "max"</code>) is the k-nearest neighbors equivalent.
<code>norm</code>	options: ("std", "NNS", NULL); NULL (default) 3 settings offered: NULL, "std", and "NNS". Selects the norm parameter in NNS.reg .
<code>method</code>	numeric options: (1, 2); Select the NNS method to include in stack. (<code>method = 1</code>) selects NNS.reg ; (<code>method = 2</code>) selects NNS.reg dimension reduction regression. Defaults to <code>method = c(1, 2)</code> , including both NNS regression methods in the stack.
<code>dim.red.method</code>	options: ("cor", "NNS.dep", "NNS.caus", "all") method for determining synthetic X* coefficients. (<code>dim.red.method = "cor"</code>) (default) uses standard linear correlation for weights. (<code>dim.red.method = "NNS.dep"</code>) uses NNS.dep for nonlinear dependence weights, while (<code>dim.red.method = "NNS.caus"</code>) uses NNS.caus for causal weights. (<code>dim.red.method = "all"</code>) averages all methods for further feature engineering.
<code>status</code>	logical; TRUE (default) Prints status update message in console.
<code>ncores</code>	integer; value specifying the number of cores to be used in the parallelized subroutine NNS.reg . If NULL (default), the number of cores to be used is equal to the number of cores of the machine - 1.

Value

Returns a vector of fitted values for the dependent variable test set for all models.

- "NNS.reg.n.best" returns the optimum "n.best" parameter for the [NNS.reg](#) multivariate regression. "SSE.reg" returns the SSE for the [NNS.reg](#) multivariate regression.

- "OBJfn.reg" returns the obj.fn for the [NNS.reg](#) regression.
- "NNS.dim.red.threshold" returns the optimum "threshold" from the [NNS.reg](#) dimension reduction regression.
- "OBJfn.dim.red" returns the obj.fn for the [NNS.reg](#) dimension reduction regression.
- "reg" returns [NNS.reg](#) output.
- "dim.red" returns [NNS.reg](#) dimension reduction regression output.
- "stack" returns the output of the stacked model.

Note

Missing data should be handled prior as well using [na.omit](#) or [complete.cases](#) on the full dataset.

If error received:

```
"Error in is.data.frame(x) : object 'RP' not found"
```

reduce the CV.size.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. (2016) "Classification Using NNS Clustering Analysis" <https://ssrn.com/abstract=2864711>

Examples

```
## Using 'iris' dataset where test set [IVs.test] is 'iris' rows 141:150.
## Not run:
NNS.stack(iris[1:140, 1:4], iris[1:140, 5], IVs.test = iris[141:150, 1:4], type = "CLASS")

## Using 'iris' dataset to determine [n.best] and [threshold] with no test set.
NNS.stack(iris[, 1:4], iris[, 5])

## Selecting NNS.reg and dimension reduction techniques.
NNS.stack(iris[1:140, 1:4], iris[1:140, 5], iris[141:150, 1:4], method = c(1, 2))
## End(Not run)
```

NNS.term.matrix

NNS Term Matrix

Description

Generates a term matrix for text classification use in [NNS.reg](#).

Usage

```
NNS.term.matrix(x, oos = NULL, names = FALSE)
```

Arguments

x	Text A two column dataset should be used. Concatenate text from original sources to comply with format. Also note the possibility of factors in "DV", so "as.numeric(as.character(...))" is used to avoid issues.
oos	Out-of-sample text dataset to be classified.
names	Column names for "IV" and "oos". Defaults to FALSE.

Value

Returns the text as independent variables "IV" and the classification as the dependent variable "DV". Out-of-sample independent variables are returned with "OOS".

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" <https://www.amazon.com/dp/1490523995>

Examples

```
x <- data.frame(cbind(c("sunny", "rainy"), c(1, -1)))
NNS.term.matrix(x)

### Concatenate Text with space separator, cbind with "DV"
x <- data.frame(cbind(c("sunny", "rainy"), c("windy", "cloudy"), c(1, -1)))
x <- data.frame(cbind(paste(x[, 1], x[, 2], sep = " "), as.numeric(as.character(x[, 3]))))
NNS.term.matrix(x)

### NYT Example
## Not run:
require(RTextTools)
data(NYTimes)

### Concatenate Columns 3 and 4 containing text, with column 5 as DV
NYT=data.frame(cbind(paste(NYTimes[, 3], NYT[, 4], sep = " "),
                        as.numeric(as.character(NYTimes[, 5]))))
NNS.term.matrix(NYT)
## End(Not run)
```

NNS.TSD

NNS TSD Test

Description

Bi-directional test of third degree stochastic dominance using lower partial moments.

Usage

```
NNS.TSD(x, y)
```

Arguments

x a numeric vector.
y a numeric vector.

Value

Returns one of the following TSD results: "X TSD Y", "Y TSD X", or "NO TSD EXISTS".

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. <http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817>.

Examples

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.TSD(x, y)
```

NNS.TSD.uni

NNS TSD Test uni-directional

Description

Uni-directional test of third degree stochastic dominance using lower partial moments used in SD Efficient Set routine.

Usage

```
NNS.TSD.uni(x, y)
```

Arguments

x a numeric vector.
y a numeric vector.

Value

Returns (1) if "X TSD Y", else (0).

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. <http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817>.

Examples

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.TSD.uni(x, y)
```

NNS.VAR	<i>NNS VAR</i>
---------	----------------

Description

Nonparametric vector autoregressive model incorporating [NNS.ARMA](#) estimates of variables into [NNS.reg](#) for a multi-variate time-series forecast.

Usage

```
NNS.VAR(variables, h, tau = 0, obj.fn = expression(sum((predicted -
  actual)^2)), objective = "min", epochs = 100, status = TRUE,
  ncores = NULL)
```

Arguments

variables	a numeric matrix or data.frame of contemporaneous time-series to forecast.
h	integer; 1 (default) Number of periods to forecast.
tau	integer; 0 (default) Number of lagged observations to consider for the time-series data.
obj.fn	expression; expression(sum((predicted - actual)^2)) (default) Sum of squared errors is the default objective function. Any expression() using the specific terms predicted and actual can be used.
objective	options: ("min", "max") "min" (default) Select whether to minimize or maximize the objective function obj.fn.
epochs	integer; 100 (default) Total number of feature combinations to run.
status	logical; TRUE (default) Prints status update message in console.
ncores	integer; value specifying the number of cores to be used in the parallelized subroutine NNS.ARMA.optim . If NULL (default), the number of cores to be used is equal to the number of cores of the machine - 1.

Value

Returns the following matrices of forecasted variables:

- "univariate" Returns the univariate [NNS.ARMA](#) forecasts.
- "multivariate" Returns the multi-variate [NNS.reg](#) forecasts.
- "ensemble" Returns the ensemble of both "univariate" and "multivariate" forecasts.

Author(s)

Fred Viole, OVVO Financial Systems

References

- Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" <https://www.amazon.com/dp/1490523995>
- Viole, F. (2019) "Multi-variate Time-Series Forecasting: Nonparametric Vector Autoregression Using NNS" <https://ssrn.com/abstract=3489550>
- Viole, F. (2019) "Forecasting Using NNS" <https://ssrn.com/abstract=3382300>
- Vinod, H. and Viole, F. (2017) "Nonparametric Regression Using Clusters" <https://link.springer.com/article/10.1007/s10614-017-9713-5>
- Vinod, H. and Viole, F. (2018) "Clustering and Curve Fitting by Line Segments" <https://www.preprints.org/manuscript/201801.0090/v1>

Examples

```
## Not run:
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100) ; z <- rnorm(100)
A <- cbind(x = x, y = y, z = z)
NNS.VAR(A, h = 12, tau = 4, status = TRUE)

## End(Not run)
```

PM.matrix

Partial Moment Matrix

Description

This function generates a co-partial moment matrix for the specified co-partial moment.

Usage

```
PM.matrix(LPM.degree, UPM.degree, target, variable, pop.adj = FALSE)
```

Arguments

- | | |
|------------|---|
| LPM.degree | integer; Degree for variable below target deviations. (degree = 0) is frequency, (degree = 1) is area. |
| UPM.degree | integer; Degree for variable above target deviations. (degree = 0) is frequency, (degree = 1) is area. |
| target | numeric; Typically the mean of Variable X for classical statistics equivalences, but does not have to be. (Vectorized) (target = "mean") will set the target as the mean of every variable. |
| variable | a numeric matrix or data.frame. |
| pop.adj | logical; FALSE (default) Adjusts the sample co-partial moment matrices for population statistics. |

Value

Matrix of partial moment quadrant values. Uncalled quadrants will return a matrix of zeros.

Note

For divergent asymmetrical "D.LPM" and "D.UPM" matrices, matrix is `D.LPM(column,row,...)`.

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" <https://www.amazon.com/dp/1490523995>

Viole, F. (2017) "Bayes' Theorem From Partial Moments" <https://ssrn.com/abstract=3457377>

Examples

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100) ; z <- rnorm(100)
A <- cbind(x,y,z)
PM.matrix(LPM.degree = 1, UPM.degree = 1, target = "mean", variable = A)

## Calling Individual Partial Moment Quadrants
cov.mtx <- PM.matrix(LPM.degree = 1, UPM.degree = 1, target = "mean", variable = A)
cov.mtx$cupm

## Full covariance matrix
cov.mtx$cov.matrix
```

UPM

Upper Partial Moment

Description

This function generates a univariate upper partial moment for any degree or target.

Usage

```
UPM(degree, target, variable)
```

Arguments

degree	integer; (degree = 0) is frequency, (degree = 1) is area.
target	numeric; Typically set to mean, but does not have to be. (Vectorized)
variable	a numeric vector.

Value

UPM of variable

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```
set.seed(123)
x <- rnorm(100)
UPM(0, mean(x), x)
```

UPM.ratio

Upper Partial Moment RATIO

Description

This function generates a standardized univariate upper partial moment for any degree or target.

Usage

```
UPM.ratio(degree, target, variable)
```

Arguments

degree	integer; (degree = 0) is frequency, (degree = 1) is area.
target	numeric; Typically set to mean, but does not have to be. (Vectorized)
variable	a numeric vector.

Value

Standardized UPM of variable

Author(s)

Fred Viole, OVVO Financial Systems

References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments"
<https://www.amazon.com/dp/1490523995>

Examples

```

set.seed(123)
x <- rnorm(100)
UPM.ratio(0, mean(x), x)

## Joint Upper CDF
## Not run:
x <- rnorm(5000) ; y <- rnorm(5000)
plot3d(x, y, Co.UPM(0, 0, sort(x), sort(y), x, y), col = "blue", xlab = "X", ylab = "Y",
zlab = "Probability", box = FALSE)

## End(Not run)

```

UPM.VaR	<i>UPM VaR</i>
---------	----------------

Description

Generates an upside value at risk (VaR) based on the Upper Partial Moment ratio

Usage

```
UPM.VaR(percentile, degree, x)
```

Arguments

percentile	numeric [0, 1]; The percentile for right-tail VaR.
degree	integer; (degree = 0) for discrete distributions, (degree = 1) for continuous distributions.
x	a numeric vector.

Value

Returns a numeric value representing the point at which "percentile" of the area of x is below.

Examples

```

set.seed(123)
x <- rnorm(100)

## For 95th percentile VaR (right-tail)
UPM.VaR(0.95, 0, x)

```

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