# Package 'NNS'

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Type Package

Title Nonlinear Nonparametric Statistics

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<b>Description</b> Nonlinear nonparametric statistics using partial moments. Partial moments are the elements of variance and asymptotically approximate the area of f(x). These robust statistics provide the basis for nonlinear analysis while retaining linear equivalences. NNS offers: Numerical integration, Numerical differentiation, Clustering, Correlation, Dependence, Causal analysis, ANOVA, Regression, Classification, Seasonality, Autoregressive modeling, Normalization and Stochastic dominance. All routines based on: Viole, F. and Nawrocki, D. (2013), Nonlinear Nonparametric Statistics: Using Partial Moments (ISBN: 1490523995).
License GPL-3
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Co.LPM

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# Description

Co.LPM

This function generates a co-lower partial moment for between two equal length variables for any degree or target.

Co-Lower Partial Moment (Lower Left Quadrant 4)

# Usage

```
Co.LPM(degree.x, degree.y, x, y, target.x = mean(x),
  target.y = mean(y))
```

# Arguments

```
degree.x integer; Degree for variable X. (degree.x = 0) is frequency, (degree.x = 1)
is area.

degree.y integer; Degree for variable Y. (degree.y = 0) is frequency, (degree.y = 1)
is area.

x a numeric vector.
y a numeric vector of equal length to x.
```

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target.x	numeric; Typically the mean of Variable X for classical statistics equivalences, but does not have to be. (Vectorized)
target.y	numeric; Typically the mean of Variable Y for classical statistics equivalences, but does not have to be. (Vectorized)

#### Value

Co-LPM of two variables

## Author(s)

Fred Viole, OVVO Financial Systems

#### References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

# **Examples**

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
Co.LPM(0, 0, x, y, mean(x), mean(y))</pre>
```

Co.UPM

Co-Upper Partial Moment (Upper Right Quadrant 1)

# **Description**

This function generates a co-upper partial moment between two equal length variables for any degree or target.

# Usage

```
Co.UPM(degree.x, degree.y, x, y, target.x = mean(x),
  target.y = mean(y))
```

# Arguments

degree.x	integer; Degree for variable X. (degree $x = 0$ ) is frequency, (degree $x = 1$ ) is area.
degree.y	integer; Degree for variable Y. (degree.y = $0$ ) is frequency, (degree.y = $1$ ) is area.
х	a numeric vector.
У	a numeric vector of equal length to x.
target.x	numeric; Typically the mean of Variable X for classical statistics equivalences, but does not have to be. (Vectorized)
target.y	numeric; Typically the mean of Variable Y for classical statistics equivalences, but does not have to be. (Vectorized)

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#### Value

Co-UPM of two variables

#### Author(s)

Fred Viole, OVVO Financial Systems

#### References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

# **Examples**

```
set.seed(123)
x<-rnorm(100) ; y<-rnorm(100)
Co.UPM(0,0,x,y,mean(x),mean(y))</pre>
```

D.LPM

Divergent-Lower Partial Moment (Lower Right Quadrant 3)

# Description

This function generates a divergent lower partial moment between two equal length variables for any degree or target.

# Usage

```
D.LPM(degree.x, degree.y, x, y, target.x = mean(x), target.y = mean(y))
```

#### **Arguments**

degree.x	integer; Degree for variable X. (degree $\cdot x = 0$ ) is frequency, (degree $\cdot x = 1$ ) is area.
degree.y	integer; Degree for variable Y. (degree.y = $0$ ) is frequency, (degree.y = $1$ ) is area.
x	a numeric vector.
У	a numeric vector of equal length to x.
target.x	numeric; Typically the mean of Variable X for classical statistics equivalences, but does not have to be. (Vectorized)
target.y	numeric; Typically the mean of Variable Y for classical statistics equivalences, but does not have to be. (Vectorized)

## Value

Divergent LPM of two variables

#### Author(s)

Fred Viole, OVVO Financial Systems

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#### References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

## **Examples**

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
D.LPM(0, 0, x, y, mean(x), mean(y))</pre>
```

D.UPM

Divergent-Upper Partial Moment (Upper Left Quadrant 2)

# **Description**

This function generates a divergent upper partial moment between two equal length variables for any degree or target.

## Usage

```
D.UPM(degree.x, degree.y, x, y, target.x = mean(x), target.y = mean(y))
```

#### **Arguments**

degree.x	integer; Degree for variable X. (degree.x = 0) is frequency, (degree.x = 1) is area.
degree.y	integer; Degree for variable Y. (degree.y = $\emptyset$ ) is frequency, (degree.y = 1) is area.
X	a numeric vector.
у	a numeric vector of equal length to x.
target.x	numeric; Typically the mean of Variable X for classical statistics equivalences, but does not have to be. (Vectorized)
target.y	numeric; Typically the mean of Variable Y for classical statistics equivalences, but does not have to be. (Vectorized)

# Value

Divergent UPM of two variables

## Author(s)

Fred Viole, OVVO Financial Systems

## References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
D.UPM(0, 0, x, y, mean(x), mean(y))</pre>
```

dy.dx

dy.dx Pa	ırtial Derivative dy/dx
----------	-------------------------

# Description

Returns the numerical partial derivate of y wrt x for a point of interest.

# Usage

```
dy.dx(x, y, order = NULL, stn = 0.99, eval.point = median(x),
  deriv.order = 1, h = 0.05, noise.reduction = "mean",
  deriv.method = "FS")
```

# Arguments

X	a numeric vector.
У	a numeric vector.
order	integer; Controls the number of partial moment quadrant means. Defaults to (order = NULL) which generates a more accurate derivative for well specified cases.
stn	numeric [0, 1]; Signal to noise parameter, sets the threshold of NNS.dep which reduces "order" when (order = NULL). Defaults to 0.99 to ensure high dependence for higher "order" and endpoint determination.
eval.point	numeric; x point to be evaluated. Defaults to (eval.point = median(x)). Set to (eval.point = "overall") to find an overall partial derivative estimate.
deriv.order	numeric options: $(1, 2)$ ; 1 (default) for first derivative. For second derivative estimate of $f(x)$ , set (deriv.order = 2).
h	numeric $[0,]$ ; Percentage step used for finite step method. Defaults to $h = .05$ representing a 5 percent step from the value of the independent variable.
noise.reductio	n
	the method of determing regression points options: ("mean", "median", "mode", "off"); In low signal to noise situations, (noise.reduction = "median") uses medians instead of means for partitions, while (noise.reduction = "mode") uses modes instead of means for partitions. (noise.reduction = "off") allows for maximum possible fit in NNS.reg. Default setting is (noise.reduction = "mean").
deriv.method	method of derivative estimation, options: ("NNS", "FS"); Determines the partial derivative from the coefficient of the NNS.reg output when (deriv.method = "NNS") or generates a partial derivative using the finite step method (deriv.method = "FS") (Defualt).

# Value

Returns the value of the partial derivative estimate for the given order.

# Note

If a vector of derivatives is required, ensure (deriv.method = "FS").

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#### Author(s)

Fred Viole, OVVO Financial Systems

#### References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

Vinod, H. and Viole, F. (2017) "Nonparametric Regression Using Clusters" https://link.springer. com/article/10.1007/s10614-017-9713-5

#### **Examples**

```
## Not run:
x \leftarrow seq(0, 2 * pi, pi / 100) ; y \leftarrow sin(x)
dy.dx(x, y, eval.point = 1.75)
## End(Not run)
# Vector of derivatives
## Not run:
dy.dx(x, y, eval.point = c(1.75, 2.5), deriv.method = "FS")
## End(Not run)
```

dy.d\_

Partial Derivative dy/d\_[wrt]

## **Description**

Returns the numerical partial derivate of y with respect to [wrt] any regressor for a point of interest. Finite difference method is used with NNS.reg estimates as f(x + h) and f(x - h) values.

# Usage

```
dy.d_{-}(x, y, wrt, eval.points = "median", order = NULL, stn = 0.99,
  h = 0.05, n.best = NULL, mixed = FALSE, plot = FALSE,
  noise.reduction = "mean")
```

# **Arguments**

stn

a numeric matrix or data frame. Х a numeric vector with compatible dimsensions to x. У integer; Selects the regressor to differentiate with respect to. wrt eval.points numeric or options: ("mean", median", "last"); Regressor points to be evaluated. (eval.points = "median") (default) to find partial derivatives at the median of every variable. Set to (eval.points = "last") to find partial derivatives at the last value of every variable. Set to (eval.points="mean") to find partial derivatives at the mean value of every variable. Set to (eval.points = "all") to find partial derivatives at every observation.

integer; NNS.reg "order", defaults to NULL. order

numeric [0, 1]; Signal to noise parameter, sets the threshold of NNS.dep which

reduces "order" when (order = NULL). Defaults to 0.99 to ensure high de-

pendence for higher "order" and endpoint determination.

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h	numeric $[0,]$ ; Percentage step used for finite step method. Defaults to h = .05 representing a 5 percent step from the value of the regressor.
n.best	integer; Sets the number of closest regression points to use in estimating finite difference points in NNS.reg. NULL (default) Uses ceiling(sqrt(ncol(x))).
mixed	logical; FALSE (default) If mixed derivative is to be evaluated, set (mixed = TRUE). Only for single valued eval.points.
plot	logical; FALSE (default) Set to (plot = TRUE) to view plot.
noise.reductio	n
	the method of determing regression points options: ("mean", "median", "mode", "off"); In low signal to noise situations, (noise.reduction = "median") uses medians instead of means for partitions, while (noise.reduction = "mode") uses modes instead of means for partitions. (noise.reduction = "off") al-

lows for maximum possible fit in NNS.reg. Default setting is (noise.reduction = "mean").

#### Value

#### Returns:

- dy.d\_(...)\$"First Derivative" the 1st derivative
- dy.d\_(...)\$"Second Derivative" the 2nd derivative
- dy.d\_(...) \*"Mixed Derivative" the mixed derivative (for two independent variables only).

Returns a vector of partial derivatives when (eval.points = "all").

## Note

For known function testing and analysis, regressors should be transformed via expand.grid to fill the dimensions with (order = "max"). Example provided below.

# Author(s)

Fred Viole, OVVO Financial Systems

## References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

```
## Not run:
set.seed(123); x_1 <- runif(100); x_2 <- runif(100); y <- x_1 ^ 2 * x_2 ^ 2
B = cbind(x_1, x_2)
## To find derivatives of y wrt 1st regressor
dy.d_(B, y, wrt = 1, eval.points = c(.5, .5))
## End(Not run)

## Known function analysis: [y = a ^ 2 * b ^ 2]
## Not run:
x_1 <- seq(0, 1, .1); x_2 <- seq(0, 1, .1)
B = expand.grid(x_1, x_2); y <- B[, 1] ^ 2 * B[, 2] ^ 2
dy.d_(B, y, wrt = 1, eval.points = c(.5, .5), order = "max")
## End(Not run)</pre>
```

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LPM

Lower Partial Moment

## **Description**

This function generates a univariate lower partial moment for any degree or target.

# Usage

```
LPM(degree, target, variable)
```

# Arguments

degree integer; (degree = 0) is frequency, (degree = 1) is area.

target numeric; Typically set to mean, but does not have to be. (Vectorized)

variable a numeric vector.

## Value

LPM of variable

#### Author(s)

Fred Viole, OVVO Financial Systems

# References

```
Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" 
http://amzn.com/1490523995
```

# **Examples**

```
set.seed(123)
x <- rnorm(100)
LPM(0, mean(x), x)</pre>
```

LPM.ratio

Lower Partial Moment RATIO

# Description

This function generates a standardized univariate lower partial moment for any degree or target.

# Usage

```
LPM.ratio(degree, target, variable)
```

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#### **Arguments**

degree integer; (degree = 0) is frequency, (degree = 1) is area.

target numeric; Typically set to mean, but does not have to be. (Vectorized)

variable a numeric vector.

## Value

Standardized LPM of variable

## Author(s)

Fred Viole, OVVO Financial Systems

#### References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

## **Examples**

```
set.seed(123)
x <- rnorm(100)
LPM.ratio(0, mean(x), x)

## Continuous CDF
LPM.ratio(1, sort(x), x)

## Joint CDF
## Not run:
x <- rnorm(5000); y <- rnorm(5000)
plot3d(x, y, Co.LPM(0, 0, sort(x), sort(y), x, y), col = "blue", xlab = "X", ylab = "Y", zlab = "Probability", box = FALSE)

## End(Not run)</pre>
```

LPM.VaR

LPM VaR

## **Description**

Generates a VaR based on the Lower Partial Moment ratio.

# Usage

```
LPM.VaR(percentile, degree, x)
```

## **Arguments**

```
percentile numeric [0, 1]; The percentile for left-tail VaR.
```

degree integer; (degree = 0) for discrete distributions, (degree = 1) for continuous

distributions.

x a numeric vector.

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#### Value

Returns a numeric value representing the point at which "percentile" of the area of x is above.

#### Author(s)

Fred Viole, OVVO Financial Systems

#### References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

## **Examples**

```
set.seed(123)
x <- rnorm(100)
## For 95th percentile VaR (left-tail)
LPM.VaR(0.95, 0, x)</pre>
```

NNS.ANOVA

NNS ANOVA

## **Description**

Analysis of variance (ANOVA) based on lower partial moment CDFs for multiple variables. Returns a degree of certainty the difference in sample means is zero, not a p-value.

# Usage

```
NNS.ANOVA(control, treatment, confidence.interval = 0.95,
  tails = "Both", pairwise = FALSE, plot = TRUE, binary = TRUE)
```

## **Arguments**

control	a numeric vector, matrix or data frame.
treatment	NULL (default) a numeric vector, matrix or data frame.
confidence.inte	erval
	numeric [0, 1]; The confidence interval surrounding the control mean when (binary = TRUE). Defaults to (confidence.interval = 0.95).
tails	options: ("Left", "Right", "Both"). tails = "Both"(Default) Selects the tail of the distribution to determine effect size.
pairwise	logical; FALSE (defualt) Returns pairwise certainty tests when set to pairwise = TRUE.
plot	logical; TRUE (default) Returns the boxplot of all variables along with grand mean identification. When (binary = TRUE), returns the boxplot of both variables along with grand mean identification and confidence interval thereof.
binary	logical; TRUE (default) Selects binary analysis between a control and treatment variable.

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#### Value

For (binary = FALSE) returns the degree certainty the difference in sample means is zero [0, 1]. For (binary = TRUE) returns:

- "Control Mean"
- "Treatment Mean"
- "Grand Mean"
- "Control CDF"
- "Treatment CDF"
- "Certainty" the certainty of the same population statistic
- "Lower Bound Effect" and "Upper Bound Effect" the effect size of the treatment for the specified confidence interval

#### Author(s)

Fred Viole, OVVO Financial Systems

# References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

#### **Examples**

```
### Binary analysis and effect size
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.ANOVA(control = x, treatment = y)

### Two variable analysis with no control variable
A <- cbind(x, y)
NNS.ANOVA(A)

### Multiple variable analysis with no control variable
set.seed(123)
x <- rnorm(100) ; y<- rnorm(100) ; z<- rnorm(100)
A <- cbind(x, y, z)
NNS.ANOVA(A)</pre>
```

NNS.ARMA

NNS ARMA

## **Description**

Autoregressive model incorporating nonlinear regressions of component series.

# Usage

```
NNS.ARMA(variable, h = 1, training.set = NULL,
seasonal.factor = TRUE, weights = NULL, best.periods = 2,
negative.values = FALSE, method = "nonlin", dynamic = FALSE,
plot = TRUE, seasonal.plot = TRUE, intervals = FALSE,
ncores = NULL)
```

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## **Arguments**

variable a numeric vector.

h integer; 1 (default) Number of periods to forecast.

training.set numeric; NULL (defualt) Sets the number of variable observations

(variable[1 : training.set]) to monitor performance of forecast over in-

sample range.

seasonal.factor

logical or integer(s); TRUE (default) Automatically selects the best seasonal lag from the seasonality test. To use weighted average of all seasonal lags set to (seasonal.factor = FALSE). Otherwise, directly input known frequency integer lag to use, i.e. (seasonal.factor = 12) for monthly data. Multiple frequency integers can also be used, i.e. (seasonal.factor = c(12, 24, 36))

weights numeric; NULL (default) sets the weights of the seasonal.factor vector when

specified as integers. If (weights = NULL) each seasonal.factor is weighted

on its NNS.seas result and number of observations it contains.

best.periods integer; [2] (default) used in conjuction with (seasonal.factor = FALSE),

uses the best.periods number of detected seasonal lags instead of ALL lags

when

(seasonal.factor = FALSE).

negative.values

logical; FALSE (default) If the variable can be negative, set to (negative.values = TRUE).

method options: ("lin", "nonlin", "both"); "nonlin" (default) To select the regression

type of the component series, select (method = "both") where both linear and nonlinear estimates are generated. To use a nonlineaer regression, set to (method = "nonlin"); to use a linear regression set to (method = "lin").

dynamic logical; FALSE (default) To update the seasonal factor with each forecast point,

set to (dynamic = TRUE). The default is (dynamic = FALSE) to retain the

original seasonal factor from the inputted variable for all ensuing h.

plot logical; TRUE (default) Returns the plot of all periods exhibiting seasonality and

the variable level reference in upper panel. Lower panel returns original data

and forecast.

seasonal.plot logical; TRUE (default) Adds the seasonality plot above the forecast. Will be set

to FALSE if no seasonality is detected or seasonal. factor is set to an integer

value.

intervals logical; FALSE (default) Plots the surrounding forecasts around the final estimate

when (intervals = TRUE) and (seasonal.factor = FALSE). There are no

other forecasts to plot when a single seasonal.factor is selected.

ncores integer; value specifying the number of cores to be used in the parallelized pro-

cedure. If NULL (default), the number of cores to be used is equal to half the

number of cores of the machine - 1.

# Value

Returns a vector of forecasts of length (h).

## Note

For monthly data series, increased accuracy may be realized from forcing seasonal factors to multiples of 12. For example, if the best periods reported are:  $\{37, 47, 71, 73\}$  use (seasonal.factor = c(36, 48, 72)).

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(seasonal.factor = FALSE) can be a very comutationally expensive exercise due to the number of seasonal periods detected.

```
If error encountered when (seasonal.factor = TRUE):
```

```
"NaNs produced Error in seq.default(length(variable)+1, 1, -lag[i]): wrong sign in 'by' argument" use the combination of (seasonal.factor = FALSE, best.periods = 1).
```

#### Author(s)

Fred Viole, OVVO Financial Systems

#### References

```
Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" 
http://amzn.com/1490523995
```

## **Examples**

```
## Nonlinear NNS.ARMA using AirPassengers monthly data and 12 period lag
## Not run:
NNS.ARMA(AirPassengers, h = 45, training.set = 100, seasonal.factor = 12, method = "nonlin")
## End(Not run)

## Linear NNS.ARMA using AirPassengers monthly data and 12, 24, and 36 period lags
## Not run:
NNS.ARMA(AirPassengers, h = 45, training.set = 120, seasonal.factor = c(12, 24, 36), method = "lin")
## End(Not run)

## Nonlinear NNS.ARMA using AirPassengers monthly data and 2 best periods lag
## Not run:
NNS.ARMA(AirPassengers, h = 45, training.set = 120, seasonal.factor = FALSE, best.periods = 2)
## End(Not run)
```

 ${\tt NNS.ARMA.optim}$ 

NNS ARMA Optimizer

# Description

Wrapper function for optimizing any combination of a given seasonal. factor vector in NNS.ARMA. Minimum sum of squared errors (forecast-actual) is used to determine optimum across all NNS.ARMA methods.

#### Usage

```
NNS.ARMA.optim(variable, training.set, seasonal.factor,
  negative.values = FALSE, obj.fn = expression(sum((predicted -
  actual)^2)), objective = "min", linear.approximation = TRUE,
  depth = 1, print.trace = TRUE, ncores = NULL, subcores = NULL)
```

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#### **Arguments**

variable a numeric vector.

training.set numeric; NULL (defualt) Sets the number of variable observations

seasonal.factor

integers; Multiple frequency integers considered for NNS.ARMA model, i.e.

(seasonal.factor = c(12, 24, 36))

negative.values

logical; FALSE (default) If the variable can be negative, set to (negative.values = TRUE).

obj.fn expression; expression(sum((predicted - actual)^2)) (default) Sum of

squared errors is the default objective function. Any expression() using the

specific terms predicted and actual can be used.

objective options: ("min", "max") "min" (default) Select whether to minimize or maxi-

mize the objective function obj.fn.

linear.approximation

logical; TRUE (default) Uses the best linear output from NNS. reg to generate a

nonlinear and mixture regression for comparison. FALSE is a more exhaustive

search over the objective space.

depth integer; depth = 1 (default) Sets the level from which further combinations are

generated containing only members from prior level's best seasonal.factors.

print.trace logical; TRUE (defualt) Prints current iteration information. Suggested as backup

in case of error, best parameters to that point still known and copyable!

ncores integer; value specifying the number of cores to be used in the parallelized pro-

cedure. If NULL (default), the number of cores to be used is equal to half the

number of cores of the machine.

subcores integer; value specifying the number of cores to be used in the parallelized pro-

cedure in the subroutine NNS.ARMA. If NULL (default), the number of cores

to be used is equal to half the number of cores of the machine - 1.

#### Value

Returns a list containing:

- \$period a vector of optimal seasonal periods
- \$weights the optimal weights of each seasonal period between an equal weight or NULL weighting
- \$obj. fn the minimum objective function value
- \$method the method identifying which NNS.ARMA method was used.

#### Note

The number of combinations will grow prohibitively large, they should be kept to a minimum when (method = "comb").

seasonal.factor containing an element too large will result in an error. Please reduce the maximum seasonal.factor.

#### Author(s)

Fred Viole, OVVO Financial Systems

NNS.boost

#### References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

#### **Examples**

```
## Nonlinear NNS.ARMA period optimization using 2 yearly lags on AirPassengers monthly data
## Not run:
nns.optims <- NNS.ARMA.optim(AirPassengers[1:132], training.set = 120,
seasonal.factor = seq(12, 24, 6))

## End(Not run)

## Then use optimal parameters in NNS.ARMA to predict 12 periods in-sample
## Not run:
NNS.ARMA(AirPassengers, h=12, training.set=132,
seasonal.factor = nns.optims$periods, method = nns.optims$method)

## End(Not run)</pre>
```

NNS.boost

NNS Boost

# Description

Ensemble method for classification using the predictions of the NNS multivariate regression NNS.reg collected from uncorrelated feature combinations.

## Usage

```
NNS.boost(IVs.train, DV.train, IVs.test, representative.sample = TRUE,
depth = NULL, n.best = 3, learner.trials = NULL, epochs = NULL,
CV.size = 0.25, threshold = NULL,
obj.fn = expression(mean(round(predicted) == as.numeric(actual))),
objective = "max", extreme = FALSE, feature.importance = TRUE,
status = TRUE, ncores = NULL, subcores = NULL)
```

## **Arguments**

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n.best integer; 3 (default) Sets the number of nearest regression points to use in weighting for multivariate regression at sqrt(# of regressors). Analogous to k in ak Nearest Neighbors algorithm. learner.trials integer; NULL (default) Sets the number of trials to obtain an accuracy threshold level. Number of observations in the training set is the default setting. integer; 2\*length(DV.train) (default) Total number of feature combinations epochs to run. CV.size numeric [0, 1]; (CV. size = .25) (default) Sets the cross-validation size. Defaults to 0.25 for a 25 percent random sampling of the training set. threshold numeric [0, 1]; NULL (default) Sets the obj. fn accuracy threshold to keep feature combinations. obj.fn expression; expression(mean(round(predicted)==as.numeric(actual))) (default) Mean accuracy is the default objective function. Any expression() using the specific terms predicted and actual can be used. options: ("min", "max") "max" (default) Select whether to minimize or maxiobjective mize the objective function obj.fn. logical; FALSE (default) Uses the maximum (minimum) threshold obtained extreme from the learner. trials, rather than the upper (lower) quintile level for maximization (minimization) objective. feature.importance logical; TRUE (default) Plots the frequency of features used in the final estimate.

status logical; TRUE (default) Prints status update message in console.

integer; value specifying the number of cores to be used in the parallelized proncores

cedure. If NULL (default), the number of cores to be used is equal to half the

number of cores of the machine.

integer; value specifying the number of cores to be used in the parallelized prosubcores

cedure in the subroutine NNS.reg. If NULL (default), the number of cores to be

used is equal to half the number of cores of the machine - 1.

#### Value

Returns a vector of fitted values for the dependent variable test set.

## Author(s)

Fred Viole, OVVO Financial Systems

## References

Viole, F. (2016) "Classification Using NNS Clustering Analysis" https://ssrn.com/abstract= 2864711

```
## Using 'iris' dataset where test set [IVs.test] is 'iris' rows 141:150.
## Not run:
a = NNS.boost(iris[1:140, 1:4], iris[1:140, 5],
IVs.test = iris[141:150, 1:4],
epochs = 100, learner.trials = 100)
## Test accuracy
```

NNS.caus

```
mean(round(a)==as.numeric(iris[141:150,5]))
## End(Not run)
```

NNS.caus

NNS Causation

# Description

Returns the causality from observational data between two variables.

# Usage

```
NNS.caus(x, y, factor.2.dummy = TRUE, tau, time.series = FALSE,
plot = FALSE)
```

#### **Arguments**

x	a numeric vector, matrix or data frame.
У	NULL (default) or a numeric vector with compatible dimsensions to x.
factor.2.dummy	logical; TRUE (default) Automatically augments variable matrix with numerical dummy variables based on the levels of factors. Includes dependent variable y.
tau	options: ("cs", "ts", integer); Number of lagged observations to consider (for time series data). Otherwise, set (tau = "cs") for cross-sectional data. (tau = "ts") automatically selects the lag of the time series data, while (tau = [integer]) specifies a time series lag.
time.series	<pre>logical; FALSE (default) If analyzing time series data with tau = [integer], select (time.series = TRUE). Not required when (tau = "ts").</pre>
plot	logical; FALSE (default) Plots the raw variables, tau normalized, and cross-normalized variables.

# Value

Returns the directional causation  $(x \longrightarrow y)$  or  $(y \longrightarrow x)$  and net quantity of association. For causal matrix, directional causation is returned as ([column variable]  $\longrightarrow$  [row variable]). Negative numbers represent causal direction attributed to [row variable].

## Author(s)

Fred Viole, OVVO Financial Systems

# References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995 NNS.cor

#### **Examples**

```
## x clearly causes y...
## Not run:
set.seed(123)
x <- rnorm(100) ; y <- x ^ 2
NNS.caus(x, y, tau = "cs")

x <- 1:100 ; y <- x^2
NNS.caus(x, y, tau = "ts", time.series = TRUE)
## End(Not run)

## Causal matrix
## Not run:
NNS.caus(data.matrix(iris), tau = 0)
## End(Not run)</pre>
```

NNS.cor

NNS Correlation

## **Description**

Returns the nonlinear correlation between two variables based on higher order partial moment matrices measured by frequency or area.

#### Usage

```
NNS.cor(x, y = NULL, order = NULL, degree = NULL)
```

## **Arguments**

x a numeric vector, matrix or data frame.

y NULL (default) or a numeric vector with compatible dimsensions to x.

order integer; Controls the level of quadrant partitioning. Defualts to (order = NULL).

Errors can generally be rectified by setting (order = 1).

degree integer; (degree = 0) is frequency based correlations, while (degree = 1)

is for area based correlations. Defaults to (degree = 0) for smaller number of

observations.

#### Value

Returns nonlinear correlation coefficient between two variables, or nonlinear correlation matrix for matrix input.

# Author(s)

Fred Viole, OVVO Financial Systems

## References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995 20 NNS.cor.hd

#### **Examples**

```
set.seed(123)
## Pairwise Correlation
x <- rnorm(100) ; y <- rnorm(100)
NNS.cor(x, y)

## Correlation Matrix
x <- rnorm(100) ; y <- rnorm(100) ; z<-rnorm(100)
B <- cbind(x, y, z)
NNS.cor(B)</pre>
```

NNS.cor.hd

NNS Co-Partial Moments Higher Dimension Correlation

## **Description**

Determines higher dimension correlation coefficients based on degree 0 co-partial moments.

# Usage

```
NNS.cor.hd(x, plot = FALSE, independence.overlay = FALSE)
```

## **Arguments**

x a numeric matrix or data frame.

plot logical; FALSE (default) Generates a 3d scatter plot with regression points using plot3d.

independence.overlay

logical; FALSE (default) Creates and overlays independent Co.LPM and Co.UPM regions to visually reference the difference in dependence from the data.frame of variables being analyzed. Under independence, the light green and red shaded areas would be occupied by green and red data points respectively.

#### Value

Returns multivariate nonlinear correlation coefficient

# Author(s)

Fred Viole, OVVO Financial Systems

## References

Viole, F. (2016) "Beyond Correlation: Using the Elements of Variance for Conditional Means and Probabilities" http://ssrn.com/abstract=2745308.

```
set.seed(123)
x <- rnorm(1000) ; y <- rnorm(1000) ; z <- rnorm(1000)
A <- data.frame(x, y, z)
NNS.cor.hd(A, plot = TRUE, independence.overlay = TRUE)</pre>
```

NNS.dep 21

NNS . dep NNS Dependence	NNS.dep
--------------------------	---------

# Description

Returns the dependence and nonlinear correlation between two variables based on higher order partial moment matrices measured by frequency or area.

# Usage

```
NNS.dep(x, y = NULL, order = NULL, degree = NULL,
    print.map = FALSE, ncores = NULL)
```

of cores of the machine - 1.

#### **Arguments**

х	a numeric vector, matrix or data frame.
у	NULL (default) or a numeric vector with compatible dimsensions to x.
order	integer; Controls the level of quadrant partitioning. Defaults to (order = NULL). Errors can generally be rectified by setting (order = 1). Will not partition further if less than 4 observations exist in a quadrant.
degree	integer; Defaults to NULL to allow number of observations to be "degree" determinant.
print.map	logical; FALSE (default) Plots quadrant means.
ncores	integer; value specifying the number of cores to be used in the parallelized procedure. If NULL (default), the number of cores to be used is equal to the number

## Value

Returns the bi-variate "Correlation" and "Dependence" or correlation / dependence matrix for matrix input.

## Author(s)

Fred Viole, OVVO Financial Systems

## References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.dep(x, y)
## Correlation / Dependence Matrix
x <- rnorm(100) ; y <- rnorm(100) ; z <- rnorm(100)
B <- cbind(x, y, z)
NNS.dep(B)</pre>
```

NNS.diff

NNS.dep.base	NNS Dependence Base	

# Description

Internal function for NNS dependence NNS.dep parallel instances.

# Usage

```
NNS.dep.base(x, y = NULL, order = NULL, degree = NULL,
    print.map = FALSE)
```

# **Arguments**

X	from NNS.part
у	from NNS.part
order	from NNS.part
degree	from NNS.part
print.map	from NNS.part

#### Value

Returns NNS dependence.

NNS.diff	NNS Numerical Differentiation	

# Description

Determines numerical derivative of a given function using projected secant lines on the y-axis. These projected points infer finite steps h, in the finite step method.

# Usage

```
NNS.diff(f, point, h = 0.1, tol = 1e-10, print.trace = FALSE)
```

# Arguments

f	an expression or call or a formula with no lhs.
point	numeric; Point to be evaluated for derivative of a given function f.
h	numeric $[0,]$ ; Initial step for secant projection. Defaults to (h = 0.1).
tol	numeric; Sets the tolerance for the stopping condition of the inferred h. Defualts to $(tol = 1e-10)$ .
print.trace	logical; FALSE (default) Displays each iteration, lower y-intercept, upper y-intercept and inferred h.

NNS.distance 23

# Value

Returns a matrix of values, intercepts, derivatives, inferred step sizes for multiple methods of estimation.

## Author(s)

Fred Viole, OVVO Financial Systems

#### References

```
Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995
```

# **Examples**

```
f <- function(x) sin(x) / x
NNS.diff(f, 4.1)</pre>
```

NNS.distance

NNS Distance

# Description

Internal function for NNS multivariate regression NNS.reg parallel instances.

# Usage

```
NNS.distance(rpm, dist.estimate, type, k)
```

# Arguments

```
\begin{tabular}{llll} $\operatorname{rpm}$ & REGRESSION.POINT.MATRIX from $\operatorname{NNS.reg}$ \\ & & & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ &
```

## Value

Returns sum of weighted distances.

24 NNS.FSD

NNS.FSD

NNS FSD Test

## **Description**

Bi-directional test of first degree stochastic dominance using lower partial moments.

## Usage

```
NNS.FSD(x, y, type = "discrete")
```

# Arguments

x a numeric vector.

y a numeric vector.

type options: ("discrete", "continuous"); "discrete" (default) selects the type of CDF.

#### Value

Returns one of the following FSD results: "X FSD Y", "Y FSD X", or "NO FSD EXISTS".

## Author(s)

Fred Viole, OVVO Financial Systems

## References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817.

Viole, F. (2017) "A Note on Stochastic Dominance." https://ssrn.com/abstract=3002675.

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.FSD(x, y)</pre>
```

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NNS.FSD.uni

NNS FSD Test uni-directional

# **Description**

Uni-directional test of first degree stochastic dominance using lower partial moments used in SD Efficient Set routine.

#### Usage

```
NNS.FSD.uni(x, y, type = "discrete")
```

## **Arguments**

```
x a numeric vector.
```

y a numeric vector.

type options: ("discrete", "continuous"); "discrete" (default) selects the type of

CDF.

# Value

```
Returns (1) if "X FSD Y", else (0).
```

# Author(s)

Fred Viole, OVVO Financial Systems

## References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817.

Viole, F. (2017) "A Note on Stochastic Dominance." https://ssrn.com/abstract=3002675.

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.FSD.uni(x, y)</pre>
```

26 NNS.part

NNS.norm

NNS Normalization

## **Description**

Normalizes a matrix of variables based on nonlinear scaling normalization method.

## Usage

```
NNS.norm(A, chart.type = NULL, linear = FALSE)
```

#### **Arguments**

A a numeric matrix or data frame.

chart.type options: ("I", "b"); NULL (default). Set (chart.type = "1") for line, (chart.type = "b")

for boxplot.

linear logical; FALSE (default) Performs a linear scaling normalization, resulting in

equal means for all variables.

# Value

Returns a data.frame of normalized values.

#### Author(s)

Fred Viole, OVVO Financial Systems

# References

```
Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" 
http://amzn.com/1490523995
```

## **Examples**

```
set.seed(123)
x <- rnorm(100) ; y<-rnorm(100)
A <- cbind(x, y)
NNS.norm(A)</pre>
```

NNS.part

NNS Partition Map

# Description

Creates partitions based on partial moment quadrant means, iteratively assigning identifications to observations based on those quadrants (unsupervised partitional and hierarchial clustering method). Basis for correlation NNS.cor, dependence NNS.dep, regression NNS.reg routines.

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#### **Usage**

```
NNS.part(x, y, Voronoi = FALSE, type = NULL, order = NULL,
  obs.req = 8, min.obs.stop = FALSE, noise.reduction = "mean")
```

#### **Arguments**

x a numeric vector.

y a numeric vector with compatible dimsensions to x.

Voronoi logical; FALSE (default) Displays a Voronoi type diagram using partial moment

quadrants.

type NULL (default) Controls the partitioning basis. Set to (type = "XONLY") for

X-axis based partitioning. Defaults to NULL for both X and Y-axis partitioning.

order integer; Number of partial moment quadrants to be generated. (order = "max")

will institute a perfect fit.

obs.req integer; (8 default) Required observations per cluster where quadrants will not

be further partitioned if observations are not greater than the entered value. Reduces minimum number of necessary observations in a quadrant to 1 when

(obs.req = 1).

min.obs.stop logical; FALSE (default) Stopping condition where quadrants will not be further

partitioned if a single cluster contains less than the entered value of obs.req.

noise.reduction

the method of determing regression points options: ("mean", "median", "mode", "off"); (noise.reduction = "median") uses medians instead of means for partitions, while (noise.reduction = "mode") uses modes instead of means

for partitions. Defaults to (noise.reduction = "mean"), while (noise.reduction = "off")

will partition quadrant to a single observation for a given (order = ...).

# Value

## Returns:

- "dt" a data.table of x and y observations with their partition assignment "quadrant" in the 3rd column and their prior partition assignment "prior.quadrant" in the 4th column.
- "regression.points" the data.table of regression points for that given (order = ...).
- "order" the order of the final partition given "min.obs.stop" stopping condition.

#### Author(s)

Fred Viole, OVVO Financial Systems

## References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.part(x, y)</pre>
```

28 NNS.PDF

```
## Data.table of observations and partitions
NNS.part(x, y, order = 1)$dt

## Regression points
NNS.part(x, y, order = 1)$regression.points

## Voronoi style plot
NNS.part(x, y, Voronoi = TRUE)

## Examine final counts by quadrant
DT = NNS.part(x, y)$dt
DT[ , counts := .N, by = quadrant]
DT
```

NNS.PDF

NNS PDF

## **Description**

This function generates an empirical PDF using continuous CDFs from LPM.ratio.

## Usage

```
NNS.PDF(variable, degree = 1, target = NULL, bins = NULL,
plot = TRUE)
```

# **Arguments**

variable a numeric vector.

degree integer; (degree = 0) is frequency, (degree = 1) (default) is area.

target a numeric range of values [a,b] where a < b. NULL (default) uses the variable

observations.

bins numeric; NULL (default) Selects number of observations as default bins.

plot logical; plots PDF.

# Value

Returns a data.table containing the intervals used and resulting PDF of the variable.

#### Author(s)

Fred Viole, OVVO Financial Systems

# References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

#### **Examples**

```
set.seed(123)
x <- rnorm(100)
NNS.PDF(x)

## Custom target range
NNS.PDF(x, target = c(-5, 5))</pre>
```

NNS.reg

NNS Regression

#### **Description**

Generates a nonlinear regression based on partial moment quadrant means.

## Usage

```
NNS.reg(x, y, factor.2.dummy = TRUE, order = NULL, stn = 0.95,
    dim.red.method = NULL, tau = NULL, type = NULL, point.est = NULL,
    location = "top", return.values = TRUE, plot = TRUE,
    plot.regions = FALSE, residual.plot = TRUE, std.errors = FALSE,
    confidence.interval = NULL, threshold = 0, n.best = NULL,
    noise.reduction = "mean", norm = NULL, dist = "L2",
    ncores = NULL, multivariate.call = FALSE)
```

#### **Arguments**

type

a vector, matrix or data frame of variables of numeric or factor data types. Х a numeric or factor vector with compatible dimsensions to x. У factor.2.dummy logical; TRUE (default) Automatically augments variable matrix with numerical dummy variables based on the levels of factors. order integer; Controls the number of partial moment quadrant means. Users are encouraged to try different (order = ...) integer settings with (noise.reduction = "off"). (order = "max") will force a limit condition perfect fit. numeric [0, 1]; Signal to noise parameter, sets the threshold of (NNS.dep) which stn reduces ("order") when (order = NULL). Defaults to 0.95 to ensure high dependence for higher ("order") and endpoint determination. dim.red.method options: ("cor", "NNS.dep", "NNS.caus", "all", NULL) method for determining synthetic X\* coefficients. Selection of a method automatically engages the dimension reduction regression. The default is NULL for full multivariate regression. (dim.red.method = "NNS.dep") uses NNS.dep for nonlinear dependence weights, while (dim.red.method = "NNS.caus") uses NNS.caus for causal weights. (dim.red.method = "cor") uses standard linear correlation for weights. (dim.red.method = "all") averages all methods for further feature engineering. options("ts", NULL); NULL(default) If (dim.red.method = "NNS.caus") or tau (dim.red.method = "all") and the regression is using time-series data, set

NULL (default). To perform a classification, set to (type = "CLASS").

(tau = "ts") for more accurate causal analysis.

point.est a numeric or factor vector with compatible dimsensions to x. Returns the fitted

value y. hat for any value of x.

location Sets the legend location within the plot, per the x and y co-ordinates used in base

graphics legend.

return.values logical; TRUE (default), set to FALSE in order to only display a regression plot

and call values as needed.

plot logical; TRUE (default) To plot regression.

plot.regions logical; FALSE (default). Generates 3d regions associated with each regression

point for multivariate regressions. Note, adds significant time to routine.

residual.plot logical; TRUE (default) To plot y.hat and Y.

std.errors logical: FALSE (default) To provide standard errors of each linear segment in the

"Fitted.xy" output.

confidence.interval

numeric [0, 1]; NULL (default) Plots the associated confidence interval with the

estimate and reports the standard error for each individual segment.

threshold numeric [0, 1]; (threshold = 0) (default) Sets the threshold for dimension

reduction of independent variables when (dim.red.method) is not NULL.

n.best integer; NULL (default) Sets the number of nearest regression points to use in

weighting for multivariate regression at sqrt(# of regressors). (n.best = "all") will select and weight all generated regression points. Analogous to k in a k Nearest Neighbors algorithm. Different values of n.best are tested us-

ing cross-validation in NNS.stack.

noise.reduction

the method of determing regression points options: ("mean", "median", "mode", "off"); In low signal:noise situations, (noise.reduction = "mean") uses means for NNS.dep restricted partitions, (noise.reduction = "median") uses medi-

ans instead of means for NNS.dep restricted partitions, while (noise.reduction = "mode") uses modes instead of means for NNS.dep restricted partitions. (noise.reduction = "off")

allows for maximum possible fit with a specific order.

norm NULL (default) the method of normalization options: ("NNS", "std"); Normalizes

x between 0 and 1 for multivariate regression when set to (norm = "std"), or

normalizes x according to NNS.norm when set to (norm = "NNS").

dist options:("L1", "L2") the method of distance calculation; Selects the distance

calculation used. dist = "L2" (default) selects the Euclidean distance and

(dist = "L1") seclects the Manhattan distance.

ncores integer; value specifying the number of cores to be used in the parallelized pro-

cedure. If NULL (default), the number of cores to be used is equal to the number

of cores of the machine - 1.

multivariate.call

Internal parameter for multivariate regressions.

# Value

# UNIVARIATE REGRESSION RETURNS THE FOLLOWING VALUES:

- "R2" provides the goodness of fit;
- "SE" returns the overall standard error of the estimate between y and y.hat;
- "Prediction.Accuracy" returns the correct rounded "Point.est" used in classifications versus the categorical y;

- "derivative" for the coefficient of the x and its applicable range;
- "Point" returns the x point(s) being evaluated;
- "Point.est" for the predicted value generated;
- "regression.points" provides the points used in the regression equation for the given order of partitions;
- "Fitted" returns a vector containing only the fitted values, y.hat;
- "Fitted.xy" returns a data.table of x,y, y.hat, and NNS.ID;

## MULTIVARIATE REGRESSION RETURNS THE FOLLOWING VALUES:

- "R2" provides the goodness of fit;
- "equation" returns the numerator of the synthetic X\* dimension reduction equation as a data.table consisting of regressor and its coefficient. Denominator is simply the length of all coefficients > 0, returned in last row of equation data.table.
- "x.star" returns the synthetic X\* as a vector;
- "rhs.partitions" returns the partition points for each regressor x;
- "RPM" provides the Regression Point Matrix, the points for each x used in the regression equation for the given order of partitions;
- "Point.est" returns the predicted value generated;
- "Fitted" returns a vector containing only the fitted values, y.hat;
- "Fitted.xy" returns a data.table of x,y, y.hat, gradient, and NNS.ID.

#### Note

Please ensure point.est is of compatible dimensions to x, error message will ensue if not compatible. Also, upon visual inspection of the data, if a highly periodic variable is observed set (stn = 0) or (order = "max") to ensure a proper fit.

Identical regressors can be used as long as they do not share the same name. For instance, NNS.reg(cbind(x, 1 \* x), y) will work as NNS.reg is not affected by multicollinearity.

NNS (>= v.0.3.4) has repurposed parameter (type = "CLASS"). (type = "CLASS") is now restricted to signifying a classification analysis for NNS.reg while (dim.red.method) enables dimension reduction regressions.

#### Author(s)

Fred Viole, OVVO Financial Systems

## References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

Vinod, H. and Viole, F. (2017) "Nonparametric Regression Using Clusters" https://link.springer.com/article/10.1007/s10614-017-9713-5

```
## Not run:
set.seed(123)
x <- rnorm(100); y <- rnorm(100)
NNS.reg(x, y)
## End(Not run)
## Manual {order} selection
## Not run:
NNS.reg(x, y, order = 2)
## End(Not run)
## Maximum {order} selection
## Not run:
NNS.reg(x, y, order = "max")
## End(Not run)
## x-only paritioning (Univariate only)
## Not run:
NNS.reg(x, y, type = "XONLY")
## End(Not run)
## For Multiple Regression:
## Not run:
x \leftarrow cbind(rnorm(100), rnorm(100), rnorm(100)); y \leftarrow rnorm(100)
NNS.reg(x, y, point.est = c(.25, .5, .75))
## End(Not run)
\#\# For Multiple Regression based on Synthetic X* (Dimension Reduction):
x <- cbind(rnorm(100), rnorm(100), rnorm(100)); y<-rnorm(100)</pre>
NNS.reg(x, y, point.est = c(.25, .5, .75), dim.red.method = "cor")
## End(Not run)
## IRIS dataset examples:
# Dimension Reduction:
## Not run:
NNS.reg(iris[,1:4], iris[,5], dim.red.method = "cor", order = 5)
## End(Not run)
# Dimension Reduction using causal weights:
## Not run:
NNS.reg(iris[,1:4], iris[,5], dim.red.method = "NNS.caus", order = 5)
## End(Not run)
# Multiple Regression:
## Not run:
NNS.reg(iris[,1:4], iris[,5], order = 2, noise.reduction = "off")
## End(Not run)
# Classification:
## Not run:
NNS.reg(iris[,1:4], iris[,5], point.est = iris[1:10, 1:4], type = "CLASS")$Point.est
## End(Not run)
## To call fitted values:
```

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```
## Not run:
x <- rnorm(100) ; y <- rnorm(100)
NNS.reg(x, y)$Fitted
## End(Not run)

## To call partial derivative (univariate regression only):
## Not run:
NNS.reg(x, y)$derivative
## End(Not run)</pre>
```

```
NNS.SD.efficient.set NNS SD Efficient Set
```

#### **Description**

Determines the set of stochastic dominant variables for various degrees.

## Usage

```
NNS.SD.efficient.set(x, degree, type = "discrete")
```

# **Arguments**

```
x a numeric matrix or data frame.

degree numeric options: (1, 2, 3); Degree of stochastic dominance test from (1, 2 or 3).

type options: ("discrete", "continuous"); "discrete" (default) selects the type of CDF.
```

# Value

Returns set of stochastic dominant variable names.

# Author(s)

Fred Viole, OVVO Financial Systems

# References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817.

Viole, F. (2017) "A Note on Stochastic Dominance." https://ssrn.com/abstract=3002675.

```
set.seed(123)
x <- rnorm(100) ; y<-rnorm(100) ; z<-rnorm(100)
A <- cbind(x, y, z)
NNS.SD.efficient.set(A, 1)</pre>
```

NNS.seas

NNS.seas

NNS Seasonality Test

## **Description**

Seasonality test based on the coefficient of variance for the variable and lagged component series. A result of 1 signifies no seasonality present.

#### Usage

```
NNS.seas(variable, plot = TRUE)
```

# **Arguments**

variable a numeric vector.

plot logical; TRUE (default) Returns the plot of all periods exhibiting seasonality and

the variable level reference.

## Value

Returns a matrix of all periods exhibiting less coefficient of variance than the variable with "all.periods"; and the single period exhibiting the least coefficient of variance versus the variable with "best.period". If no seasonality is detected, NNS.seas will return ("No Seasonality Detected").

#### Author(s)

Fred Viole, OVVO Financial Systems

## References

```
Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" 
http://amzn.com/1490523995
```

```
set.seed(123)
x <- rnorm(100)
## To call strongest period based on coefficient of variance:
NNS.seas(x)$best.period</pre>
```

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NNS.SSD

NNS SSD Test

## **Description**

Bi-directional test of second degree stochastic dominance using lower partial moments.

## Usage

```
NNS.SSD(x, y)
```

# **Arguments**

x a numeric vector.y a numeric vector.

## Value

Returns one of the following SSD results: "X SSD Y", "Y SSD X", or "NO SSD EXISTS".

#### Author(s)

Fred Viole, OVVO Financial Systems

#### References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817.

# **Examples**

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.SSD(x, y)</pre>
```

NNS.SSD.uni

NNS SSD Test uni-directional

# Description

Uni-directional test of second degree stochastic dominance using lower partial moments used in SD Efficient Set routine.

# Usage

```
NNS.SSD.uni(x, y)
```

NNS.stack

#### **Arguments**

```
x a numeric vector.y a numeric vector.
```

#### Value

```
Returns (1) if "X SSD Y", else (0).
```

## Author(s)

Fred Viole, OVVO Financial Systems

#### References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817.

## **Examples**

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.SSD.uni(x, y)</pre>
```

NNS.stack

NNS Stack

# Description

Prediction model using the predictions of the NNS base models NNS.reg as features (i.e. meta-features) for the stacked model.

#### Usage

```
NNS.stack(IVs.train, DV.train, IVs.test = NULL,
  obj.fn = expression(sum((predicted - actual)^2)), objective = "min",
  CV.size = NULL, folds = 5, order = NULL, norm = NULL,
  method = c(1, 2), dim.red.method = "cor", ncores = NULL)
```

## **Arguments**

IVs.train	a vector, matrix or data frame of variables of numeric or factor data types.
DV.train	a numeric or factor vector with compatible dimsensions to (IVs.train).
IVs.test	a vector, matrix or data frame of variables of numeric or factor data types with compatible dimsensions to (IVs.train).
obj.fn	expression; expression(sum((predicted - actual)^2)) (default) Sum of squared errors is the default objective function. Any expression() using the specific terms predicted and actual can be used.
objective	options: ("min", "max") "min" (default) Select whether to minimize or maximize the objective function obj. fn.

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CV.size	numeric $[0, 1]$ ; NULL (default) Sets the cross-validation size if (IVs.test = NULL). Defaults to $0.25$ for a $25$ percent random sampling of the training set under (CV.size = NULL).
folds	integer; folds = 5 (default) Select the number of cross-validation folds.
order	integer; NULL (default) Sets the order for NNS.reg, where (order = 'max') is the $k$ -nearest neighbors equivalent.
norm	options: ("std", "NNS", NULL); NULL (default) 3 settings offered: NULL, "std", and "NNS". Selects the norm parameter in NNS.reg.
method	numeric options: $(1, 2)$ ; Select the NNS method to include in stack. (method = 1) selects NNS.reg; (method = 2) selects NNS.reg dimension reduction regression. Defaults to method = $c(1, 2)$ , including both NNS regression methods in the stack.
dim.red.method	options: ("cor", "NNS.dep", "NNS.caus", "all") method for determining synthetic X* coefficients. (dim.red.method = "cor") (default) uses standard linear correlation for weights. (dim.red.method = "NNS.dep") uses NNS.dep for nonlinear dependence weights, while (dim.red.method = "NNS.caus") uses NNS.caus for causal weights. (dim.red.method = "all") averages all methods for further feature engineering.
ncores	integer; value specifying the number of cores to be used in the parallelized subroutine NNS.reg. If NULL (default), the number of cores to be used is equal to half the number of cores of the machine - 1.

#### Value

Returns a vector of fitted values for the dependent variable test set for all models.

- "NNS.reg.n.best" returns the optimum "n.best" paramater for the NNS.reg multivariate regression. "SSE.reg" returns the SSE for the NNS.reg multivariate regression.
- "OBJfn.reg" returns the obj.fn for the NNS.reg regression.
- "NNS.dim.red.threshold" returns the optimum "threshold" from the NNS.reg dimension reduction regression.
- "OBJfn.dim.red" returns the obj.fn for the NNS.reg dimension reduction regression.
- "reg" returns NNS.reg output.
- "dim. red" returns NNS.reg dimension reduction regression output.
- "stack" returns the output of the stacked model.

#### Note

If character variables are used, transform them first to factors using as.factor, or data.matrix to ensure overall dataset is numeric. A multifunction sapply can also be applied to the overall dataset: data <- sapply(data,function(x){as.factor(x); as.numeric(x)}). Then run NNS.stack with transformed variables.

Missing data should be handled prior as well using na.omit or complete.cases on the full dataset.

If error received:

```
"Error in is.data.frame(x) : object 'RP' not found" reduce the CV.size.
```

38 NNS.term.matrix

#### Author(s)

Fred Viole, OVVO Financial Systems

#### References

```
Viole, F. (2016) "Classification Using NNS Clustering Analysis" https://ssrn.com/abstract=2864711
```

## **Examples**

```
## Using 'iris' dataset where test set [IVs.test] is 'iris' rows 141:150.
## Not run:
NNS.stack(iris[1:140, 1:4], iris[1:140, 5], IVs.test = iris[141:150, 1:4])
## End(Not run)

## Using 'iris' dataset to determine [n.best] and [threshold] with no test set.
## Not run:
NNS.stack(iris[ , 1:4], iris[ , 5])
## End(Not run)

## Selecting NNS.reg and dimension reduction techniques.
## Not run:
NNS.stack(iris[1:140, 1:4], iris[1:140, 5], iris[141:150, 1:4], method = c(1, 2))
## End(Not run)
```

NNS.term.matrix

NNS Term Matrix

# Description

Generates a term matrix for text classification use in NNS.reg.

# Usage

```
NNS.term.matrix(x, oos = NULL, names = FALSE)
```

# Arguments

X	Text A two column dataset should be used. Concatenate text from original
	sources to comply with format. Also note the possiblity of factors in "DV", so "as.numeric(as.character())" is used to avoid issues.
oos	Out-of-sample text dataset to be classified.
names	Column names for "IV" and "oos". Defaults to FALSE.

## Value

Returns the text as independent variables "IV" and the classification as the dependent variable "DV". Out-of-sample independent variables are returned with "OOS".

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#### References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

## **Examples**

NNS.TSD

NNS TSD Test

## **Description**

Bi-directional test of third degree stochastic dominance using lower partial moments.

# Usage

```
NNS.TSD(x, y)
```

# **Arguments**

```
x a numeric vector.
y a numeric vector.
```

## Value

Returns one of the following TSD results: "X TSD Y", "Y TSD X", or "NO TSD EXISTS".

# Author(s)

Fred Viole, OVVO Financial Systems

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#### References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817.

## **Examples**

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.TSD(x, y)</pre>
```

NNS.TSD.uni

NNS TSD Test uni-directional

# Description

Uni-directional test of third degree stochastic dominance using lower partial moments used in SD Efficient Set routine.

#### Usage

```
NNS.TSD.uni(x, y)
```

## **Arguments**

```
x a numeric vector.
y a numeric vector.
```

# Value

```
Returns (1) if "X TSD Y", else (0).
```

# Author(s)

Fred Viole, OVVO Financial Systems

#### References

Viole, F. and Nawrocki, D. (2016) "LPM Density Functions for the Computation of the SD Efficient Set." Journal of Mathematical Finance, 6, 105-126. http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=63817.

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100)
NNS.TSD.uni(x, y)</pre>
```

PM.matrix 41

PM.matrix	Partial Moment Matrix	
-----------	-----------------------	--

# **Description**

This function generates a co-partial moment matrix for the specified co-partial moment.

# Usage

```
PM.matrix(LPM.degree, UPM.degree, target, variable, pop.adj = FALSE)
```

#### **Arguments**

LPM.degree	integer; Degree for variable below target deviations. (degree = 0) is frequency, (degree = 1) is area.
UPM.degree	integer; Degree for variable above target deviations. (degree = $\emptyset$ ) is frequency, (degree = 1) is area.
target	numeric; Typically the mean of Variable X for classical statistics equivalences, but does not have to be. (Vectorized) (target = "mean") will set the target as the mean of every variable.
variable	a numeric matrix or data.frame.
pop.adj	logical; FALSE (default) Adjusts the sample co-partial moment matrices for population statistics.

## Value

Matrix of partial moment quadrant values. Uncalled quadrants will return a matrix of zeros.

# Note

For divergent asymmetical "D.LPM" and "D.UPM" matrices, matrix is D.LPM(column, row, ...).

# Author(s)

Fred Viole, OVVO Financial Systems

## References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

```
set.seed(123)
x <- rnorm(100) ; y <- rnorm(100) ; z <- rnorm(100)
A <- cbind(x,y,z)
PM.matrix(LPM.degree = 1, UPM.degree = 1, target = "mean", variable = A)
## Calling Individual Partial Moment Quadrants
cov.mtx = PM.matrix(LPM.degree = 1, UPM.degree = 1, target = "mean", variable = A)
cov.mtx$cupm</pre>
```

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```
## Full covariance matrix
cov.mtx$matrix
```

UPM

Upper Partial Moment

# Description

This function generates a univariate upper partial moment for any degree or target.

# Usage

```
UPM(degree, target, variable)
```

# Arguments

degree integer; (degree = 0) is frequency, (degree = 1) is area.

target numeric; Typically set to mean, but does not have to be. (Vectorized)

variable a numeric vector.

# Value

UPM of variable

# Author(s)

Fred Viole, OVVO Financial Systems

## References

```
Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" 
http://amzn.com/1490523995
```

```
set.seed(123)
x <- rnorm(100)
UPM(0, mean(x), x)</pre>
```

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UPM.ratio

Upper Partial Moment RATIO

## **Description**

This function generates a standardized univariate upper partial moment for any degree or target.

## Usage

```
UPM.ratio(degree, target, variable)
```

# Arguments

```
degree integer; (degree = 0) is frequency, (degree = 1) is area.
```

target numeric; Typically set to mean, but does not have to be. (Vectorized)

variable a numeric vector.

#### Value

Standardized UPM of variable

# Author(s)

Fred Viole, OVVO Financial Systems

## References

Viole, F. and Nawrocki, D. (2013) "Nonlinear Nonparametric Statistics: Using Partial Moments" http://amzn.com/1490523995

```
set.seed(123)
x <- rnorm(100)
UPM.ratio(0, mean(x), x)

## Joint Upper CDF
## Not run:
x <- rnorm(5000); y <- rnorm(5000)
plot3d(x, y, Co.UPM(0, 0, sort(x), sort(y), x, y), col = "blue", xlab = "X", ylab = "Y",
zlab = "Probability", box = FALSE)

## End(Not run)</pre>
```

44 UPM.VaR

# **Description**

Generates an upside VaR based on the Upper Partial Moment ratio

# Usage

```
UPM.VaR(percentile, degree, x)
```

## **Arguments**

percentile numeric [0, 1]; The percentile for right-tail VaR.

degree integer; (degree = 0) for discrete distributions, (degree = 1) for continuous

distributions.

x a numeric vector.

## Value

Returns a numeric value representing the point at which "percentile" of the area of x is below.

```
set.seed(123)
x <- rnorm(100)
## For 95th percentile VaR (right-tail)
UPM.VaR(0.95, 0, x)</pre>
```

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