Continuous CDFs and ANOVA with NNS

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1 Introduction

The following document provides a hands-on tutorial on discrete and continuous CDFs using partial moments and an extension to ANOVA analysis.

1.1 Install NNS

We need the latest version of NNS available on GitHub.

```
require(devtools); install_github('OVVO-Financial/NNS',ref = "NNS-Beta-Version")
require(NNS)
```

2 CDFs

Cumulative distribution functions (CDFs) represent the probability a variable X will take a value less than or equal to x.

$$F_X(x) = P(\leq x)$$

2.1 Empirical CDF

The empirical CDF is a simple construct, provided in the base package of R. We can generate an empirical CDF with the `ecdf` function and create a function `(P)` to return the CDF of a given value of X.

```
set.seed(123);x=rnorm(100)
ecdf(x)
```

```
## Empirical CDF
## Call: ecdf(x)
##  x[1:100] = -2.3092, -1.9666, -1.6867,  ...,  2.169, 2.1873
```

```
P=ecdf(x)
P(0);P(1)
```

```
## [1] 0.48
```

```
## [1] 0.83
```

2.1.1 Lower Partial Moment CDF (`LPM.CDF`)

The empirical CDF and Lower Partial Moment CDF (`LPM.CDF`) are identical when the degree term of the `LPM.CDF` is set to zero.

Using the same targets from our `ecdf` example above (0,1) we can compare `LPM.CDF`s.

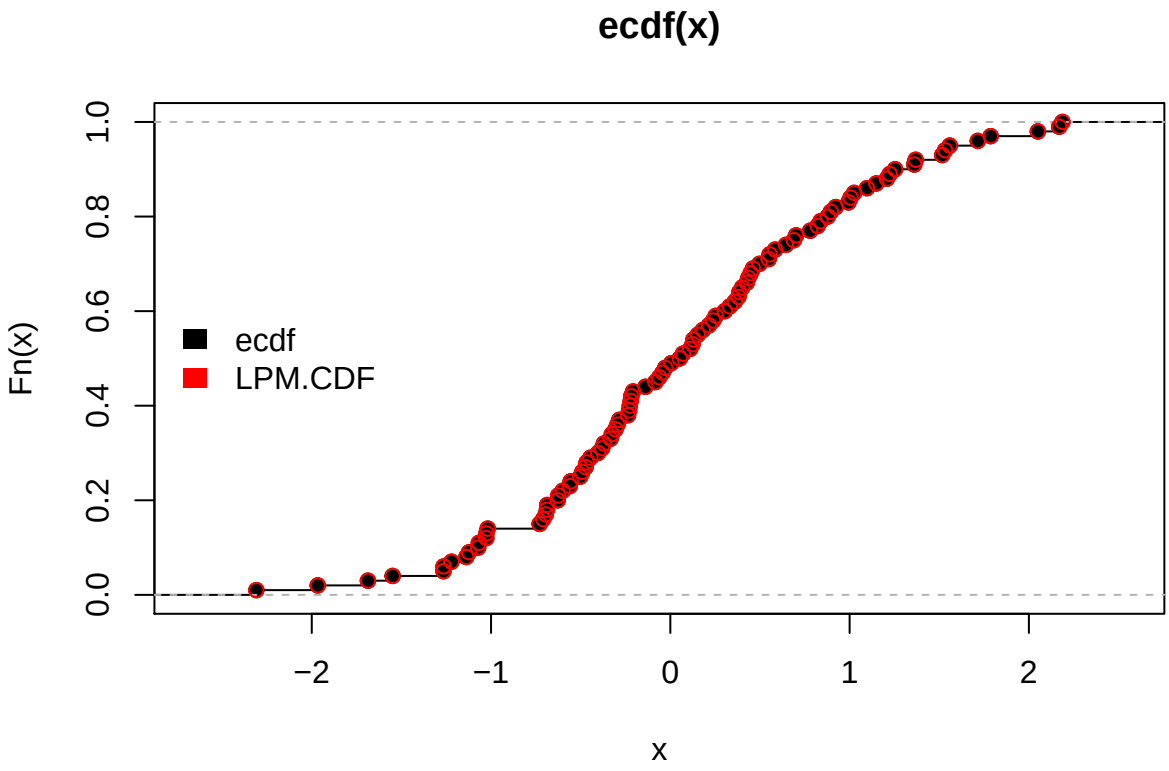
```
LPM(0,0,x);LPM(0,1,x)
```

```
## [1] 0.48
```

```
## [1] 0.83
```

Calculating the probability for every value in X, we can plot both methods visualizing their identical results. `ecdf` function in black and `LPM.CDF` in red.

```
plot(ecdf(x))
LPM.CDF=numeric()
for(i in 1:length(x)){LPM.CDF[i]=LPM(0,sort(x)[i],x)}
points(sort(x),LPM.CDF,col='red')
legend('left',legend = c('ecdf','LPM.CDF'),fill=c('black','red'),border=NA,bty='n')
```



2.1.2 Degree 1 LPM CDF (`LPM.1.CDF`)

We already noted how the `LPM.CDF` is equivalent to the empirical CDF when the LPM degree is set to zero. However, if we wish to consider the area below the point when generating a probability, set the degree equal to 1, for any target (t) creating `LPM.1.CDF`.

Degree 0 LPM:

$$LPM(0,t,X) = \frac{1}{N}\sum_{n=1}^{N}[max(t-X_n),0]^0$$

Degree 1 LPM:

$$LPM(1,t,X) = \frac{1}{N}\sum_{n=1}^{N}[max(t-X_n),0]^1$$

`LPM.CDF` is equivalent to the following form for any target (t) and variable X:

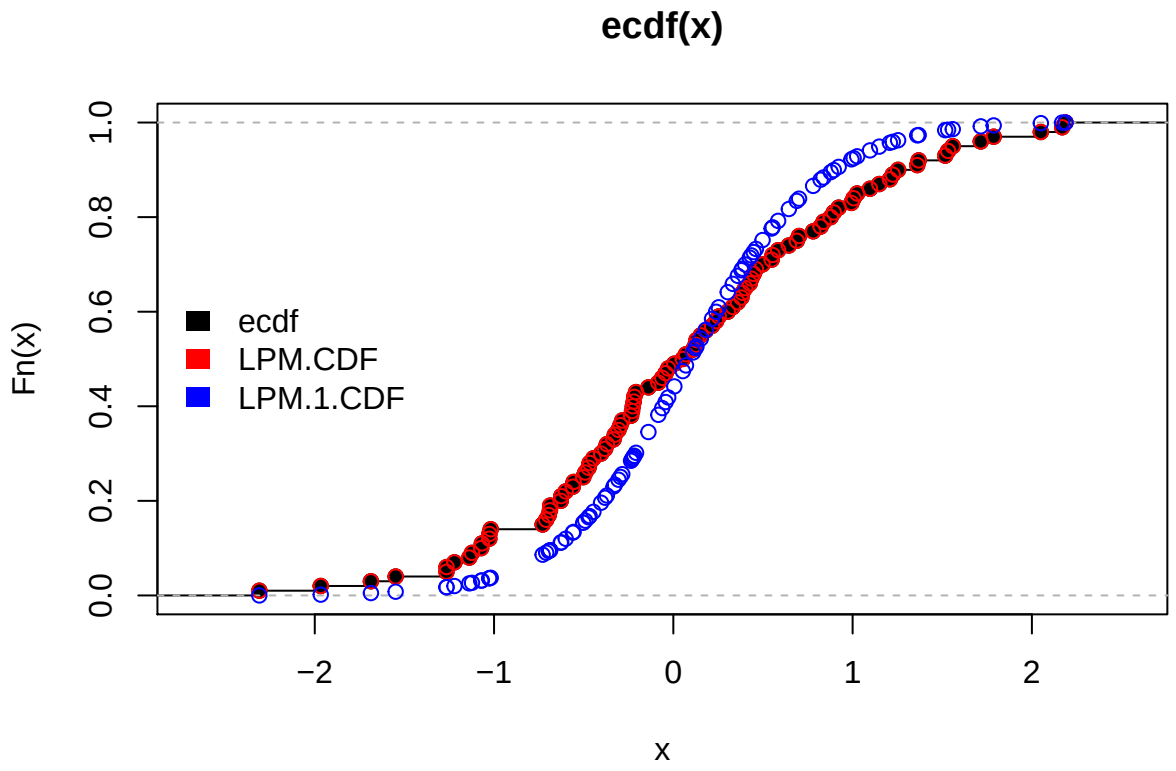
$$LPM(0,t,X) = \frac{LPM(0,t,X)}{LPM(0,t,X) + UPM(0,t,X)}$$

And `LPM.1.CDF` is equivalent to the following form for any target (t) and variable X:

$$LPM(1,t,X) = \frac{LPM(1,t,X)}{LPM(1,t,X) + UPM(1,t,X)}$$

Visualizing these CDFs together reveals some interesting properties.

```
plot(ecdf(x))
LPM.CDF=numeric()
for(i in 1:length(x)){LPM.CDF[i]=LPM(0,sort(x)[i],x)}
points(sort(x),LPM.CDF,col='red')
LPM.1.CDF=numeric()
for(i in 1:length(x)){LPM.1.CDF[i]=LPM(1,sort(x)[i],x)/
(LPM(1,sort(x)[i],x)+UPM(1,sort(x)[i],x))}
points(sort(x),LPM.1.CDF,col='blue')
legend('left',legend = c('ecdf','LPM.CDF','LPM.1.CDF'),fill=c('black','red','blue'),
border=NA,bty='n')
```

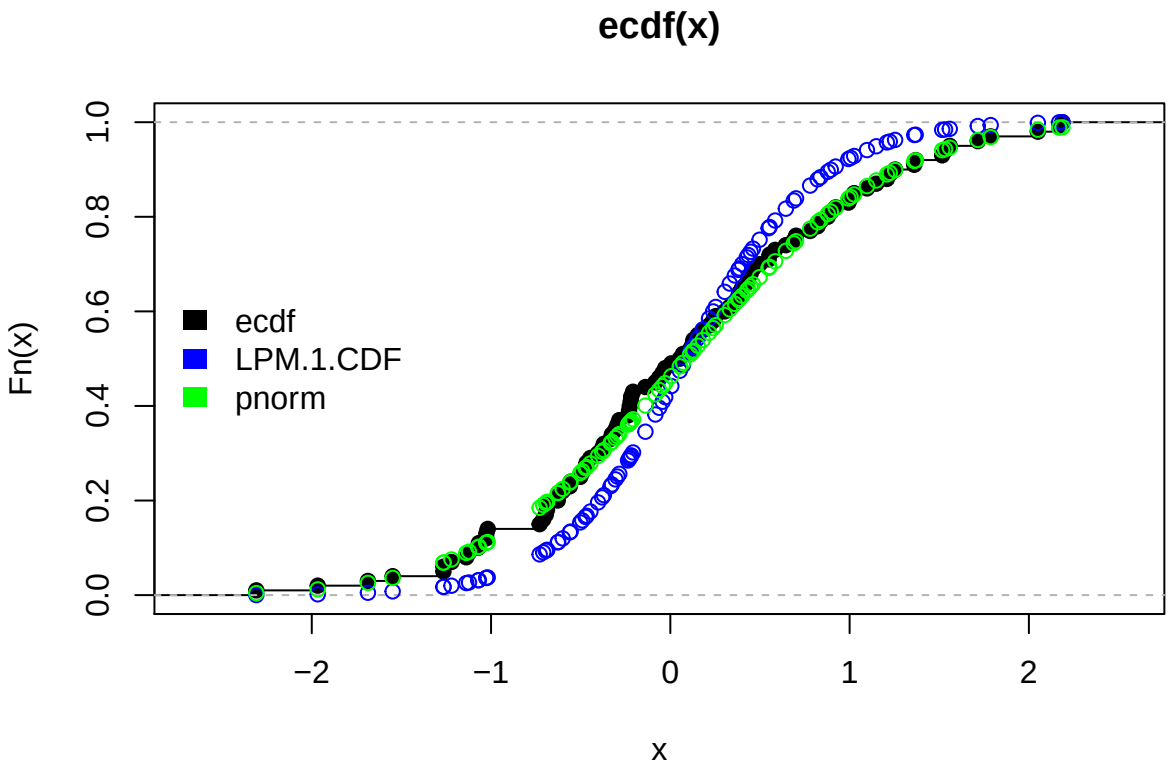


3 Comparing to Continuous CDF

R offers the `prnorm` function which returns the integral from -∞ to x where x is a Z-score.

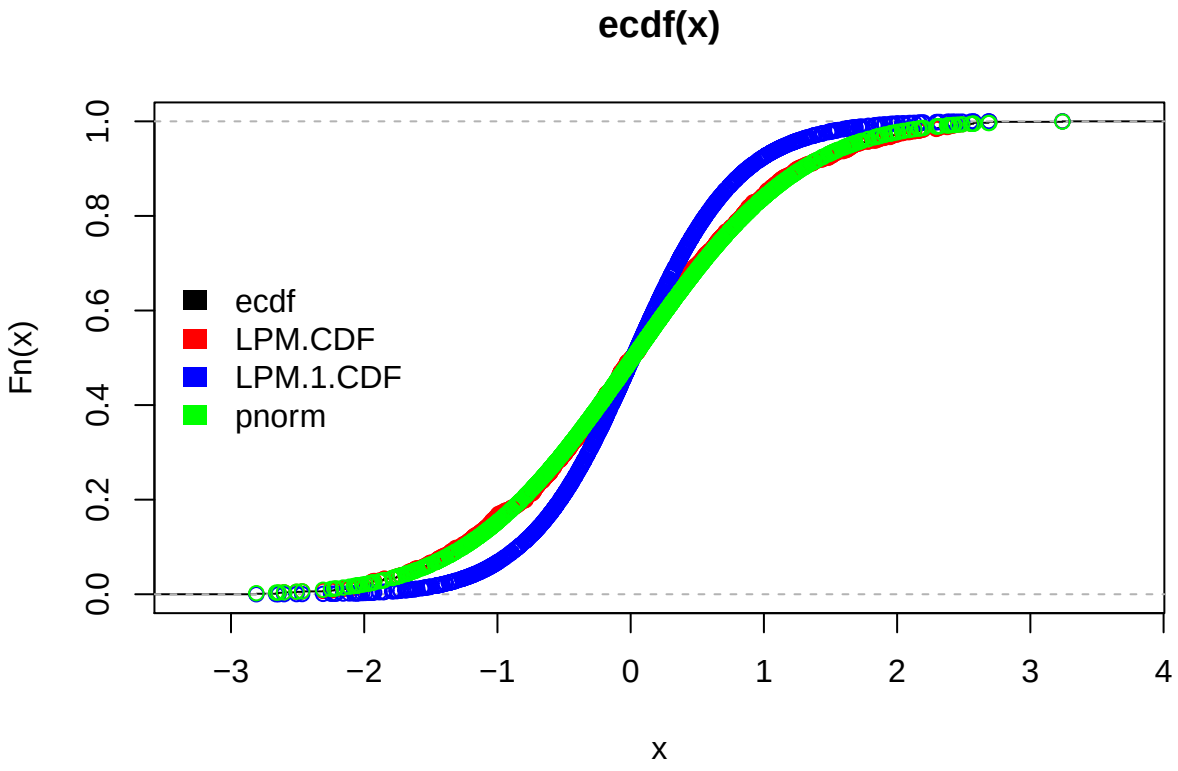
Adding this to our previous CDF estimates, we can see how `LPM.1.CDF` is more "S" shaped than the others.

```
plot(ecdf(x))
points(sort(x),LPM.1.CDF,col='blue')
points(sort(x),pnorm(sort(x),mean=mean(x),sd=sd(x)),col='green')
legend('left',legend = c('ecdf','LPM.1.CDF','pnorm'),fill=c('black','blue','green'),
border=NA,bty='n')
```



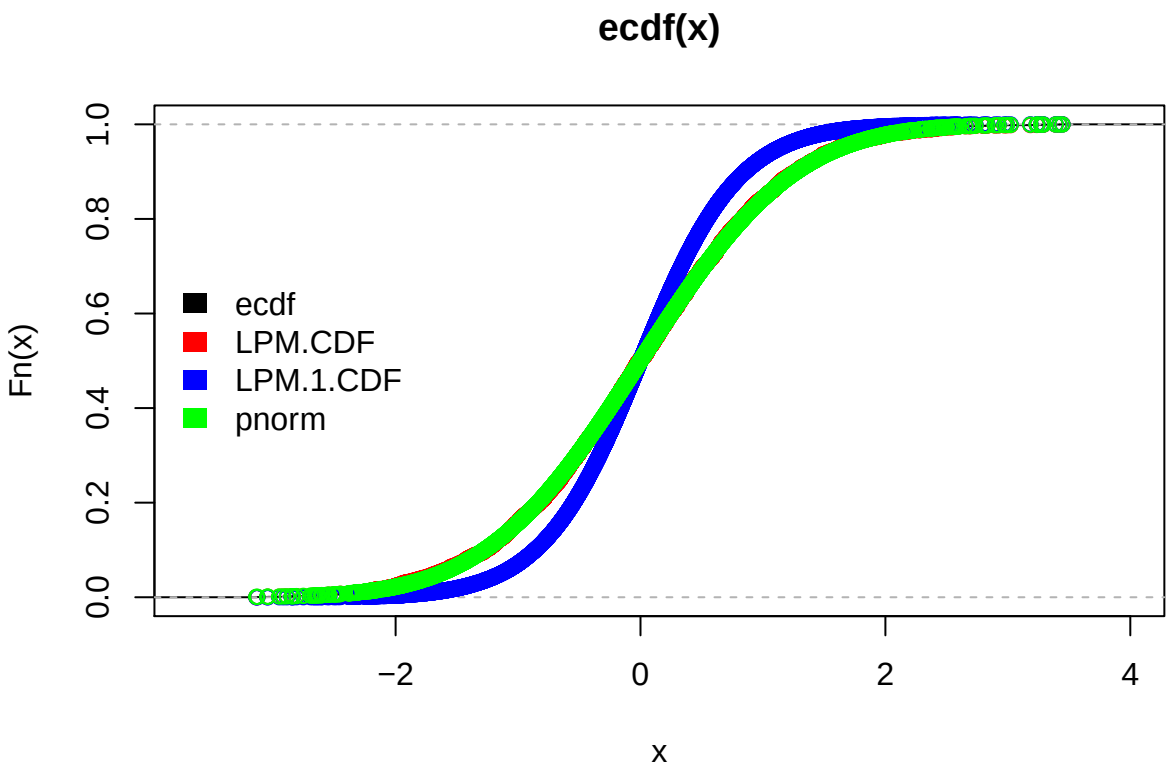
Extending the number of observations we can see how these functions behave. The `ecdf` and the `pnorm` are converging while `LPM.1.CDF` remains distinct:

```
set.seed(123);x=rnorm(1000)
plot(ecdf(x))
LPM.CDF=numeric()
for(i in 1:length(x)){LPM.CDF[i]=LPM(0,sort(x)[i],x)}
points(sort(x),LPM.CDF,col='red')
LPM.1.CDF=numeric()
for(i in 1:length(x)){LPM.1.CDF[i]=LPM.ratio(1,sort(x)[i],x)}
points(sort(x),LPM.1.CDF,col='blue')
points(sort(x),pnorm(sort(x),mean=mean(x),sd=sd(x)),col='green')
legend('left',legend = c('ecdf','LPM.CDF','LPM.1.CDF','pnorm'),fill=c('black','red',
'blue','green'),border=NA,bty='n')
```



Even more observations...

```
set.seed(123);x=rnorm(5000)
plot(ecdf(x))
LPM.CDF=numeric()
for(i in 1:length(x)){LPM.CDF[i]=LPM(0,sort(x)[i],x)}
points(sort(x),LPM.CDF,col='red')
LPM.1.CDF=numeric()
for(i in 1:length(x)){LPM.1.CDF[i]=LPM.ratio(1,sort(x)[i],x)}
points(sort(x),LPM.1.CDF,col='blue')
points(sort(x),pnorm(sort(x),mean=mean(x),sd=sd(x)),col='green')
legend('left',legend = c('ecdf','LPM.CDF','LPM.1.CDF','pnorm'),fill=c('black','red',
'blue','green'),border=NA,bty='n')
```



4 Properties

4.1 Smoothness / Steepness

One property that immediately presents itself is the smoothness of `LPM.1.CDF` especially for smaller sample sizes. Remember, this too is an empirical CDF, however, it is an area based probability and these area considerations are directly responsible for its distinct smoothness.

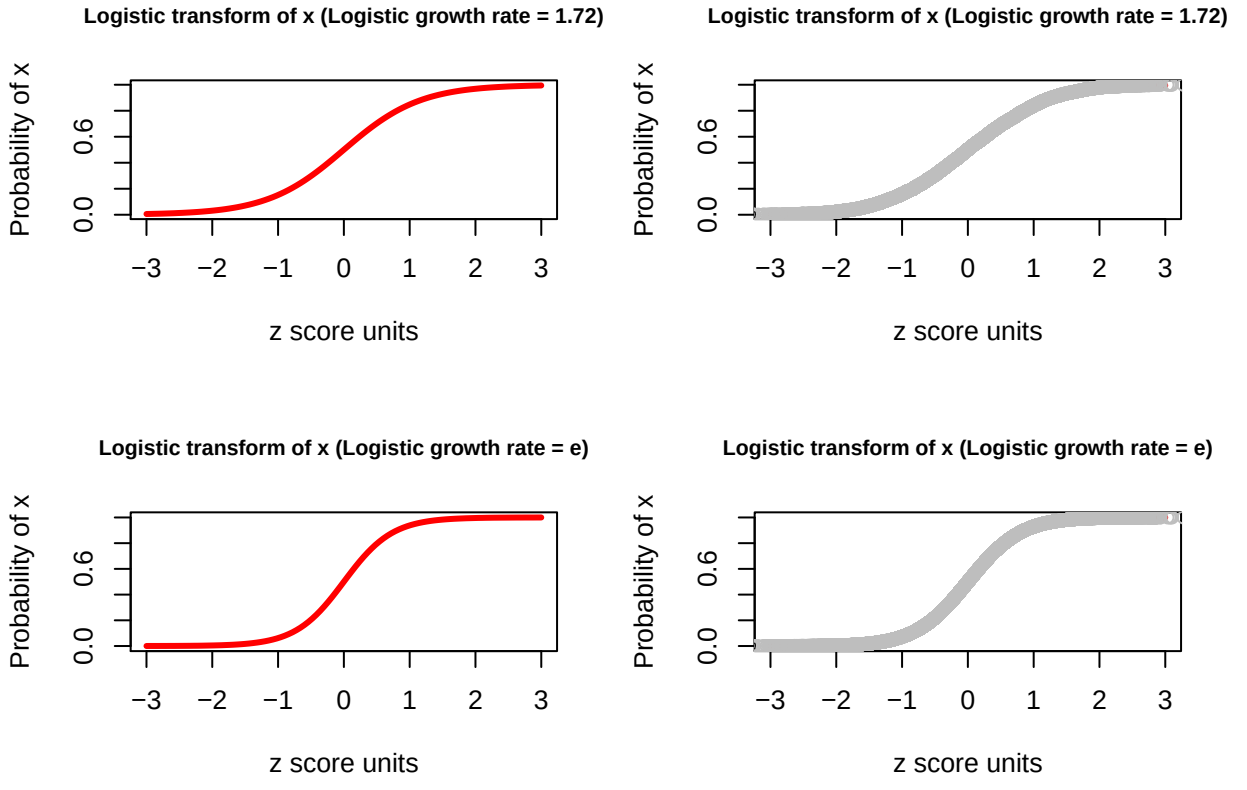
Another property is the steepness. This too can be attributed to the `LPM.1` being an area based statistic and having the corresponding Logistic growth rate equal to e. Below is a comparison of the discrete `LPM.CDF` and the continuous `LPM.1.CDF` (in grey) with the respective Logistic transofrmation growth rates of 1.72 for the Normal approximation, and e (in red).

```
require(NNS); require(psych)
```

```
## Loading required package: psych
```

```
x=rnorm(10000)

par(mfrow=c(2,2))


#Logistic growth rate equal to 1.72
curve(logistic(x,a=1.72),-3,3,ylab="Probability of x",
      main="Logistic transform of x (Logistic growth rate = 1.72)",
      xlab="z score units", col='red', lwd=3,cex.main=0.85)
```

```r
#Redraw for comparison
curve(logistic(x,a=1.72),-3,3,ylab="Probability of x",
      main="Logistic transform of x (Logistic growth rate = 1.72)",
      xlab="z score units", col='red',lwd=3,cex.main=0.85)
#Discrete Degee 0 LPM CDF
points(sort(x),LPM(0,sort(x),x),col='grey')



#Logistic growth rate equal to e
curve(logistic(x,a=exp(1)),-3,3,ylab="Probability of x",
      main="Logistic transform of x (Logistic growth rate = e)",
      xlab="z score units", col='red', lwd=3,cex.main=0.85)

#Redraw for comparison
curve(logistic(x,a=exp(1)),-3,3,ylab="Probability of x",
      main="Logistic transform of x (Logistic growth rate = e)",
      xlab="z score units", col='red', lwd=3,cex.main=0.85)
#Continuous Degree 1 LPM CDF
points(sort(x),LPM.ratio(1,sort(x),x),col='grey')
```




```r
par(mfrow=(c(1,1)))
```

4.2 Mean Target

For every observation of every type of distribution, `LPM.1.CDF` is equal to 0.5 when the mean is used as a target.

$$\frac{LPM(1, \mu, X)}{LPM(1, \mu, X) + UPM(1, \mu, X)} = 0.5$$

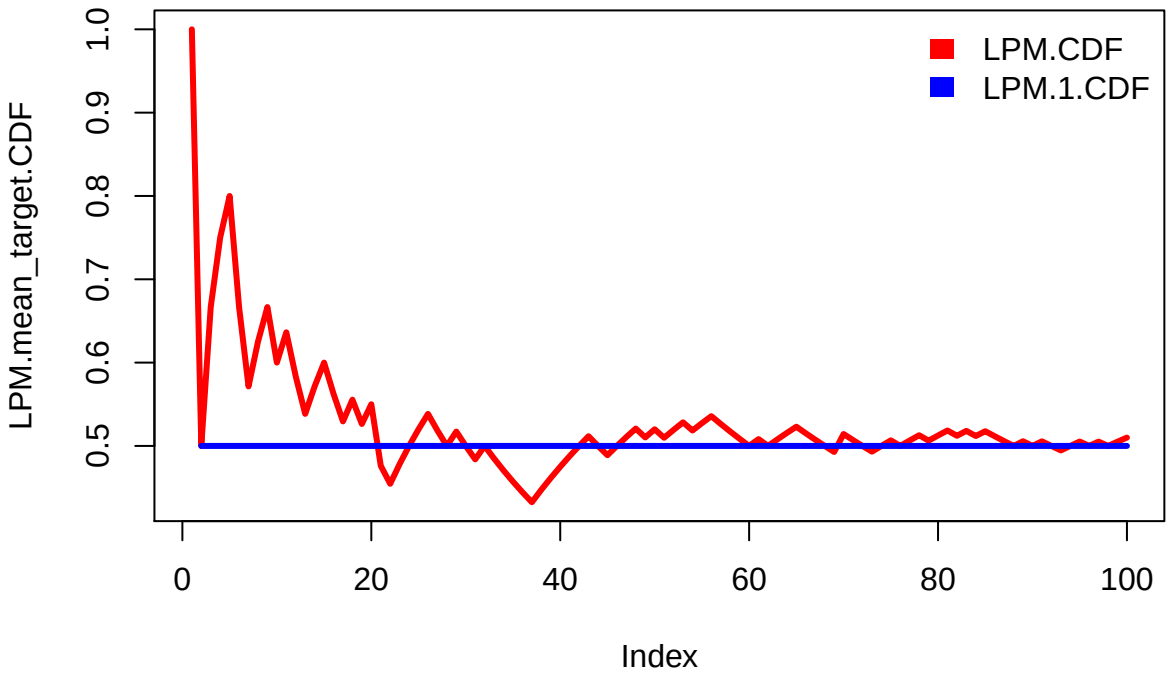
This bears repeating, "***For every observation of every type of distribution***". This is a very special property that can be used to compare means of distributions without underlying assumptions by using their CDFs, and serves as the basis of the `NNS.ANOVA` method.

5 Distribution Examples

The following examples highlight the `LPM.1.CDF` stability from a mean target.

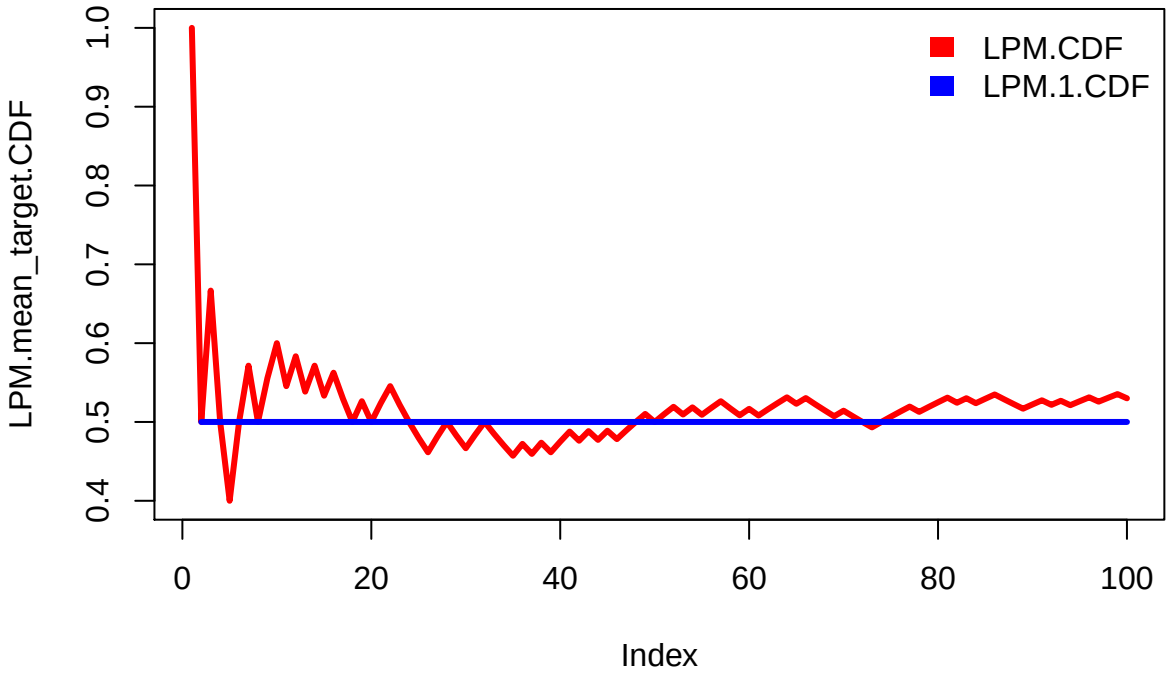
5.1 Normal Distribution

```
set.seed(123);x=rnorm(100)
LPM.mean_target.CDF=numeric();LPM.1.mean_target.CDF=numeric()
for(i in 1:length(x)){
LPM.mean_target.CDF[i]=LPM(0,mean(x[1:i]),x[1:i]);
LPM.1.mean_target.CDF[i]=LPM.ratio(1,mean(x[1:i]),x[1:i])}
plot(LPM.mean_target.CDF,col='red',type = 'l',lwd=3)
lines((1:100),LPM.1.mean_target.CDF,col='blue',lwd=3)
legend('topright',legend = c('LPM.CDF','LPM.1.CDF'),fill=c('red','blue'),
border=NA,bty='n')
```



5.2 Uniform Distribution

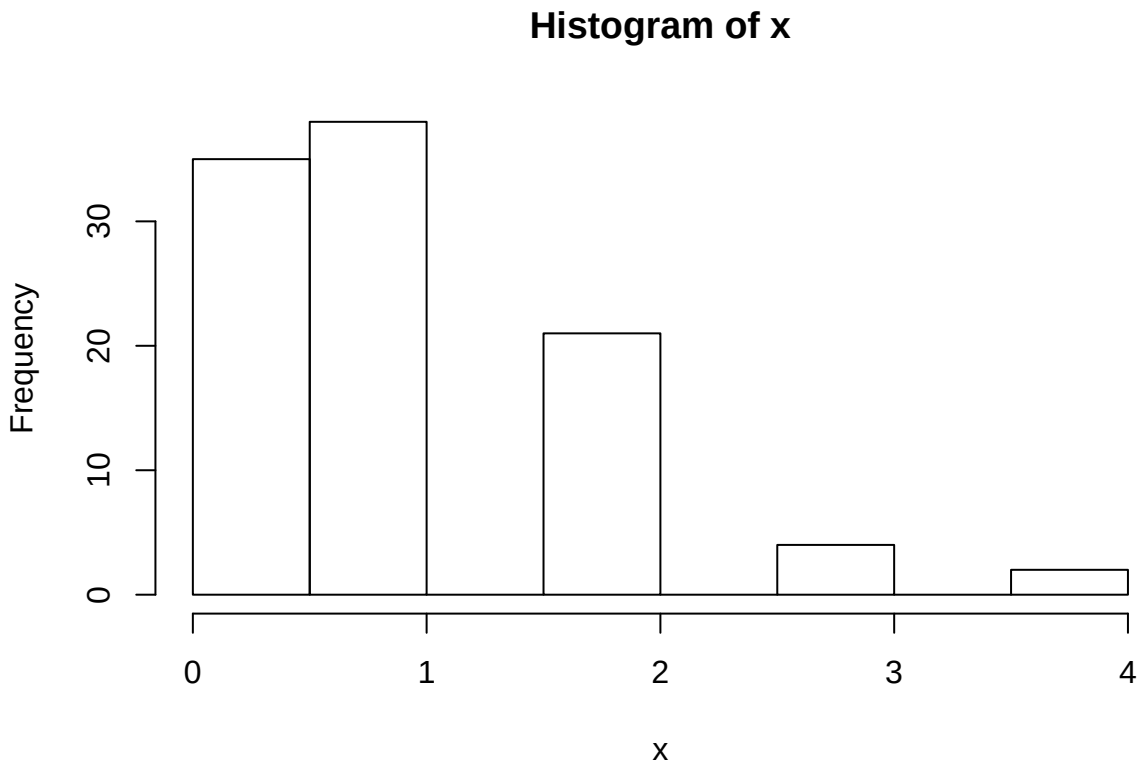
```
set.seed(123);x=runif(100)
LPM.mean_target.CDF=numeric();LPM.1.mean_target.CDF=numeric()
for(i in 1:length(x)){
LPM.mean_target.CDF[i]=LPM(0,mean(x[1:i]),x[1:i]);
LPM.1.mean_target.CDF[i]=LPM.ratio(1,mean(x[1:i]),x[1:i])}
plot(LPM.mean_target.CDF,col='red',type = 'l',lwd=3)
lines((1:100),LPM.1.mean_target.CDF,col='blue',lwd=3)
legend('topright',legend = c('LPM.CDF','LPM.1.CDF'),fill=c('red','blue'),
border=NA,bty='n')
```



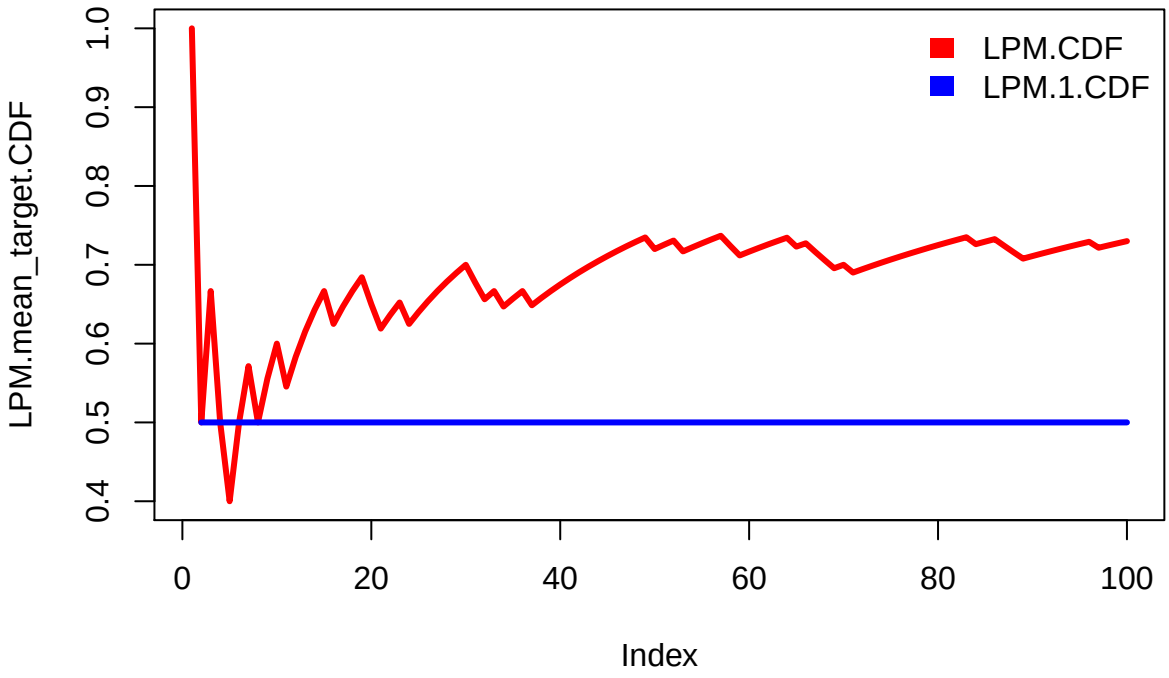
5.3 Poisson Distribution

This example demonstrates the non-association between frequency and area per the LPM CDFs from a mean target ($\lambda = 1$) in this distribution.

```
set.seed(123);x=rpois(100,lambda = 1)
hist(x)
```




```
set.seed(123);x=rpois(100,lambda = 1)
LPM.mean_target.CDF=numeric();LPM.1.mean_target.CDF=numeric()
for(i in 1:length(x)){
LPM.mean_target.CDF[i]=LPM(0,mean(x[1:i]),x[1:i]);
LPM.1.mean_target.CDF[i]=LPM.ratio(1,mean(x[1:i]),x[1:i])}
plot(LPM.mean_target.CDF,col='red',type = 'l',lwd=3)
lines((1:100),LPM.1.mean_target.CDF,col='blue',lwd=3)
legend('topright',legend = c('LPM.CDF','LPM.1.CDF'),fill=c('red','blue'),
border=NA,bty='n')
```


LPM.CDF
LPM.1.CDF
LPM.mean_target.CDF
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0
20
40
60
80
100
Index

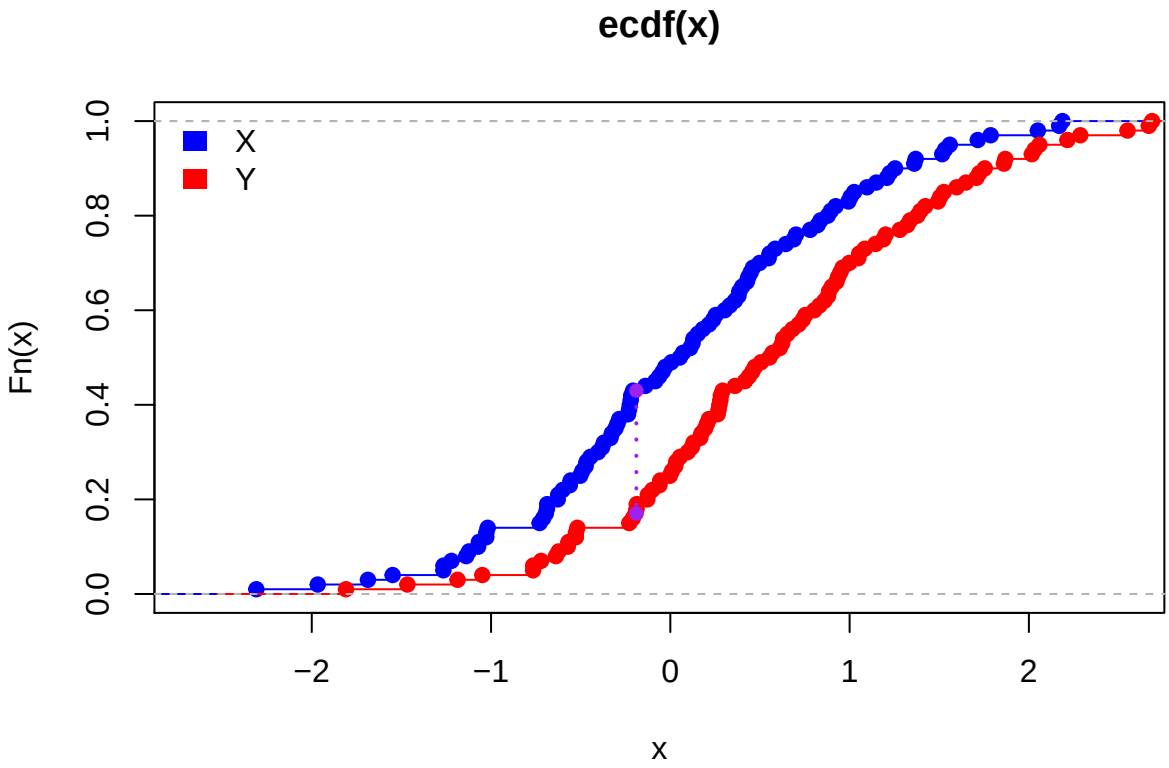
6 ANOVA Analysis

Using our previously derived information, we can effectively compare two means and ascertain a degree of certainty $[0, 1]$ whether the difference in means is zero via their CDFs. This avoids any and all distributional assumptions and requirements for the underlying data.

6.1 Kolmogorov-Smirnov (KS)

The KS statistic measures the maximum distance between the two empirical CDFs of two given samples in testing whether the samples were drawn from the same distribution. KS shown in purple below...

```
set.seed(123);x=rnorm(100);y=x+.5
plot(ecdf(x),col='blue')
lines(ecdf(y),col='red')
legend('topleft',legend = c('X','Y'),fill=c('blue','red'),border=NA,bty='n')

points(c(x0, x0), c(y0, y1), pch=16, col="purple")
segments(x0, y0, x0, y1, col="purple", lty="dotted",lwd=2)
```



6.2 NNS ANOVA

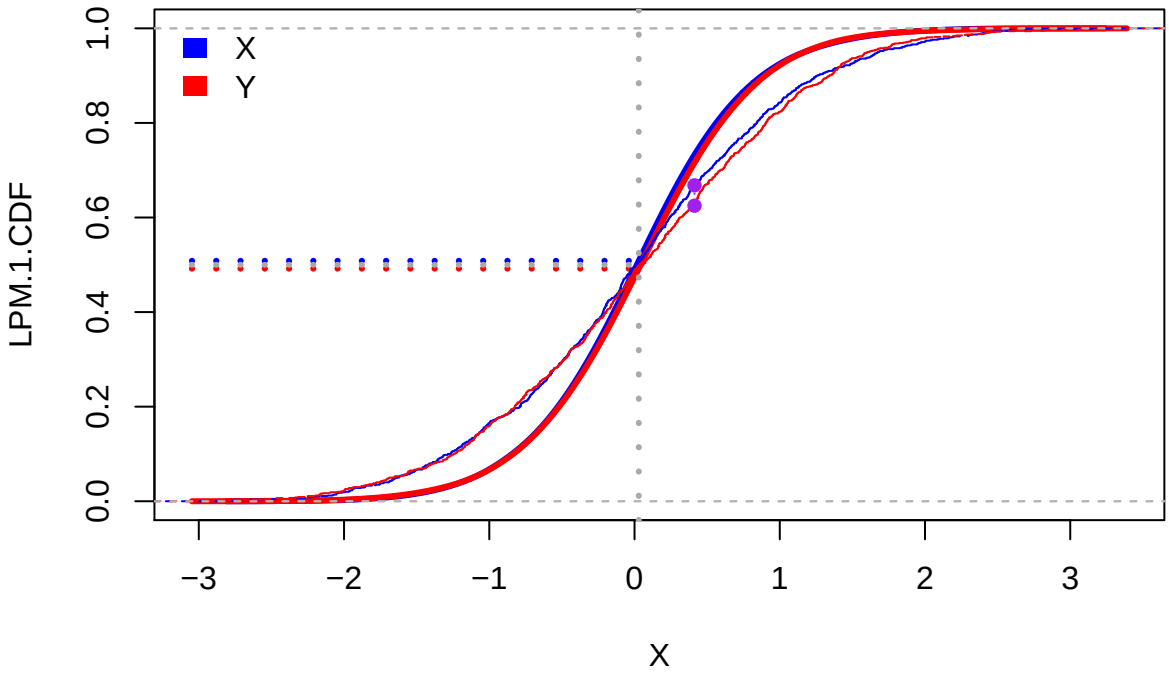
NNS also analyzes the difference in CDFs of the two samples, but it is performed at the grand mean ($\frac{\mu_x+\mu_y}{2}$) between distributions, which can be generalized to multiple distributions. This specific grand mean point analysis coupled with the `LPM.1.CDF` is what distinguishes NNS from KS.

For some more related information on the NNS certainty statistic, it is approximately equal to the average difference of the $\frac{\texttt{upper area}}{\texttt{lower area}}$ of each distribution from the grand mean. These $\frac{UPM}{LPM}$ measures (a more intuitive measure of skewness) are presented for reference.

6.2.1 Same Population

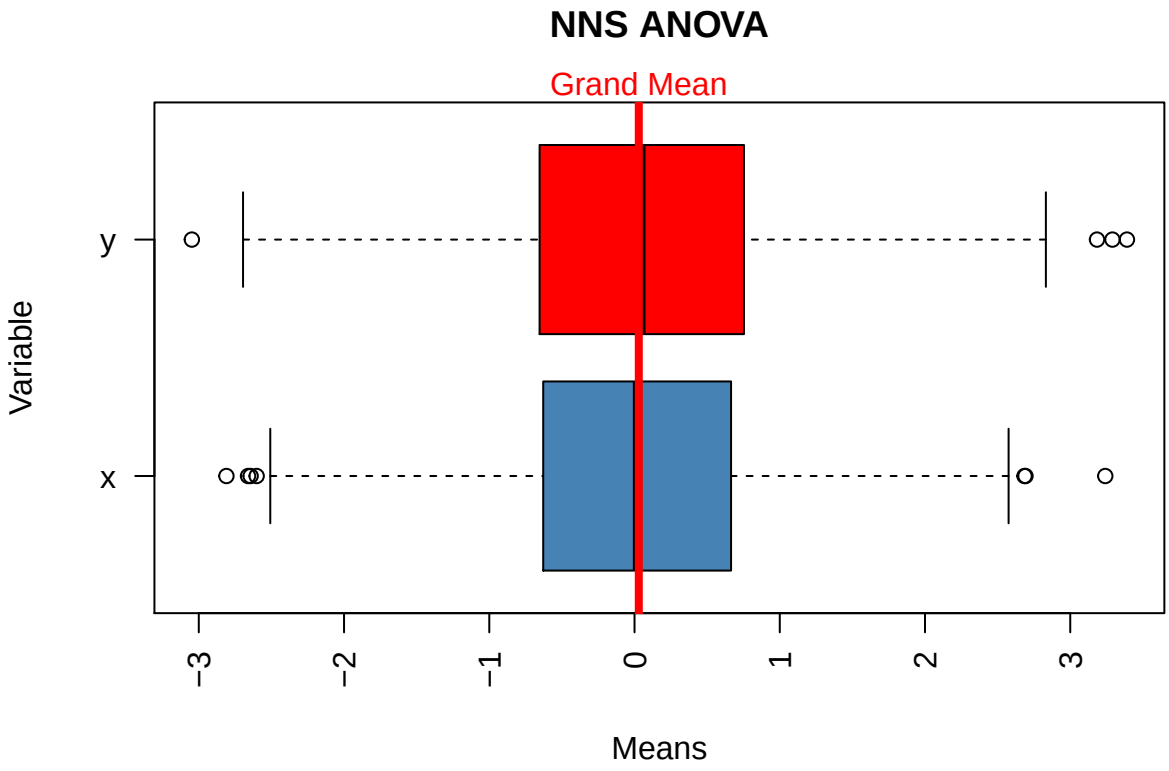
If both means were equal and the samples were from the same population, the difference between CDFs at the grand mean would be nil. ***We know unequivocally that the `LPM.1.CDF` will equal 0.5 from the mean of a given sample. Thus two samples with identical means will both have `LPM.1.CDF` equal to 0.5 if they are from the same population.*** The `LPM.1.CDF` lines are the thick blue and red lines (for X and Y respectively), while the ECDF lines for the KS test are the thin lines.

```
set.seed(123);x=rnorm(1000);y=rnorm(1000)
LPM.1.CDF.x=numeric();LPM.1.CDF.y=numeric()
for(i in 1:length(x)){LPM.1.CDF.x[i]=LPM.ratio(1,sort(x)[i],x)
LPM.1.CDF.y[i]=LPM.ratio(1,sort(y)[i],y)}
plot(sort(x),LPM.1.CDF.x,col='blue',type='l',lwd=3,
     xlim=c(min(y),max(y)),xlab='X',ylab="LPM.1.CDF")
points(sort(y),LPM.1.CDF.y,col='red',type='l',lwd=3)
abline(v=mean(c(mean(x),mean(y))),col='darkgrey',lty=3,lwd=3)
target=mean(c(mean(x),mean(y)))
segments(x0=min(y),y0=LPM(1,target,y)/(LPM(1,target,y)+UPM(1,target,y)),
         x1=target,y1=LPM(1,target,y)/(LPM(1,target,y)+UPM(1,target,y)),
           col='red',lty=3,lwd=3)
segments(x0=min(y),y0=LPM(1,target,x)/(LPM(1,target,x)+UPM(1,target,x)),
         x1=target,y1=LPM(1,target,x)/(LPM(1,target,x)+UPM(1,target,x)),
           col='blue',lty=3,lwd=3)
segments(x0=min(y),y0=.5,
         x1=target,y1=.5,
           col='darkgrey',lty=3,lwd=3)

#KS line based off of ECDFs not LPM.1.CDF
lines(ecdf(x),lty=2,col='blue');lines(ecdf(y),lty=2,col='red')
cdf1 <- ecdf(x);cdf2 <- ecdf(y)
minMax <- seq(min(x, y), max(x, y), length.out=length(x))
x0<- minMax[which( abs(cdf1(minMax) - cdf2(minMax)) == max(abs(cdf1(minMax) - cdf2(minMax))) )]
y0 <- cdf1(x0)
y1 <- cdf2(x0)
points(c(x0, x0), c(y0, y1), pch=16, col="purple")
segments(x0, y0, x0, y1, col="purple", lty="dotted")
legend('topleft',legend = c('X','Y'),fill=c('blue','red'),border=NA,bty='n')
```



Comparing NNS CDF analysis (degree of certainty of the null hypothesis that the difference in means is zero) with a t-test (and its infamous p-value) and a ks.test:

```
NNS.ANOVA(cbind(x,y))
```




```
## Certainty
## 0.9672646
```

```
# Partial moments skew
UPM(1,target,x)/LPM(1,target,x)
```

```
## [1] 0.9670253
```

```
UPM(1,target,y)/LPM(1,target,y)
```

```
## [1] 1.033024
```

```
# Average difference in partial moments skew
(UPM(1,target,x)/LPM(1,target,x)-UPM(1,target,y)/LPM(1,target,y))/2
```

```
## [1] -0.03299957
```

```
t.test(x,y)
```

```
##
##  Welch Two Sample t-test
##
## data:  x and y
## t = -0.5885, df = 1997.4, p-value = 0.5563
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.1141064  0.0614316
## sample estimates:
##  mean of x  mean of y
```

```
## 0.01612787 0.04246525
```

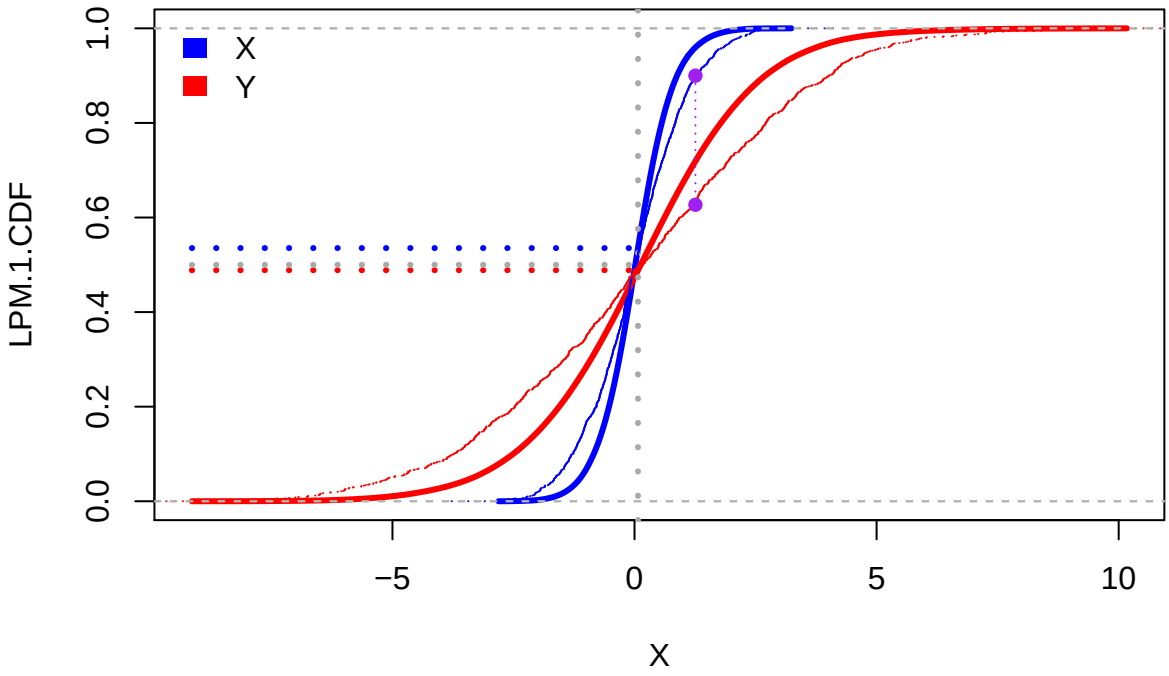
```
ks.test(x,y)
```

```
##
##  Two-sample Kolmogorov-Smirnov test
##
## data:  x and y
## D = 0.043, p-value = 0.3136
## alternative hypothesis: two-sided
```

6.2.2 Different Populations

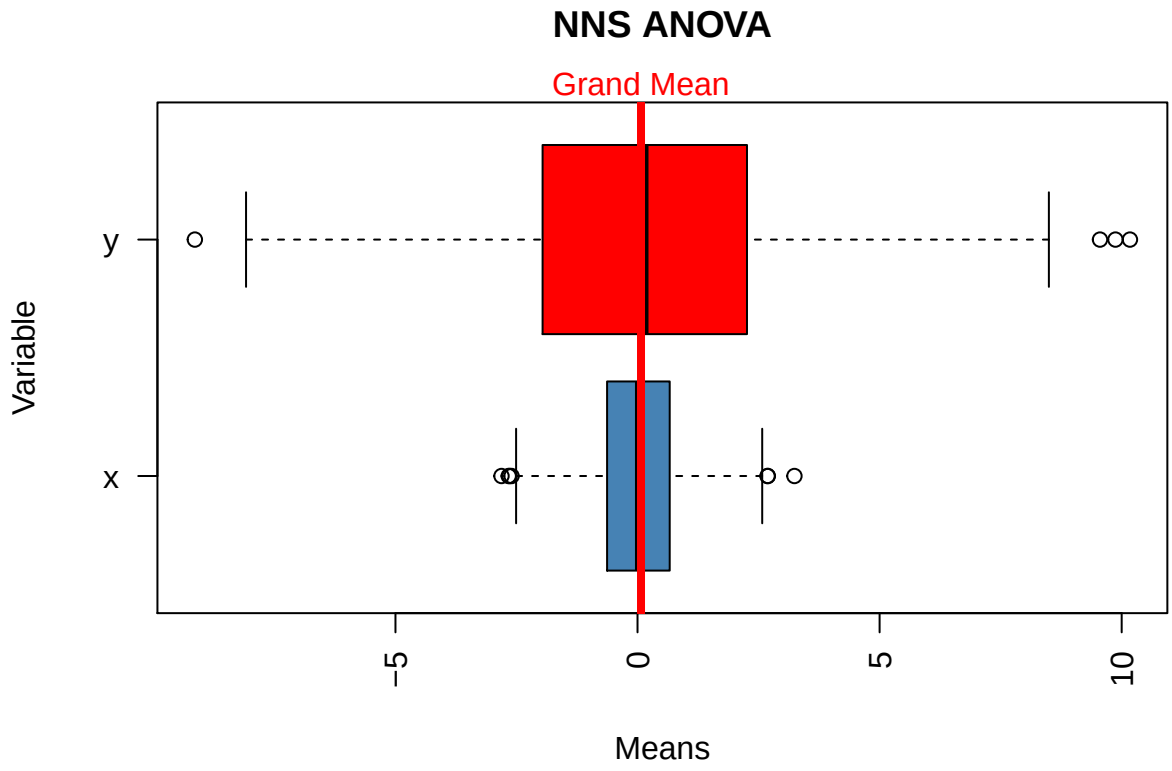
If both means were equal and the samples were from the different populations, the difference between CDFs at the grand mean would be evident.

```
set.seed(123);x=rnorm(1000);y=rnorm(1000,sd=3)
LPM.1.CDF.x=numeric();LPM.1.CDF.y=numeric()
for(i in 1:length(x)){LPM.1.CDF.x[i]=LPM.ratio(1,sort(x)[i],x)
LPM.1.CDF.y[i]=LPM.ratio(1,sort(y)[i],y)}
plot(sort(x),LPM.1.CDF.x,col='blue',type='l',lwd=3,
     xlim=c(min(y),max(y)),xlab='X',ylab="LPM.1.CDF")
points(sort(y),LPM.1.CDF.y,col='red',type='l',lwd=3)
abline(v=mean(c(mean(x),mean(y))),col='darkgrey',lty=3,lwd=3)
target=mean(c(mean(x),mean(y)))
segments(x0=min(y),y0=LPM(1,target,y)/(LPM(1,target,y)+UPM(1,target,y)),
         x1=target,y1=LPM(1,target,y)/(LPM(1,target,y)+UPM(1,target,y)),
           col='red',lty=3,lwd=3)
segments(x0=min(y),y0=LPM(1,target,x)/(LPM(1,target,x)+UPM(1,target,x)),
         x1=target,y1=LPM(1,target,x)/(LPM(1,target,x)+UPM(1,target,x)),
           col='blue',lty=3,lwd=3)
segments(x0=min(y),y0=.5,
         x1=target,y1=.5,
           col='darkgrey',lty=3,lwd=3)
#KS line
lines(ecdf(x),lty=3,col='blue');lines(ecdf(y),lty=3,col='red')
cdf1 <- ecdf(x);cdf2 <- ecdf(y)
minMax <- seq(min(x, y), max(x, y), length.out=length(x))
x0<- minMax[which( abs(cdf1(minMax) - cdf2(minMax)) == max(abs(cdf1(minMax) - cdf2(minMax))) )]
y0 <- cdf1(x0)
y1 <- cdf2(x0)
points(c(x0, x0), c(y0, y1), pch=16, col="purple")
segments(x0, y0, x0, y1, col="purple", lty="dotted")
legend('topleft',legend = c('X','Y'),fill=c('blue','red'),border=NA,bty='n')
```



Comparing NNS CDF analysis with a t-test and a KS-test:

```
NNS.ANOVA(cbind(x,y))
```




```
## Certainty
## 0.9085942
```

```
# Partial moments skew
UPM(1,target,x)/LPM(1,target,x)
```

```
## [1] 0.8678981
```

```
UPM(1,target,y)/LPM(1,target,y)
```

```
## [1] 1.046818
```

```
# Average difference in partial moments skew
(UPM(1,target,x)/LPM(1,target,x)-UPM(1,target,y)/LPM(1,target,y))/2
```

```
## [1] -0.08945998
```

```
t.test(x,y)
```

```
##
##  Welch Two Sample t-test
##
## data:  x and y
## t = -1.104, df = 1210.7, p-value = 0.2698
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.30900852  0.08647273
## sample estimates:
##  mean of x  mean of y
```

```
## 0.01612787 0.12739576
```

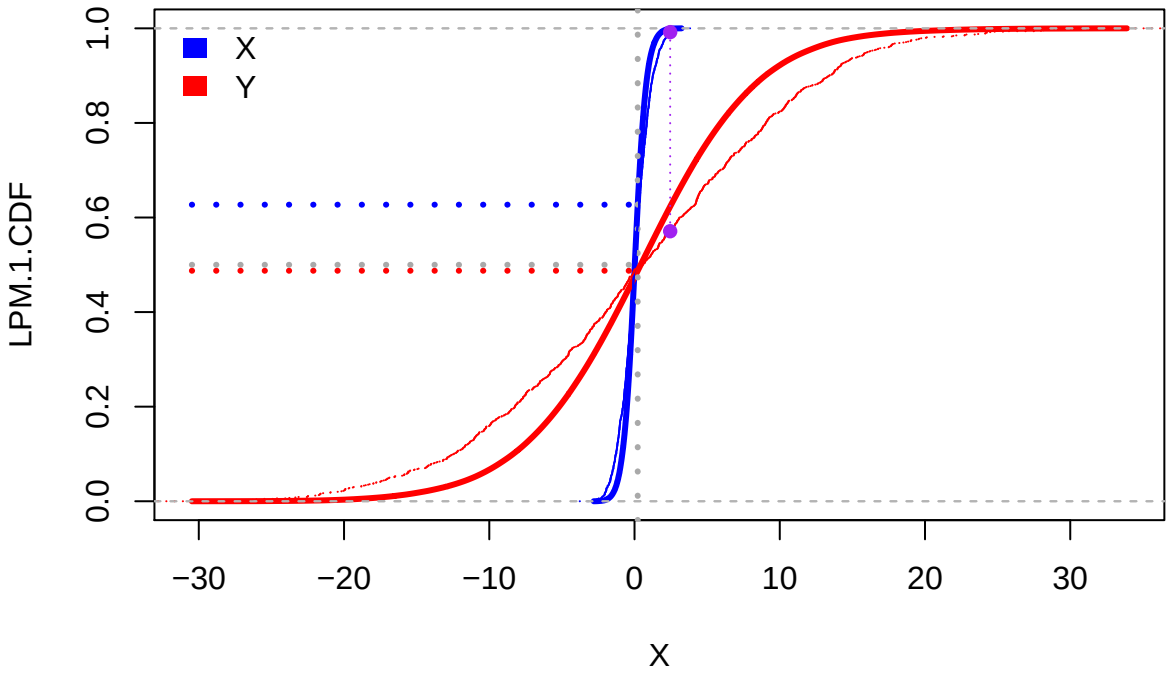
```
ks.test(x,y)
```

```
##
##  Two-sample Kolmogorov-Smirnov test
##
## data:  x and y
## D = 0.273, p-value < 2.2e-16
## alternative hypothesis: two-sided
```

6.2.3 VERY Different Populations

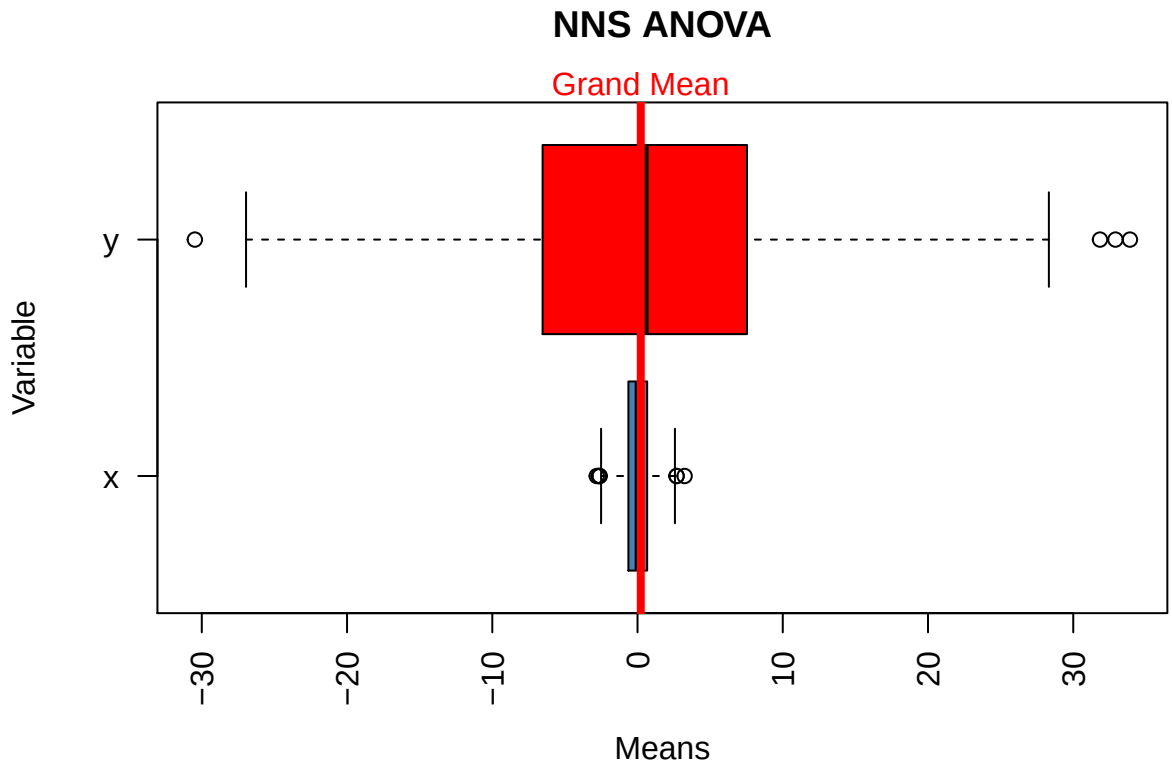
Note the difference between the NNS certainty vs. the t-test p-value...

```
set.seed(123);x=rnorm(1000);y=rnorm(1000,sd=10)
LPM.1.CDF.x=numeric();LPM.1.CDF.y=numeric()
for(i in 1:length(x)){LPM.1.CDF.x[i]=LPM.ratio(1,sort(x)[i],x)
LPM.1.CDF.y[i]=LPM.ratio(1,sort(y)[i],y)}
plot(sort(x),LPM.1.CDF.x,col='blue',type='l',lwd=3,
     xlim=c(min(y),max(y)),xlab='X',ylab="LPM.1.CDF")
points(sort(y),LPM.1.CDF.y,col='red',type='l',lwd=3)
abline(v=mean(c(mean(x),mean(y))),col='darkgrey',lty=3,lwd=3)
target=mean(c(mean(x),mean(y)))
segments(x0=min(y),y0=LPM(1,target,y)/(LPM(1,target,y)+UPM(1,target,y)),
         x1=target,y1=LPM(1,target,y)/(LPM(1,target,y)+UPM(1,target,y)),
           col='red',lty=3,lwd=3)
segments(x0=min(y),y0=LPM(1,target,x)/(LPM(1,target,x)+UPM(1,target,x)),
         x1=target,y1=LPM(1,target,x)/(LPM(1,target,x)+UPM(1,target,x)),
           col='blue',lty=3,lwd=3)
segments(x0=min(y),y0=.5,
         x1=target,y1=.5,
           col='darkgrey',lty=3,lwd=3)
#KS line
lines(ecdf(x),lty=3,col='blue');lines(ecdf(y),lty=3,col='red')
cdf1 <- ecdf(x);cdf2 <- ecdf(y)
minMax <- seq(min(x, y), max(x, y), length.out=length(x))
x0<- minMax[which( abs(cdf1(minMax) - cdf2(minMax)) == max(abs(cdf1(minMax) - cdf2(minMax))) )]
y0 <- cdf1(x0)
y1 <- cdf2(x0)
points(c(x0, x0), c(y0, y1), pch=16, col="purple")
segments(x0, y0, x0, y1, col="purple", lty="dotted")
legend('topleft',legend = c('X','Y'),fill=c('blue','red'),border=NA,bty='n')
```



Comparing NNS CDF analysis with a t-test and KS-test:

```
NNS.ANOVA(cbind(x,y))
```




```
## Certainty
## 0.7401294
```

```
# Partial moments skew
UPM(1,target,x)/LPM(1,target,x)
```

```
## [1] 0.5946536
```

```
UPM(1,target,y)/LPM(1,target,y)
```

```
## [1] 1.05169
```

```
# Average difference in partial moments skew
(UPM(1,target,x)/LPM(1,target,x)-UPM(1,target,y)/LPM(1,target,y))/2
```

```
## [1] -0.228518
```

```
t.test(x,y)
```

```
##
##  Welch Two Sample t-test
##
## data:  x and y
## t = -1.2734, df = 1018.3, p-value = 0.2032
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -1.0380753  0.2210259
## sample estimates:
##  mean of x  mean of y
```

```
## 0.01612787 0.42465253
```

```
ks.test(x,y)
```

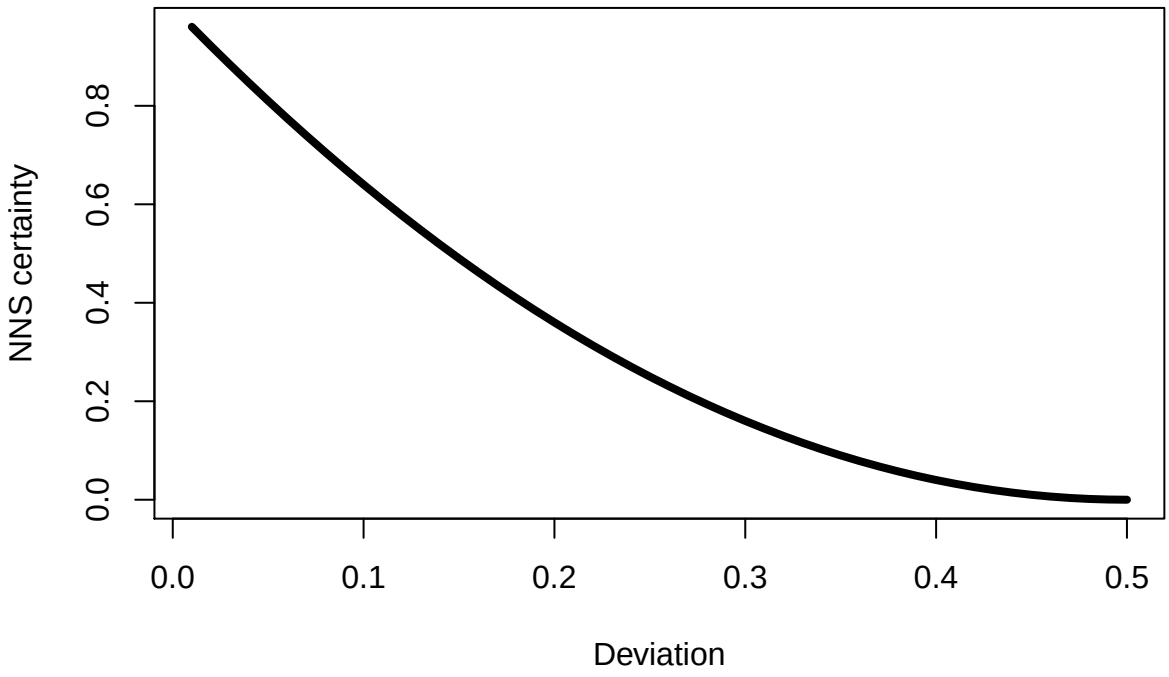
```
##
##  Two-sample Kolmogorov-Smirnov test
##
## data:  x and y
## D = 0.422, p-value < 2.2e-16
## alternative hypothesis: two-sided
```

Of course t-tests are only informed by differences in means, while NNS and KS consider variances and other distribution characteristics. However, as we see, KS can evaluate differences far removed from the means while NNS simultaneously considers sample means explicitly.

7 NNS Certainties

Finally a note on the NNS certainty statistic. NNS uses the squared sum of absolute `LPM.1.CDF` deviations from the 0.5 null hypothesis. The range of these deviations is [0,.5] (representing a target below the sample range and above the sample range respectively.). The associated certainties over the range of possible deviations is clearly bound within the interval [0,1].

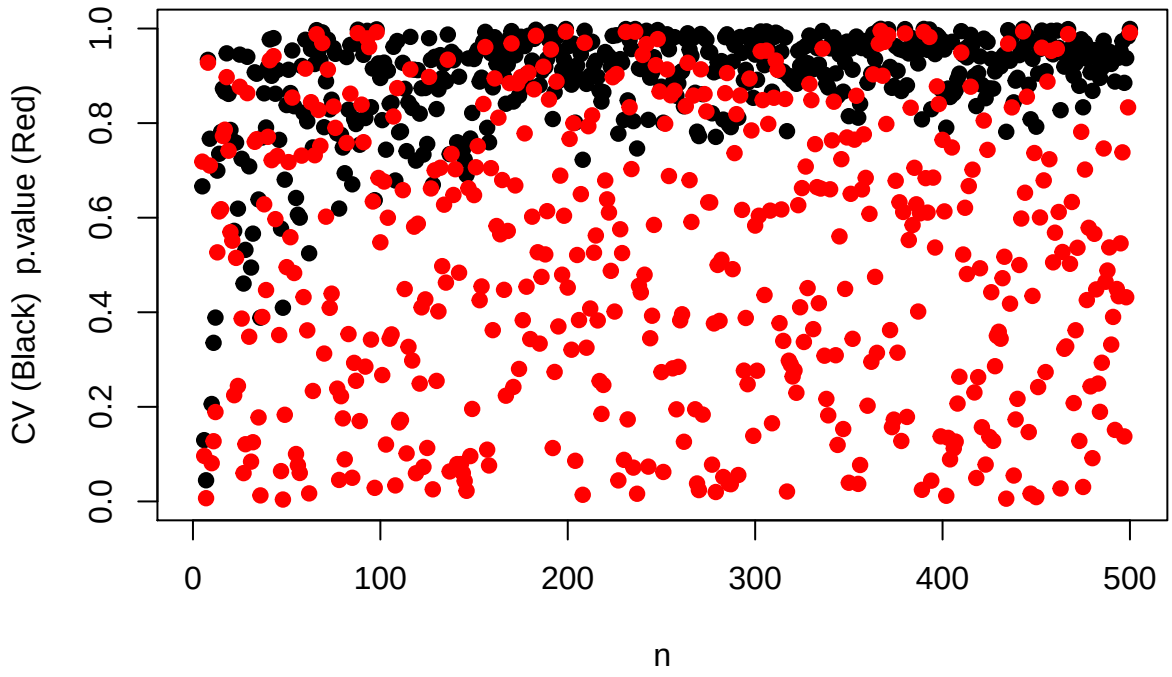
```
deviation=seq(.01,.5,.01)
plot(deviation,(0.5-deviation)^2/.25,type='l',
     lwd=4,xlab="Deviation",ylab="NNS certainty")
```



7.1 NNS Certainties vs. p values

Below is a visualization of the stability of the NNS certainty statistic vs. the corresponding p value. The first example creates two standard normal distributions ranging in size from 5 to 500 observations.

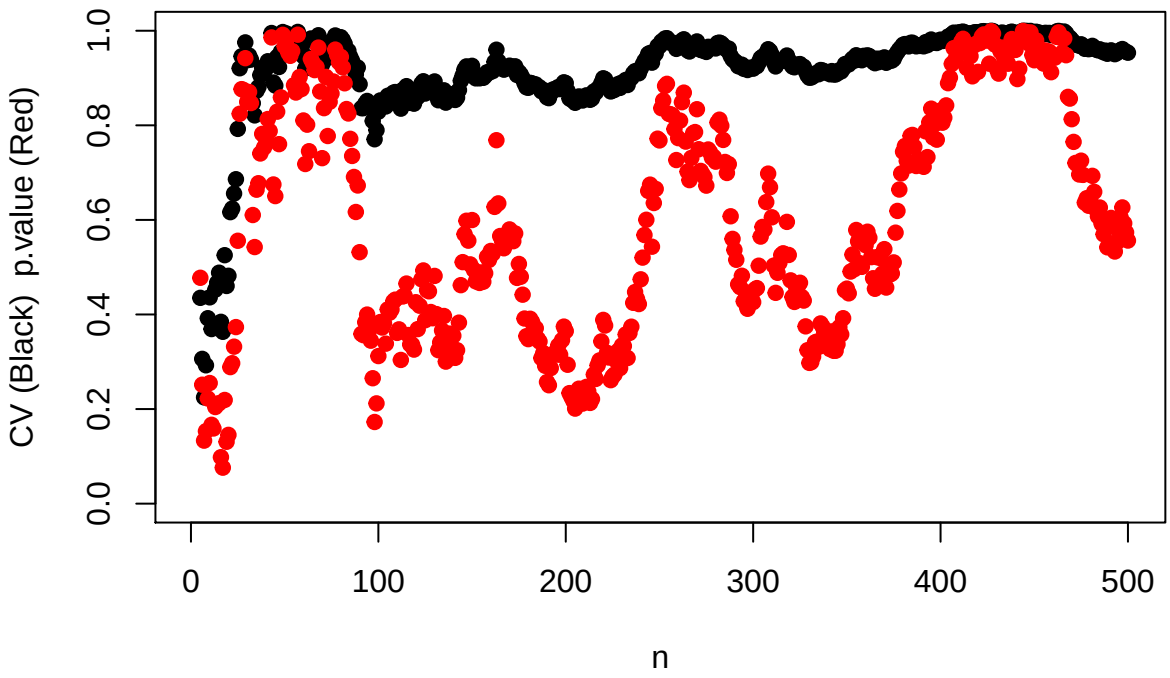
```
set.seed(123)
certainties=numeric();p.values=numeric()
for(i in 5:500){
  x1=rnorm(i);x2=rnorm(i)
  certainties[i]=NNS.ANOVA(x1,x2,plot=FALSE)$Certainty
  p.values[i]=t.test(x1,x2)$p.value
}

plot(certainties,pch=19,xlab='n',ylab="CV (Black)  p.value (Red)",ylim=c(0,1))
points(p.values,col='red',pch=19)
```



Finally, we isolate a single distribution and increase the number of observations from 5 to 500. Again, note the stability of the NNS certainty statistic...

```
set.seed(123)
certainties=numeric();p.values=numeric()
x1=rnorm(500);x2=rnorm(500)
for(i in 5:500){
  certainties[i]=NNS.ANOVA(x1[1:i],x2[1:i],plot = FALSE)$Certainty
  p.values[i]=t.test(x1[1:i],x2[1:i])$p.value
}
```

```r
plot(certainties,pch=19,xlab='n',ylab="CV (Black)  p.value (Red)",ylim=c(0,1))
points(p.values,col='red',pch=19)
```



8 NNS Certainties vs. p values and the role of 'Power'

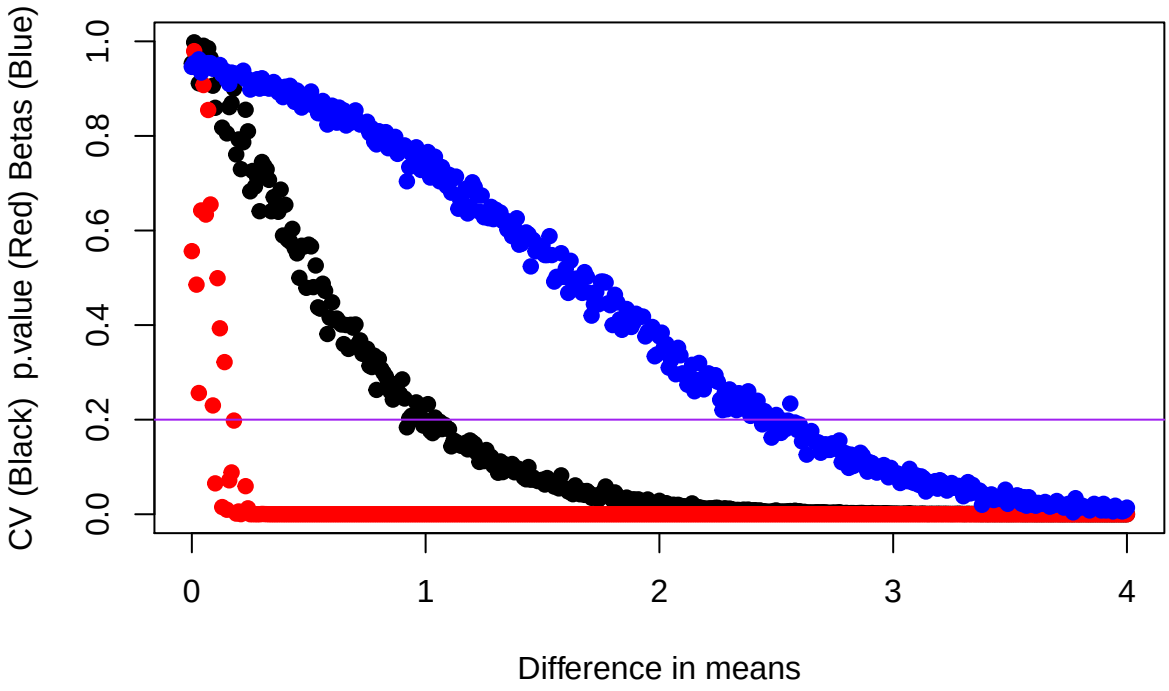
One obvious culprit for the difference in NNS certainties and p values is the power, or probability of a type II error. From the critical α level selected ($\alpha = 0.05$) in the reference distribution (X_1), we can easily calculate the probability below this level of the other sampled distribution (X_2).

For instance, if the sample mean of X_2 is greater than the mean of X_1, the β is simply the `LPM.0.CDF` of X_2 from this α target, defined in 2.1.1. Thus, the $(1 - \beta)$ power measure is the `UPM.0.CDF` of X_2 from this target. Stated more clearly, the `UPM.0.CDF` is the probability above a target in a distribution.

Let's demonstrate the role of power in both tests' assessment of the significance in the difference in means. We will keep one sample $N(0, 1)$ as our reference distribution X_1, and sample another distribution with an increasing mean from 0:4 by increments of 0.01, calling this distribution X_2.

We will then compute the associated β for X_2 at $\alpha = 0.05$ and determine the correlation between NNS certainties and p values to this β value (we are using the empirical 5% α level, not the standard z score of 1.645, even though our results do not differ materially). As β decreases, the power increases and the NNS certainty and p value should decrease as well...

```
set.seed(123)
certainties=numeric();p.values=numeric();betas=numeric()
x1=rnorm(500,mean=0,sd=1)

# alpha = 0.05 in X1
alpha=UPM.VaR(.95,0,x1)

for(i in seq(0,4,.01)){
  x2=rnorm(500,mean=i,sd=1)
  certainties[which(i==seq(0,4,.01))]=NNS.ANOVA(x1,x2,plot=FALSE)$Certainty
  p.values[which(i==seq(0,4,.01))]=t.test(x1,x2)$p.value
  betas[which(i==seq(0,4,.01))]=LPM(0,1.645,x2)
}

plot(seq(0,4,.01),certainties,pch=19,ylab="CV (Black)  p.value (Red) Betas (Blue)",
     ylim=c(0,1),xlab="Difference in means")
points(seq(0,4,.01),p.values,col='red',pch=19)
points(seq(0,4,.01),betas,col='blue',pch=19)
abline(h=0.2,col='purple')
```



We can visualize how the power increases (via decreasing β) and indeed NNS certainties and p values decrease. What is concerning however, is the rate at which the p values decrease. The purple horizontal line highlights 0.2, and $\beta < 0.2$ translates to a power greater than 80%. Why does the p value completely ignore the power of the test? NNS certainty does a much better job of conveying this information.

8.1 Power Correlations

Using a linear correlation measure, we quantify the relationship between β and the two measures of significance in the difference of means:

```
cor(betas,p.values)
```

```
## [1] 0.2921888
```

```
cor(betas,certainties)
```

```
## [1] 0.8166771
```

NNS certainty has a much stronger correlation to the power of the test than the p value, confirming our earlier suspicions. Given the relevant power information, are p values too hastily declaring a significant difference in means?

8.2 Confidence Intervals

The the 95% confidence interval of the mean of X_1 spans the interval [-1.96,1.96]. Given this range, how certain can we be of a difference in means of two normally distributed unit variance distributions equal to 0.5

being significantly different? In fact, if we reverse the procedure and ask what percentage confidence interval does a difference in means of 0.5 represent, we get an alarming result. We use the `uniroot` command in R to find the associated percentage of the `qnorm` value representing a ± 0.5 difference in means between two distributions.

```
f=function(x) qnorm(x,mean=0,sd=1)+.5
uniroot(f,tol=0.0001,c(.3,.5))
```

```
## $root
## [1] 0.3085423
##
## $f.root
## [1] 1.345481e-05
##
## $iter
## [1] 3
##
## $init.it
## [1] NA
##
## $estim.prec
## [1] 5e-05
```

Our result shows that a difference in means of 0.5 has an associated confidence interval of 61.70846% on our reference distribution X_1. The p value is significant at a much lower difference in means, ignoring the much lower confidence interval.

9 Remarks

In summary, the well documented shortfalls of the p value are warranted. It is a test that ignores too much information regarding the power of the test and the associated confidence interval of the difference in means. NNS certainty captures this information with its use of CDFs based on partial moments.

This is merely an introductory work and by no means exhaustive. A further discussion comparing the underlying assumptions to p values to that of NNS is presented in the References section.

10 References

To learn more about NNS statistics and their theoretical foundations, see "*Nonlinear Nonparametric Statistics: Using Partial Moments*" available on Amazon: http://a.co/5bpHvUg

More hands-on NNS examples posted on GitHub:

https://github.com/OVVO-Financial/NNS/tree/NNS-Beta-Version/examples

Questions or comments welcomed at: ovvo.financial.systems@gmail.com